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METHODS FOR SUBSURFACE PARAMETER ESTIMATION IN FULL WAVEFIELD INVERSION AND REVERSE-TIME MIGRATION

CROSS-REFERENCE TO RELATED APPLICATION

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[0001] This application claims the benefit of U.S. Provisional Application No. 61/303,148 filed February 10, 2010, entitled Methods for Subsurface Parameter Estimation in Full Wavefield Inversion and Reverse-Time Migration, which is incorporated by reference, in its entirety, for all purposes.

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FIELD OF THE INVENTION

[0002] This invention relates generally to the field of geophysical prospecting and, more particularly, to seismic data processing. Specifically, the invention is a method for subsurface parameter estimation in full wave field inversion and reverse-time migration.

BACKGROUND OF THE INVENTION

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[0003] Full wavefield inversion (FWI) in exploration seismic processing relies on the calculation of the gradient of an objective function with respect to the subsurface model parameters [12]. An objective function E is usually given as an L_2 norm as

$$E = \frac{1}{2} \iiint |p(\mathbf{r}_g, \mathbf{r}_s; t) - p_b(\mathbf{r}_g, \mathbf{r}_s; t)|^2 dt dS_g dS_s, \quad (1)$$

where p and p_b are the measured pressure, i.e. seismic amplitude, and the modeled pressure in the background subsurface model at the receiver location $\mathbf{r}_g$ for a shot located at $\mathbf{r}_s$. In iterative inversion processes, the *background* medium is typically the medium resulting from the previous inversion cycle. In non-iterative inversion processes or migrations, the background medium is typically derived using conventional seismic processing techniques such as migration velocity analysis. The objective function is integrated over all time t , and the surfaces S_g and S_s that are defined by the spread of the receivers and the shots. We define $K_d(\mathbf{r}) = K(\mathbf{r}) - K_b(\mathbf{r})$ and $\rho_d(\mathbf{r}) = \rho(\mathbf{r}) - \rho_b(\mathbf{r})$, where $K(\mathbf{r})$ and $\rho(\mathbf{r})$ are the true bulk modulus and density, and $K_b(\mathbf{r})$ and $\rho_b(\mathbf{r})$ are the bulk modulus and the density of the background model at the subsurface location $\mathbf{r}$. We also define the difference between the measured and the modeled pressure to be $p_d(\mathbf{r}_g, \mathbf{r}_s; t) = p(\mathbf{r}_g, \mathbf{r}_s; t) - p_b(\mathbf{r}_g, \mathbf{r}_s; t)$.

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[0004] The measured pressure p , satisfies the wave equation

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$$\rho \nabla \cdot \left(\frac{1}{\rho} \nabla p \right) - \frac{\rho}{K} \ddot{p} = -q(t) \delta(\mathbf{r} - \mathbf{r}_s), \quad (2)$$

or

$$(\rho_b + \rho_d) \nabla \cdot \left(\frac{1}{\rho_b + \rho_d} \nabla (p_b + p_d) \right) - \frac{\rho_b + \rho_d}{K_b + K_d} (\ddot{p}_b + \ddot{p}_d) = -q(t) \delta(\mathbf{r} - \mathbf{r}_s), \quad (3)$$

where $q(t)$ is the source signature. By expanding the perturbation terms and keeping only the
 5 1st order Born approximation terms, one can derive the Born scattering equation for the pressure p_d ,

$$\rho_b \nabla \cdot \left(\frac{1}{\rho_b} \nabla p_d \right) - \frac{\rho_b}{K_b} \ddot{p}_d = - \left[\frac{\rho_b K_d}{K_b^2} \ddot{p}_b - \rho_b \nabla \cdot \left(\frac{\rho_d}{\rho_b^2} \nabla p_b \right) \right], \quad (4)$$

and so p_d satisfies

$$p_d(\mathbf{r}_g, \mathbf{r}_s; t) = \int \left[\rho_b(\mathbf{r}') \frac{K_d(\mathbf{r}')}{K_b^2(\mathbf{r}')} \ddot{p}_b(\mathbf{r}', \mathbf{r}_s; t) - \rho_b(\mathbf{r}') \nabla \cdot \left(\frac{\rho_d(\mathbf{r}')}{\rho_b^2(\mathbf{r}')} \nabla p_b(\mathbf{r}', \mathbf{r}_s; t) \right) \right] * g_b(\mathbf{r}_g, \mathbf{r}'; t) dV', \quad (5)$$

10 where V' is the volume spanned by $\mathbf{r}'$, and g_b is the Green's function in the background medium.

[0005] One can derive the equations for the gradients of p_b using Eq. (5) and by considering the fractional change δp_b due to fractional change δK_b and $\delta \rho_b$ over an infinitesimal volume dV ,

$$15 \quad \frac{\partial p_b}{\partial K_b(\mathbf{r})} = \frac{\rho_b(\mathbf{r}) dV}{K_b^2(\mathbf{r})} g_b(\mathbf{r}_g, \mathbf{r}; t) * \ddot{p}_b(\mathbf{r}, \mathbf{r}_s; t), \quad (6)$$

and

$$\frac{\partial p_b}{\partial \rho_b(\mathbf{r})} = F^{-1} \left\{ \frac{dV}{\rho_b(\mathbf{r})} \nabla G_b(\mathbf{r}_g, \mathbf{r}; f) \cdot \nabla P_b(\mathbf{r}, \mathbf{r}_s; f) \right\}, \quad (7)$$

where $P_b = F \{p_b\}$, $P_d = F \{p_d\}$, $G_b = F \{g_b\}$, and F and F^{-1} are the Fourier transform and the inverse Fourier transform operators.

20 [0006] By using Eqs. 6 and 7, and using the reciprocity relationship $\rho_b(\mathbf{r}) G_b(\mathbf{r}_g, \mathbf{r}) = \rho_b(\mathbf{r}_g) G_b(\mathbf{r}, \mathbf{r}_g)$,

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$$\begin{aligned}
\frac{\partial E}{\partial K_b(\mathbf{r})} &= - \iiint p_d \frac{\partial p_b}{\partial K_b(\mathbf{r})} dt dS_g dS_s \\
&= - \frac{\rho_b(\mathbf{r}) dV}{K_b^2(\mathbf{r})} \iiint (i2\pi f)^2 P_b(\mathbf{r}, \mathbf{r}_s; f) G_b(\mathbf{r}_g, \mathbf{r}; f) P_d^*(\mathbf{r}_g, \mathbf{r}_s; f) df dS_g dS_s \quad (8) \\
&= - \frac{\rho_b(\mathbf{r}) dV}{K_b^2(\mathbf{r})} \iint \dot{p}_b(\mathbf{r}, \mathbf{r}_s; t) \int \frac{\rho_b(\mathbf{r}_g)}{\rho_b(\mathbf{r})} g_b(\mathbf{r}, \mathbf{r}_g; -t) * \dot{p}_d(\mathbf{r}_g, \mathbf{r}_s; t) dS_g dt dS_s,
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial E}{\partial \rho_b(\mathbf{r})} &= - \iiint p_d \frac{\partial p_b}{\partial \rho_b(\mathbf{r})} dt dS_g dS_s \\
&= - \frac{dV}{\rho_b(\mathbf{r})} \iiint P_d^*(\mathbf{r}_g, \mathbf{r}_s; f) \nabla P_b(\mathbf{r}, \mathbf{r}_s; f) \cdot \nabla G_b(\mathbf{r}_g, \mathbf{r}; f) df dS_g dS_s \quad (9) \\
&= - \frac{dV}{\rho_b(\mathbf{r})} \iint \nabla p_b(\mathbf{r}, \mathbf{r}_s; t) \cdot \left[\int \nabla \left(g_b(\mathbf{r}, \mathbf{r}_g; -t) * \frac{\rho_b(\mathbf{r}_g)}{\rho_b(\mathbf{r})} p_d(\mathbf{r}_g, \mathbf{r}_s; t) \right) dS_g \right] dt dS_s.
\end{aligned}$$

One can then use Eqs. 8 and 9 to perform full wavefield inversion in an iterative manner.

- 5 [0007] Reverse-time migration (RTM) is based on techniques similar to gradient computation in FWI, where the forward propagated field is cross-correlated with the time-reversed received field. By doing so, RTM overcomes limitations of ray-based migration techniques such as Kirchhoff migration. In RTM, the migrated image field M at subsurface location $\mathbf{r}$ is given as

$$10 \quad M(\mathbf{r}) = \iint p_b(\mathbf{r}, \mathbf{r}_s; t) \int g_b(\mathbf{r}, \mathbf{r}_g; -t) * p(\mathbf{r}_g, \mathbf{r}_s; t) dS_g dt dS_s, \quad (10)$$

which is very similar to the gradient equation 8 of FWI.

- [0008] While Eqs. 8 and 9 provide the framework for inverting data into subsurface models, the convergence of the inversion process often is very slow. Also, RTM using Eq. 10 suffers weak amplitude in the deep section due to spreading of the wavefield. Many attempts
15 have been made to improve the convergence of FWI or improve the amplitude of the reverse-time migration by using the Hessian of the objective function [9], i.e., a second derivative of the objective function. Computation of the Hessian, however, is not only prohibitively expensive in computational resources, but it requires prohibitively large storage space for a realistic 3-D inversion problem. Furthermore, FWI using the full Hessian matrix may result
20 in suboptimal inversion [2].

- [0009] One may be able to perform more stable inversion by lumping non-diagonal terms of the Hessian into the diagonal terms [2]. These, however, still require computation of

the full Hessian matrix or at least a few off-diagonal terms of the Hessian matrix, which can be costly computations. While one may choose to use the diagonal of the Hessian only [11], this is valid only in the high frequency asymptotic regime with infinite aperture [1, 7].

[0010] Plessix and Mulder tried to overcome these difficulties by first computing an approximate diagonal Hessian, then by scaling these by $z^\alpha v_p^\beta$, where z is the depth and v_p is the compressional wave velocity [7]. From numerical experiment, they have determined that the best scaling parameter is $z^{0.5} v_p^{0.5}$. This approach, however, does not provide quantitative inversion of the subsurface medium parameters with correct units, since only approximate scaling has been applied. Furthermore, this approach was applied to RTM where only variations in compressional wave velocity is considered, and so may not be applicable to FWI where other elastic parameters such as density and shear wave velocity vary in space.

SUMMARY OF THE INVENTION

[0011] In one embodiment, the invention is a method for determining a model of a physical property in a subsurface region from inversion of seismic data, acquired from a seismic survey of the subsurface region, or from reverse time migration of seismic images from the seismic data, said method comprising determining a seismic resolution volume for the physical property and using it as a multiplicative scale factor in computations performed on a computer to either

(a) convert a gradient of data misfit in an inversion, or

(b) compensate reverse-time migrated seismic images,

to obtain the model of the physical property or an update to an assumed model.

[0012] In some embodiments of the inventive method, the gradient of data misfit or the reverse time migrated seismic images are multiplied by additional scale factors besides seismic resolution volume, wherein the additional scale factors include a source illumination factor, a receiver illumination factor, and a background medium properties factor. This results in a model of the physical property or an update to an assumed model having correct units.

[0013] It will be obvious to those who work in the technical field that in any practical application of the invention, inversion or migration of seismic data must be performed on a computer specifically programmed to carry out that operation.

BRIEF DESCRIPTION OF THE DRAWINGS

[0014] The present invention and its advantages will be better understood by referring to the following detailed description and the attached drawings in which:

5 [0015] Fig. 1 is a flowchart showing basic steps in one embodiment of the present inventive method;

[0016] Figs. 2-5 pertain to a first example application of the present invention, where Fig. 2 shows the gradient of the objective function with respect to the bulk modulus in $\text{Pa m}^4/\text{s}$, computed using Eq. 8;

10 [0017] Fig. 3 shows the bulk modulus update $\langle K_d(\mathbf{r}) \rangle$ in Pa computed using Eq. 18 and the gradient in Fig. 2;

[0018] Fig. 4 shows the bulk modulus update $\langle K_d(\mathbf{r}) \rangle$ in Pa computed using Eq. 24 and the gradient in Fig. 2;

[0019] Fig. 5 shows the gradient of the objective function with respect to the density in $\text{Pa}^2 \text{ m}^7/\text{s kg}$, computed using Eq. 9;

15 [0020] Figs. 6 and 7 pertain to a second example application of the present invention, where Fig. 6 shows the density update $\langle \rho_d(\mathbf{r}) \rangle$ in kg/m^3 computed using Eq. 28 and the gradient in Fig. 5; and

[0021] Fig. 7 shows the density update $\langle \rho_d(\mathbf{r}) \rangle$ in kg/m^3 computed using Eq. 34 and the gradient in Fig. 5.

20 [0022] The invention will be described in connection with example embodiments. However, to the extent that the following detailed description is specific to a particular embodiment or a particular use of the invention, this is intended to be illustrative only, and is not to be construed as limiting the scope of the invention. On the contrary, it is intended to cover all alternatives, modifications and equivalents that may be included within the scope of
25 the invention, as defined by the appended claims.

DETAILED DESCRIPTION OF EXAMPLE EMBODIMENTS

[0023] We derive inversion equations for K_d and ρ_d in the present invention using Eqs. 8 and 9. This is done by first taking advantage of the fact that p_d in Eqs. 8 and 9 can

also be expanded using the Born approximation in Eq. 5. Neglecting the crosstalk components between K_d and ρ_d , Eqs. 8 and 9 can then be approximated as

$$\frac{\partial E}{\partial K_b(\mathbf{r})} \approx -\frac{\rho_b(\mathbf{r})dV}{K_b^2(\mathbf{r})} \iiint \frac{\rho_b(\mathbf{r}')K_d(\mathbf{r}')}{K_b^2(\mathbf{r}')} \left[g_b(\mathbf{r}_g, \mathbf{r}'; t) * \ddot{p}_b(\mathbf{r}', \mathbf{r}_s; t) \right] \times \left[g_b(\mathbf{r}_g, \mathbf{r}; t) * \ddot{p}_b(\mathbf{r}, \mathbf{r}_s; t) \right] dt dS_g dS_s dV', \quad (11)$$

and

$$\frac{\partial E}{\partial \rho_b(\mathbf{r})} \approx -\frac{dV}{\rho_b(\mathbf{r})} \iiint \frac{\rho_d(\mathbf{r}')}{\rho_b(\mathbf{r}')} \left[\nabla' P_b^*(\mathbf{r}', \mathbf{r}_s; f) \cdot \nabla' G_b^*(\mathbf{r}_g, \mathbf{r}'; f) \right] \times \left[\nabla P_b(\mathbf{r}, \mathbf{r}_s; f) \cdot \nabla G_b(\mathbf{r}_g, \mathbf{r}; f) \right] df dS_g dS_s dV'. \quad (12)$$

By changing the orders of the integral, Eq. 11 can be rewritten in the frequency domain as

$$\frac{\partial E}{\partial K_b(\mathbf{r})} \approx -\frac{\rho_b(\mathbf{r})dV}{K_b^2(\mathbf{r})} \iint (2\pi f)^4 \frac{\rho_b(\mathbf{r}')dV K_d(\mathbf{r}')}{K_b^2(\mathbf{r}')} \times \left\{ \int \frac{\rho_b(\mathbf{r}_g)}{\rho_b(\mathbf{r})} G_b(\mathbf{r}, \mathbf{r}_g; f) G_b^*(\mathbf{r}_g, \mathbf{r}'; f) dS_g \right\} \left\{ \int P_b(\mathbf{r}, \mathbf{r}_s; f) P_b^*(\mathbf{r}', \mathbf{r}_s; f) dS_s \right\} dV' df. \quad (13)$$

The first integral term

$$\int \frac{\rho_b(\mathbf{r}_g)}{\rho_b(\mathbf{r})} G_b(\mathbf{r}, \mathbf{r}_g; f) G_b^*(\mathbf{r}_g, \mathbf{r}'; f) dS_g \quad (14)$$

in Eq. 13 is the approximation to the time reversal backpropagation for a field generated by an impulse source at $\mathbf{r}'$, measured over the surface S_g , and then backpropagated to $\mathbf{r}$ (See, e.g., Refs. [8, 3]). The wavefield due to this term propagates back towards the impulse source location at $\mathbf{r}'$, and behaves similar to the spatial delta function $\delta(\mathbf{r}-\mathbf{r}')$ when $t=0$, if the integral surface S_g embraces the point $\mathbf{r}'$. This wavefield is correlated with the wavefield due to the second term, $\int P_b(\mathbf{r}, \mathbf{r}_s; f) P_b^*(\mathbf{r}', \mathbf{r}_s; f) dS_s$, to form the gradient near $\mathbf{r}=\mathbf{r}'$. The correlation of the first and the second term then decays rapidly near $\mathbf{r}=\mathbf{r}'$. The present invention recognizes that the zone in which the amplitude of the correlation term is not negligible is determined by the seismic resolution of the survey. In the present invention, we make an approximation that

$$\int (2\pi f)^4 \left\{ \int \frac{\rho_b(\mathbf{r}_g)}{\rho_b(\mathbf{r})} G_b(\mathbf{r}, \mathbf{r}_g; f) G_b^*(\mathbf{r}_g, \mathbf{r}'; f) dS_g \right\} \left\{ \int P_b(\mathbf{r}, \mathbf{r}_s; f) P_b^*(\mathbf{r}', \mathbf{r}_s; f) dS_s \right\} df \quad (15)$$

$$\approx I_K(\mathbf{r}) V_K(\mathbf{r}) \delta(\mathbf{r} - \mathbf{r}'),$$

where

$$I_K(\mathbf{r}) = \int (2\pi f)^4 \left\{ \int \frac{\rho_b(\mathbf{r}_g)}{\rho_b(\mathbf{r})} G_b(\mathbf{r}, \mathbf{r}_g; f) G_b^*(\mathbf{r}_g, \mathbf{r}; f) dS_g \right\} \quad (16)$$

$$\times \left\{ \int P_b(\mathbf{r}, \mathbf{r}_s; f) P_b^*(\mathbf{r}, \mathbf{r}_s; f) dS_s \right\} df,$$

and $V_K(\mathbf{r})$ is the seismic resolution at the subsurface location $\mathbf{r}$. Equation 15 is equivalent to the mass lumping of the Gauss-Newton Hessian matrix by assuming that the non-diagonal components are equal to the diagonal components when the non-diagonal components are within the seismic resolution volume of the diagonal component, and those outside the resolution volume zero. In other words, Eq. 15 is equivalent to implicitly counting the number of non-diagonal components N_i of the Gauss-Newton Hessian matrix in each i -th row that are significant in amplitude by using the seismic resolution volume of the survey, then multiplying the diagonal component of the i -th row by N_i .

[0024] Seismic resolution volume may be thought of as the minimal volume at $\mathbf{r}$ that a seismic imaging system can resolve under given seismic data acquisition parameters. Two small targets that are within one seismic resolution volume of each other are usually not resolved and appear as one target in the seismic imaging system. The resolution volumes for different medium parameters are different due to the difference in the radiation pattern. For example, the targets due to a bulk modulus perturbation yield a monopole radiation pattern, while those due to a density perturbation yield a dipole radiation pattern. Seismic resolution volume $V_K(\mathbf{r})$ for bulk modulus can be computed, for example, using a relatively inexpensive ray approximation [6, 4]. Persons who work in the technical field may know other ways to estimate the resolution volume. For example, one may be able to empirically estimate the resolution volume by distributing point targets in the background medium, and by investigating spread of the targets in the seismic image. If the background medium contains discontinuity in velocity due to iterative nature of the inversion, the background medium may need to be smoothed for ray tracing. One may also make a simplifying assumption that the wavenumber coverage is uniform. The seismic resolution volume in this case is a sphere with radius $\sigma \approx (5/18\pi)^{0.5} v_p(\mathbf{r}) / f_p$, where f_p is the peak frequency [6]. One

may also employ an approximation, $\sigma \approx v_p(\mathbf{r})T/4 = v_p(\mathbf{r})/4B$, where T and B are the effective time duration and the effective bandwidth of the source waveform, following the radar resolution equation [5].

[0025] Equation 11 can then be simplified using Eq. 15 as

$$5 \quad \frac{\partial E}{\partial K_b(\mathbf{r})} \approx -\frac{\rho_b^2(\mathbf{r})dV}{K_b^4(\mathbf{r})} \langle K_d(\mathbf{r}) \rangle I_K(\mathbf{r}) V_K(\mathbf{r}), \quad (17)$$

and so

$$\langle K_d(\mathbf{r}) \rangle \approx -\frac{K_b^4(\mathbf{r})}{\rho_b^2(\mathbf{r})dV} \frac{1}{I_K(\mathbf{r}) V_K(\mathbf{r})} \frac{\partial E}{\partial K_b(\mathbf{r})}, \quad (18)$$

where $\langle K_d(\mathbf{r}) \rangle$ is the spatial average of K_d over the seismic resolution at spatial location $\mathbf{r}$.

[0026] Equation 16 can be further simplified if we use free-space Green's function

$$10 \quad G(\mathbf{r}_g, \mathbf{r}; f) = \frac{1}{4\pi |\mathbf{r} - \mathbf{r}_g|} e^{ik|\mathbf{r} - \mathbf{r}_g|}, \quad (19)$$

and assume that S_g subtends over half the solid angle. Equation 16 then simplifies to

$$I_K(\mathbf{r}) \approx I_{K,s}(\mathbf{r}) I_{K,g}(\mathbf{r}), \quad (20)$$

where

$$I_{K,s}(\mathbf{r}) = \int |\ddot{p}_b(\mathbf{r}, \mathbf{r}_s; t)|^2 dt, \quad (21)$$

15 and

$$I_{K,g}(\mathbf{r}) = \int \frac{\rho_b(\mathbf{r}_g)}{\rho_b(\mathbf{r})} G_b(\mathbf{r}, \mathbf{r}_g; f) G_b^*(\mathbf{r}_g, \mathbf{r}; f) dS_g \approx \frac{1}{8\pi} \frac{\rho_b(\mathbf{r}_g)}{\rho_b(\mathbf{r})}. \quad (22)$$

The term $I_{K,s}(\mathbf{r})$ may be recognized as the source illumination in the background model, and $I_{K,g}(\mathbf{r})$ can be understood to be the receiver illumination. One may also be able to vary the solid angle of integral at each subsurface location $\mathbf{r}$ following the survey geometry. Equation

20 11 then becomes

$$\frac{\partial E}{\partial K_b(\mathbf{r})} \approx -\frac{\rho_b^2(\mathbf{r})dV}{K_b^4(\mathbf{r})} \langle K_d(\mathbf{r}) \rangle I_{K,s}(\mathbf{r}) I_{K,g}(\mathbf{r}) V_K(\mathbf{r}) \quad (23)$$

and

$$\langle K_d(\mathbf{r}) \rangle \approx -\frac{K_b^4(\mathbf{r})}{\rho_b^2(\mathbf{r})dV} \frac{1}{I_{K,s}(\mathbf{r})I_{K,g}(\mathbf{r})V_K(\mathbf{r})} \frac{\partial E}{\partial K_b(\mathbf{r})}. \quad (24)$$

[0027] Equations 18 and 24 show that one can convert the gradient $\partial E / \partial K_b(\mathbf{r})$ into a medium parameter $\langle K_d(\mathbf{r}) \rangle$ by scaling the gradient by the source and receiver illumination, resolution volume, and the background medium properties. If the inversion process is not iterative, one should be able to use Eq. 24 for parameter inversion. If the inversion process is iterative, one can use $\langle K_d(\mathbf{r}) \rangle$ in Eq. 24 as a preconditioned gradient for optimization techniques such as steepest descent, conjugate gradient (CG), or Newton CG method. It is important to note that Eqs. (18) and (24) yield bulk modulus with the correct units, i.e. are dimensionally correct, because all terms have been taken into account, and none have been neglected to simplify the computation as is the case with some published approaches. The published approaches that neglect one or more of the terms source illumination, receiver illumination, background medium properties and seismic resolution volume, will not result in the correct units, and therefore will need some sort of ad hoc fix up before they can be used for iterative or non-iterative inversion.

15 [0028] For density gradient, we make an assumption similar to that in Eq. 15,

$$\begin{aligned} \iint \left[\nabla' P_b^*(\mathbf{r}', \mathbf{r}_s; f) \cdot \nabla' G_b^*(\mathbf{r}_g, \mathbf{r}'; f) \right] \\ \times \left[\nabla P_b(\mathbf{r}, \mathbf{r}_s; f) \cdot \nabla G_b(\mathbf{r}_g, \mathbf{r}; f) \right] dS_g dS_s df \approx I_\rho(\mathbf{r}) V_\rho(\mathbf{r}) \delta(\mathbf{r} - \mathbf{r}'), \end{aligned} \quad (25)$$

where

$$I_\rho(\mathbf{r}) = \iiint \left| \nabla P_b(\mathbf{r}, \mathbf{r}_s; f) \cdot \nabla G_b(\mathbf{r}_g, \mathbf{r}; f) \right|^2 dS_g dS_s df, \quad (26)$$

and $V_\rho(\mathbf{r})$ is the seismic resolution for density ρ at subsurface location $\mathbf{r}$. The resolution volume $V_\rho(\mathbf{r})$ differs from $V_K(\mathbf{r})$, since the wavenumbers are missing when the incident and the scattered field are nearly orthogonal to each other. This is due to the dipole radiation pattern of the density perturbation, as discussed previously. As was done for $V_K(\mathbf{r})$, one can employ ray tracing to compute the resolution volume $V_\rho(\mathbf{r})$ while accounting for these missing near-orthogonal wavenumbers. Alternatively, one may be able assume $V_K(\mathbf{r}) \approx V_\rho(\mathbf{r})$ by neglecting the difference in the wavenumber coverage.

[0029] The gradient equation 12 can then be rewritten as

$$\frac{\partial E}{\partial \rho_b(\mathbf{r})} \approx -\frac{dV}{\rho_b^2(\mathbf{r})} \langle \rho_d(\mathbf{r}) \rangle I_\rho(\mathbf{r}) V_\rho(\mathbf{r}), \quad (27)$$

and so

$$\langle \rho_d(\mathbf{r}) \rangle \approx -\frac{\rho_b^2(\mathbf{r})}{dV} \frac{1}{I_\rho(\mathbf{r}) V_\rho(\mathbf{r})} \frac{\partial E}{\partial \rho_b(\mathbf{r})}, \quad (28)$$

where $\langle \rho_d(\mathbf{r}) \rangle$ is the spatial average of $\rho_d(\mathbf{r})$ over the seismic resolution volume $V_\rho(\mathbf{r})$.

- 5 [0030] We can further simplify Eq. 25 by using the vector identity $(a \cdot b)(c \cdot d) = (a \cdot d)(b \cdot c) + (a \times c) \cdot (b \times d)$ to obtain

$$\begin{aligned} & \left[\nabla' P_b^*(\mathbf{r}', \mathbf{r}_s; f) \cdot \nabla' G_b^*(\mathbf{r}_g, \mathbf{r}'; f) \right] \left[\nabla P_b(\mathbf{r}, \mathbf{r}_s; f) \cdot \nabla G_b(\mathbf{r}_g, \mathbf{r}; f) \right] \\ &= \left[\nabla' P_b^*(\mathbf{r}', \mathbf{r}_s; f) \cdot \nabla P_b(\mathbf{r}, \mathbf{r}_s; f) \right] \left[\nabla' G_b^*(\mathbf{r}_g, \mathbf{r}'; f) \cdot \nabla G_b(\mathbf{r}_g, \mathbf{r}; f) \right] \\ &+ \left[\nabla' P_b^*(\mathbf{r}', \mathbf{r}_s; f) \times \nabla G_b(\mathbf{r}_g, \mathbf{r}; f) \right] \left[\nabla' G_b^*(\mathbf{r}_g, \mathbf{r}'; f) \times \nabla P_b(\mathbf{r}, \mathbf{r}_s; f) \right]. \end{aligned} \quad (29)$$

- The second term in the right-hand side of Eq. 29 is the correction term for the dipole radiation pattern of the scattered field, and so it reaches a maximum when ∇P_b and ∇G_b are
10 orthogonal to each other. Neglecting this correction term,

$$\begin{aligned} & \left[\nabla' P_b^*(\mathbf{r}', \mathbf{r}_s; f) \cdot \nabla' G_b^*(\mathbf{r}_g, \mathbf{r}'; f) \right] \left[\nabla P_b(\mathbf{r}, \mathbf{r}_s; f) \cdot \nabla G_b(\mathbf{r}_g, \mathbf{r}; f) \right] \\ &\approx \left[\nabla' P_b^*(\mathbf{r}', \mathbf{r}_s; f) \cdot \nabla P_b(\mathbf{r}, \mathbf{r}_s; f) \right] \left[\nabla' G_b^*(\mathbf{r}_g, \mathbf{r}'; f) \cdot \nabla G_b(\mathbf{r}_g, \mathbf{r}; f) \right]. \end{aligned} \quad (30)$$

Then Eq. 12 can be approximated as

$$\begin{aligned} \frac{\partial E}{\partial \rho_b(\mathbf{r})} \approx & -\frac{dV}{\rho_b(\mathbf{r})} \iint \frac{\rho_d(\mathbf{r}')}{\rho_b(\mathbf{r}')} \left\{ \int \left[\nabla' G_b^*(\mathbf{r}_g, \mathbf{r}'; f) \cdot \nabla G_b(\mathbf{r}_g, \mathbf{r}; f) \right] dS_g \right\} \\ & \times \left\{ \int \left[\nabla' P_b^*(\mathbf{r}', \mathbf{r}_s; f) \cdot \nabla P_b(\mathbf{r}, \mathbf{r}_s; f) \right] dS_s \right\} df dV' \end{aligned} \quad (31)$$

We can approximate the integral over S_g in Eq. 26 by using free-space Green's function,

$$15 \quad \int \nabla G_b^*(\mathbf{r}_g, \mathbf{r}; f) \cdot \nabla G_b(\mathbf{r}_g, \mathbf{r}; f) dS_g \approx \frac{\rho_b(\mathbf{r}_g)}{\rho_b(\mathbf{r})} \int \nabla G_b^*(\mathbf{r}_g, \mathbf{r}; f) \cdot \nabla G_b(\mathbf{r}, \mathbf{r}_g; f) dS_g \approx \frac{\rho_b(\mathbf{r}_g)}{\rho_b(\mathbf{r})} \frac{(2\pi f)^2}{8\pi v_p^2(\mathbf{r})}, \quad (32)$$

The integral was performed over half the solid angle under the assumption that $\nabla(\rho_b(\mathbf{r}_g)/\rho_b(\mathbf{r})) \approx 0$, and $\rho_b(\mathbf{r}_g)$ is constant along S_g .

- [0031] The gradient equation above can then be rewritten as

$$\frac{\partial E}{\partial \rho_b(\mathbf{r})} \approx -\frac{dV}{\rho_b^2(\mathbf{r})} \langle \rho_d(\mathbf{r}) \rangle I_{\rho,s}(\mathbf{r}) I_{\rho,g}(\mathbf{r}) V_\rho(\mathbf{r}) \quad (33)$$

and so

$$\langle \rho_d(\mathbf{r}) \rangle \approx -\frac{\rho_b^2(\mathbf{r})}{dV} \frac{1}{I_{\rho,s}(\mathbf{r}) I_{\rho,g}(\mathbf{r}) V_\rho(\mathbf{r})} \frac{\partial E}{\partial \rho_b(\mathbf{r})}, \quad (34)$$

where

$$I_{\rho,s}(\mathbf{r}) = \int |\nabla \dot{p}(\mathbf{r}, \mathbf{r}_s; t)|^2 dt dS_s, \quad (35)$$

and

$$I_{\rho,g}(\mathbf{r}) = \frac{\rho_b(\mathbf{r}_g)}{\rho_b(\mathbf{r})} \frac{1}{8\pi v_p^2(\mathbf{r})}. \quad (36)$$

As was the case with $\langle K_d(\mathbf{r}) \rangle$, Eqs. 28 or 34 can be used as an inversion formula for non-iterative inversion, or as a preconditioned gradient equation for iterative inversion. It is important to note that these equations yield density with the correct units, i.e. are dimensionally correct, because all terms have been taken into account, and none have been neglected to simplify the computation as is the case with some published approaches. The same is true for Eqs. 18 and 24 for bulk modulus. The published approaches that neglect one or more of the terms source illumination, receiver illumination, background medium properties and seismic resolution volume, will not result in the correct units, and therefore will need some sort of ad hoc fix up before they can be used for iterative or non-iterative inversion.

[0032] Since the first iteration of FWI is similar to RTM, the method provided here can be applied to analyze the amplitude term in RTM with little modification. Seismic migration including RTM is typically used to image the structure of the subsurface, and so amplitude information in the migrated image is often discarded. We show that the RTM amplitude, when properly scaled using the method provided here, represents the difference between the true compressional wave velocity of the subsurface and the velocity of the background model.

[0033] We note that the RTM equation 10 is missing the double derivative of the incident field in Eq. 8. This double derivative represents that high frequency components scatter more efficiently than the low frequency components in the classical Rayleigh

scattering regime (Refs. [10, 13]). Therefore, we can consider Eq. 10 as the gradient computation operation in Eq. 8, partially neglecting the frequency dependence of the scattered field,

$$\begin{aligned}\frac{\partial E}{\partial K_b(\mathbf{r})} &\approx -\frac{\rho_b dV (i2\pi f_c)^2}{K_b^2(\mathbf{r})} \iiint P_b(\mathbf{r}, \mathbf{r}_s; f) G_b(\mathbf{r}, \mathbf{r}_g; f) P^*(\mathbf{r}_g, \mathbf{r}_s; f) df dS_g dS_s \\ &\approx -\frac{\rho_b dV (i2\pi f_c)^2}{K_b^2(\mathbf{r})} M(\mathbf{r}),\end{aligned}\quad (37)$$

- 5 where f_c is the center frequency of the source waveform. Frequency dependence is partially neglected because, while the frequency dependence in the forward field p_b has been neglected, the frequency dependence implicit in the received field p_s cannot be neglected. Spatial variation of density is usually not considered in RTM, and so ρ_b is assumed to be constant in Eq. 37.

- 10 **[0034]** One can now apply the same approximation used to derive Eqs. 17 and 23 to Eq. 10,

$$\frac{\partial E}{\partial K_b(\mathbf{r})} \approx -\frac{\rho_b^2 I_{K,g}(\mathbf{r}) V_K(\mathbf{r}) dV \langle K_d(\mathbf{r}) \rangle (i2\pi f_c)^2}{K_b^4(\mathbf{r})} \iint (i2\pi f)^2 P_b(\mathbf{r}, \mathbf{r}_s; f) P_b^*(\mathbf{r}, \mathbf{r}_s; f) df dS_s, \quad (38)$$

which, together with Eq. 37, yields

$$15 \quad \langle K_d(\mathbf{r}) \rangle \approx \frac{K_b^2(\mathbf{r})}{\rho_b I_{K,g}(\mathbf{r}) V_K(\mathbf{r})} \frac{M(\mathbf{r})}{\iint |\dot{p}_b(\mathbf{r}, \mathbf{r}_s; t)|^2 dt dS_s}. \quad (39)$$

Equation 39 enables quantitative analysis of the amplitude in the reverse-time migrated image. More specifically, it enables inversion of the amplitude into the difference bulk modulus of the subsurface.

- [0035]** Figure 1 is a flowchart showing basic steps in one embodiment of the present
 20 inventive method. In step **103**, the gradient of an objective function is computed using an input seismic record (**101**) and information about the background subsurface medium (**102**). At step **104**, the source and receiver illumination in the background model is computed. At step **105**, the seismic resolution volume is computed using the velocities of the background model. At step **106**, the gradient from step **103** is converted into the difference subsurface
 25 model parameters using the source and receiver illuminations from step **104**, seismic resolution volume from step **105**, and the background subsurface model (**102**). If an iterative

inversion process is to be performed, at step 107 the difference subsurface model parameters from step 106 are used as preconditioned gradients for the iterative inversion process.

Examples

[0036] We consider the case of a 30 m $\times$ 30 m $\times$ 30 m “perfect” Born scatterer in a homogeneous medium with $K_b = 9$ MPa and $\rho_b = 1000$ kg/m³. The target is centered at $(x, y, z) = (0, 0, 250$ m), where x and y are the two horizontal coordinates and z is the depth. The target may be seen in Figs. 2-7 as a 3 $\times$ 3 array of small squares located in the center of each diagram. We assume co-located sources and receivers in the -500 m $\leq x \leq 500$ m and -500 m $\leq y \leq 500$ m interval with the 10-m spacing in both x and y directions. We assume that the source wavelet has a uniform amplitude of 1 Pa/Hz at 1 m in the 1 to 51 Hz frequency band.

[0037] In the first example, we assume that the target has the bulk modulus perturbation given as $K_d = 900$ kPa. Figure 2 shows the gradient $\partial E / \partial K_b(\mathbf{r})$ along the $y = 0$ plane using Eq. 8. The scattered field p_d in Eq. 8 has been computed using Eq. 5. The gradient in Fig. 2 has units of Pa m⁴ s, and so cannot be directly related to K_d . As described in the “Background” section above, this is a difficulty encountered in some of the published attempts to compute a model update from the gradient of the objective function.

[0038] Figure 3 shows $\langle K_d(\mathbf{r}) \rangle$ using Eq. 18 of the present invention. It has been assumed that the seismic resolution volume $V_K(\mathbf{r})$ is a sphere with radius $\sigma = v_p(\mathbf{r}) / 4B = 15$ m. One can see that Fig. 3 is the smeared image of the target, since $\langle K_d(\mathbf{r}) \rangle$ is the averaged property over the seismic resolution volume. Figure 4 is $\langle K_d(\mathbf{r}) \rangle$ when the less rigorous Eq. 24 is employed. One can see that $\langle K_d(\mathbf{r}) \rangle$ in Figs. 3 and 4 are in good agreement with each other. The value of $\langle K_d(\mathbf{r}) \rangle$ at the center of the target in Fig. 3 is 752 kPa, and that in Fig. 4 is 735 kPa, both of which are within 20% of the true value of 900 kPa.

[0039] The second example is the case where the target has a density perturbation of $\rho_d = 100$ kg/m³. Figure 5 shows the gradient $\partial E / \partial \rho_b(\mathbf{r})$ along the $y = 0$ plane using Eq. 9. The scattered field in Eq. 9 has been computed using Eq. 5. The gradient in Fig. 5 has units of Pa² m⁷ s/kg. As with the first example, the different units prevent the gradient from

being directly related to a density update, again illustrating the problem encountered in published methods.

[0040] Figure 6 shows $\langle \rho_d(\mathbf{r}) \rangle$ using Eq. 28. The seismic resolution volume $V_\rho(\mathbf{r})$ has been assumed to be identical to $V_k(\mathbf{r})$. Figure 7 is $\langle \rho_d(\mathbf{r}) \rangle$ when Eq. 34 is employed.

5 Estimation of $\langle \rho_d(\mathbf{r}) \rangle$ using Eq. 34 results in less accurate inversion than that using Eq. 28 due to the neglected dipole illumination term in Eq. 31.

[0041] The foregoing patent application is directed to particular embodiments of the present invention for the purpose of illustrating it. It will be apparent, however, to one skilled in the art, that many modifications and variations to the embodiments described herein
10 are possible. All such modifications and variations are intended to be within the scope of the present invention, as defined in the appended claims. Persons skilled in the art will readily recognize that in practical applications of the invention, at least some of the steps in the present inventive method are performed on or with the aid of a computer, i.e. the invention is computer implemented.

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The invention claimed is:

1. A method for determining a model of a physical property in a subsurface region from inversion of seismic data, acquired from a seismic survey of the subsurface region, or from reverse time migration of seismic images from the seismic data, said method comprising determining a seismic resolution volume for the physical property and using it as a multiplicative scale factor in computations performed on a computer to either convert a gradient of data misfit in an inversion, or compensate reverse-time migrated seismic images, to obtain the model of the physical property or an update to an assumed model.
2. The method of claim 1, further comprising multiplying the gradient of data misfit or the reverse time migrated seismic images by additional scale factors including a source illumination factor, a receiver illumination factor, and a background medium properties factor to then obtain the model of the physical property or an update to an assumed model.
3. The method of claim 1, wherein the seismic resolution volume is determined by ray tracing using velocity of a background medium model and using an assumed function of frequency for a seismic wavelet.
4. The method of claim 2, wherein the model is determined from inversion of seismic data, said method further comprising: assuming an initial model of the subsurface region specifying a model parameter at discrete cell locations in the subsurface region; forming a mathematical objective function to measure misfit between measured seismic data and model-calculated seismic data; selecting a mathematical relationship that gives an adjustment, i.e. update, to the initial model that would reduce the misfit, said mathematical relationship relating said adjustment to a scaled gradient of the objective function, said gradient being with respect to said model parameter, the scaling comprising the four scale factors, i.e. the seismic resolution volume factor, the source illumination factor, the receiver illumination factor, and the background medium properties factor, all of which appear in the mathematical relationship as multiplicative factors that scale the gradient of the objective function to yield the adjustment to the model parameter; and using a computer to compute the adjustment from the mathematical relationship, and then updating the initial model with the computed adjustment.
5. The method of claim 4, wherein the physical property, i.e. the model parameter, is either bulk modulus or density, or a combination of bulk modulus and density.
6. The method of claim 4, wherein the mathematical relationship depends upon the physical property.
7. The method of claim 4, wherein the background medium properties factor comprises bulk modulus raised to the fourth power divided by density squared when the physical property is bulk modulus, and comprises density squared when the physical property is density.
8. The method of claim 4, wherein the receiver illumination factor when the physical property is bulk modulus is approximated by $(1/8\pi)(\rho_b(r_g)/\rho_b(r))$, where $\rho_b(r)$ is background density at location r and r_g is the receiver's location; and the receiver illumination factor when the physical property is density, $I_{\rho,g}(r)$, is approximated by:

$$I_{\rho,g}(r) = \frac{\rho_b(r_g)}{\rho_b(r)} \frac{1}{8\pi v_p^2(r)}.$$

where $v_p(r)$ is velocity at location r .

9. The method of claim 4, further comprising repeating the method for at least one iteration, where in the initial model is replaced by the updated model from the previous iteration.

10. The method of claim 9, wherein the objective function's functional form and the mathematical relationship's functional form are unchanged from one iteration to a next iteration.
11. The method of claim 4, wherein the adjustment to the initial model is computed by minimizing the objective function using the objective function's Hessian resulting in a Hessian matrix, wherein off-diagonal elements of the Hessian matrix are ignored when outside of a seismic resolution volume.
12. The method of claim 11, wherein off-diagonal elements of the Hessian matrix within a seismic resolution volume are assumed to be equal to a corresponding diagonal element, resulting in computing only diagonal elements.
13. The method of claim 2, wherein the physical property is bulk modulus and the receiver illumination factor is approximated by an integral of a Green's function in free space over a surface defined by the survey's receiver spread.
14. The method of claim 4, wherein the physical property is density and the receiver illumination factor is approximated by an integral of a gradient of a Green's function in free space over a surface defined by the survey's receiver spread.
15. The method of claim 1, wherein the seismic resolution volume is approximated as a sphere based on an assumption of uniform wavenumber coverage.
16. The method of claim 4, further comprising using the updated model to precondition the gradient in an iterative optimization technique.
17. The method of claim 2, wherein the model is determined from reverse time migration of seismic images from the seismic data.
18. The method of claim 17, wherein the physical property is bulk modulus.
19. The method of claim 17, wherein the background medium properties factor comprises bulk modulus squared divided by density.
20. The method of claim 17, wherein the receiver illumination factor is approximated by $(1/8\pi)(\rho_b(r_o)/\rho_b(r))$, where $\rho_b(r)$ is background density at location r and r_o is the receiver's location.
21. The method of claim 17, wherein the migrated images seismic amplitudes are converted into difference bulk modulus or difference compressional wave velocity.