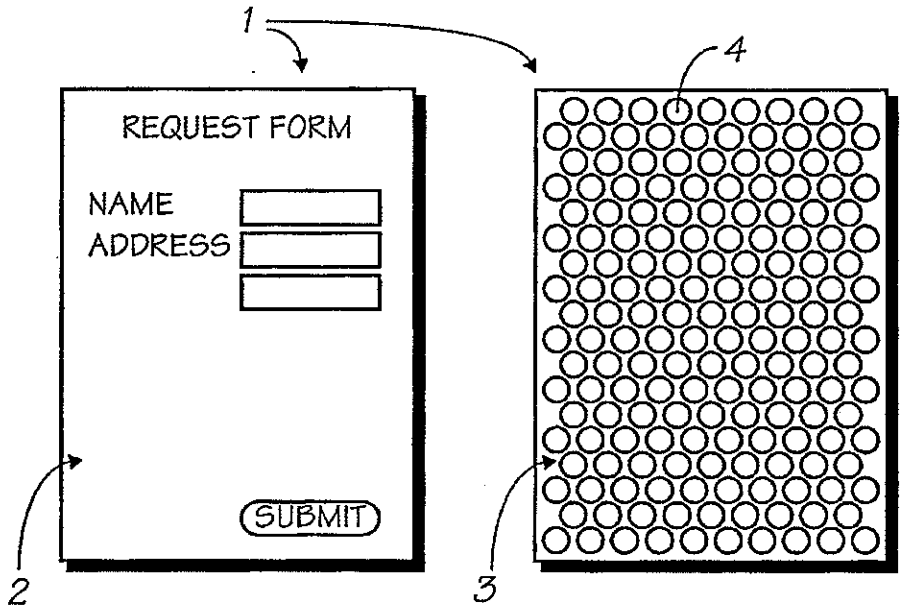

1
REQUEST FORM
NAME
ADDRESS
SUBMIT
2
3
4

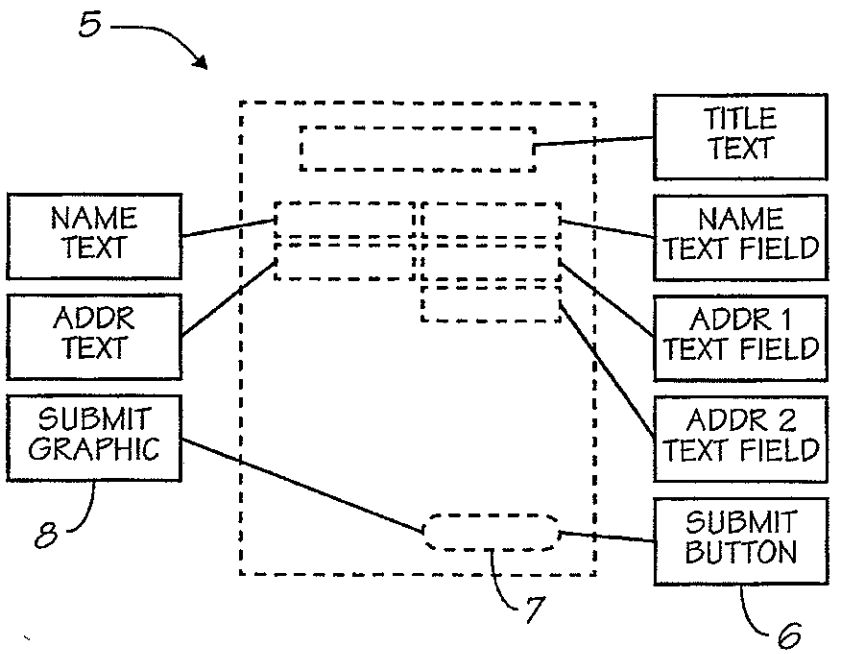

5
TITLE TEXT
NAME TEXT
NAME TEXT FIELD
ADDR TEXT
ADDR 1 TEXT FIELD
SUBMIT GRAPHIC
ADDR 2 TEXT FIELD
SUBMIT BUTTON
8
7
6

FIG. 1

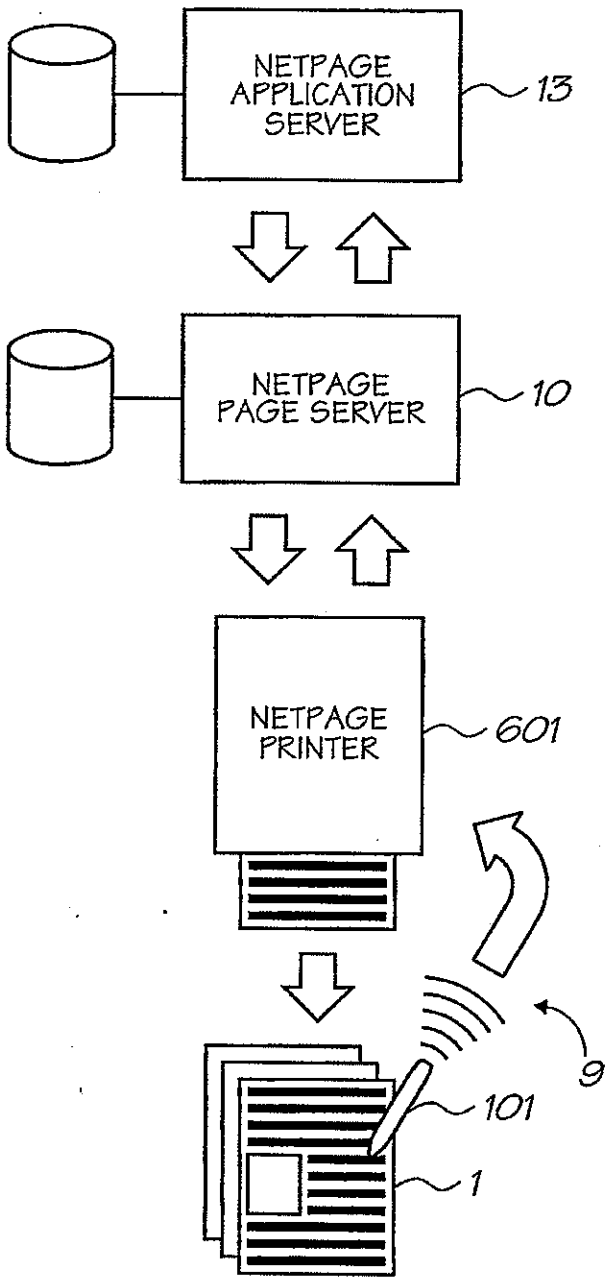

NETPAGE APPLICATION SERVER
13
NETPAGE PAGE SERVER
10
NETPAGE PRINTER
601
9
101
1

FIG. 2

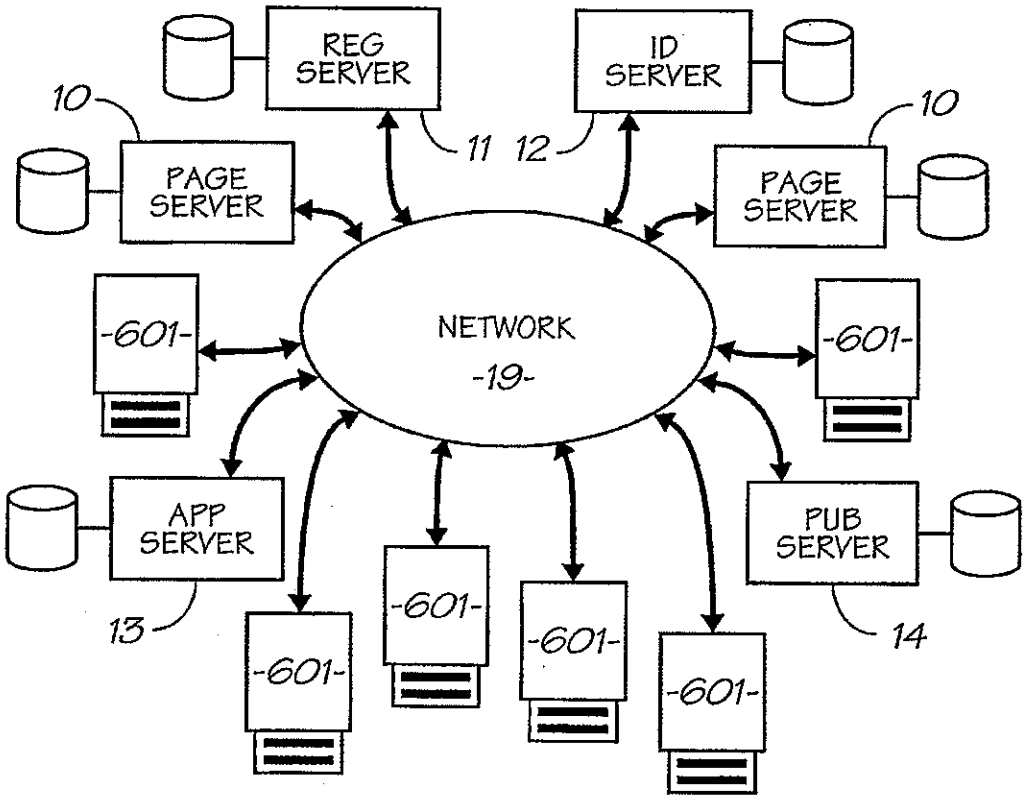

REG SERVER
ID SERVER
10
PAGE SERVER
11
12
10
PAGE SERVER
-601-
NETWORK
-19-
-601-
APP SERVER
PUB SERVER
13
14
-601-
-601-
-601-
-601-

FIG. 3

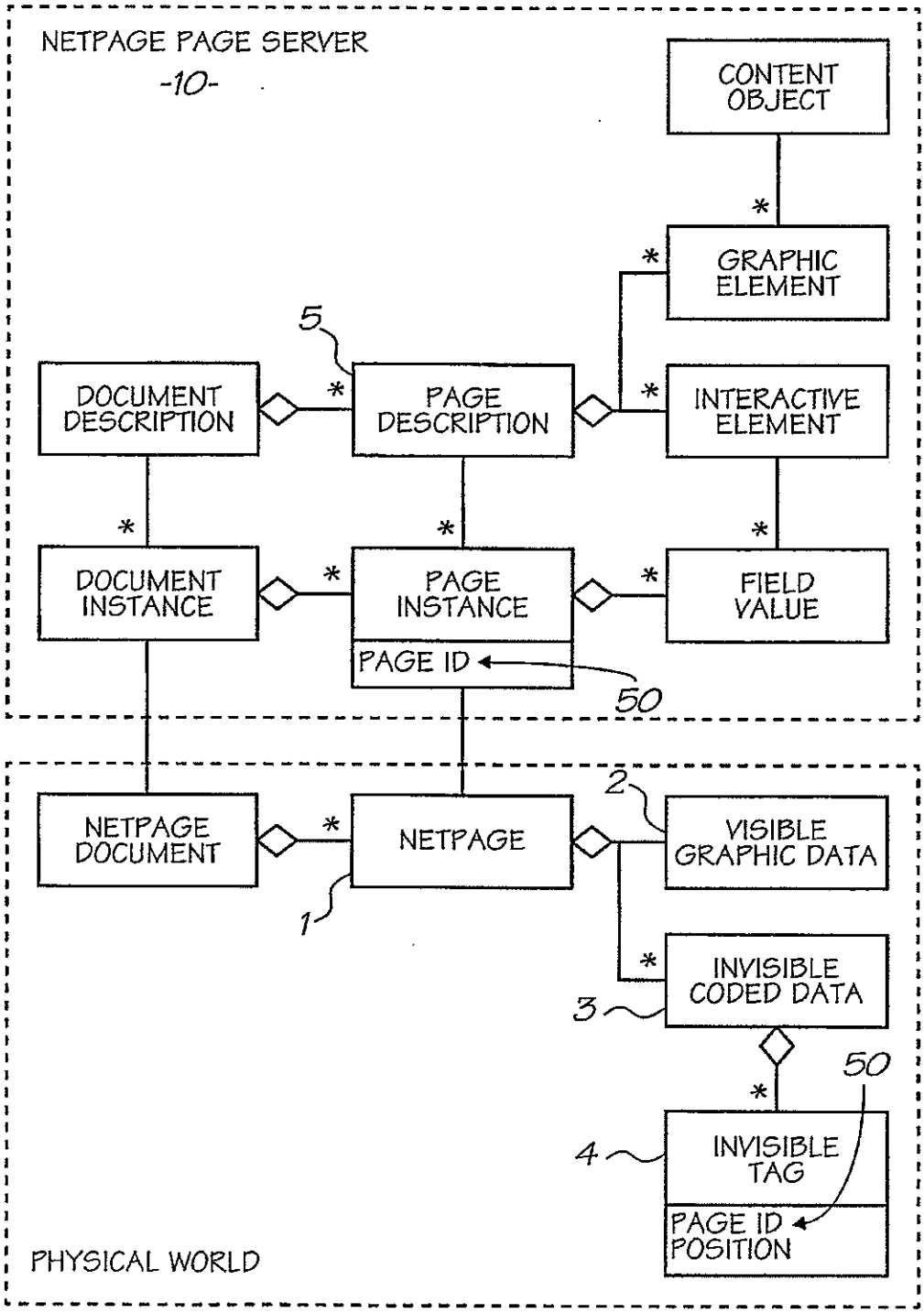

NETPAGE PAGE SERVER
-10-
CONTENT OBJECT
GRAPHIC ELEMENT
INTERACTIVE ELEMENT
FIELD VALUE
5
DOCUMENT DESCRIPTION
PAGE DESCRIPTION
DOCUMENT INSTANCE
PAGE INSTANCE
PAGE ID
50
NETPAGE DOCUMENT
NETPAGE
1
2
VISIBLE GRAPHIC DATA
INVISIBLE CODED DATA
3
50
4
INVISIBLE TAG
PAGE ID POSITION
PHYSICAL WORLD

FIG. 4

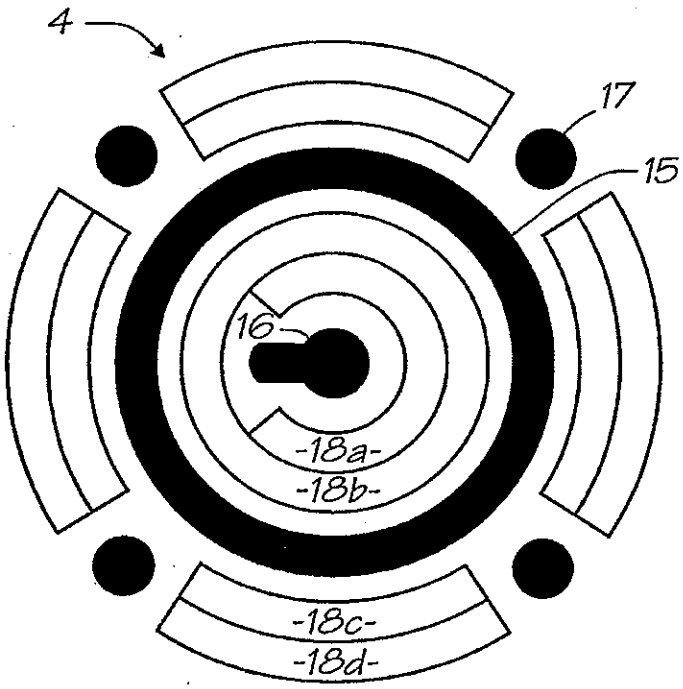


FIG. 5

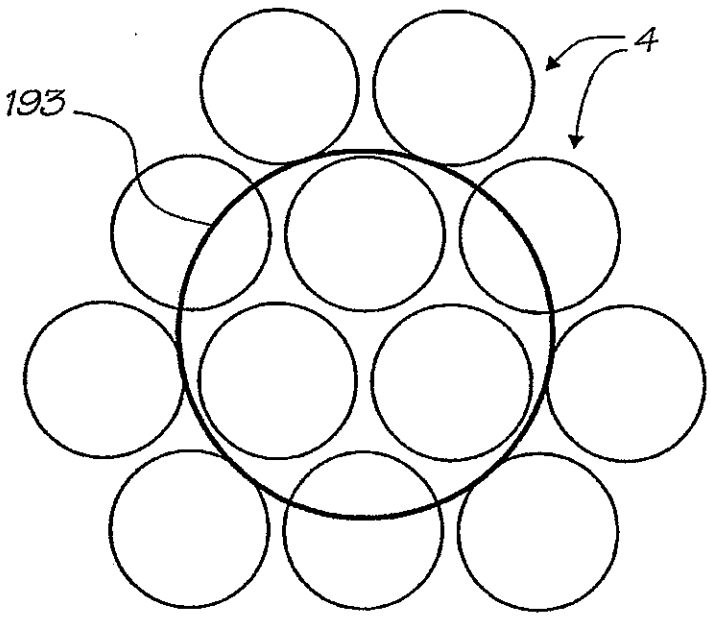


FIG. 6

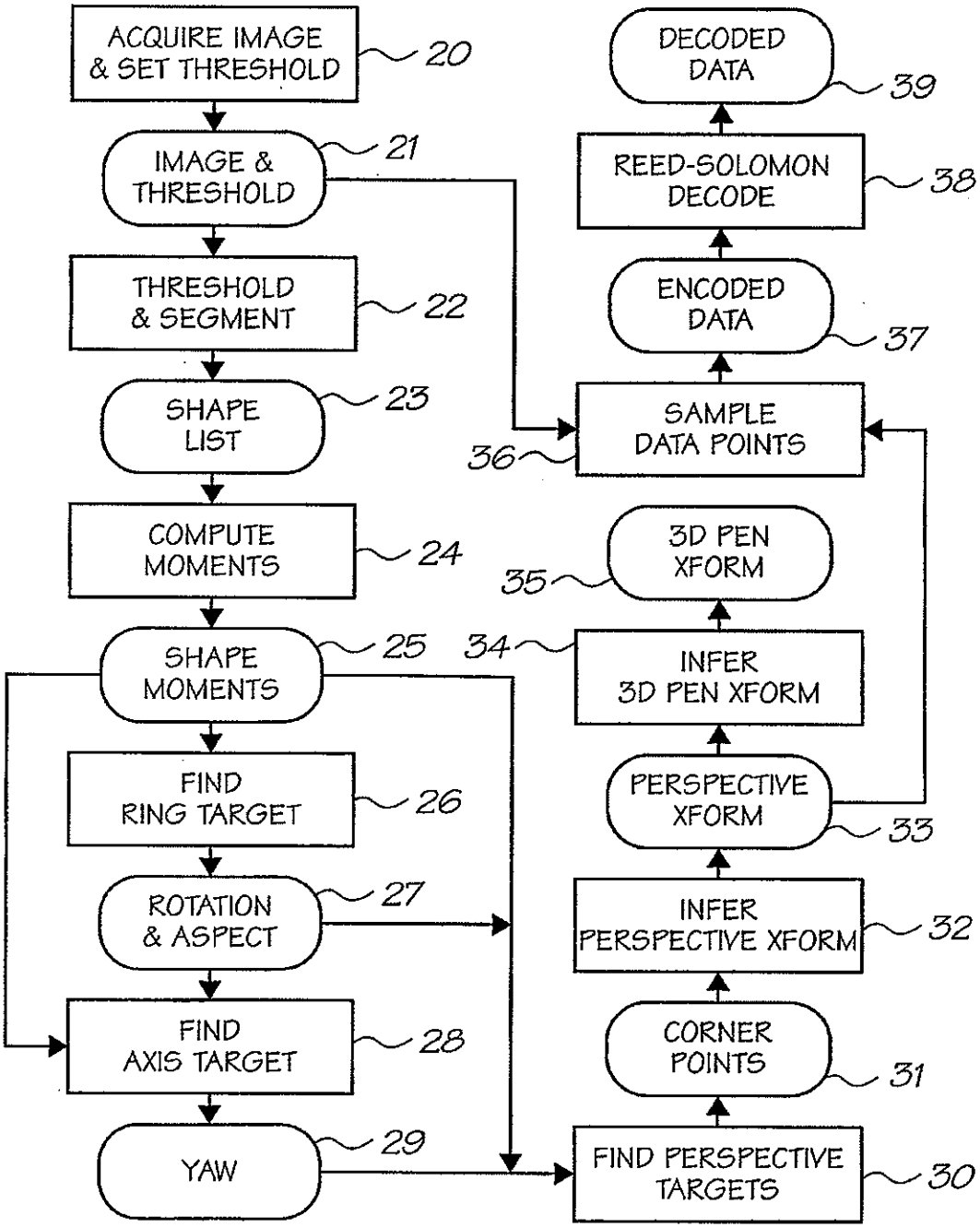

ACQUIRE IMAGE & SET THRESHOLD
20
IMAGE & THRESHOLD
21
THRESHOLD & SEGMENT
22
SHAPE LIST
23
COMPUTE MOMENTS
24
SHAPE MOMENTS
25
FIND RING TARGET
26
ROTATION & ASPECT
27
FIND AXIS TARGET
28
YAW
29
FIND PERSPECTIVE TARGETS
30
CORNER POINTS
31
INFER PERSPECTIVE XFORM
32
PERSPECTIVE XFORM
33
INFER 3D PEN XFORM
34
3D PEN XFORM
35
SAMPLE DATA POINTS
36
ENCODED DATA
37
REED-SOLOMON DECODE
38
DECODED DATA
39

FIG. 7

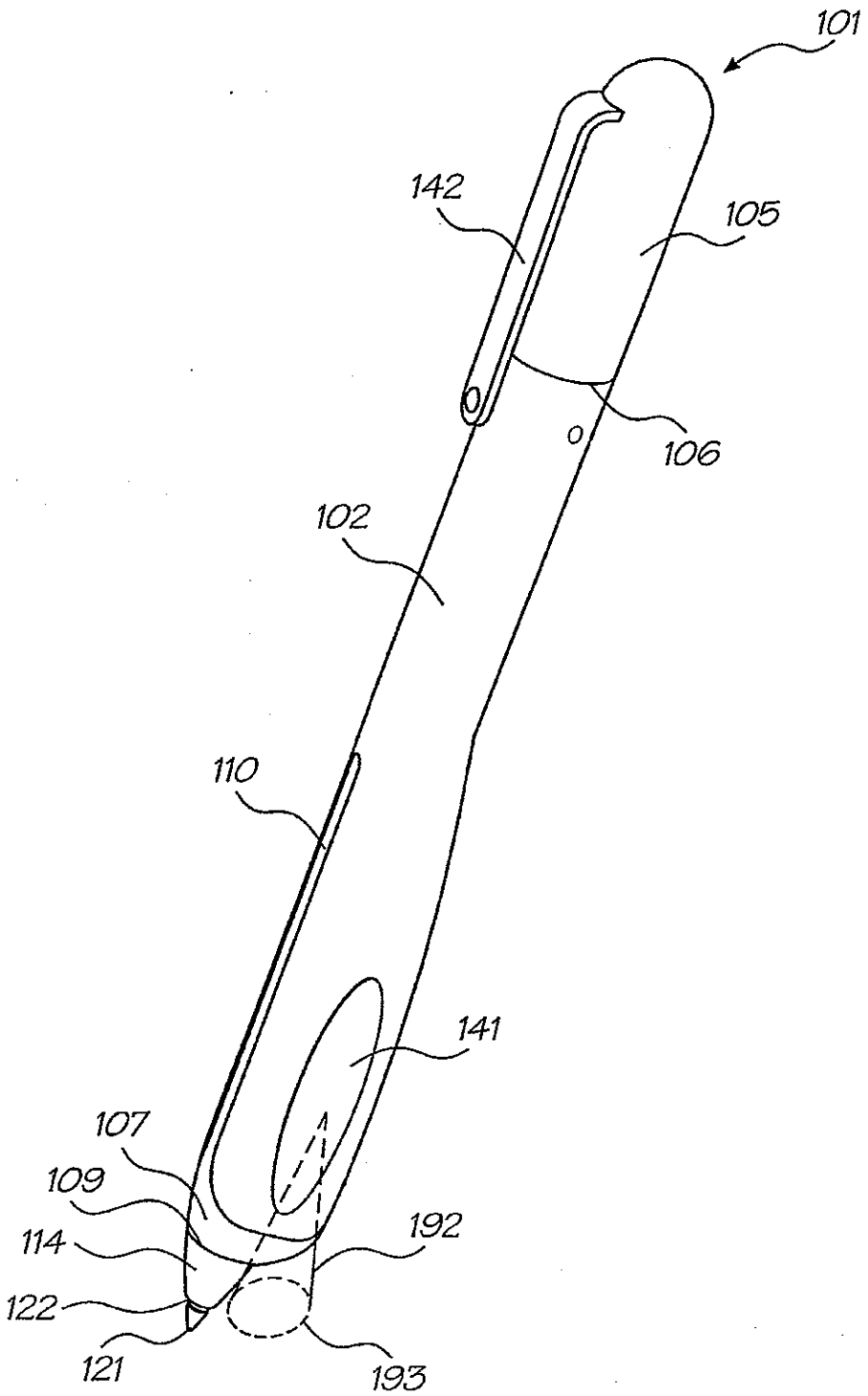


FIG. 8

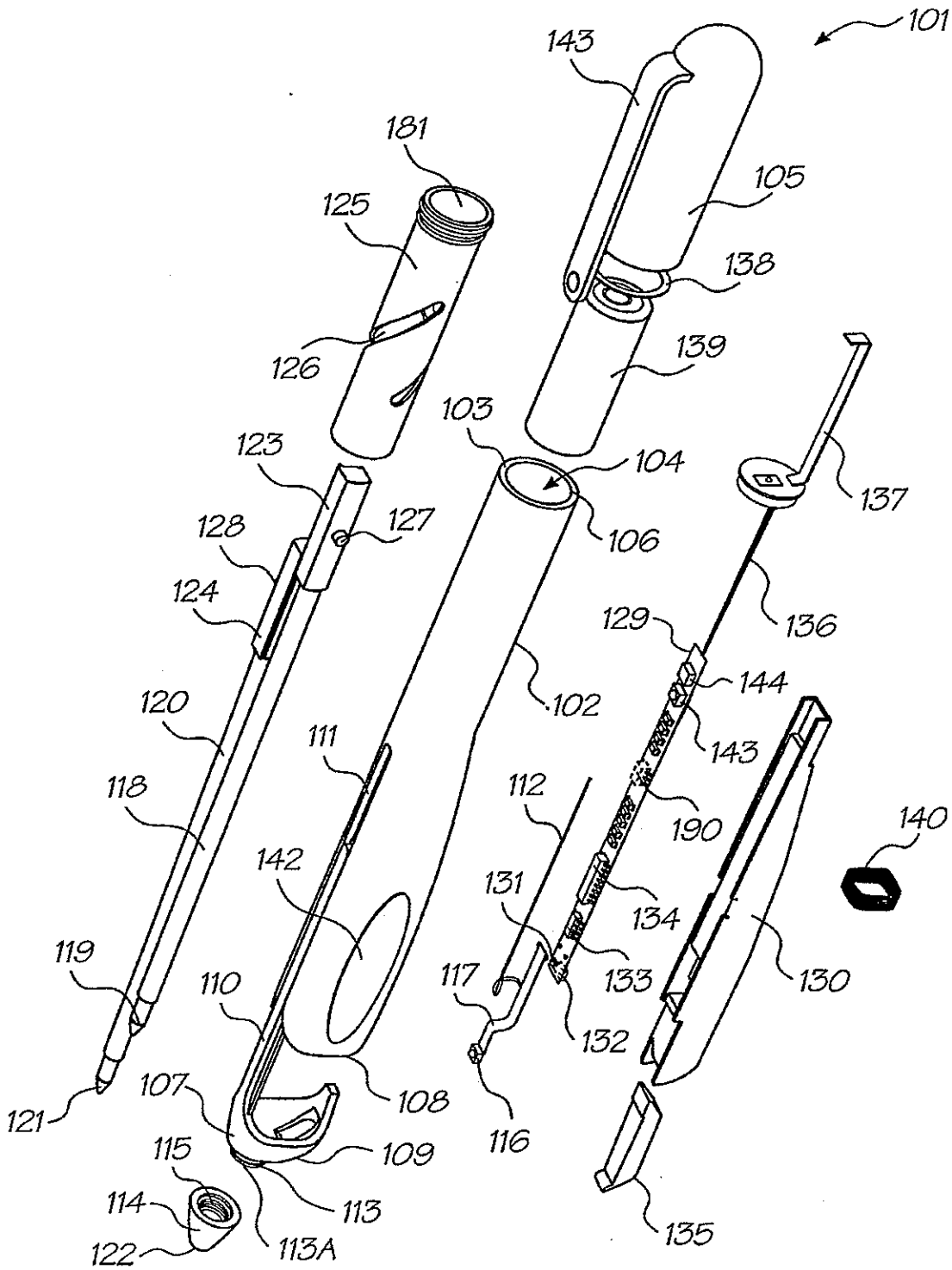

101
143
181
105
125
138
126
139
103
104
123
137
128
127
106
124
129
136
102
120
144
111
143
118
112
190
140
142
131
134
119
133
130
110
117
132
107
108
116
121
109
115
113
114
113A
122
135

FIG. 9

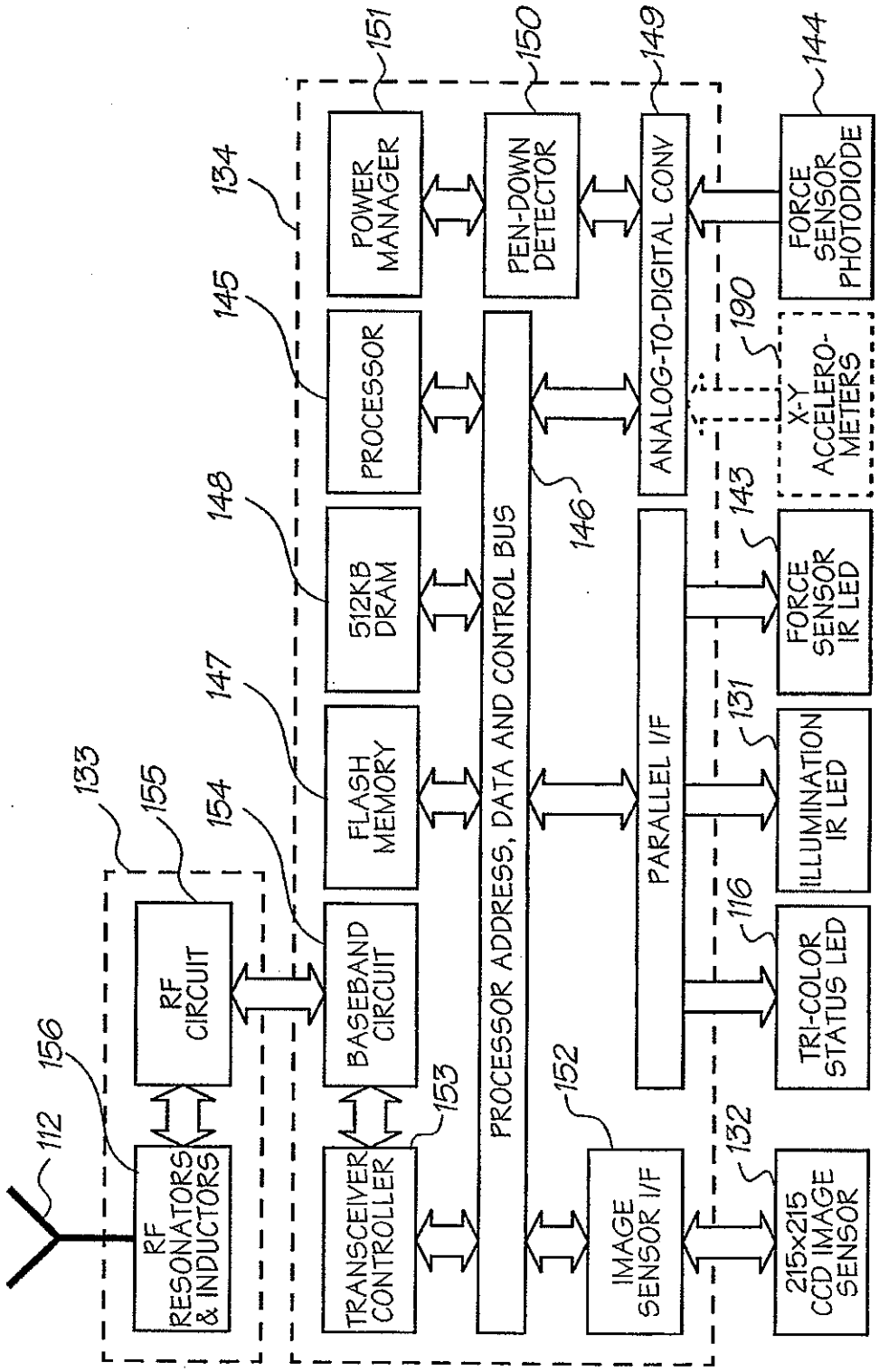

112
156
133
155
154
147
148
145
134
151
150
149
144
190
143
146
131
116
153
152
132
RF RESONATORS & INDUCTORS
RF CIRCUIT
BASEBAND CIRCUIT
FLASH MEMORY
512KB DRAM
PROCESSOR
POWER MANAGER
PEN-DOWN DETECTOR
ANALOG-TO-DIGITAL CONV
FORCE SENSOR PHOTODIODE
X-Y ACCELERO-METERS
FORCE SENSOR IR LED
ILLUMINATION IR LED
TRI-COLOR STATUS LED
TRANSCEIVER CONTROLLER
PROCESSOR ADDRESS, DATA AND CONTROL BUS
PARALLEL I/F
IMAGE SENSOR I/F
215x215 CCD IMAGE SENSOR

FIG. 10

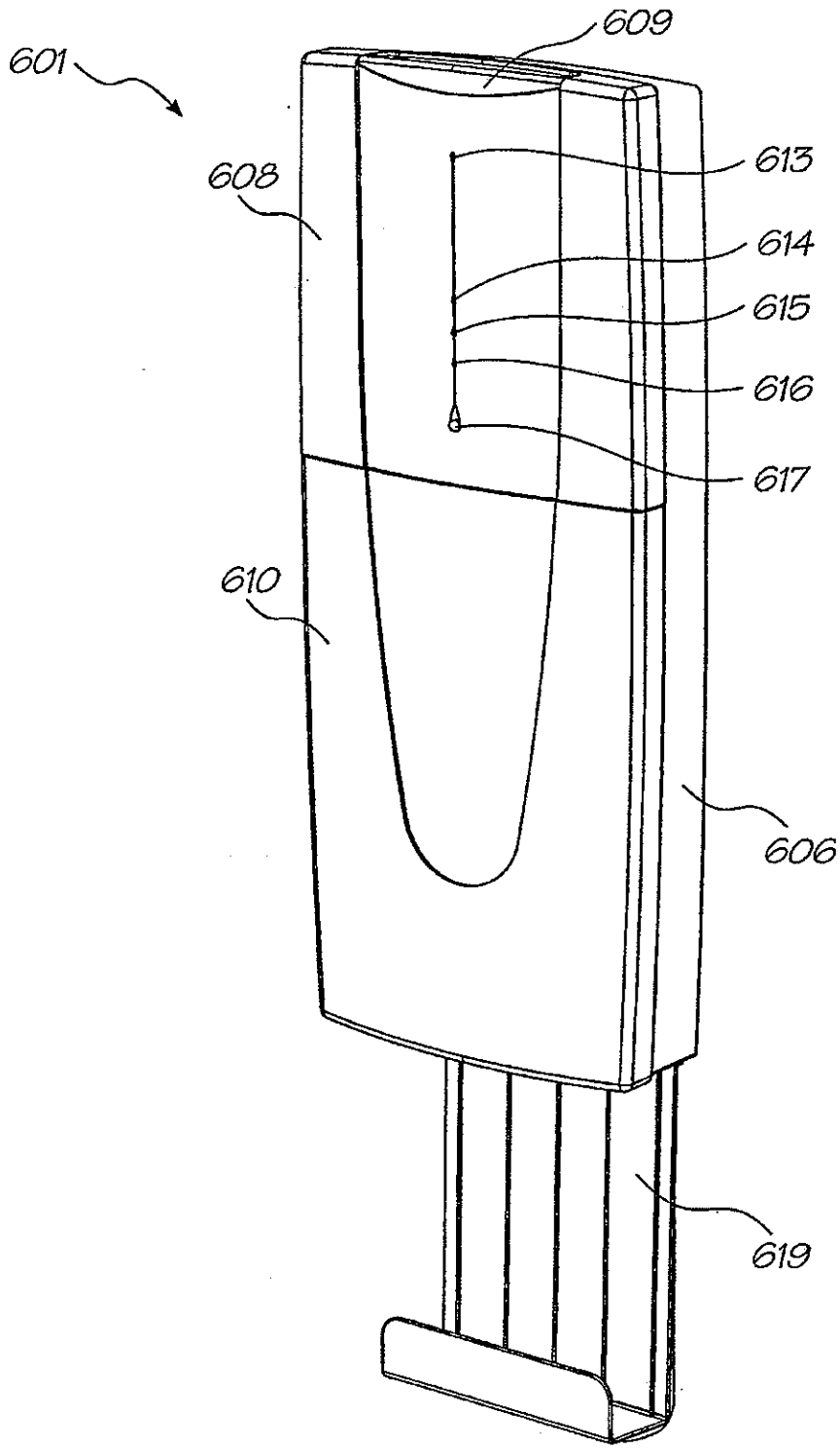


FIG. 11

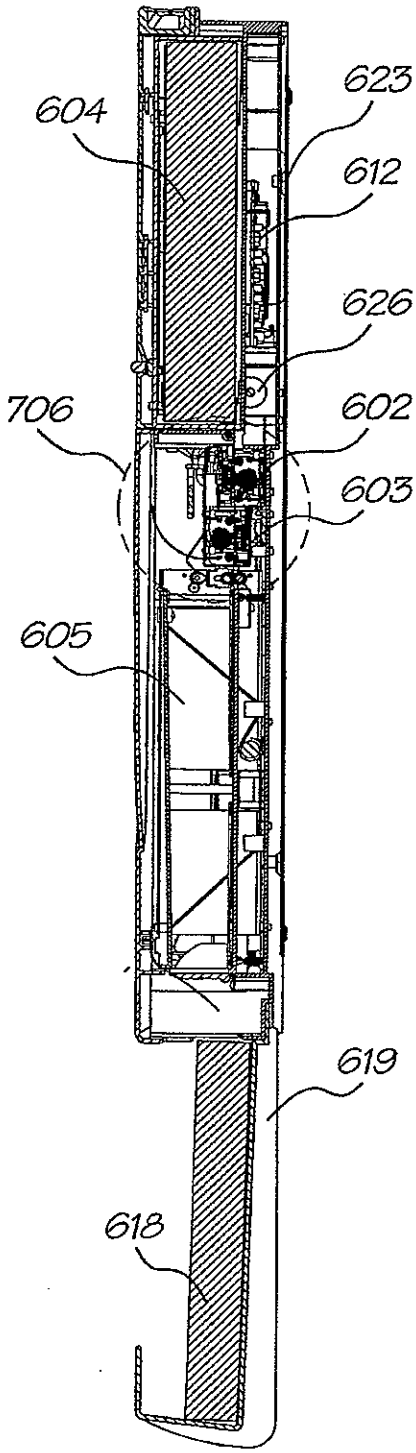


FIG. 12

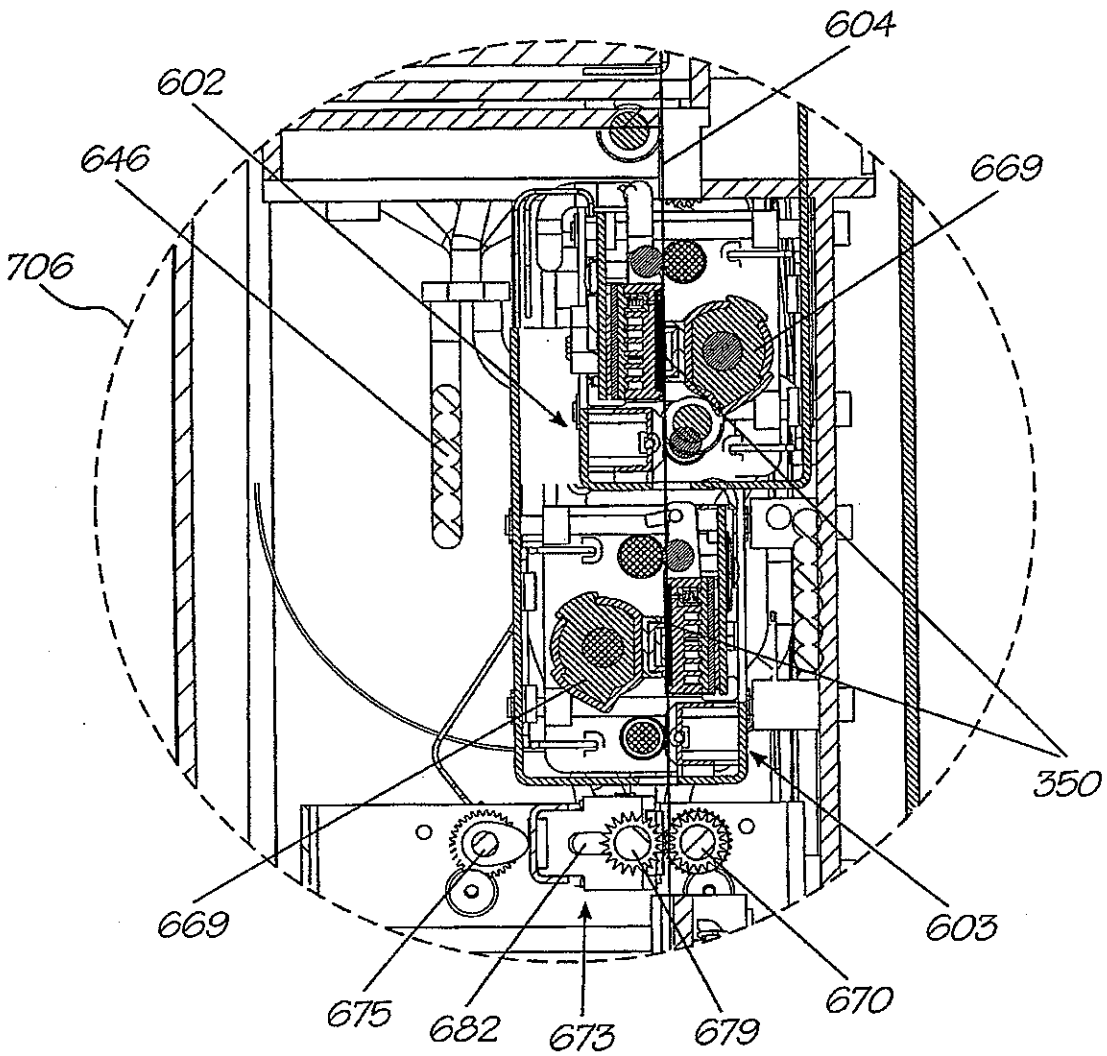


FIG. 12a

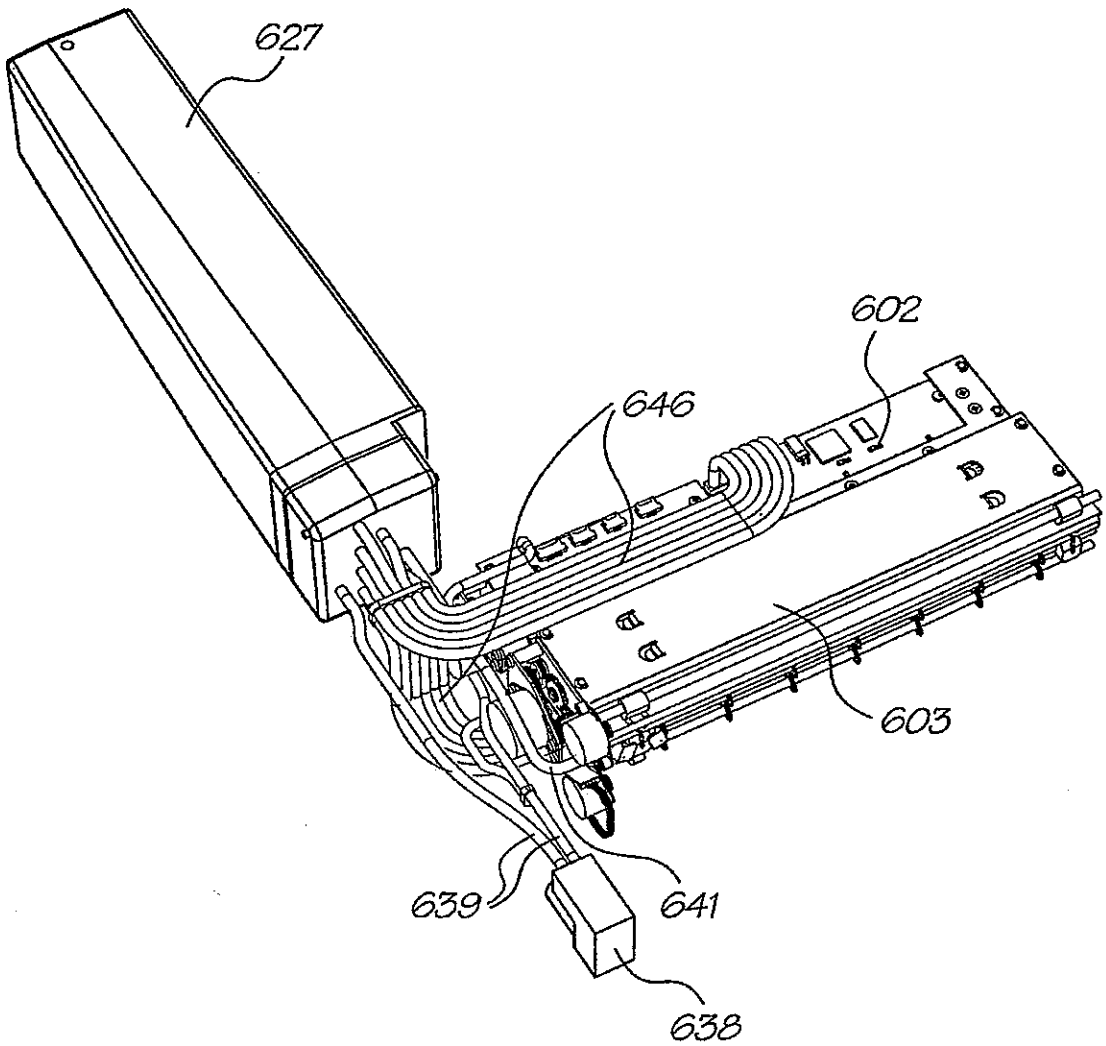


FIG. 13

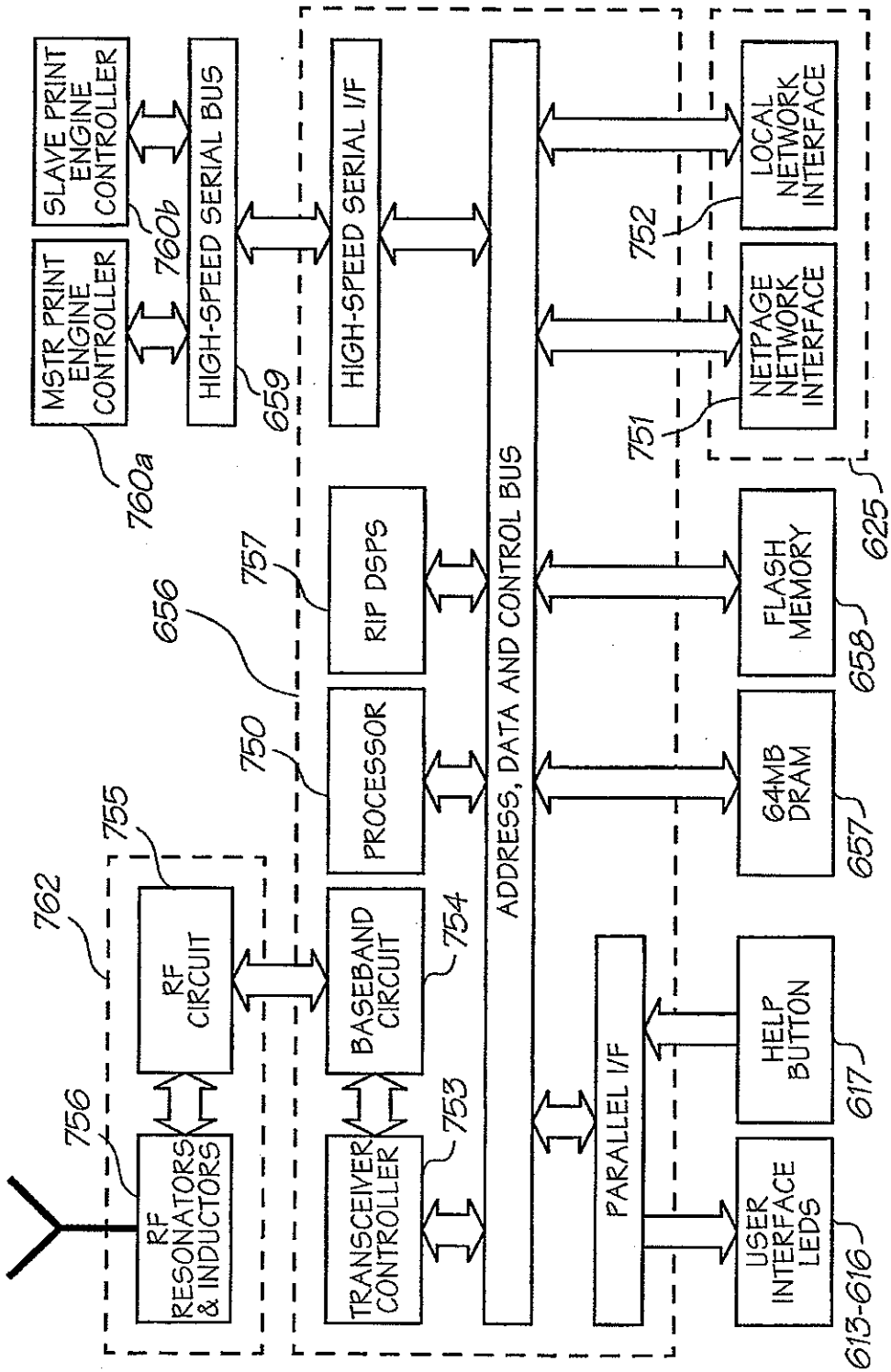

SLAVE PRINT ENGINE CONTROLLER
MSTR PRINT ENGINE CONTROLLER
760b
760a
HIGH-SPEED SERIAL BUS
659
HIGH-SPEED SERIAL I/F
LOCAL NETWORK INTERFACE
752
NETPAGE NETWORK INTERFACE
751
625
656
757
RIP DSPS
FLASH MEMORY
658
750
PROCESSOR
64MB DRAM
657
ADDRESS, DATA AND CONTROL BUS
755
762
RF CIRCUIT
BASEBAND CIRCUIT
754
756
RF RESONATORS & INDUCTORS
TRANSCEIVER CONTROLLER
753
PARALLEL I/F
HELP BUTTON
617
USER INTERFACE LEDS
613-616

FIG. 14

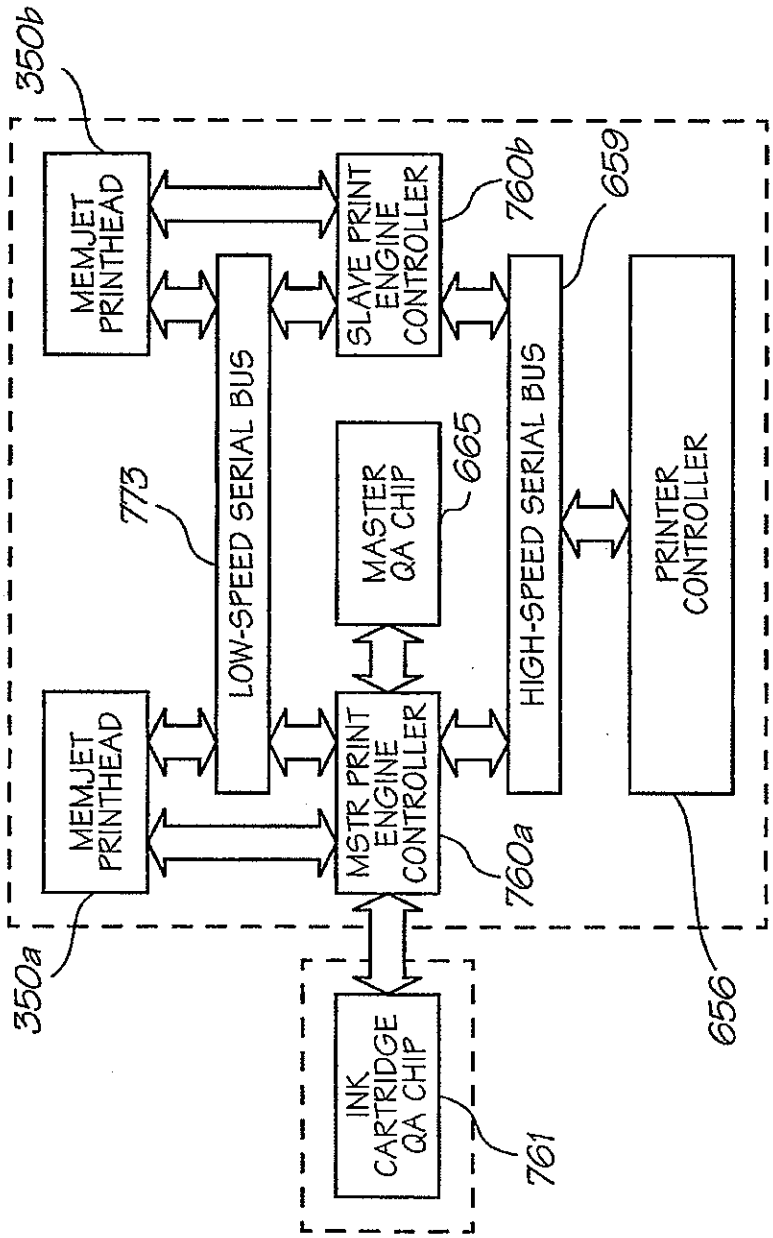

350b
MEMJET PRINTHEAD
SLAVE PRINT ENGINE CONTROLLER
760b
659
773
LOW-SPEED SERIAL BUS
MASTER QA CHIP
665
HIGH-SPEED SERIAL BUS
PRINTER CONTROLLER
MEMJET PRINTHEAD
MSTR PRINT ENGINE CONTROLLER
760a
350a
656
INK CARTRIDGE QA CHIP
761

FIG. 15

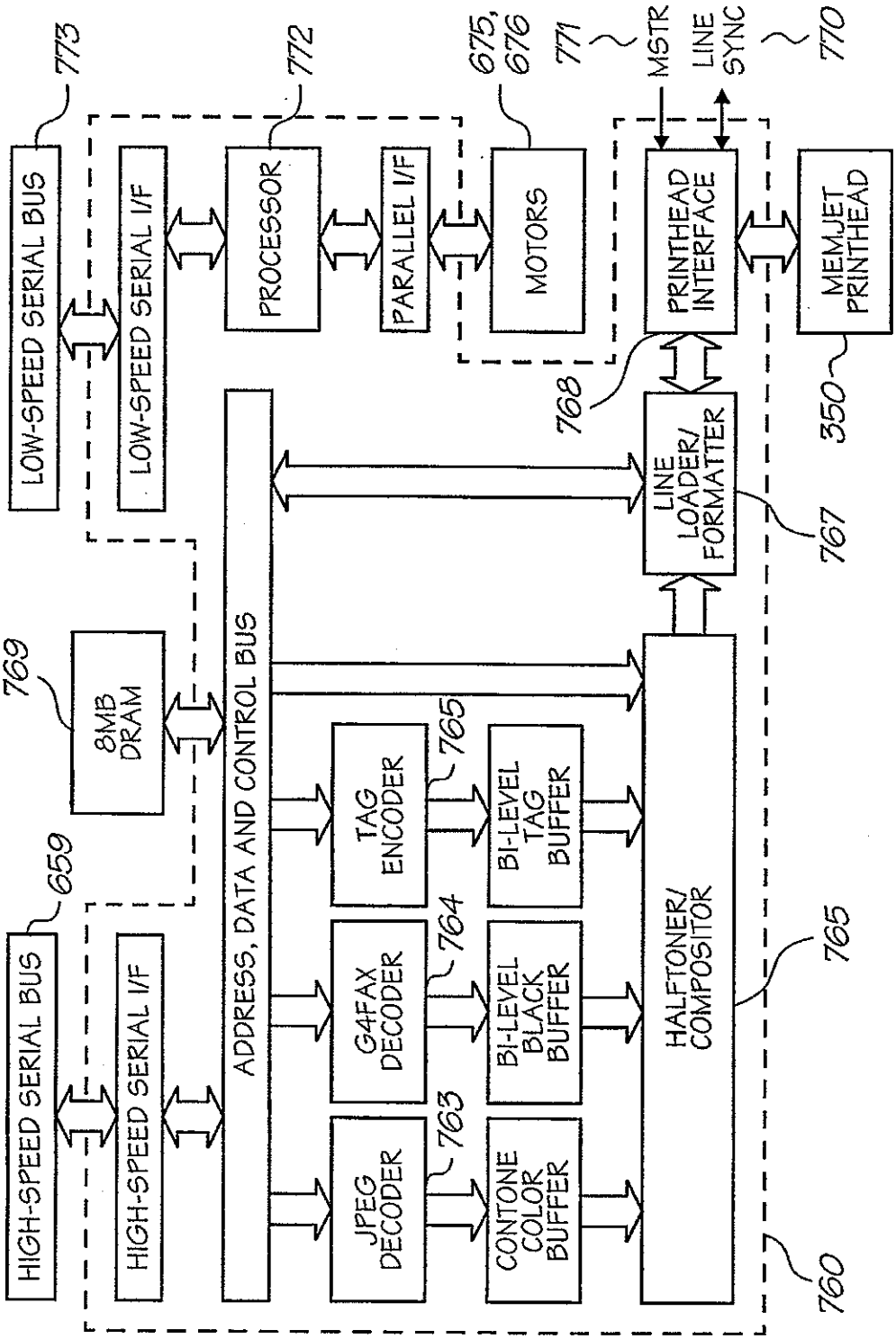

773
772
675, 676
771
MSTR
LINE SYNC
770
LOW-SPEED SERIAL BUS
LOW-SPEED SERIAL I/F
PROCESSOR
PARALLEL I/F
MOTORS
PRINTHEAD INTERFACE
MEMJET PRINTHEAD
768
350
LINE LOADER/ FORMATTER
767
769
8MB DRAM
ADDRESS, DATA AND CONTROL BUS
765
TAG ENCODER
BI-LEVEL TAG BUFFER
659
764
G4FAX DECODER
BI-LEVEL BLACK BUFFER
HIGH-SPEED SERIAL BUS
HIGH-SPEED SERIAL I/F
763
JPEG DECODER
CONTONE COLOR BUFFER
HALFTONER/ COMPOSITOR
765
760

FIG. 16

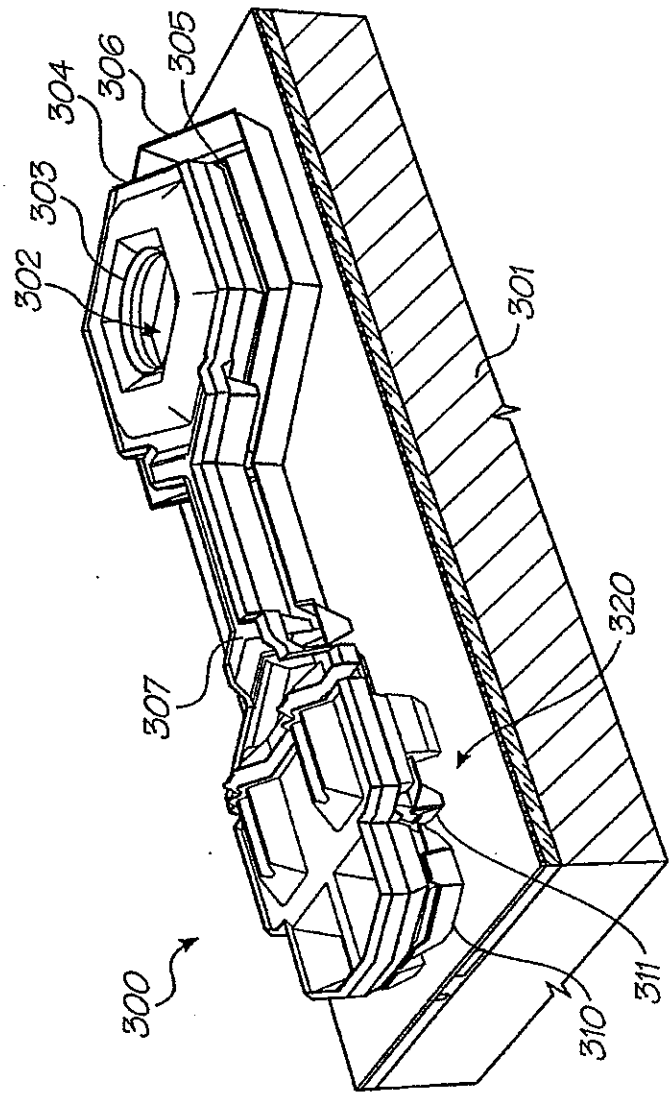


FIG. 17

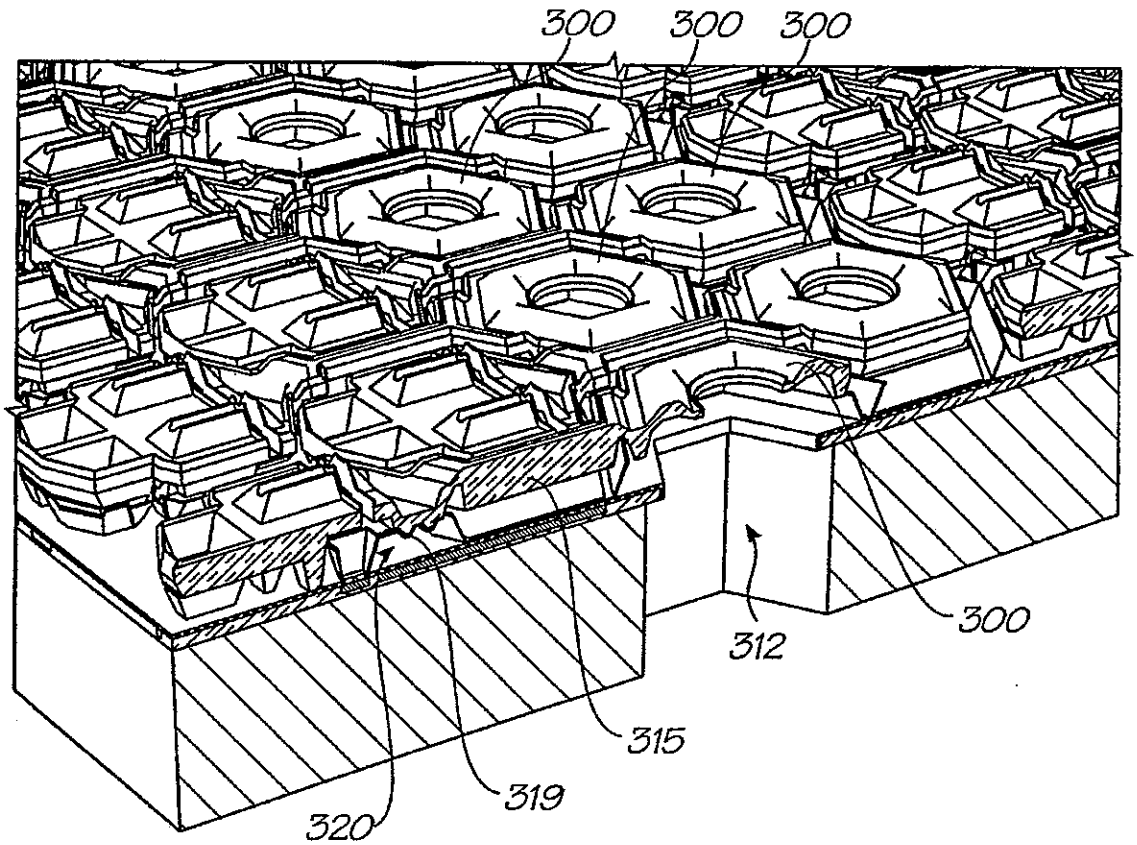


FIG. 18

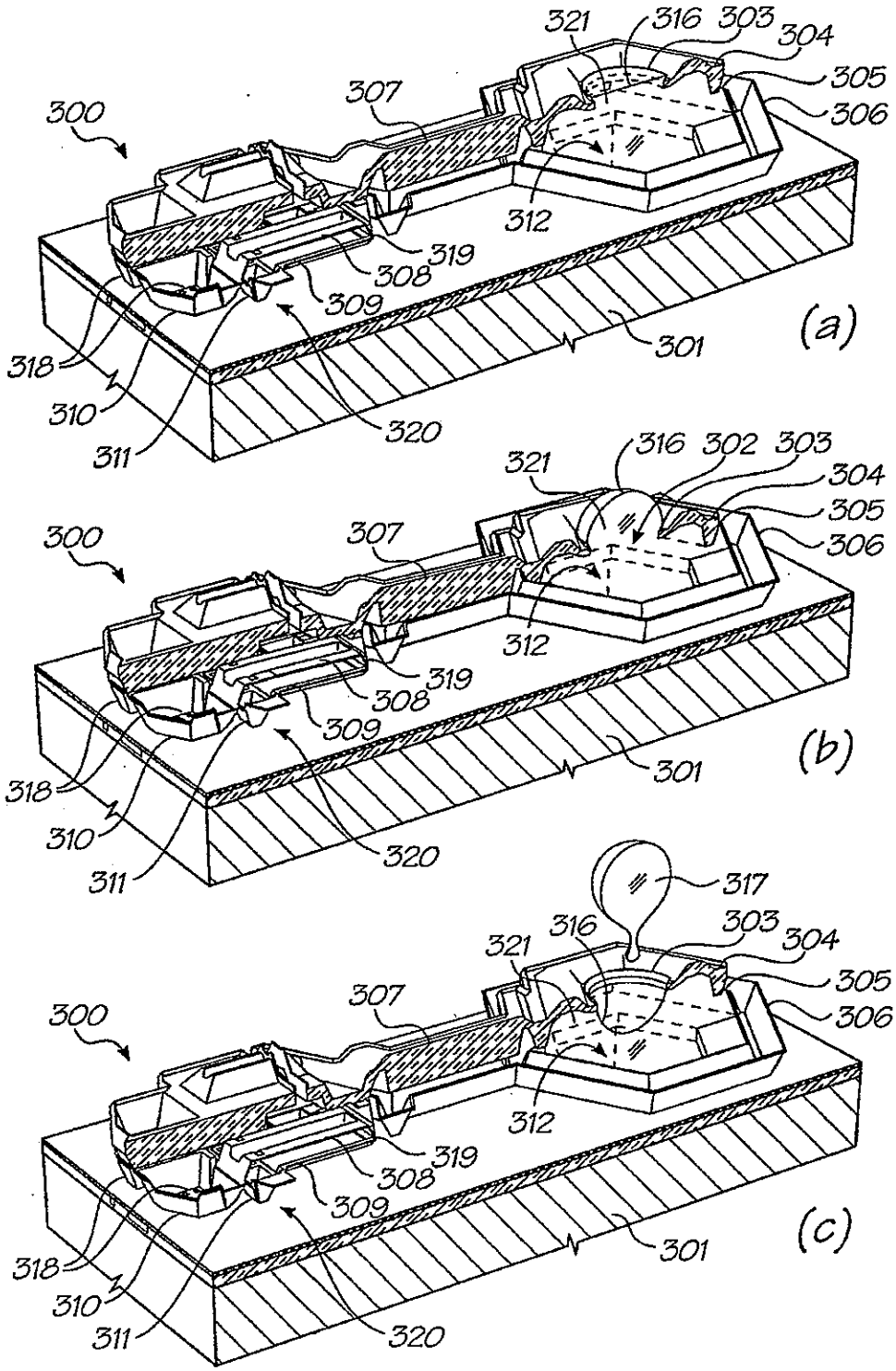

321
316
303
304
305
306
307
300
312
319
308
309
318
310
311
320
301
(a)
316
302
303
304
305
306
321
307
300
312
319
308
309
318
310
311
320
301
(b)
317
316
303
304
321
305
307
306
300
312
319
308
309
318
310
311
320
301
(c)

FIG. 19

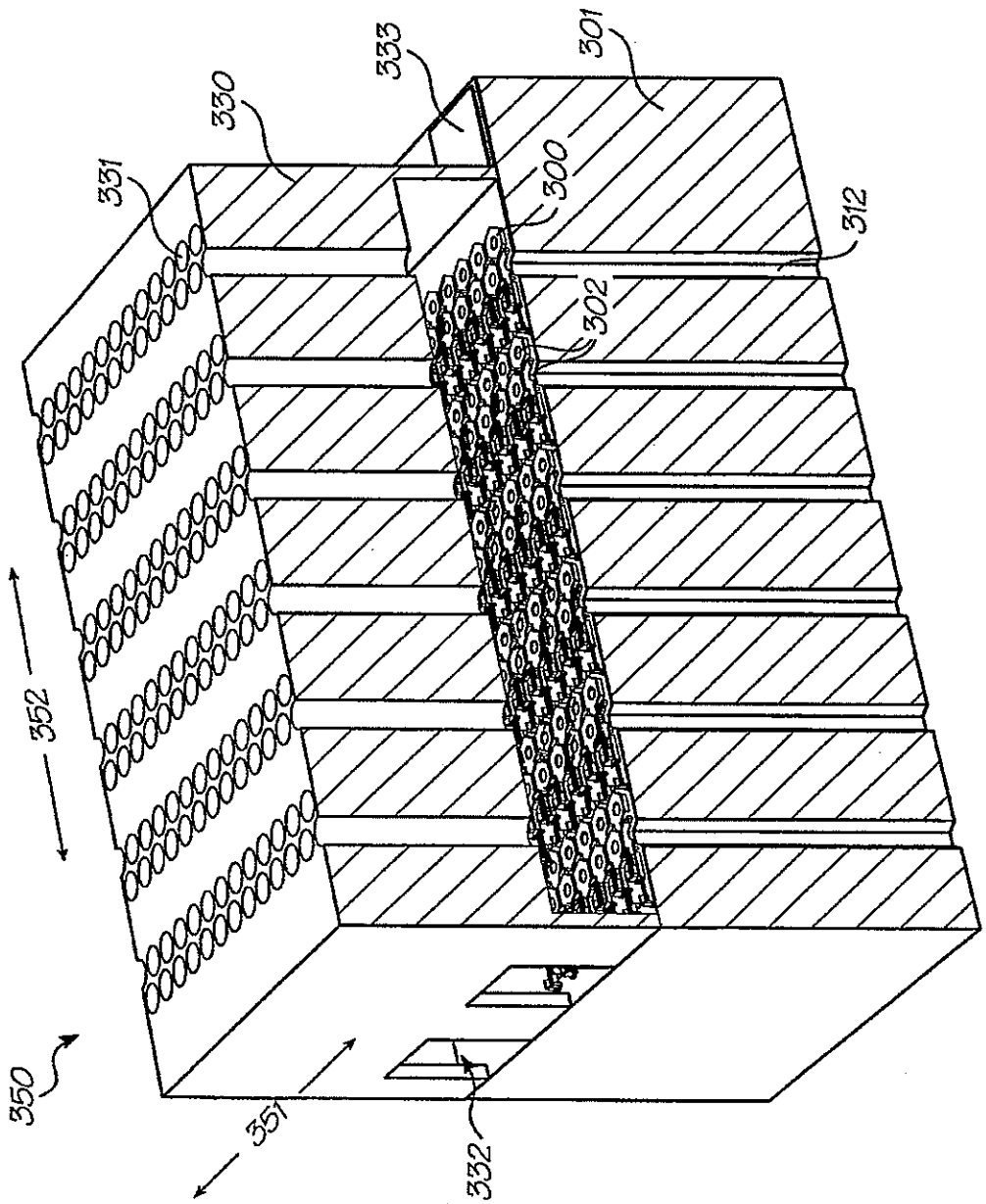


FIG. 20

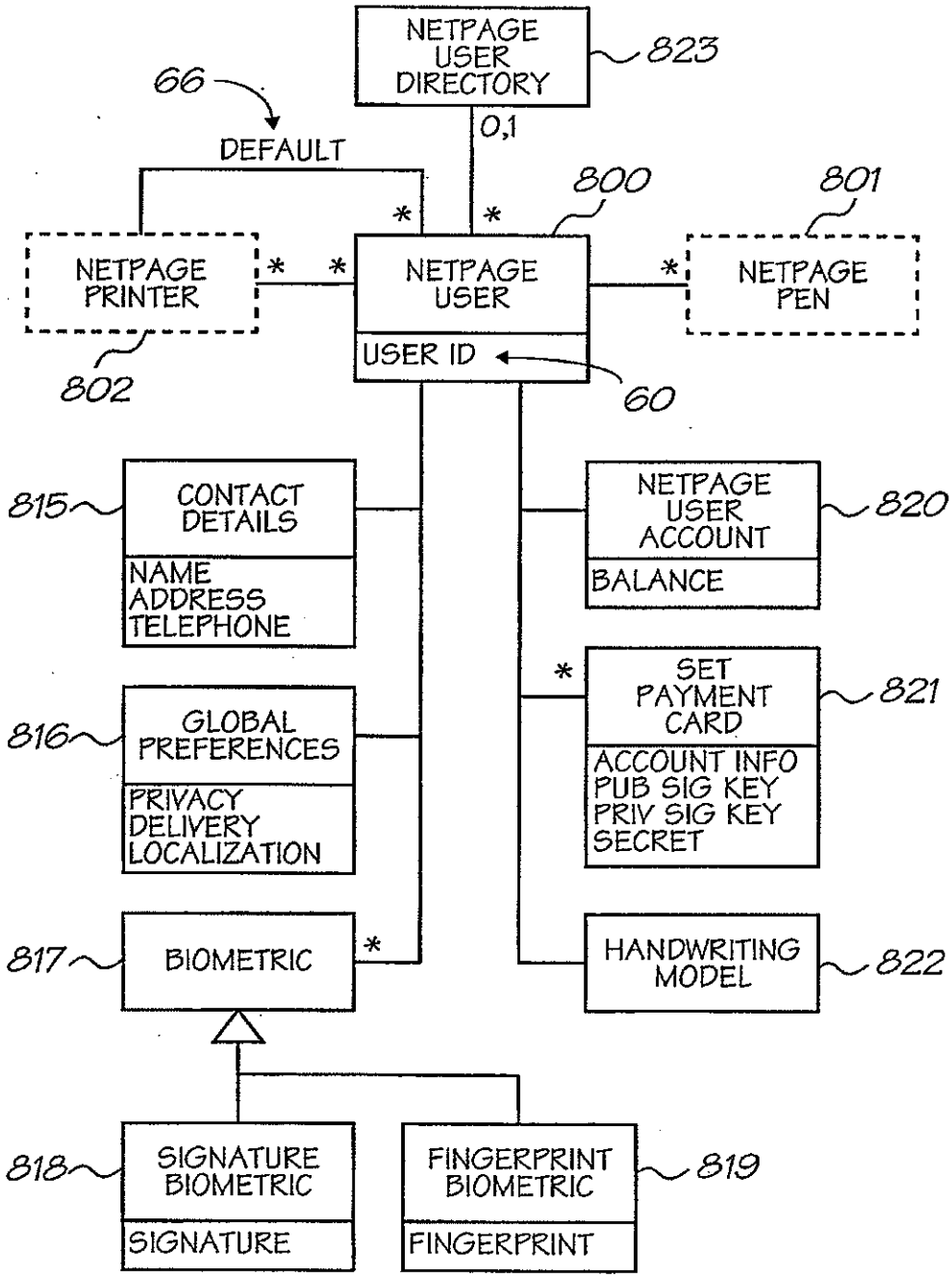

NETPAGE USER DIRECTORY
823
66
DEFAULT
0,1
800
801
NETPAGE PRINTER
NETPAGE USER
NETPAGE PEN
USER ID
60
802
815
CONTACT DETAILS
NAME
ADDRESS
TELEPHONE
NETPAGE USER ACCOUNT
820
BALANCE
816
GLOBAL PREFERENCES
PRIVACY
DELIVERY
LOCALIZATION
SET PAYMENT CARD
821
ACCOUNT INFO
PUB SIG KEY
PRIV SIG KEY
SECRET
817
BIOMETRIC
HANDWRITING MODEL
822
818
SIGNATURE BIOMETRIC
SIGNATURE
FINGERPRINT BIOMETRIC
819
FINGERPRINT

FIG. 21

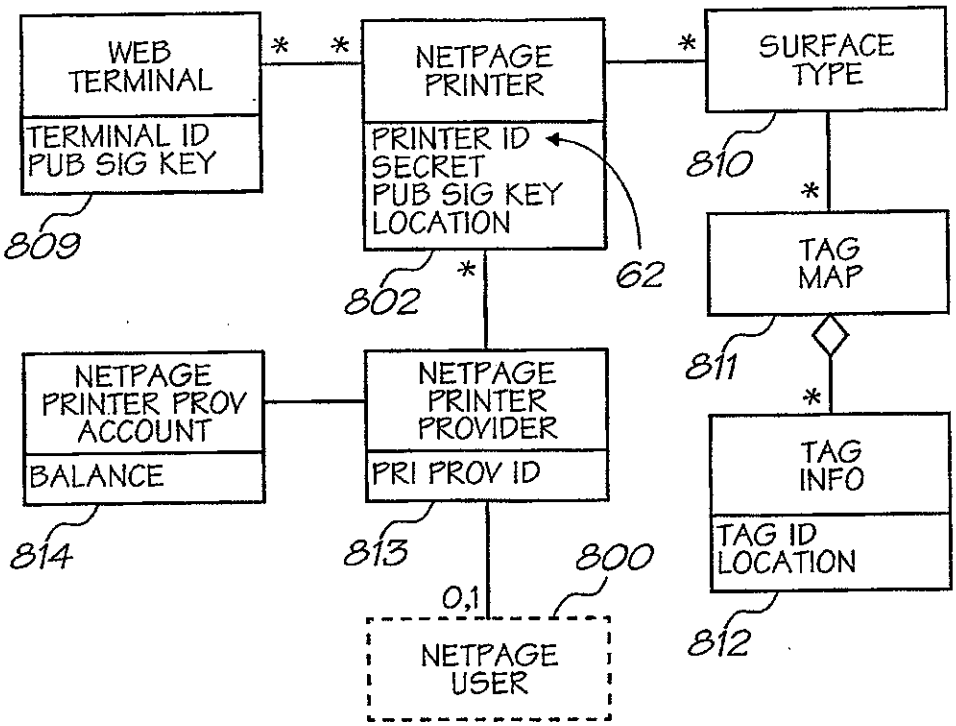

WEB TERMINAL
TERMINAL ID
PUB SIG KEY
809
NETPAGE PRINTER
PRINTER ID
SECRET
PUB SIG KEY
LOCATION
802
62
SURFACE TYPE
810
TAG MAP
811
TAG INFO
TAG ID
LOCATION
812
NETPAGE PRINTER PROV ACCOUNT
BALANCE
814
NETPAGE PRINTER PROVIDER
PRI PROV ID
813
800
0,1
NETPAGE USER
*

FIG. 22

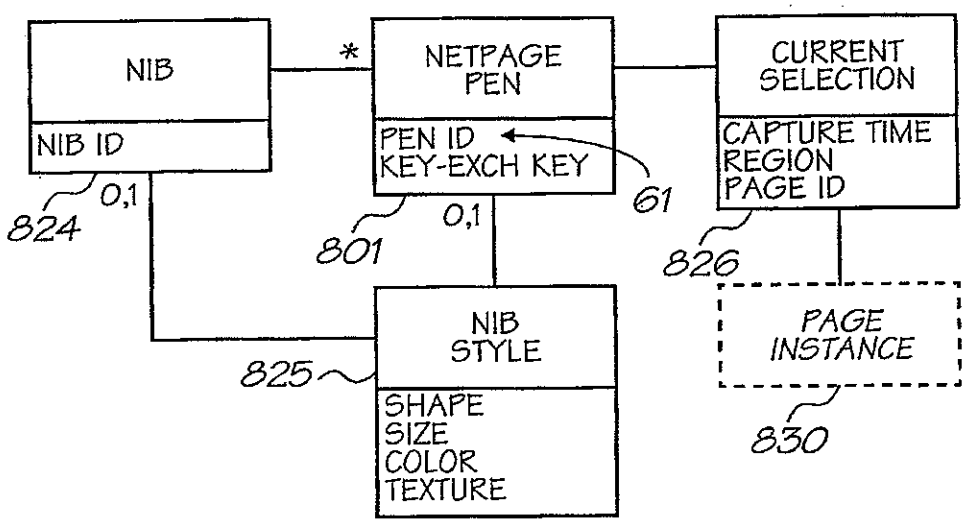

NIB
NIB ID
824
0,1
NETPAGE PEN
PEN ID
KEY-EXCH KEY
801
61
0,1
CURRENT SELECTION
CAPTURE TIME
REGION
PAGE ID
826
NIB STYLE
825
SHAPE
SIZE
COLOR
TEXTURE
PAGE INSTANCE
830
*

FIG. 23

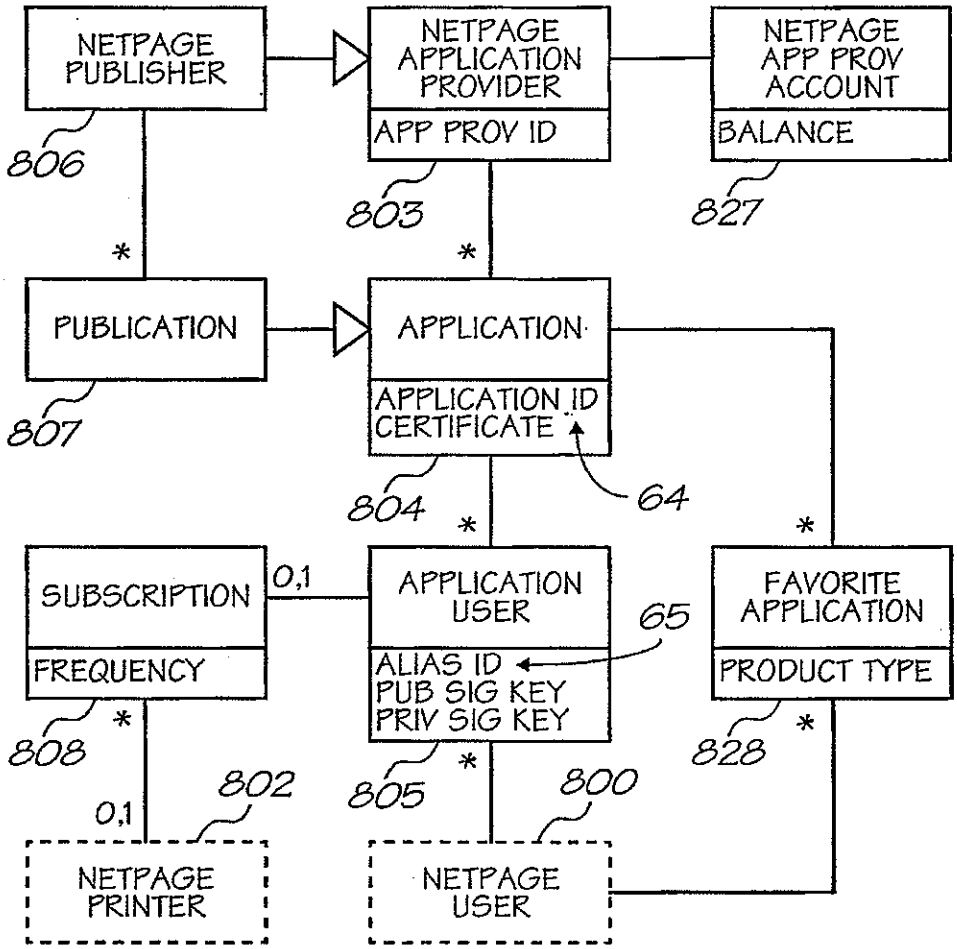

NETPAGE PUBLISHER
NETPAGE APPLICATION PROVIDER
APP PROV ID
NETPAGE APP PROV ACCOUNT
BALANCE
806
803
827
*
*
PUBLICATION
APPLICATION
APPLICATION ID
CERTIFICATE
807
804
64
*
*
SUBSCRIPTION
0,1
APPLICATION USER
65
FAVORITE APPLICATION
FREQUENCY
ALIAS ID
PUB SIG KEY
PRIV SIG KEY
PRODUCT TYPE
808
*
805
*
828
*
0,1
802
800
NETPAGE PRINTER
NETPAGE USER

FIG. 24

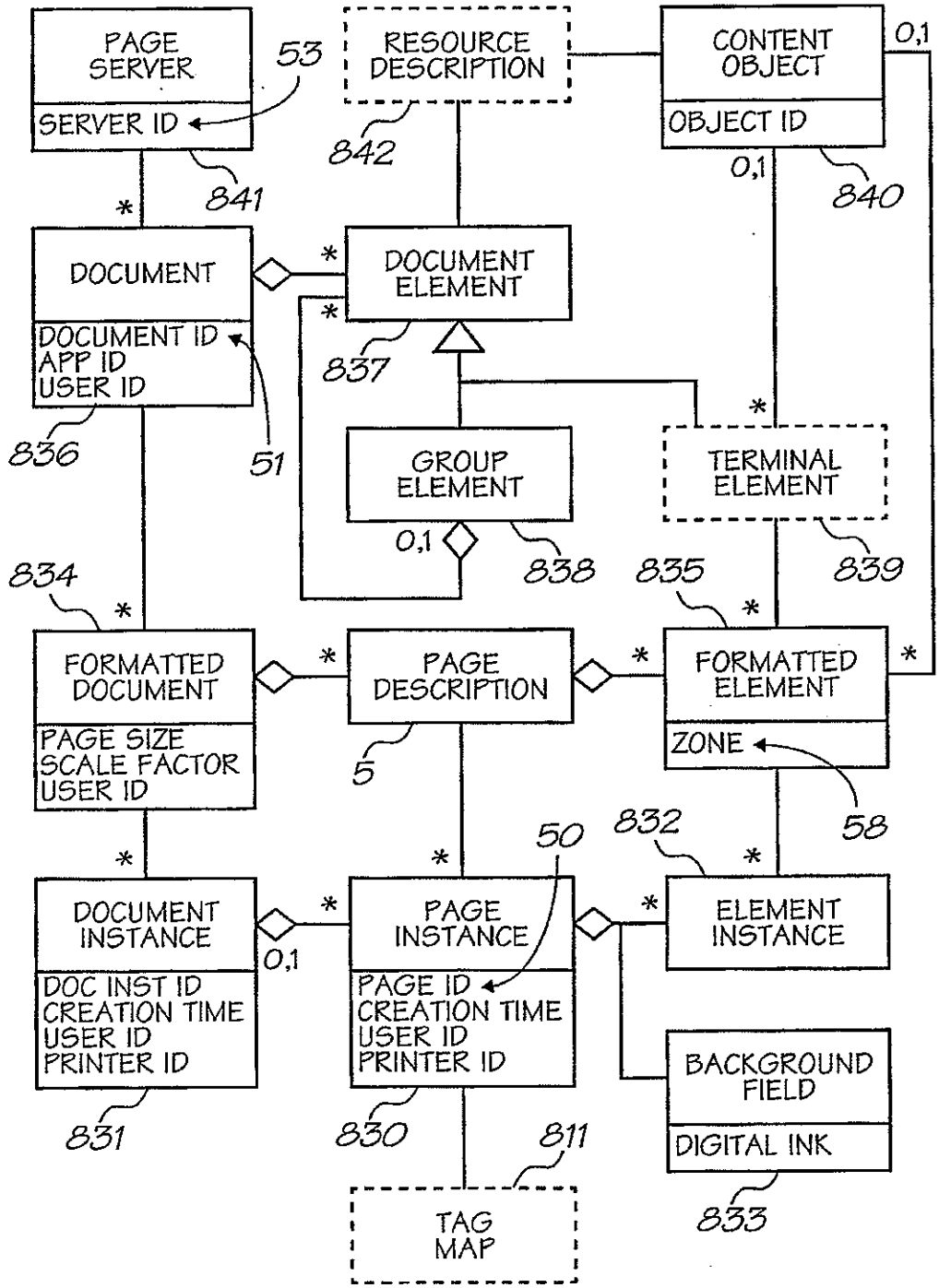

PAGE SERVER
SERVER ID
53
841
RESOURCE DESCRIPTION
842
CONTENT OBJECT
OBJECT ID
0,1
840
DOCUMENT
DOCUMENT ID
APP ID
USER ID
836
51
DOCUMENT ELEMENT
837
GROUP ELEMENT
0,1
838
TERMINAL ELEMENT
839
834
835
FORMATTED DOCUMENT
PAGE SIZE
SCALE FACTOR
USER ID
PAGE DESCRIPTION
5
FORMATTED ELEMENT
ZONE
58
832
50
DOCUMENT INSTANCE
0,1
831
PAGE INSTANCE
PAGE ID
CREATION TIME
USER ID
PRINTER ID
830
ELEMENT INSTANCE
BACKGROUND FIELD
DIGITAL INK
833
811
TAG MAP

FIG. 25

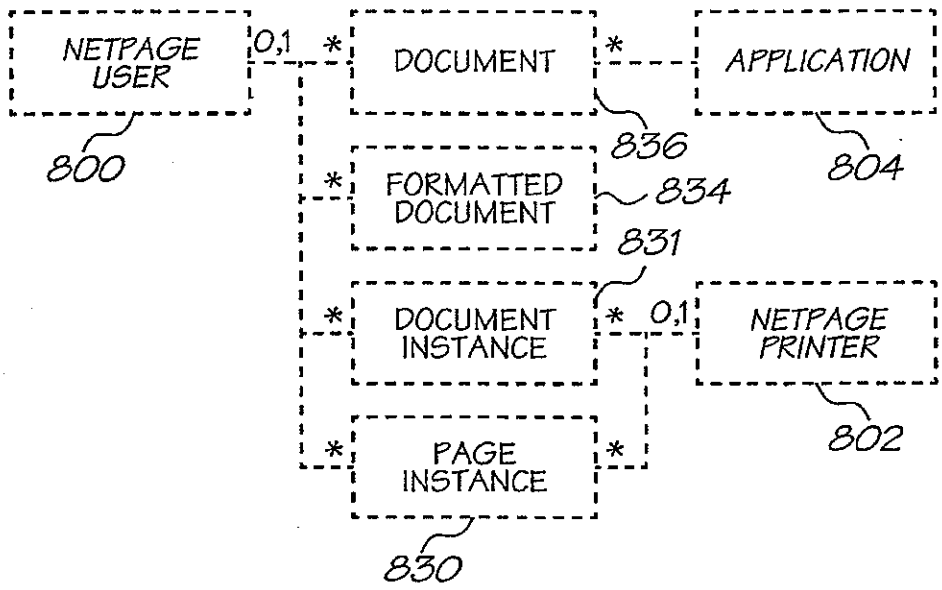

NETPAGE USER
0,1
*
DOCUMENT
*
APPLICATION
800
836
804
*
FORMATTED DOCUMENT
834
831
*
DOCUMENT INSTANCE
*
0,1
NETPAGE PRINTER
802
*
PAGE INSTANCE
*
830

FIG. 26

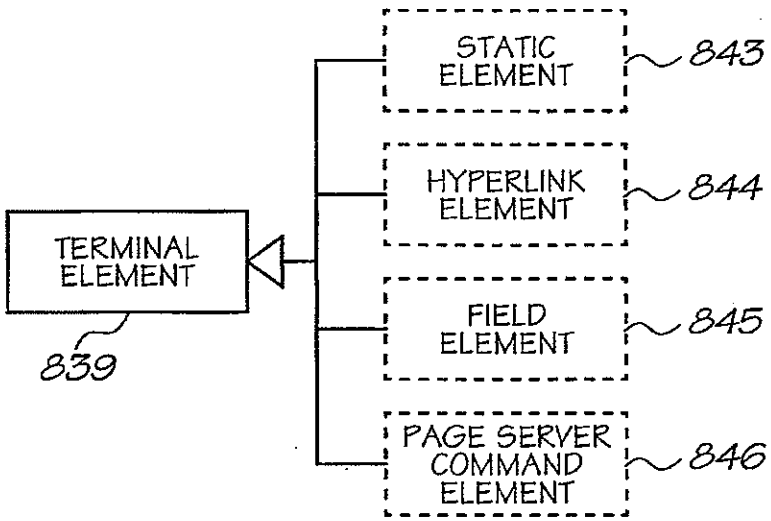

STATIC ELEMENT
843
HYPERLINK ELEMENT
844
TERMINAL ELEMENT
FIELD ELEMENT
845
839
PAGE SERVER COMMAND ELEMENT
846

FIG. 27

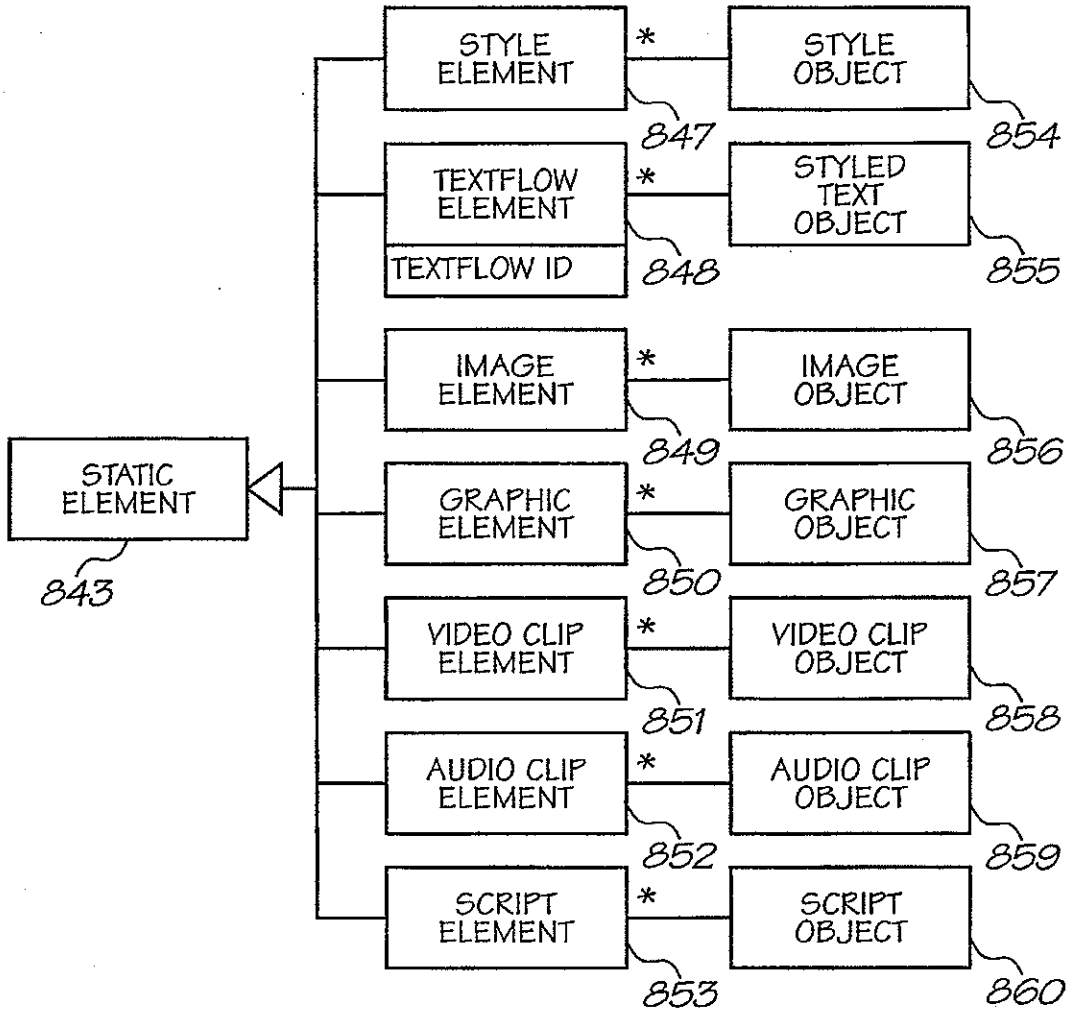

STATIC ELEMENT
843
STYLE ELEMENT
847
STYLE OBJECT
854
TEXTFLOW ELEMENT
TEXTFLOW ID
848
STYLED TEXT OBJECT
855
IMAGE ELEMENT
849
IMAGE OBJECT
856
GRAPHIC ELEMENT
850
GRAPHIC OBJECT
857
VIDEO CLIP ELEMENT
851
VIDEO CLIP OBJECT
858
AUDIO CLIP ELEMENT
852
AUDIO CLIP OBJECT
859
SCRIPT ELEMENT
853
SCRIPT OBJECT
860
*

FIG. 28

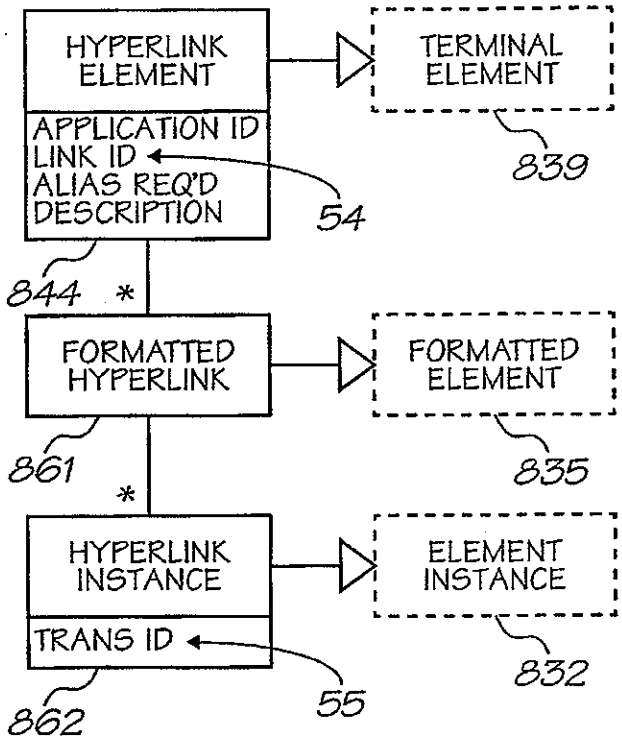

HYPERLINK ELEMENT
APPLICATION ID
LINK ID
ALIAS REQ'D
DESCRIPTION
54
844
*
TERMINAL ELEMENT
839
FORMATTED HYPERLINK
861
FORMATTED ELEMENT
835
*
HYPERLINK INSTANCE
TRANS ID
55
862
ELEMENT INSTANCE
832

FIG. 29

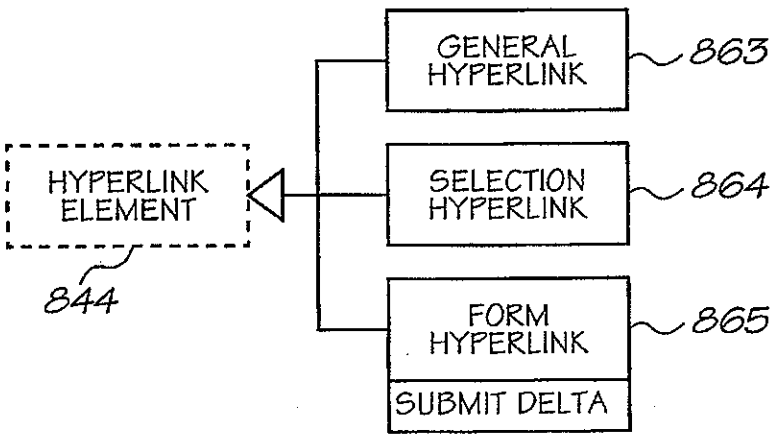

HYPERLINK ELEMENT
844
GENERAL HYPERLINK
863
SELECTION HYPERLINK
864
FORM HYPERLINK
SUBMIT DELTA
865

FIG. 30


GROUP ELEMENT
HYPERLINKED GROUP
HYPERLINK ELEMENT
838
866
844

FIG. 31

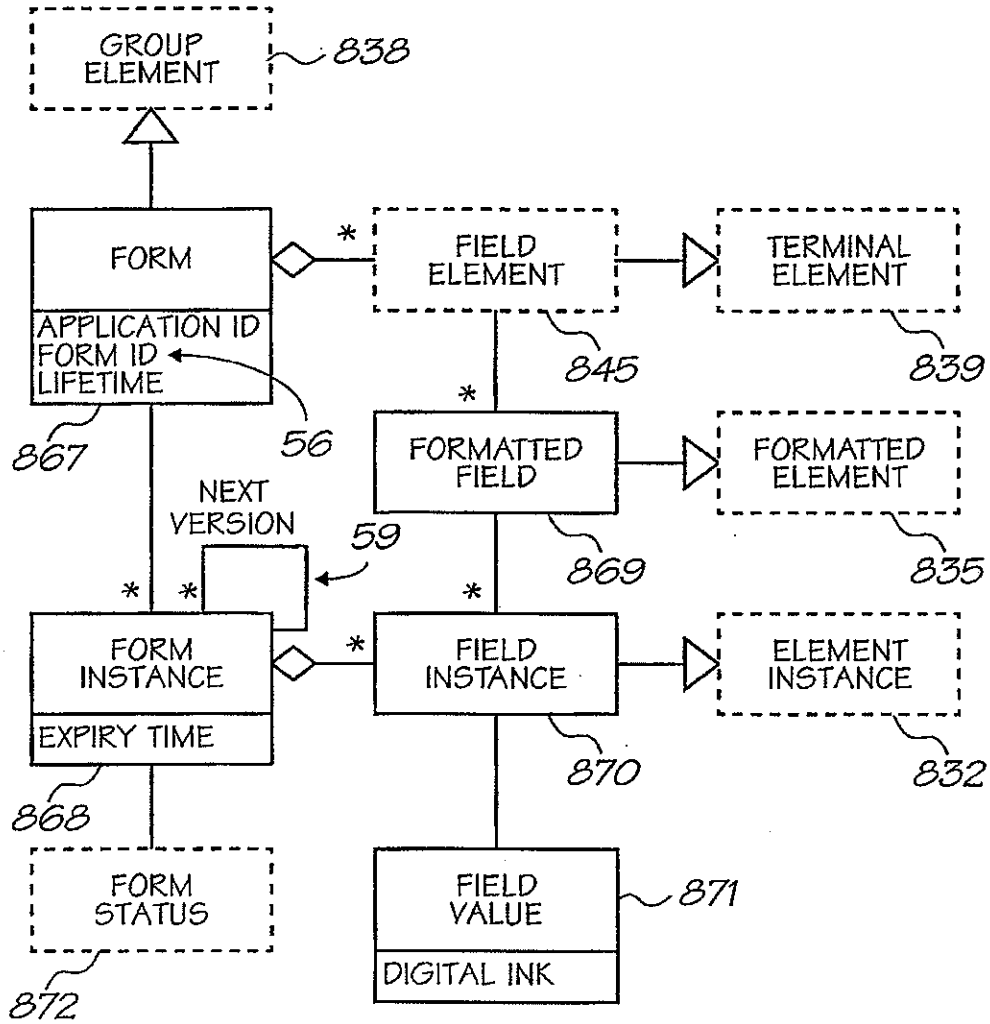

GROUP ELEMENT
838
FORM
APPLICATION ID
FORM ID
LIFETIME
867
56
*
FIELD ELEMENT
TERMINAL ELEMENT
845
839
*
FORMATTED FIELD
FORMATTED ELEMENT
869
835
NEXT VERSION
59
* *
*
FORM INSTANCE
EXPIRY TIME
868
*
FIELD INSTANCE
ELEMENT INSTANCE
870
832
FORM STATUS
872
FIELD VALUE
DIGITAL INK
871

FIG. 32

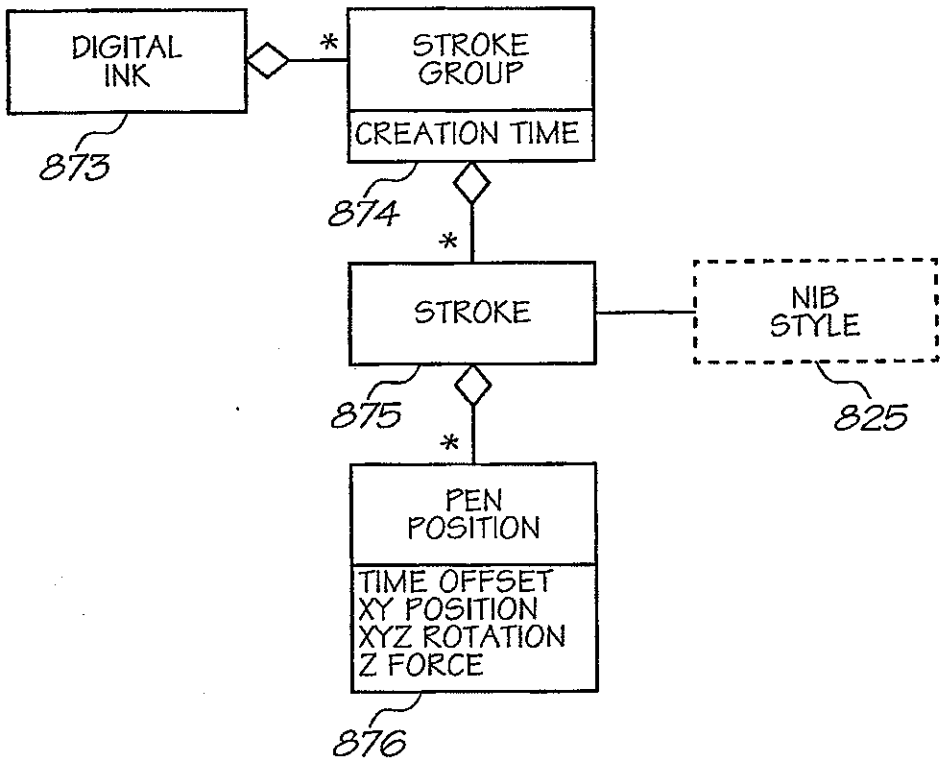

DIGITAL INK
873
*
STROKE GROUP
CREATION TIME
874
*
STROKE
875
NIB STYLE
825
*
PEN POSITION
TIME OFFSET
XY POSITION
XYZ ROTATION
Z FORCE
876

FIG. 33

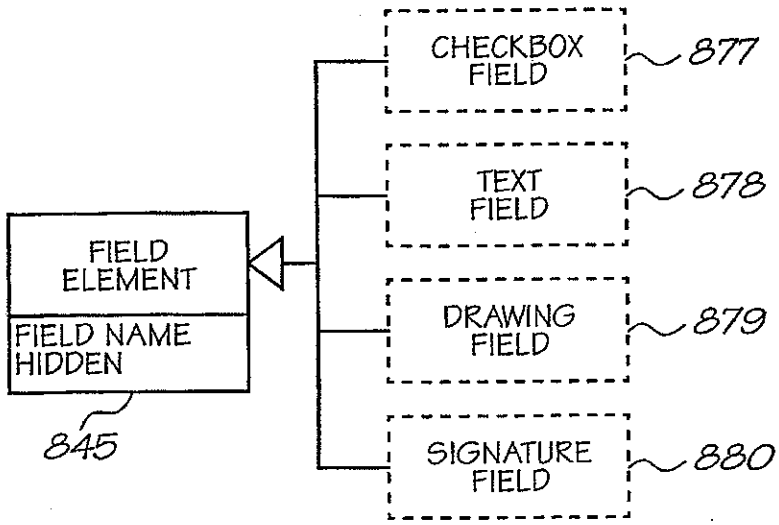

FIELD ELEMENT
FIELD NAME
HIDDEN
845
CHECKBOX FIELD
877
TEXT FIELD
878
DRAWING FIELD
879
SIGNATURE FIELD
880

FIG. 34

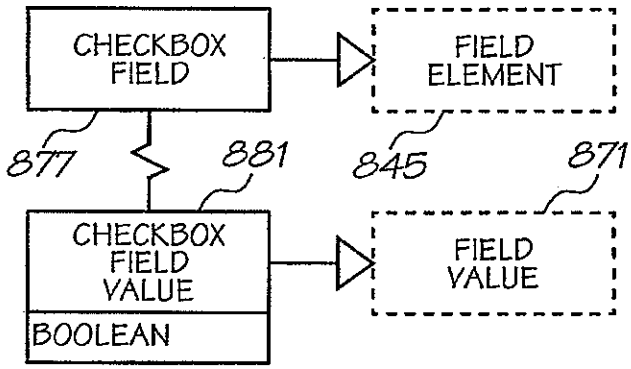


FIG. 35

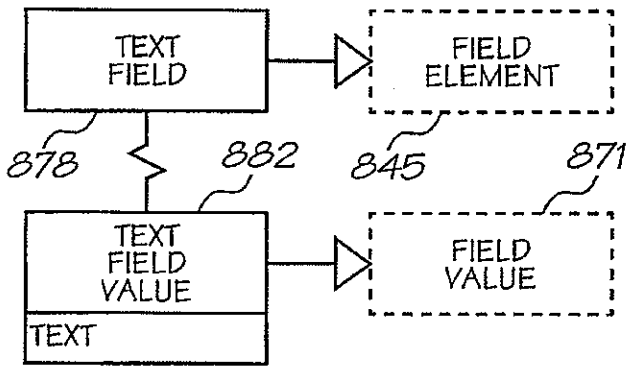


FIG. 36

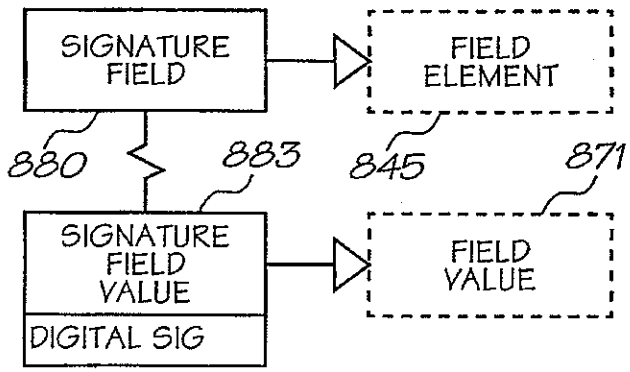


FIG. 37

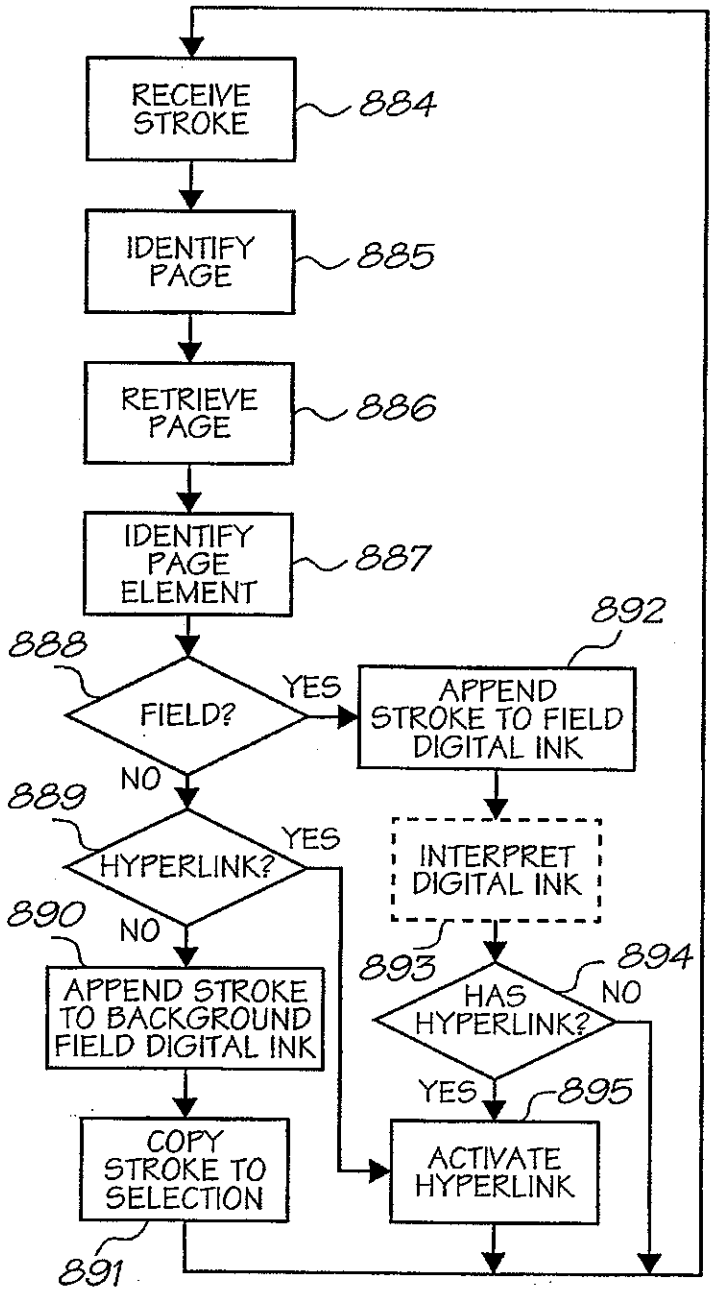

RECEIVE STROKE
884
IDENTIFY PAGE
885
RETRIEVE PAGE
886
IDENTIFY PAGE ELEMENT
887
888
FIELD?
YES
892
APPEND STROKE TO FIELD DIGITAL INK
NO
889
HYPERLINK?
YES
INTERPRET DIGITAL INK
890
NO
893
894
HAS HYPERLINK?
NO
APPEND STROKE TO BACKGROUND FIELD DIGITAL INK
YES
895
COPY STROKE TO SELECTION
ACTIVATE HYPERLINK
891

FIG. 38

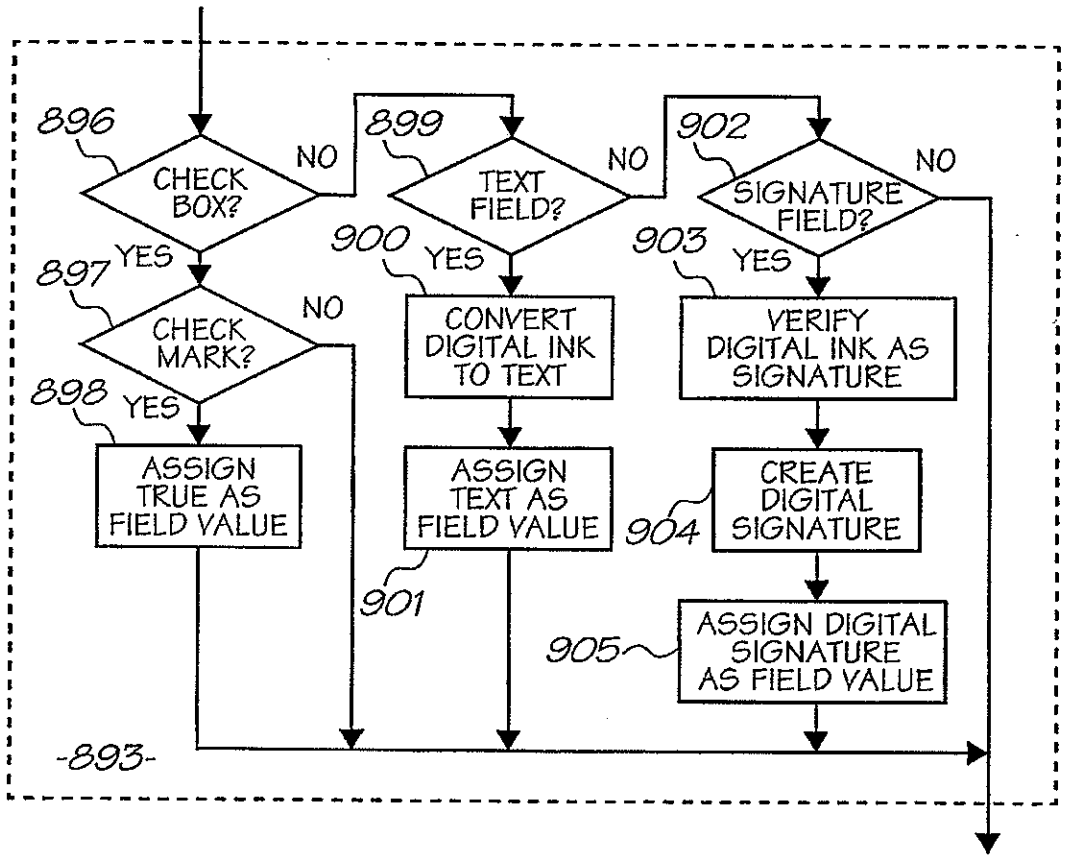

896
CHECK BOX?
NO
899
TEXT FIELD?
NO
902
SIGNATURE FIELD?
NO
897
YES
CHECK MARK?
NO
900
YES
CONVERT DIGITAL INK TO TEXT
903
YES
VERIFY DIGITAL INK AS SIGNATURE
898
YES
ASSIGN TRUE AS FIELD VALUE
ASSIGN TEXT AS FIELD VALUE
901
904
CREATE DIGITAL SIGNATURE
905
ASSIGN DIGITAL SIGNATURE AS FIELD VALUE
-893-

FIG. 38a

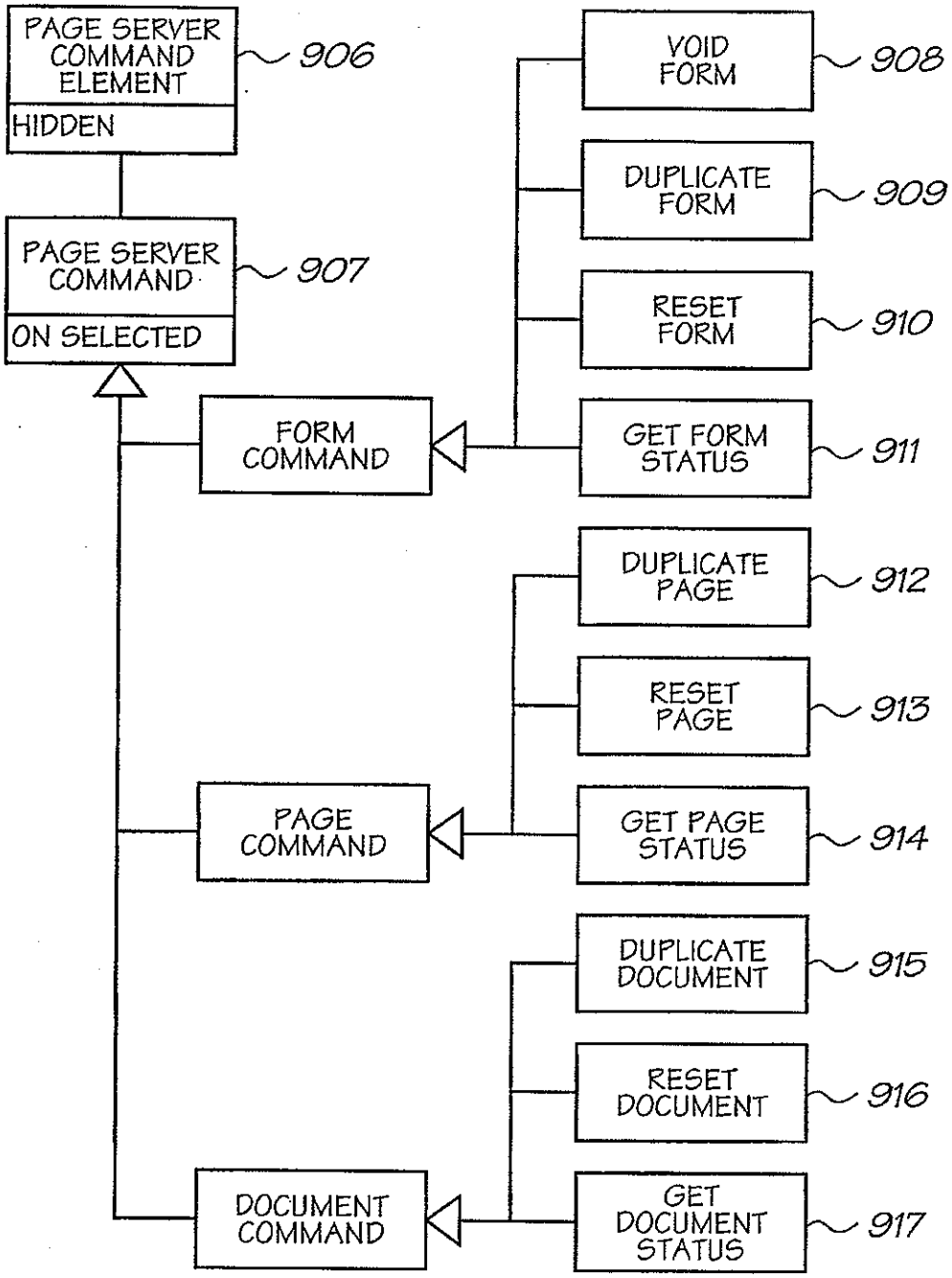

PAGE SERVER COMMAND ELEMENT
HIDDEN
906
PAGE SERVER COMMAND
ON SELECTED
907
FORM COMMAND
VOID FORM
908
DUPLICATE FORM
909
RESET FORM
910
GET FORM STATUS
911
PAGE COMMAND
DUPLICATE PAGE
912
RESET PAGE
913
GET PAGE STATUS
914
DOCUMENT COMMAND
DUPLICATE DOCUMENT
915
RESET DOCUMENT
916
GET DOCUMENT STATUS
917

FIG. 39

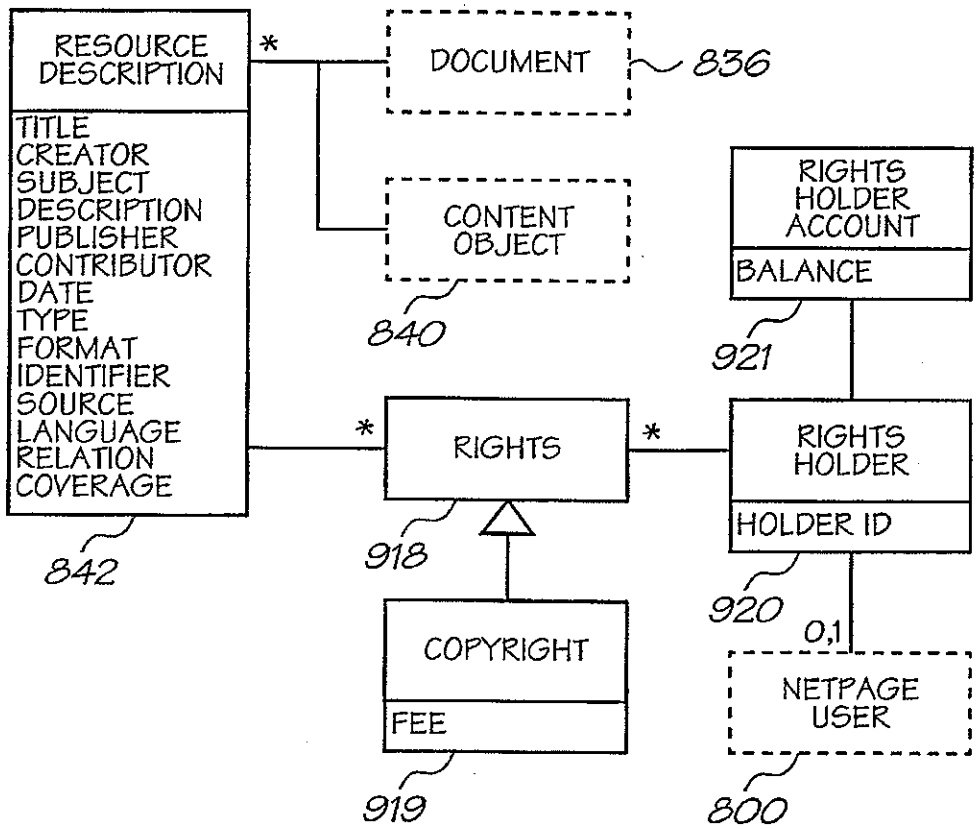

RESOURCE DESCRIPTION
TITLE
CREATOR
SUBJECT
DESCRIPTION
PUBLISHER
CONTRIBUTOR
DATE
TYPE
FORMAT
IDENTIFIER
SOURCE
LANGUAGE
RELATION
COVERAGE
842
*
DOCUMENT
836
CONTENT OBJECT
840
RIGHTS HOLDER ACCOUNT
BALANCE
921
*
RIGHTS
918
*
RIGHTS HOLDER
HOLDER ID
920
0,1
COPYRIGHT
FEE
919
NETPAGE USER
800

FIG. 40

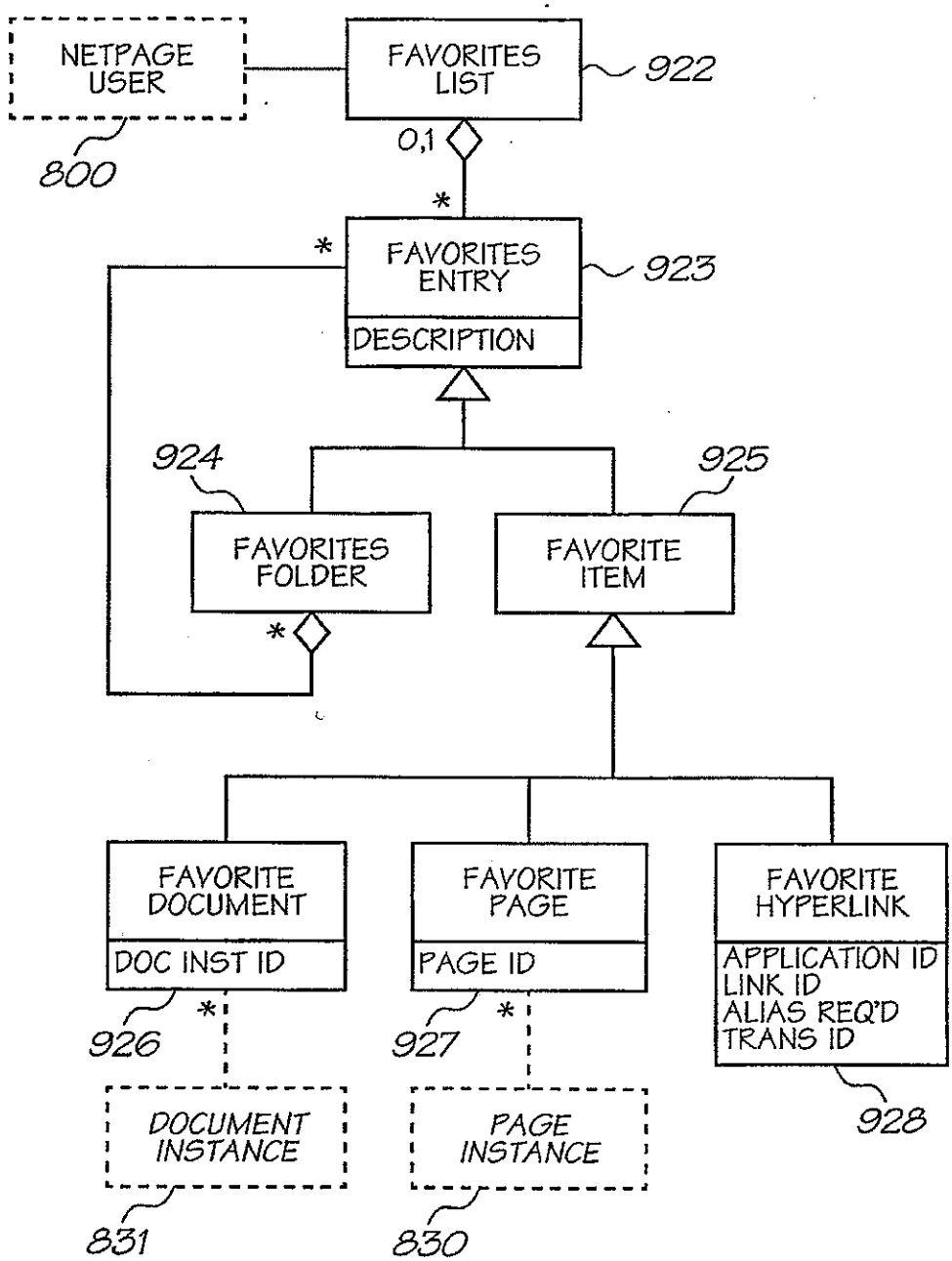

NETPAGE USER
800
FAVORITES LIST
922
0,1
*
*
FAVORITES ENTRY
923
DESCRIPTION
924
925
FAVORITES FOLDER
FAVORITE ITEM
*
FAVORITE DOCUMENT
DOC INST ID
FAVORITE PAGE
PAGE ID
FAVORITE HYPERLINK
APPLICATION ID
LINK ID
ALIAS REQ'D
TRANS ID
926
927
928
DOCUMENT INSTANCE
PAGE INSTANCE
831
830

FIG. 41

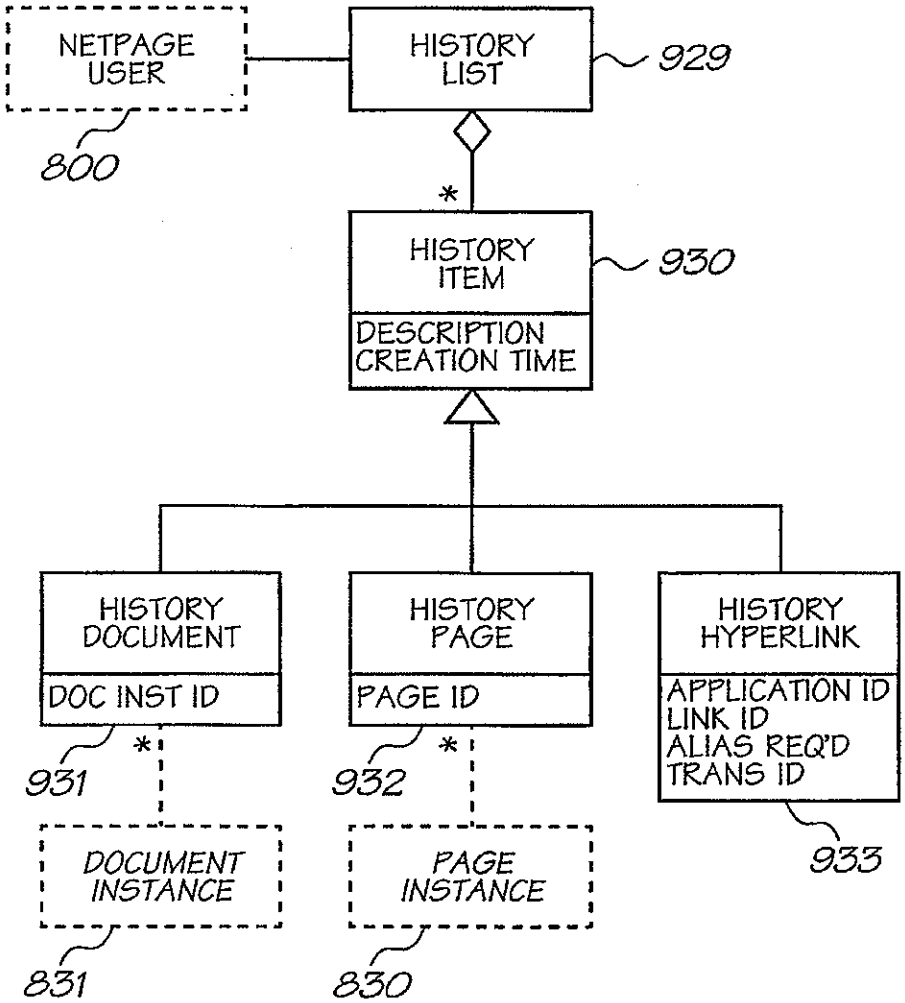

NETPAGE USER
800
HISTORY LIST
929
*
HISTORY ITEM
930
DESCRIPTION
CREATION TIME
HISTORY DOCUMENT
DOC INST ID
931
*
DOCUMENT INSTANCE
831
HISTORY PAGE
PAGE ID
932
*
PAGE INSTANCE
830
HISTORY HYPERLINK
APPLICATION ID
LINK ID
ALIAS REQ'D
TRANS ID
933

FIG. 42

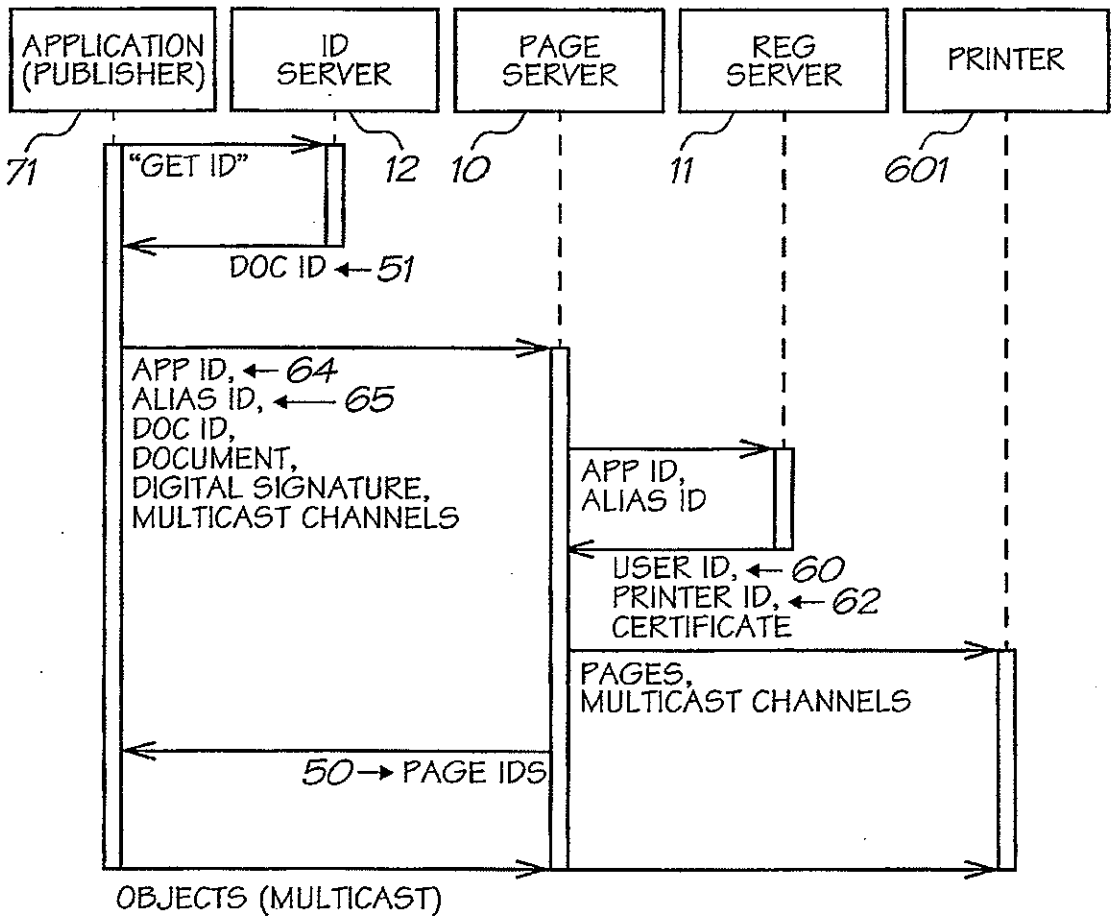

APPLICATION (PUBLISHER)
ID SERVER
PAGE SERVER
REG SERVER
PRINTER
71
12
10
11
601
"GET ID"
DOC ID ← 51
APP ID, ← 64
ALIAS ID, ← 65
DOC ID,
DOCUMENT,
DIGITAL SIGNATURE,
MULTICAST CHANNELS
APP ID,
ALIAS ID
USER ID, ← 60
PRINTER ID, ← 62
CERTIFICATE
PAGES,
MULTICAST CHANNELS
50 → PAGE IDS
OBJECTS (MULTICAST)

FIG. 43

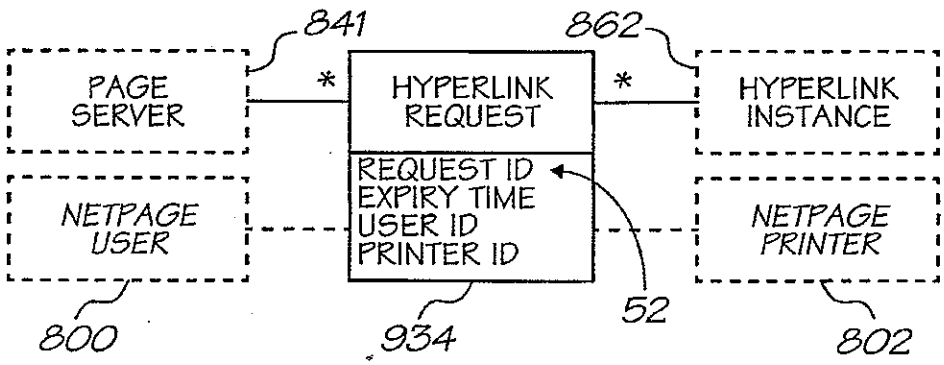

841
862
PAGE SERVER
*
HYPERLINK REQUEST
*
HYPERLINK INSTANCE
REQUEST ID
EXPIRY TIME
USER ID
PRINTER ID
NETPAGE USER
NETPAGE PRINTER
800
934
52
802

FIG. 44

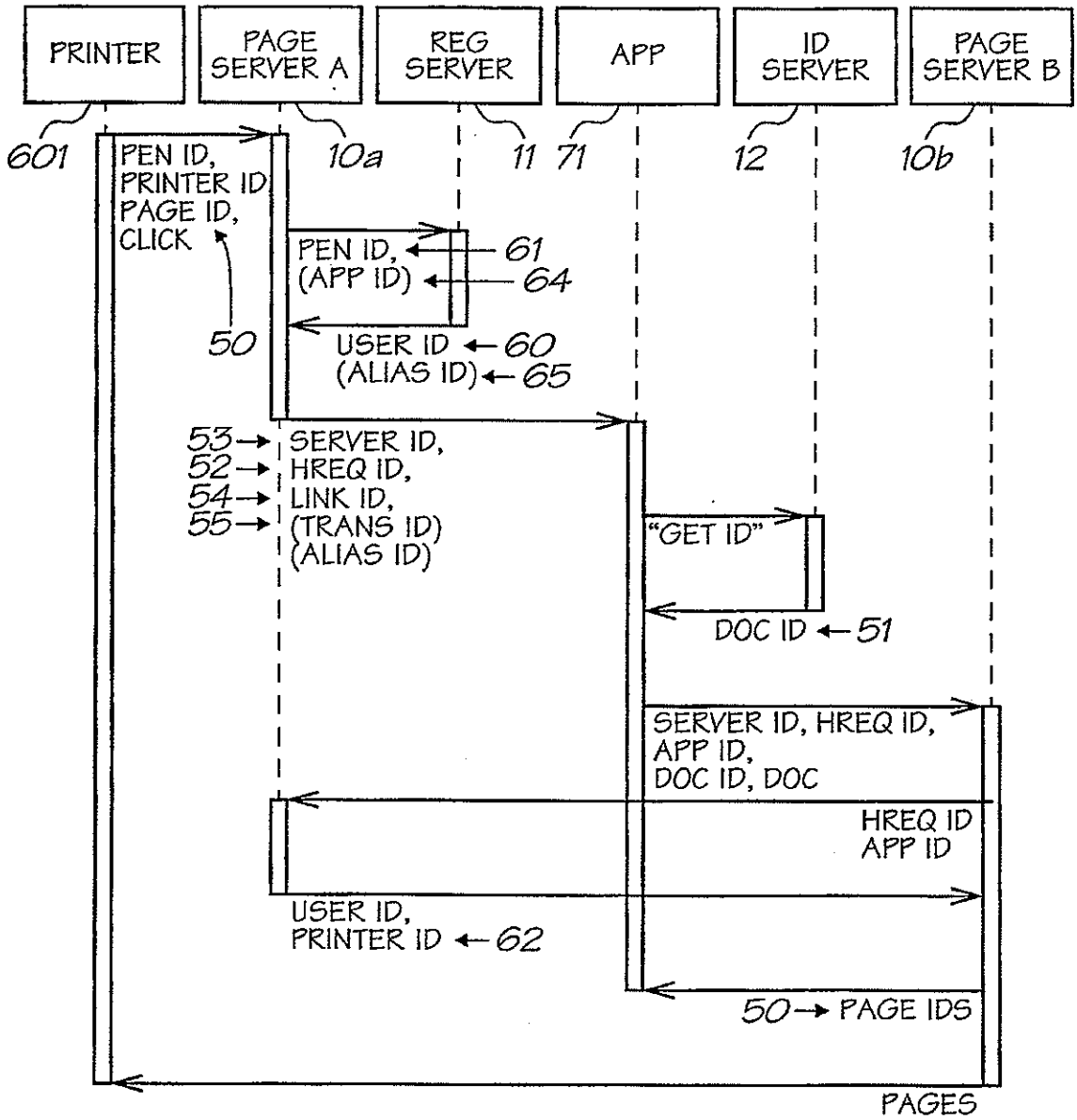

PRINTER
PAGE SERVER A
REG SERVER
APP
ID SERVER
PAGE SERVER B
601
10a
11
71
12
10b
PEN ID,
PRINTER ID
PAGE ID,
CLICK
50
PEN ID, 61
(APP ID) 64
USER ID 60
(ALIAS ID) 65
53 SERVER ID,
52 HREQ ID,
54 LINK ID,
55 (TRANS ID)
(ALIAS ID)
"GET ID"
DOC ID 51
SERVER ID, HREQ ID,
APP ID,
DOC ID, DOC
HREQ ID
APP ID
USER ID,
PRINTER ID 62
50 PAGE IDS
PAGES

FIG. 45

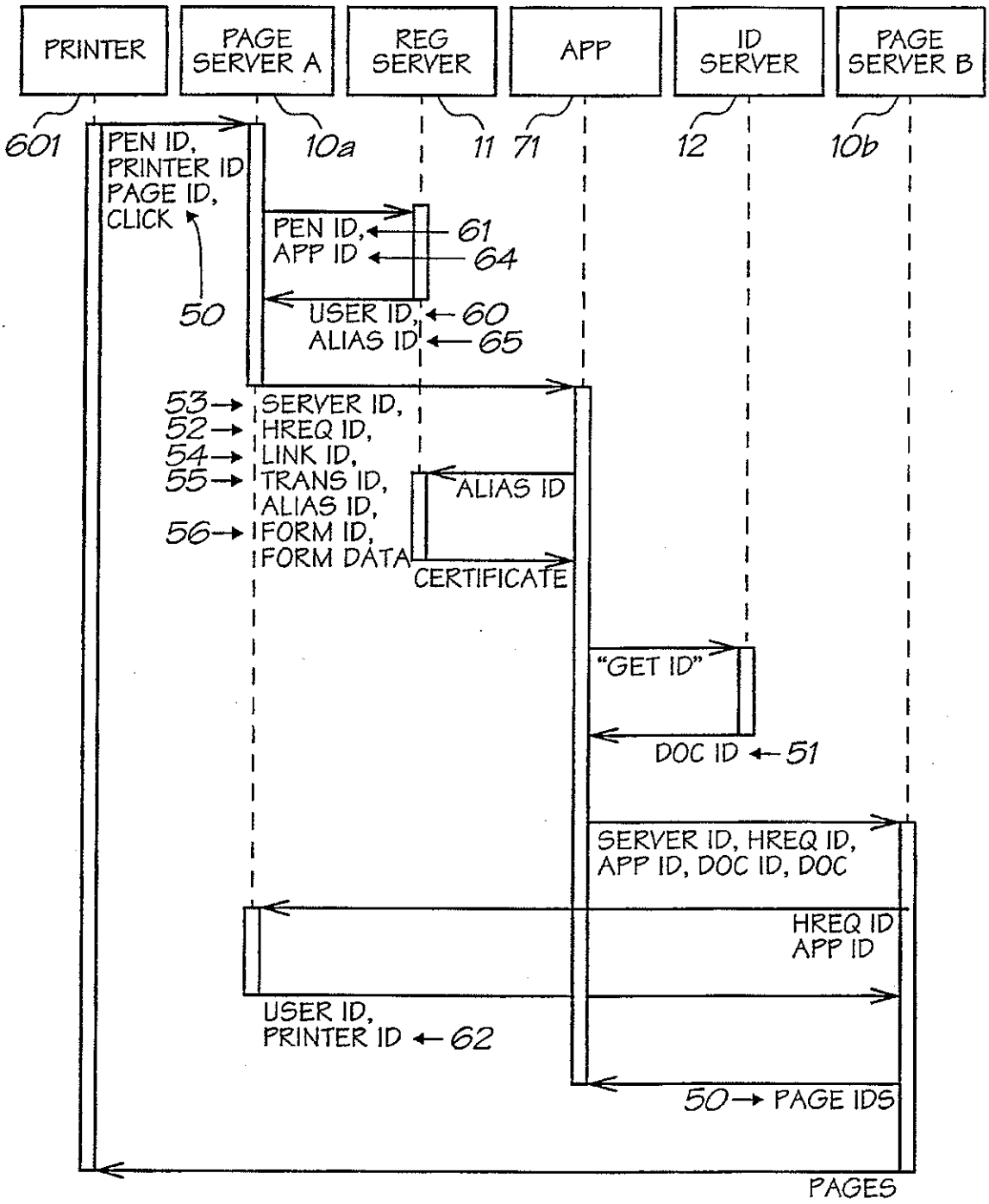

PRINTER
PAGE SERVER A
REG SERVER
APP
ID SERVER
PAGE SERVER B
601
10a
11
71
12
10b
PEN ID,
PRINTER ID
PAGE ID,
CLICK
50
PEN ID, 61
APP ID 64
USER ID, 60
ALIAS ID 65
53 SERVER ID,
52 HREQ ID,
54 LINK ID,
55 TRANS ID,
ALIAS ID,
56 FORM ID,
FORM DATA
ALIAS ID
CERTIFICATE
"GET ID"
DOC ID 51
SERVER ID, HREQ ID,
APP ID, DOC ID, DOC
HREQ ID
APP ID
USER ID,
PRINTER ID 62
50 PAGE IDS
PAGES

FIG. 46

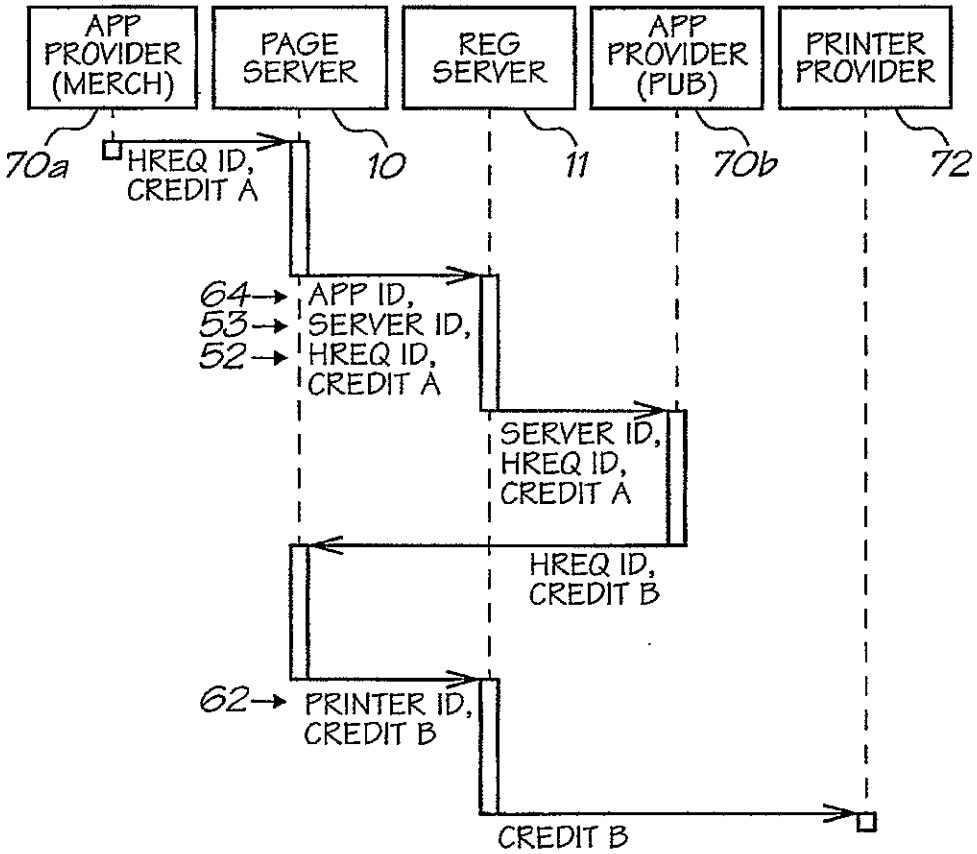

APP PROVIDER (MERCH)
PAGE SERVER
REG SERVER
APP PROVIDER (PUB)
PRINTER PROVIDER
70a
10
11
70b
72
HREQ ID, CREDIT A
64
53
52
APP ID, SERVER ID, HREQ ID, CREDIT A
SERVER ID, HREQ ID, CREDIT A
HREQ ID, CREDIT B
62
PRINTER ID, CREDIT B
CREDIT B

FIG. 47

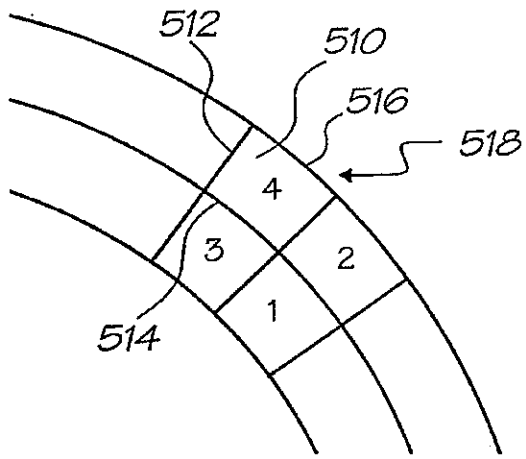


FIG. 48

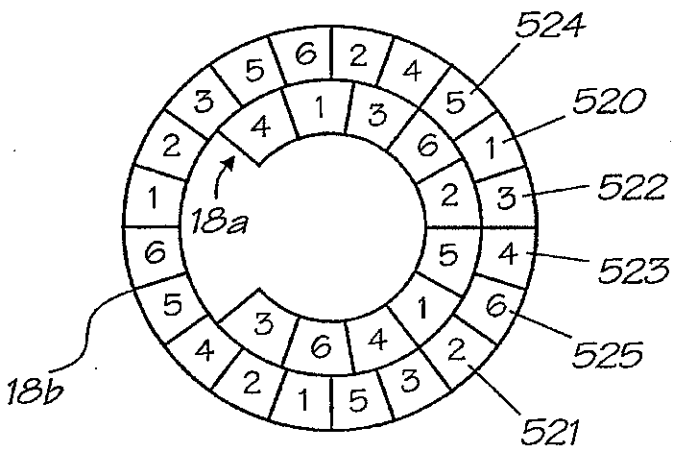


FIG. 49

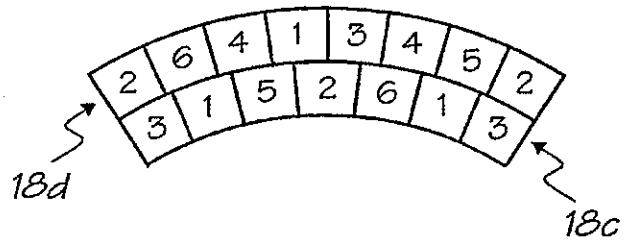


FIG. 50

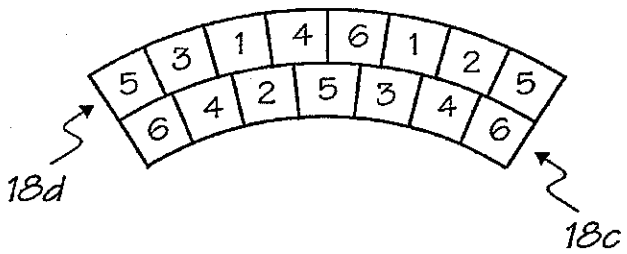


FIG. 51

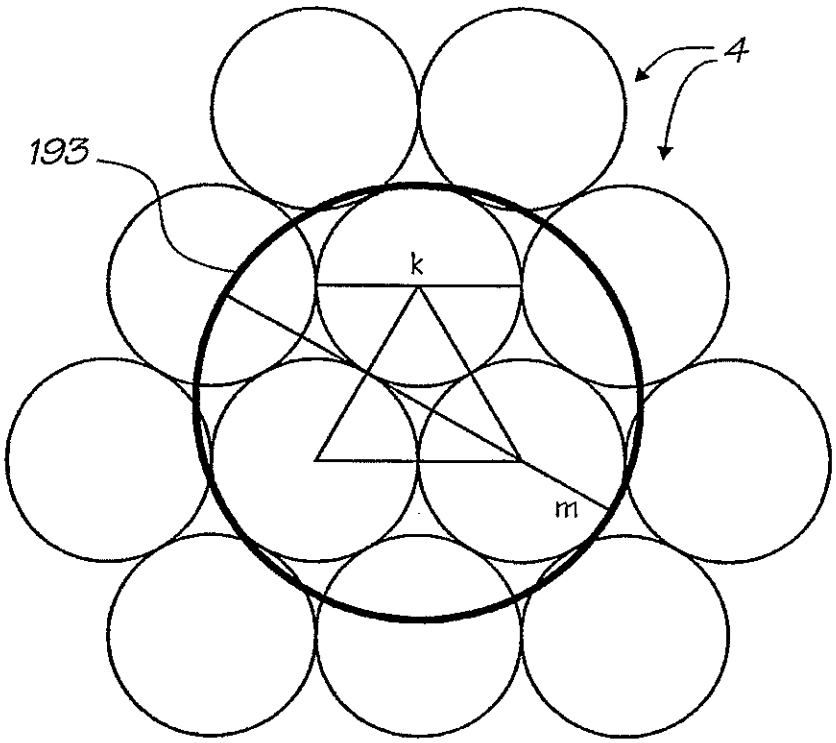


FIG. 52

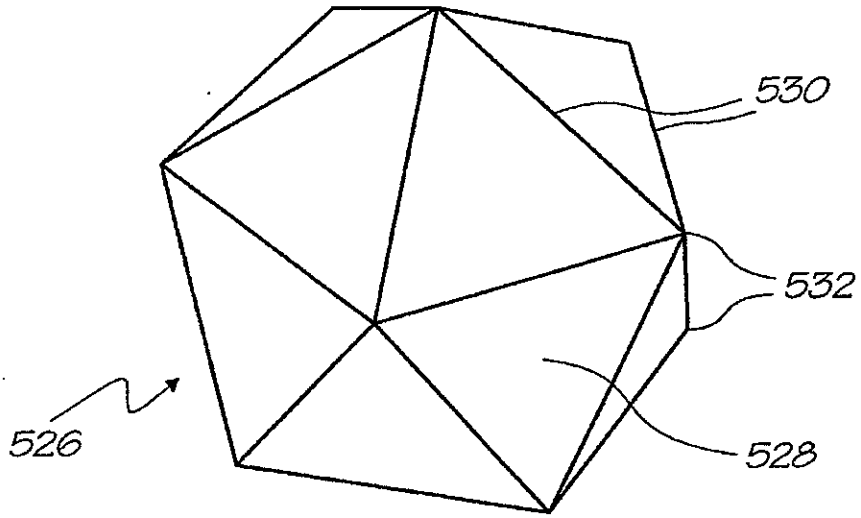


FIG. 53

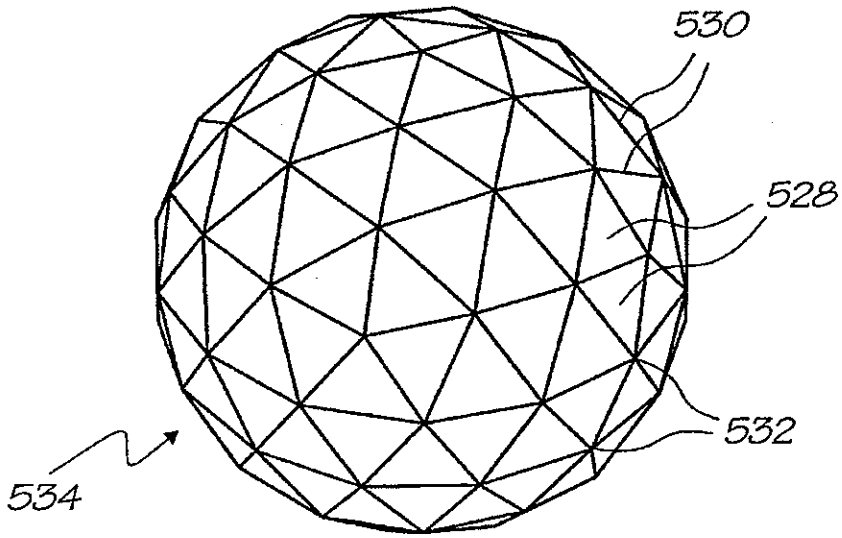


FIG. 54

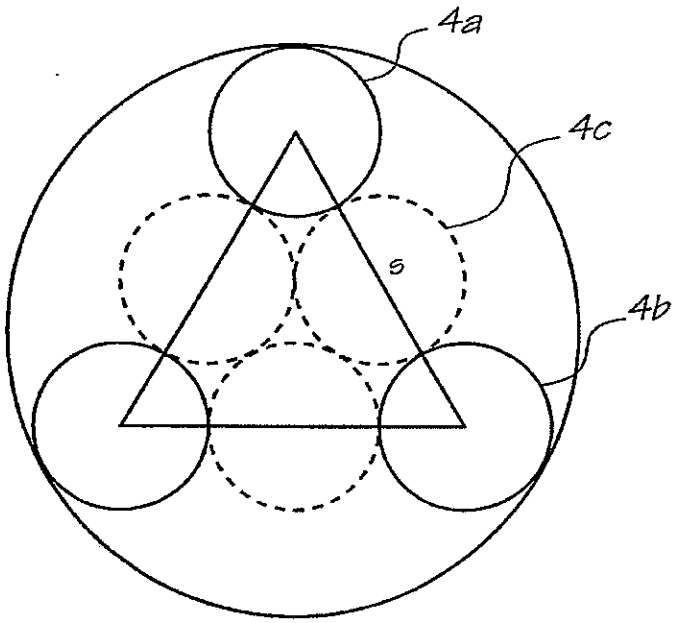


FIG. 55

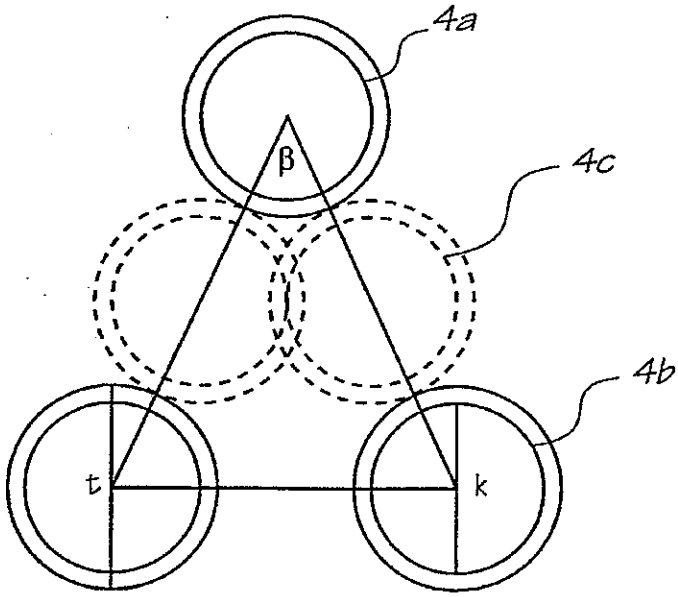


FIG. 56

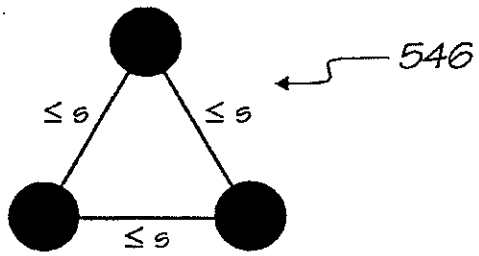


FIG. 57

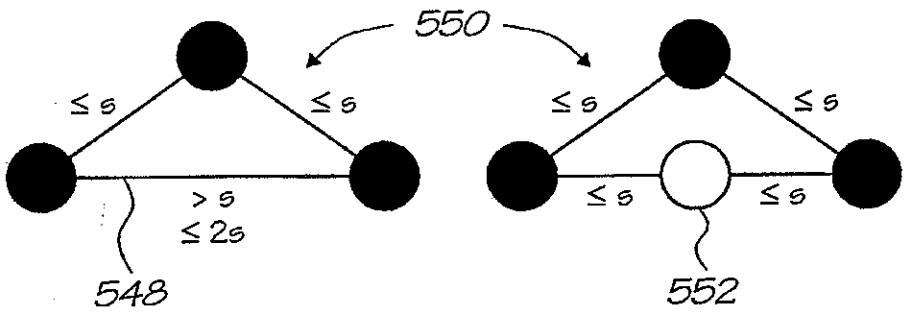


FIG. 58

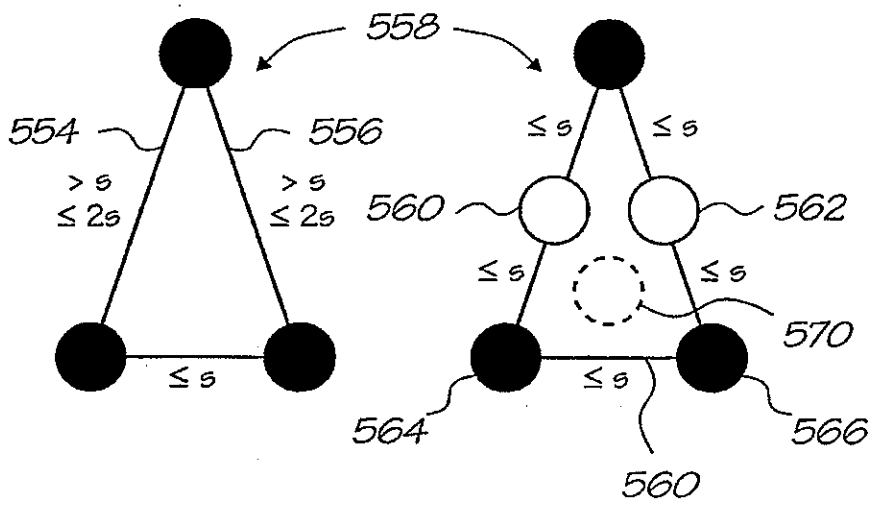

558
554
556
> s
≤ 2s
> s
≤ 2s
≤ s
≤ s
≤ s
560
562
≤ s
≤ s
570
≤ s
564
566
560

FIG. 59

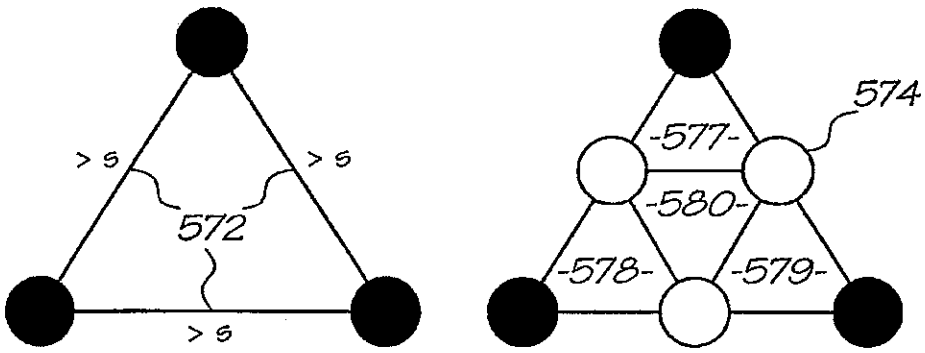

> s
> s
572
> s
574
-577-
-580-
-578-
-579-

FIG. 60

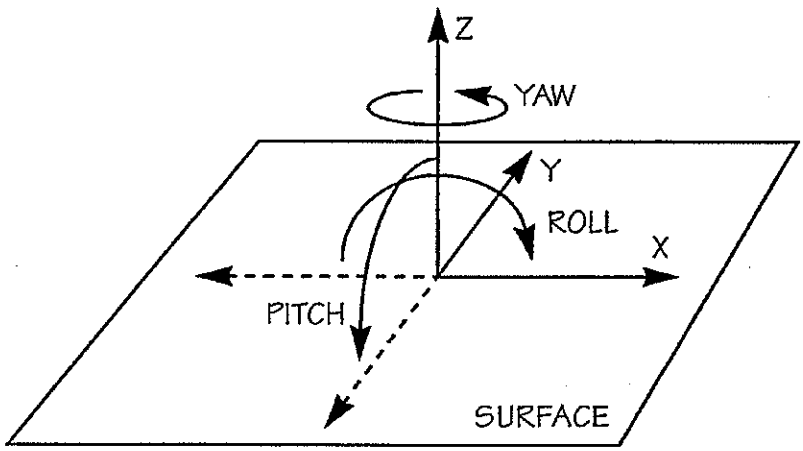


FIG. 61

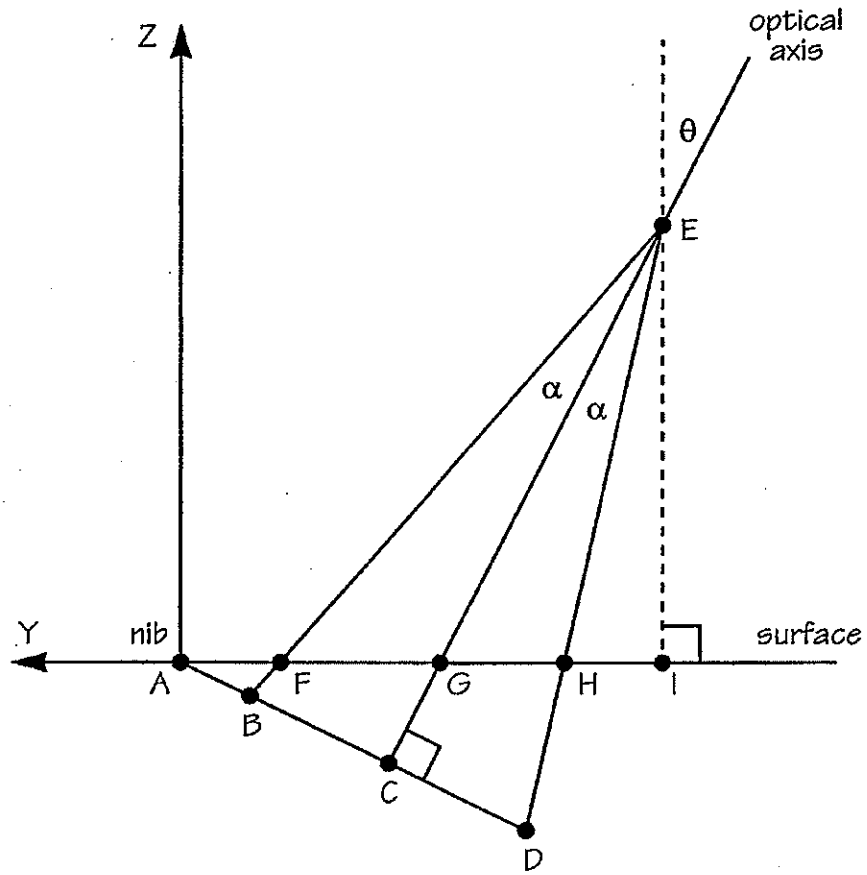


FIG. 62

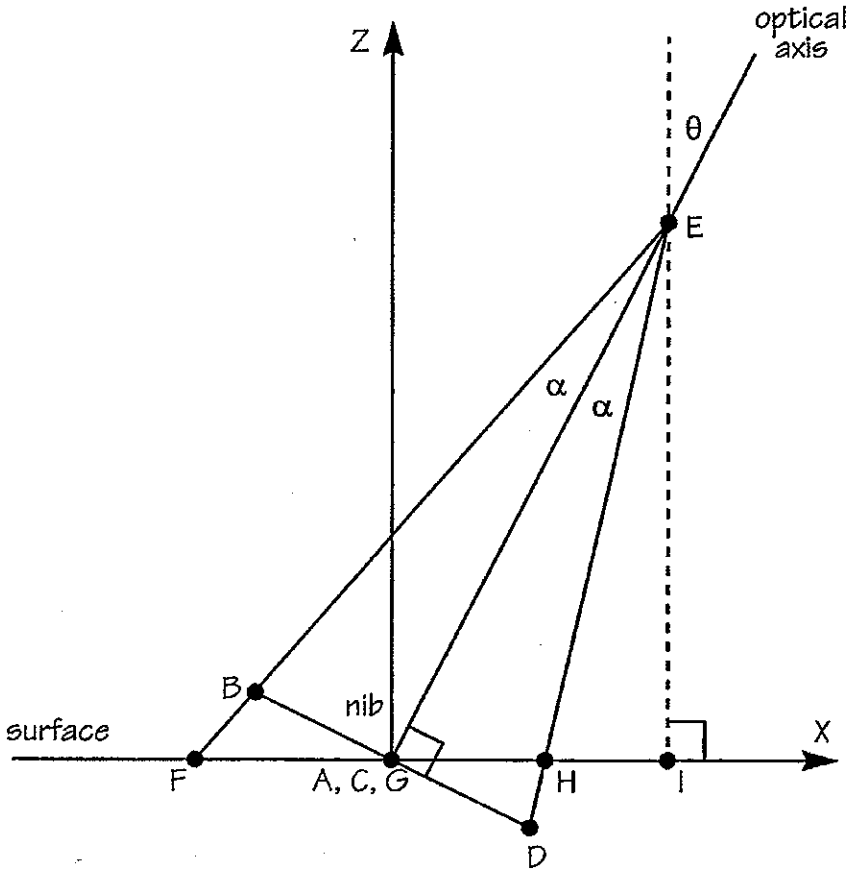


FIG. 63

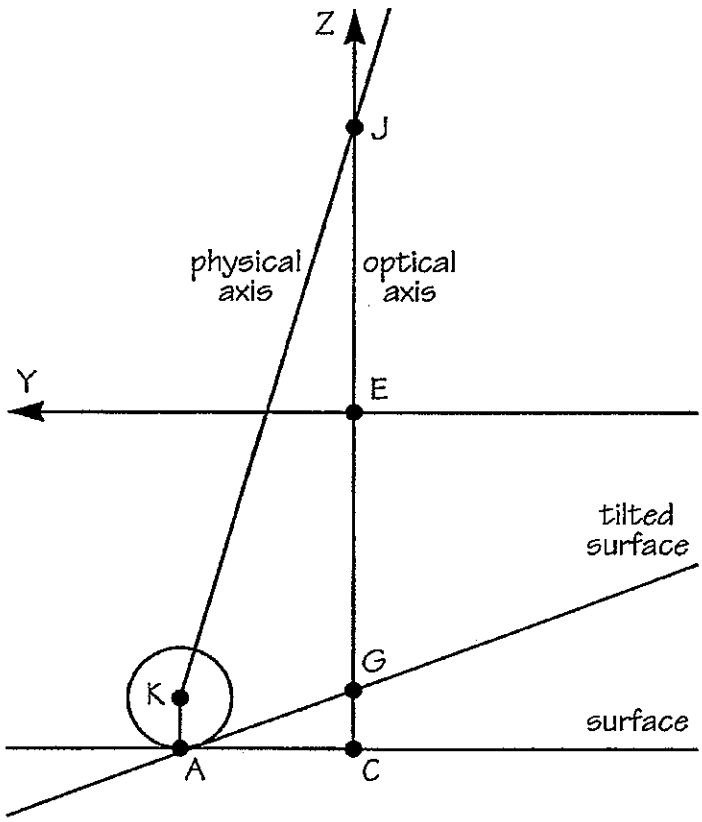

Z
J
physical axis
optical axis
Y
E
tilted surface
G
K
surface
A
C

FIG. 64

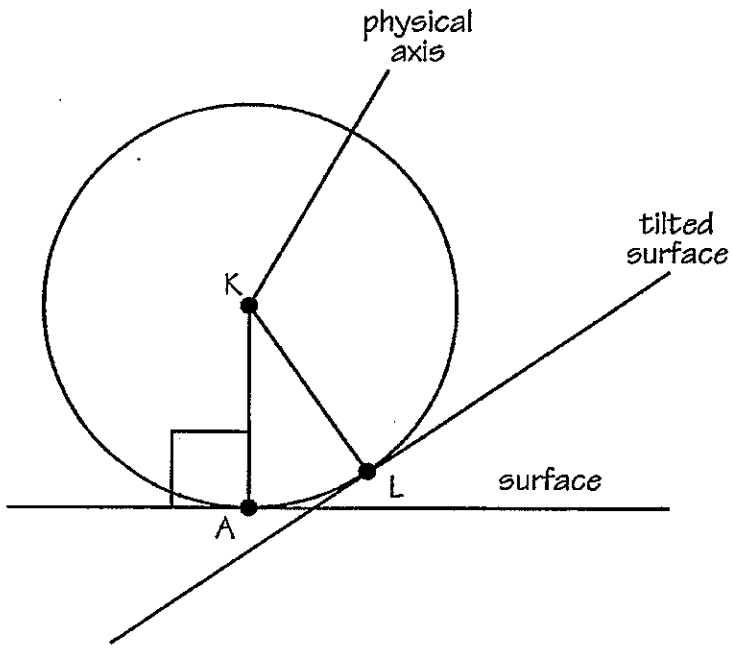

physical
axis
tilted
surface
K
L
surface
A

FIG. 65

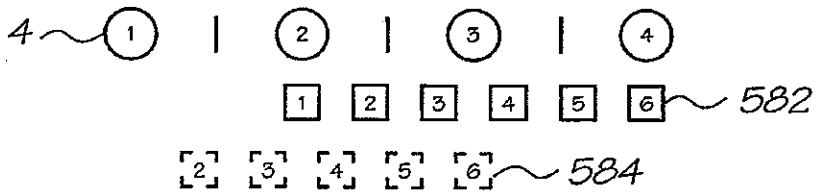

4
1
2
3
4
1
2
3
4
5
6
582
2
3
4
5
6
584

FIG. 66

EQ NUMBER	EQUATION
1	$m = k\left(\frac{2}{\sqrt{3}} + 1\right)$
2	$m = \frac{2s}{\sqrt{3}} + k$
3	$u = k\left(\frac{2}{\sqrt{3}} - 1\right)$
4	$m = \frac{7k}{3}$
5	$\theta = 2\arcsin\left(\frac{2}{\sqrt{10+2\sqrt{5}}}\right) \cong 63.4° \cong 1.11$ radians
6	$n = 10v^2 + 2 = 10\left\lfloor \frac{\theta r}{K} \right\rfloor^2 + 2$
7	$r \le \frac{K}{\theta}\left\lfloor \sqrt{\frac{n-2}{10}} \right\rfloor$
8	$s \ge 2k$
9	$\beta = 2\arcsin\frac{k}{2t}$
10	$s \ge 2t$

FIG. 67

EQ NUMBER	EQUATION
11	$\sin\theta = \cos\phi\cos\psi$
12	$2D\tan\alpha$
13	$S = T + D\tan\alpha$
14	$d = D - S\tan\theta$
15	$d\cos\theta(\tan(\theta+\alpha) - \tan(\theta-\alpha))$
16	$d\cos\theta(\tan(\theta+\alpha) - \tan(\theta-\alpha)) \geq m$
17	$\frac{d\cos\theta}{\cos(\lvert\theta\rvert+\alpha)}$
18	$\cos(\lvert\theta\rvert+\alpha)$
19	$\omega = \frac{fd\cos\theta}{\cos^2(\lvert\theta\rvert+\alpha)}$
20	$\omega_0 = \frac{fD}{\cos^2\alpha}$

FIG. 68

EQ NUMBER	EQUATION
21	$q = 2\alpha' n\omega \cos^2(\lvert\theta'\rvert + \alpha')$
22	$q = \frac{2\alpha' nfd\cos\theta\cos^2(\lvert\theta'\rvert + \alpha')}{\cos^2(\lvert\theta\rvert + \alpha)}$
23	$q = \frac{2\alpha nfd\cos\theta\cos^2\alpha}{\cos^2(\lvert\theta\rvert + \alpha)}$
24	$q = \frac{2\alpha nfd}{\cos\theta(1 - \tan\lvert\theta\rvert\tan\alpha)^2}$
25	$q_0 = 2\alpha nfD$
26	$q = 2\alpha nfd\cos\theta$

FIG. 69

EQ NUMBER	EQUATION
27	$P_{sensed} = \begin{pmatrix} 0 \\ 0 \\ -d \end{pmatrix}$
28	$P_{pivot} = \begin{pmatrix} 0 \\ S \\ R-D \end{pmatrix}$
29	$\vec{V}_{pivot-sensed} = P_{pivot} - P_{sensed} = \begin{bmatrix} 0 \\ S \\ R-D+d \end{bmatrix}$
30	$\vec{N} = M \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$
31	$\vec{V}_{contact-pivot} = R\frac{\vec{N}}{\left\|\vec{N}\right\|}$
32	$\vec{V}_{contact-sensed} = \vec{V}_{contact-pivot} - \vec{V}_{pivot-sensed}$
33	$\vec{V}_{contact-tag} = \vec{V}_{sensed-tag} + M^{-1}\vec{V}_{contact-sensed}$
34	$P_{contact} = P_{tag} + \vec{V}_{contact-tag}$

FIG. 70

EQ NUMBER	EQUATION
35	$\gamma_{pen} = -\gamma_{tag}$
36	$\phi_{pen} = -\phi_{tag} - \phi_{sensor}$
37	$\psi_{pen} = -\psi_{tag}$
38	$\lambda_S < \frac{\lambda_T}{2}$
39	$P_{i+1} - P_i = \lambda_S$
40	$\left\|\lambda_T - (P_{i+1} - P_i)\right\| = \lambda_S$
41	$\lambda_S = \frac{\lambda_T}{2}$

FIG. 71

EQ NUM	EQUATION
42	$M_1 = T_{xy} = \begin{bmatrix} 1 & 0 & 0 & A \\ 0 & 1 & 0 & B \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
43	$R_z = \begin{bmatrix} C & -D & 0 & 0 \\ D & C & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
44	$M_2 = R_z M_1 = \begin{bmatrix} C & -D & 0 & AC-BD \\ D & C & 0 & AD+BC \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

FIG. 72

EQ NUM	EQUATION
45	$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & E & -F & 0 \\ 0 & F & E & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
46	$M_3 = R_x M_2 = \begin{bmatrix} C & -D & 0 & AC-BD \\ DE & CE & -F & ADE+BCE \\ DF & CF & E & ADF+BCF \\ 0 & 0 & 0 & 1 \end{bmatrix}$
47	$R_y = \begin{bmatrix} G & 0 & H & 0 \\ 0 & 1 & 0 & 0 \\ -H & 0 & G & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

FIG. 73

EQ NUM	EQUATION
48	$M_4 = R_y M_3 = \begin{bmatrix} CG + DFH & CFH - DG & EH & GK + HL \\ DE & CE & -F & ADE + BCE \\ DFG - CH & DH + CFG & EG & GL - HK \\ 0 & 0 & 0 & 1 \end{bmatrix}$
49	$K = AC - BD$
50	$L = ADF + BCF$
51	$T_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & I \\ 0 & 0 & 0 & 1 \end{bmatrix}$

FIG. 74

EQ NUM	EQUATION
52	$M_5 = T_z M_4 = \begin{bmatrix} CG+DFH & CFH-DG & EH & GK+HL \\ DE & CE & -F & ADE+BCE \\ DFG-CH & DH+CFG & EG & GL-HK+I \\ 0 & 0 & 0 & 1 \end{bmatrix}$
53	$M_p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & J & 1 \end{bmatrix}$
54	$M_6 = M_p M_5 = \begin{bmatrix} CG+DFH & CFH-DG & EH & GK+HL \\ DE & CE & -F & ADE+BCE \\ 0 & 0 & 0 & 0 \\ J(DFG-CH) & J(DH+CFG) & EGJ & J(GL-HK+I)+1 \end{bmatrix}$

FIG. 75

EQ NUM	EQUATION
55	$M_s = \begin{bmatrix} S & 0 & 0 & 0 \\ 0 & S & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
56	$M_7 = M_s M_6 = \begin{bmatrix} S(CG+DFH) & S(CFH-DG) & SEH & S(GK+HL) \\ SDE & SCE & -SF & S(ADE+BCE) \\ 0 & 0 & 0 & 0 \\ J(DFG-CH) & J(DH+CFG) & EGJ & J(GL-HK+I)+1 \end{bmatrix}$
57	$P_1 = M_7 \begin{bmatrix} x \\ y \\ 0 \\ 1 \end{bmatrix}$

FIG. 76

EQ NUM	EQUATION
58	$P_1 = \begin{bmatrix} S(CG + DFH)x + S(CFH - DG)y + S(GK + HL) \\ SDEx + SCEy + S(ADE + BCE) \\ 0 \\ J(DFG - CH)x + J(DH + CFG)y + J(GL - HK + I) + 1 \end{bmatrix}$
59	$P_1 = \begin{bmatrix} S(CG + DFH)x + S(CFH - DG)y + S(ACG - BDG + ADFH + BCFH) \\ SDEx + SCEy + S(ADE + BCE) \\ 0 \\ J(DFG - CH)x + J(DH + CFG)y + J(BDH - ACH + ADFG + BCFG + I) + 1 \end{bmatrix}$

FIG. 77

EQ NUMBER	EQUATION
60	$M_{p2Dinf} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}$
61	$M_{p2D} = \begin{bmatrix} ai & bi & ci \\ di & ei & fi \\ gi & hi & i \end{bmatrix}$
62	$P_2 = M_{per2D} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$
63	$P_2 = \begin{bmatrix} aix + biy + ci \\ dix + eiy + fi \\ gix + hiy + i \end{bmatrix}$

FIG. 78

EQ NUMBER	EQUATION
64	$\frac{ai}{S} = CG + DFH$
65	$\frac{bi}{S} = CFH - DG$
66	$\frac{ci}{S} = ACG - BDG + ADFH + BCFH$
67	$\frac{di}{S} = DE$
68	$\frac{ei}{S} = CE$
69	$\frac{fi}{S} = ADE + BCE$
70	$gi = J(DFG - CH)$
71	$hi = J(DH + CFG)$
72	$i = J(BDH - ACH + ADFG + BCFG + I) + 1$
73	$\sin^2\theta + \cos^2\theta = 1$
74	$\theta = \arctan(\sin\theta, \cos\theta)$

FIG. 79

EQ NUMBER	EQUATION
75	$\frac{ci}{S} = A(CG + DFH) + B(CFH - DG)$
76	$c = Aa + Bb$
77	$\frac{fi}{S} = A(DE) + B(CE)$
78	$f = Ad + Be$
79	$i = AJ(DFG - CH) + BJ(DH + CFG) + IJ + 1$
80	$i = Agi + Bhi + IJ + 1$
81	$A = \frac{c - Bb}{a}$
82	$A = \frac{f - Be}{d}$
83	$\frac{c - Bb}{a} = \frac{f - Be}{d}$
84	$cd - Bbd = af - Bae$
85	$B(ae - bd) = af - cd$
86	$B = \frac{af - cd}{ae - bd}$

FIG. 80

EQ NUMBER	EQUATION
87	$A = \frac{f-\left(\frac{af-cd}{ae-bd}\right)e}{d}$
88	$A = \frac{f(ae-bd)-e(af-cd)}{d(ae-bd)}$
89	$A = \frac{aef-bdf-aef+cde}{d(ae-bd)}$
90	$A = \frac{-bdf+cde}{d(ae-bd)}$
91	$A = \frac{-bf+ce}{ae-bd}$
92	$C = \frac{ei}{ES}$
93	$D = \frac{di}{ES}$
94	$a = \frac{e}{E}G + \frac{d}{E}FH$
95	$b = \frac{-d}{E}G + \frac{e}{E}FH$
96	$g = \frac{-e}{ES}H + \frac{d}{ES}FG$
97	$h = \frac{d}{ES}H + \frac{e}{ES}FG$
98	$E = \frac{eG+dFH}{a}$
99	$E = \frac{-dG+eFH}{b}$

FIG. 81

EQ NUMBER	EQUATION
100	$ES = \frac{-eH + dFG}{g}$
101	$ES = \frac{dH + eFG}{h}$
102	$\frac{eG + dFH}{a} = \frac{-dG + eFH}{b}$
103	$\frac{FH}{G} = \frac{ad + be}{ae - bd}$
104	$\frac{-eH + dFG}{g} = \frac{dH + eFG}{h}$
105	$\frac{FG}{H} = \frac{dg + eh}{dh - eg}$
106	$\frac{FH}{G} \times \frac{FG}{H} = \frac{(ad + be)(dg + eh)}{(ae - bd)(dh - eg)} = F^2$
107	$F = \pm\sqrt{\frac{(ad + be)(dg + eh)}{(ae - bd)(dh - eg)}}$
108	$E = \pm\sqrt{1 - F^2}$
109	$\phi = \arctan(E, F)$
110	$\frac{1}{F} \times \frac{FH}{G} = \frac{H}{G} = \left(\pm\sqrt{\frac{(ae - bd)(dh - eg)}{(ad + be)(dg + eh)}}\right) \times \frac{(ad + be)}{(ae - bd)}$ $= \pm\sqrt{\frac{(ad + be)(dh - eg)}{(ae - bd)(dg + eh)}}$
111	$\frac{H}{G} = \frac{H}{\pm\sqrt{1 - H^2}} = \frac{1}{\pm\sqrt{\frac{1}{H^2} - 1}}$

FIG. 82

EQ NUMBER	EQUATION
112	$H = \frac{1}{\pm\sqrt{\frac{1}{\left(\frac{H}{G}\right)^2}+1}}$
113	$G = \pm\sqrt{1-H^2}$
114	$\psi = \arctan(G, H)$
115	$\left(\frac{ei}{ES}\right)^2 + \left(\frac{di}{ES}\right)^2 = 1$
116	$\frac{S}{i} = \frac{\pm\sqrt{d^2+e^2}}{E}$
117	$C = \frac{e}{E} \times \frac{E}{\pm\sqrt{d^2+e^2}}$
118	$C = \frac{e}{\pm\sqrt{d^2+e^2}}$
119	$D = \frac{d}{E} \times \frac{E}{\pm\sqrt{d^2+e^2}}$
120	$D = \frac{d}{\pm\sqrt{d^2+e^2}}$

FIG. 83

EQ NUMBER	EQUATION
121	$i(1 - Ag - Bh) = IJ + 1$
122	$\text{sign}(i) = -\text{sign}(1 - Ag - Bh)$
123	$\gamma = \arctan(D, C)$
124	$S = \frac{di}{DE}$
125	$S = \frac{ei}{CE}$
126	$\text{sign}(FH) = \text{sign}\left(\frac{ad + be}{ae - bd}\right)$
127	$J = \left\lvert \frac{gi}{-CH + \text{sign}(FH)DFG} \right\rvert$
128	$J = \left\lvert \frac{hi}{DH + \text{sign}(FH)CFG} \right\rvert$
129	$I = \frac{(i - Agi - Bhi - 1)}{J}$

FIG. 84

case	C,D	E,F	G,H	ai	bi	ci	di	ei	fi	gi	hi
1a	±1,0	1,0	1,0	±S	0	±AS	0	±S	±BS	0	0
1b	0,±1	1,0	1,0	0	±(-S)	±(-BS)	±S	0	±AS	0	0
1c	C,D	1,0	1,0	CS	-DS	Aai+Bbi	DS	CS	BS	0	0
2a	±1,0	E,F	1,0	±S	0	±AS	0	±ES	Bei	0	±FJ
2b	0,±1	E,F	1,0	0	±(-S)	±(-BS)	±ES	0	Adi	±FJ	0
2c	C,D	E,F	1,0	CS	-DS	Aai+Bbi	DES	CES	Adi+Bei	DFJ	CFJ
3a	±1,0	1,0	G,H	±GS	0	Aai	0	±S	±BS	±(-HJ)	0
3b	0,±1	1,0	G,H	0	±(-GS)	Bbi	±S	0	±AS	0	±HJ
3c	C,D	1,0	G,H	CGS	-DGS	Aai+Bbi	DS	CS	Adi+Bei	-CHJ	DHJ
4a	±1,0	E,F	G,H	±GS	±FHS	Aai+Bbi	0	±ES	Bei	±(-HJ)	±FGJ
4b	0,±1	E,F	G,H	±FHS	±(-GS)	Aai+Bbi	±ES	0	Adi	±FGJ	±HJ
4c	C,D	E,F	G,H	CGS+ DFHS	-DGS+ CFHS	Aai+Bbi	DES	CES	Adi+Bei	-CHJ+ DFGJ	DHJ+ CFGJ

FIG. 85

description	case	condition	handling
zero pitch & zero roll	1	$g = h = 0$	$E \leftarrow 1$ $F \leftarrow 0$ $G \leftarrow 1$ $H \leftarrow 0$
zero roll	2a	$b = d = g = 0$	$E \leftarrow \frac{e}{a}$ $\frac{FJ}{S} = \frac{h}{a}$
	2b	$a = e = h = 0$	$E \leftarrow \frac{-d}{b}$ $\frac{FJ}{S} = \frac{-g}{b}$
	2c	$\frac{a}{b} = \frac{-h}{g}$	handle via 2a or 2b
	2		$G \leftarrow 1$ $H \leftarrow 0$ $F \leftarrow \text{sign}\left(\frac{FJ}{S}\right)\sqrt{1-E^2}$

FIG. 86

description	case	condition	handling
zero pitch	3a	$b = d = h = 0$	$G \leftarrow \frac{a}{e}$ $\frac{HJ}{S} = \frac{-g}{e}$
	3b	$a = e = g = 0$	$G \leftarrow \frac{-b}{d}$ $\frac{HJ}{S} = \frac{h}{d}$
	3c	$\frac{a}{b} = \frac{g}{h}$	handle via 3a or 3b
	3		$E \leftarrow 1$ $F \leftarrow 0$ $H = \text{sign}\left(\frac{HJ}{S}\right)\sqrt{1-G^2}$
non-zero pitch & non-zero roll	4	$(g \neq 0) \wedge (h \neq 0)$	handle via general solution

FIG. 87