area in the rock formation where non-linear interactions resulted in the returned energy in the borehole.

7. The system according to claim 1, wherein an axis of the first acoustic source is arranged at an angle relative to an axis of the second acoustic source.

8. The system according to claim 1, wherein the ratio of the second frequency to the first frequency is selected to maximize a mixing coefficient of the non-linear mixing process within the non-linear mixing zone, using selection rules governing non-linear interaction of acoustic waves.

9. The system according to claim 1, further comprising an encoder configured to encode the first acoustic wave, the second acoustic wave or both, with a time or frequency varying attribute or both.

10. The system according to claim 1, further comprising a storage device configured to record data corresponding to the received third shear acoustic wave, and a processor, arranged to process machine-readable instructions, that when executed cause the processor to estimate a compressional and shear velocity ratio according to selection rules governing the frequency ratio and convergence angle relationship, or non-linear properties of the rock formation around the borehole using the recorded data from the storage device and by repeating the generating of the first acoustic wave at the first frequency, the generating of the second acoustic wave at the second frequency, the receiving of the third acoustic wave returning to the borehole and identifying a location of the intersection volume based on an arrangement of the first and the second acoustic sources and a plurality of orientations of the first acoustic wave and the second acoustic wave.

11. The system according to claim 1, further comprising a storage device configured to record data corresponding to the received third shear acoustic wave, and a processor, arranged to process machine-readable instructions, that when executed cause the processor to calculate a three-dimensional image of non-linear properties of the rock formation or a three-dimensional image of compressional and shear velocity ratio, using the recorded data from the storage device by repeating the generating of

17. The method according to claim 12, further comprising analyzing the returning energy and determining the location of the mixing area in the rock formation where non-linear interactions resulted in the returned third acoustic wave in the borehole.

18. The method according to claim 12, further comprising arranging the axial orientation of the first acoustic source at an angle relative to an axial orientation of the second acoustic source.

19. The method according to claim 12, further comprising encoding, using an encoder, the first acoustic wave, the second acoustic wave or both with a time or frequency varying attribute, or both.

20. The method according to claim 12, further comprising calculating a location of the intersection zone based on an arrangement of the first and the second acoustic sources and a direction of the returned energy.

21. The method according to claim 20, further comprising estimating a compressional velocity to shear velocity ratio or non-linear properties of rock formation around the borehole or both, using data recorded by repeating the generating and receiving at a plurality of orientations of the first acoustic wave and the second acoustic wave.

22. The method according to claim 12, further comprising determining a mixing coefficient of the non-linear mixing process within the non-linear mixing zone using selection rules governing non-linear interaction of acoustic waves, and selecting a ratio of the second frequency and the first frequency so as to maximize the mixing coefficient.

~~23. A system for investigating rock formations outside a borehole using coded signals~~ from acoustic sources, the system comprising:

a first acoustic source configured to generate a first acoustic signal comprising a first ~~plurality of pulses~~ arranged as a time sequence, the first plurality of pulses

being separated in time, each pulse comprising a first modulated signal at a central frequency, wherein central frequencies of two consecutive pulses are different;

a second acoustic source configured to generate a second acoustic signal comprising a second plurality of pulses arranged as a time sequence, the second plurality of pulses being separated in time, wherein a separation in time between centers of two consecutive pulses is the same as a separation in time between centers of two corresponding pulses in the first plurality of pulses, wherein a start time difference is provided between a start time of a broadcast of the second plurality of pulses and a start time of a broadcast of the first plurality of pulses, wherein each pulse comprises a second modulated signal, and a central frequency of the second modulated signal within each pulse in the second plurality of pulses is a selected fraction of the central frequency of the first modulated signal within each corresponding pulse in the first plurality of pulses;

wherein the first acoustic source and the second acoustic source are arranged within the borehole and are controllable such that trajectories of the first and the second acoustic signals intersect in an intersection volume outside the borehole;

a receiver arranged within the borehole, the receiver being configured to detect a received signal returning to the borehole, the received signal including a signal being generated by a non-linear mixing process from the first acoustic signal and the second acoustic signal in a non-linear mixing zone within the intersection volume; and

a processor configured to perform data processing on the received signal to extract the signal generated by the non-linear mixing process over noise or over signals generated by a linear interaction process, or both.

24. The system according to claim 23, wherein a separation in time between centers of two consecutive pulses in the first plurality of pulses is greater than a time duration of each pulse.

25. The system according to claim 23, wherein the processor is configured to extract signals generated by the non-linear mixing process using waveform recognition.

26. The system according to claim 23, wherein the processor is configured to extract signals generated by the non-linear mixing process using band pass filtering based on forecast properties of the non-linear signal according to selection rules of non-linear mixing in the region where the non-linear mixing process occurs.

27. The system according to claim 23, wherein the processor is configured to extract signals generated by the non-linear mixing process by correlating the detected signal with a template signal designed according to forecast properties of the signal according to the selection rules of non-linear mixing in the region where the non-linear mixing process occurs.

28. The system according to claim 23, wherein the signal generated by a non-linear mixing process comprises a third plurality of pulses arranged in a time sequence and separated in time, wherein a separation in time between centers of two consecutive pulses is the same as the separation in time between centers of two consecutive pulses in the first plurality of pulses,

wherein each pulse in the third plurality of pulses comprises a third modulated signal having a third central frequency that is equal to a difference between the first central frequency and the second central frequency in each corresponding pulse of the first plurality of pulses and the second plurality of pulses,

wherein an arrival time at the receiver of each pulse of the third plurality of pulses is time delayed relative to a generation of a corresponding pulse of the first plurality of pulses by a total of the travel time from the first acoustic source to a center of the mixing zone and the travel time from the center of the mixing zone to the receiver.

29. The system according to claim 23, wherein the processor is configured to control the first source and the second source to repeat the generating of the first signal, the generating of the second signal, to control the receiver to repeat the detecting of the received signal returning to the borehole and repeat the performing of data processing on the received signal to extract the signal generated by the non-linear mixing process over noise or over signals generated by a linear interaction process, or

both, for a range of start time differences and a range of frequency fractions d to obtain signals generated at a plurality of mixing zones.

30. The system according to claim 23, wherein the signal generated by non-linear interaction at the mixing zone is generated and detected only if the start time difference between the second plurality of pulses of the second acoustic source and the first plurality of pulses of the first acoustic source is equal to the difference in travel time between the travel time from the first acoustic source to the center of the mixing zone and the travel time between the second acoustic source to the center of the mixing zone.

31. The system according to claim 23, wherein increasing a number of the first plurality of pulses and increasing a number of the second plurality of pulses enhances discrimination of the signal generated by the non-linear mixing process over noise or over signals generated by a linear interaction process, or both.

32. The system according to claim 23, wherein the first signal comprises a sum of a plurality of pulses, wherein each pulse has a signal amplitude equal to the product of an envelope function and a modulated signal function.

33. The system according to claim 23, wherein the first signal $u_1(t)$ is in the form

$$u_1(t) = \sum_n E1_n(t - T_n) * \exp\left(i\,2\pi * f_n * \left(t - T_n\right)\right) * \exp(i\zeta_n)$$

where:

$E1n(t-T_n)$ corresponds to an envelope of an nth pulse in the first plurality of pulses, the envelope having a positive or negative amplitude,

$\exp(i2\pi * f_n * (t-T_n))$ corresponds to the first modulated signal within the nth pulse in the first plurality of pulses,

f_n is a central frequency of the modulated signal within the nth pulse,

t is a signal time,

T_n is a time when the nth pulse is generated, and

ζ_n is a phase of each pulse n and $\exp(i\zeta_n)$ is a phase term of each pulse n.

34. The system according to claim 23, wherein the second signal comprises a sum of a plurality of pulses, wherein each pulse has a signal amplitude equal to the product of an envelope function and a modulated signal function.

35. The system according to claim 23, wherein the second signal $u_2(t-\delta)$ is in the form:

$$u_2(t-\delta) = \sum_n E2_n(t-(T_n+\delta)) * \exp(i\,2\pi * d * f_n * (t-(T_n+\delta))) * \exp($$

where:

E2n(t-(T_n+δ)) corresponds to an envelope of an nth pulse in the second plurality of pulses,

exp (i2π * d * f_n * (t-(T_n+δ))) corresponds to the second modulated signal within the nth pulse in the second plurality of pulses,

d * fn is a frequency of the modulated signal within the nth pulse in the second plurality of pulses,

d is a selected frequency ratio,

t is the signal time,

(T_n + δ) is a time when the nth pulse in the second plurality of pulses is broadcast,

δ is the start time difference between the start time of the broadcast of the second acoustic signal and the start time of the broadcast of the first acoustic signal, and

ζ_n is a phase of each pulse n and $\exp(i\zeta_n)$ is a phase term of each pulse n.

36. The system according to claim 23, wherein the signal generated by a non-linear mixing process is proportional to a sum over a number of a third plurality of pulses of a product of an envelope function for each pulse by a modulated signal function for each pulse.

37. The system according to claim 23, wherein the signal $u_3(t)$ being generated by a non-linear mixing process is in the form:

$$u_3(t) \propto \sum_n E3_n\left(t-(T_n+T)\right) * \exp\left(i2\pi * (1-d) * f_n * \left(t-(T_n+T)\right)\right) * \exp(i\zeta_n)$$

where:

$E3_n\left(t-(T_n+T)\right)$ is an envelope of a pulse in a third plurality of pulses of the signal u3 generated by non-linear mixing of the first signal and the second signal,

$\exp(i2\pi * (1-d) * f_n * (t-(T_n+T)))$ corresponds to a third modulated signal within the nth pulse in the third plurality of pulses,

$(1-d) * fn$ is a frequency of the modulated signal within the nth pulse in the third plurality of pulses which is a difference between the central frequency fn of the first modulated signal within the nth pulse of the first plurality of pulses and the central frequency $d * fn$ of the second modulated signal within the nth pulse of the second plurality of pulses,

d is a selected frequency ratio,

t is the signal time,

$(T_n + T)$ is a time when the nth pulse in the third plurality of pulses arrives at the receiver,

T is the total travel time from the first acoustic source to a center of the mixing zone and from the center of the mixing zone to the receiver, and

ζ_n is a phase of each pulse n and $\exp(i\zeta_n)$ is a phase term of each pulse n.

38. The system according to claim 23, wherein the processor is configured to extract signals generated by the non-linear mixing process by correlating the detected signal with a template signal wherein the template signal comprises a sum of a plurality of pulses wherein each pulse has a signal amplitude equal to the product of an envelope function and a modulated signal function.

39. The system according to claim 23, wherein the processor is configured to extract signals generated by the non-linear mixing process by correlating the detected signal with a template signal, wherein the template u_s is in the form:

$$u_s(t) = \sum_n W_n(t - T_n) * \exp(i2\pi * g(f_n) * (t - T_n)) * \exp(i\zeta_n)$$

where:

$W_n(t\text{-}T_n)$ is an amplitude envelope of an nth pulse in the template signal,

exp ($i2\pi * g(f_n) * (t\text{-}T_n)$) corresponds to modulated signal within the nth pulse in the plurality of pulses of the template signal,

$g(f_n)$ is a selected function of the modulated signal within the nth pulse in the plurality of pulses of the template,

t is a propagation time,

T_n is a time when the nth pulse in the plurality of pulses is simulated to arrive at the receiver, and

ζ_n is a phase of each pulse n and $\exp(i\zeta_n)$ is a phase term of each pulse n.

40. The system according to claim 23,

wherein the processor is configured to extract signals generated by the non-linear mixing process by correlating the detected signal with a template signal designed according to forecast properties of the signal according to the selection rules of non-linear interaction of acoustic waves,

wherein the processor is further configured to repeat for a plurality of start time differences δ between the second plurality of pulses and the first plurality of pulses and for a plurality of frequency ratios d between the central frequency of the modulated signal within each pulse in the second plurality of pulses and the central frequency of the modulated signal with each corresponding pulse in the first plurality of pulses, and the correlation between the template signal and the detected signal to yield, for each start time difference δ and for each frequency ratio d, a correlated signal M(t, δ, d) which contains a non-linear interaction signal.

41. The system according to claim 40, wherein the processor is further configured to search all values of arrival time t, start time difference δ and frequency ratio d in the signal M(t, δ, d) to determine a value of arrival time T_{NL}, start time difference δ_{NL} and frequency ratio d_{NL} for the presence of a correlated band-limited spike signal corresponding to the occurrence of a signal generated by the non-linear mixing process.

42. The system according to claim 41, wherein the determined start time difference δ_{NL}, where the band-limited spike signal generated by the non-linear process occurs in the correlated signal M(t, δ, d), is substantially equal to a time difference between a travel time from the first acoustic source to a center of the mixing zone and a travel time from the second acoustic source to the center of the mixing zone.

43. The system according to claim 41, wherein the determined arrival time T_{NL}, where the band-limited spike signal generated by the non-linear mixing process occurs in the correlated signal M (t, δ, d), is substantially equal to a total travel time of the travel time from the first acoustic source to a center of the mixing zone and the travel time from the center of the mixing zone to the receiver.

44. A method for investigating rock formations outside a borehole, the method comprising:

generating, by a first acoustic source, a first acoustic signal comprising a first plurality of pulses arranged as a time sequence, the first plurality of pulses being separated in time, each pulse comprising a first modulated signal at a central frequency, wherein central frequencies of two consecutive pulses are different;

generating, by a second acoustic source, a second acoustic signal comprising a second plurality of pulses arranged as a time sequence, the second plurality of pulses being separated in time, wherein a separation in time between centers of two consecutive pulses is the same as a separation in time between centers of two corresponding pulses in the first plurality of pulses, wherein a start time difference is provided between a start time of a broadcast of the second plurality of pulses and a start time of a broadcast of the first plurality of pulses, wherein each pulse comprises a second modulated signal and a central frequency of the second modulated signal within each pulse in the second plurality of pulses is a selected fraction of the central frequency of the first modulated signal within each corresponding pulse in the first plurality of pulses;

wherein the first acoustic source and the second acoustic source are arranged within the borehole and are controllable such that trajectories of the first and the second acoustic signals intersect in an intersection volume outside the borehole;

receiving, by a receiver arranged within the borehole, a detected signal returning to the borehole, the detected signal including a signal being generated by a non-linear mixing process from the first acoustic signal and the second acoustic signal in a non-linear mixing zone within the intersection volume; and

performing, by a processor, data processing on the received signal to extract the signal generated by the non-linear mixing process over noise or over signals generated by a linear interaction process, or both.

45. The method according to claim 44, wherein generating the first acoustic signal comprises generating the first acoustic signal such that a separation in time between the centers of two consecutive pulses in the first plurality of pulses is greater than a time duration of each pulse.

46. The method according to claim 44, further comprising extracting, by the processor, signals generated by the non-linear mixing process using waveform recognition.

47. The method according to claim 44, further comprising extracting, by the processor, signals generated by the non-linear mixing process using band pass filtering based on forecast properties of the non-linear signal according to the selection rules of non-linear mixing in the region where the non-linear mixing process occurs.

48. The method according to claim 44, further comprising extracting, by the processor, signals generated by the non-linear mixing process by correlating the detected signal with a template signal designed according to forecast properties of the signal according to selection rules of non-linear mixing in the region where the non-linear mixing process occurs.

49. The method according to claim 44, wherein the signal generated by the non-linear mixing process comprises a third plurality of pulses arranged in a time sequence and separated in time, wherein a separation in time between centers of two consecutive pulses is the same as the separation in time between centers of two consecutive pulses in the first plurality of pulses,

wherein each pulse in the third plurality of pulses comprises a third modulated signal having a third central frequency that is equal to a difference between the first central frequency and the second central frequency in each corresponding pulse of the first plurality of pulses and the second plurality of pulses,

wherein an arrival time at the receiver of each pulse of the third plurality of pulses is time delayed relative to a generation of a corresponding pulse of the first plurality of pulses by a total of the travel time from the first acoustic source to a center of the mixing zone and the travel time from the center of the mixing zone to the receiver.

50. The method according to claim 44, further comprising repeating the generating of the first signal, the generating of the second signal, to repeat the

detecting by the receiver of the received signal returning to the borehole and repeating the performing of data processing on the received signal to extract the signal generated by the non-linear mixing process over noise or over signals generated by a linear interaction process, or both, for a range of start time differences and a range of frequency fractions d to obtain signals generated at a plurality of mixing zones.

51. The method according to claim 44, further comprising detecting the signal generated by non-linear interaction at the mixing zone only if the start time difference between the second plurality of pulses of the second acoustic source and the first plurality of pulses of the first acoustic source is equal to the difference in travel time between the travel time from the first acoustic source to the center of the mixing zone and the travel time between the second acoustic source to the center of the mixing zone.

52. The method according to claim 44, further comprising increasing a number of the first plurality of pulses and increasing a number of the second plurality of pulses to enhance discrimination of the signal generated by the non-linear mixing process over noise or over signals generated by a linear interaction process, or both.

53. The method according to claim 44, wherein generating the first signal comprises generating a first signal that is a sum of a plurality of pulses, wherein each pulse has a signal amplitude equal to the product of an envelope function and a modulated signal function.

54. The method according to claim 44, wherein generating the first signal comprises generating a first signal u_1 (t) is in the form

$$u_1(t) = \sum_n E1_n(t - T_n) * \exp\left(i\,2\pi * f_n * \left(t - T_n\right)\right) * \exp(i\zeta_n)$$

where:

$E1n(t-T_n)$ corresponds to an envelope of an nth pulse in the first plurality of pulses, the envelope having a positive or negative amplitude,

exp ($i2\pi * f_n * (t-T_n)$) corresponds to the first modulated signal within the nth pulse in the first plurality of pulses,

f_n is a central frequency of the modulated signal within the nth pulse,

t is a signal time,

T_n is a time when the nth pulse is generated, and

ζ_n is a phase of each pulse n and $\exp(i\zeta_n)$ is a phase term of each pulse n.

55. The method according to claim 44, wherein generating the second signal comprises generating a second signal that is a sum of a plurality of pulses, wherein each pulse has a signal amplitude equal to the product of an envelope function and a modulated signal function.

56. The method according to claim 44, wherein generating the second signal comprises generating a second signal $u_2(t-\delta)$ is in the form:

$$u_2(t-\delta) = \sum_n E2_n\left(t-(T_n+\delta)\right) * \exp(i2\pi * d * f_n * (t-(T_n+\delta))) * \exp($$

where:

E2n(t-(T_n+δ)) corresponds to an envelope of an nth pulse in the second plurality of pulses,

exp ($i2\pi * d * f_n * (t-(T_n+\delta))$) corresponds to the second modulated signal within the nth pulse in the second plurality of pulses,

$d * fn$ is a frequency of the modulated signal within the nth pulse in the second plurality of pulses,

d is a selected frequency ratio,

t is the signal time,

(T_n + δ) is a time when the nth pulse in the second plurality of pulses is broadcast,

δ is the start time difference between the start time of the broadcast of the second acoustic signal and the start time of the broadcast of the first acoustic signal, and

ζ_n is a phase of each pulse n and $\exp(i\zeta_n)$ is a phase term of each pulse n.

57. The method according to claim 44, wherein the signal generated by the non-linear mixing process comprises a sum of a plurality of pulses, wherein each pulse has a signal amplitude equal to the product of an envelope function and a modulated signal function.

58. The method according to claim 44, wherein the signal $u_3(t)$ being generated by a non-linear mixing process is in the form:

$$u_3(t) \propto \sum_n E3_n(t-(T_n+T)) * \exp(i2\pi * (1-d) * f_n * (t-(T_n+T))) * \exp(i\zeta_n)$$

where:

$E3_n(t-(T_n+T))$ is an envelope of a pulse in a third plurality of pulses of the signal u3 generated by non-linear mixing of the first signal and the second signal,

$\exp(i2\pi * (1-d) * f_n * (t-(T_n+T)))$ corresponds to a third modulated signal within the nth pulse in the third plurality of pulses,

$(1-d) * fn$ is a frequency of the modulated signal within the nth pulse in the third plurality of pulses which is a difference between the central frequency fn of the first modulated signal within the nth pulse of the first plurality of pulses and the central frequency $d * fn$ of the second modulated signal within the nth pulse of the second plurality of pulses,

d is a selected frequency ratio,

t is the signal time,

(T_n + T) is a time when the nth pulse in the third plurality of pulses arrives at the receiver,

T is the total travel time from the first acoustic source to a center of the mixing zone and from the center of the mixing zone to the receiver, and

ζ_n is a phase of each pulse n and $\exp(i\zeta_n)$ is a phase term of each pulse n.

59. The method according to claim 44, further comprising extracting, by the processor, signals generated by the non-linear mixing process by correlating the detected signal with a template signal wherein the template signal comprises a sum of a plurality of pulses, wherein each pulse has a signal amplitude equal to the product of an envelope function and a modulated signal function.

60. The method according to claim 44, further comprising extracting, by the processor signals generated by the non-linear mixing process by correlating the detected signal with a template signal, wherein the template u_s is in the form:

$$u_s(t) = \sum_n W_n(t - T_n) * \exp(i2\pi * g(f_n) * (t - T_n)) * \exp(i\zeta_n)$$

Where:

$W_n(t\text{-}T_n)$ is an amplitude envelope of an nth pulse in the template signal,

$\exp(i2\pi * g(f_n) * (t\text{-}T_n))$ corresponds to modulated signal within the nth pulse in the plurality of pulses of the template signal,

$g(f_n)$ is a selected function of the modulated signal within the nth pulse in the plurality of pulses of the template,

t is a propagation time,

T_n is a time when the nth pulse in the plurality of pulses is simulated to arrive at the receiver, and

ζ_n is a phase of each pulse n and $\exp(i\zeta_n)$ is a phase term of each pulse n.

61. The method according to claim 44, further comprising:

extracting, by the processor, signals generated by the non-linear mixing process, by correlating the detected signal with a template signal designed according to forecast properties of the signal according to the selection rules of non-linear interaction of acoustic waves, and repeating for a plurality of start time differences δ between the second plurality of pulses and the first plurality of pulses and for a plurality of frequency ratios d between the central frequency of the modulated signal within each pulse in the second plurality of pulses and the central frequency of the modulated signal with each corresponding pulse in the first plurality of pulses, the correlating between the template signal and the detected signal to yield, for each start time difference δ and for each frequency ratio d, a correlated signal M(t, δ, d) which contains a non-linear interaction signal.

62. The method according to claim 61, further comprising searching, by the processor, all values of arrival time t, start time difference δ and frequency ratio d in the signal M(t, δ, d) to determine a value of arrival time T_{NL}, start time difference δ_{NL} and frequency ratio d_{NL} for the presence of a correlated band-limited spike signal corresponding to the occurrence of a signal generated by the non-linear mixing process.

63. The method according to claim 62, wherein the determined start time difference δ_{NL} where the band-limited spike signal generated by the non-linear mixing process occurs in the correlated signal M(t, δ, d) is substantially equal to a time difference between a travel time from the first acoustic source to a center of the mixing zone and a travel time from the second acoustic source to the center of the mixing zone.

64. The method according to claim 62, wherein the determined arrival time T_{NL} where the band-limited spike signal generated by the non-linear mixing process occurs in the correlated signal M(t, δ, d) is substantially equal to a total travel time of the travel time from the first acoustic source to a center of the mixing zone and the travel time from the center of the mixing zone to the receiver.

65. A system for investigating rock formation outside a borehole, the system comprising:

a first acoustic source configured to generate a first acoustic signal at a first frequency;

a second acoustic source configured to generate a second acoustic signal at a second frequency;

wherein the first acoustic source and the second acoustic source are arranged and positioned within the borehole and are controllable such that the first and the second acoustic signals intersect in an intersection volume outside the borehole and a start time difference is provided between the second acoustic signal and the first acoustic signal;

a receiver arranged within the borehole, the receiver being configured to receive a detected signal returning to the borehole having a frequency equal to a difference between the first frequency and the second frequency, the detected signal being generated by a non-linear mixing process from the first acoustic signal and the second acoustic signal in a non-linear mixing zone within the intersection volume; and

a recording system configured to record the detected signal and store the detected signal in a storage device and to record measurement parameters including a position of the first acoustic source, a position of the second acoustic source, a position of the receiver, elevation angle and azimuth angle of the first acoustic signal and elevation angle and azimuth angle of the second acoustic signal.

66. The system according to claim 65, further comprising a processor configured to scan a range of start time differences between the start time of a broadcast of the first acoustic signal and the start time of a broadcast of the second acoustic signal so that the range of start time differences includes a time difference for which the first acoustic signal and the second acoustic signal arrive substantially simultaneously at the mixing zone.

67. The system according to claim 65, further comprising hardware or software or both, configured to control a phase of the first acoustic signal or a phase of the second acoustic signal, or both, such that received signals from the interaction of the first acoustic signal and the second acoustic signal have opposite polarity.

68. The system according to claim 67, wherein the received signals from the interaction of the first acoustic signal and the second acoustic signal having opposite polarities are combined to enhance the signal generated by the non-linear interaction over noise or signals originating from a linear interaction or both.

69. The system according to claim 65, wherein at least a portion of the detected signal generated by the non-linear interaction is proportional to a product of amplitudes of the first acoustic signal and the second acoustic signal.

70. The system according to claim 65, further comprising a time variant band-pass filtering device configured to apply a time-variant frequency band-pass filter to the detected signal so as to keep a narrow frequency bandwidth around an expected frequency bandwidth of the signal generated by non-linear interaction, the narrow frequency bandwidth being selected around the difference between the first frequency and the second frequency.

71. The system according to claim 65, wherein:

the first acoustic signal comprises a first plurality of pulses arranged as a time sequence, the first plurality of pulses being separated in time, each pulse comprising a first modulated signal at a central frequency, wherein central frequencies of two consecutive pulses are different;

the second acoustic signal comprises a second plurality of pulses arranged as a time sequence, the second plurality of pulses being separated in time, wherein a separation in time between centers of two consecutive pulses is the same as a separation in time between centers of two corresponding pulses in the first plurality of

pulses, wherein a start time difference is provided between a start time of a broadcast of the second plurality of pulses and a start time of a broadcast of the first plurality of pulses, wherein each pulse comprises a second modulated signal, and wherein a central frequency of the second modulated signal within each pulse in the second plurality of pulses is a selected fraction of the central frequency of the first modulated signal within each corresponding pulse in the first plurality of pulses; and

the start time difference is controlled such that the first acoustic signal and the second acoustic signal intersect in the mixing zone.

72. The system according to claim 71, wherein a separation in time between centers of two consecutive pulses in the first plurality of pulses is greater than a time duration of each pulse.

73. The system according to claim 71, further comprising a processor configured to correlate the detected signal with a template signal comprising a plurality of pulses, the plurality of pulses being arranged in a time sequence and separated in time, wherein a separation in time between centers of two consecutive pulses is the same as the separation in time between centers of two consecutive pulses in the first plurality of pulses, wherein each pulse in the plurality of pulses comprises a modulated signal having a central frequency that is equal to a difference between a central frequency of the first modulated signal and a central frequency of the second modulated signal in each corresponding pulse of the first plurality of pulses and the second plurality of pulses, to obtain a correlated signal containing the signal generated by non-linear interaction in the non-linear mixing zone.

74. The system according claim 73, further comprising a plurality of receivers configured to receive a plurality of detected signals, wherein the processor is configured to apply multi-dimensional filtering, or time moveout analysis and stacking or both, to the plurality of detected signals that are received by the plurality of receivers.

75. The system of claim 73, wherein the receiver comprises a hydrophone or multi-component detectors, or both, and wherein the detected signal comprises a

pressure signal detected by the hydrophone or multi-component signals detected by the multi-component detectors or both.

76. The system of claim 75, wherein the processor is further configured to perform hodogram analysis on the detected signal.

77. The system of claim 76, wherein the processor is further configured to apply a hodogram analysis to the three detected component signals corresponding to vertical shear SV, horizontal shear SH and compressional P signals.

78. The system of claim 77, wherein the processor is further configured to apply multi-dimensional filtering, or time moveout analysis and stacking, or both, to the three detected component signals that are received by the three detectors, or to apply multi-dimensional filtering, or time moveout analysis and stacking, or both, to the pressure signal detected by the hydrophone.

79. The system of claim 77, further comprising a time variant band-pass filtering device configured to apply a time-variant frequency band pass filter to the detected signal so as to keep a narrow frequency bandwidth around an expected frequency bandwidth of the signal generated by non-linear interaction, the narrow frequency bandwidth being selected around the difference between the first frequency and the second frequency.

80. The system of claim 79, wherein the processor is further configured to control the first acoustic source, the second acoustic source and the receiver, and repeat a plurality of times the generating of the first acoustic signal, the generating of the second acoustic signal, the receiving of the detected signal, the applying of the time variant frequency band pass filter, the correlating of the detected signal with the template signal, the applying of the hodogram analysis, and the combining of signals to improve signal to noise ratio.

81. The system of claim 79, wherein the processor is further configured to vary the start time difference and repeat a plurality of times the generating of the first

acoustic signal, the generating of the second acoustic signal, the receiving of the detected signal, the applying of the time variant frequency band pass filter, the correlating of the detected signal with the template signal, and the applying of the hodogram analysis for each of three detected signals to separate different propagation signal modes including compressional mode P, vertical shear mode SV and horizontal shear mode SH.

82. The system of claim 79, wherein the processor is further configured to vary the start time difference and repeat a plurality of times the generating of the first signal, the generating of the second signal, the receiving of the detected signal, the applying of the time variant frequency band pass filter, and the correlating of the signal detected by a hydrophone with the template signal.

83. The system according to claim 79, wherein the processor is further configured to control the first acoustic source, the second acoustic source and the receiver to scan a plurality of azimuth and elevation angles of the first acoustic signal, a plurality of azimuth and elevation angles of the second acoustic signal, or a position of the receiver, and repeating a plurality of times the generating of the first acoustic signal, the generating of the second acoustic signal, the receiving of the detected signal, the applying of the time-variant frequency band pass filter to obtain a filtered signal, the correlating of the filtered signal with the template signal, and the applying of the hodogram analysis on the three detected component signals to separate the component signals to different propagation signal modes including compressional mode P, vertical shear mode SV and horizontal shear mode SH.

84. The system according to claim 83, wherein the processor is further configured to repeat varying the start time difference between the start time of a broadcast of the first acoustic signal and the start time of a broadcast of the second acoustic signal, to repeat varying the frequency ratio between the first frequency and the second frequency, and to repeat scanning the plurality of azimuth and elevation angles of the first acoustic signal, the plurality of azimuth and elevation angles of the second acoustic signal, and the position of the receiver to obtain a plurality of signal measurements.

85. The system according to claim 84, further comprising a storage device in communication with the processor, the storage device being configured to store the obtained plurality of signal measurements.

86. A method of investigating rock formation outside a borehole comprising:

generating, by a first acoustic source, a first signal at a first frequency;

generating, by a second acoustic source, a second signal at a second frequency;

wherein the first acoustic source and the second acoustic source are arranged and positioned within the borehole and are controllable such that the first and the second acoustic signals intersect in an intersection volume outside the borehole and a start time difference is provided between the second acoustic signal and the first acoustic signal;

receiving, by a receiver, a detected signal returning to the borehole having a frequency equal to a difference between the first frequency and the second frequency, the detected signal being generated by a non-linear mixing process from the first acoustic signal and the second acoustic signal in a non-linear mixing zone within the intersection volume; and

recording the detected signal and storing the detected signal in a storage device and recording measurement parameters including a position of the first acoustic source, a position of the second acoustic source, a position of the receiver, elevation angle and azimuth angle of the first acoustic signal and elevation angle and azimuth angle of the second acoustic signal.

87. The method according to claim 86, further comprising scanning a range of start time differences between the start time of a broadcast of the first acoustic signal and the start time of a broadcast of the second acoustic signal so that the range of start time differences includes a time difference for which the first acoustic signal and the second acoustic signal arrive substantially simultaneously at the center of the mixing zone.

88. The method according to claim 86, further comprising controlling a phase of the first acoustic signal or a phase of the second acoustic signal, or both such that received signals from the interaction of the first acoustic signal and the second acoustic signal have opposite polarity.

89. The method according to claim 88, further comprising combining received signals from the interaction of the first acoustic signal and the second acoustic signal having opposite polarities to enhance the signal generated by the non-linear interaction over noise or signals originating from linear interaction, or both.

90. The method according to claim 86, wherein at least a portion of the detected signal generated by the non-linear interaction is proportional to a product of amplitudes of the first acoustic signal and the second acoustic signal.

91. The method according to claim 86, further comprising applying, by a time variant band-pass filter, a time-variant frequency band-pass filter to the detected signal so as to keep a narrow frequency bandwidth around an expected frequency bandwidth of the signal generated by non-linear interaction, the narrow frequency bandwidth being selected around the difference between the first frequency and the second frequency.

92. The method according to claim 86, wherein:

generating the first acoustic signal comprises generating a first plurality of pulses arranged as a time sequence, the first plurality of pulses being separated in time, each pulse comprising a first modulated signal at a central frequency, wherein central frequencies of two consecutive pulses are different;

generating the second acoustic signal comprises generating a second plurality of pulses arranged as a time sequence, the second plurality of pulses being separated in time, wherein a separation in time between centers of two consecutive pulses is the same as a separation in time between centers of two corresponding pulses in the first

plurality of pulses, wherein a start time difference is provided between a start time of a broadcast of the second plurality of pulses and a start time of a broadcast of the first plurality of pulses, wherein each pulse comprises a second modulated signal, and wherein a central frequency of the second modulated signal within each pulse in the second plurality of pulses is a selected fraction of the central frequency of the first modulated signal within each corresponding pulse in the first plurality of pulses; and

the start time difference is controlled such that the first acoustic signal and the second acoustic signal intersect in the mixing zone .

93. The method according to claim 92, wherein a separation in time between centers of two consecutive pulses in the first plurality of pulses is greater than a time duration of each pulse.

94. The method according to claim 92, further comprising

correlating, by a processor, the detected signal with a template signal comprising a plurality of pulses, the plurality of pulses being arranged in a time sequence and separated in time, wherein a separation in time between centers of two consecutive pulses is the same as the separation in time between centers of two consecutive pulses in the first plurality of pulses, wherein each pulse in the plurality of pulses comprises a modulated signal having a central frequency that is equal to a difference between a central frequency of the first modulated signal and a central frequency of the second modulated signal in each corresponding pulse of the first plurality of pulses and the second plurality of pulses, to obtain a correlated signal containing the signal generated by non-linear interaction in the non-linear mixing zone.

95. The method according claim 94, further comprising receiving a plurality of detected signals at a plurality of receivers and applying, by the processor, multi-dimensional filtering, or time moveout analysis and stacking, or both, to the plurality of detected signals that are received by the plurality of receivers.

96. The method according to claim 94, further comprising detecting a pressure signal by a hydrophone or detecting multi-component signals by multi-component detectors, or both.

97. The method according to claim 94, further comprising detecting each of three components signals of the received signal by each of three detectors of the receiver.

98. The method according to claim 97, further comprising performing, by the processor, hodogram analysis on the detected signal.

99. The method according to claim 98, wherein performing hodogram analysis comprises applying a hodogram analysis to the three detected component signals corresponding to vertical shear SV, horizontal shear SH and compressional P signals.

100. The method according to claim 99, further comprising applying multi-dimensional filtering, or time moveout analysis and stacking, or both, to the three detected component signals that are received by the three detectors, or applying multi-dimensional filtering, or time moveout analysis and stacking, or both, to the pressure signal detected by the hydrophone.

101. The method according to claim 99, applying a time variant frequency band pass filter to the detected signal so as to keep a narrow frequency bandwidth around an expected frequency bandwidth of the signal generated by non-linear interaction, the narrow frequency bandwidth being selected around the difference between the first frequency and the second frequency.

102. The method according claim 101, further comprising repeating a plurality of times the generating of the first acoustic signal, the generating of the second acoustic

signal, the receiving of the detected signal, the applying of the time variant frequency band pass filter, the correlating of the detected signal with the template signal, the applying of the hodogram analysis, and the combining of signals to improve signal to noise ratio.

103. The method according claim 101 further comprising varying the start time difference and repeating a plurality of times the generating of the first acoustic signal, the generating of the second acoustic signal, the receiving of the detected signal, the applying of the time variant frequency band pass filter, the correlating of the detected signal with the template signal, and the applying of the hodogram analysis for each of three detected signals to separate different propagation signal modes including compressional mode P, vertical shear mode SV and horizontal shear mode SH.

104. The method according to claim 103, varying the start time difference and repeating a plurality of times the generating of the first signal, the generating of the second signal, the receiving of the detected signal , and the applying of the time variant frequency band pass filter, and the correlating of the signal detected by the hydrophone with the template signal.

105. The method of claim 97, further comprising scanning a plurality of azimuth and elevation angles of the first acoustic signal, a plurality of azimuth and elevation angles of the second acoustic signal, or a position of the receiver, and repeating a plurality of times the generating of the first acoustic signal, the generating of the second acoustic signal, the receiving of the detected signal, the applying of the time variant frequency band pass filter to obtain a filtered signal, the correlating of the filtered signal with the template signal, and the applying of the hodogram analysis on the three detected component signals to separate the component signals to different propagation signal modes including compressional mode P, vertical shear mode SV and horizontal shear mode SH.

106. The method according to claim 105, further comprising repeating the varying the start time difference between the start time of a broadcast of the first acoustic signal and the start time of a broadcast of the second acoustic signal, and repeating the varying the frequency ratio between the first frequency and the second frequency, and repeating the scanning the plurality of azimuth and elevation angles of the first acoustic signal, the plurality of azimuth and elevation angles of the second acoustic signal, and the position of the receiver to obtain a plurality of signal measurements.

107. The method according to claim 106, further comprising storing in a storage device in communication with the processor, the obtained plurality of signal measurements.

108. A method of generating a three-dimensional image of a rock formation comprising:

generating a first acoustic signal by a first acoustic source, wherein the first acoustic signal comprises a first plurality of pulses arranged in a time sequence, the first plurality of pulses being separated in time, each pulse comprising a first modulated signal at a central frequency, wherein central frequencies of two consecutive pulses are different;

generating a second acoustic signal by a second acoustic source, wherein the second acoustic signal comprises a second plurality of pulses arranged as a time sequence, the second plurality of pulses being separated in time, wherein a separation in time between centers of two consecutive pulses is the same as a separation in time between centers of two corresponding pulses in the first plurality of pulses, wherein a start time difference is provided between a start time of a broadcast of the second plurality of pulses and a start time of a broadcast of the first plurality of pulses, wherein each pulse comprises a modulated signal and wherein a central frequency of the modulated signal within each pulse in the second plurality of pulses is a selected fraction of the central frequency of the modulated signal within each corresponding pulse in the first plurality of pulses;

wherein the first acoustic source and the second acoustic source are arranged within the borehole and are controllable such that trajectories of the first and the second acoustic signals intersect in an intersection volume outside the borehole;

receiving, by a receiver arranged within the borehole, a detected signal returning to the borehole, the detected signal including a signal being generated by a non-linear mixing process from the first acoustic signal and the second acoustic signal in a non-linear mixing zone within the intersection volume;

performing, by a processor, data processing on the received signal to extract the signal generated by the non-linear mixing process over noise or over signals generated by a linear interaction process, or both; and

generating a three dimensional image of a propagation compressional velocity, a propagation of shear velocity, a ratio of compressional velocity and shear velocity, or non-linear properties of a rock formation, or any combination of two or more thereof based on the signal generated by the non-linear mixing process.

109. The method according to claim 108, wherein generating the first acoustic signal comprises generating the plurality of pulses such that a separation in time between centers of two consecutive pulses in the first plurality of pulses is greater than a time duration of each pulse.

110. The method according to claim 108, wherein generating the three dimensional image of the propagation compressional velocity, the propagation of shear velocity, the ratio of compressional velocity and shear velocity, or the non-linear properties of the rock formation, or any combination of two or more thereof is performed by using Kirchhoff imaging, beam imaging or wave equation imaging.

111. The method according to claim 110, further comprising determining a value of the propagation compressional velocity or a value of the shear velocity or both using tomographic velocity inversion or full wave form inversion or by iterative imaging in combination with tomographic velocity inversion.

112. The method according to claim 108, wherein generating the three dimensional image of the propagation compressional velocity, the propagation of shear velocity, the ratio of compressional velocity to shear velocity, or the non-linear properties of a rock formation, or any combination of two or more thereof is performed using an analysis or mapping method comprising:

estimating an initial propagating velocity model using well logs in a borehole and assumptions about lateral continuity of a rock formation away from the borehole;

calculating a first travel time of the first acoustic signal from the first acoustic source to a center of the mixing zone;

calculating a second travel time of the second acoustic signal from the second acoustic source to the center of the mixing zone;

calculating a third travel time between the center of the mixing zone and a receiver configured to receive a detected signal returning to the borehole having a frequency equal to a difference between the first frequency and the second frequency, the detected signal being generated by a non-linear mixing process from the first acoustic signal and the second acoustic signal in the mixing zone;

calculating an arrival time Tp corresponding to a total travel time of a signal returning to the borehole by adding the first travel time and the third travel time, and calculating a start time difference δp between the first travel time and the second travel time;

extracting from the signal received at the receiver signals generated by the non-linear mixing process to yield for each start time difference δ and for each frequency ratio d, a correlated signal $M(t, \delta, d)$, which is a function of arrival time t, the start time difference δ and the frequency ratio d, and which contains non-linear interaction signals;

searching all values of arrival time t, start time difference δ, and frequency ratio d in the correlated signal $M(t, \delta, d)$ to determine a value of arrival time T_{NL}, start time difference δ_{NL} and frequency ratio d_{NL} for which a band-limited spike signal generated by the non-linear mixing process occurs;

comparing the calculated arrival time Tp with a measured arrival time T_{NL} and comparing the calculated start time difference δp with a measured start time difference δ_{NL};

determining whether the calculated arrival time Tp is different from the measured arrival time T_{NL} and the calculated start time difference δp is different from the measured start time difference δ_{NL} and, if so, updating the propagating velocity model using a difference between the measured and the calculated arrival times and a difference between the calculated and the measured start time differences; and

repeating iteratively the above steps until the calculated arrival time corresponding to the total travel time is substantially equal to a measured arrival time, or the calculated start time difference is substantially equal to a measured start time difference, or both, for a range of values of the position of the first acoustic source, the position of the second acoustic source, the position of the receiver, the azimuth angle of the first acoustic signal, the azimuth angle of the second acoustic signal, the elevation angle of the first acoustic signal, or the elevation angle of the second acoustic signal, or any combination of two or more thereof, to obtain a propagating velocity Vp and propagating velocity Vs that results in the calculated arrival time being substantially equal to the measured arrival time or the calculated time difference being substantially equal to the measured start time difference, or both; and

generating a three dimensional image of a propagation compressional velocity, a propagation of shear velocity, a ratio of compressional velocity to shear velocity, or non-linear properties, or any combination of two or more thereof.

113. The method according to claim 108, wherein performing, by a processor, data processing on the received signal to extract the signal generated by the non-linear mixing process comprises extracting signals generated by the non-linear mixing process by correlating the detected signal with a template signal designed according to forecast properties of the signal according to the selection rules for non-collinear mixing in a non-linear medium, and repeating for a plurality of start time differences δ between the second plurality of pulses and the first plurality of pulses and for a plurality of frequency ratios d between the central frequency of the modulated signal

within each pulse in the second plurality of pulses and the central frequency of the modulated signal within each corresponding pulse in the first plurality of pulses, the correlating between the template signal and the detected signal to yield for each start time difference δ and each frequency ratio d a correlated signal M(t, δ, d) which contains a non-linear interaction signal.

114. The method according to claim 112, further comprising calculating, based on a propagation velocity model, trajectories of the first acoustic signal emitted by a first acoustic source and the second acoustic signal emitted by the second acoustic source using the position of the first acoustic source, the elevation angle and the azimuth angle of the first acoustic signal and the position of the second acoustic source, the elevation angle and the azimuth angle of the second acoustic signal.

115. The method according to claim 114, further comprising determining a location of a mixing zone where the first acoustic signal and the second acoustic signal interact non-linearly based on the position of the first acoustic source, the elevation angle and the azimuth angle of the first acoustic signal and the position of the second acoustic source, the elevation angle and the azimuth angle of the second acoustic signal.

116. The method according to claim 115, further comprising calculating spatial coordinates of the mixing zone where the two acoustic signals interact non-linearly, calculating a convergence angle between the first acoustic signal from the first acoustic source and the second acoustic signal from the second acoustic source based on the elevation angle and the azimuth angle of the first acoustic signal, and the position of the first acoustic source, and the elevation angle and the azimuth angle of the second acoustic signal, and a position of the second acoustic source.

117. The method according to claim 116, further comprising mapping the correlated signal $M(T_{NL}, \delta_{NL}, d_{NL})$ to the spatial coordinates of the mixing zone corresponding to the non-linear properties of the rock formation at the mixing zone.

118. The method according to claim 117, further comprising calculating the velocity ratio between the compressional velocity and the shear velocity at the mixing zone from the frequency ratio d_{NL} and the convergence angle using a selection rule relationship between velocity ratio of compressional velocity and shear velocity, frequency ratio and angle of convergence and mapping the calculated velocity ratio to the spatial coordinates of the mixing zone.

119. The method of claim 118, further comprising repeating mapping the correlated signal $M(T_{NL}, \delta_{NL}, d_{NL})$ to the spatial coordinates of the mixing zone, or mapping the calculated velocity ratio to the spatial coordinates of the mixing zone, or both, for a range of values of the position of the first acoustic source, the position of the second acoustic source, the position of the receiver, the azimuth angle of the first acoustic signal, the azimuth angle of the second acoustic signal, the elevation angle of the first acoustic signal, or the elevation angle of the second acoustic signal, or any combination of two or more thereof, to obtain a three dimensional image of a strength of the non-linear properties or a three dimensional image of the velocity ratio, or both, by combining the measured values through repeated measurements.

120. The method according to claim 108, further comprising determining a value of the propagation compressional velocity, a value of the propagation of shear velocity, or a value of the ratio of compressional velocity and shear velocity, or any combination of two or more thereof.

121. The method according to claim 108, wherein determining the value of propagation compressional velocity, the value of propagation of shear velocity, the value of ratio of compressional velocity and shear velocity, or any combination of two or more thereof is performed using a mapping method comprising:

estimating an initial propagating velocity model using well logs in a borehole and assumptions about lateral continuity of a the rock formation away from the borehole;

calculating a first travel time of the first acoustic signal from the first acoustic source to a center of the mixing zone;

calculating a second travel time of the second acoustic signal from the second acoustic source to the center of the mixing zone;

calculating a third travel time between the center of the mixing zone and a receiver configured to receive a detected signal returning to the borehole having a frequency equal to a difference between the first frequency and the second frequency, the detected signal being generated by a non-linear mixing process from the first acoustic signal and the second acoustic signal in a mixing zone;

calculating an arrival time Tp corresponding to a total travel time of a signal returning to the borehole by adding the first travel time and the third travel time, and calculating a start time difference δp between the first travel time and the second travel time;

extracting from the signal received at the receiver, signals generated by the non-linear mixing process to yield for each start time difference δ and for each frequency ratio d, a correlated signal $M(t, \delta, d)$, which is a function of arrival time t, the start time difference δ and the frequency ratio d, and which contains non-linear interaction signals;

searching all values of arrival time t, start time difference δ, and frequency ratio d in the correlated signal $M(t, \delta, d)$ to determine a value of arrival time T_{NL}, start time difference δ_{NL} and frequency ratio d_{NL} for which a signal generated by the non-linear mixing process occurs;

comparing the calculated arrival time Tp with a measured arrival time T_{NL} and comparing the calculated start time difference δp with a measured start time difference δ_{NL};

determining whether the calculated arrival time Tp is different from the measured arrival time T_{NL} and the calculated time difference δp is different from the measured start time difference δ_{NL} and, if so, updating the propagating velocity model using a difference between the measured and the calculated arrival times and a difference between the calculated and the measured start time differences; and

repeating iteratively the above steps until the calculated arrival time corresponding to the total travel time is substantially equal to a measured arrival time, or the calculated time difference is substantially equal to a measured start time difference, or both, for a range of values of the position of the first acoustic source, the position of the second acoustic source, the position of the receiver, the azimuth angle of the first acoustic signal, the azimuth angle of the second acoustic signal, the elevation angle of the first acoustic signal, or the elevation angle of the second acoustic signal, or any combination of two or more thereof, to obtain a propagating velocity Vp and propagating velocity Vs that result in the calculated arrival time being substantially equal to the measured arrival time or the calculated start time difference being substantially equal to the measured start time difference, or both.

122. The method according to claim 121, further comprising calculating based on a propagation velocity model trajectories of the first acoustic signal having the first frequency emitted by the first acoustic source and the second acoustic signal having the second frequency emitted by the second acoustic source using a position of the first acoustic source, the elevation angle and the azimuth angle of the first acoustic signal and the position of the second acoustic source, the elevation angle and the azimuth angle of the second acoustic signal, and the position of the receiver.

123. The method according to claim 122, further comprising determining a location of a mixing zone where the first acoustic signal and the second acoustic signal interact non-linearly based on the position of the first acoustic source, the elevation angle and the azimuth angle of the first acoustic signal and the position of the second acoustic source, the elevation angle and the azimuth angle of the second acoustic signal.

124. The method according to claim 123, further comprising calculating spatial coordinates of the mixing zone where the two acoustic signals interact non-linearly, calculating a convergence angle between the first acoustic signal from the first acoustic source and the second acoustic signal from the second acoustic source based on the elevation angle and the azimuth angle of the first acoustic signal and the

position of the first acoustic source, and the elevation angle and the azimuth angle of the second acoustic signal and the position of the second acoustic source.

125. The method according to claim 124, further comprising calculating the velocity ratio between the compressional velocity and the shear velocity at the mixing zone from the frequency ratio d_{NL} and the convergence angle using a selection rule relationship between velocity ratio of compressional velocity and shear velocity, frequency ratio and angle of convergence and mapping the calculated velocity ratio to the spatial coordinates of the mixing zone.

126. The method of claim 125, further comprising repeating mapping the calculated velocity ratio to the spatial coordinate of the mixing zone for a range of values of the position of the first acoustic source, the position of the second acoustic source, the position of the receiver, the azimuth angle of the first acoustic signal, the azimuth angle of the second acoustic signal, the elevation angle of the first acoustic signal, or the elevation angle of the second acoustic signal, or any combination of two or more thereof, to obtain the value of the velocity ratio for a volume of the rock formation around the borehole.

127. A method of generating a three-dimensional image of non-linear properties of a rock formation, the method comprising:

reading, by a processor, measurement parameters including a position of a first acoustic source configured to generate a first acoustic signal at a first frequency, a position of a second acoustic source configured to generate a second acoustic signal at a second frequency, a position of a receiver configured to receive a detected signal returning to a borehole from a mixing zone within the rock formation at the intersection of the first acoustic signal and the second acoustic signal, and elevation and azimuth angles of the first and second acoustic signals;

reading, by the processor, a correlated signal $M(t, \delta, d)$, which is a function of arrival time t, start time difference δ and frequency ratio d between the first frequency and the second frequency, the correlated signal $M(t, \delta, d)$ containing signals generated

by a non-linear mixing process from a first acoustic signal and a second acoustic signal at a mixing zone within the rock formation;

reading, by the processor, an initial propagation compressional and shear velocity model;

calculating, by the processor, a first travel time of the first acoustic signal from the first acoustic source to a center of the mixing zone;

calculating, by the processor, a second travel time of the second acoustic signal from the second acoustic source to the center of the mixing zone;

calculating, by the processor, a third travel time between the center of the mixing zone and the receiver configured to receive the detected signal returning to the borehole having a frequency equal to a difference between the first frequency and the second frequency, the detected signal being generated by a non-linear mixing process from the first acoustic signal and the second acoustic signal in the mixing zone;

calculating, by the processor, an arrival time Tp corresponding to a total travel time of a signal returning to the borehole by adding the first travel time and the third travel time, and calculating a start time difference δ_p between the first travel time and the second travel time;

searching all values of arrival time t, start time difference δ, and the frequency ratio d in the correlated signal $M(t, \delta, d)$ to determine for which value of arrival T_{NL}, start time difference δ_{NL} and frequency ratio d_{NL} a band-limited spike signal $M(T_{NL},\delta_{NL},d_{NL})$ generated by the non-linear mixing process occurs; and

calculating, by the processor, trajectories of acoustic propagation using an elevation angle of the first acoustic signal, and a position of the first acoustic source, and an elevation angle of the second acoustic signal, and a position of the second acoustic source based on the propagation compressional and shear velocity model;

mapping the band-limited spike signal $M(T_{NL},\delta_{NL},d_{NL})$ generated by the non-linear mixing process to spatial coordinates of an interaction mixing zone where the first acoustic signal and the second acoustic signal intersect; and

repeating the mapping of the band-limited spike signal $M(T_{NL},\delta_{NL},d_{NL})$ to the spatial coordinates of the interaction mixing zone for a plurality of values of the

position of the first acoustic source, the position of the second acoustic source, the position of the receiver, an elevation angle of the first acoustic signal, an azimuth angle of the first acoustic signal, an elevation angle of the second acoustic signal, or an azimuth angle of the second acoustic signal, or any combination of two or more thereof, and generating a three dimensional image of a strength of the non-linear mixing process.

128. The method according to claim 127, further comprising determining a location of the mixing zone where the first acoustic signal and the second acoustic signal intersect using the position of the first acoustic source, the elevation angle and the azimuth angle of the first acoustic signal and the position of the second acoustic source, the elevation angle and the azimuth angle of the second acoustic signal.

129. A method of generating a three-dimensional image of a propagation compressional velocity, a propagation of shear velocity, a ratio of compressional velocity to shear velocity of a rock formation, or any combination of two or more thereof, the method comprising:

reading, by a processor, measurement parameters including a location of a first acoustic source configured to generate a first acoustic signal at a first frequency, a position of second acoustic source configured to generate a second acoustic signal at a second frequency, a position of a receiver configured to receive a detected signal returning to a borehole from a mixing zone within the rock formation at the intersection of the first acoustic signal and the second acoustic signal, and elevation and azimuth angles of the first and second acoustic signals;

reading, by the processor, a correlated signal $M(t, \delta, d)$, which is a function of arrival time t, start time difference δ and frequency ratio d between the first frequency and the second frequency, the correlated signal $M(t, \delta, d)$ containing signals generated by a non-linear mixing process from a first acoustic signal and a second acoustic signal at a mixing zone within the rock formation;

reading, by the processor, an initial propagation compressional and shear velocity model;

estimating, by the processor, an initial propagating velocity model using well logs in a borehole and assumptions about lateral continuity of the rock formation away from the borehole;

calculating, by the processor, a first travel time of the first acoustic signal from the first acoustic source to a center of the mixing zone;

calculating, by the processor, a second travel time of the second acoustic signal from the second acoustic source to the center of the mixing zone;

calculating, by the processor, a third travel time between the center of the mixing zone and the receiver configured to receive the detected signal returning to the borehole having a frequency equal to a difference between the first frequency and the second frequency, the detected signal being generated by a non-linear mixing process from the first acoustic signal and the second acoustic signal in mixing zone;

calculating, by the processor, an arrival time Tp corresponding to a total travel time of a signal returning to the borehole by adding the first travel time and the third travel time, and calculating a start time difference δ_p between the first travel time and the second travel time;

searching all values of arrival time t, start time difference δ, and the frequency ratio d in the correlated signal $M(t, \delta, d)$ to determine for which value of arrival time T_{NL}, start time difference δ_{NL} and frequency ratio d_{NL} a band-limited spike signal $M(T_{NL},\delta_{NL},d_{NL})$ generated by the non-linear mixing process occurs;

comparing the calculated arrival time Tp with a measured arrival time T_{NL} or comparing the calculated start time difference δ_p with a measured start time difference δ_{NL};

updating the propagating velocity model by tomographic velocity inversion or full wave form inversion using a difference between the measured and the calculated arrival times if the calculated arrival time is different from the measured arrival time, or using a difference between the calculated and the measured start time differences if the calculated start time difference is different from the measured start time difference, or both;

repeating iteratively the above steps until the calculated arrival time is substantially equal to the measured arrival time or the calculated start time difference is substantially equal to the measured start time difference, or both, for a range of values of the position of the first acoustic source, the position of the second acoustic source, the position of the receiver, the azimuth angle of the first acoustic signal, the azimuth angle of the second acoustic signal, the elevation angle of the first acoustic signal, or the elevation angle of the second acoustic signal, or any combination of two or more thereof, to obtain a propagating velocity Vp and propagating velocity Vs that results in the calculated arrival time being substantially equal to the measured arrival time or the calculated start time difference being substantially equal to the measured start time difference, or both; and

generating a three dimensional image of a propagation compressional velocity, a propagation of shear velocity, a ratio of compressional velocity to shear velocity, or any combination of two or more thereof.

130. The method according to claim 129, further comprising determining a value of the propagating compressional velocity, a value of the propagating shear velocity, or a value of the velocity ratio of the compressional velocity and the shear velocity, or any combination of two or more thereof.

131. The method according to claim 129, further comprising calculating spatial coordinates of the mixing zone where the two acoustic signals intersect, calculating a convergence angle between the first acoustic signal from the first acoustic source and the second acoustic signal from the second acoustic source based on the elevation angle and the azimuth angle of the first acoustic signal, and the position of the first acoustic source, and the elevation angle and the azimuth angle of the second acoustic signal, and the position of the second acoustic source.

132. The method according to claim 131, further comprising calculating the velocity ratio of compressional velocity and shear velocity at the mixing zone from the measured frequency ratio d_{NL} and the calculated the convergence angle using a

selection rule relationship between velocity ratio of compressional velocity and shear velocity, frequency ratio and angle of convergence.

133. The method of claim 132, further comprising repeating mapping the calculated velocity ratio to the spatial coordinate of the mixing zone for a range of values of the position of the first acoustic source, the position of the second acoustic source, the position of the receiver, the azimuth angle of the first acoustic signal, the azimuth angle of the second acoustic signal, the elevation angle of the first acoustic signal, or the elevation angle of the second acoustic signal, or any combination of two or more thereof to obtain a three dimensional image of the velocity ratio.

134. A system for generating a three-dimensional image of a rock formation comprising:

a first acoustic source configured to generate a first acoustic signal, wherein the first acoustic signal comprises a first plurality of pulses arranged in a time sequence, the first plurality of pulses being separated in time, each pulse comprising a first modulated signal at a central frequency, wherein central frequencies of two consecutive pulses are different;

a second acoustic source configured to generate a second acoustic signal by a second acoustic source, wherein the second acoustic signal comprises a second plurality of pulses arranged as a time sequence, the second plurality of pulses being separated in time, wherein a separation in time between centers of two consecutive pulses is the same as a separation in time between centers of two corresponding pulses in the first plurality of pulses, wherein a start time difference is provided between a start time of a broadcast of the second plurality of pulses and a start time of a broadcast of the first plurality of pulses, wherein each pulse comprises a modulated signal, and wherein a central frequency of the modulated signal within each pulse in the second plurality of pulses is a selected fraction of the central frequency of the modulated signal within each corresponding pulse in the first plurality of pulses;

wherein the first acoustic source and the second acoustic source are arranged within the borehole and are controllable such that trajectories of the first and the second acoustic signals intersect in an intersection volume outside the borehole;

a receiver arranged within the borehole, the receiver being configured to detect a received signal returning to the borehole, the received signal including a signal being generated by a non-linear mixing process from the first acoustic signal and the second acoustic signal in a non-linear mixing zone within the intersection volume;

a first processor configured to perform data processing on the received signal to extract the signal generated by the non-linear mixing process over noise or over signals generated by a linear interaction process, or both; and

a second processor configured to generate a three dimensional image of a propagation compressional velocity, a propagation of shear velocity, a ratio of compressional velocity and shear velocity, or non-linear properties of a rock formation, or any combination of two or more thereof based on the signal generated by the non-linear mixing process.

135. The system according to claim 134, wherein a separation in time between centers of two consecutive pulses in the first plurality of pulses is greater than a time duration of each pulse.

136. The system according to claim 134, wherein the second processor is configured to generate the three dimensional image of the propagation compressional velocity, the propagation of shear velocity, the ratio of compressional velocity and shear velocity, or the non-linear properties of the rock formation, or any combination of two or more thereof by using Kirchhoff imaging, beam imaging or wave equation imaging.

137. The system according to claim 136, wherein the second processor is configured to determine a value of the propagation compressional velocity or a value of the shear velocity or both using tomographic velocity inversion or full wave form inversion or by iterative imaging in combination with tomographic velocity inversion.

138. The system according to claim 134, wherein the second processor is configured to:

estimate an initial propagating velocity model using well logs in a borehole and assumptions about lateral continuity of a rock formation away from the borehole;

calculate a first travel time of the first acoustic signal from the first acoustic source to a center of the mixing zone;

calculate a second travel time of the second acoustic signal from the second acoustic source to the center of the mixing zone;

calculate a third travel time between the center of the mixing zone and a receiver configured to receive a detected signal returning to the borehole having a frequency equal to a difference between the first frequency and the second frequency, the detected signal being generated by a non-linear mixing process from the first acoustic signal and the second acoustic signal in the mixing zone;

calculate an arrival time Tp corresponding to a total travel time of a signal returning to the borehole by adding the first travel time and the third travel time, and calculating a start time difference δ_p between the first travel time and the second travel time;

extract from the signal received at the receiver, signals generated by the non-linear mixing process to yield for each start time difference δ and for each frequency ratio a correlated signal $M(t, \delta, d)$, which is a function of arrival time t, the start time difference δ and the frequency ratio d, and which contains non-linear interaction signals;

search all values of arrival time t, start time difference δ, and frequency ratio d in the correlated signal $M(t, \delta, d)$ to determine a value of arrival time T_{NL}, start time difference δ_{NL} and frequency ratio d_{NL} for which a band-limited spike signal generated by the non-linear mixing process occurs;

compare the calculated arrival time Tp with a measured arrival time T_{NL} and compare the calculated start time difference δp with a measured start time difference δ_{NL};

determine whether the calculated arrival time Tp is different from the measured arrival time T_{NL} and the calculated start time difference δp is different from the measured start time difference δ_{NL} and, if so, update the propagating velocity model using a difference between the measured and the calculated arrival times and a difference between the calculated and the measured start time differences; and

repeat iteratively the above steps until the calculated arrival time corresponding to the total travel time is substantially equal to a measured arrival time or the calculated start time difference is substantially equal to a measured start time difference, or both, for a range of values of the position of the first acoustic source, the position of the second acoustic source, the position of the receiver, the azimuth angle of the first acoustic signal, the azimuth angle of the second acoustic signal, the elevation angle of the first acoustic signal, or the elevation angle of the second acoustic signal, or any combination of two or more thereof, to obtain a propagating velocity Vp and propagating velocity Vs that results in the calculated arrival time being substantially equal to the measured arrival time or the calculated time difference being substantially equal to the measured start time difference, or both; and

generate a three dimensional image of a propagation compressional velocity, a propagation of shear velocity, a ratio of compressional velocity to shear velocity, or non-linear properties, or any combination of two or more thereof.

139. The system according to claim 134, wherein the first processor is configured to extract signals generated by the non-linear mixing process by correlating the detected signal with a template signal designed according to forecast properties of the signal according to the selection rules for non-collinear mixing in a non-linear medium, and repeating for a plurality of start time differences δ between the second plurality of pulses and the first plurality of pulses and for a plurality of frequency ratios d between the central frequency of the modulated signal within each pulse in the second plurality of pulses and the central frequency of the modulated signal within each corresponding pulse in the first plurality of pulses, the correlating between the template signal and the detected signal, to yield for each start time difference δ and each frequency ratio d a correlated signal M(t, δ, d) which contains a non-linear interaction signal.

140. The system according to claim 138, wherein the second processor is configured to calculate, based on a propagation velocity model, trajectories of the first acoustic signal emitted by a first acoustic source and the second acoustic signal emitted by the second acoustic source using the position of the first acoustic source, the elevation angle and the azimuth angle of the first acoustic signal and the position of the second acoustic source, the elevation angle and the azimuth angle of the second acoustic signal.

141. The system according to claim 140, wherein the second processor is configured to determine a location of a mixing zone where the first acoustic signal and the second acoustic signal interact non-linearly based on the position of the first acoustic source, the elevation angle and the azimuth angle of the first acoustic signal and the position of the second acoustic source, the elevation angle and the azimuth angle of the second acoustic signal.

142. The system according to claim 141, wherein the second processor is configured to calculate spatial coordinates of the mixing zone where the two acoustic signals interact non-linearly, calculating a convergence angle between the first acoustic signal from the first acoustic source and the second acoustic signal from the second acoustic source based on the elevation angle and the azimuth angle of the first acoustic signal, and the position of the first acoustic source, and the elevation angle and the azimuth angle of the second acoustic signal, and a position of the second acoustic source.

143. The system according to claim 142, wherein the second processor is configured to map the correlated signal $M(T_{NL}, \delta_{NL}, d_{NL})$ to the spatial coordinates of the mixing zone corresponding to the non-linear properties of the rock formation at the mixing zone.

144. The system according to claim 143, wherein the second processor is configured to calculate the velocity ratio between the compressional velocity and the shear velocity at the mixing zone from the frequency ratio d_{NL} and the convergence angle using a selection rule relationship between velocity ratio of compressional velocity and shear velocity, frequency ratio and angle of convergence and mapping the calculated velocity ratio to the spatial coordinates of the mixing zone.

145. The system of claim 144, wherein the second processor is configured to map the correlated signal $M(T_{NL}, \delta_{NL}, d_{NL})$ to the spatial coordinates of the mixing zone or map the calculated velocity ratio to the spatial coordinate of the mixing zone, or both, for a range of values of the position of the first acoustic source, the position of the second acoustic source, the position of the receiver, the azimuth angle of the first acoustic signal, the azimuth angle of the second acoustic signal, the elevation angle of the first acoustic signal, or the elevation angle of the second acoustic signal, or any combination of two or more thereof, to obtain a three dimensional image of a strength of the non-linear properties or a three dimensional image of the velocity ratio, or both, by combining the measured values through repeated measurements.

146. The system according to claim 134, wherein the second processor is configured to determine a value of the propagation compressional velocity, a value of the propagation of shear velocity, or a value of the ratio of compressional velocity and shear velocity, or any combination of two or more thereof.

147. The system according to claim 146, wherein the second processor is configured to:

estimate an initial propagating velocity [illegible] the rock formation away from the borehole and assumptions about lateral continuity of [illegible] borehole;

calculate a first travel time of the first acoustic signal from the first acoustic source to a center of the mixing zone;

calculate a second travel time of the second acoustic signal from the second acoustic source to the center of the mixing zone;

calculate a third travel time between the center of the mixing zone and a receiver configured to receive a detected signal returning to the borehole having a frequency equal to a difference between the first frequency and the second frequency, the detected signal being generated by a non-linear mixing process from the first acoustic signal and the second acoustic signal in a mixing zone;

calculate an arrival time Tp corresponding to a total travel time of a signal returning to the borehole by adding the first travel time and the third travel time, and calculating a start time difference δp between the first travel time and the second travel time;

extract from the signal received at the receiver, signals generated by the non-linear mixing process, to yield for each start time difference δ and for each frequency ratio d, a correlated signal M(t, δ, d) which is a function of arrival time t, the start time difference δ and the frequency ratio d, and which contains non-linear interaction signals;

search all values of arrival time t, start time difference δ, and frequency ratio d in the correlated signal M(t, δ, d) to determine a value of arrival time T_{NL}, start time difference δ_{NL} and frequency ratio d_{NL} for which a signal generated by the non-linear mixing process occurs;

compare the calculated arrival time Tp with a measured arrival time T_{NL} and compare the calculated start time difference δ_p with a measured start time difference δ_{NL};

determine whether the calculated arrival time Tp is different from the measured arrival time T_{NL} and the calculated time difference δp is different from the measured start time difference δ_{NL} and, if so, update the propagating velocity model using a difference between the measured and the calculated arrival times and a difference between the calculated and the measured start time differences; and

repeat iteratively the above steps until the calculated arrival time corresponding to the total travel time is substantially equal to a measured arrival time,

or the calculated time difference is substantially equal to a measured start time difference, or both, for a range of values of the position of the first acoustic source, the position of the second acoustic source, the position of the receiver, the azimuth angle of the first acoustic signal, the azimuth angle of the second acoustic signal, the elevation angle of the first acoustic signal, or the elevation angle of the second acoustic signal, or any combination of two or more thereof, to obtain a propagating velocity Vp and propagating velocity Vs that result in the calculated arrival time being substantially equal to the measured arrival time, or the calculated start time difference being substantially equal to the measured start time difference, or both.

148. The system according to claim 147, wherein the second processor is configured to calculate, based on a propagation velocity model, trajectories of the first acoustic signal having the first frequency emitted by the first acoustic source and the second acoustic signal having the second frequency emitted by the second acoustic source using the position of the first acoustic source, the elevation angle and the azimuth angle of the first acoustic signal and the position of the second acoustic source, the elevation angle and the azimuth angle of the second acoustic signal, and the position of the receiver.

149. The system according to claim 148, wherein the second processor is configured to calculate spatial coordinates of the mixing zone where the two acoustic signals interact non-linearly, calculate a convergence angle between the first acoustic signal from the first acoustic source and the second acoustic signal from the second acoustic source based on the elevation angle and the azimuth angle of the first acoustic signal, and the position of the first acoustic source, and the elevation angle and the azimuth angle of the second acoustic signal, and the position of the second acoustic source.

150. The system according to claim 149, wherein the second processor is configured to calculate the velocity ratio between the compressional velocity and the shear velocity at the mixing zone from the frequency ratio d_{NL} and the convergence

angle using a selection rule relationship between velocity ratio of compressional velocity and shear velocity, frequency ratio and angle of convergence and to map the calculated velocity ratio to the spatial coordinates of the mixing zone.

151. The system of claim 150, wherein the second processor is configured to repeat mapping the calculated velocity ratio to the spatial coordinates of the mixing zone for a range of values of the position of the first acoustic source, the position of the second acoustic source, the position of the receiver, the azimuth angle of the first acoustic signal, the azimuth angle of the second acoustic signal, the elevation angle of the first acoustic signal, or the elevation angle of the second acoustic signal, or any combination of two or more thereof, to obtain the value of the velocity ratio for a volume of the rock formation around the borehole.

152. A system for generating a three-dimensional image of non-linear properties of a rock formation, comprising a processor configured to:

read measurement parameters including a position of a first acoustic source configured to generate a first acoustic signal at a first frequency, a position of a second acoustic source configured to generate a second acoustic signal at a second frequency, a position of a receiver configured to receive a detected signal returning to a borehole from a mixing zone within the rock formation at the intersection of the first acoustic signal and the second acoustic signal, and elevation and azimuth angles of the first and second acoustic signals;

read a correlated signal M(t, δ, d), which is a function of arrival time t, start time difference δ and frequency ratio d between the first frequency and the second frequency, the correlated signal M(t, δ, d) containing signals generated by a non-linear mixing process from a first acoustic signal and a second acoustic signal at a mixing zone within the rock formation;

read an initial propagation compressional and shear velocity model;

calculate a first travel time of the first acoustic signal from the first acoustic source to a center of the mixing zone;

calculate a second travel time of the second acoustic signal from the second acoustic source to the center of the mixing zone;

calculate a third travel time between the center of the mixing zone and the receiver configured to receive the detected signal returning to the borehole having a frequency equal to a difference between the first frequency and the second frequency, the detected signal being generated by a non-linear mixing process from the first acoustic signal and the second acoustic signal in the mixing zone;

calculate an arrival time corresponding to a total travel time Tp of a signal returning to the borehole by adding the first travel time and the third travel time, and calculating a start time difference δ_p between the first travel time and the second travel time;

search all values of arrival time t, start time difference δ, and the frequency ratio d in the correlated signal $M(t, \delta, d)$ to determine for which value of arrival T_{NL}, start time difference δ_{NL} and frequency ratio d_{NL} a band-limited spike signal $M(T_{NL},\delta_{NL},d_{NL})$ generated by the non-linear mixing process occurs; and

calculate trajectories of acoustic propagation using an elevation angle of the first acoustic signal, and a position of the first acoustic source, and an elevation angle of the second acoustic signal, and a position of the second acoustic source based on the propagation compressional and shear velocity model;

map the band-limited spike signal $M(T_{NL},\delta_{NL},d_{NL})$ generated by the non-linear mixing process to spatial coordinates of an interaction mixing zone where the first acoustic signal and the second acoustic signal intersect; and

repeat the mapping of the band-limited spike signal $M(T_{NL},\delta_{NL},d_{NL})$ to the spatial coordinates of the interaction mixing zone for a plurality of values of the position of the first acoustic source, the position of the second acoustic source, the position of the receiver, an elevation angle of the first acoustic signal, an azimuth angle of the first acoustic signal, an elevation angle of the second acoustic signal, or an azimuth angle of the second acoustic signal, or any combination of two or more thereof, and generating a three dimensional image of a strength of the non-linear mixing process.

153. The system according to claim 152, further comprising determining a location of the mixing zone where the first acoustic signal and the second acoustic signal intersect using the position of the first acoustic source, the elevation angle and the azimuth angle of the first acoustic signal and the position of the second acoustic source, the elevation angle and the azimuth angle of the second acoustic signal.

154. A system for generating a three-dimensional image of a propagation compressional velocity, a propagation of shear velocity, a ratio of compressional velocity to shear velocity of a rock formation, or any combination of two or more thereof, the system comprising a processor configured to:

read measurement parameters including a location of a first acoustic source configured to generate a first acoustic signal at a first frequency, a position of second acoustic source configured to generate a second acoustic signal at a second frequency, a position of a receiver configured to receive a detected signal returning to a borehole from a mixing zone within the rock formation at the intersection of the first acoustic signal and the second acoustic signal, and elevation and azimuth angles of the first and second acoustic signals;

read a correlated signal M(t, δ, d), which is a function of arrival time t, start time difference δ and frequency ratio d between the first frequency and the second frequency, the correlated signal M(t, δ, d) containing signals generated by a non-linear mixing process from a first acoustic signal and a second acoustic signal at a mixing zone within the rock formation;

read an initial propagation compressional and shear velocity model;

estimate an initial propagating velocity model using well logs in a borehole and assumptions about lateral continuity of the rock formation away from the borehole;

calculate a first travel time of the first acoustic signal from the first acoustic source to a center of the mixing zone;

calculate a second travel time of the second acoustic signal from the second acoustic source to the center of the mixing zone;

calculate a third travel time between the center of the mixing zone and the receiver configured to receive the detected signal returning to the borehole having a frequency equal to a difference between the first frequency and the second frequency, the detected signal being generated by a non-linear mixing process from the first acoustic signal and the second acoustic signal in mixing zone;

calculate an arrival time Tp corresponding to a total travel time of a signal returning to the borehole by adding the first travel time and the third travel time, and calculating a start time difference δ_p between the first travel time and the second travel time;

search all values of arrival time t, start time difference δ, and the frequency ratio d in the correlated signal M(t, δ, d) to determine for which value of arrival time T_{NL}, start time difference δ_{NL} and frequency ratio d_{NL} a band-limited spike signal $M(T_{NL},\delta_{NL},d_{NL})$ generated by the non-linear mixing process occurs;

compare the calculated arrival time Tp with a measured arrival time T_{NL} or comparing the calculated start time difference δ_p with a measured start time difference δ_{NL};

update the propagating velocity model by tomographic velocity inversion or full wave form inversion using a difference between the measured and the calculated arrival times if the calculated arrival time is different from the measured arrival time, or using a difference between the calculated and the measured start time differences if the calculated start time difference is different from the measured start time difference, or both;

repeat iteratively the above steps until the calculated arrival time is substantially equal to the measured arrival time or the calculated start time difference is substantially equal to the measured start time difference, or both, for a range of values of the position of the first acoustic source, the position of the second acoustic source, the position of the receiver, the azimuth angle of the first acoustic signal, the azimuth angle of the second acoustic signal, the elevation angle of the first acoustic signal, or the elevation angle of the second acoustic signal, or any combination of two or more thereof, to obtain a propagating velocity Vp and propagating velocity Vs that results in the calculated arrival time being substantially equal to the measured arrival

time or the calculated start time difference being substantially equal to the measured start time difference, or both; and

generate a three dimensional image of a propagation compressional velocity, a propagation of shear velocity, a ratio of compressional velocity to shear velocity, or any combination of two or more thereof.

155. The system according to claim 154, wherein the processor is configured to determine a value of the propagating compressional velocity, a value of the propagating shear velocity, or a value of the velocity ratio of the compressional velocity and the shear velocity, or any combination of two or more thereof.

156. The system according to claim 154, wherein the processor is configured to calculate spatial coordinates of the mixing zone where the two acoustic signals intersect, calculating a convergence angle between the first acoustic signal from the first acoustic source and the second acoustic signal from the second acoustic source based on the elevation angle and the azimuth angle of the first acoustic signal, and the position of the first acoustic source, and the elevation angle and the azimuth angle of the second acoustic signal, and the position of the second acoustic source.

157. The system according to claim 156, wherein the processor is configured to calculate the velocity ratio of compressional velocity and shear velocity at the mixing zone from the measured frequency ratio d_{NL} and the calculated the convergence angle using a selection rule relationship between velocity ratio of compressional velocity and shear velocity, frequency ratio and angle of convergence.

158. The method of claim 157, further comprising repeating mapping the calculated velocity ratio to the spatial coordinate of the mixing zone for a range of values of the position of the first acoustic source, the position of the second acoustic source, the position of the receiver, the azimuth angle of the first acoustic signal, the azimuth angle of the second acoustic signal, the elevation angle of the first acoustic signal, or

the elevation angle of the second acoustic signal, or any combination of two or more thereof to obtain a three dimensional image of the velocity ratio.

159. A method of investigating rock formations outside a borehole, the method comprising:

generating, by a first acoustic source disposed within a borehole, a first conical acoustic signal, the first conical acoustic signal being broadcast at a first frequency;

generating, by a second acoustic source disposed within the borehole, a second conical acoustic signal, the second conical acoustic signal being broadcast at a second frequency different from the first frequency, the first frequency and the second frequency being in the frequency range between approximately 500 Hz and 500 kHz,

configuring the first acoustic source and the second acoustic source so that the first conical acoustic signal and the second conical acoustic signal intersect in a desired intersection volume outside the borehole; and

receiving, by a receiver disposed within the borehole, a detected signal returning to the borehole having a frequency equal to a difference between the first frequency and the second frequency, the detected signal being generated by a non-linear mixing process from the first conical acoustic signal and the second conical acoustic signal in a non-linear mixing zone within the intersection volume;

recording the detected signal and storing the detected signal in a storage device; and recording, in the storage device, measurement parameters including a position of the first acoustic source, a position of the second acoustic source, a position of the receiver, and a cone angle and a cone axis orientation of the first signal and a cone angle and a cone axis orientation of second conical acoustic signal.

160. The method according to claim 159, wherein generating the first conical acoustic signal and generating the second conical acoustic signal comprise generating the first conical acoustic signal and the second conical acoustic signal while the first acoustic source and the second acoustic are located in a single borehole.

161. The method according to claim 159, wherein the borehole is not straight or the borehole includes sidetracks such that the first conical acoustic signal and the second conical acoustic signal have axes of symmetry that are not collinear.

162. The method according to claim 159, wherein generating the first conical acoustic signal comprises generating a first acoustic conical signal comprising a first plurality of pulses arranged in a coded time sequence, the first plurality of pulses being separated in time, each pulse comprising a first modulated signal at a central frequency, wherein central frequencies of two consecutive pulses are different.

163. The method according to claim 160, wherein a separation in time between centers of two consecutive pulses in the first plurality of pulses is greater than a time duration of each pulse.

164. The method according to claim 159, wherein generating the second conical acoustic signal comprises generating a second plurality of pulses arranged in a coded time sequence, the second plurality of pulses being separated in time, wherein a separation in time between centers of two consecutive pulses is the same as a separation in time between centers of two corresponding pulses in the first plurality of pulses, wherein a start time difference δ is provided between a start time of the broadcast of the second plurality of pulses and a start time of the broadcast of the first plurality of pulses, wherein each pulse comprises a modulated signal, and a central frequency of the modulated signal within each pulse in the second plurality of pulses is a selected fraction frequency ratio d of the central frequency of the modulated signal within each corresponding pulse in the first plurality of pulses.

165. The method according to claim 164, further comprising repeating the generating of the first acoustic signal, the generating of the second acoustic signal and the receiving of the detected signal for a plurality of start time differences δ between the first acoustic signal and the second acoustic signal and for a plurality of frequency ratios d between the first frequency and the second frequency.

166. The method according claim 159, wherein generating the first conical acoustic signal and generating the second conical acoustic signal comprises controlling a phase of the first conical acoustic signal or a phase of the second conical acoustic signal, or both such that the broadcast signals have opposite polarity.

167. The method according to claim 166, further comprising adding the signal detected at the receiver, generated by the mixing process from the first conical acoustic signal and the second conical acoustic signal, to the signal detected at the receiver, generated by the mixing process from the first conical acoustic signal having opposite polarity and the second acoustic signal having opposite polarity, in order to enhance non-linear interaction signals and minimize linear interaction signals and noise.

168. The method according claim 159, further comprising receiving a plurality of detected signals by a plurality of detectors and applying multi-dimensional filtering, or time moveout analysis and stacking, or both, to the plurality of detected signals that are received by the plurality of detectors.

169. The method according to claim 168, wherein receiving the detected signal by the plurality of detectors comprises receiving component detected signals of each of the detected signals by a plurality of component detectors of each of the plurality of detectors, and applying multi-dimensional filtering, or time moveout analysis and stacking, or both, to the plurality of detected component signals that are received on the plurality of component detectors of each of the plurality of receivers.

170. The method according to claim 159, further comprising repeating the generating of the first conical acoustic signal, the generating of the second conical acoustic signal and the receiving of the detected signal for different positions of the first acoustic source, different positions of the second acoustic source, different positions of the receiver, broadcast cone angles of the first acoustic source, or broadcast cone angles of the second acoustic source, or any combination of two or more thereof.

171. The method according to claim 159, further comprising extracting and enhancing the signal generated by the non-linear mixing process and suppressing signals generated by a linear interaction process or noise, or both, from the recorded detected signal generated by non-linear interaction in the rock formation from the first and second acoustic sources.

172. The method according to claim 159, wherein generating the first conical acoustic signal comprises generating a first plurality of pulses arranged in a coded time sequence, the first plurality of pulses being separated in time, each pulse comprising a first modulated signal at a central frequency, wherein central frequencies of two consecutive pulses are different,

wherein generating the second conical acoustic signal comprises generating a second plurality of pulses arranged in a coded time sequence, the second plurality of pulses being separated in time, wherein a separation in time between centers of two consecutive pulses is the same as a separation in time between centers of two corresponding pulses in the first plurality of pulses, wherein a start time difference δ is provided between a start time of the broadcast of the second plurality of pulses and a start time of the broadcast of the first plurality of pulses, wherein each pulse comprises a second modulated signal, and a central frequency of the second modulated signal within each pulse in the second plurality of pulses is a selected fraction frequency ratio d of the central frequency of the first modulated signal within each corresponding pulse in the first plurality of pulses, and

reading the recorded data and correlating, using a processor, the detected signal with a template signal comprising a plurality of pulses arranged in a time sequence, wherein a separation in time between centers of two consecutive pulses is substantially the same as a separation in time between centers of two consecutive pulses in the first plurality of pulses, wherein each pulse in the plurality of pulses comprises a modulated signal having a central frequency that is equal to a difference between a central frequency of the first modulated signal and a central frequency of the second modulated signal in each corresponding pulse of the first plurality of pulses and the second plurality of pulses.

173. The method according to claim 172, wherein a separation in time between centers of two consecutive pulses in the first plurality of pulses is greater than a time duration of each pulse.

174. The method according to claim 172, repeating for a plurality of start time differences δ and for a plurality of frequency ratios d between the first frequency and the second frequency, the correlating between the template signal and the detected signal to yield a correlated signal $M(t, \delta, d)$ which contains non-linear interaction signals.

175. The method according to claim 174, further comprising applying a time-variant frequency band pass signal to the detected signal so as to keep a narrow frequency band around an expected frequency bandwidth of the detected signal, the narrow frequency bandwidth being selected around the difference between the first frequency and the second frequency of each pulse.

176. The method according to claim 172, further comprising performing, using a processor, a hodogram analysis of the detected signal.

177. The method of claim 159, further comprising scanning a frequency ratio of the first frequency to the second frequency or scanning a range of start time difference, or both, so as to control a position of the mixing zone within the intersection volume in accordance with selection rules for non-linear interaction of acoustic signals within the rock formation.

178. The method of claim 159, further comprising: positioning the first acoustic source in a first axial orientation within the borehole and positioning the second acoustic source in a second axial orientation within the borehole, the first axial orientation and the second axial orientation being non-coaxial orientations or non-parallel.

179. The method of claim 178, further comprising controlling the frequency ratio of the first frequency and the second frequency and the start time difference to select a specific mixing zone within the intersection volume located at a locus of the intersection of the first conical acoustic signal and the second conical acoustic signal.

180. The method of claim 179, further comprising discriminating between mixing zones located on the locus of the intersection of the first conical broadcast and the second conical broadcast by a total of a travel time between the first acoustic source to the center of the mixing zone and from the center of the mixing zone to the receiver, or a convergence angle between the first conical acoustic signal and the second conical acoustic signal to the mixing zone according to non-linear interaction selection rule relationship between convergence angle and frequency ratio of the first and second acoustic signals, or both.

181. The method of claim 159, further comprising positioning the first acoustic source within a first borehole and positioning the second acoustic source within a second borehole and positioning the receiver within either the first borehole or the second borehole, or a third borehole.

182. The method of claim 181, further comprising selecting the cone angles or cone axis orientation or both such that the first conical acoustic signal and the second conical acoustic signal intersect at a targeted region in the rock formation surrounding the borehole.

183. A method of determining a value of propagation compressional or shear velocity in a volume of rock formation surrounding a borehole, or both, by travel time tomographic inversion, the method comprising:

estimating an initial propagating velocity model using well logs in a borehole and assumptions about lateral continuity of a rock formation away from the borehole;

calculating, by a processor, spatial coordinates of mixing zones from positions

of a first acoustic source broadcasting a first conical acoustic signal and a second acoustic source broadcasting a second conical acoustic signal and the elevation angles of the first and second conical conical acoustic signals;

calculating, by the processor, total travel times from the first acoustic source to the centers of the mixing zones and from the centers of the mixing zones to the receiver;

calculating, by the processor, differences in travel times from the first acoustic source to centers of the mixing zones and from second acoustic source to the centers of the mixing zones;

reading, by the processor, a correlated signal M(t, δ, d), which is a function of arrival time t, the start time difference δ between the second acoustic signal and the first acoustic signal and the frequency ratio d between a frequency of the first acoustic signal and a frequency of the second acoustic signal, and which contains non-linear interaction signals;

scanning, by the processor, a range of values of arrival time t, start time difference δ and frequency ratio d in the signal M(t, δ, d) to determine a plurality of values for the arrival time T_{NL}, a plurality of values for the start time difference δ_{NL}, a plurality of values for the frequency ratio d_{NL} for which the measured arrival time T_{NL}, the start time difference δ_{NL} and the frequency ratio d_{NL} correspond to a band-limited spike signal $M(T_{NL},\delta_{NL},d_{NL})$ generated by the non-linear mixing process;

comparing, by the processor, the calculated arrival times with measured arrival times T_{NL} or comparing the calculated start time differences with measured start time differences δ_{NL}, or both;

updating the propagating velocity model by tomographic velocity inversion or full wave form inversion using the differences between the measured and the calculated arrival times and the differences between the calculated and the measured start time differences for all receiver, first source, second source and elevation of the first conical acoustic signal and elevation of the second conical acoustic signal, if the calculated arrival times are different from the measured arrival times and the calculated start time difference are different from the measured start time differences,

repeating iteratively the above steps for a plurality of detected signals obtained for a plurality of positions of the first acoustic source, a plurality of positions of the second acoustic source, a plurality of elevation angles of the first conical acoustic signal and a plurality of elevation angles of the second conical acoustic signal until the calculated arrival time corresponding to the total travel time is substantially equal to a measured arrival time or the calculated start time difference is substantially equal to a measured start time difference, or both, to obtain a propagating velocity Vp and propagating velocity Vs that results in the calculated arrival times being substantially equal to the measured arrival times and the calculated start time differences being substantially equal to the measured start time differences.

184. The method of claim 183, further comprising calculating trajectories of acoustic propagation and travel times, based on the propagating velocity model, using ray tracing or other acoustic numerical modeling techniques.

185. The method according to claim 183, further comprising generating a three dimensional image of non-linear properties or a three-dimensional image of velocity ratio of compressional velocity and shear velocity, or any combination of two or more thereof from the recorded signal data by mapping, Kirchhoff imaging, beam imaging or wave equation imaging or determining propagation compressional or shear velocity or both, using tomographic velocity inversion or full wave form inversion or iterative imaging in combination with tomographic velocity inversion.

186. A method of generating a three dimensional image of non-linear properties, the method comprising:

reading, by a processor, measurement parameters including a position of the first acoustic source configured to generate a first conical acoustic signal at a first frequency, a position of second acoustic source configured to generate a second conical acoustic signal at a second frequency, a position of a receiver configured to receive a detected signal returning to a borehole from a mixing zone within the rock formation at the intersection of the first conical acoustic signal and the second conical

acoustic signal, and cone angle and conical axis orientation of the first conical acoustic signal and the second conical acoustic signal;

reading, by the processor, a correlated signal M(t, δ, d) which contains non-linear interaction signals, which is a function of arrival time t, the start time difference δ between a start time of a broadcast of the first conical acoustic signal and a start time of a broadcast of second conical acoustic signal and the frequency ratio d between the first frequency and the second frequency;

reading, by the processor, an initial propagation compressional and shear velocity model;

calculating, by the processor, based on the propagation compressional and shear velocity model, trajectories of acoustic propagation, using a position of the first acoustic source and a position of the second acoustic source, and a cone angle and axial orientation of the first conical acoustic signal and a cone angle and axial orientation of the second conical acoustic signal;

calculating spatial coordinates of an intersection volume zone where the first conical acoustic signal and the second conical acoustic signal intersect;

searching, by the processor, the correlated signal M(t, δ, d) in a range of arrival time t, start time difference δ and frequency ratio d to determine a plurality of values for the arrival time T_{NL}, a plurality of values for the start time difference δ_{NL}, a plurality of values for the frequency ratio d_{NL} for which the measured arrival time T_{NL}, the start time difference δ_{NL} and the frequency ratio d_{NL} correspond to a band-limited spike signal $M(T_{NL},\delta_{NL},d_{NL})$ generated by the non-linear mixing process; and

mapping the band-limited spike signals $M(T_{NL},\delta_{NL},d_{NL})$ generated by the non-linear mixing process to spatial coordinates of the interaction mixing zones where the first acoustic conical broadcast signal and the second acoustic conical broadcast signal intersect; and

repeating the mapping of the band-limited spike signals $M(T_{NL},\delta_{NL},d_{NL})$ to the spatial coordinates of the interaction mixing zones for a plurality of values of the position of the first acoustic source, the position of the second acoustic source, the position of the receiver, a cone angle and axial orientation of the first conical acoustic signal or a cone angle and axial orientation of the second conical acoustic signal, or

any combination of two or more thereof, and combining mapped correlated signals to obtain a three dimensional image of a strength of the non-linear mixing process.

187. A method of generating a three dimensional image of a velocity ratio of compressional velocity and shear velocity, the method comprising:

reading, by a processor, measurement parameters including a position of the first acoustic source configured to generate a first conical acoustic signal at a first frequency, a position of second acoustic source configured to generate a second conical acoustic signal at a second frequency, a position of a receiver configured to receive a detected signal returning to a borehole from a mixing zone within the rock formation at the intersection of the first conical acoustic signal and the second conical acoustic signal, and cone angle and conical axis orientation of the first conical acoustic signal and the second conical acoustic signal;

reading, by the processor, a correlated signal M(t, δ, d) which contains non-linear interaction signals, which is a function of arrival time t, the start time difference δ between a start time of a broadcast of the first conical acoustic signal and a start time of a broadcast of second conical acoustic signal and the frequency ratio d between the first frequency and the second frequency;

reading, by the processor, an initial propagation compressional and shear velocity model;

calculating, by the processor, based on the propagation compressional and shear velocity model, trajectories of acoustic propagation, using a position of the first acoustic source and a position of the second acoustic source, and a cone angle and axial orientation of the first conical acoustic signal and a cone angle and axial orientation of the second conical acoustic signal;

calculating spatial coordinates of an intersection volume zone where the first conical acoustic signal and the second conical acoustic signal intersect;

calculating a convergence angle between the first conical acoustic signal and the second conical acoustic signal based on the cone angle and the axial orientation of the first conical acoustic signal and the cone angle and the axial orientation of the

second conical acoustic signal, and a position of the first acoustic source, and a position of the receiver;

searching, by the processor, the correlated signal M(t, δ, d) in a range of values of arrival time t, start time difference δ and frequency ratio d to determine the plurality of values for the arrival time T_{NL}, the plurality of values for the start time difference δ_{NL}, the plurality of values for the frequency ratio d_{NL} for which the measured arrival time T_{NL}, the start time difference δ_{NL} and the frequency ratio d_{NL} correspond to a band-limited spike signal $M(T_{NL},\delta_{NL},d_{NL})$ generated by the non-linear mixing process;

calculating the velocity ratios using the frequency ratios d_{NL} resulting in a detected non-linear signal and calculating the convergence angle using a selection rule relationship between the frequency ratio and the angle of convergence;

mapping the calculated velocity ratios to the spatial coordinates of the mixing zones;

repeating the mapping of the velocity ratios to the spatial coordinates of the mixing zone for a plurality of values of the position of the first acoustic source, the position of the second acoustic source, the position of the receiver, the cone angle and the axial orientation of the first conical acoustic signal or the cone angle and the axial orientation of the second conical acoustic signal, or any combination of two or more thereof, and combining mapped correlated signals to obtain a three dimensional image of the velocity ratio.

188. A system for investigating rock formations outside a borehole, the system comprising:

a first acoustic source disposed within a borehole, the first acoustic source being configured to generate a first conical acoustic signal, the first conical acoustic signal being broadcast at a first frequency;

a second acoustic source disposed within the borehole, the second acoustic source being configured to generate a second conical acoustic signal, the second conical acoustic signal being broadcast at a second frequency different from the first

frequency, the first frequency and the second frequency being in the frequency range between approximately 500 Hz and 500 kHz,

wherein the first acoustic source and the second acoustic source are configured such that the first conical acoustic signal and the second conical acoustic signal intersect in a desired intersection volume outside the borehole; and

a receiver disposed within the borehole, the receiver being configured to receive a detected signal returning to the borehole having a frequency equal to a difference between the first frequency and the second frequency, the detected signal being generated by a non-linear mixing process from the first conical acoustic signal and the second conical acoustic signal in a non-linear mixing zone within the intersection volume;

a storage device configured to record the detected signal and store the detected signal and record and store measurement parameters including a position of the first acoustic source, a position of the second acoustic source, a position of the receiver, and a cone angle and a cone axis orientation of the first conical acoustic signal, and a cone angle and a cone axis orientation of second conical acoustic signal.

189. The system according to claim 188, wherein the first acoustic source generating first conical acoustic signal generating and the second acoustic source generating the second conical acoustic signal are located in a single borehole.

190. The system according to claim 188, wherein the borehole is not straight or the borehole includes sidetracks such that the first conical acoustic signal and the second conical acoustic signal have axes of symmetry that are not collinear.

191. The system according to claim 188, wherein the first conical acoustic signal comprises a first plurality of pulses arranged in a coded time sequence, the first plurality of pulses being separated in time, each pulse comprising a first modulated signal at a central frequency, wherein central frequencies of two consecutive pulses are different.

192. The system according to claim 191, wherein a separation in time between centers of two consecutive pulses in the first plurality of pulses is greater than a time duration of each pulse.

193. The system according to claim 188, wherein the second conical acoustic signal comprises a second plurality of pulses arranged in a coded time sequence, the second plurality of pulses being separated in time, wherein a separation in time between centers of two consecutive pulses is the same as a separation in time between centers of two corresponding pulses in the first plurality of pulses, wherein a start time difference δ is provided between a start time of the broadcast of the second plurality of pulses and a start time of the broadcast of the first plurality of pulses, wherein each pulse comprises a modulated signal, and a central frequency of the modulated signal within each pulse in the second plurality of pulses is a selected fraction frequency ratio d of the central frequency of the modulated signal within each corresponding pulse in the first plurality of pulses.

194. The system according to claim 193, further comprising a processor configured to control the first acoustic source and the second acoustic source and the receiver to repeat a generating of the first acoustic signal, a generating of the second acoustic signal and a receiving of the detected signal for a plurality of start time differences δ between the first acoustic signal and the second acoustic signal and for a plurality of frequency ratios d between the first frequency and the second frequency.

195. The system according claim 188, further comprising a phase controller configured to control a phase of the first conical acoustic signal or a phase of the second conical acoustic signal, or both such that the first conical acoustic signal and the second conical acoustic signal have opposite polarity.

196. The system according to claim 195, further comprising a processor configured to add the signal detected at the receiver, generated by the mixing process from the first conical acoustic signal and the second conical acoustic signal, to the signal detected at the receiver, generated by the mixing process from the first conical acoustic signal having opposite polarity and the second acoustic signal having

opposite polarity, in order to enhance non-linear interaction signals and minimize linear interaction signals and noise.

197. The system according claim 188, further comprising a plurality of detectors configured to receive a plurality of detected signals and applying multi-dimensional filtering, or time moveout analysis and stacking, or both, to the plurality of detected signals that are received by the plurality of detectors.

198. The system according to claim 197, wherein the detector comprises a plurality of component detectors wherein the detected signal comprises component detected signals by the component detectors and further comprising a processor configured to apply multi-dimensional filtering, or time moveout analysis and stacking, or both, to the plurality of detected component signals that are received on the plurality of component detectors.

199. The system according to claim 188, further comprising a processor configured to control the first acoustic source, the second acoustic source and the receiver to repeat the generating of the first conical acoustic signal, the generating of the second conical acoustic signal and the receiving of the detected signal for different positions of the first acoustic source, different positions of the second acoustic source, different positions of the receiver, broadcast cone angles of the first acoustic source, or broadcast cone angles of the second acoustic source, or any combination of two or more thereof.

200. The system according to claim 188, further comprising a processor configured to extract and enhance the signal generated by the non-linear mixing process and suppress signals generated by a linear interaction process or noise, or both, from the recorded detected signal generated by non-linear interaction in the rock formation from the first and second acoustic sources.

201. The system according to claim 188, wherein the first conical acoustic signal comprises a first plurality of pulses arranged in a coded time sequence, the first plurality of pulses being separated in time, each pulse comprising a first modulated

signal at a central frequency, wherein central frequencies of two consecutive pulses are different,

wherein the second conical acoustic signal comprises a second plurality of pulses arranged in a coded time sequence, the second plurality of pulses being separated in time, wherein a separation in time between centers of two consecutive pulses is the same as a separation in time between centers of two corresponding pulses in the first plurality of pulses, wherein a start time difference δ is provided between a start time of the broadcast of the second plurality of pulses and a start time of the broadcast of the first plurality of pulses, wherein each pulse comprises a second modulated signal, and a central frequency of the second modulated signal within each pulse in the second plurality of pulses is a selected fraction frequency ratio d of the central frequency of the first modulated signal within each corresponding pulse in the first plurality of pulses, and

further comprising a processor configured to read the recorded data and correlate the detected signal with a template signal comprising a plurality of pulses arranged in a time sequence, wherein a separation in time between centers of two consecutive pulses is substantially the same as a separation in time between centers of two consecutive pulses in the first plurality of pulses, wherein each pulse in the plurality of pulses comprises a modulated signal having a central frequency that is equal to a difference between a central frequency of the first modulated signal and a central frequency of the second modulated signal in each corresponding pulse of the first plurality of pulses and the second plurality of pulses.

202. The system according to claim 201, wherein a separation in time between centers of two consecutive pulses in the first plurality of pulses is greater than a time duration of each pulse.

203. The system according to claim 201, further comprising a processor configured to repeat for a plurality of start time differences δ and for a plurality of frequency ratios d between the first frequency and the second frequency, the correlation between the template signal and the detected signal to yield a correlated signal M(t, δ, d) which contains non-linear interaction signals.

204. The system according to claim 203, further comprising a processor configured to apply a time-variant frequency band pass signal to the detected signal so as to keep a narrow frequency band around an expected frequency bandwidth of the detected signal, the narrow frequency bandwidth being selected around the difference between the first frequency and the second frequency of each pulse.

205. The system according to claim 201, further comprising a processor configured to perform a hodogram analysis of the detected signal.

206. The system of claim 188, further comprising a processor configured to scan a frequency ratio of the first frequency to the second frequency, or scan a range of start time difference, or both, so as to control a position of the mixing zone within the intersection volume in accordance with selection rules for non-linear interaction of acoustic signals within the rock formation.

207. The system of claim 188, wherein the first acoustic source is positioned in a first axial orientation within the borehole and the second acoustic source is positioned in a second axial orientation within the borehole, the first axial orientation and the second axial orientation being non-coaxial orientations or non-parallel.

208. The system of claim 207, further comprising a processor configured to control the frequency ratio of the first frequency and the second frequency and a start time difference to select a specific mixing zone within the intersection volume located at a locus of the intersection of the first conical acoustic signal and the second conical acoustic signal.

209. The system of claim 208, further comprising a processor configured to discriminate between mixing zones located on the locus of the intersection of the first conical broadcast and the second conical broadcast by a total of a travel time between the first acoustic source to the center of the mixing zone and from the center of the mixing zone to the receiver, or a convergence angle between the first conical acoustic

signal and the second conical acoustic signal to the mixing zone according to non-linear interaction selection rule relationship between convergence angle and frequency ratio of the first and second acoustic signals, or both.

210. The system of claim 188, wherein the first acoustic source is positioned within a first borehole and the second acoustic source is positioned within a second borehole and the receiver is positioned within either the first borehole or the second borehole, or a third borehole.

211. The system of claim 210, wherein the cone angles or cone axis orientation or both are selected such that the first conical acoustic signal and the second conical acoustic signal intersect at a targeted region in the rock formation surrounding the borehole.

212. A system for determining the value of propagation compressional or shear velocity in a volume of rock formation surrounding a borehole, or both, by travel time tomographic inversion, the system comprising a processor configured to:

estimate an initial propagating velocity model using well logs in a borehole and assumptions about lateral continuity of a rock formation away from the borehole;

calculate spatial coordinates of mixing zones from the positions of a first acoustic source broadcasting a first acoustic signal in a form of a first conical signal broadcast and a second acoustic source broadcasting a second acoustic signal in a form of a second conical signal broadcast and the elevation angles of the first and second conical broadcasts;

calculate total travel times from the first acoustic source to the centers of the mixing zones and from the centers of the mixing zones to the receiver;

calculate the differences in travel times from the first acoustic source to the centers of the mixing zones and from second acoustic source to the centers of the mixing zones;

read a correlated signal M(t, δ, d), which is a function of arrival time t, the start time difference δ between the second acoustic signal and the first acoustic signal and the frequency ratio d between a frequency of the first acoustic signal and a frequency of the second acoustic signal, and which contains non-linear interaction signals;

scan all values of arrival time t, start time difference δ and frequency ratio d in the signal M(t, δ, d) to determine the pluralities of values for the arrival time T_{NL} , the start time difference δ_{NL} , the frequency ratio d_{NL} and $M(T_{NL},\delta_{NL},d_{NL})$ for which the measured arrival time T_{NL}, the start time difference δ_{NL} and the frequency ratio d_{NL} correspond to a band-limited spike signal $M(T_{NL},\delta_{NL},d_{NL})$ generated by the non-linear mixing process; ;

compare, by the processor, the calculated arrival times with measured arrival times T_{NL} or comparing the calculated start time differences with measured start time differences δ_{NL}, or both;

update the propagating velocity model by tomographic velocity inversion or full wave form inversion using the differences between the measured and the calculated arrival times and the differences between the calculated and the measured start time differences for all receiver, first source, second source and broadcast elevation configurations, if the calculated arrival times are different from the measured arrival times and the calculated start time difference are different from the measured start time differences,

repeat iteratively the above steps for a plurality of detected signals obtained for a plurality of positions of the first acoustic source, a plurality of positions of the second acoustic source, a plurality of broadcast elevation angles of the first acoustic signal and the second acoustic signal until the calculated arrival time corresponding to the total travel time is substantially equal to a measured arrival time or the calculated start time difference is substantially equal to a measured start time difference, or both, to obtain a propagating velocity Vp and propagating velocity Vs that results in the calculated arrival times being substantially equal to the measured arrival times and the calculated start time differences being substantially equal to the measured start time differences.

213. The system of claim 212, further comprising a processor configured to calculate trajectories of acoustic propagation and travel times, based on the propagating velocity model, using ray tracing or other acoustic numerical modeling techniques.

214. The system according to claim 212, further comprising a processor configured to generate a three dimensional image of non-linear properties or a three dimensional image of velocity ratio of compressional velocity and shear velocity, or any combination of two or more thereof from the recorded signal data by mapping, Kirchhoff imaging, beam imaging or wave equation imaging or determining propagation compressional or shear velocity or both, using tomographic velocity inversion or full wave form inversion or iterative imaging in combination with tomographic velocity inversion.

215. A system for generating a three dimensional image of non-linear properties, the system comprising a processor configured to:

read measurement parameters including a position of the first acoustic source configured to generate a first conical acoustic signal at a first frequency, a position of second acoustic source configured to generate a second conical acoustic signal at a second frequency, a position of a receiver configured to receive a detected signal returning to a borehole from a mixing zone within the rock formation at the intersection of the first conical acoustic signal and the second conical acoustic signal, and cone angle and conical axis orientation of the first conical acoustic signal and the second conical acoustic signal;

read a correlated signal M(t, δ, d) which contains non-linear interaction signals, which is a function of arrival time t, the start time difference δ between a start time of a broadcast of the first conical acoustic signal and a start time of a broadcast of second conical acoustic signal and the frequency ratio d between the first frequency and the second frequency;

read an initial propagation compressional and shear velocity model;

calculate, based on the propagation compressional and shear velocity model, trajectories of acoustic propagation, using a position of the first acoustic source and a position of the second acoustic source, and a cone angle and axial orientation of the first conical acoustic signal and a cone angle and axial orientation of the second conical acoustic signal;

calculate spatial coordinates of an intersection volume zone where the first conical acoustic signal and the second conical acoustic signal intersect;

search the correlated signal M(t, δ, d) in a range of arrival time t, start time difference δ and frequency ratio d to determine a plurality of values for the arrival time T_{NL}, a plurality of values for the start time difference δ_{NL}, a plurality of values for the frequency ratio d_{NL} for which the measured arrival time T_{NL}, the start time difference δ_{NL} and the frequency ratio d_{NL} correspond to a band-limited spike signal $M(T_{NL},\delta_{NL},d_{NL})$ generated by the non-linear mixing process;

map the band-limited spike signals $M(T_{NL},\delta_{NL},d_{NL})$ generated by the non-linear mixing process to spatial coordinates of the interaction mixing zones where the first acoustic conical broadcast signal and the second acoustic conical broadcast signal intersect; and

repeat the mapping of the band-limited spike signals $M(T_{NL},\delta_{NL},d_{NL})$ to the spatial coordinates of the interaction mixing zones for a plurality of values of the position of the first acoustic source, the position of the second acoustic source, the position of the receiver, a cone angle and axial orientation of the first conical acoustic signal or a cone angle and axial orientation of the second conical acoustic signal, or any combination of two or more thereof, and combining mapped correlated signals to obtain a three dimensional image of a strength of the non-linear mixing process.

216. A system for generating a three dimensional image of velocity ratio of compressional velocity and shear velocity, the system comprising a processor configured to:

read measurement parameters including a position of the first acoustic source configured to generate a first conical acoustic signal at a first frequency, a position of second acoustic source configured to generate a second conical acoustic signal at a

second frequency, a position of a receiver configured to receive a detected signal returning to a borehole from a mixing zone within the rock formation at the intersection of the first conical acoustic signal and the second conical acoustic signal, and cone angle and conical axis orientation of the first conical acoustic signal and the second conical acoustic signal;

read a correlated signal M(t, δ, d) which contains non-linear interaction signals, which is a function of arrival time t, the start time difference δ between a start time of a broadcast of the first conical acoustic signal and a start time of a broadcast of second conical acoustic signal and the frequency ratio d between the first frequency and the second frequency;

read an initial propagation compressional and shear velocity model;

calculate based on the propagation compressional and shear velocity model, trajectories of acoustic propagation, using a position of the first acoustic source and a position of the second acoustic source, and a cone angle and axial orientation of the first conical acoustic signal and a cone angle and axial orientation of the second conical acoustic signal;

calculate spatial coordinates of an intersection volume zone where the first conical acoustic signal and the second conical acoustic signal intersect;

calculate a convergence angle between the first conical acoustic signal and the second conical acoustic signal based on the cone angle and the axial orientation of the first conical acoustic signal and the cone angle and the axial orientation of the second conical acoustic signal, and a position of the first acoustic source, and a position of the receiver;

search the correlated signal M(t, δ, d) in a range of values of arrival time t, start time difference δ and frequency ratio d to determine the plurality of values for the arrival time T_{NL}, the plurality of values for the start time difference δ_{NL}, the plurality of values for the frequency ratio d_{NL} for which the measured arrival time T_{NL}, the start time difference δ_{NL} and the frequency ratio d_{NL} correspond to a band-limited spike signal $M(T_{NL},\delta_{NL},d_{NL})$ generated by the non-linear mixing process; ;

calculate the velocity ratios using the frequency ratios d_{NL} resulting in a detected non-linear signal and the convergence angle using a selection rule relationship between the frequency ratio and the angle of convergence;

map the calculated velocity ratios to the spatial coordinates of the mixing zones;

repeating the mapping of the velocity ratios to the spatial coordinates of the mixing zone for a plurality of values of the position of the first acoustic source, the position of the second acoustic source, the position of the receiver, the cone angle and the axial orientation of the first conical acoustic signal or the cone angle and the axial orientation of the second conical acoustic signal, or any combination of two or more thereof, and combining mapped correlated signals to obtain a three dimensional image of the velocity ratio.

217. A system for investigating non-linear properties of a rock formation around a borehole, comprising:

a first sub-system configured to perform data acquisition, control and recording of data;

a second subsystem in communication with the first sub-system and configured to perform non-linearity and velocity preliminary imaging;

a third subsystem in communication with the first subsystem and configured to emit controlled acoustic broadcasts and receive acoustic energy;

a fourth subsystem in communication with the first subsystem and the third subsystem and configured to generate a source signal directed towards the rock formation; and

a fifth subsystem in communication with the third subsystem and the fourth subsystem and configured to perform detection of signals representative of the non-linear properties of the rock formation.

218. The system according to claim 217, wherein the first subsystem comprises an acquisition modeling module configured to receive inputs from the second subsystem and operator inputs.

219. The system according to claim 218, wherein the first subsystem comprises a data acquisition controller configured to receive operator inputs and inputs from an acquisition design module.

220. The system according to claim 219, wherein the first subsystem comprises a data pre-processing and enhancement module and a data storage device, the data pre-processing and enhancement module configured to read data from the data storage device and input data into the second subsystem.

221. The system according to claim 217, wherein the second subsystem comprises an initial velocity model module configured to provide an initial velocity model derived from logs of compressional and shear slowness and information about formation lateral continuity to the first subsystem.

222. The system according to claim 221, wherein the second subsystem comprises an image initializing module configured to initialize volumes of propagation compressional and shear velocity values, velocity ratio images and images of non-linearity which are associated with amplitudes of a measured signal generated by the non-linear interaction at a mixing zone.

223. The system according to claim 217, wherein the third subsystem comprises a first acoustic source, a second acoustic source, one or more receivers, a tool deployment and transport module, a tool mechanical controller, a directional controller for controlling an orientation of the first acoustic signal, and a directional controller for controlling an orientation of the second acoustic signal.

224. The system according to claim 223, wherein the third system is configured to receive from the first subsystem commands to control the directional controllers within the third subsystem to control azimuth and elevation angles of the first acoustic signal, the second acoustic signal, or both.

225. The system according to claim 223, wherein acquisition record parameters pertaining to tool configuration and broadcast geometry are collected from the third subsystem and are recorded in a data storage device within the first subsystem.

226. The system according to claim 217, wherein the fourth subsystem comprises a signal generator configured to generate a first signal, a signal generator configured to generate a second acoustic signal, a frequency multiplier and start time difference module, and a coded signal generator.

227. The system according to claim 226, wherein the fourth subsystem is configured to receive input commands from the first subsystem.

228. The system according to claim 227, wherein the fourth subsystem is configured to receive input commands from the first subsystem so as to generate pulse sequences using the coded signal generator module.

229. The system according to claim 228, wherein the coded signal generated by the coded signal generator module is input into a first signal control and amplifier module to input a first signal at a first frequency to the first acoustic source, wherein the coded signal generated by the coded signal generator module is input into frequency multiplier and start time difference module and then input into a second signal control and amplifier module to provide a second signal to the second acoustic source having a start time difference relative to the first signal and having a second frequency which is a selected fraction of the first frequency.

230. The system according to claim 229, wherein the start time difference and the frequency fraction and are stored as record broadcast information in a data storage device within the first sub-system.

231. The system according to claim 217, wherein the fifth subsystem comprises a receiver module configured to receive a signal from one or more receivers, a non-linear signal enhancement module configured to enhance the signal received by the signal receiver module, a template signal generator module configured to generate a template signal, and a signal correlation module configured to correlate the received signal with the template signal.

232. The system according to claim 231, wherein signals from the receiver module are processed by the non-linear signal enhancement module to enhance a content of non-linear origin and reduce noise, wherein the signals from the receiver module and enhanced signals are stored in a data storage device within the first sub-system.

233. The system according to claim 231, wherein the template generator is configured to generate a template signal, wherein the correlation module is configured to correlate the template signal with the received signal so as to extract a correlated signal the correlated signal identifying a signal generated at an intersection of a first signal from a first acoustic source and a second signal from a second acoustic in a non-linear mixing zone within the rock formation around the borehole, where t corresponds to signal time, d to a frequency ratio of the first frequency and the second frequency, δ is a time delay between the first signal and the second signal.

234. The system according to claim 233, wherein the correlated signal is stored in a data storage device within the first subsystem.

235. The system according to claim 234, wherein a data acquisition controller within the first subsystem is configured to repeat recording within the storage device

of the first subsystem and repeat extracting the correlated data from a plurality of values of the position of the first acoustic source, the position of the second acoustic source, the position of the receiver, an elevation angle of the first acoustic signal, or an elevation angle of the second acoustic signal, or any combination of two or more thereof.

236. The system according to claim 235, further comprising a hodogram analysis module configured to process stored data within the storage device to further enhance the correlated data.

237. The system according to claim 217, further comprising a sixth subsystem in communication with the second subsystem and configured to perform imaging of non-linear properties of the rock formation and determining acoustic signal velocity within the rock formation.

238. The system according to claim 237, wherein the sixth subsystem comprises sub-system a data pre-processing and enhancement module, a velocity model iteration module, an imaging iteration module, an output image module for velocity ratio images and/or non-linearity images, and an output velocity module for outputting determined values of compressional velocity Vp, shear velocity Vs, or velocity ratio between the compressional velocity and the shear velocity, or any combination of two or more thereof.