

HYBRID METHOD FOR FULL WAVEFORM INVERSION USING SIMULTANEOUS AND SEQUENTIAL SOURCE METHOD

CROSS REFERENCE TO RELATED APPLICATIONS

5 [0001] This application claims the benefit of U.S. Provisional Application 61/386,828 filed on September 27, 2010, which is incorporated herein by reference in all jurisdictions that allow it. This application is related to that particular application entitled "Simultaneous Source Encoding and Source Separation as a Practical Solution for Full Wavefield Inversion" and claiming the benefit of U.S. Provisional Application 61/386,831 filed on September 27,
10 2010 and U.S. Patent Application 12/903,749 filed on October 13, 2010. These related applications are also incorporated by reference herein in all jurisdictions that all it.

FIELD OF THE INVENTION

[0002] The invention relates generally to the field of geophysical prospecting, and more particularly to geophysical data processing. Specifically, the invention is a method for
15 inversion of data acquired from multiple geophysical sources such as seismic sources, involving geophysical simulation that computes the data from many simultaneously-active geophysical sources in one execution of the simulation.

BACKGROUND OF THE INVENTION

[0003] Geophysical inversion [1,2] attempts to find a model of subsurface properties that
20 optimally explains observed data and satisfies geological and geophysical constraints. There are a large number of well known methods of geophysical inversion. These well known methods fall into one of two categories, iterative inversion and non-iterative inversion. The following are definitions of what is commonly meant by each of the two categories:

[0004] Non-iterative inversion – inversion that is accomplished by assuming some simple
25 background model and updating the model based on the input data. This method does not use the updated model as input to another step of inversion. For the case of seismic data these methods are commonly referred to as imaging, migration, diffraction tomography or Born inversion.

[0005] Iterative inversion – inversion involving repetitious improvement of the
30 subsurface properties model such that a model is found that satisfactorily explains the observed data. If the inversion converges, then the final model will better explain the observed data and will more closely approximate the actual subsurface properties. Iterative

inversion usually produces a more accurate model than non-iterative inversion, but is much more expensive to compute.

[0006] Iterative inversion is generally preferred over non-iterative inversion, because it yields more accurate subsurface parameter models. Unfortunately, iterative inversion is so computationally expensive that it is impractical to apply it to many problems of interest. This high computational expense is the result of the fact that all inversion techniques require many compute intensive simulations. The compute time of any individual simulation is proportional to the number of sources to be inverted, and typically there are large numbers of sources in geophysical data, where the term *source* as used in the preceding refers to an activation location of a source apparatus. The problem is exacerbated for iterative inversion, because the number of simulations that must be computed is proportional to the number of iterations in the inversion, and the number of iterations required is typically on the order of hundreds to thousands.

[0007] The most commonly employed iterative inversion method employed in geophysics is cost function optimization. Cost function optimization involves iterative minimization or maximization of the value, with respect to the model M , of a cost function $S(M)$ which is a measure of the misfit between the calculated and observed data (this is also sometimes referred to as the objective function), where the calculated data are simulated with a computer using the current geophysical properties model and the physics governing propagation of the source signal in a medium represented by a given geophysical properties model. The simulation computations may be done by any of several numerical methods including but not limited to finite difference, finite element or ray tracing. The simulation computations can be performed in either the frequency or time domain.

[0008] Cost function optimization methods are either local or global [3]. Global methods simply involve computing the cost function $S(M)$ for a population of models $\{M_1, M_2, M_3, \dots\}$ and selecting a set of one or more models from that population that approximately minimize $S(M)$. If further improvement is desired this new selected set of models can then be used as a basis to generate a new population of models that can be again tested relative to the cost function $S(M)$. For global methods each model in the test population can be considered to be an iteration, or at a higher level each set of populations tested can be considered an iteration. Well known global inversion methods include Monte Carlo, simulated annealing, genetic and evolution algorithms.

[0009] Unfortunately global optimization methods typically converge extremely slowly and therefore most geophysical inversions are based on local cost function optimization.

Algorithm 1 summarizes local cost function optimization.

- | | |
|---|--|
| 5 | 1. selecting a starting model,
2. computing the gradient of the cost function $S(M)$ with respect to the parameters that describe the model,
3. searching for an updated model that is a perturbation of the starting model in the negative gradient direction that better explains the observed data. |
|---|--|

Algorithm 1 – Algorithm for performing local cost function optimization.

10 [0010] This procedure is iterated by using the new updated model as the starting model for another gradient search. The process continues until an updated model is found that satisfactorily explains the observed data. Commonly used local cost function inversion methods include gradient search, conjugate gradients and Newton's method.

15 [0011] Local cost function optimization of seismic data in the acoustic approximation is a common geophysical inversion task, and is generally illustrative of other types of geophysical inversion. When inverting seismic data in the acoustic approximation the cost function can be written as:

$$S(M) = \sum_{g=1}^{N_g} \sum_{r=1}^{N_r} \sum_{t=1}^{N_t} W(\psi_{calc}(M, r, t, w_g) - \psi_{obs}(r, t, w_g)) \quad (\text{Eqn. 1})$$

where:

20 S = cost function,

M = vector of N parameters, $(m_1, m_2, \dots, m_N)$ describing the subsurface model,

g = gather index,

w_g = source function for gather g which is a function of spatial coordinates and time, for a point source this is a delta function of the spatial coordinates,

25 N_g = number of gathers,

r = receiver index within gather,

N_r = number of receivers in a gather,

t = time sample index within a trace,

N_t = number of time samples,

W = minimization criteria function (we usually choose $W(x)=x^2$ which is the least squares (L2) criteria),

ψ_{calc} = calculated seismic pressure data from the model M,

5 ψ_{obs} = measured seismic pressure data.

[0012] The gathers can be any type of gather that can be simulated in one run of a seismic forward modeling program. Usually the gathers correspond to a seismic shot, although the shots can be more general than point sources. For point sources the gather index g corresponds to the location of individual point sources. For plane wave sources g would
 10 correspond to different plane wave propagation directions. This generalized source data, ψ_{obs} , can either be acquired in the field or can be synthesized from data acquired using point sources. The calculated data ψ_{calc} on the other hand can usually be computed directly by using a generalized source function when forward modeling. For many types of forward modeling, including finite difference modeling, the computation time needed for a
 15 generalized source is roughly equal to the computation time needed for a point source.

[0013] Equation(1) can be simplified to:

$$S(M) = \sum_{g=1}^{N_g} W(\delta(M, w_g)) \quad (\text{Eqn. 2})$$

where the sum over receivers and time samples is now implied and,

$$20 \quad \delta(M, w_g) = \psi_{calc}(M, w_g) - \psi_{obs}(w_g) \quad (\text{Eqn. 3})$$

[0014] Inversion attempts to update the model M such that $S(M)$ is a minimum. This can be accomplished by local cost function optimization which updates the given model $M^{(k)}$ as follows:

$$25 \quad M^{(k+1)} = M^{(k)} - \alpha^{(k)} \nabla_M S(M) \quad (\text{Eqn. 4})$$

where k is the iteration number, α is the scalar size of the model update, and $\nabla_M S(M)$ is the gradient of the misfit function, taken with respect to the model parameters. The model perturbations, or the values by which the model is updated, are calculated by multiplication

of the gradient of the objective function with a step length α , which must be repeatedly calculated.

[0015] From equation (2), the following equation can be derived for the gradient of the cost function:

$$\nabla_M S(M) = \sum_{g=1}^{N_g} \nabla_M W(\delta(M, w_g)) \quad (\text{Eqn. 5})$$

[0016] So to compute the gradient of the cost function one must separately compute the gradient of each gather's contribution to the cost function, then sum those contributions. Therefore, the computational effort required for computing $\nabla_M S(M)$ is N_g times the compute effort required to determine the contribution of a single gather to the gradient. For geophysical problems, N_g usually corresponds to the number of geophysical sources and is on the order of 10,000 to 100,000, greatly magnifying the cost of computing $\nabla_M S(M)$.

[0017] Note that computation of $\nabla_M W(\delta)$ requires computation of the derivative of $W(\delta)$ with respect to each of the N model parameters m_i . Since for geophysical problems N is usually very large (usually more than one million), this computation can be extremely time consuming if it had to be performed for each individual model parameter. Fortunately, the adjoint method can be used to efficiently perform this computation for all model parameters at once [1]. The adjoint method for the least squares objective function and a gridded model parameterization is summarized by the following algorithm:

1. Compute forward simulation of the data using the current model and the gather signature w_g as the source to get $\psi_{calc}(M^{(k)}, w_g)$,
2. Subtract the observed data from the simulated data giving $\delta(M^{(k)}, w_g)$,
3. Compute the reverse simulation (i.e. backwards in time) using $\delta(M^{(k)}, w_g)$ as the source producing $\psi_{adjoint}(M^{(k)}, w_g)$,
4. Compute the integral over time of the product of $\psi_{calc}(M^{(k)}, w_g)$ and $\psi_{adjoint}(M^{(k)}, w_g)$ to get $\nabla_M W(\delta(M, w_g))$.

Algorithm 2 – Algorithm for computing the least-squares cost-function gradient of a gridded model using the adjoint method.

[0018] While computation of the gradients using the adjoint method is efficient relative to other methods, it is still very costly. In particular the adjoint methods requires two

simulations, one forward in time and one backward in time, and for geophysical problems these simulations are usually very compute intensive. Also, as discussed above, this adjoint method computation must be performed for each measured data gather individually, increasing the compute cost by a factor of N_g .

5 **[0019]** The compute cost of all categories of inversion can be reduced by inverting data from combinations of the sources, rather than inverting the sources individually. This may be called simultaneous source inversion. Several types of source combination are known including: coherently sum closely spaced sources to produce an effective source that produces a wavefront of some desired shape (e.g. a plane wave), sum widely spaced sources,
10 or fully or partially stacking the data before inversion.

[0020] The compute cost reduction gained by inverting combined sources is at least partly offset by the fact that inversion of the combined data usually produces a less accurate inverted model. This loss in accuracy is due to the fact that information is lost when the individual sources are summed, and therefore the summed data does not constrain the
15 inverted model as strongly as the unsummed data. This loss of information during summation can be minimized by encoding each shot record before summing. Encoding before combination preserves significantly more information in the simultaneous source data, and therefore better constrains the inversion [4]. Encoding also allows combination of closely spaced sources, thus allowing more sources to be combined for a given computational region.
20 Various encoding schemes can be used with this technique including time shift encoding and random phase encoding. The remainder of this Background section briefly reviews various published geophysical simultaneous source techniques, both encoded and non-encoded.

[0021] Van Manen [6] suggests using the seismic interferometry method to speed up forward simulation. Seismic interferometry works by placing sources everywhere on the
25 boundary of the region of interest. These sources are modeled individually and the wavefield at all locations for which a Green's function is desired is recorded. The Green's function between any two recorded locations can then be computed by cross-correlating the traces acquired at the two recorded locations and summing over all the boundary sources. If the data to be inverted have a large number of sources and receivers that are within the region of
30 interest (as opposed to having one or the other on the boundary), then this is a very efficient method for computing the desired Green's functions. However, for the seismic data case it is rare that both the source and receiver for the data to be inverted are within the region of

interest. Therefore, this improvement has very limited applicability to the seismic inversion problem.

[0022] Berkhout [7] and Zhang [8] suggest that inversion in general can be improved by inverting non-encoded simultaneous sources that are summed coherently to produce some
5 desired wave front within some region of the subsurface. For example, point source data could be summed with time shifts that are a linear function of the source location to produce a down-going plane wave at some particular angle with respect to the surface. This technique could be applied to all categories of inversion. A problem with this method is that coherent summation of the source gathers necessarily reduces the amount of information in the data.
10 So for example, summation to produce a plane wave removes all the information in the seismic data related to travel time versus source-receiver offset. This information is critical for updating the slowly varying background velocity model, and therefore Berkhout's method is not well constrained. To overcome this problem many different coherent sums of the data (e.g. many plane waves with different propagation directions) could be inverted, but then
15 efficiency is lost since the cost of inversion is proportional to the number of different sums inverted. Herein, such coherently summed sources are called generalized sources. Therefore, a generalized source can either be a point source or a sum of point sources that produces a wave front of some desired shape.

[0023] Van Riel [9] suggests inversion by non-encoded stacking or partial stacking (with
20 respect to source-receiver offset) of the input seismic data, then defining a cost function with respect to this stacked data which will be optimized. Thus, this publication suggests improving cost function based inversion using non-encoded simultaneous sources. As was true of the Berkhout's [6] simultaneous source inversion method, the stacking suggested by this method reduces the amount of information in the data to be inverted and therefore the
25 inversion is less well constrained than it would have been with the original data.

[0024] Mora [10] proposes inverting data that is the sum of widely spaced sources. Thus, this publication suggests improving the efficiency of inversion using non-encoded simultaneous source simulation. Summing widely spaced sources has the advantage of preserving much more information than the coherent sum proposed by Berkhout. However,
30 summation of widely spaced sources implies that the aperture (model region inverted) that must be used in the inversion must be increased to accommodate all the widely spaced sources. Since the compute time is proportional to the area of this aperture, Mora's method

does not produce as much efficiency gain as could be achieved if the summed sources were near each other.

[0025] Ober [11] suggests speeding up seismic migration, a special case of non-iterative inversion, by using simultaneous encoded sources. After testing various coding methods, 5 Ober found that the resulting migrated images had significantly reduced signal-to-noise ratio due to the fact that broad band encoding functions are necessarily only approximately orthogonal. Thus, when summing more than 16 shots, the quality of the inversion was not satisfactory. Since non-iterative inversion is not very costly to begin with, and since high signal-to-noise ratio inversion is desired, this technique is not widely practiced in the 10 geophysical industry.

[0026] Ikelle [12] suggests a method for fast forward simulation by simultaneously simulating point sources that are activated (in the simulation) at varying time intervals. A method is also discussed for decoding these time-shifted simultaneous-source simulated data back into the separate simulations that would have been obtained from the individual point 15 sources. These decoded data could then be used as part of any conventional inversion procedure. A problem with Ikelle's method is that the proposed decoding method will produce separated data having noise levels proportional to the difference between data from adjacent sources. This noise will become significant for subsurface models that are not laterally constant, for example from models containing dipping reflectors. Furthermore, this 20 noise will grow in proportion to the number of simultaneous sources. Due to these difficulties, Ikelle's simultaneous source approach may result in unacceptable levels of noise if used in inverting a subsurface that is not laterally constant.

[0027] Source encoding proposed by Krebs et al. in PCT Patent Application Publication No. WO 2008/042081, which is incorporated herein by reference in all jurisdictions that 25 allow it, is a very cost effective method to invert full wave field data. (The same approach of simultaneous inversion of an encoded gather will work for receivers, either via source-receiver reciprocity or by encoding the actual receiver locations in common-source gathers of data.) For fixed receivers, the forward and adjoint computations only need to be performed for a single effective source; see PCT Patent Application Publication No. WO 2009/117174, 30 which is incorporated herein by reference in all jurisdictions that allow it. Given the fact that hundreds of shots are recorded for typical 2D acquisition geometries, and thousands in the case of 3D surveys, computational savings from this method are quite significant. In practice,

a fixed receiver assumption is not strictly valid for most common field data acquisition geometries. In the case of marine streamer data, both sources and receivers move for every new shot. Even in surveys where the locations of receivers are fixed, the practice often is that not all receivers are “listening” to every shot, and the receivers that are listening can vary from shot-to-shot. This also violates the “fixed-receiver assumption.” In addition, due to logistical problems, it is difficult to record data close to the source, and this means that near-offset data are typically missing. This is true for both marine and land surveys. Both of these factors mean that for a simultaneous source gather, every receiver location will be missing data for some source shots. In summary, in simultaneous encoded-source inversion, for a given simultaneous encoded gather, data are required at all receiver locations for every shot, and this may be referred to as the fixed-receiver assumption of simultaneous encoded-source inversion. In WO 08/042081, some of the disclosed embodiments may work better than others when the fixed receiver assumption is not satisfied. Therefore, it would be advantageous to have an accommodation or adjustment to straightforward application of simultaneous encoded sources (and/or receivers) inversion that would enhance its performance when the fixed receiver assumption is compromised. The present invention provides ways of doing this. Haber et al. [15] also describe an approach to the problem of moving receivers in simultaneous encoded source inversion using a stochastic optimization method, and apply it to a direct current resistivity problem.

20

SUMMARY OF THE INVENTION

[0028] In one embodiment, the invention is a computer-implemented method for full-wavefield inversion of measured geophysical data to determine a physical properties model for a subsurface region, comprising: (a) using a computer to invert a selected shallow time window of arrivals from the measured geophysical data by simultaneous encoded sources and/or receivers inversion to obtain a first physical properties model for the subsurface region; (b) using a computer to invert the measured geophysical data, or a selected deep time window of arrivals from the measured geophysical data, by iterative sequential source inversion, which may use only a sparse sampling of the measured data, to obtain a second physical properties model for the subsurface region, wherein the first physical properties model is used as a starting model and a set of source locations is used to update the second physical properties model in the iterative sequential source inversion; and (c) outputting,

30

displaying, or saving to data storage the second physical properties model of the subsurface region.

[0029] In another embodiment, the invention is a computer-implemented method for full-wavefield inversion of measured geophysical data to determine a physical properties model for a subsurface region, comprising: (a) using a computer to invert a selected shallow time window of arrivals from the measured geophysical data by simultaneous encoded sources and/or receivers inversion to obtain a first physical properties model for the subsurface region; (b) using the first physical properties model to simulate, using a computer, synthetic data for longer offsets corresponding to arrivals from deeper than said shallow time window; (c) using a computer to invert the measured geophysical data, wherein the data with longer offsets are augmented, said inversion being simultaneous encoded-sources and/or encoded-receivers inversion, to obtain a second physical properties model of the subsurface region, wherein said augmented data with longer offsets are the sum of the synthetic data for longer offsets and the measured data at the longer offsets; and (d) outputting, displaying, or saving to data storage the second physical properties model of the subsurface region.

BRIEF DESCRIPTION OF THE DRAWINGS

[0030] Due to patent rule restrictions on use of color, the attached drawings are black and white reproductions of color originals. Copies of this patent application or publication with color drawings may be obtained from the U.S. Patent and Trademark Office (U.S. Application No. 12/903,744) upon request and payment of the necessary fee.

[0031] The present invention and its advantages will be better understood by referring to the following detailed description and the attached drawings in which:

Fig. 1 is a schematic diagram showing the data window that can be used for simultaneous source inversion;

Fig. 2 is a flowchart showing basic steps in one embodiment of the present inventive method, wherein simultaneous source encoding is used in a shallow time window and sparse sequential source inversion is used for deeper windows in Approach I, and in Approach II the shallow model is used to compute the missing data traces that are first encoded to carry out simultaneous source inversion;

Fig. 3 shows the “true” velocity model used in the example to generate the measured data;

Fig. 4 shows two shot gathers of sequential source data computed using the velocity model in Fig. 3 and encoded using binary encoding described in WO 2008/042081, indicating a shallow time window of 0-3 seconds;

Fig. 5 shows the starting velocity model for the full wavefield inversion in the test example;

- 5 Fig. 6 shows the model obtained from simultaneous source inversion by inverting the low frequency data (peak frequency of 7 Hz) from the shallow window (0-3 seconds); as shown in Fig. 4, the fixed receiver assumption is valid for the time window 0-3 seconds;

- Fig. 7 shows the model obtained using simultaneous source inversion of low frequency data (peak frequency of 7 Hz) with the data window from 0-4 seconds; since the fixed receiver
10 assumption is no longer valid, artifacts can clearly be seen in the inverted model;

Fig. 8 shows the model obtained using simultaneous source inversion of low frequency data (peak frequency of 7 Hz) with the data window from 0-5 seconds; since the fixed receiver assumption is no longer valid, artifacts can clearly be seen in the inverted model;

- Fig. 9 shows the model obtained in the test example by using sequential source inversion of
15 low frequency data (peak frequency of 7 Hz) with the data window from 0-4 seconds using a sparse set of sources with shot separation of 1.2 km; since sequential sources are used, the model does not have any artifacts compared to Fig. 7;

- Fig. 10 shows the model obtained in the test example using sequential source inversion of
20 low frequency data (peak frequency of 7 Hz) with the data window from 0-5 seconds using a sparse set of sources with shot separation of 1.2 km; since sequential sources are used, the model does not have any artifacts compared to Fig. 8;

- Fig. 11 shows the model obtained in the test example using sequential source inversion of full
band frequency data (peak frequency of 40 Hz) with the data window from 0-5 seconds using
a sparse set of sources with shot separation of 1.2 km; since sequential sources are used, the
25 model does not have any artifacts compared to Fig. 8;

Fig. 12 shows a near and far-offset limited shot gather computed in the test example using the true model in Fig. 3; the nearest available offset is 200 m and farthest available offset is 5000 m;

Fig. 13 shows the near-offset limited shot gather in Fig. 12 filled with the trace from nearest available offset at $x = 200$;

Fig. 14 illustrates inversion using the encoded data with near-offset limited shot gathers shown in Fig. 12, the model shown at iteration 5 obtained with simultaneous source inversion with the low frequency data (peak frequency of 7 Hz);

Fig. 15 illustrates inversion using the encoded data with near-offset limited shot gathers shown in Fig. 12, the model shown at iteration 10 obtained with simultaneous source inversion with the low frequency data (peak frequency of 7 Hz); it clearly shows that the model has diverged with significant artifacts;

Fig. 16 illustrates inversion using the encoded data with near-offset filled shot gathers shown in Fig. 13 (since the near-offset filling was approximate the gradient in the water layer is muted to reduce the impact of the approximation on the simultaneous source inversion), the model shown at iteration 5 obtained with simultaneous source inversion with the low frequency data (peak frequency of 7 Hz); and

Fig. 17 illustrates inversion using the encoded data with near-offset filled shot gathers shown in Fig. 13 (since the near-offset filling was approximate the gradient in the water layer is muted to reduce the impact of the approximation on the simultaneous source inversion), the model shown at iteration 200 obtained with simultaneous source inversion with the low frequency data (peak frequency of 7 Hz).

[0032] The invention will be described in connection with example embodiments. However, to the extent that the following detailed description is specific to a particular embodiment or a particular use of the invention, this is intended to be illustrative only, and is not to be construed as limiting the scope of the invention. On the contrary, it is intended to cover all alternatives, modifications and equivalents that may be included within the scope of the invention, as defined by the appended claims.

DETAILED DESCRIPTION OF EXAMPLE EMBODIMENTS

[0033] One embodiment of the present invention is a hybrid combination of simultaneous encoded source inversion with traditional sequential source inversion. This embodiment uses simultaneous source encoding for the shallower time-window of the data and uses sparse

sequential sources to invert the deeper part of the data. Krebs et al. [5,16] show that the encoded simultaneous source cost function can be computed more efficiently than conventional cost functions while still providing accurate inversions. The simultaneous source cost function is defined here as (compare with equation (2) above):

$$S_{sim}(M) = \sum_{G=1}^{N_G} W \left(\delta \left(M, \sum_{g \in G} c_g \otimes w_g \right) \right) \quad (\text{Eqn. 6})$$

where a summation over receivers and time samples is implied as in Eqn. (2), and:

$\sum_{g=1}^{N_g} = \sum_{G=1}^{N_G} \sum_{g \in G}$ defines a sum over gathers by sub groups of gathers,

S_{sim} = cost function for simultaneous source data,

G = the groups of simultaneous generalized sources, and

10 N_G = the number of groups,

c_g = functions of time that are convolved ($\otimes$) with each gather's source signature to encode the gathers, these encoding functions are chosen to be approximately orthogonal with respect to some appropriate operation depending on the weighting function W . When W is the L2-Norm the appropriate operation is cross-correlation.

15 **[0034]** The outer summation in Eqn. (6) is over groups of simultaneous generalized sources corresponding to the gather type (e.g. points sources for common shot gathers). The inner summation, over g , is over the gathers that are grouped for simultaneous computation. For some forward modeling methods, such as finite difference modeling, the computation of the forward model for summed generalized sources (the inner sum over $g \in G$) can be performed in the same amount of time as the computation for a single source. Therefore, as shown in Krebs et. al. [5] $\delta(M, \sum c_g \otimes w_g)$ can be computed very efficiently using Algorithm 3.

25

1. Simulate $\psi_{calc}(M, \sum c_g \otimes w_g)$ using a single run of the simulator using $\sum c_g \otimes w_g$ as the source,
2. Convolve each measured data gather with the c_g encoding functions, then sum the resulting encoded gathers (i.e. $\sum c_g \otimes \psi_{obs}(w_g)$),
3. Subtract the result of step 2 from the result of step 1.

Algorithm 3 – Algorithm for computing the encoded simultaneous-source cost function.

[0035] Again as shown in Krebs et al. [5] this algorithm can compute $S_{sim}(M)$ a factor of N_g/N_G times faster than $S(M)$ from Eqn. (2).

5 [0036] In fixed receiver geometry the shot gathers have complete receiver coverage. For the shallower window this can be achieved if the near offsets can be populated. The shallower windows are defined as corresponding to times smaller than the arrival times for the faster modes at the longest offsets. The size of the shallow window, over which the encoded simultaneous source approach is applicable, depends on the near-surface velocity and the maximum available offsets. Figure 1 is a schematic diagram showing the data window that can be used for simultaneous source inversion. The lines with smaller slopes define the bottom of the window for a faster shallow-velocity case (100). The lines with larger slopes define the bottom of the window for a slower shallow-velocity case (101). The vertical axis is time and the horizontal axis is source-receiver offset, with zero offset for the selected receiver being in the middle of the diagram, with positive offsets to the right and negative offsets to the left. The schematic shown in Fig. 1 suggests that for slow near-surface velocities the usable window (101) is longer compared to the case when the near-surface velocities are faster (100).

[0037] The negative offsets in the shot gathers may be filled using reciprocity (201). The missing near-offsets may be filled with estimated values, for example from neighboring traces (201). Filling the near-offsets in the shallow window will provide complete receiver coverage to use simultaneous source encoding. Another alternative is to simply eliminate all receivers that were a near offset for some shot. There are various prior approaches to fill in the near-offset data such as reconstruction methods typically used in surface-related multiple attenuation. This makes the shallow windowed shot gathers (202) conform to the fixed-receiver assumption preferable for simultaneous source encoding (203). Therefore, for the shallow window, one can generate simultaneous-source encoded data (204) and invert (205) for a subsurface model (206) that fits these data. Since the computation time is significantly less by several orders of magnitude, a more sophisticated simulation algorithm (e.g. elastic full wavefield inversion) can potentially be used if necessary. Following the simultaneous source inversion with the shallow windowed data, the next step is use the shallow model as a starting model (207) to invert the deeper window data (208) since the fixed receiver assumption breaks down for deeper windows.

[0038] The present inventive method includes two embodiments for inverting the deeper window data. A flowchart of these hybrid approaches is presented in Fig. 2. In the first approach shown in Fig. 2, traditional sequential-shot techniques are used to invert (209) the deeper window data (208). However, a sparser set of sources is sufficient to update the deeper part of the model (210) and may therefore be used to advantage. How many sources are required will depend on the spatial wavelength of the model and the maximum available offsets. From a physics point of view, the resolution of the subsurface degrades with depth. For example, in offset-limited data the contribution in the deeper window is dominantly from reflections rather than transmission. Thus, depending on the desired scale length of the model that needs to be updated, the choice of how many sequential sources are necessary can be problem-dependent. For example, if the goal is just to obtain a smooth velocity model that better explains the kinematics (event traveltimes), then the user of the invention can choose a sparse set of sources for inversion. If the goal is to obtain a high resolution target-oriented model update, then the user can include more sources above the target region and sparse sources everywhere else.

[0039] In the second approach (Approach II in Fig. 2), advantage is made of the fact that the shallow model (206) resulting from the shallow window inversion should be satisfactory for predicting long offset data (211) at times larger than the shallow window. Therefore, the shallow model obtained from simultaneous inversion is used to compute the data at the larger missing offsets(211). The advantage of this approach is that with the available far-offset traces obtained using forward modeling, the fixed receiver assumption can be applied. Thus with the available data along with the computed far offsets, the shot gathers can be encoded (212) for deeper window (213) inversion (214), as indicated in Fig. 2.

[0040] The present invention's hybrid approach is not only applicable to streamer data, but also to a variety of other acquisition geometries where the fixed-receiver assumption breaks down. For example, ocean bottom cable (OBC) acquisitions are typically patch-based (fixed receivers for a subset of shots), and they do not conform to the idealized situation where all receivers are fixed and recording for all shots. Similarly for land acquisitions due to logistical problems the fixed receiver geometry is difficult to achieve.

[0041] Many types of encoding functions c_g can be used in equation (6) including but not limited to:

- Linear, random, chirp and modified-chirp frequency-dependent phase encoding as presented in Romero et al. [13];
- The frequency independent phase encoding as presented in Jing et al. [14];
- Random time shift encoding;
- 5 • Frequency division multiplexing (FDMA), time division multiplexing (TDMA) and code division multiplexing (CDMA) used in telecommunications.

[0042] Some of these encoding techniques will work better than others depending upon the application, and some can be combined. In particular, good results have been obtained
 10 using frequency dependent random phase encoding and also by combining frequency independent encoding of nearby sources with frequency dependent random phase encoding for more widely separated sources. An indication of the relative merits of different encodings can be obtained by running test inversions with each set of encoding functions to determine which converges faster.

15 [0043] It should be noted that the *simultaneous encoded-source technique* can be used for many types of inversion cost function. In particular it could be used for cost functions based on other norms than L2 discussed above. It could also be used on more sophisticated cost functions than the one presented in Equation 2, including regularized cost functions. Finally, the simultaneous encoded-source method could be used with any type of global or local cost
 20 *function inversion method including Monte Carlo, simulated annealing, genetic algorithm, evolution algorithm, gradient line search, conjugate gradients and Newton's method.*

The present inventive method can also be used in conjunction with various types of generalized source techniques, such as those suggested by Berkhout [7]. In this case, rather than encoding different point source gather signatures, one would encode the signatures for
 25 different synthesized plane waves.

[0044] Some variations on the embodiment described above include:

- The c_g encoding functions can be changed for each iteration of the inversion. In at least some instances this leads to faster convergence of the inversion.
- In some cases (e.g., when the source sampling is denser than the receiver
 30 sampling) it may be advantageous to use reciprocity to treat the actual

receivers as computational sources, and encode the receivers instead of the sources.

- 5 • This invention is not limited to single-component point receivers. For example, the receivers could be receiver arrays or they could be multi-component receivers.
- 10 • The method may be improved by optimizing the encoding to yield the highest quality inversion. For example the encoding functions could be optimized to reduce the number of local minima in the cost function. The encoding functions could be optimized either by manual inspection of tests performed using different encoding functions or using an automated optimization procedure.
- 15 • Acquisition of simultaneous encoded-source data could result in significant geophysical data acquisition cost savings.
- For marine seismic data surveys, it would be very efficient to acquire encoded source data from multiple simultaneously operating marine vibrators that operate continuously while in motion.
- 20 • As indicated above, the encoding process in the present invention may be performed in the field acquisition of the data, for example where the pilot signals of multiple simultaneously operating vibrators are encoded with different encoding functions. In the attached claims, steps referring to encoding geophysical data, or to geophysical data from encoded sources, or to obtaining encoded gathers of geophysical data will be understood to include obtaining data already encoded in the field acquisition process, except where the context clearly indicates that encoding is occurring in a data processing
- 25 step.
- Other definitions for the cost function may be used, including the use of a different norm (e.g. L1 norm (absolute value) instead of L2 norm), and additional terms to regularize and stabilize the inversion (e.g. terms that would penalize models that aren't smooth or models that are not sparse).

Example

[0045] Figures 3-11 present a synthetic example of inverting constant-density acoustic seismic data using the hybrid approach of this invention when the fixed receiver assumption is violated. The results are compared with the simultaneous source inversion where the fixed receiver assumption is valid.

[0046] Figure 3 is the true velocity model, i.e. the velocity model that will be used to generate synthetic data. The model has 500 m of water depth and the reservoir is depth of 3 km.

[0047] Figure 4 shows an example of two far-offset limited representative shot gathers encoded using binary encoding described in PCT Patent Application Publication No. WO 2008/042081. The offset-limited nature of the shot gathers that is evident from the figure makes the fixed-receiver assumption invalid. However, it can also be seen that the fixed-receiver assumption *is* valid for the shallow time window from 0 to about 3 seconds. Figure 4 shows that the receivers for the left shot between $x = 800$ and $x = 2000$ do not have any contribution from the right shot, i.e. energy from the right shot does not reach those receivers before 3 seconds. Similarly the receivers for the right shot between $x = 2800$ and $x = 4200$ do not have any contribution from left shot in the shallow time window between 0 and 3 seconds. The maximum far-offset used in this example is 5 km. The data are generated using 80 m shot spacing and receiver separation is 10 m. This example uses near-offset data computed using the model. To generate the starting velocity model for inversion, the common depth point (CDP) gathers are generated with the 5-km offset-limited sequential shots, and Kirchhoff depth migration is used to flatten the common image gathers. The starting velocity model shown in Fig. 5 is obtained using Kirchhoff depth migration. Since the depth migration fits the kinematic part of the data, the starting model is a smooth model.

[0048] The starting model in Fig. 5 is used next to invert encoded data using simultaneous source inversion. Since full wavefield inversion is highly nonlinear problem, time-frequency windowing is typically necessary to make the problem well-posed for stable convergence to the desired solution.

[0049] Figure 6 shows the model obtained by inverting the low frequency data (with peak frequency of 7 Hz). Since the fixed receiver assumption is not violated up to 3 seconds of the data (shown in Figure 4), the recovered model achieved stable convergence with simultaneous source inversion. For the time windowed data up to 3 s, the maximum offset is

about 5 km. Thus the inversion recovers the shallower part of the model well compared to the deeper part. Figures 7 and 8 show the result of inverting the data with time window of 4 s and 5 s, respectively. It is clear from the results that the inverted model has artifacts since the encoded data in these time windows do not have the large offset data and as a result the incorrect gradient is computed for model updating. Because the fixed receiver assumption is violated in these time windows, the simultaneous source inversion produces artifacts in the model.

[0050] The model obtained from shallow window simultaneous source inversion (Fig. 5) is used as the starting model for the sequential source inversion for the deeper time window 4s. A sparse set of sources (20 sources out of 383 sources used for encoding the data) is used for the 4 s window inversion. The resulting model is shown in Fig. 9. Using the model from Fig. 9 as starting model, the 5 s windowed data are inverted. For the 5 s data inversion, only 10 sequential sources are used. The resulting model is shown in Fig. 10. For offset-limited data, the aperture becomes limited with depth, hence a sparse set of sources are sufficient to obtain a reasonable model. Using sparse sources provides a significant decrease in the computational effort.

[0051] Figure 11 shows the result of inverting the 5 s data window with full band data (peak frequency of 40Hz) with 10 sequential sources.

[0052] In the example presented above, the measured near-offset data were used. In practice, the marine streamer data typically have missing near-offsets. In the next example, offset data up to 200 m are eliminated. The goal here is to show the impact of the missing near offsets in the simultaneous inversion and its importance to be included in the encoded data.

[0053] Figure 12 shows a representative shot gather with missing offsets that are encoded for simultaneous inversion. Since there is missing near-offset information in the encoded data, the simultaneous inversion not only produces artifacts in the recovered model, but it is also difficult to fit the data. The recovered model at iteration 5 and 50 is shown in Figs. 14 and 15, respectively. The inverted model has significant artifacts and the solution has completely diverged for the low frequency data inversion. Figure 15 shows that due to missing near-offset information in the encoded data, there are large velocity artifacts in the water layer. To improve the solution, data interpolation or regularization can be used to fill in the near-offset traces. This is a common procedure for preparing data for multiple elimination

problems. Another factor that leads to the artifacts in the inversion is the missing large energy data due to direct arrivals. Typically the near-offset data with direct arrivals have significant energy, therefore from a data fitting point of view it has large contribution to the data misfit. The missing information in the measured encoded data exacerbates the problem and causes the inversion to diverge, resulting in significant artifacts. Recognizing this aspect of data fitting, the problem can be circumvented to an extent by masking or muting the gradient in the shallow part of the model that is commonly associated with water velocity. This is demonstrated in the next example.

[0054] Instead of interpolating the near-offsets traces that is a common procedure for multiple-elimination, the nearest available trace at 200 m is used and multiple copies are made to populate the missing near-offset traces, with this trace shown in Fig. 13. The data are then encoded using binary encoding for simultaneous inversion. During the simultaneous inversion of the low frequency encoded data for the shallow 3 s time windowed data, the gradient in the water layer is muted so that the error accumulated in the water layer does not propagate for the model updating in the deeper part and cause the inversion to diverge.

[0055] Figures 16 and 17 show the inversion result for iteration 5 and 200, respectively, using simultaneous source inversion of the low frequency data with gradient muting in the water layer. Figure 17 clearly shows that the inversion is stable and the recovered model is similar to the inversion in Figure 5. The results should further improve if the near-offsets are interpolated using data regularization methods such as radon interpolation. All of the examples presented demonstrate the approach "I" described in Figure 2.

[0056] The foregoing application is directed to particular embodiments of the present invention for the purpose of illustrating it. It will be apparent, however, to one skilled in the art, that many modifications and variations to the embodiments described herein are possible. All such modifications and variations are intended to be within the scope of the present invention, as defined in the appended claims. Persons skilled in the art will readily recognize that in preferred embodiments of the invention, at least some of the steps in the present inventive method are performed on a computer, i.e. the invention is computer implemented. In such cases, the resulting updated physical properties model may either be downloaded, displayed, or saved to computer storage.

References

1. Tarantola, A., "Inversion of seismic reflection data in the acoustic approximation," *Geophysics* **49**, 1259-1266 (1984).
2. Sirgue, L., and Pratt G. "Efficient waveform inversion and imaging: A strategy for
5 selecting temporal frequencies," *Geophysics* **69**, 231-248 (2004).
3. Fallat, M. R., Dosso, S. E., "Geoacoustic inversion via local, global, and hybrid algorithms," *Journal of the Acoustical Society of America* **105**, 3219-3230 (1999).
4. Hinkley, D. and Krebs, J., "Gradient computation for simultaneous source inversion," PCT Patent Application Publication No. WO 2009/117174.
- 10 5. Krebs, J. R., Anderson, J. A., Neelamani, R., Hinkley, D., Jing, C., Dickens, T., Krohn, C., Traynin, P., "Iterative inversion of data from simultaneous geophysical sources," PCT Patent Application Publication No. WO 2008/042081.
6. Van Manen, D. J., Robertsson, J.O.A., Curtis, A., "Making wave by time reversal," *SEG International Exposition and 75th Annual Meeting Expanded Abstracts*, 1763-1766
15 (2005).
7. Berkhout, A. J., "Areal shot record technology," *Journal of Seismic Exploration* **1**, 251-264 (1992).
8. Zhang, Y., Sun, J., Notfors, C., Gray, S. H., Cherris, L., Young, J., "Delayed-shot 3D depth migration," *Geophysics* **70**, E21-E28 (2005).
- 20 9. Van Riel, P., and Hendrik, W. J. D., "Method of estimating elastic and compositional parameters from seismic and echo-acoustic data," U.S. Patent No. 6,876,928 (2005).
10. Mora, P., "Nonlinear two-dimensional elastic inversion of multi-offset seismic data," *Geophysics* **52**, 1211-1228 (1987).
11. Ober, C. C., Romero, L. A., Ghiglia, D. C., "Method of Migrating Seismic Records,"
25 U.S. Patent No. 6,021,094 (2000).
12. Ikelle, L. T., "Multi-shooting approach to seismic modeling and acquisition," U.S. Patent No. 6,327,537 (2001).
13. Romero, L. A., Ghiglia, D. C., Ober, C. C., Morton, S. A., "Phase encoding of shot records in prestack migration," *Geophysics* **65**, 426-436 (2000).

14. Jing X., Finn, C. J., Dickens, T. A., Willen, D. E., "Encoding multiple shot gathers in prestack migration," *SEG International Exposition and 70th Annual Meeting Expanded Abstracts*, 786-789 (2000).
15. Haber, E., Chung M. and Herrmann, "An effective method for parameter estimation
5 with PDE constraints with multiple right hand sides," Preprint - UBC
<http://www.math.ubc.ca/~haber/pubs/PdeOptStochV5.pdf> (2010).
16. Jerome R. Krebs, John E. Anderson, David Hinkley, Ramesh Neelamani, Sunwoong Lee, Anatoly Baumstein, and Martin-Daniel Lacasse, "Full-wavefield seismic inversion using encoded sources," *Geophysics* **74**-6, WCC177-WCC188 (2009).

CLAIMS

1. A computer-implemented method for full-wavefield inversion of measured geophysical data to determine a physical properties model for a subsurface region, comprising:

using a computer to invert a selected shallow time window of arrivals from the measured geophysical data by simultaneous encoded sources and/or receivers inversion to obtain a first physical properties model for the subsurface region;

using a computer to invert the measured geophysical data, or a selected deep time window of arrivals from the measured geophysical data, by iterative sequential source inversion to obtain a second physical properties model for the subsurface region, wherein the first physical properties model is used as a starting model and a set of source locations is used to update the second physical properties model in the iterative sequential source inversion; and

outputting, displaying, or saving to data storage the second physical properties model of the subsurface region.

2. The method of claim 1, wherein the shallow time window consists of arrival times smaller than arrival times for fastest propagation modes at longest offsets, wherein offset is source-receiver spacing.

3. The method of claim 1, wherein how many source locations are used to update the second physical properties model in the iterative sequential source inversion is determined based at least in part on spatial wavelength of the second physical properties model and maximum available offsets in the measured geophysical data.

4. The method of claim 1, wherein the set of source locations used to update the second physical properties model in the iterative sequential source inversion consists of fewer source locations than are represented in the measured geophysical data or are used in the simultaneous encoded sources and/or receivers inversion.

5. The method of claim 1, wherein:

missing data for negative offsets in the selected shallow time window of arrivals are filled from positive offset data using source-receiver reciprocity; and

missing near-offset data in the selected shallow time window of arrivals are provided with estimated values, or, alternatively, receivers that were a near offset for some source shot are not considered in the method.

6. The method of claim 1, wherein the measured geophysical data were acquired in a survey using non-stationary receivers.

7. The method of claim 1, further comprising using iterative inversion in the simultaneous encoded sources and/or receivers inversion, wherein the sources and or receivers are encoded using a selected set of encoding functions, and wherein a different set of encoding functions is selected for at least one of the iterations.

8. The method of claim 1, wherein the measured geophysical data are seismic data.

9. The method of claim 1, wherein the simultaneous encoded sources and/or receivers inversion comprises:

(a) obtaining a group of two or more encoded gathers of the measured geophysical data for the shallow time window, wherein each gather is associated with a single generalized source or with a single receiver, and wherein each gather is encoded with a different encoding signature selected from a set of non-equivalent encoding signatures;

(b) summing the encoded gathers in the group by summing all data records in each gather that correspond to a single receiver, or to a single source if receiver gathers are used, and repeating for each different receiver or source, resulting in a simultaneous encoded gather;

(c) assuming a physical properties model of the subsurface region, said assumed model providing values of at least one physical property at locations in the subsurface region;

(d) calculating an update to the assumed physical properties model that is more consistent with the simultaneous encoded gather from step (b), said calculation involving

one or more encoded simultaneous source forward or reverse simulation operations that use the assumed physical properties model and encoded source signatures using the same encoding functions used to encode corresponding gathers of measured data, wherein an entire simultaneous encoded gather is simulated in a single simulation operation;

(e) repeating step (d) at least one more iteration, using the updated physical properties model from the previous iteration of step (d) as the assumed model to produce a further updated physical properties model of the subsurface region that is more consistent with a corresponding simultaneous encoded gather of measured data, using the same encoding signatures for source signatures in the simulation as were used in forming the corresponding simultaneous encoded gather of measured data; and

(f) taking the further updated physical properties model for the first physical properties model.

10. The method of claim 9, further comprising obtaining at least one additional group of two or more encoded gathers of the measured geophysical data as in step (a), and performing step (b) for each additional group, then accumulating the corresponding updates to the physical properties model from step (d), wherein the updated physical properties model to be used in step (e) is based on the accumulated updates.

11. The method of claim 9, wherein said encoded gathers of measured data are encoded by temporally convolving all traces from a gather with the encoding function selected for the gather.

12. The method of claim 9, wherein the two or more encoded gathers of measured data are obtained by obtaining gathers of data from a geophysical survey in which data are acquired from a plurality of simultaneously operating, uniquely encoded source devices.

13. The method of claim 1, wherein the simultaneous encoded sources and/or receivers inversion comprises:

(a) obtaining a group of two or more encoded gathers of the measured geophysical data for the shallow time window, wherein each gather is associated with a single generalized source or with a single receiver, and wherein each gather is encoded with a different encoding function selected from a set of non-equivalent encoding functions;

(b) summing the encoded gathers in the group by summing all data records in each gather that correspond to a single receiver, or to a single source if receiver gathers are used, and repeating for each different receiver or source, resulting in a simultaneous encoded gather;

(c) assuming a physical properties model of the subsurface region, said assumed model providing values of at least one physical property at locations in the subsurface region;

(d) inverting the group of two or more encoded gathers, one encoded gather at a time, all data traces in each encoded gathered being inverted simultaneously, using the assumed physical properties model as an initial model and iteratively updating said model to minimize a cost function measuring degree of misfit between model-simulated data and the measured geophysical data in the encoded gathers to generate an updated physical properties model, wherein model adjustments are made using a gradient of the cost function with respect to at least one model parameter, which gradient is computed from a time integration of a product of encoded simultaneous-source data simulated forward in time and encoded simultaneous-source data simulated backward in time; and

(e) taking the updated physical properties model as the first physical properties model.

14. The method of claim 13, wherein said encoded gathers of measured data are encoded by temporally convolving all traces from a gather with the encoding signature selected for the gather.

15. The method of claim 13, wherein the two or more encoded gathers of measured data are obtained by obtaining gathers of data from a geophysical survey in which data are acquired from a plurality of simultaneously operating, uniquely encoded source devices.

16. A method for producing hydrocarbons from a subsurface region, comprising:

performing a geophysical survey of the subsurface region resulting in measured geophysical data;

processing the measured geophysical data on a computer by a method of claim 1 to generate a physical properties model of the subsurface region;

assessing hydrocarbon potential of the subsurface region using the physical properties model; and

drilling a well into the subsurface region based at least in part on the assessment of hydrocarbon potential, and producing hydrocarbons from the well.

17. A computer-implemented method for full-wavefield inversion of measured geophysical data to determine a physical properties model for a subsurface region, comprising:

using a computer to invert a selected shallow time window of arrivals from the measured geophysical data by simultaneous encoded sources and/or receivers inversion to obtain a first physical properties model for the subsurface region;

using the first physical properties model to simulate, using a computer, synthetic data for longer offsets corresponding to arrivals from deeper than said shallow time window;

using a computer to invert the measured geophysical data, wherein the data with longer offsets are augmented, said inversion being simultaneous encoded-sources and/or encoded-receivers inversion, to obtain a second physical properties model of the subsurface region, wherein said augmented data with longer offsets are the sum of the synthetic data for longer offsets and the measured data at the longer offsets; and

outputting, displaying, or saving to data storage the second physical properties model of the subsurface region.

18. The method of claim 17k, wherein the measured geophysical data were acquired in a survey using non-stationary receivers.

19. The method of claim 17, wherein:

all simultaneous encoded source inversions are iterative, the sources are encoded using a selected set of encoding functions, and

a different set of encoding functions is selected for at least one of the iterations.

20. A method for producing hydrocarbons from a subsurface region, comprising:
- performing a geophysical survey of the subsurface region resulting in measured geophysical data;
 - processing the measured geophysical data on a computer by a method of claim 1 to generate a physical properties model of the subsurface region;
 - assessing hydrocarbon potential of the subsurface region using the physical properties model; and
 - drilling a well into the subsurface region based at least in part on the assessment of hydrocarbon potential, and producing hydrocarbons from the well.
21. A computer program product, comprising a non-transitory computer usable medium having a computer readable program code embodied therein, said computer readable program code adapted to be executed to implement a method for full-wavefield inversion of measured geophysical data to determine a physical properties model for a subsurface region, said method comprising:
- inverting a selected shallow time window of arrivals from the measured geophysical data by simultaneous encoded sources and/or receivers inversion to obtain a first physical properties model for the subsurface region;
 - inverting the measured geophysical data, or a selected deep time window of arrivals from the measured geophysical data, by iterative sequential source inversion to obtain a second physical properties model for the subsurface region, wherein the first physical properties model is used as a starting model and a set of source locations is used to update the second physical properties model in the iterative sequential source inversion; and
 - outputting, displaying, or saving to data storage the second physical properties model of the subsurface region.
22. A computer program product, comprising a non-transitory computer usable medium having a computer readable program code embodied therein, said computer readable program code adapted to be executed to implement a method for full-wavefield inversion

of measured geophysical data to determine a physical properties model for a subsurface region, said method comprising:

inverting a selected shallow time window of arrivals from the measured geophysical data by simultaneous encoded sources and/or receivers inversion to obtain a first physical properties model for the subsurface region;

using the first physical properties model to simulate synthetic data for longer offsets corresponding to arrivals from deeper than said shallow time window;

inverting the measured geophysical data, wherein the data with longer offsets are augmented, said inversion being simultaneous encoded-sources and/or encoded-receivers inversion, to obtain a second physical properties model of the subsurface region, wherein said augmented data with longer offsets are the sum of the synthetic data for longer offsets and the measured data at the longer offsets; and

outputting, displaying, or saving to data storage the second physical properties model of the subsurface region.