

**COMBINED DIGITAL-ANALOG TWO-BEAM VIBRATING
ACCELEROMETER HAVING LINEAR OUTPUT ACROSS COMPLETE
RANGE**

FIELD OF THE INVENTION

This invention relates to Vibrating Beam Accelerometers.

PRIOR ART

Prior art references considered to be relevant as a background to the invention are listed below and their contents are incorporated herein by reference. Acknowledgement of the references herein is not to be inferred as meaning that these are in any way relevant to the patentability of the invention disclosed herein. Each reference is identified by a number enclosed in square brackets and accordingly the prior art will be referred to throughout the specification by numbers enclosed in square brackets.

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10 BACKGROUND OF THE INVENTION

Vibrating Beam Accelerometers are based on the fact that the resonant frequency of a vibrating beam attached to a seismic-mass changes when subjected to an external acceleration. The beam that is pulled vibrates at a higher frequency while the beam that is pushed vibrates at a lower frequency. This action is similar to that of a violin string, which increases pitch with an increase in tension, and decreases pitch with a decrease in tension (or relative compression). Typically, two vibrating beams are used in a differential arrangement in order to reduce common parasitic sensitivities e.g. temperature, pressure or aging. The two beams operate in a push-pull mode whereby when one beam is pushed and is subjected to compression, the other beam is pulled and subjected to tension. During operation, the two beams constantly change roles and, as is described in the literature, for each beam there is a measured vibrating frequency and it can be shown that they vibrate at two frequencies: one slightly higher corresponding to pull-mode and the other slightly lower corresponding to push-mode.

The beams are commonly based on piezoelectric material such as quartz that generates an electric charge in response to mechanical stress. When the beams are compressed or stretched, they produce an electric voltage across their surfaces. Conversely, an electric field applied across the beams will induce vibration. To this end, an AC voltage is commonly applied across a respective pair of electrodes deposited on opposing surfaces of each beam, thereby inducing a steady-state vibration of the beams whose amplitudes are measured as analog signals across respective electrodes deposited on opposing surfaces of both beams. The two beams vibrate at respective frequencies whose difference may be used to measure acceleration as is well-known in the literature.

However, the use of two beams coupled by a seismic mass may lead to the presence of a “blind zone” where the sensor does not produce a useful output in the presence of an external acceleration. It is widely believed that this blind zone is caused by the “lock-in” phenomenon whereby the resonant frequency of both beams becomes
5 “locked” (i.e. both beams vibrate with the same frequency) and therefore the output acceleration (which is proportional to the frequency difference) becomes zero even in the presence of an external acceleration. This effect limits the range of accelerations for which the sensor can be used.

Our co-pending IL 315720 filed September 18, 2024 discloses that a principal
10 cause of lock-in is that conventional approaches to measuring the vibration frequency of the beams measure only the dominant frequency, i.e. the frequency whose amplitude is higher and, as a result, ignore the less dominant frequency whose amplitude is lower. Throughout the present disclosure, we will refer to these frequencies as the dominant and sub-dominant frequencies. In contrast, IL 315720 detects both the dominant and sub-
15 dominant frequencies, which, when properly taken into account, can determine the lock-in boundary within which the amplitudes of the dominant and sub-dominant frequencies cross over. The frequencies within the lock-in boundary can then be selected to avoid the effect of lock-in. Lock-in is avoided by measuring also the sub-dominant frequencies in the lock-in zone and computing acceleration in the lock-in zone as a function of the
20 difference between the dominant frequency of one beam and the sub-dominant frequency of the other beam. It is found that below a specific acceleration threshold, the relationship between frequency difference and acceleration is non-linear.

Specifically, for each of the beams there are corresponding first and second frequencies (f_- , f_+) at which the respective amplitude is dominant. The lock-in zone is bound
25 by respective crossover points for each beam where the respective amplitudes (A_- , A_+) at both the first and second frequencies (f_- , f_+) are the same and which define a crossover region within which the respective dominant amplitudes of the first and second frequencies cross over such that the frequency for which amplitude is higher outside the crossover region has a smaller amplitude within the crossover region and *vice versa*.

30 An alternative approach is taken in our co-pending IL 318446 filed January 15, 2025, which avoids the problem of lock-in by computing acceleration as a function not of vibration frequency but rather of vibration amplitude, which is measured directly for each beam as an analog signal. It is shown that acceleration may be derived as a linear

function of amplitude across the full range of acceleration. To the extent that the accelerometer must be responsive across the complete range, the ideal linear function involves measurement of both the dominant and sub-dominant amplitudes for each beam. This requires measurement of the vibration amplitudes (in dimensions of length) at both the
5 dominant and sub-dominant frequencies. However and highly significantly, within the lock-in zone an alternative function may be based on measurement of only the dominant or only the sub-dominant amplitudes. The alternative function also provides good linearity inside the lock-in zone albeit not quite as accurate as the ideal function.

Summarizing the combined teachings of IL 315720 and IL 318446, we can state
10 as follows. IL 315720 discloses an accelerometer having a linear characteristic outside of the lock-in zone, wherein acceleration is a linear function of the difference between the measured vibration frequencies of the two beams. But further refinement is required inside the lock-in zone where the dominant frequencies of the two beams are identical. On the other hand, IL 318446 teaches an accelerometer having a linear characteristic
15 inside the lock-in zone, wherein acceleration is a linear function of the measured vibration amplitudes of the two beams.

For the sake of abundant clarity, it is noted that although reference is made to our co-pending applications IL 315720 and IL 318446, neither application nor their subject-matter has been published prior to the effective filing date of the present application.
20 Therefore, while the foregoing discussion relating to lock-in and dominant and sub-dominant frequencies provides important background for the present disclosure, it is not prior art.

SUMMARY OF THE INVENTION

In accordance with a broad aspect of the present invention, there is provided a
25 method for determining acceleration using a vibrating beam accelerometer, wherein outside the lock-in zone, acceleration is derived as a function of the difference between the measured vibration frequencies of the two beams and inside the lock-in zone, acceleration is derived as a function of the measured vibration amplitudes of the two beams. In some embodiments, the measured frequencies and amplitudes for both cases
30 correspond to the dominant frequencies and amplitudes within the respective zones, i.e. frequency outside the lock-in zone and amplitude inside the lock-in zone. Consequently,

provided we know the boundary of the lock-in zone and can thereby determine for any measured acceleration whether we are inside or outside the lock-in zone, we can apply the appropriate transformation based on a direct measurement of frequency difference or amplitude. This avoids the needs for the supplementary processing required in IL 315720
5 and IL 318446 in order to measure sub-dominant frequencies and amplitudes.

BRIEF DESCRIPTION OF THE DRAWINGS

In order to understand the invention and to see how it may be carried out in practice, embodiments will now be described, by way of non-limiting example only, with reference to the accompanying drawings, in which:

- 10 **Fig. 1** shows schematically a simple differential vibrating beam accelerometer;
 Fig. 2 is the equivalent spring-mass model of the device shown in Fig. 1;
 Fig. 3 is a graphical representation of the amplitude-acceleration characteristic for the device as derived using the model of Fig. 2;
 Figs. 4 and 5 show graphically amplitude-acceleration characteristics of the device
15 shown in Fig. 1 both inside and outside the lock-in zone as functions of the ratio of vibration amplitudes for different combinations of dominant frequency amplitudes;
 Fig. 6 shows graphically amplitude-acceleration curves for the device as derived using the model of Fig. 2 according to three different embodiments of the invention;
 Figs. 7, 8 and 9 show graphically the deviation from linearity for each of the three
20 curves in Fig. 6;
 Figs. 10 and 11 show the dominant and sub-dominant frequencies of Beam 1 and Beam 2, respectively both of which may be measured according to different embodiments of the invention;
 Fig. 12 is a graphical representation of the frequency-acceleration characteristic
25 for the device as derived using the model of Fig. 2;
 Fig. 13 shows graphically the relationship between the vibration amplitudes of both beams inside and outside of the lock-in zone;
 Fig. 14 is a graphical representation showing corrected frequency-difference ($\Delta f(t)$) versus acceleration using an algorithm according to an embodiment of the
30 invention; and

Figs. 15 and 16 are flow charts showing alternative methods for determining the boundary of the lock-in zone during calibration of the accelerometer.

DETAILED DESCRIPTION OF EMBODIMENTS

A proper understanding of the invention requires a detailed analysis of the vibrating beam accelerometer, much of which will be familiar to those skilled in the art. For the sake of completeness, we present a model of a vibrating beam accelerometer and derive equations that define the boundaries of the lock-in zone.

Fig. 1 shows schematically a simple vibrating beam accelerometer that can be analyzed mathematically using the equivalent spring-mass model shown in Fig. 2. For illustrative purposes and sake of simplicity, in the following explanation the model shown in Fig 2 considers the case of equal beams, it being understood that the invention defined by the appended claims encompasses any practical implementation of the vibrating beam accelerometer that can be reduced to the same or similar mathematical model including the case where the beams are not equal. Thus, Fig. 1 shows an accelerometer 10 comprising a pair of collinear or parallel beams 11, 11' each anchored at one end to a respective support 12, 12' and commonly coupled at their opposite ends to a seismic mass 13. The beams 11, 11' may be formed of piezoelectric material such as quartz that generates an electric charge in response to mechanical stress. When the beams are compressed or stretched, they produce an electric voltage across their surfaces. Conversely, an electric field applied across the beams will induce vibration. To this end, an AC voltage 14 is commonly applied across a respective pair of electrodes 15, 15' deposited on opposing surfaces of each beam, thereby inducing a steady-state vibration of the beams whose amplitudes are measured as analog signals across respective electrodes 16, 16' deposited on opposing surfaces of both beams.

When the structure is subjected to an external acceleration, a force component parallel to the beams' longitudinal axes shifts the seismic mass axially to and fro, thereby pushing one of the beams while pulling the other, thereby giving rise to slight differences in their respective resonant frequencies. Specifically, when using conventional methods for measuring the frequency, the shorter compressed beam will vibrate at a lower frequency, which we shall denote as f_- compared to the longer stretched beam, whose resonant frequency f_+ is higher. In a practical implementation, the beams are used in flexural mode as sensitive elements, and vibration is maintained by means of an oscillator

circuit denoted schematically by the AC voltage 14. For so long as the beams are subjected to axial vibration, they will continually elongate and foreshorten in anti-phase, and their resonant frequencies will likewise decrease and increase in anti-phase.

The applied acceleration induces tensile or compressive stresses into each beam which modify the resonant frequency, measurement of which is thus indicative of the applied acceleration.

The equations of motion that describe the dynamics of the model of Fig. 2 are:

$$\begin{aligned} m_0 \ddot{x}_1 + k^+(x_1 - X) &= m_0 \Gamma \\ M \ddot{X} + k^+(X - x_1) + k^-(X - x_2) &= M \Gamma \\ m_0 \ddot{x}_2 + k^-(x_2 - X) &= m_0 \Gamma \end{aligned} \quad (1)$$

where Γ is the external applied acceleration.

$$\begin{aligned} k^+ &= k_0(1 + \lambda \Gamma) \\ k^- &= k_0(1 - \lambda \Gamma) \end{aligned} \quad (2)$$

It is clear from Eq. (2) that k^+ and k^- are functions of Γ . However, in all the following equations, we shall omit the dependency $k^+(\Gamma)$ and $k^-(\Gamma)$ on Γ in order to avoid the equations becoming unwieldy and refer simply to k^+ and k^- .

Dividing by m_0 and M , respectively, leads to:

$$\begin{aligned} \ddot{x}_1 + \frac{k^+}{m_0}(x_1 - X) &= \Gamma \\ \ddot{X} + \frac{k^+}{M}(X - x_1) + \frac{k^-}{M}(X - x_2) &= \Gamma \\ \ddot{x}_2 + \frac{k^-}{m_0}(x_2 - X) &= \Gamma \end{aligned} \quad (3)$$

Subtracting the first and third equations from the second leads to:

$$\begin{aligned} \ddot{X} - \ddot{x}_1 + \left(\frac{k^+}{M} + \frac{k^+}{m_0} \right) (X - x_1) + \frac{k^-}{M} (X - x_2) &= 0 \\ \ddot{X} - \ddot{x}_2 + \frac{k^+}{M} (X - x_1) + \left(\frac{k^-}{M} + \frac{k^-}{m_0} \right) (X - x_2) &= 0 \end{aligned} \quad (4)$$

We define new variables y_1 and y_2 as follows:

$$\begin{aligned} y_1 &\equiv X - x_1 \\ y_2 &\equiv X - x_2 \end{aligned}$$

Substituting in Eq. (4) yields:

$$\begin{aligned}
\ddot{y}_1 + \left(\frac{k^+}{M} + \frac{k^+}{m_0}\right)y_1 + \frac{k^-}{M}y_2 &= 0 \\
\ddot{y}_2 + \frac{k^+}{M}y_1 + \left(\frac{k^-}{M} + \frac{k^-}{m_0}\right)y_2 &= 0
\end{aligned} \tag{5}$$

We define new variables y_3 and y_4 as follows:

$$\begin{aligned}
y_3 &\equiv \dot{y}_1 \\
y_4 &\equiv \dot{y}_2
\end{aligned} \tag{6}$$

Substituting in Eq. (5) gives:

$$\begin{aligned}
\dot{y}_1 &= y_3 \\
\dot{y}_2 &= y_4 \\
\dot{y}_3 &= -\left(\frac{k^+}{M} + \frac{k^+}{m_0}\right)y_1 - \frac{k^-}{M}y_2 \\
\dot{y}_4 &= -\frac{k^+}{M}y_1 - \left(\frac{k^-}{M} + \frac{k^-}{m_0}\right)y_2
\end{aligned} \tag{7}$$

Equation (7) can be more conveniently written in matrix form:

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\left(\frac{k^+}{M} + \frac{k^+}{m_0}\right) & -\frac{k^-}{M} & 0 & 0 \\ -\frac{k^+}{M} & -\left(\frac{k^-}{M} + \frac{k^-}{m_0}\right) & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} \tag{8}$$

5

We define:

$$M_0 \equiv \frac{m_0 M}{M + m_0}$$

We can re-write Eq. (8) as follows:

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k^+}{M_0} & -\frac{k^-}{M} & 0 & 0 \\ -\frac{k^+}{M} & -\frac{k^-}{M_0} & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} \tag{9}$$

Eq. (9) has a solution of the form:

$$\begin{aligned}
y_1(t) &= 2c_1 \cos \omega_+ t - 2e_1 \sin \omega_+ t + 2c_2 \cos \omega_- t - 2e_2 \sin \omega_- t \\
y_2(t) &= -(2c_1 \cos \omega_+ t - 2e_1 \sin \omega_+ t) \frac{M}{k^-} \left(\frac{k^+}{M_0} - \omega_+^2 \right) \\
&\quad - (2c_2 \cos \omega_- t - 2e_2 \sin \omega_- t) \frac{M}{k^-} \left(\frac{k^+}{M_0} - \omega_-^2 \right) \\
y_3(t) &= -(2c_1 \sin \omega_+ t + 2e_1 \cos \omega_+ t) \omega_+ \\
&\quad - (2c_2 \sin \omega_- t + 2e_2 \cos \omega_- t) \omega_- \\
y_4(t) &= (2c_1 \sin \omega_+ t + 2e_1 \cos \omega_+ t) \frac{\omega_+ M}{k^-} \left(\frac{k^+}{M_0} - \omega_+^2 \right) \\
&\quad + (2c_2 \sin \omega_- t + 2e_2 \cos \omega_- t) \frac{\omega_- M}{k^-} \left(\frac{k^+}{M_0} - \omega_-^2 \right)
\end{aligned} \tag{10}$$

and

$$\begin{aligned}
c_1 &= \frac{y_1(0)}{2} - \frac{y_1(0)}{2} \left(\frac{\frac{k^+}{M_0} - \omega_+^2}{\omega_-^2 - \omega_+^2} \right) - \frac{y_2(0)k^-}{2M(\omega_-^2 - \omega_+^2)} \\
e_1 &= -\frac{y_3(0)}{2\omega_+} + \frac{y_3(0)}{2\omega_+} \left(\frac{\frac{k^+}{M_0} - \omega_+^2}{\omega_-^2 - \omega_+^2} \right) + \frac{y_4(0)k^-}{2M\omega_+(\omega_-^2 - \omega_+^2)} \\
c_2 &= \frac{y_1(0)}{2} \left(\frac{\frac{k^+}{M_0} - \omega_+^2}{\omega_-^2 - \omega_+^2} \right) + \frac{y_2(0)k^-}{2M(\omega_-^2 - \omega_+^2)} \\
e_2 &= -\frac{y_3(0)}{2\omega_-} \left(\frac{\frac{k^+}{M_0} - \omega_+^2}{\omega_-^2 - \omega_+^2} \right) - \frac{y_4(0)k^-}{2M\omega_-(\omega_-^2 - \omega_+^2)}
\end{aligned} \tag{11}$$

where:

$$\begin{aligned}
\omega_- &\equiv \sqrt{k_0 \left[\frac{1}{M_0} - \sqrt{\frac{1}{M^2} + \lambda^2 \Gamma^2 \left(\frac{1}{M_0^2} - \frac{1}{M^2} \right)} \right]} \\
\omega_+ &\equiv \sqrt{k_0 \left[\frac{1}{M_0} + \sqrt{\frac{1}{M^2} + \lambda^2 \Gamma^2 \left(\frac{1}{M_0^2} - \frac{1}{M^2} \right)} \right]}
\end{aligned} \tag{12}$$

The above equations form the basis for the derivation of acceleration based on both amplitude and frequency of vibration of the two beams. However, since it is well-known that outside of the lock-in zone, acceleration is a near linear function of the difference in measured frequencies, we shall confine our description to amplitude measurement within the lock-in zone. In this context, it will of course be appreciated that we need to know the boundary of the lock-in zone before we can determine whether acceleration is computed as a function of frequency difference or amplitude. We will address this issue in due course.

Calculation of acceleration from vibration amplitudes

The vibration amplitudes of each beam can be calculated from Eq. (10) as:

$$\begin{aligned}
 A_{1+} &= 2\sqrt{c_1^2 + e_1^2} \\
 A_{1-} &= 2\sqrt{c_1^2 + e_2^2} \\
 A_{2+} &= 2\sqrt{(c_1^2 + e_1^2) \left(\frac{M}{k^-}\right)^2 \left(\frac{k^+}{M_0} - \omega_+^2\right)^2} \\
 A_{2-} &= 2\sqrt{(c_1^2 + e_2^2) \left(\frac{M}{k^-}\right)^2 \left(\frac{k^+}{M_0} - \omega_-^2\right)^2}
 \end{aligned} \tag{13}$$

For the sake of abundant clarity, we reiterate that in Eq. (13) the dependency of k^- and k^+ as well ω_- and ω_+ on Γ is omitted for the sake of simplicity. Note that each beam vibrates at two frequencies given by ω_+ and ω_- with respective vibration amplitudes $A_{1+}(\Gamma)$, $A_{1-}(\Gamma)$ and $A_{2+}(\Gamma)$, $A_{2-}(\Gamma)$. As far as we are aware, there is no mention in the prior art of the effect of the amplitude of the *non-dominant* frequencies in the determination of the lock-in zone. Likewise, there is no discussion of the possible influence of the non-dominant amplitudes on the amplitude-acceleration characteristic inside or outside the lock-in zone.

Thus, Zhang *et al.* [15], [16] and [17] show that within the lock-in zone the ratio $\left(\frac{A_{2+}(\Gamma)}{A_{1+}(\Gamma)}\right)$ of dominant vibration frequency amplitudes for the two beams is sensitive to acceleration, but is non-linear. Similar results are shown by HaLevy *et al.* [18], Fig. 2c of which plots the ratio between the measured dominant amplitudes of the two beams and shows what appears to be a near linear relationship in the lock-in zone. However, the

apparent linearity is deceptive owing to a highly compressed scale. Moreover, it is seen that outside of the lock-in zone, there is almost no sensitivity at all.

Fig. 3 shows graphically the amplitude-acceleration characteristic for the device derived using Eq. (13). It is seen that there are particular accelerations (denoted by Γ_1^{LI} and Γ_2^{LI}) for which the amplitudes of both vibrating frequencies for the same beam intersect, and define the opposite boundaries of the lock-in zone. For accelerations $\Gamma < \Gamma_1^{LI}$, we see that the dominant amplitudes are A_{1-} and A_{2+} ; for $\Gamma_1^{LI} < \Gamma < \Gamma_2^{LI}$ the dominant amplitudes are A_{1+} and A_{2+} ; and finally for $\Gamma_1^{LI} > \Gamma_2^{LI}$ the dominant amplitudes are A_{1+} and A_{2-} . In order to avoid the need to refer to the frequency having the dominant amplitude, we will denote this frequency as the “dominant frequency”, it being understood that the dominant frequency may be f_- or f_+ . For the sake of clarity, it should be noted that positive acceleration denotes acceleration in the direction of increasing x in Fig. 2, while negative acceleration denotes deceleration in the direction of x or acceleration in the opposite direction. It is seen that the opposing boundaries of the lock-in zone occur where $A_{1-} = A_{1+}$ and $A_{2-} = A_{2+}$, i.e. the respective amplitudes of the dominant and sub-dominant frequencies for each beam are equal. The numbers 1 and 2 refer to the beams 1 and 2 referenced 11 and 11', respectively in Fig. 1. In order to distinguish between the four frequencies (two for each of the two beams), the amplitude-acceleration curves are shown using different colors. For those jurisdictions where colored drawings are admissible, the four curves appear as follows:

Beam	Frequency	Color	Amplitude
Beam 1	f_+	Red	A_{1+}
	f_-	Blue	A_{1-}
Beam 2	f_+	Green	A_{2+}
	f_-	Light blue	A_{2-}

Table 1: Color key for frequency curves

In addition, the curves are labeled according to their amplitudes as shown in the table, thus allowing identification for those for whom the curves appear in greyscale. From the figure we see that the vibration amplitudes of both beams change in a continuous way.

It is seen that the crossover points determine the negative and positive accelerations at the extremities of the lock-in zone. It is also seen that at the crossover points, the

frequency that was previously dominant becomes sub-dominant and *vice versa*. Specifically, for positive accelerations outside the lock-in zone, the dominant frequency for Beam 1 is f_{1+} (shown in red) and this remains the dominant frequency also inside the lock-in zone. For negative accelerations outside the lock-in zone, the only measured frequency for Beam 1 is f_{1-} (shown in blue). What this means in practice is that if the acceleration to which Beam 1 is subjected is higher than $-2g$, the only measured frequency will be f_{1+} , while for accelerations less than $-2g$, the only measured frequency will be f_{1-} . It can likewise be seen that if Beam 2 is subjected to positive accelerations greater than $+2g$ i.e. outside the lock-in zone, the only measured frequency will be f_{2+} , while for accelerations less than $+2g$, the only measured frequency will be f_{2-} . Of particular note is the fact that inside the lock-in zone, the amplitudes for Beams 1 and 2 are A_{1+} (shown in red) and A_{2+} (shown in green) respectively, corresponding to the vibration amplitudes for the dominant frequencies f_{1+} and f_{2+} , the significance of which will become apparent shortly with reference to Eq. (14) below.

Figs. 4 and 5 show respective amplitude ratios for the dominant frequencies within the lock-in zone given by $\left(\frac{A_{1+}(f)}{A_{2+}(f)}\right)$ and $\left(\frac{A_{2+}(f)}{A_{1+}(f)}\right)$ at an expanded scale, showing that while indeed the amplitude ratios are sensitive to acceleration inside the lock-in zone as shown in the prior art, the relationship is certainly not linear. We also show in these figures in dotted line, amplitude acceleration characteristics outside of the lock-in zone based on the ratio between the respective dominant amplitudes. Thus, Fig. 4 plots $\left(\frac{A_{1-}(f)}{A_{2+}(f)}\right)$ for negative accelerations and $\left(\frac{A_{1+}(f)}{A_{2-}(f)}\right)$ for positive accelerations, these being the ratios of dominant amplitudes for Beam 1 to Beam 2 in the respective ranges. Thus, it is seen from Fig. 3 that for negative accelerations outside the lock-in zone, the dominant amplitude for Beam 1 is A_{1-} (blue) and for Beam 2 is A_{2+} (green) while for positive accelerations outside the lock-in zone, the dominant amplitude for Beam 1 is A_{1+} (red) and for Beam 2 is A_{2-} (light blue). Similarly, Fig. 5 plots $\left(\frac{A_{2+}(f)}{A_{1-}(f)}\right)$ for negative accelerations and $\left(\frac{A_{2-}(f)}{A_{1+}(f)}\right)$ for positive accelerations, these being the ratios of dominant amplitudes for Beam 2 to Beam 1 in the respective ranges. It can be seen that while amplitude ratios based on the measured dominant amplitudes outside of the lock-in zone are, at least to some extent, sensitive to acceleration, the relationships are also not linear.

Before proceeding further, it may be useful at this point to summarize the above findings because they establish the essential building blocks for the present invention. We show that for each beam there are two vibration frequencies. The one whose amplitude is higher is called the dominant frequency; the other is called the sub-dominant frequency.

5 Over the complete range of the accelerometer, the two frequencies may exchange roles at one of the boundaries of the lock-in zone, whereby the frequency that was dominant becomes sub-dominant and *vice versa*. The manner in which happens is shown graphically in Fig. 3. By definition, the vibration frequency whose amplitude is measured directly is always the dominant frequency and we refer to its amplitude as the dominant amplitude.

10 We also acknowledge that within the lock-in zone the ratio of the respective dominant amplitudes for the two beams is sensitive to acceleration, but the relationship is not linear.

In one aspect, the present invention sets out to achieve greater linearity across the complete range, while determining acceleration based on measurement of dominant frequency and amplitude only. Thus in accordance with some embodiments, within the

15 lock-in zone, we compute acceleration as a function of measured dominant amplitudes of vibration for the two beams, while outside of the lock-in zone, we compute acceleration as a function of the difference in measured dominant frequencies of vibration for the two beams. For the sake of abundant clarity, the term ‘measured’ in this context means that the dominant frequencies and amplitudes are measured *directly* across the output electrodes 16,

20 16’ (shown in Fig. 1). In contrast, the sub-dominant frequencies and amplitudes can only be derived by further processing of the signals across the output electrodes and are therefore measured only *indirectly*. The ability to obtain a linear characteristic across the complete range of the accelerometer based only on direct measurement of the dominant frequencies and amplitudes represents a significant benefit over known approaches including those

25 taught in our co-pending applications IL 315720 and IL 318446 both of which *require* additional processing for the determination of either sub-dominant frequencies or amplitudes in order to achieve linearity across the complete range. This requirement is obviated according to one aspect of the present invention, which allows acceleration to be determined as a linear function of either amplitude or differential frequency across the

30 complete range based only on the measurement of dominant amplitude and frequency, as appropriate.

This being said, it remains feasible to compute acceleration also inside the lock-in zone as a linear function of sub-dominant amplitudes and therefore for the sake of full disclosure, we shall relate to all possibilities. To this end, we define amplitude-acceleration characteristics $A^+(\Gamma)$ and $A^-(\Gamma)$ for the dominant and sub-dominant amplitudes as follows:

$$\begin{aligned} A^+(\Gamma) &\equiv \frac{A_{1+}(\Gamma)}{A_{2+}(\Gamma)} - \frac{A_{2+}(\Gamma)}{A_{1+}(\Gamma)} \\ A^-(\Gamma) &\equiv \frac{A_{2-}(\Gamma)}{A_{1-}(\Gamma)} - \frac{A_{1-}(\Gamma)}{A_{2-}(\Gamma)} \end{aligned} \quad (14)$$

Using these equations, we construct an effective amplitude $A^0(\Gamma)$ that varies as a function of acceleration:

$$A^0(\Gamma) = A^-(\Gamma) + A^+(\Gamma) \quad (15)$$

By substituting the values $A^+(\Gamma)$ and $A^-(\Gamma)$ from Eq. (14) into Eq. (15), we can re-write Eq. (15) as:

$$A^0(\Gamma) = \frac{A_{2-}(\Gamma)}{A_{1-}(\Gamma)} - \frac{A_{1-}(\Gamma)}{A_{2-}(\Gamma)} - \frac{A_{2+}(\Gamma)}{A_{1+}(\Gamma)} + \frac{A_{1+}(\Gamma)}{A_{2+}(\Gamma)} \quad (16)$$

Re-ordering terms in Eq. (16) gives:

$$A^0(\Gamma) = \left(\frac{A_{2-}(\Gamma)}{A_{1-}(\Gamma)} - \frac{A_{2+}(\Gamma)}{A_{1+}(\Gamma)} \right) - \left(\frac{A_{1-}(\Gamma)}{A_{2-}(\Gamma)} - \frac{A_{1+}(\Gamma)}{A_{2+}(\Gamma)} \right) \quad (17)$$

Eq. (17) can be expressed physically as:

$$\begin{aligned} A^0(\Gamma) &= \text{Amplitude ratio of Beam 2: Beam 1} \\ &\quad - \text{Amplitude ratio of Beam 1: Beam 2} \end{aligned} \quad (18)$$

Eq. (17) specifies that the acceleration-dependent amplitude $A^0(\Gamma)$ is the difference between the respective amplitude ratios for f^- to f^+ and f^+ to f^- for Beam 2 to Beam 1 less the difference between the same respective amplitude ratios for of Beam 1 to Beam 2. In both cases, the amplitudes of the frequencies f^- and f^+ will be either dominant or sub-dominant as shown in Fig. 3 depending on whether the associated acceleration is inside or outside the lock-in zone. Of particular significance is that, as can be seen from Fig. 3, within the lock-in zone the dominant amplitudes always correspond to $A_{1+}(\Gamma)$ and $A_{2+}(\Gamma)$ for Beams 1 and 2, respectively.

Using Equations 13, 14 and 15, it can be shown that the “amplitude ratio – acceleration” relationship has an exact mathematical form in terms of the physical parameters of the system:

$$A^0(\Gamma) = \frac{4M\lambda}{M_0} \sum_{n=0}^{\infty} \lambda^{2n} \Gamma^{2n+1} \quad (19)$$

The term $\frac{m_0 M}{M+m_0}$ is referred to as the effective mass of the beam and is denoted by

- 5 M_0 . It is seen from Eq. (19) that the non-linear terms depend as a power of $\lambda\Gamma$. It will also be recalled from Eq. (2) that the spring constants k^+ and k^- are equal to $k_0(1 \pm \lambda\Gamma)$. λ is the ratio of seismic mass to the Euler buckling load and is made low by design. Since the spring constant k^- cannot be negative, it follows that $\lambda\Gamma$ must be less than one. To simplify Eq. (19), we shall assume that $\lambda\Gamma \ll 1$ so that we can ignore terms in $\lambda^{2n}\Gamma^{2n+1}$, such that
- 10 Eq. (19) can be approximated to:

$$A^0(\Gamma) = \frac{4M\lambda}{M_0} \Gamma \quad (20)$$

A less accurate “amplitude ratio–acceleration” relationship for $\lambda\Gamma \ll 1$ can be obtained from Eq. (14):

$$A^+(\Gamma) \approx \frac{2M\lambda}{M_0} \Gamma \quad (21a)$$

$$A^-(\Gamma) \approx \frac{2M\lambda}{M_0} \Gamma \quad (21b)$$

It thus follows that acceleration is approximately a linear function of the measured amplitude of vibration of the two beams, both for dominant and sub-dominant amplitudes.

- 15 In general, we can write the output acceleration as:

$$\Gamma_{output} = \sum_{n=0} \frac{K_n}{K_1} \Gamma^n \quad (22)$$

where K_1 is the scale factor and the terms K_n accounts for the nonlinearity of the scale factor. In practice K_2 is taken as a measure of the scale factor nonlinearity. These terms are calculated by fitting the accelerometer output using a polynomial function.

- 20 What emerges from the foregoing analysis is that within the lock-in zone, we can use Eq. (21a) to compute acceleration as a linear function of the measured amplitudes of vibration of the two beams because these will always correspond to the dominant

amplitudes within the lock-in zone. Alternatively, Eq. (21b) can be used based on the sub-dominant amplitudes, albeit at the cost of additional signal processing. Likewise, Eq. (20) can be used based on the combined measurements of dominant and sub-dominant amplitudes.

5 **Example**

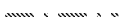
Accelerations are commonly represented in units of the local value of the gravity constant g , which was approximated to 10 ms^{-2} for the purpose of simulation. By way of example, consider a concrete example having a seismic mass $M = 2 \times 10^{-5} \text{ kg}$, beams of mass $m_0 = 2 \times 10^{-8} \text{ kg}$, spring constant $k_0 = 3850 \text{ kg s}^{-2}$, and $\lambda = \frac{0.0005}{g}$. The
10 initial conditions are $y_1(0) = y_2(0) = 1 \mu\text{m}$ and $y_3(0) = y_4(0) = 0$. The effective mass of the beam M_0 may be calculated as follows:

$$\begin{aligned} M_0 &= \frac{m_0 M}{M + m_0} \\ &= \frac{2 \times 10^{-8} \text{ Kg} \times 2 \times 10^{-5} \text{ Kg}}{2 \times 10^{-8} \text{ Kg} + 2 \times 10^{-5} \text{ Kg}} \\ &= 1.998002 \times 10^{-8} \text{ Kg} \end{aligned} \quad (23)$$

Substituting these values into Eqs. (20) and (21), we calculate the theoretical scales factors (SF) as follows:

$$\begin{aligned} SF^0 &= 4 \times \frac{2 \times 10^{-5} \text{ Kg}}{1.998002 \times 10^{-8} \text{ Kg}} \times \frac{0.0005}{g} = \frac{2.002}{g} \\ SF^+ &= 2 \times \frac{-2 \times 10^{-5} \text{ Kg}}{1.998002 \times 10^{-8} \text{ Kg}} \times \frac{0.0005}{g} = -\frac{1.001}{g} \\ SF^- &= 2 \times \frac{2 \times 10^{-5} \text{ Kg}}{1.998002 \times 10^{-8} \text{ Kg}} \times \frac{0.0005}{g} = \frac{1.001}{g} \end{aligned} \quad (24)$$

We use Eqs. (14) and (15) to calculate the new “amplitude-ratio” acceleration
15 relationships corresponding to $A^+(\Gamma)$, $A^-(\Gamma)$, and $A^0(\Gamma)$ as shown in Fig. 6. In order to distinguish between the three graphs, the amplitude-acceleration curves are shown using different colors and different line styles. For those jurisdictions where colored drawings are admissible, the three curves appear as follows:

Amplitude	Color	Line description	Line style
$A^0(\Gamma)$	Blue	Chain dot	

Amplitude	Color	Line description	Line style
$A^-(\Gamma)$	Black	Long Dash	— — —
$A^+(\Gamma)$	Red	Solid	~~~~~

Table 2: Color key for amplitude-acceleration curves

In addition, the curves are labeled according to their amplitudes as shown in the table, thus allowing identification for those for whom the curves appear in greyscale. From the figure we see that the acceleration varies substantially linearly as a function of measured vibration amplitudes across the complete range and that within the lock-in zone only the measured dominant amplitudes are required.

Although the amplitude-acceleration relationships shown in Fig. 6 appear to be linear, they are only approximately linear. In practice, in order to estimate the nonlinearity of the scale factor, we plot the actual accelerations as a function of measured amplitudes and then use a least-squares fit or any other suitable technique to estimate a polynomial fit of the resulting curves of the form:

$$P(\Gamma) = K_0 + K_1\Gamma + K_2\Gamma^2 + K_3\Gamma^3 \quad (25)$$

where K_0 refers to the accelerometer bias, K_1 is the scale factor and K_2 and K_3 characterize the nonlinearity of the amplitude-ratio acceleration relationship. From this polynomial fit the measured polynomial coefficients are estimated as follows:

$$\begin{aligned}
 P^0(\Gamma) &= 4.494 \times 10^{-13} + 2.002\Gamma + 8.571 \times 10^{-14}\Gamma^2 \\
 &\quad + 5.005 \times 10^{-7}\Gamma^3 \\
 P^+(\Gamma) &= 1.346 \times 10^{-6} + 0.992\Gamma - 6.728 \times 10^{-8}\Gamma^2 \\
 &\quad - 3.440 \times 10^{-5}\Gamma^3 \\
 P^-(\Gamma) &= -1.346 \times 10^{-6} + 1.003\Gamma + 6.728 \times 10^{-8}\Gamma^2 \\
 &\quad + 3.490 \times 10^{-5}\Gamma^3
 \end{aligned} \quad (26)$$

From Eq. (26) we see several interesting facts. First, the scale factors obtained from the polynomial fit are in close agreement with the theoretical one calculated in Eq. (24). Table 3 summarizes the results:

Scale Factor K_1	Eq. (24) [1/g]	Eq. (26) [1/g]
SF^0	2.002	2.002
SF^+	1.001	0.992
SF^-	1.001	1.003

Table 3: Scale factors calculated from Eq. (24) and Eq. (26)

The ratio $\frac{K_0}{K_1}$ is the bias value (it should be zero for the ideal case).

	Bias K_0/K_1 [μg]
Bias ⁰	2.245×10^{-7}
Bias ⁺	1.357
Bias ⁻	1.342

5

Table 4: Bias values in units of μg obtained from Eq. (26).

It can be seen that the bias for the implementation defined by Eq. (19) is almost zero. But the biases defined by Eq. (21) have a small value, albeit not negligible. Finally, we compute the nonlinear coefficients of order 2 and 3 as follows:

SF Error	K_2/K_1 [$\mu\text{g}/\text{g}^2$]	K_3/K_1 [$\mu\text{g}/\text{g}^3$]
SF^0	4.371×10^{-8}	0.250
SF^+	6.782×10^{-2}	34.677
SF^-	6.708×10^{-2}	34.796

10

Table 5: Nonlinear coefficients of order 2 and 3 expressed in terms of the scale factor K_1

Again, the nonlinear coefficient $\frac{K_2}{K_1}$ is very small or even negligible for SF^0 .

However the term $\frac{K_2}{K_1}$ is several orders of magnitude larger for SF^+ and SF^- . The same is

15 true for the term $\frac{K_3}{K_1}$.

Figs. 7, 8 and 9 show graphically the deviation from linearity for each of the three curves in Fig. 6. Each curve plots the deviation of the amplitude-acceleration characteristic from the respective best linear fit for each curve. It is seen the maximum deviation in acceleration from the ideal linear value for $A^-(\Gamma)$ and $A^+(\Gamma)$ is approximately 0.007g i.e. approximately 0.07 m/s² while for $A^0(\Gamma)$ there is almost zero deviation for accelerations in the lock-in zone. This means that there is a price to pay for computing acceleration in the lock-in zone based only on the measured dominant vibration amplitudes since the result is prone to a slight inaccuracy. On the other hand, it avoids the need to determine the amplitudes of the sub-dominant amplitudes as is necessary for $A^0(\Gamma)$ and $A^-(\Gamma)$. There would appear to be little incentive to compute acceleration inside the lock-in zone based only on $A^-(\Gamma)$ since, as can be seen from Fig. 9, it is subject to the same deviation as $A^+(\Gamma)$, while requiring further processing to determine the sub-dominant amplitudes. Nevertheless, it remains possible to compute acceleration inside the lock-in zone from the sub-dominant amplitudes. More significantly, if the slight inaccuracy in computing acceleration based only on the measured dominant amplitudes is unacceptable, both dominant and sub-dominant amplitudes can be determined, and acceleration can then be computed based on $A^0(\Gamma)$ in accordance with Eq. (20). Since, $A^0(\Gamma)$ is a function of $A^+(\Gamma)$ and $A^-(\Gamma)$ as shown in Eqs. (16) and (17), this requires measurement of both the dominant and sub-dominant amplitudes, and computation of $A^0(\Gamma)$. The corresponding acceleration is then determined from the corresponding amplitude-acceleration shown in Fig. 6. This, of course is applicable across the complete range of the accelerometer even though its applicability for the present invention is principally confined to within the lock-in zone. In any event, it is reiterated that computing acceleration based on $A^0(\Gamma)$ is required only if the slight inaccuracy of the Eq. (14) is unacceptable.

The above description with reference to Figs. 6 to 9 relates to computation of the amplitude-acceleration characteristic across the whole range of the accelerometer, i.e. both outside and inside the lock-in range. Fig. 6 shows three different linear amplitude-acceleration characteristics, two of which corresponding to $A^-(\Gamma)$ and $A^+(\Gamma)$ are based on Eq. (21) are valid for $\lambda\Gamma \ll 1$, while the other characteristic corresponding to $A^0(\Gamma)$ makes no such assumption. However, the present invention in all its aspects is limited to a hybrid accelerometer which computes acceleration inside a defined range that includes the lock-in zone based on amplitude measurements and computes acceleration outside the defined range based on differential frequency measurements.

In summary, within the lock-in zone we obtain a substantially linear characteristic, which, in one embodiment, is based on direct measurement of the vibration amplitude $A^+(f)$ relating to the dominant frequency. This means that inside the lock-in zone, the vibration amplitudes as measured across the electrodes 16 and 16' in Fig. 1 are sufficient
 5 for deriving the acceleration using Eq. (14). In such an embodiment, there is no need to determine the vibration amplitudes A_{1-} and A_{2-} for the sub-dominant frequencies f_{1-} (shown in blue) and f_{2-} (shown in pale blue). Outside of the lock-in zone, acceleration is computed as a function of the difference in vibration frequency of the two beams in known manner.

10 It is well-known that lock-in occurs when the frequencies of vibration whose amplitudes are dominant are identical for the two beams. Since conventional differential two-beam accelerometers determine acceleration as a function of frequency difference and only the dominant frequencies are measured, lock-in occurs when the measured frequencies of the two beams are the same because the frequency difference is then zero. Our co-pending
 15 IL 315720 shows that for each of the beams there are corresponding first and second frequencies (f_- , f_+) at which the respective amplitude is dominant. The lock-in zone is bound by respective crossover points for each beam where the respective amplitudes (A_- , A_+) at both the first and second frequencies (f_- , f_+) are the same and which define a crossover region within which the respective dominant amplitudes of the first and second frequencies
 20 cross over such that the frequency for which amplitude is higher outside the crossover region has a smaller amplitude within the crossover region and *vice versa*.

In order to know when we need to measure the amplitude of the sub-dominant frequency, we need to know the boundaries of the lock-in zone. This may be determined in the factory and made known to the end-user by the manufacturer. However, for the
 25 sake of completeness, we will now describe how to derive the lock-in zone before detailing how to measure the correct amplitudes depending on whether a given acceleration falls inside or outside of the lock-in zone.

Derivation of the Lock-In Zone

From Eq. (12) we see that both beams vibrate at the Eigen-frequencies f_- and f_+ ,
 30 where $\omega = 2\pi f$. This means that while a single beam attached to a seismic mass vibrates at a single frequency, two beams attached to the same mass will vibrate at a combination of the two Eigen-frequencies f_- and f_+ .

In real world applications, the beams are put in vibration with initial zero velocity i.e. $y_3(0) = y_4(0) = 0$. That is, the beams are taken out of equilibrium by pulling the beam along the transversal axis i.e. $y_1(0) = a$ and $y_2(0) = b$. Equations (10) and (13) may then be simplified as follows:

$$\begin{aligned} y_1(t) &= A_{1+} \cos \omega_+ t + A_{1-} \cos \omega_- t \\ y_2(t) &= A_{2+} \cos \omega_+ t + A_{2-} \cos \omega_- t \end{aligned} \quad (27)$$

5 We define:

$$\begin{aligned} A_{1+}(\Gamma) &= 2c_1 \\ A_{1-}(\Gamma) &= 2c_2 \\ A_{2+}(\Gamma) &= 2 \sqrt{c_1^2 \left(\frac{M}{k^-}\right)^2 \left(\frac{k^+}{M_0} - \omega_+^2\right)^2} \\ A_{2-}(\Gamma) &= 2 \sqrt{c_2^2 \left(\frac{M}{k^-}\right)^2 \left(\frac{k^+}{M_0} - \omega_-^2\right)^2} \end{aligned} \quad (28)$$

where c_1 and c_2 are given by:

$$\begin{aligned} c_1 &= \frac{a}{2} - \frac{a}{2} \left(\frac{\frac{k^+}{M_0} - \omega_+^2}{\omega_-^2 - \omega_+^2} \right) - \frac{bk^-}{2M(\omega_-^2 - \omega_+^2)} \\ c_2 &= \frac{a}{2} \left(\frac{\frac{k^+}{M_0} - \omega_+^2}{\omega_-^2 - \omega_+^2} \right) + \frac{bk^-}{2M(\omega_-^2 - \omega_+^2)} \end{aligned} \quad (29)$$

The amplitudes will change with the value of the external acceleration Γ . The lock-in boundary is defined by the following condition:

$$\begin{aligned} A_{1+}(\Gamma^{LI}) &= A_{1-}(\Gamma^{LI}) \\ A_{2+}(\Gamma^{LI}) &= A_{2-}(\Gamma^{LI}) \end{aligned} \quad (30)$$

We solve conditions (30) for Γ^{LI} to obtain:

$$\begin{aligned} \Gamma_1^{LI} &= \frac{-bm_0}{\lambda[aM + m_0(a-b)]} \\ \Gamma_2^{LI} &= \frac{am_0}{\lambda[bM + m_0(b-a)]} \end{aligned} \quad (31)$$

10 For the special case where $a = b$, i.e. both beams are subjected to the same initial transverse displacement:

$$\begin{aligned} \Gamma_1^{LI} &= -\frac{m_0}{\lambda M} = -\frac{\Delta\omega(\Gamma = 0)}{\eta} \\ \Gamma_2^{LI} &= +\frac{m_0}{\lambda M} = \frac{\Delta\omega(\Gamma = 0)}{\eta} \end{aligned} \quad (32)$$

Equation (32) conforms to the range of the lock-in zone measured by Le Traon, 2006 ([1]). It may be shown that if the masses of the two beams are unequal, then the lock-in range will not be centered at zero frequency, but will be centered either in a positive or negative acceleration position depending on the sizes of the two beams. For small differences between the masses of the beams, Eq. (32) remains a very good approximation. It will be appreciated that Eq. (32) specifies the negative and positive accelerations at the boundary of the lock-in zone. In practice, we need to know not the frequencies at the boundary of the lock-in zone but rather the vibration amplitudes because we measure amplitude in order to determine acceleration.

With this in mind, we now revert to Eq. (14) showing the amplitude-acceleration characteristics $A^-(\Gamma)$ and $A^+(\Gamma)$ for the sub-dominant and dominant frequencies, and to the effective amplitude $A^0(\Gamma)$ defined in Eq. (17). In order to compute $A^+(\Gamma)$, we need to measure the vibration amplitude of frequency f_+ . But as is clear from Fig. 3, f_+ will be the dominant or sub-dominant frequency according to whether we are inside or outside the lock-in zone. The same, of course, is true for $A^-(\Gamma)$ which requires measurement of the vibration amplitude of frequency f_- , and for $A^0(\Gamma)$ which requires measurement of the respective amplitudes of both f_+ and f_- . However, within the lock-in zone, the directly measured amplitudes $A^+(\Gamma)$ are always those of the dominant frequency.

Consequently, if we know for a fact that the measured acceleration falls within the lock-in zone, then we can measure the dominant amplitudes directly. In other words, within the lock-in zone, the measured vibration amplitude will always be equal to $A^+(\Gamma)$ allowing acceleration to be directly computed from Eq. (21) as follows:

$$A^+(\Gamma) \approx \frac{2M\lambda}{M_0} \Gamma \quad (21a)$$

Alternatively, as noted above, we could determine the sub-dominant amplitudes corresponding to $A^-(\Gamma)$ and then derive the acceleration from Eq. (21) as follows:

$$A^-(\Gamma) \approx \frac{2M\lambda}{M_0} \Gamma \quad (21b)$$

Likewise, having determined the sub-dominant amplitudes corresponding to $A^-(\Gamma)$, we could then derive the acceleration from Eq. (17) as follows:

$$A^0(\Gamma) = \left(\frac{A_{2-}(\Gamma)}{A_{1-}(\Gamma)} - \frac{A_{2+}(\Gamma)}{A_{1+}(\Gamma)} \right) - \left(\frac{A_{1-}(\Gamma)}{A_{2-}(\Gamma)} - \frac{A_{1+}(\Gamma)}{A_{2+}(\Gamma)} \right) \quad (17)$$

For accelerations outside the lock-in zone, we measure not vibration amplitudes of the two beams but rather their vibration frequencies in conventional manner, and compute acceleration as a function of the difference in frequencies for the two beams. To this end, we will revert to Eq. (12) relating to the frequency-acceleration characteristics, it being reiterated that the dependency of ω_- and ω_+ on Γ is omitted for the sake of simplicity. The differential frequency $\Delta\omega = \omega_+ - \omega_-$ is given by:

$$\Delta\omega = \sqrt{2 \frac{k_0}{m_0}} \sqrt{1 + \frac{m_0}{M} - \sqrt{(1 - \lambda^2 \Gamma^2) \left(1 + 2 \frac{m_0}{M}\right)}} \quad (33)$$

It will be recalled that at any instant, the beams will vibrate at slightly different frequencies (f_- , f_+). Specifically, one beam will vibrate at a slightly higher frequency f_+ corresponding to pull-mode and the other slightly lower f_- corresponding to push-mode. It is these two frequencies that are denoted by Eq. (12), such that the difference between the measured frequencies of the two beams $\Delta\omega$ is given by Eq. (33), since $\omega = 2\pi f$.

For the case that $m_0 \ll M$ and $\lambda\Gamma \ll 1$, we can neglect $2 \frac{m_0}{M} \lambda^2 \Gamma^2$ and Eq. (33) can be approximated to:

$$\Delta\omega \approx \sqrt{2 \frac{k_0}{m_0}} \sqrt{1 + \frac{m_0}{M} - \sqrt{1 + 2 \frac{m_0}{M} - \lambda^2 \Gamma^2}} \quad (34)$$

Using the partial Taylor expansion for $\sqrt{1+x} \approx 1 + \frac{1}{2}x - \frac{1}{8}x^2$ where $x = 2 \frac{m_0}{M} - \lambda^2 \Gamma^2$ and neglecting terms in $4 \frac{m_0}{M} \lambda^2 \Gamma^2$ and $\lambda^4 \Gamma^4$:

$$\begin{aligned} \Delta\omega &\approx \sqrt{2 \frac{k_0}{m_0}} \sqrt{1 + \frac{m_0}{M} - 1 - \frac{1}{2} \left(2 \frac{m_0}{M} - \lambda^2 \Gamma^2\right) + \frac{1}{8} \cdot 4 \left(\frac{m_0}{M}\right)^2} \\ &\approx \sqrt{2 \frac{k_0}{m_0}} \sqrt{1 + \frac{m_0}{M} - 1 - \frac{m_0}{M} + \frac{1}{2} \lambda^2 \Gamma^2 + \frac{1}{2} \left(\frac{m_0}{M}\right)^2} \end{aligned} \quad (35)$$

This leads to:

25

$$\Delta\omega \approx \sqrt{\frac{k_0}{m_0}} \sqrt{\lambda^2 \Gamma^2 + \frac{m_0^2}{M^2}} \quad (36)$$

and for $\Gamma = 0$:

$$\Delta\omega(0) \approx \sqrt{\frac{k_0}{m_0}} \frac{m_0}{M} \quad (37)$$

and for $\Gamma \gg 0$:

$$\Delta\omega(\Gamma) \approx \lambda \sqrt{\frac{k_0}{m_0}} \Gamma = \eta \Gamma \quad (38)$$

where $\eta \equiv \lambda \sqrt{\frac{k_0}{m_0}} = \lambda \omega_0$ is the scale factor.

Example

- 5 By way of example, consider a concrete example having a seismic mass $M = 2 \times 10^{-5} kg$, beams of mass $m_0 = 2 \times 10^{-8} kg$, spring constant $k_0 = 3850 kg s^{-2}$, and $\lambda = \frac{0.0005}{g}$. The initial conditions are $y_1(0) = y_2(0) = 1 \mu m$ and $y_3(0) = y_4(0) = 0$. We use Eq. (10) to calculate the output signals $y_1(t)$ and $y_2(t)$.

- 10 A frequency counter based on signal zero-cross in a given time interval is used in order to measure the vibrating frequency of each beam. Note that a frequency counter gives a single output frequency, which always corresponds to the dominant frequency. Provided that the measured acceleration is outside the lock-in zone, the frequency difference Δf is proportional to the applied external acceleration. The vibration amplitudes of each beam can be calculated from Eq. (10) as:

$$\begin{aligned} A_{1+} &= 2 \sqrt{c_1^2 + e_1^2} \\ A_{1-} &= 2 \sqrt{c_1^2 + e_2^2} \\ A_{2+} &= 2 \sqrt{(c_1^2 + e_1^2) \left(\frac{M}{k^-}\right)^2 \left(\frac{k^+}{M_0} - \omega_+^2\right)^2} \\ A_{2-} &= 2 \sqrt{(c_1^2 + e_2^2) \left(\frac{M}{k^-}\right)^2 \left(\frac{k^+}{M_0} - \omega_-^2\right)^2} \end{aligned} \quad (39)$$

Again, for the sake of abundant clarity, we reiterate that in Eq. 39 the dependency of k^- and k^+ as well ω_- and ω_+ on Γ is omitted for the sake of simplicity.

Fig. 10 depicts the frequency-acceleration characteristic for Beam 1, where both the dominant and sub-dominant frequencies are shown. The dominant frequencies are shown in full red and blue lines for f_{1+} and f_{1-} , respectively. For the sake of completeness, also seen in red and blue dotted lines are the respective sub-dominant frequencies. Likewise, Fig. 11 depicts the frequency-acceleration characteristic for Beam 2, where both the dominant and sub-dominant frequencies are shown. The dominant frequencies are shown in full green and pale blue lines for f_{2+} and f_{2-} , respectively. Likewise, also seen in green and pale blue dotted lines are the respective sub-dominant frequencies. It is seen that the frequency-acceleration characteristics for both beams are identical, as is to be expected for identical beams.

It is also seen that outside the lock-in zone, the dominant and sub-dominant frequencies for each beam form a continuum for both frequencies, i.e. f_+ and f_- . Thus, referring to Fig. 10 relating to Beam 1, by way of example, the full red curve representing f_{1+} , stops at the negative boundary of the lock-in zone at an acceleration of $-2g$, and at exactly this point for accelerations less than $-2g$, the sub-dominant frequency shown in dotted red line takes over seamlessly. The same is true in reverse for the blue curves representing f_{1-} , which are mirror images of the f_{1+} curves about a horizontal axis. The same is true for Beam 2, except that the respective curves are complementary to those of Beam 1. In other words, f_{2+} is the dominant frequency and extends for negative accelerations, stopping at the positive boundary of the lock-in zone at an acceleration of $+2g$. At exactly this point for accelerations greater than $+2g$, the sub-dominant frequency shown in dotted green line takes over seamlessly. Similar but complementary observations can be made concerning the sub-dominant frequencies.

It emerges from Figs. 10 and 11 that if we know the boundaries of the lock-in zone, we can use these figures to determine the crossover frequencies at the two boundaries of the lock-in zone, i.e. the respective upper frequency where the dominant frequency intersects the boundary of the lock-in zone. For a balanced system whose two beams are identical, the frequency-acceleration curves are symmetrical about the vertical frequency axis i.e. for zero acceleration. In this case, the dominant crossover frequencies of Beam 1 will be identical and so, too, will the dominant crossover frequencies of Beam 2. It can be seen that for both beams, the crossover frequency is approximately 69,910 Hz. For a non-balanced system, the crossover frequencies at opposing boundaries of the lock-

in zone may be different, but the principle is the same. In both Figs. 10 and 11, the lower frequency characteristics shown predominantly in dotted line, because they represent for the most part sub-dominant frequencies, also intersect the boundaries of the lock-in zone at identical frequencies. It can be seen that for both beams, the sub-dominant crossover
5 frequency is approximately 69,810 Hz. If we consider only the upper frequency-acceleration characteristics, we see that within the lock-in zone the dominant frequencies dip, i.e. they are lower than the crossover frequencies. This means that if the measured dominant frequency for Beam 1 is higher than the crossover frequency, it can be inferred that we are not in the lock-in zone. In this case, we need to measure the sub-dominant amplitude
10 of one or other of the beams. One way to do this is to sample the signals across the electrodes 16, 16' for each beam over a given time period, and convert from the time domain to the frequency domain using, for example but not limited to, FFT as described above. This allows determination of both f_+ and f_- and their respective amplitudes.

Conversely, if the measured dominant frequency for Beam 1 is lower than the
15 crossover frequency, it can be inferred that we are in the lock-in zone, in which case, we do not need to measure the sub-dominant amplitudes. The same is true for Beam 2 as can be seen in Fig. 11. The dominant frequencies can be measured using a zero-cross frequency counter in conventional manner, or using any other suitable method. The vibration amplitude of the beam vibrating at the dominant frequency is measured directly.
20 To determine the vibration amplitude associated with the sub-dominant frequency, we can use FFT as described earlier. Of course, we can use FFT in all cases to determine the respective vibration amplitude associated with the dominant or sub-dominant frequency.

For the sake of completeness, although the previous analysis relates to a balanced system wherein the masses of the two beams are the same, it may be shown that for an
25 accelerometer having beams of different masses the lock-in range will not be centered at zero frequency, but will be centered either in a positive or negative acceleration position depending on the masses of the two beams. This does not affect the manner in which the boundaries of the lock-in zone are determined and therefore permits the same principles of the present invention to be applied inside and outside of the lock-in zone.

30 Fig. 12 shows on a common axis the measured output frequencies of the two beams as measured in conventional accelerometers, these being, of course, by definition the dominant frequencies because only these are measured using conventional approaches. For those jurisdictions where colored drawings are admissible, the frequency curves for Beam 1 are blue and for Beam 2 they are orange. It is seen that by using the

signal zero-cross detection method, for the range of accelerations $\Gamma_1^{LI}, \Gamma_2^{LI}$ the frequency outputs are the same. When seen in color, the blue and orange curves for the dominant frequencies overlap, i.e. the two dominant frequencies f_{1+} and f_{2+} overlap in the lock-in zone. We reiterate that by ‘dominant frequencies’ we mean the frequencies whose amplitudes are dominant. This can be explained by the fact that this method detects only the dominant frequency of each beam. Since we have already established with reference to Fig. 3 that the dominant amplitudes in this range are A_{1+} and A_{2+} the detected frequency will be f_+ . It may be observed that the measured frequencies in the lock-in zone are f_- or f_+ depending on the initial conditions. For instance, for the case where $a = b$, the lock-in frequency is f_+ , while for $a = -b$, the lock-in frequency is f_- . It can also be seen that boundaries of the lock-in zone are $[-2, 2]g$.

Fig. 13 summarizes the relationship between the vibration amplitudes of both beams inside and outside the lock-in zone. The colors of the amplitude-acceleration curves for the two beams are the same as Fig. 3 and, for those viewing the curves in greyscale, can be deciphered according to Table I.

As seen in the figure, outside the lock-in zone, both dominant amplitudes have different associated frequencies for each beam. For negative accelerations, the dominant amplitude for beam 1 is A_{1-} with corresponding frequency f_{1-} and for beam 2 is A_{2+} with corresponding frequency f_{2+} . If we subtract the frequency of the second beam from that of the first beam, this leads to $\Delta f = f_{1-} - f_{2+}$. For positive accelerations, outside the lock-in range, the dominant amplitude for beam 1 is A_{1+} with corresponding frequency f_{1+} and for beam 2 is A_{2-} with corresponding frequency f_{2-} . Again, subtracting the frequency of the second beam from that of the first beam leads to $\Delta f = f_{1+} - f_{2-}$.

Fig. 14 depicts the frequency difference Δf as a function of the external acceleration, Γ when Δf is computed according to the above procedure. It is seen that acceleration is zero when the measured frequency difference is zero. However, when the acceleration is zero, the frequency difference is discontinuous and non-linear but, in accordance with the present disclosure, this is of no concern because inside the lock-in zone acceleration is computed based on vibration amplitude rather than frequency. It is also seen that a major part of the frequency-acceleration characteristic is approximately linear. Specifically, when the absolute value of differential frequency exceeds 200 Hz, corresponding to an acceleration of approximately $\pm 5 g$, acceleration is substantially a linear function of the differential frequency. This avoids the need to store the complete frequency-acceleration

characteristic and facilitates fast computation of acceleration for much of the range once the differential frequency has been measured. Furthermore, since, in accordance with the present disclosure, the linear frequency-acceleration characteristic is used only outside of the lock-in zone, the differential frequency can easily be calculated as the difference
 5 between the dominant frequencies of the two beams, both of which are measured directly across the output electrodes 16, 16'. Consequently, determination of differential frequency and hence acceleration are fast and require very little processing.

It follows from the foregoing description that we need to know in advance or determine *in situ* the boundaries of the lock-in zone. For accelerations outside the lock-in
 10 zone, we can then measure the dominant vibration frequencies for both beams, and compute the frequency-acceleration function as shown, for example, in Eq. (38). Alternatively, albeit at the expense of additional processing, we can measure the sub-dominant vibration frequencies for both beams since the frequency-acceleration characteristic shown in Fig. 14 is the same for dominant and sub-dominant frequencies. That this is true can be seen
 15 empirically from Figs. 10 and 11 since the two frequency-acceleration characteristics for each beam are identical within the lock-in zone and complementary (i.e. anti-symmetrical) outside of the lock-in zone. For example, it can be seen from Fig. 10 that at an acceleration of 3g, the dominant frequency for Beam 1 is f_{1+} (solid line) and equal to approximately 69,925 Hz. Likewise, it can be seen from Fig. 11 that at the same acceleration, the
 20 dominant frequency for Beam 2 is f_{2-} (solid line) and equal to approximately 69,800 Hz. Similarly, it can be seen from Fig. 10 that at an acceleration of 3g, the sub-dominant frequency for Beam 1 is f_{1-} (dotted line) and equal to approximately 69,800 Hz, while Fig. 11 shows that at the same acceleration, the sub-dominant frequency for Beam 2 is f_{2+} (dotted line) and equal to approximately 69,925 Hz. It is thus seen that the absolute
 25 difference between the frequencies is the same regardless of whether dominant or sub-dominant frequencies are measured.

To measure the sub-dominant amplitude or the sub-dominant frequency, we need to isolate the sub-dominant frequency component of the output signal. Any frequency measurement that provides discrimination between both harmonics, i.e. f_+ and f_- may be
 30 employed. For instance, the Fast Fourier Transform (FFT) may be considered the oldest and best known and plots in the frequency domain the frequency spikes showing the dominant and sub-dominant frequency, whose respective amplitudes are given by the

height of their spikes. The Least Square Method and other estimators such as Kalman and extended Kalman filters are described in the literature. We can also use signal cancellation by determining the dominant frequency using a Phase Lock Loop or a frequency counter, synthesize an equivalent sinusoidal signal of identical frequency and then subtract the synthetic signal from the original to derive the sub-dominant signal. Once the dominant and sub-dominant frequencies are known, their respective amplitudes can be computed using Eq. (27) or can be determined by Fourier transformation as explained above. Without loss of generality, any method that can detect the harmonic components of a given signal can be used.

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Practical Implementations

Our discussion so far has been largely analytical. We have shown that for a two-beam accelerometer or indeed any accelerometer that can be modelled mathematically in the form of a system having two beams operating in a differential push-pull mode, each beam will vibrate with two frequencies, one of which is of higher amplitude and to which we therefore refer as the dominant frequency; and the other of which is of lower amplitude and to which we refer as the non-dominant frequency. We have further established that there is a respective point for both beams during both negative and positive acceleration, where the amplitudes of both frequencies for each beam are identical. We call these the crossover points and they define the extent or boundary of the lock-in range. To avoid doubt, the amplitudes of the respective frequencies at the crossover points may be different for the two beams if the beams are not identical. But regardless of the physical properties of the beams such as mass and stiffness, which may give rise to differences in the amplitudes of the two frequencies for each beam at their respective crossover points, it is always the case that the respective crossover points for both beams define opposite extremes of the lock-in range.

In an alternative implementation, we can determine the crossover frequency as explained previously, i.e. the upper frequency where the dominant frequency intersects the boundary of the lock-in zone. Once we know this, we can assess based on whether at any given acceleration to be measured, the measured frequency is larger or smaller than the crossover frequency within any specified tolerance threshold. The threshold establishes a predefined range which includes within it the lock-in zone but may extend beyond the lock-

30

in zone. For example, with reference to Fig. 10, it can be seen that if the acceleration is $\pm 3g$, the dominant frequency is approximately 69,925 Hz. We could define a range bounded by this frequency for both positive and negative accelerations, such that if the measured dominant frequency is less than 69,925Hz, we compute acceleration based only on the measured amplitudes. This can be done in accordance with Eq. (17), which holds true for the complete range of the accelerometer. Alternatively, we can adopt a two-pronged approach whereby if the measured dominant frequency is smaller than a predetermined crossover frequency at opposing boundaries of the lock-in range, we can compute acceleration based on the measured dominant amplitudes of both beams, or of the derived sub-dominant amplitudes of both beams. This allows us to compute $A^+(f)$ or $A^-(f)$ using Eq. (14) after which we can determine the acceleration using Eqs. (21a) and (21b), respectively. Otherwise, if the dominant frequency is greater than the predetermined crossover frequency at opposing boundaries of the lock-in range, this means we are outside of the lock-in range. However, since we are still within the defined range where acceleration is computed as a function of amplitude rather than frequency, the effective amplitude $A^0(f)$ must be computed based on the measured dominant and sub-dominant amplitudes of the two beams using Eq. (17), after which we can determine the acceleration using Eq. (20).

Conversely, if the measured dominant frequency exceeds 69,925Hz, we compute acceleration as a function of the difference $\Delta\omega(f)$ between the measured frequencies of the two beams in accordance with Eq. (39).

In its simplest and most direct implementation, the predefined range is bound by the crossover frequencies at the exact boundaries of the lock-in range. In this case, if the measured frequency is smaller than the crossover frequency, then we are definitely inside the lock-in zone and acceleration may be computed as a function of the respective measured (i.e. dominant) vibration amplitudes of the two beams, most simply using the upper equation in Eq. (14). Conversely, if the measured frequency is larger than the crossover frequency, acceleration is computed as a function of $\Delta\omega(f)$ divided by the scale factor. In either case, if the defined range is set exactly at the boundaries of the lock-in zone, we can avoid measuring the sub-dominant frequency or amplitude altogether, thereby saving resources, and achieving faster computation.

All embodiments of the present invention require pre-calibration to determine the boundary of the lock-in range and crossover frequencies. This can be done by a

manufacturer, for example, or by the end-user prior to actual use by subjecting the accelerometer to a range of accelerations and measuring corresponding amplitudes such that for each beam there are corresponding first and second frequencies (f^- , f^+) whose amplitudes vary with acceleration at which the respective amplitude is dominant. The
 5 respective crossover points for each beam are determined where the respective amplitudes (A^- , A^+) at both the first and second frequencies (f^- , f^+) are the same. It will thus be apparent that pre-calibration to determine the boundary of the lock-in zone serves to facilitate subsequent fast measurement of acceleration, particularly for those
 10 embodiments that require direct measurement of only the dominant frequencies or amplitudes of vibration. In an alternative implementation, we can measure the vibration frequencies for both beams. If the measured frequencies for both beams are the same, then we are inside the lock-in zone, in which case we need only concern ourselves with the measured amplitudes for both beams. Conversely, if the measured frequencies for both beams are different, we are outside the lock-in zone, in which case acceleration may
 15 be computed as a function of the difference between the measured frequencies.

Fig. 12 shows on a common axis the output frequencies of the two beams as measured in conventional accelerometers, these being by definition the dominant frequencies because only these are measured using conventional approaches. For those jurisdictions where colored drawings are admissible, the frequency curves for Beam 1
 20 are blue and for Beam 2 they are orange. It is seen that by using the signal zero-cross detection method, for the range of accelerations Γ_1^{LI} , Γ_2^{LI} the frequency outputs are the same. When seen in color, the blue and orange curves for the dominant frequencies overlap, i.e. the two dominant frequencies f_{1+} and f_{2+} overlap in the lock-in zone. We reiterate that by “dominant frequencies” we mean the frequencies whose amplitudes are
 25 dominant. This can be explained by the fact that this method detects only the dominant frequency of each beam. Since we have already established with reference to Fig. 3 that the dominant amplitudes in this range are A_{1+} and A_{2+} the detected frequency will be f^+ . It may be observed that the measured frequencies in the lock-in zone are f^- or f^+ depending on the initial conditions. For instance, for the case where $a = b$, the lock-in
 30 frequency is f^+ , while for $a = -b$, the lock-in frequency is f^- . It can also be seen that boundaries of the lock-in zone are $[-2, 2] g$.

Figs. 13 and 14 are flow charts showing alternative methods for determining the boundary of the lock-in zone during calibration of the accelerometer based on the above-

mentioned characteristics of the lock-in zone. In both cases, to simplify the flow charts only the positive boundary of the lock-in zone is calibrated and an identical procedure is performed to determine the negative boundary. Thus, in Fig. 13 the device is subjected to varying accelerations typically increasing positively from zero within a given range. The boundary is established by measuring the acceleration at which the amplitudes of the dominant and sub-dominant frequencies are equal.

In Fig. 14, the device is subjected to varying accelerations extending beyond the expected acceleration at the crossover boundary and decreasing in absolute value until the respective dominant frequencies of both beams are first equal. At this point, we have hit the boundary of the lock-in zone. If we were to decrease positive acceleration or increase negative acceleration (i.e. make it less negative), we would remain inside the lock-in zone and hit the opposite boundary when the measured dominant frequencies of the two beams are different. We mention this as yet a further alternative approach that could be adopted to determine the boundaries of the lock-in zone. In both cases, the crossover points correspond to the positive and negative accelerations having highest maximum absolute value for which the dominant or measured vibration frequencies of the two beams is equal.

Finally, for the sake of completeness, although we have not discussed damping, it can be shown that the same principles apply also to damped oscillation of the accelerometer. In a practical system, damping will occur but the AC voltage 14 applied to the respective pairs of electrodes 15 and 15' across the two beams may be adjusted using a feedback control to counter the effects of damping.

Conclusion

The present invention differs from known approaches in that it employs measurement of vibration amplitudes within a predefined range that includes the lock-in zone and of vibration frequencies outside the predefined range. In the case where the predefined range is set at the exact boundary of the lock-in zone, the measured amplitudes and frequencies may correspond to the dominant frequencies and the measured values are used "as is", there being no requirement to determine sub-dominant amplitudes or frequencies.

CLAIMS:

1. A method for determining acceleration using a vibrating beam accelerometer (10) that can be modelled mathematically as a system having two beams (11, 11') operating in a differential push-pull mode, such that each of the two beams vibrates at corresponding first and second frequencies of vibration (f_+ , f_-) at which the respective amplitude (A_+ , A_-) is dominant outside a predetermined lock-in zone within which the respective dominant frequencies (f_1 , f_2) of the first and second beam are equal, said method comprising:

- (a) inside a predefined range that includes the lock-in zone, deriving acceleration as a function of measured vibration amplitudes of the two beams; and
- (b) outside the predefined range, deriving acceleration as a function of a difference between the measured vibration frequencies of the two beams.

2. The method according to claim 1, comprising:

- (a) subjecting the accelerometer (10) to an acceleration and measuring the respective dominant frequency of vibration (f_1 , f_2) for each beam;
- (b) if the dominant frequency exceeds a predetermined crossover frequency at opposing boundaries of the predefined range:
 - i) measuring a corresponding differential frequency Δf equal to a difference in the respective measured dominant frequency for each beam; and
 - ii) determining acceleration (Γ) from the measured differential frequency Δf and a pre-calibrated frequency-acceleration characteristic;
- (c) if the dominant frequency is smaller than the crossover frequency at opposing boundaries of the predefined range:
 - i) computing acceleration (Γ) as a function, $A(\Gamma)$ such that:

$$A(\Gamma) = \varphi\left(\frac{\alpha}{\beta}\right) + \varphi\left(\frac{\beta}{\alpha}\right)$$

where: $A(\Gamma)$ is an effective amplitude from which acceleration is derived according to a known approximately linear function,

φ is a function of measured or derived amplitudes of vibration α and β for the respective beams or of functions thereof, such that:

if the dominant frequency is smaller than a predetermined cross-over frequency at opposing boundaries of the lock-in range, α and β are either the respective dominant amplitudes or the respective sub-dominant amplitudes of the two beams, else

if the dominant frequency is greater than the predetermined crossover frequency at opposing boundaries of the lock-in range, α and β are functions of both the dominant and sub-dominant amplitudes of the two beams.

3. The method according to claim **2**, wherein:

the predefined range is set at the boundary of the lock-in zone;

α is the amplitude (A1+) of the dominant frequency of vibration of Beam 1 inside the lock-in zone; and

β is the amplitude (A2+) of the dominant frequency of vibration of Beam 2 inside the lock-in zone.

4. The method according to claim **3**, wherein acceleration (Γ) is a solution to the following equation:

$$A^+(\Gamma) = k \cdot \Gamma$$

where: $A^+(\Gamma)$ is a function of the amplitudes of vibration for both beams at respective dominant frequencies of vibration (f_+) for each beam, and

k is a constant.

5. The method according to claim **4**, wherein $A^+(\Gamma)$ is given by:

$$A^+(\Gamma) \equiv \frac{A_{1+}(\Gamma)}{A_{2+}(\Gamma)} - \frac{A_{2+}(\Gamma)}{A_{1+}(\Gamma)}$$

6. The method according to claim **2**, wherein:

the predefined range is set at the boundary of the lock-in zone;

α is the amplitude (A1-) of the sub-dominant frequency of vibration of Beam 1 inside the lock-in zone; and

β is the amplitude (A_{2-}) of the sub-dominant frequency of vibration of Beam 2 inside the lock-in zone.

7. The method according to claim 6, wherein acceleration (Γ) is a solution to the following equation:

$$A^-(\Gamma) = k \cdot \Gamma$$

where: $A^-(\Gamma)$ is a function of the amplitudes of vibration for both beams at respective sub-dominant frequencies of vibration (f_-) for each beam, and k is a constant.

8. The method according to claim 7, wherein $A^-(\Gamma)$ is given by:

$$A^-(\Gamma) \equiv \frac{A_{2-}(\Gamma)}{A_{1-}(\Gamma)} - \frac{A_{1-}(\Gamma)}{A_{2-}(\Gamma)}$$

9. The method according to claim 4 or 7, wherein the constant k is given by:

$$k = \frac{2M\lambda}{M_0}$$

where: M is the mass of the seismic mass,

$$M_0 = \frac{m_0 M}{M + m_0},$$

m_0 is the mass of each beam, λ is a scale factor such that $k^\pm(\Gamma) = k_0(1 \pm \lambda\Gamma)$, k_0 is the spring stiffness at acceleration = 0, and $k^\pm(\Gamma)$ is the spring stiffness at acceleration = Γ .

10. The method according to claim 1 or 2, wherein acceleration Γ inside the predefined range is a solution to the following equation:

$$A^0(\Gamma) = \frac{4M\lambda}{M_0} \sum_{n=0}^{\infty} \lambda^{2n} \Gamma^{2n+1}$$

where: $A^0(\Gamma)$ is a function of the amplitudes of vibration for both beams at both the first and second frequencies of vibration (f_-, f_+) for each beam, M is the mass of the seismic mass,

$$M_0 = \frac{m_0 M}{M + m_0},$$

m_0 is the mass of each beam,

λ is a scale factor such that $k^\pm(\Gamma) = k_0(1 \pm \lambda\Gamma)$,

k_0 is the spring stiffness at acceleration = 0, and

$k^\pm(\Gamma)$ is the spring stiffness at acceleration = Γ .

- 5 **11.** The method according to claim **10**, wherein $A^0(\Gamma)$ is given by:

$$A^0(\Gamma) = \left(\frac{A_{2-}(\Gamma)}{A_{1-}(\Gamma)} - \frac{A_{2+}(\Gamma)}{A_{1+}(\Gamma)} \right) - \left(\frac{A_{1-}(\Gamma)}{A_{2-}(\Gamma)} - \frac{A_{1+}(\Gamma)}{A_{2+}(\Gamma)} \right)$$

where: $A_{1+}(\Gamma)$ and $A_{2+}(\Gamma)$ are the respective amplitudes of the first vibration frequencies (f_+) of Beams 1 and 2;

10 $A_{1-}(\Gamma)$ and $A_{2-}(\Gamma)$ are the respective amplitudes of the second vibration frequencies (f_-) of Beams 1 and 2.

- 12.** The method according to claim **10** or **11**, wherein $\lambda\Gamma \ll 1$ and acceleration Γ inside the predefined range is an approximate solution to the following equation:

$$A^0(\Gamma) = \frac{4M\lambda}{M_0}\Gamma$$

- 15 **13.** The method according to claim **12**, wherein the acceleration $A(\Gamma)$ inside the lock-in zone is approximated to:

$$A(\Gamma) \approx \frac{2M\lambda}{M_0}\Gamma$$

where: $A(\Gamma) = A^+(\Gamma)$ or $A^-(\Gamma)$, i.e. functions of the dominant or sub-dominant amplitude, respectively,

M is the mass of the seismic mass,

20 Γ is the acceleration,

$$M_0 = \frac{m_0 M}{M + m_0},$$

m_0 is the mass of each beam,

λ is a scale factor such that $k^\pm(\Gamma) = k_0(1 \pm \lambda\Gamma)$,

k_0 is the spring stiffness at acceleration = 0, and

25 $k^\pm(\Gamma)$ is the spring stiffness at acceleration = Γ .

14. The method for determining acceleration according to any one of claims **2** to **13**, wherein the pre-calibrated frequency-acceleration characteristic is a linear function.

15. The method according to claim **14**, wherein the linear function is:

$$\Delta\omega \approx \lambda \sqrt{\frac{k_0}{m_0}} \Gamma$$

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where: $\omega=2\pi f$,

Δf = measured frequency difference;

Γ = acceleration;

m_0 = mass of each beam;

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k_0 = stiffness of each beam; and

λ = a known scale factor.

16. The method according to any one of claims **1** to **13**, further including calibrating the vibrating beam accelerometer by:

- i)** obtaining for each of the beams a respective pair of amplitude-acceleration characteristics by subjecting the accelerometer to a range of known accelerations and measuring corresponding signal amplitudes at each acceleration such that for each beam there are corresponding first and second frequencies (f^- , f^+) at which the respective amplitude is dominant at different accelerations throughout said range; and
- ii)** determining respective crossover points for each beam where at each crossover point the respective amplitudes (A^- , A^+) at both the first and second frequencies (f^- , f^+) are the same and equal to A_1 and A_2 for the respective beams.

17. The method according to any one of claims **1** to **13**, further including calibrating the vibrating beam accelerometer by:

- i)** subjecting the accelerometer to a range of known positive and negative accelerations such that for each beam there are corresponding first and second frequencies (f^- , f^+) one of which is dominant and one of which is sub-dominant;

- ii) for each known acceleration measuring the respective dominant frequency of each beam; and
- iii) determining the respective crossover points at which the dominant vibration frequencies of the two beams are equal and the absolute values of positive
5 and negative accelerations for both beams are maximum.

18. The method according to any one of claims **1** to **13**, further including measuring the respective dominant vibration frequencies of the two beams, and establishing that acceleration is inside the lock-in zone when the measured frequencies are equal for both beams.