

METHOD OF TEACHING MATH PROBLEM-SOLVING

FIELD OF THE INVENTION

The present invention relates to teaching methods in the general subject area of mathematics, and more particularly to methods and training aids for
5 teaching students, especially children, to analyse and solve elementary math word problems.

BACKGROUND OF THE INVENTION

Math problem-solving is widely considered to be an important and valuable skill. An important subset of problems utilised in the teaching of mathematics
10 may broadly be classified as word problems. As the name suggests, word problems are mathematical problems that are expressed in the form of problem statements in substantially plain language, as opposed, for example, to problems that are expressed in the form of mathematical expressions or equations. The solution of word problems requires the student to identify the relevant problem
15 elements and values from within a problem statement, in order to identify the relevant mathematical operations that must be carried out to arrive at a solution to the problem.

A conventional approach to mathematics teaching structures the subject into a series of topics. One such topic may be, for example, "fractions", and
20 within this topic area students would be expected to master skills such as basic arithmetic (*ie* addition, subtraction, multiplication and division) of fractions, and so forth. According to the conventional topic-oriented approach, word problems may be set which require students to apply the new skills developed in association with the topic in order to arrive at a solution. It will be appreciated, however, that
25 solving word problems involves a two-step process. Firstly, it is necessary for the student to analyse the word problem, and formulate a corresponding math problem that can be solved utilising previously acquired mathematical skills. Only then is the student able to apply those skills in order to arrive at a solution. The latter step may be straightforward for a student who has successfully acquired the
30 necessary skills in the relevant topic area or areas. In practice, however, conventional approaches to math teaching are less effective in providing students with the skills necessary to address the first step in the solution of word problems.

The most common approach to teaching word problem-solving is repetition. In particular, with teaching structured around math topics, word problems are presented as being incidental to the topics, and as a means for putting into practice newly acquired skills within each topic area. Students may
5 be taught the use of various different approaches, heuristics and methods for solving different types of word problems, and by presenting students with large numbers of different word problems it is hoped that somehow they will learn to recognise different situations and arrangements of information within such problems, thereby increasing the range of word problems that they are able to
10 solve. As a result, students develop an ad hoc, rather than structured, approach to the solution of word problems in general. This is a very inefficient method of learning, with widely varying outcomes for different students, which has inadvertently turned problem-solving into a tortuous chore for teachers and students alike.

15 Amongst the undesirable outcomes of the conventional approach to teaching word problem-solving, students may come to see it as nothing more than a necessary means for achieving success in math examinations. Students who have difficulty in analysing word problems, and formulating corresponding approaches to solutions, may develop a fear of word problems and a general
20 feeling of disdain towards problem-solving.

By way of example, consider the word problem “If Ali eats $\frac{1}{4}$ of a cake, Bob eats $\frac{1}{4}$ of the rest, and Chen eats the remainder, how much of the cake does Chen eat?” This problem relates to the math topic of “fractions”, and the skills required of the student to solve the problem are “subtraction of fractions” and
25 “multiplication of fractions”, according to a conventional teaching approach. It might therefore seem fairly straightforward for a student who has mastered these calculation skills to be able to solve the word problem. However, even students who have mastered both subtraction and multiplication of fractions may be unable to successfully analyse the word problem, and thereby arrive at the correct
30 mathematical expressions leading to the solution.

The above example illustrates that, in accordance with conventional teaching methods, word problems are typically provided as examples of the application of skills associated with selected math topics, however there is no

formal, structured teaching of the skills necessary to analyse, formulate and solve word problems generally. In other words, students are not taught problem-solving skills, rather it is presumed that by exposure to a variety of word problems associated with different topics, students will somehow develop the necessary problem-solving skills over time. In fact, in the absence of a structured approach, the problem-solving ability of students may vary greatly, and there is accordingly a failure to realise, across a wide range of students, the many benefits that may be achieved, throughout a lifetime, resulting from a mastery of problem-solving.

While a comprehensive structure has been developed for the teaching of math topics, there is no corresponding, established, structured approach to the teaching of math problem-solving, and the solution of word problems in particular. There is therefore an ongoing need for the development of improved, structured approaches for the teaching of math problem-solving. It is accordingly an object of the present invention to overcome, or at least to mitigate, this failing of conventional teaching approaches.

SUMMARY OF THE INVENTION

In one aspect, the present invention provides a method of teaching a student to solve math word problems of the type comprising one or more problem statements including problem elements and problem values, the method including the steps of:

defining a plurality of elementary concepts relating to possible relationships between problem elements and/or problem values;

providing a plurality of visual aids associated with said elementary concepts, wherein each said visual aid enables a corresponding relationship between problem elements and/or problem values to be represented in diagrammatic form;

teaching the student a plurality of fundamental math skills underlying said plurality of elementary concepts;

presenting the student with one or more math problems comprising math problem statements requiring application of at least a common one of said elementary concepts; and

teaching the solution of each of said plurality of math problems to the student by in each case identifying problem elements and problem values within

the corresponding problem statements, selecting from said plurality of elementary concepts those which define relevant relationships between the identified problem elements and problem values, and demonstrating the problem solution by use of the visual aids associated with each said elementary concept and by
5 corresponding application of one or more of said fundamental math skills.

Advantageously, the teaching method of the present invention is structured around a plurality of elementary concepts rather than around a progression of topics and/or topic areas. By focusing upon the identification of problem elements and problem values within a word problem, and the relationships
10 therebetween, rather than upon conventional math topics, a structured approach to problem-solving may be developed. The method further provides visual aids that, in accordance with a preferred approach, are not merely diagrams for the passive display of data, but rather provide working platforms enabling the dynamic relationships between problem elements and problem values within word
15 problems to be organised, tracked, processed and managed visually. Students are further taught fundamental skills, independent of particular topic areas, that underlie the elementary concepts and corresponding visual aids. A particularly advantageous feature of the teaching method is that it provides students with a structured approach to problem-solving in general, and can therefore provide
20 improved learning outcomes and efficiency as compared with conventional topic-based teaching, while being complementary to this conventional approach.

According to preferred embodiments, the plurality of elementary concepts includes a Relativity Concept, a Basic Comparison Concept, a Redistribution Concept, a Constants and Changes Concept, a Dual Variables Concept, and a
25 Systematic Testing Concept. While this list of elementary concepts may not be exhaustive, and further concepts may become apparent, it is presently believed that these six elementary concepts are the most important, and may be sufficient, at least at a primary/elementary school level.

In accordance with preferred embodiments of the teaching method, the
30 visual aids are also known as "schemas". In accordance with preferred embodiments, the visual aids include a Relativity Schema, associated with the Relativity Concept, a Basic Comparison Schema, associated with the Basic Comparison Concept, a Redistribution Schema, associated with the

Redistribution Concept, and a Multiplicity Schema, associated with the Constants and Changes, Dual Variables, and Systematic Testing Concepts. The visual aids, or schemas, are themselves preferably represented using a plurality of visual symbols. It is particularly preferred that visual symbols be provided
 5 denoting matching or equality relationships between values, differences between values, operations performed upon values (*eg* addition, subtraction, multiplication or division), sums of values, sequenced stages, and splitting of values.

In accordance with preferred embodiments of the teaching method, the fundamental math skills are also known as “scaffolds” (in a sense that they
 10 provide support for the structure of the teaching approach). Preferably, the fundamental math skills, or scaffolds, include math language, reciprocity, multiples, comparing gain and loss, restating and rearranging, sets property, and direct comparison.

Preferably, the math language scaffold includes “more than/less
 15 than/difference”, “proportion of”, “proportion more than/proportion less than”, and “sum/total/altogether”. Further advantageous features and details of the preferred elementary concepts, schemas and scaffolds will become apparent from the detailed description of a preferred embodiment of the teaching method, and the examples, provided herein.

20 In another aspect, the present invention provides a method of solving a math problem of the type comprising one or more problem statements including problem elements and problem values, the method including:

identifying problem elements and problem values included within the problem statement;

25 selecting from a plurality of predetermined elementary concepts relating to possible relationships between problem elements and/or problem values those elementary concepts which define relevant relationships between the identified problem elements and problem values of the problem statement;

using one or more predetermined visual aids associated with said plurality
 30 of predetermined elementary concepts to construct a visual representation of corresponding identified relationships between the problem elements and/or problem values;

applying one or more predetermined fundamental math skills to the identified relationships depicted in the visual representation to arrive at a solution to the math problem; and

providing the solution as an output.

5 In yet another aspect, the present invention provides a system for solving math problems of the type comprising one or more problem statements including problem elements and problem values, the system including:

a plurality of visual aids comprising a set of relational symbols, said visual aids being adapted for the purpose of representing visually a plurality of
10 predetermined elementary concepts relating to possible relationships between problem elements and/or problem values;

means for presenting and arranging said visual aids in accordance with elementary concepts selected from said predetermined elementary concepts, and which define relevant relationships between identified problem elements and
15 problem values of the math problem; and

wherein a solution to the math problem is determined by applying one or more predetermined fundamental math skills to the relationships depicted in a visual representation constructed using said presenting and arranging means.

Further preferred features and advantages of the invention will be apparent
20 to those skilled in the relevant art from the following description of a preferred embodiment of the invention, which should not be considered to be limiting of the scope of the invention as defined in the preceding statements, or in the claims appended hereto.

BRIEF DESCRIPTION OF THE DRAWINGS

25 Preferred embodiments of the invention are described with reference to the accompanying drawings, in which:

Figure 1 is a block diagram illustrating a schematic system for the solution of math word problems according to the preferred embodiment of the invention;

Figure 2 is a flowchart illustrating a method of problem solution according
30 to the preferred embodiment of the invention; and

Figure 3 is a flowchart illustrating a teaching method according to the preferred embodiment of the invention.

DETAILED DESCRIPTION OF PREFERRED EMBODIMENT

Figure 1 is a block diagram illustrating a schematic system 100 for the solution of math word problems 102, in accordance with the preferred embodiment of the present invention. The schematic system 100 includes a number of interrelated elements, which together provide a structured framework for the solution, and teaching, of problem-solving.

Elementary math problem concepts 104 are central to the schematic system 100. By comparison with conventional structured systems and methods, elementary math problem concepts 104 displace math topics as the organising principle of the system 100. As has been noted above, math topics relate to particular areas of math learning (such as “fractions”), and have traditionally provided the primary organising principle for the teaching of mathematics. Mastery of each area, or topic, enables students to perform the necessary operations and calculations pertaining to the topic.

By contrast, math problem concepts 104 relate to particular kinds of interrelationships between the subjects, elements or values in math word problems. In accordance with embodiments of the present invention, mastery of the varied structures that these interrelationships may take enables students to comprehend, manage and solve a variety of math word problems.

It should be understood that it is not an object of the present invention to replace the teaching, understanding and application of math topics. Rather, math topics remain an important aspect of math teaching, and continue as an important aspect of math word problems. For example, students will be unable ultimately to arrive at a numerical solution to a math word problem involving the manipulation of fractions, unless they are able to perform the necessary operations and calculations on fractions. However, the introduction of math problem concepts 104 as a central aspect in the solution of math word problems, and associated teaching methods, refocuses the process on the comprehension and systematic formulation and solution of word problems. Mastery of the interrelationships in word problems, utilising elementary math problem concepts 104, enables students to comprehend and manage word problems before performing the necessary operations and calculations required to arrive at a specific numerical solution.

This focus on comprehension and management of word problems in general, rather than upon the operations and calculations required to arrive at numerical results, is a novel and unique contribution realised in embodiments of the present invention. Through drawing a clear distinction between topics and
5 problem concepts, students are able to work towards achieving mastery over math word problems.

A further component of the schematic system 100 is that of scaffolds 106. Scaffolds 106 deal with fundamental reasoning and math skills that enable students to comprehend the relationships in math word problems. While many of
10 the fundamental reasoning and math skills may appear obvious to those having already developed basic mathematical competence, students are disadvantaged in their efforts to solve math word problems if they do not consciously learn the necessary basic skills, or if they are not aware of the usefulness of these skills in problem-solving. Accordingly, the organisation of a set of fundamental reasoning
15 and math skills into the scaffolds 106 of the schematic system 100 is a key feature enabling the construction of a comprehensive structured system for elementary math word problem solving.

The various scaffolds 106 of the schematic system 100 are described in greater detail below, however it is noted at this point that the scaffolds 106
20 incorporate a specific math language 108 which is useful in the overall development and operation of the schematic system 100.

The system 100 also includes a set of schemas 110. The schemas 110 of the preferred embodiments are visual presentation techniques that enable students to organise, track, process and manage the relationships in math word
25 problems. Each of the schemas 110, discussed and exemplified in greater detail below, enable the visual or diagrammatic presentation of one or more of the elementary math problem concepts 104. It is preferred to use a particular set of relational symbols 112 within the schemas 110. These specific symbols 112 utilised in the preferred embodiment are described and exemplified below,
30 however it is noted that the particular set of signs or symbols disclosed herein is not unique, and persons skilled in the relevant art will appreciate that alternatives are available, or may be devised.

In accordance with the methods enabled by the schematic system 100, word problems 102 may be analysed using the schemas 110 embodying the elementary math problem concepts 104 in order to build up a dynamic visual representation 114 of the relationships between various problem elements and problem values within the word problems 102. The dynamic aspect of this process is significant, since it should be understood that the schemas 110 are not merely diagrams or visual aids for the passive display of data, but rather provide working platforms for the dynamic relationships between elements and values in math word problems to be organised, tracked, processed and managed visually.

This process is clarified below by reference to a number of worked examples.

Application of the scaffolds 106 to the problem elements and problem values depicted in the dynamic visual representation 114, built up from word problems 102, utilising the schemas 110, enables solutions 118 to be found.

The discussion now turns to a more detailed description of the scaffolds 106 and elementary math problem concepts 104 of the preferred embodiment, followed by an explanation of the schemas 110 by reference to a number of worked examples. Thereafter, the overall problem solution method is summarised with reference to Figure 2, and a corresponding teaching method is described with reference to Figure 3.

20 Scaffolds

As shown in Figure 1, the preferred embodiment of the schematic system 100 includes seven distinct scaffolds 106, or fundamental reasoning and math skills. These are math language 108, reciprocity, multiples, comparing gain and loss, restating and rearranging, the sets property, and direct comparison.

The math language scaffold 108 itself consists of four distinct elements, namely “more than/less than/difference”, “proportion of”, “proportion more than/proportion less than”, and “sum/total/altogether”. The elements of math language 108 are most easily described by reference to examples.

“More than/less than/difference” is used to represent specific numerical relationships between values or elements. Within the system 100, it is preferred to use a smallest common unit amongst elements within a math word problem as a basis for comparison, and to deal with positive operations wherever possible,

since experience suggests that beginning students have great difficulty with negation, and negative values, than with positive values. By way of example, if Ali has \$2.00 more than Bob, then Bob has the smaller value, the difference is \$2.00, and the relationship can be expressed as:

$$\text{Ali} \longrightarrow 1P + \$2$$

5

$$\text{Bob} \longrightarrow 1P$$

Conversely, if Ali has \$2.00 less than Bob, then Ali has the small value, the difference is again \$2.00, and the relationship can be expressed as:

$$\text{Ali} \longrightarrow 1P$$

$$\text{Bob} \longrightarrow 1P + \$2$$

10 It should be understood that in the above examples, and throughout this specification, elements or values indicated by notation such as the letter “P” above, are not variables of conventional algebra, but rather are symbolic placeholders utilised to assist in representing concepts, relationships and/or values visually.

15 “Proportion of” relates to ratios or proportional relationships. By way of example, if Ali has twice Bob’s amount, Ali:Bob is 2:1. As a further example, if Ali has $\frac{3}{4}$ of Bob’s amount, Ali:Bob is 3:4.

20 “Proportion more than/proportion less than” relates to differences expressed as ratios or proportional relations. By way of example, if Ali has twice more than Bob’s amount, Ali:Bob is 3:1 (*ie* 2 + 1:1). Similarly, if Ali has $\frac{3}{4}$ more than Bob’s amount, Ali:Bob is 7:4 (*ie* 3 + 4:4).

25 “Sum/total/altogether” relates to sums or totals (*ie* the three terms have equivalent meaning). By way of example, if Ali has \$2.00, Bob has \$3.00 and Chen has \$4.00, Ali and Bob have a total of \$5.00, Ali and Chen have a total of \$6.00, Bob and Chen have a total of \$7.00, and Ali, Bob and Chen altogether have a total of \$9.00.

Turning now to the remaining scaffolds 106, reciprocity relates to reciprocal relations amongst problem elements or values. At the level of fundamental reasoning, reciprocity should be understood as a symbolic manipulation, and should not be confused with the algebraic manipulation familiar

to persons already skilled in mathematics, and more particularly in the topic area of “algebra”. An example of reciprocity is that if $1 + 2 = 3$, then $3 - 1 = 2$ and $3 - 2 = 1$. Similarly, if $2 \times 3 = 6$, then $6 \div 2 = 3$ and $6 \div 3 = 2$.

The next scaffold 106 is that of multiples. Multiples are especially useful in assisting students to convert proportions, and especially fractions and/or decimal values, into whole numbers. Experience suggests that beginning students are more comfortable in dealing with whole numbers than with proportions or fractional numbers. By way of example, multiples may be applied to establish that if Ali has a number of marbles, being divisible by both 2 and 3, then the number is also divisible by the lowest common multiple of 6.

The scaffold of comparing gain and loss relates to tracking changes independently of absolute values. Thus, for example, the scaffold of comparing gain and loss requires that if Ali gains \$12.00, and Bob gains \$12.00, then the difference between their respective amounts remains constant. Alternatively, if Ali gains \$12.00 and Bob loses \$12.00, then their total remains constant. If Ali gains \$12.00, Bob gains \$8.00, and as a result their respective amounts become equal, then their original difference is equal to the difference in their gains. If Ali gains \$12.00, Bob loses \$8.00, and their values become equal, then the original difference in their respective amounts is the sum of the gain and loss.

Turning now to the scaffold of restating and rearranging, restating refers to the breaking up of larger values, while rearranging refers to the arranging of equal values to enable students to make an obvious match between values. An example of restating is as follows:

If $\star\star\star + 12 = 27$, it can be restated & rearranged as :

$\star\star\star + 12$ $= \quad 15 \quad + 12$

The pupil can then understand that $\star\star\star = 15$

Similarly, an example of rearranging is as follows:

If $\star\star\star + \text{(((}} = 14$ and $\star\star\star\star\star\star + \text{(((((((} = 36$,

it can be restated & rearranged as :

$\star\star\star + \text{(((} + \star\star\star + \text{(((} + \text{(((}$
$= \qquad 14 \qquad + \qquad 14 \qquad + \qquad 8$

The pupil can then understand that $\text{(((} = 8$

It is again important to understand that at the level of fundamental reasoning and math skills, the scaffold of restating and rearranging relates to symbolic, rather than algebraic, manipulation. Nonetheless, students who develop a solid understanding of restating and rearranging at an early stage in their mathematical learning may be better equipped to understand the rationale behind algebraic manipulations, such as change in operations, substitution and elimination, for the solution of equations encountered later in their mathematical education.

The sets property included within the scaffolds 106 of the schematic system 100 corresponds with the distributive property of addition. By way of example, if a Box contains three Apples and four Pears, then:

$$1B \text{ --- } 3A + 4P$$

$$2B \text{ --- } 6A + 8P$$

$$3B \text{ --- } 9A + 12P$$

As a further example, if five Boxes contain 30 Apples and 50 Pears, then:

$$5B \text{ --- } 30A + 50P$$

$$1B \text{ --- } 6A + 10P$$

$$2B \text{ --- } 12A + 20P$$

$$3B \text{ --- } 18A + 30P$$

The scaffold of direct comparison embodies the notion that, for example, we can know how much six Apples cost when eight Apples cost \$12.00. By knowing that six Apples is $\frac{6}{8}$ of eight Apples. That is, direct comparison can be established by drawing directly upon the math language term of "proportion of".

The tables below illustrate direct comparison of this type for this specific example, and in the more general case.

8 Apples --- \$12 6 Apples --- $\frac{6}{8} \times \\$12$	\$12 --- 8 Apples $\frac{6}{8} \times \\$12$ --- 6 Apples
$\frac{6}{8} \times \\$12$ --- 6 Apples \$12 --- 8 Apples	6 Apples --- $\frac{6}{8} \times \\$12$ 8 Apples --- \$12

B --- C A --- $\frac{A}{B} \times C$	C --- B $\frac{A}{B} \times C$ --- A
$\frac{A}{B} \times C$ --- A C --- B	A --- $\frac{A}{B} \times C$ B --- C

5 Elementary Math Problem Concepts

In the schematic system 100, the elementary math problem concepts 104 consist of the Relativity Concept, the Basic Comparison Concept, the Redistribution Concept, the Constants and Changes Concept, the Dual Variables Concept, and the Systematic Testing Concept.

- 10 The Relativity Concept involves establishing the relative values and/or locations of problem elements (*eg* subjects or stages). As will be demonstrated by the examples presented later, the Relativity Concept can generally best be presented linearly to enable students to obtain a visual understanding of the relative values and locations of subjects or stages. The Relativity Concept is
- 15 most commonly embodied in differences, addition or subtraction operations, and/or sequenced stages.

The Basic Comparison Concept involves establishing a basis for comparison to compare problem elements (*eg* subjects) with static values, *ie* values that are not undergoing a distribution or change. As noted above in relation to the term “more than/less than/difference” of the math language scaffold, for ease of comparison and to enable students to avoid dealing with negation, or even double negation, it is often preferable to use the smallest common unit within a problem as a basis for comparison. In a number of the examples set out later, the symbol “1P”, or similar, is often used to represent the smallest common unit. Central to the Basic Comparison Concept are relationships founded in the math language scaffold, such as “more than”, “less than”, “difference”, “proportion of”, “proportion more than”, “proportion less than”, “total” and “partial total”. As a result, mastery by students of the Basic Comparison Concept is most desirable within the preferred embodiment, since these relationships will also feature in word problems involving the various other concepts.

The Redistribution Concept involves a unidirectional distribution (or flow) of values from a single source problem element to two or more generations of subject problem elements. The Redistribution Concept is embodied in the notion that the value associated with the source is the sum of the values that the subjects receive from it, whether directly or indirectly. While redistribution involves a “movement” of values, it is not a change concept insofar as the total value distributed between the source and subject elements is constant, and the primary concern is with the static value each subject element finally receives from the source element.

By contrast, the Constants and Changes Concept involves changes in some or all of the subject element values which may result in changes in their sum values, or in the difference between their values. Identifying a constant is extremely useful, since it amounts to establishing an equality relationship in terms of conventional algebra. The Constants and Changes Concept may usefully draw on the scaffolds of comparing gain and loss, multiples, restating and rearranging, and the sets property.

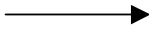
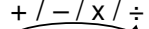
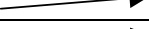
As implied by its name, the Dual Variables Concept involves a set of items grouped according to two variables. The value of the whole set can be found

either by adding the value of each variable, or by adding the values associated with individual elements.

The Systematic Testing Concept involves a methodical (as distinguished from random) testing of various combinations of subject values to arrive at one that meets a given criteria.

Schemas

As noted previously, the schemas 110 of the schematic system 100 relate to visual presentation techniques for tracking, processing and managing the relationships in word problems. The schemas 110 employ relational symbols 112 for this purpose. The table below shows one possible selection of relational symbols, or signs, that is presently preferred for use with the schemas 110 of the schematic system 100.

Signs Used	What They Denote
	This denotes a matching or equality relationship
	This denotes a difference
	This denotes an operation
	This denotes a sum (except for sequenced stages where it shows the conditions for each stage)
	This denotes a sum of values (usually used when the above is not suitable)
	This denotes a splitting of values (mainly used for redistribution schema)

The schemas 110 consist of the Relativity Schema, associated with the Relativity Concept, the Basic Comparison Schema, associated with the Basic Comparison Concept, the Redistribution Schema, associated with the Redistribution Concept, and the Multiplicity Schema, associated with the Constants and Changes, Dual Variables and Systematic Testing Concepts.

The schemas 110 and Elementary Math Problem Concepts 104, as well as the relationships therebetween, are perhaps best explained by reference to specific examples. Accordingly, a number of such examples are set out below.

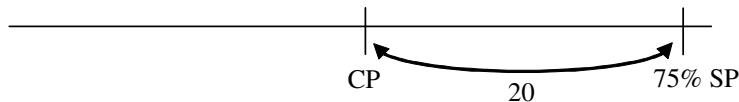
Example 1

This example relates to the Relativity Concept, and utilises the Relativity Schema. In particular, it involves an ordering of subjects. Consider the following word problem:

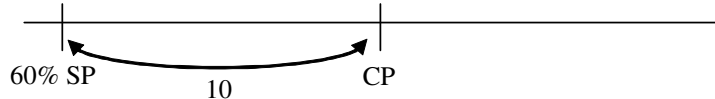
- 5 If Andy sells a bag at a discount of 25%, he makes a profit of \$20.00. If he sells the bag at a discount of 40%, he makes a loss of \$10.00. What is the cost price of the bag?

The problem elements and problem values within this word problem may be represented using the Relativity Schema as follows:

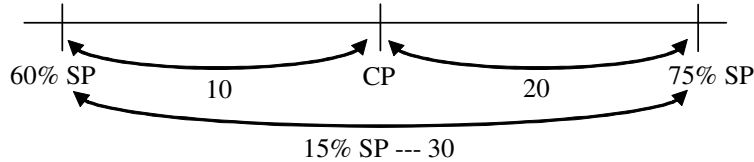
(a) Using 'If Andy sells a bag at a **discount of 25%**, he makes a **profit of \$20**'...



(b) Using 'If he sells it at a **discount of 40%**, he makes a **loss of \$10**'...



(c) We shall finally see the following...



10

Applying the relevant scaffolds, a solution may be found as follows:

15% SP ----- 30
60% SP ----- 120

15

CP --- 120 + 10 --- 130

Accordingly, the cost price of the bag is \$130.00.

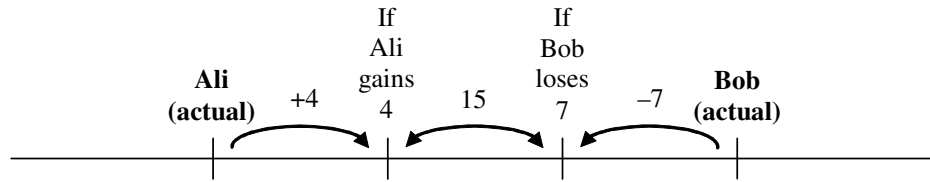
Example 2

- 20 This example again demonstrates the Relativity Schema. Consider the following word problem:

If Ali gets \$2.00 more and Bob spends \$7.00, Bob has \$15.00 more than Ali. If Ali gives Bob \$3.00, Bob has five times the amount that Ali has. Find Ali's amount.

The Relativity Schema may be applied to the problem elements and problem values as follows:

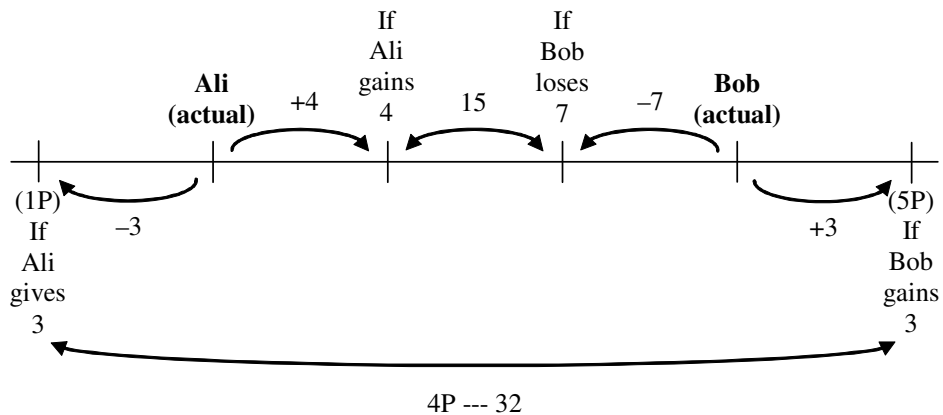
(a) Using 'If Ali gets \$4 more and Bob spends \$7, Bob has \$15 more than Ali'...



(b) Using 'If Ali gives Bob \$3, Bob has 5 times of what Ali has'...



(c) We shall finally see the following...



Applying the relevant scaffolds enables a solution to be found as follows:

5

4P --- 32
1P --- 8

Ali --- $8 + 3$ --- 11

10

Accordingly, Ali has \$11.00.

Example 3

This example demonstrates the Basic Comparison Schema, and relates to the Basic Comparison Concept. In particular, the example involves a basic comparison not involving proportion. Consider the following word problem:

- 5 Bob has \$3.00 more than Ali while Chen has \$7.00 less than Ali. They have \$35.00 altogether. Find Ali's amount.

The Basic Comparison Schema may be applied to the problem elements and problem values as follows:

(a) Using '**Chen had \$7 less than Ali**' (since Chen has the smallest amount among the 3)...

Ali $\longrightarrow$ $1P + 7$

Bob $\longrightarrow$

Chen $\longrightarrow$ $1P$

(b) Using '**Bob has \$3 more than Ali**'...

Ali $\longrightarrow$ $1P + 7$

Bob $\longrightarrow$ $1P + 10$

Chen $\longrightarrow$

(c) Using '**They have \$35 altogether**', we shall finally see the following...

35 { Ali $\longrightarrow$ $1P + 7$
 Bob $\longrightarrow$ $1P + 10$
 Chen $\longrightarrow$ $1P$

- 10 Applying the relevant scaffolds, a solution may be found as follows:

$3P \text{ --- } 35 - 7 - 10 \text{ --- } 18$

$1P \text{ ----- } 6$

Ali $\text{--- } 6 + 7 \text{ ----- } 13$

- 15

Accordingly, Ali's amount is \$13.00.

Example 4

This example again involves the Basic Comparison Schema, and once again requires basic comparison not involving proportion, but this time involving restating and rearranging. Consider the following word problem:

- 5 Ali wanted to save some money to buy a bicycle. In the first week, his savings were \$6.00 less than $\frac{1}{8}$ of the price of the bicycle. In the second week, he saved \$30.00 more than in the first week. In the third week, his savings were \$9.00 more than his total savings in the first two weeks. He was still short by \$7.00. Find the price of the bicycle.

- 10 The Basic Comparison Schema may be applied to the problem elements and problem values as follows:

(a) Using 'In the **first week**, his **savings was \$6 less than $\frac{1}{8}$ the price of the bicycle**'...

$$\frac{1}{8} \text{ Bicycle} \text{ --- } 1P + 6$$

$$\text{Bicycle} \text{ ----- } 8P + 48$$

$$8P + 48 \left\{ \begin{array}{l} 1^{\text{st}} \text{ wk} \longrightarrow 1P \\ 2^{\text{nd}} \text{ wk} \longrightarrow \\ 3^{\text{rd}} \text{ wk} \longrightarrow \\ \text{Short} \longrightarrow \end{array} \right.$$

(b) Let's now deal with 'In the second week, he saved \$30 more than in the first week'.

$$8P + 48 \left\{ \begin{array}{l} 1^{\text{st}} \text{ wk} \longrightarrow 1P \\ 2^{\text{nd}} \text{ wk} \longrightarrow 1P + 30 \\ 3^{\text{rd}} \text{ wk} \longrightarrow \\ \text{Short} \longrightarrow \end{array} \right.$$

(c) Using 'In the **third week**, his savings was **\$9 more than his total savings in the first two weeks**'...

$$8P + 48 \left\{ \begin{array}{l} 1^{\text{st}} \text{ wk} \longrightarrow 1P \\ 2^{\text{nd}} \text{ wk} \longrightarrow 1P + 30 \\ 3^{\text{rd}} \text{ wk} \longrightarrow 2P + 39 \\ \text{Short} \longrightarrow \end{array} \right.$$

(d) Using 'He was **still short of \$7**', we shall finally see the following...

$$8P + 48 \left\{ \begin{array}{l} 1^{\text{st}} \text{ wk} \longrightarrow 1P \\ 2^{\text{nd}} \text{ wk} \longrightarrow 1P + 30 \\ 3^{\text{rd}} \text{ wk} \longrightarrow 2P + 39 \\ \text{Short} \longrightarrow 7 \end{array} \right.$$

Applying the relevant scaffolds, a solution may be found as follows:

$$\text{Bicycle} \text{ ----- } 8P + 48 \text{ --- } 1P + 1P + 30 + 2P + 39 + 7 \text{ --- } 4P + 76$$

5

Using **restating & rearranging**, $4P \text{ --- } 28$

$$8P \text{ ----- } 56$$

10

$$\text{Bicycle} \text{ --- } 56 + 48 \text{ ----- } 104$$

Accordingly, the price of the bicycle is \$104.00.

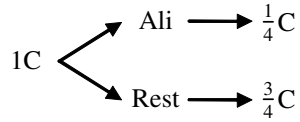
Example 5

15 This example involves the Redistribution Schema, and Redistribution Concept. Consider the following word problem:

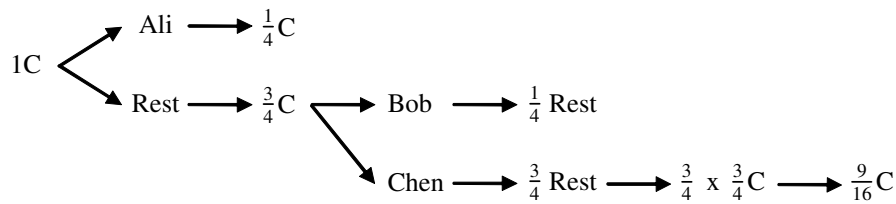
Ali eats $\frac{1}{4}$ of cake, Bob eats $\frac{1}{4}$ of the rest, and Chen eats the remainder. How much of the cake does Chen eat?

The Redistribution Schema may be applied to the problem elements and problem values as follows:

(a) Using 'Ali eats $\frac{1}{4}$ of a cake'...



(b) Using 'Bob eats $\frac{1}{4}$ of the rest and Chen eats the remainder', we shall finally see the following...



Accordingly, it may be seen that the solution is that Chen eats $\frac{9}{16}$ of the cake.

Example 6

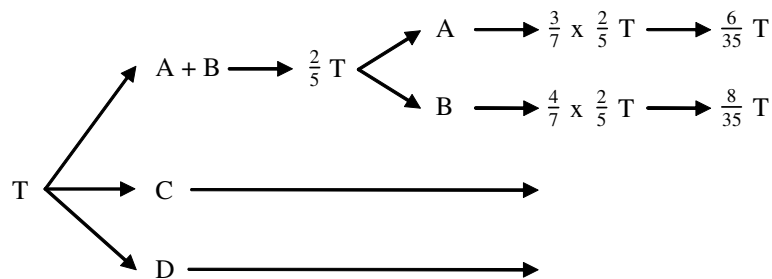
- 5 This example again involves the Redistribution Schema and Redistribution Concept. Consider the following word problem:

Ali, Bob, Chen and Deepak received some marbles. Ali and Bob received $\frac{2}{5}$ of the marbles in the ratio 3:4. Chen received 30 marbles more than Bob. Deepak received 20 marbles less than Chen. How many marbles did Chen

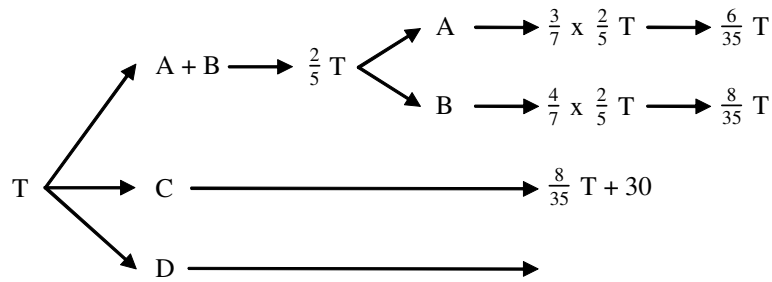
- 10 receive?

The Redistribution Schema may be applied to the problem elements and problem values as follows:

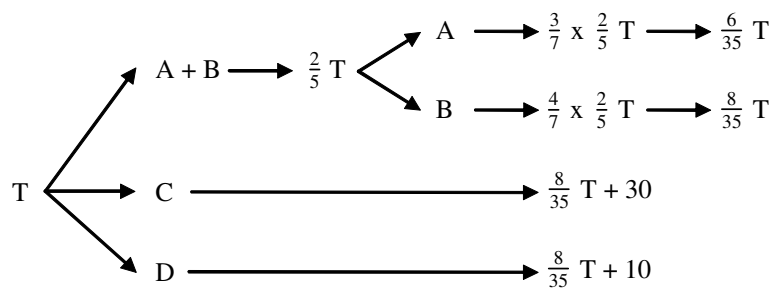
(a) Using 'Ali and Bob received $\frac{2}{5}$ of the marbles in the ratio 3 : 4'...



(b) Using 'Chen received 30 marbles more than Bob'...



(c) Using 'Deepak received 20 marbles less than Chen', we shall finally see the following...



Applying the relevant scaffolds, a solution may be found as follows:

$$\frac{5}{35} T \text{ --- } 40$$

$$\frac{8}{35} T \text{ --- } \frac{8}{5} \times 40 \text{ --- } 64$$

$$\text{Chen --- } 64 + 30 \text{ --- } 94$$

Accordingly, the solution is that Chen received 94 marbles.

Example 7

This example demonstrates the use of the Multiplicity Schema, in association with the Constants and Changes Concept. Consider the following word problem:

The ratio of the marbles in Box A to those in Box B is 2:5. When some marbles are transferred from Box B to Box A, the ratio is 3:7. What fraction of the marbles in Box B has been transferred out?

Applying the Multiplicity Schema with the Constants and Changes Concept, the problem elements and problem values may be represented as follows:

(a) Using what is given, we shall be able to see the following (since the total is constant)...

	(7U) (10S)	
	A	B
At First	2U	5U
Later	3S	7S

Diagram showing a transition from 'At First' to 'Later' for two boxes, A and B. Box A starts with 2U and ends with 3S, indicated by a downward arrow with a '+' sign. Box B starts with 5U and ends with 7S, indicated by a downward arrow with a '-' sign. A bracket above the columns A and B spans the top two rows, labeled (7U) (10S).

(b) Treating the total sum as 70P...

7U --- 70P
 1U --- 10P
 2U --- 20P
 5U --- 50P

	70P	
	A	B
At First	2U (20P)	5U (50P)
Later	3S	7S

Diagram showing a transition from 'At First' to 'Later' for two boxes, A and B. Box A starts with 2U (20P) and ends with 3S, indicated by a downward arrow with a '+' sign. Box B starts with 5U (50P) and ends with 7S, indicated by a downward arrow with a '-' sign. A bracket above the columns A and B spans the top two rows, labeled 70P.

(c) We shall finally see the following...

10S --- 70P
 1S ---- 7P
 3S ---- 21P
 7S ---- 49P

50P - 49P --- 1P

	70P	
	A	B
At First	(20P)	(50P)
Later	+ 1P 3S (21P)	- 1P 7S (49P)

Diagram showing a transition from 'At First' to 'Later' for two boxes, A and B. Box A starts with (20P) and ends with + 1P, 3S (21P), indicated by a downward arrow with a '+' sign. Box B starts with (50P) and ends with - 1P, 7S (49P), indicated by a downward arrow with a '-' sign. A bracket above the columns A and B spans the top two rows, labeled 70P.

Accordingly, the solution is found to be that $\frac{1}{50}$ of the marbles in Box B has been transferred out.

Example 8

This example again applies the Multiplicity Schema and the Constants and Changes Concept. Consider the following word problem:

The ratio of the marbles in Box A to those in Box B is 2:5. When 87 marbles are added to Box A, the ratio is 3:7. Find the original number of marbles in Box A.

- Applying the Multiplicity Schema and the Constants and Changes
- 5 Concept, the problem elements and problem values may be represented as follows:

(a) Using what is given, we shall be able to see the following...

	A	B
At First	2U	5U
Later	3S	7S

(b) Treating the constant value of Box B as 35P...

	A	B
At First	2U	5U (35P)
Later	3S	7S (35P)

(c) We shall finally see the following...

$$5U \text{ --- } 35P$$

$$2U \text{ --- } \frac{2}{5} \times 35P \text{ --- } 14P$$

$$7S \text{ --- } 35P$$

$$3S \text{ --- } \frac{3}{7} \times 35P \text{ --- } 15P$$

$$15P - 14P \text{ ----- } 1P \text{ ----- } 87 \text{ marbles}$$

	A	B
At First	2U (14P)	5U (35P)
Later	3S (15P)	7S (35P)

Applying the relevant scaffolds, a solution may be found as follows:

$$\text{Box A (original) --- } 14P \text{ ----- } 1218 \text{ marbles}$$

Accordingly, the solution is that Box A originally held 1,218 marbles.

Example 9

This example again applies the Multiplicity Schema and the Constants and Changes Concept, to a case involving constant difference. Consider the following word problem:

The ratio of the marbles in Box A to those in Box B is 2:5. When an equal number of marbles is added to each box, the ratio is 5:11. Express the marbles added as a fraction of the original number of marbles in the boxes.

Applying the Multiplicity Schema and the Constants and the Changes Concept, the problem elements and problem values may be represented as follows:

(a) Using what is given, we shall be able to see the following...

	A	B
At First	2U	5U
Later (16P)	5P	11P

Diagram showing the transition from 'At First' to 'Later' for both boxes A and B. For Box A, 2U becomes 5P. For Box B, 5U becomes 11P. Curved arrows with a '+' sign indicate the addition of marbles to each box.

(b) Understanding the constant difference as 3U as well as 6P...

3U --- 6P
7U --- 14P

5P + 11P --- 16P

	A	B
At First (7U) (14P)	2U	5U
Later (16P)	5P	11P

Diagram showing the transition from 'At First' to 'Later' for both boxes A and B. For Box A, 2U becomes 5P. For Box B, 5U becomes 11P. Curved arrows with a '+' sign indicate the addition of marbles to each box. A double-headed curved arrow between the 'At First' and 'Later' rows is labeled (3U) (6P), indicating the constant difference.

Applying the relevant scaffolds, a solution may be found as follows:

Marbles added --- $16P - 14P$ --- $2P$

Accordingly the number of marbles added is $\frac{2}{14}$, or equivalently $\frac{1}{7}$, of the original number of marbles in the boxes.

Example 10

This example applies the Multiplicity Schema in combination with the Dual Variables Concept. Consider the following word problem:

- 5 The ratio of the number of pupils in Class A to the number of pupils in Class B is 2:1. The ratio of boys in Class A to boys in Class B is 1:4. The ratio of girls in Class B to boys in Class A is 3:2. What fraction of the total number of pupils in the two classes are the girls in Class A.

Applying the Multiplicity Schema and the Dual Variables Concept, the problem elements and problem values may be represented as follows:

(a) Using 'The ratio of **boys in Class A : boys in Class B** is **1 : 4**' ...

Since the number of boys in Class A is divisible by 1 and 2, let's use the lowest common **multiple** of 2 for it.

	Class A	Class B
Boys	2P	8P
Girls		

(b) Using 'The ratio of **girls in Class B : boys in Class A** is **3 : 2**' ...

	Class A	Class B
Boys	2P	8P
Girls		3P

(c) Using 'The ratio of the **number of pupils in Class A** : the **number of pupils in Class B** is **2 : 1**'...

Class B --- 1U ----- 11P

Class A --- 2U ----- 22P

	Class A (2U) (22P)	Class B (1U) (11P)
Boys	2P	8P
Girls		3P

(d) We shall finally see the following...

Girls in Class A --- 22P – 2P --- 20P

	Class A (22P)	Class B (11P)
Boys	2P	8P
Girls	20P	3P

Applying the relevant scaffolds, the solution may be found as follows:

Total ----- 3U ----- 33P

- 5 Accordingly, the solution is found that the girls in Class A are $\frac{20}{33}$ of the pupils in the two classes.

Example 11

This example once again applies the Multiplicity Schema and the Dual Variables Concept. Consider the following word problem:

- 10 A box contains balls and bats that are either red or blue. The ratio of balls to bats is 2:1. The ratio of red to blue items is 3:2. The ratio of red bats to blue bats is 1:4. What fraction of the items in the box are red balls?

Applying the Multiplicity Schema and the Dual Variables Concept, the problem elements and problem values may be represented as follows:

(a) Using 'The ratio of **red bats : blue bats** is **1 : 4**'...

	Balls	Bats
Red		1P
Blue		4P

(b) Using 'The ratio of **balls : bats** is **2 : 1**'...

Bats ---- 1U ---- 5P
 Balls --- 2U ---- 10P
 Total --- 3U ---- 15P

	Balls (2U) (10P)	Bats (1U) (5P)
Red		1P
Blue		4P

(c) Using 'The ratio of **red : blue** items is **3 : 2**'...

Total --- 5S ---- 15P
 1S ----- 3P
 Blue ---- 2S ---- 6P
 Red ---- 3S ---- 9P

	Balls (10P)	Bats (5P)
Red (3S) (9P)		1P
Blue (2S) (6P)		4P

(d) We shall finally see the following...

Red balls --- 9P – 1P --- 8P

	Balls (10P)	Bats (5P)
Red (9P)	8P	1P
Blue (6P)		4P

The solution can therefore be seen to be that the red balls are $\frac{8}{15}$ of the items in the box.

Example 12

This example again applies the Multiplicity Schema with the Dual Variables
5 Concept. Consider the following word problem:

There are some passengers on a train. The ratio of adults to children is 5:1. The ratio of males to females is 1:2. The ratio of girls to men is 1:4. What fraction of the passengers on the train are women?

Applying the Multiplicity Schema and the Dual Variables Concept, the
10 problem elements and problem values may be represented as follows:

(a) Using 'The ratio of **adults : children** on it is **5 : 1**' as well as 'The ratio of **males : females** is **1 : 2**'...

Since the total is divisible by 6 and 3, let's use the lowest common **multiple** of 6 for it.

	Adults (5S)	Children (1S)
Male (2S)	Men	Boys
Female (4S)	Women	Girls

(b) Using 'The ratio of **girls : men** is **1: 4**'...

	Adults (5S)	Children (1S)
Male (2S)	Men (4U)	Boys
Female (4S)	Women	Girls (1U)

(c) Treating the number of boys to be 1P...

	Adults (5S)	Children (1S)
Male (2S) (1P + 4U)	Men (4U)	Boys (1P)
Female (4S)	Women	Girls (1U)

(d) We shall finally see the following...

Since $1S \text{ --- } 1P + 1U$
 Male $\text{--- } 2S \text{ --- } 2P + 2U$

So $2P + 2U \text{ --- } 1P + 4U$

Using **restating & rearranging**, $1P \text{ --- } 2U$

$1S \text{ --- } 1P + 1U \text{ --- } 2U + 1U \text{ --- } 3U$
 Female $\text{--- } 4S \text{ --- } 12U$

Women $\text{--- } 12U - 1U \text{ --- } 11U$

	Adults (5S)	Children (1S)
Male (2S) (6U)	Men (4U)	Boys (2U)
Female (4S) (12U)	Women (11U)	Girls (1U)

Applying the relevant scaffolds, a solution may be found as follows:

Total $\text{--- } 6S \text{ --- } 18U$

- 5 Accordingly, it can be seen that women make up $\frac{11}{18}$ of the passengers on the train, which is the solution.

Example 13

This final example illustrates the Multiplicity Schema applied with the Systematic Testing Concept. Consider the following word problem:

- 10 Driver Ali earned 20c for every vase he delivered successfully to Shop A. However, he is required to compensate his company \$1.00 for every vase he breaks and therefore is unable to deliver. If Ali finally earned \$37.20 for the 300 vases he was supposed to deliver to the shop, how many vases did he successfully deliver?

- 15 Applying the Multiplicity Schema and the Systematic Testing Concept, the problem elements and problem values may be represented as follows:

(a) Using what is given, we shall be able to see the following...

$$\begin{aligned} 300 \times 20\text{¢} & \text{-----} \$60 \\ 299 \times 20\text{¢} & \text{-----} \$59.80 \\ \$60 - \$59.80 & \text{-----} \$1.20 \end{aligned}$$

Successful (+ Amount)	Broke (- Amount)	Final Amount
300 (\$60)	0 (\$0)	\$60 (x)
299 (\$59.80)	1 (\$1)	\$58.80 (x)

Arrows indicate transitions: from 300 to 299 (labeled -1), from 0 to 1 (labeled +1), and from \$60 to \$58.80 (labeled - \$1.20).

(b) As we continue, we shall be able to see an emerging pattern...

$$\begin{aligned} 298 \times 20\text{¢} & \text{-----} \$59.60 \\ \$58.80 - \$57.60 & \text{---} \$1.20 \end{aligned}$$

Successful (+ Amount)	Broke (- Amount)	Final Amount
300 (\$60)	0 (\$0)	\$60 (x)
299 (\$59.80)	1 (\$1)	\$58.80 (x)
298 (\$59.60)	2 (\$2)	\$57.60 (x)

Arrows indicate transitions: from 300 to 299 (labeled -1), from 1 to 2 (labeled +1), and from \$58.80 to \$57.60 (labeled - \$1.20).

(c) We shall finally see the following...

$$\begin{aligned} \$60 - \$37.20 & \text{-----} \$22.80 \\ \$22.80 \div \$1.20 & \text{-----} 19 \\ 300 - 19 & \text{---} 281 \\ 281 \times 20\text{¢} - 19 \times \$1 & \text{---} \$37.20 \end{aligned}$$

Successful (+ Amount)	Broke (- Amount)	Final Amount
300 (\$60)	0 (\$0)	\$60 (x)
299 (\$59.80)	1 (\$1)	\$58.80 (x)
298 (\$59.60)	2 (\$2)	\$57.60 (x)
281 (\$56.20)	19 (\$19)	\$37.20 (?)

Arrows indicate transitions: from 300 to 299 (labeled -1), from 1 to 2 (labeled +1), from 2 to 19 (labeled -19), and from \$58.80 to \$37.20 (labeled - \$22.80).

By noting the emerging pattern, it soon becomes apparent through
5 systematic testing using the visual aid of the Multiplicity Schema that driver Ali
successfully delivered 281 vases, breaking 19.

Problem Solution Method

From the foregoing, it is possible to generalise a method for solving mathematical word problems in accordance with the preferred embodiment of the present invention. Figure 2 is a flowchart 200 illustrating this method.

5 Given a word problem 202, the method involves identifying problem elements and problem values (step 204), the selection of relevant elementary math problem concepts (step 206), and the construction of a visual representation using relevant Schemas (step 208).

10 A number of points should be noted in relation to the steps 204, 206, 208, as will be apparent from the foregoing examples. Firstly, problem elements are typically the “actors”, such as items, objects and subjects appearing in the one or more statements making up the word problem 202. The problem values are (known or unknown) values associated with the problem elements.

15 The identification of problem elements and/or values, and the selection of the relevant elementary math problem concept or concepts required to analyse the problem need not occur in the order shown.

20 Furthermore, the construction of the visual representation using the relevant schemas, at step 208, is a dynamic process which may be ongoing as the problem elements and values are identified, and the relevant elementary math problem concepts are selected. This aspect of the process is illustrated in the flowchart 200 by allowing the steps 204, 206, 208 to be repeated in a loop, in accordance with the decision step 210. In particular, the steps 204, 206, 208 will be repeated until all of the relevant problem elements and problem values have been utilised in the construction of a complete visual representation of the
25 problem.

30 Once the visual representation is complete, at step 212, the relevant scaffolds are applied, if necessary, in order to arrive at the final output solution 214. As will be appreciated from a number of the examples above, in some cases the explicit application of particular scaffolds will not be necessary, since the solution may be apparent from inspection of the complete visual representation of the problem. Nonetheless, there will generally be an application of fundamental mathematical reasoning and skills, however trivial, in identifying

the information representing the solution within the complete visual representation.

Teaching Method

Furthermore, the foregoing description of the elementary math problem
5 concepts 104, schemas 110, scaffolds 106, and the various examples, imply a
general framework for the structured teaching of math problem solving. Figure 3
shows a flowchart 300 illustrating a teaching method in accordance with a
preferred embodiment of the invention.

The first step 302 in the teaching method is to define the elementary math
10 problem concepts, such as the concepts 104 included within the preferred
embodiment of the schematic system 100. Additionally, at step 304, students
should be provided with appropriate visual aids, such as the schemas 110 and
corresponding relational symbols 112. Students must also be taught the
scaffolds, as indicated in step 306, which provide the underlying fundamental
15 reasoning and math skills for solving mathematical problems. As will be
appreciated, the specific order in which the steps 302, 304 and 306 are
established is not important. Initially, while it may be possible and appropriate to
develop specific methods for teaching the scaffolds at step 306, a variety of
teaching methods are known and any appropriate and effective method may be
20 employed.

As in topic-based learning, students may be taught appropriate skills by
way of example and practice. However, in contrast to conventional topic-based
teaching, the development of examples and practical experience is organised
around the elementary math problem concepts 104, rather than around math
25 topic areas. At step 308, therefore, students are presented with example word
problems, structured such that the examples are arranged to include at least one
common elementary math problem concept which is to be taught, exemplified or
practised.

At steps 310, the general method of problem solution is taught, in relation
30 to the selected elementary math problem concept, and associated schemas,
utilising an embodiment of the general method represented by the flowchart 200.

The decision step 312 facilitates repetition of the teaching step 310 until all of the relevant examples or practice problems have been completed.

Computer Implementation

While a preferred embodiment of the invention has been described above
5 in general terms, with reference to Figures 1 to 3 and by way of Examples 1 to 13, it will be apparent to persons skilled in the art that numerous practical implementations of problem solving methods and teaching methods may be provided within the scope of the invention. For example, the invention may be
10 facilitated by a human teacher, and with the use of appropriate visual aids, including textbooks, workbooks, flashcards, blackboards/whiteboards and the like.

However, the systematic structured approach to teaching, learning and math problem-solving provided by embodiments of the present invention may be
15 well-suited to computer implementation, either as a complete teaching/learning tool or through the employment of computer-aided learning within the classroom. In one possible approach, example problems (such as Examples 1 to 13 above) may be implemented in the form of an interactive computer program, enabling students to build up the dynamic visual representations associated with the
20 schemas employed in the solution of each problem on a computer display using computer input devices, such as a keyboard and/or pointing device.

A computer-implemented system providing a graphical user interface within which students are able to construct visual representations corresponding with any given work problem, utilising the finite available set of schemas 110 and
25 relational symbols 112 is entirely feasible. Computer software products for performing vastly more sophisticated mathematical manipulations, used by advanced practitioners such as engineers and scientists, have been available commercially for some time. A similar approach could therefore be employed in order to provide a computer software product for use in the teaching and practice
30 of the schematic system 100.

Accordingly, while a particular embodiment of the present invention has been described herein, by way of example, it will be understood that the invention

is not so limited, but rather that the scope of the invention is defined by the claims appended hereto.

CLAIMS:

1. A method of teaching a student to solve math word problems of the type comprising one or more problem statements including problem elements and problem values, the method including the steps of:
 - 5 defining a plurality of elementary concepts relating to possible relationships between problem elements and/or problem values;
providing a plurality of visual aids associated with said elementary concepts, wherein each said visual aid enables a corresponding relationship between problem elements and/or problem values to be represented in
10 diagrammatic form;
teaching the student a plurality of fundamental math skills underlying said plurality of elementary concepts;
presenting the student with one or more math problems comprising math problem statements requiring application of at least a common one of said
15 elementary concepts; and
teaching the solution of each of said plurality of math problems to the student by in each case identifying problem elements and problem values within the corresponding problem statements, selecting from said plurality of elementary concepts those which define relevant relationships between the identified problem
20 elements and problem values, and demonstrating the problem solution by use of the visual aids associated with each said elementary concept and by corresponding application of one or more of said fundamental math skills.
2. The method of teaching of claim 1 wherein the plurality of elementary concepts includes a Relativity Concept, a Basic Comparison Concept, a
25 Redistribution Concept, a Constants and Changes Concept, a Dual Variables Concept, and a Systematic Testing Concept.
3. The method of teaching of claim 2 wherein the plurality of visual aids includes a Relativity Schema, associated with the Relativity Concept, a Basic Comparison Schema, associated with the Basic Comparison Concept, a
30 Redistribution Schema, associated with the Redistribution Concept, and a

Multiplicity Schema, associated with the Constants and Changes, Dual Variables, and Systematic Testing Concepts.

4. The method of teaching of claim 3 wherein the visual aids are represented using a plurality of visual symbols.
- 5 5. The method of teaching of claim 4 wherein the plurality of visual symbols includes visual symbols denoting matching or equality relationships between values, differences between values, operations performed upon values, sums of values, sequenced stages, and splitting of values.
- 10 6. The method of teaching of claim 1 wherein the plurality of fundamental math skills includes maths language, reciprocity, multiples, comparing gain and loss, restating and rearranging, a sets property, and direct comparison.
7. The method of teaching of claim 6 wherein said math language includes “more than/less than/difference”, “proportion of”, “proportion more than/proportion less than”, and “sum/total/altogether”.
- 15 8. A method of solving a math problem of the type comprising one or more problem statements including problem elements and problem values, the method including:
 - identifying problem elements and problem values included within the problem statement;
 - 20 selecting from a plurality of predetermined elementary concepts relating to possible relationships between problem elements and/or problem values those elementary concepts which define relevant relationships between the identified problem elements and problem values of the problem statement;
 - using one or more predetermined visual aids associated with said plurality
 - 25 of predetermined elementary concepts to construct a visual representation of corresponding identified relationships between the problem elements and/or problem values;

applying one or more predetermined fundamental math skills to the identified relationships depicted in the visual representation to arrive at a solution to the math problem; and

providing the solution as an output.

- 5 9. The method of solving a math problem of claim 8 wherein the plurality of elementary concepts includes a Relativity Concept, a Basic Comparison Concept, a Redistribution Concept, a Constants and Changes Concept, a Dual Variables Concept, and a Systematic Testing Concept.

- 10 10. The method of solving a math problem of claim 9 wherein the plurality of visual aids includes a Relativity Schema, associated with the Relativity Concept, a Basic Comparison Schema, associated with the Basic Comparison Concept, a Redistribution Schema, associated with the Redistribution Concept, and a Multiplicity Schema, associated with the Constants and Changes, Dual Variables, and Systematic Testing Concepts.

- 15 11. The method of solving a math problem of claim 8 wherein the plurality of fundamental math skills includes math language, reciprocity, multiples, comparing gain and loss, restating and rearranging, a sets property, and direct comparison.

- 20 12. A system for solving math problems of the type comprising one or more problem statements including problem elements and problem values, the system including:

a plurality of visual aids comprising a set of relational symbols, said visual aids being adapted for the purpose of representing visually a plurality of predetermined elementary concepts relating to possible relationships between problem elements and/or problem values;

- 25 means for presenting and arranging said visual aids in accordance with elementary concepts selected from said predetermined elementary concepts, and which define relevant relationships between identified problem elements and problem values of the math problem; and

wherein a solution to the math problem is determined by applying one or more predetermined fundamental math skills to the relationships depicted in a visual representation constructed using said presenting and arranging means.

13. The system for solving math problems of claim 12 wherein the visual aids
5 include one or more of a textbook, a workbook, a set of flashcards, a blackboard or a whiteboard.

14. The system for solving math problems of claim 12 which includes a
computer having a graphical display, and wherein the computer is loaded with
executable instruction code for displaying said visual aids on the graphical
10 display, and for enabling a user to present and arrange the visual aids in
accordance with said selected elementary concepts using a graphical user
interface provided on the graphical display.