An event is a tuple $e = \{d, h, t, c, s\}$ where:

$d = data(e)$ = the "payload" data, which may include transactions.
$h = hashes(e)$ = a list of hashes of the event's parents, self-parent first.
$t = time(e)$ = creator's claimed date and time of the event's creation.
$c = creator(e)$ = creator's ID number.
$s = sig(e)$ = creator's digital signature of {d,h,t,c}.

n = the number of members in the population
$m = 1 + \lfloor 2n/3 \rfloor$
$first$ = the unique event that has no parents
E = the set of all events
$\mathbb{T}$ = set of all possible $(time, date)$ pairs
$\mathbb{B} = \{true, false\}$
$\mathbb{N} = \{0, 1, 2, ...\}$

$$
\begin{aligned}
\text{ancestor} &: E \times E \to B \\
\text{selfAncestor} &: E \times E \to B \\
\text{see} &: E \times E \to B \\
\text{stronglySee} &: E \times E \to B \\
\text{parentRound} &: E \to \mathbb{N} \\
\text{witness} &: E \to \mathbb{B} \\
\text{round} &: E \to \mathbb{N} \\
\text{roundDiff} &: E \times E \to \mathbb{I} \\
\text{votes} &: E \times E \times \mathbb{B} \to \mathbb{N} \\
\text{voteFraction} &: E \times E \to \mathbb{R} \\
\text{vote} &: E \times E \to \mathbb{B} \\
\text{decide} &: E \times E \to \mathbb{B} \\
\text{allFamous} &: \mathbb{I} \to 2^E \\
\text{famous} &: E \to \mathbb{B} \\
\text{roundReceived} &: E \to \mathbb{N} \\
\text{timeReceived} &: E \to \mathbb{T}
\end{aligned}
$$

FIG. 16a

$$
\begin{array}{lcl}
\text{ancestor}(x,y) & = & (x=y) \vee (\exists z \in \text{parents}(x) : \text{ancestor}(z,y)) \\
\text{selfAncestor}(x,y) & = & \text{ancestor}(x,y) \wedge ((\text{selfParent}(x)=y) \vee \text{selfAncestor}(\text{selfParent}(x),y)) \\
\text{see}(x,y) & = & \text{ancestor}(x,y) \wedge \neg(\exists a,b,c \in E : \\
 & & (\text{ancestor}(y,a) \wedge \text{ancestor}(y,b) \wedge c \in \text{parents}(x) \wedge c \in \text{parents}(b))) \wedge \\
 & & \text{creator}(a)=\text{creator}(b)=\text{creator}(c) \\
\text{stronglySee}(x,y) & = & \text{see}(x,y) \wedge (\exists S \in 2^E : (|S| \geq m) \wedge (z \in S \iff (\text{see}(x,z) \wedge \text{see}(z,y)))) \\
\text{parentRound}(x) & = & \begin{cases} 0 & \text{if } x = first \\ \max_{y \in parents(x)} round(y) & \text{otherwise} \end{cases} \\
\text{witness}(x) & = & \exists S \in 2^E : (|S| \geq m \wedge \\
 & & (\forall y \in S : (\text{round}(y) = \text{parentRound}(x) \wedge \text{stronglySee}(x,y)))) \\
\text{round}(x) & = & \begin{cases} 1+\text{parentRound}(x) & \text{if witness}(x) \\ \text{parentRound}(x) & \text{otherwise} \end{cases} \\
\text{roundDiff}(x,y) & = & \text{round}(x) - \text{round}(y) \\
\text{votes}(x,y,v) & = & |\{z \in E \mid \text{see}(x,y) \wedge \text{roundDiff}(x,z)=1 \wedge \\
 & & \text{stronglySee}(x,z) \wedge \text{vote}(z,y)=v\}| \\
\text{voteFraction}(x,y) & = & \text{votes}(x,true)/(\text{votes}(x,true)+\text{votes}(x,false)) \\
\text{vote}(x,y) & = & \begin{cases} \text{see}(x,y) & \text{if roundDiff}(x,y)=1 \\ (\text{voteFraction}(x,y) >= 1/2) & \text{if } (\text{roundDiff}(x,y) \bmod 5 \neq 1) \vee \\ & |\text{voteFraction}(x,y) - 1/2| > 1/6 \\ (1 = \text{LSB}(\text{signature}(x))) & \text{otherwise} \end{cases} \\
\text{decide}(x,y) & = & \text{vote}(x,y) \wedge (\text{roundDiff}(x,y) \bmod 5 \neq 1) \wedge (\text{voteFraction}(x,y) > 2/3) \\
\text{allFamous}(r) & = & \{x \in E \mid \text{famous}(x) \wedge \text{round}(x)=4\} \\
\text{famous}(x) & = & \text{witness}(x) \wedge \exists y \in E : \text{decide}(y,x) \\
\text{roundReceived}(x) & = & \min_{r \in \mathbb{N}}(|\{y \in E| \text{ round}(y)=r \wedge \text{famous}(y) \wedge \text{see}(y,x)\}| \ / \\
 & & |\{y \in E| \text{ round}(y)=r \wedge \text{famous}(y)\}| >= 1/2) \\
\text{timeReceived}(x) & = & \text{median}(\{\text{time}(y) | y \in E \wedge \text{see}(y,x) \wedge \\
 & & (\exists z \in E : \text{round}(z)=\text{roundReceived}(x) \wedge \text{selfAncestor}(z,y)) \wedge \\
 & & \neg(\exists w \in E : \text{selfAncestor}(y,w) \wedge \text{see}(w,x))))
\end{array}
$$

FIG. 16b

An event is a tuple $e = \{d, h, t, c, s\}$ where:

$d = data(e)$ = the "payload" data, which may include transactions.
$h = hashes(e)$ = a list of hashes of the event's parents, self-parent first.
$t = time(e)$ = creator's claimed date and time of the event's creation.
$i = creator(e)$ = creator's ID number.
$s = sig(e)$ = creator's digital signature of {d,h,t,c}.
n = the number of members in the population
c = frequency of coin rounds (e.g., $c = 6$)
E = (the set of all events) $\cup$ $\{\varnothing\}$
$\mathbb{T}$ = set of all possible $(time, date)$ pairs
$\mathbb{B}$ = $\{true, false\}$
$\mathbb{N}$ = $\{0, 1, 2, ...\}$

$$
\begin{aligned}
\text{parents} &: E \to 2^E \\
\text{selfParent} &: E \to E \\
\text{ancestor} &: E \times E \to \mathbb{B} \\
\text{selfAncestor} &: E \times E \to \mathbb{B} \\
\text{see} &: E \times E \to \mathbb{B} \\
\text{stronglySee} &: E \times E \to \mathbb{B} \\
\text{parentRound} &: E \to \mathbb{N} \\
\text{roundInc} &: E \to \mathbb{B} \\
\text{round} &: E \to \mathbb{N} \\
\text{witness} &: E \to \mathbb{B} \\
\text{roundDiff} &: E \times E \to \mathbb{I} \\
\text{votes} &: E \times E \times \mathbb{B} \to \mathbb{N} \\
\text{fractTrue} &: E \times E \to \mathbb{R} \\
\text{decide} &: E \times E \to \mathbb{B} \\
\text{vote} &: E \times E \to \mathbb{B} \\
\text{famous} &: E \to \mathbb{B} \\
\text{roundReceived} &: E \to \mathbb{N} \\
\text{timeReceived} &: E \to \mathbb{T}
\end{aligned}
$$

FIG. 17a

$$
\begin{aligned}
\text{parents}(x) &= \text{set of parents of event x} \\
\text{selfParent}(x) &= \text{the self-parent of event x, or } \varnothing \text{ if none} \\
\text{ancestor}(x,y) &= (x \neq \varnothing) \wedge ((x = y) \vee (\exists z \in \text{parents}(x) : \text{ancestor}(z,y))) \\
\text{selfAncestor}(x,y) &= (x \neq \varnothing) \wedge ((x = y) \vee \text{selfAncestor}(\text{selfParent}(x), y)) \\
\text{see}(x,y) &= \text{ancestor}(x,y) \wedge \neg(\exists a,b \in E : \text{creator}(y) = \text{creator}(a) = \text{creator}(b) \wedge \\
&\quad \text{ancestor}(x,a) \wedge \text{ancestor}(x,b) \wedge \neg \text{selfAncestor}(a,b) \wedge \neg \text{selfAncestor}(b,a)) \\
\text{stronglySee}(x,y) &= \text{see}(x,y) \wedge (\exists S \in 2^E : (|S| > 2n/3) \wedge (z \in S \iff (\text{see}(x,z) \wedge \text{see}(z,y)))) \\
\text{parentRound}(x) &= \max(\{0\} \cup \{\text{round}(y) | y \in \text{parents}(x)\}) \\
\text{roundInc}(x) &= \exists S \in 2^E : (|S| > 2n/3 \wedge \\
&\quad (\forall y \in S : (\text{round}(y) = \text{parentRound}(x) \wedge \text{stronglySee}(x,y)))) \\
\text{round}(x) &= \text{parentRound}(x) + \begin{cases} 1 & \text{if roundInc}(x) \\ 0 & \text{otherwise} \end{cases} \\
\text{witness}(x) &= (\text{selfParent}(x) = \varnothing) \vee (\text{round}(x) > \text{round}(\text{selfParent}(x))) \\
\text{roundDiff}(x,y) &= \text{round}(x) - \text{round}(y) \\
\text{votes}(x,y,v) &= |\{z \in \text{E} \mid \text{roundDiff}(x,z) = 1 \wedge \text{stronglySee}(x,z) \wedge \text{vote}(z,y) = v\}| \\
\text{fractTrue}(x,y) &= \frac{\text{votes}(x,y,true)}{(\text{votes}(x,y,true) + \text{votes}(x,y,false))} \\
\text{decide}(x,y) &= (x \neq \varnothing) \wedge (\text{roundDiff}(x,y) > 1) \wedge (\text{decide}(\text{selfParent}(x),y) \vee \\
&\quad (\text{witness}(x) \wedge (\text{roundDiff}(x,y) \mod c \neq 1) \wedge \neg(\tfrac{1}{3} \leq \text{fractTrue}(x,y) \leq \tfrac{2n}{3}))) \\
\text{vote}(x,y) &= \begin{cases} \text{vote}(\text{selfParent}(x),y) & \text{if } (\neg \text{witness}(x)) \vee \text{decide}(\text{selfParent}(x),y) \\ 1 = \text{middleBit}(\text{signature}(x)) & \text{if witness}(x) \\ & \quad \wedge \neg \text{decide}(\text{selfParent}(x),y) \\ & \quad \wedge (\text{roundDiff}(x,y) \neq 1) \\ & \quad \wedge (\text{roundDiff}(x,y) \mod c = 1) \\ \text{fractTrue}(x,y) \geq \frac{1}{2} & \text{otherwise} \end{cases} \\
\text{famous}(x) &= \text{witness}(x) \wedge \exists y \in E : \text{decide}(y,x) \wedge \text{vote}(y,x) \\
\text{roundReceived}(x) &= \min_{r \in \mathbb{N}} \frac{|\{y \in E | (\text{round}(y) = r) \wedge \text{famous}(y) \wedge \text{see}(y,x)\}|}{|\{y \in E | (\text{round}(y) = r) \wedge \text{famous}(y)\}|} \geq 1/2) \\
\text{timeReceived}(x) &= \text{median}(\{\text{time}(y) | y \in E \wedge \text{see}(y,x) \wedge \\
&\quad (\exists z \in E : \text{round}(z) = \text{roundReceived}(x) \wedge \text{selfAncestor}(z,y)\}) \wedge \\
&\quad \neg(\exists w \in E : \text{selfAncestor}(y,w) \wedge \text{see}(w,x))))
\end{aligned}
$$

FIG. 17b