

1099,483



PATENT SPECIFICATION

DRAWINGS ATTACHED

1099,483

Date of Application and filing Complete Specification: March 30, 1965.

No. 13468/65.

Application made in United States of America (No. 356,776) on April 2, 1964.

Complete Specification Published: Jan. 17, 1968.

© Crown Copyright 1968.

Index at acceptance:—H4 D(G1G1, G1G2, G3B, G3P, G3R, G4A5, G7P); G4 G(2B1, 2X, 3A1, 3A2, 3AX, 3B1, 3B5)

Int. Cl.:—G 01 v 1/36

COMPLETE SPECIFICATION

Optimum Horizontal Stacking

We, TEXAS INSTRUMENTS INCORPORATED, a Corporation organised according to the laws of the State of Delaware, United States of America, of 13500 North Central Expressway, Dallas, Texas, United States of America, do hereby declare the invention, for which we pray that a patent may be granted to us, and the method by which it is to be performed, to be particularly described in and by the following statement:—

This invention relates to seismic exploration and, more particularly, to multiple removal by optimum horizontal stacking of a plurality of seismic signals related to common depth points.

In geophysical prospecting, it is common practice to use an array of seismometers to detect the seismic disturbance from an explosion detonated at or below the surface of the earth. Seismometer outputs have been summed to produce a composite trace for enhancing the seismic signal containing subsurface structure information. The purpose is to give prominence to the important features of the received seismic signal and reduce or remove those undesired features which obscure the important parts of the signal. Both direct and weighted summations of seismometer array outputs have been used. Frequency filtering has been applied to the sum output according to the single channel Weiner least-mean-square-error theory. Special array geometries have been used in an attempt to enhance the seismic data.

Prior techniques, however, do not possess the capability of operating in an optimum manner. The Applicants have found (a) that highly useful treatment of seismic data is achieved when a plurality of signals related to the same deflection point on a subsurface reflector are produced, (b) that they can be so treated as to reduce multiple reflections.

In accordance with the present invention, there is generated a plurality of signals representative of seismic waves detected after travel to and from the same subsurface reflecting point over a like plurality of paths all of different lengths and in which multiple reflections have an apparent velocity which differs from the apparent velocity of primary reflections in dependence upon the geometries of and velocity profiles along said paths. After applying normal moveout correction to align the primary reflections in each of the said plurality of signals, separate filters are applied to the said plurality of signals to alter the time distribution of the multiple reflections of each of said signals relative to the primary reflections therein in dependence upon the geometries of the respective paths of the associated seismic waves to and from said point and upon the velocity profile along said path. The aligned and filtered signals are then combined to produce a composite signal in which said primary reflections are enhanced relative to said multiple reflections.

The seismic signals can be obtained from a seismometer array and be representative of a seismic disturbance or from a seismometer receiving energy from a plurality of spatially separated shots.

The degree of modification of the time distribution of the multiple reflections in each signal can itself be modified as a function of record time. From each of the seismic wave derived signals, there can be generated a signal in which a time distribution of the multiple reflections is modified in dependence on the geometry of its path and upon the velocity along that path.

A method according to this invention is capable of operating in an optimum manner over the entire duration of the desired seismic signal to reduce multiple reflections even though the signals have overlapping frequency spectra. For example, desired and undesired signals having overlapping frequency spectra with different apparent horizontal velocities are separated by use of a signal processor having an overall response for passing a desired signal of one apparent velocity and suppressing or rejecting an undesired signal of a different velocity.

By use of the invention, it is possible to enhance the important parts of a received seismic signal and substantially reduce or remove undesired parts of the signal which obscure the important parts. Signals from a seismometer array can be processed in accordance with the invention so that when combined, the resultant output is the best estimate in the least-mean-square-error sense of the desired seismic signal.

By way of example, the invention will be described in greater detail with reference to the accompanying drawings in which:

FIGURE 1 illustrates a continuous reflection with triple subsurface coverage;

FIGURE 2 illustrates multiple attenuation in accordance with the present invention;

FIGURE 3 diagrammatically illustrates the filter units of FIGURE 2;

FIGURE 4 illustrates the impulse response of the filter of FIGURE 3;

FIGURE 5 illustrates moveout difference of the signal shown in FIGURE 2;

FIGURE 6 illustrates the selection of a plurality of time intervals for optimum filter control;

FIGURE 7 is a mathematical model of the multiple reflection problem; and

FIGURE 8 is a system for the solution of Equation 3.

The identification and elimination of multiple reflections has been one of the most perplexing noise problems in reflection seismograph operations. Shallow multiple generators and problems attendant their presence have largely been overcome through systems currently in use. However, multiple reflections in deep sections present a different problem.

It has been found that if there is an increase in the average velocity with depth, even deep primaries and multiples may be separated on the basis of their normal moveout differences. Conventionally, this difference is exploited by shooting multiple subsurface coverage. Static and dynamic corrections are then applied. The signals reflected from common depth points with different shot-detector distances are then combined or stacked. Such a procedure is described in *Geophysics*, Vol. XXVII, No. 6, Part 2, Dec. 1962, pp. 927-938. Because of an excess in the multiple normal moveout over the moveout of the primary energy, the multiple energy at the output of the stacking system is partially cancelled.

The following description relates to a horizontal stack which is optimized through the use of filters peculiarly related to each seismic signal. The filters are employed prior to stacking, but in connection therewith. This results in a drastic reduction of multiple energy. It permits greater attenuation over a wide frequency band with the use of fewer detector stations. It also provides more effective utilization of the excess multiple moveout as compared with primary moveout. Finally, it preserves high frequency signal components usually lost in conventional stacking where excess subsurface coverage is necessary to achieve a similar degree of multiple reduction.

FIGURE 1 illustrates one of several field procedures which may be employed when a three-channel optimum horizontal stacking procedure is to be carried out. The circles 1-24 between the shot points A-1 and A-2 represent the detector locations. Generally at each detector location a group of seismic detectors will be planted in a predetermined array. The depth point dots represent points of reflection at a given depth on a subsurface reflecting horizon.

In practice, a first shot is detonated at shot point A-1 which is located beyond the left end of the first group by a distance equal to one-half the distance between group centers. A second shot is then fired at shot point A-2. First and second reproducible seismograms are thus obtained for shot points A-1 and A-2. Following this, one-third of the detector groups, namely detector groups 1-8, are then moved to the opposite end of the spread to form a new detector array. The new array is established as illustrated between the shot points B-1 and B-2. At this time a third shot is detonated at point B-1. Following this, a fourth shot is detonated at point B-2. The third

and fourth seismograms are thus recorded. Thereafter, the detectors originally at group positions 9—16 are moved to the other end of the spread. The new array is as indicated between shot point C—1 and shot point C—2. Thereafter, shots are successively detonated at shot points C—1 and C—2. By this means, each subsurface reflecting point is covered three times. The subsurface zone treated by 300% subsurface coverage is indicated by the shaded area of Figure 1. This procedure is continued for multiple subsurface coverage of an entire traverse of which the segments shown in Figure 1 are but a part.

For each subsurface reflecting point, three detector signals may be obtained from three of the above six seismograms. For example, consider the reflection point 9 in the line of A—1 depth points. This reflection point is sampled first by a shot from A—2 travelling along ray paths 11. The reflected energy is detected at group position 9. Second, a signal including energy travelling along ray paths 10 arrives at detector location 17 following detonation of a shot at point B—1. Path 10 differs from path 11, being a slightly longer path. Third, a signal including energy travelling along path 12 is generated in response to the arrival at detector location 1 of energy from a shot detonated at shot point C—1. Path 12 is much shorter than either paths 10 or 11.

The three paths 10—12 to the common subsurface reflection point are of different lengths. Consequently for those cases where there is an increase in average velocity with depth over the section through which the seismic waves travel, there will be a difference between the normal moveout of the multiple reflections as compared with primary reflections.

Such a result is illustrated in FIGURE 2 where the primary and one multiple from each of three signals are illustrated. Channel X1 represents a limited time segment or gate of the signal from shot C—1. The signal on channel X2 includes the signal from shot A—2 on the next longest path. The channel X3 includes the signal from shot B—1, the signal from the longest traveled path.

In Figure 2 it will be noted that the primary reflections are aligned in time. It is to be understood that this alignment ordinarily is brought about by normal moveout correction to accommodate for differences in the lengths of the travel paths of FIGURE 1 over which the primary energy traveled. With the primary energy thus corrected for normal moveout, the multiple reflections 30, 31 and 32 have a residual moveout. It is this difference which permits some attenuation of the multiple energy merely by mixing the signals on channels X1—X3 without further treatment as in the above disclosure from *Geophysics*. A typical straight stacking operation is illustrated by the trace 33 of FIGURE 2. The three signals of FIGURE 2, were merely added. The primary reflection 34 is perfectly preserved. The attenuation of the multiple reflection by a straight three-channel stack is only about 5.4 db. To achieve more complete attenuation, it would be necessary to record 600% to 1200% subsurface coverage.

Filters Y1, Y2, and Y3 are included in the channel leading to the summing unit and thus operate on the signal before the signals are stacked. As indicated, the primary reflection 36 on the output trace 37 appears in perfect preservation. The energy of the multiple 38 is down about 19 db.

FIGURE 3 illustrates one embodiment of the filtering system shown in FIGURE 2. In this embodiment it will be understood that the three channels X1—X3 are on a magnetic medium 29. A plurality of pick-up heads 40—44 are positioned along the channel X1 at predetermined intervals. For seismic operations in the frequency range of up to 300 or 400 cycles, the spacing may be of the order of two milliseconds. The magnetic medium on which channels X1—X3 are recorded is moved past the pick-up heads 40—41 in the direction of arrow 45.

The time-spaced representations of the signal recorded in channel X1 appear at the outputs of the detectors 40—44. Detector 40 is connected by way of a resistor 40a to the armature of a switch 51 which in turn is connected by way of channel 52 to a summing amplifier 53. In a similar manner, the detector 41 is connected by way of resistor 41a, switch 55 and channel 56 to a second input of amplifier 53. Similarly, the detectors 42—44 are connected to the amplifier 53. The values of the resistors 40a—44a, etc. are made dependent upon the multiple velocity function, the spread geometry, and the record time. Detectors 70—74 are operative to detect signals on channel X2 as a time domain filter operation. Similarly detectors 80—84 are operable to detect signals on channel X3.

The system thus far described is a representation of three time domain filters, Y1, Y2, and Y3. For a given time gate from the record, the values of resistors 40a—84, will be such that the multiple energy on the signal on the three channels will be so

treated as to time and amplitude on the output line 60 that the multiple energy will be eliminated when mixed with signal energy on the output line 62 from amplifier 61 and with the signal energy on line 64 from amplifier 63. Lines 60, 62 and 64 are connected to the inputs of a summing unit 65 so that the signal on output line 66 is the signal represented by trace 37 of FIGURE 2.

While the number of pickup units 40—44 illustrated in FIGURE 3 is limited, to five for each of the channels, it will be understood that a time domain filter ordinarily may require a greater number.

It is now necessary to adjust the filters of FIGURE 3 and particularly the values of the resistors 40a—84a in the three filters to proper values in order to duplicate or approximate the impulse responses shown in FIGURE 4. The values of the resistors are determined by utilizing acoustic velocity data for the particular formations through which the ray paths 10—12 extend or through formations of the same geologic section.

Where a well is available velocity data can be obtained from a continuous velocity log of the section through which the seismic waves pass. Such information may be obtained generally in the manner disclosed in British Patent Specification No. 774,098.

The multiple velocity function may also be obtained where a borehole is not available for logging purposes by shooting an expanding spread program as described in *Geophysics*, Vol. XXVII, No. 6, Part II, December 1962, p 981 et seq. Both the primary and multiple velocity profiles may be obtained in accordance with this technique. The velocity information is then used in order to provide information for establishing the values of the resistors so that the time domain filters of FIGURE 3 will have the desired response.

For the specific cases illustrated in FIGURES 1 and 2, the impulse responses for the three time domain filters of FIGURE 3 are illustrated in FIGURE 4. The impulse response for the filter on channel X1 is shown by the curve 86. Similarly the curves 87 and 88 represent the impulse response for the filters on channels X2 and X3 respectively.

From either a velocity log or from an expanding spread section strong reflectors at substantial depths which cause multiples can readily be identified. From the geometry of such reflectors and with the knowledge of the section above the reflectors there can be computed or plotted, for each shot-detector spacing, data from FIGURE 2. More particularly, curve 90 represents the difference in the time occurrence of the multiples on channel X2 of FIGURE 2. The primary and the multiples 30, 31 and 32 occur at a record time of about three seconds.

The difference between the multiple arrival time of multiples 30 and 31 on channels X1 and X2 is about fifteen milliseconds. The difference between the arrival time of multiples 30 and 32 on channels X1 and X3 is about seventeen milliseconds.

Such data can be readily plotted from the velocity profile so that the curves 90 and 91 may be completed from early record time to late record time. Multiple arrivals for any particular groups having ray paths 10—12 is shown in Figure 1. The ray path 12 for the channel X1 has a separation between the shot and detector group of 110' as illustrated in connection with plot 5A. The separation between the shot A—2 and the group location 9 is much longer, being 3410'. Similarly, the shot-detector group separation for shot B—1 and group location 17 is 3630'.

Plot 5B shows data for trace group 4 where the shot-detector group locations are spaced 770', 2750' and 4290' respectively.

Plot 5C shows the shot-detector group spacings for trace group 8 in which the shot-detector group locations are spaced 1650', 1870', and 5170' respectively. The foregoing description has dealt primarily with trace group 1 for which the data is plotted in plot 5A. The following description will be limited to trace group 1 with the understanding that similar treatment will be applied for each of the other seven trace groups. It will be readily recognized that for the procedure shown in FIGURE 1 involving 24 detector groups with 300% subsurface coverage, there will be eight separate groups of three traces per group. The eight groups repeat themselves as the spread locations and shot point locations are moved along the seismic traverse.

In order to understand the design of the filters it will be helpful at this point to consider the mathematical model for signal and noise pertinent to the multiple reflection problem as shown in the diagram of FIGURE 7. In FIGURE 7 where X1, X2, and X3 again represent the three traces having common subsurface reflection points in one of the stack groups, S_1 , S_2 , and S_3 represent the relative arrival times of a signal (primary) on the three channels, and ΔS_i , $i = 1, 2, 3$ are the range of relative arrival times that signal (primary) may have due to various errors, e.g. static errors, normal moveout errors.

The N_1 , N_2 and N_3 represent the relative arrival times of coherent noise (multiple) on the channels, and ΔN_i , $i = 1, 2, 3$, are the range of relative arrival times that noise may have due to the change of multiple normal moveout with various factors, e.g. record time, static errors. It is understood that these two types of arrivals (primary and multiple) may occur anywhere on the record any number of times independent of one another.

The three filters Y_1 , Y_2 and Y_3 physically embodied in analog form in FIGURE 3 are designed on the least-mean-square-error criteria according to the matrix equation:

$$[S_{ij}^*(f) + N_{ij}^*(f)] [Y_j(f)] = [S_{i0}^*(f)] \quad (1)$$

$$i = 1, 2, 3 \dots n,$$

$$j = 1, 2, 3 \dots n.$$

The matrix elements $S_{ij}(f)$ and $N_{ij}(f)$ of Equation 1 are the cross spectral densities between channels i and j for signal and noise respectively. The elements of the right hand column vector $S_{i0}(f)$, are the cross spectral densities between the desired output signal and the signal in channels $i = 1, 2, \dots, n$ respectively, assuming no correlation between signal and noise. The term "cross spectral density" relates to the cross power spectra of signal or noise, as the case may be, and is obtained in respect of signals on channels i and j from Equation 4, as will be explained. Similar expressions can be derived from Equation 9 in respect of noise on channels i and j and from Equation 11 in respect of the desired output signal and the signal on channels $i = 1, 2 = n$ respectively.

Equation 1 may be rewritten in the following form:

$$\begin{bmatrix} [S_{11}(f) + N_{11}(f)] & [S_{12}(f) + N_{12}(f)] & [S_{13}(f) + N_{13}(f)] \\ [S_{21}(f) + N_{21}(f)] & [S_{22}(f) + N_{22}(f)] & [S_{23}(f) + N_{23}(f)] \\ [S_{31}(f) + N_{31}(f)] & [S_{32}(f) + N_{32}(f)] & [S_{33}(f) + N_{33}(f)] \end{bmatrix} \begin{bmatrix} Y_1(f) \\ Y_2(f) \\ Y_3(f) \end{bmatrix} = \begin{bmatrix} S_{10}(f) \\ S_{20}(f) \\ S_{30}(f) \end{bmatrix} \quad (2)$$

The terms along the main diagonal in Equation 2 are auto-power spectra as $i = j$. The desired primary and undesired multiple signals to be evaluated by Equations 1 and 2 may be the signal $S(t)$ [desired] and noise $N(t)$ [undesired] where $N(t) = C(t) + R(t)$

$C(t)$ = multiple reflection $M(t)$

$R(t)$ = random noise $n(t)$

The signal and noise models to be evaluated by Equations 1 and 2 are two signals distinguished by their different relative delays on the three channels.

Equation 2 may be inverted to the following form by any one of several well-known techniques, of which the Crout reduction technique is exemplary. By this means the unknown quantities Y_1 , Y_2 and Y_3 may be evaluated for each frequency of interest.

$$\begin{bmatrix} Y_1(f) \\ Y_2(f) \\ Y_3(f) \end{bmatrix} = \begin{bmatrix} M_{11}(f) & M_{12} & M_{13} \\ M_{21}(f) & M_{22} & M_{23} \\ M_{31}(f) & \dots & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} S_{10}(f) \\ S_{20}(f) \\ S_{30}(f) \end{bmatrix} \quad (3)$$

where $M_{11}(f)$ etc. are the elements resulting from inversion of the first member of Equation 2.

The matrix elements in Equation 2 may be determined on a theoretical basis under the following assumptions.

1) The signal and noise are uncorrelated with each other on a given channel and between channels.

2) The signal (or noise) waveform is assumed to be the same on all channels except for arrival times.

The signal may be modeled in terms of its space correlation properties. The autocorrelation function of a signal $\rho_{kk}^s(\tau)$ is assumed identical for all channels (k being a running number). The cross-correlation between signals on channels i and j is given by:

$$\rho_{ij}^s(\tau) = \frac{1}{2T-T} \int_{-T}^T S_i(t) S_j(t-\tau) dt = \rho_{kk}^s(\tau + \alpha_i - \alpha_j) \quad (4)$$

where

α_i = relative delay time of signal on channel i

α_j = relative delay time of signal on channel j

The parameters α_i and α_j represent the moveout that a desired signal may have between channels i and j . They are in practice subject to errors resulting for example from poor static error correction, incorrect normal moveout correction, and they may be expected to vary over some range. We may treat the α_i 's as random variables with uniform probability over some range, and compute the average correlations as:

$$\langle \rho_{ij}^s(\tau) \rangle = \int_{S_i - \Delta S_i}^{S_i + \Delta S_i} d\alpha_i \int_{S_j - \Delta S_j}^{S_j + \Delta S_j} d\alpha_j p(\alpha_i, \alpha_j) \rho_{kk}^s(\tau + \alpha_j - \alpha_i) \quad (5)$$

where $p(\alpha_i, \alpha_j)$ is the joint probability distribution function. We make the assumption that static and other errors are statistically independent between channels hence

$$p(\alpha_i, \alpha_j) = p(\alpha_i) p(\alpha_j)$$

and that the distribution is uniform

$$p(\alpha_i) p(\alpha_j) = \frac{1}{2\Delta S_i} \cdot \frac{1}{2\Delta S_j}$$

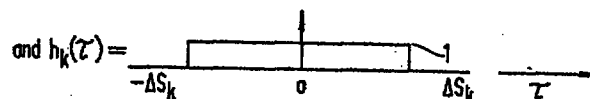
The range of moveout between channels i and j for the signal is

$$(S_i \pm \Delta S_i) - (S_j \pm \Delta S_j)$$

The average in Equation 2 may also be written as a double convolution of $\rho_{kk}^s(\tau)$ with two "box car" functions

$$\langle \rho_{ij}^s(\tau) \rangle = \frac{\left[\rho_{kk}^s(\tau) \otimes h_i(\tau - S_i) \right] \otimes h_j(\tau + S_j)}{4\Delta S_i \Delta S_j} \quad (6)$$

where
* = convolution



The signal auto and cross power terms may be obtained from Equation 6 by Fourier transform

$$S_{ij}^*(f) = \int_{-\infty}^{+\infty} \langle \rho_{ij}^s(\tau) \rangle e^{i2\pi f \tau} d\tau \quad (7)$$

$$S_{ij}^*(f) = \phi_s(f) \frac{\sin 2\pi f \Delta S_i}{2\pi f \Delta S_i} \frac{\sin 2\pi f \Delta S_j}{2\pi f \Delta S_j} \cdot i2\pi f(S_i - S_j) \quad (8)$$

for $i \neq j$

where $\phi_s(f)$ is the Fourier transform of $\rho_{kk}^s(\tau)$.

$S_{ii}^*(f) = \phi_s(f)$ [Let $\alpha_i = \alpha_1$ in Equation 2] for $i = j$.

For the model illustrated in FIGURE 7 each of the auto power functions $S_{ij}^*(f)$ may be considered to be a constant. More particularly S_1 , S_2 and S_3 of FIGURE 7 are in time alignment so that exponential term of Equation 8 is unity. All of the remainder of the terms on the right-hand side of Equation 8 are constants. However, special consideration must be given to the terms ΔS_i and ΔS_j . These terms may also be considered to be constant and of the magnitude illustrated in FIGURE 7. Thus, each of the auto power functions of the signal as required by Equation 2 will be constant for a given frequency.

By similar analysis, the matrix elements for coherent noise are:

$$N_{ij}^*(f) = \phi_n(f) \frac{\sin 2\pi f \Delta N_i}{2\pi f \Delta N_i} \frac{\sin 2\pi f \Delta N_j}{2\pi f \Delta N_j} \cdot i2\pi f(N_i - N_j) \quad (9)$$

for $i \neq j$

where $\phi_n(f)$ is the Fourier transform of the coherent noise autocorrelation $\rho_{kk}^n(\tau)$.

In the foregoing equation, ΔN_i and ΔN_j may be considered to be the constant values or functions illustrated in FIGURE 7. However, the exponential term of Equation 9 varies in dependence upon spread geometry and record time. It has the value of the time differences illustrated in FIGURE 5, for example, the value at the given record time of three seconds from the curves of plot 5A, FIGURE 5. The value of the parenthetical portion of the exponential ($N_i - N_j$) would be zero for the first element. For the second element, ($N_1 - N_2$) would be fifteen milliseconds. For the third element, ($N_1 - N_3$) would be seventeen milliseconds. Thus, by substituting into the Equations 8 and 9, all the data necessary for solution to Equation 2 is obtained. The data comes from the velocity functions such as plotted in FIGURE 5 and from the assumed values of error illustrated in FIGURE 7.

$$N_{ii}^*(f) = \phi_n(f) + \phi_R(f) \quad i = j \quad (10)$$

where $\phi_R(f)$ is the random noise power uncorrelated between channels.

Finally, the elements of the column vector on the right-hand side of matrix Equation 1 may be obtained in a similar fashion as:

$$S_o^*(f) = \phi_s(f) \frac{\sin 2\pi f \Delta S_i}{2\pi f \Delta S_i} \frac{\sin 2\pi f \Delta S_o}{2\pi f \Delta S_o} \cdot i2\pi f(S_i - S_o) \quad (11)$$

for $i \neq o$

where the parameter "o" represents the reference position in the array at which the signal is to be estimated. If "o" corresponds to one of the seismometer positions $x_d = x_i$; then that term of the column vector simply becomes,

$$S_{oo}^{*(n)} = \phi_s(f) \text{ for } i = o \quad (12)$$

Having generated analytic expression for the matrix elements in Equation 1, the matrix is inverted for the Y_i , $i = 1, 2, 3$. Since the $Y(f)$ are computed in the frequency domain they are in turn inverted to the time domain by Fourier transform to determine the impulse response of the filters as shown in FIGURE 4. The impulse responses of the filters are then convolved with the common depth point traces for which they are

designed, and stacked for multiple attenuation as shown in FIGURE 3.

FIGURE 8 illustrates one system for evaluation of the impulse response of the filters Y1, Y2, and Y3 by carrying out the operations as indicated in Equation 3.

Variable voltage sources 100, 101 and 102 provide variable unidirectional voltages which may be adjusted in magnitude in accordance with the desired values of the functions S_{10} , S_{20} , and S_{30} respectively. Sources 105, 106 and 107 similarly are adjustable voltage sources which will provide voltages representative of the values M_{11} , M_{21} and M_{31} respectively, the latter being represented mathematically in Equation 3.

Sources 110, 111 and 112 are provided for representing the values M_{12} , M_{22} , M_{32} . Sources 115, 116 and 117 are provided to supply voltages representative of the elements M_{13} , M_{23} and M_{33} .

Multipliers 120, 121 and 122 are also provided. The source 100 is connected to one input terminal of each of the multipliers 120—122. The source 105 is connected to the second input terminal of multiplier 120. The source 106 is connected to the second input terminal of multiplier 121. The source 107 is connected to the second input terminal of multiplier 122. The outputs of the multipliers 120—122 are summed in a network 123 with the sum being applied to a meter-recorder unit 124 to produce a record 125. For each setting of the sources 100, 105, 106 and 107, one point on the curve 126 will be plotted. The point will represent the particular frequency at which the values of the Equation 3 are determined. Thus, an amplitude frequency plot is formed on record 125. This plot characterizes the filter Y1. The Y1 filter information is then applied to a unit 127 to provide a Fourier transform on chart 128. The function recorded on chart 128 represents the impulse response of the filter Y1 at the record time selected for the data for the values to be used in the exponential of Equation 9. In a similar manner, charts 129 and 130 are produced which represent respectively the impulse response of the filter Y2 and filter Y3 respectively.

The impulse responses of the filters Y1, Y2 and Y3 are illustrated in detail in FIGURE 4. The impulse response as plotted in FIGURE 4 is then employed to set or adjust the values of the resistors 40a—84a of the unit of FIGURE 3. More particularly for channel X1 at Time=0, the magnitude of the resistor 42a is made proportional to the ordinate 42a'. The value of resistor 72a is proportional to the ordinate 72'. The value of resistor 82a is proportional to the ordinate 82a'.

Similarly, the resistor 41a is proportional to the ordinate 41a'; resistor 71a to ordinate 71a' (with provision being made though not shown for polarity reversal); resistor 81a to ordinate 81a'; resistor 43a to ordinate 43a'; resistor 73a to ordinate 73a'; and resistor 83a to ordinate 83a'. In a similar manner, each of the resistors in the filters will be adjusted to magnitudes and polarities which are dependent upon the respective impulse response curves of FIGURE 4. The filters will then serve to eliminate multiples so long as the data, such as represented in the plot 5A of FIGURE 5 is representative of the multiple excess moveout.

In FIGURE 6 it will be noted that the curves are segmented. Consistent therewith, the filters will be changed as to their respective impulse responses as a function of record time. This may be accomplished in FIGURE 3 by actuating linkage 89 at 2.5 seconds, 4 seconds, and 7 seconds record time as indicated in Figure 6. The filter switches successively couple one set of resistors and then another in each of the filter banks. The values of the selected resistors are selected in dependence upon functions such as in FIGURE 6 to provide for multiple removal in each of the record intervals.

It will be appreciated that the foregoing description of an analog type of apparatus for carrying out the invention may be substituted by a digital computer which is programmed to carry out the operations mathematically and graphically described above. The input data will comprise the seismic reflection data and the design data for the optimum filters. Three filters for each of eight different sets will be employed.

It is to be understood that the invention is not limited to the specific field technique illustrated in FIGURE 1. For example, it may be of the type described in the above-identified article from *Geophysics* dealing with expanding spreads. In such case, the expanding spread data could be rendered free from multiples. Other field procedures may also be employed where more than but single coverage of the subsurface is accomplished.

WHAT WE CLAIM IS:—

1. A method for reducing multiple reflections in seismic signals including the steps of generating a plurality of signals representative of seismic waves detected after travel to and from the same subsurface reflecting point over a like plurality of paths all of different lengths and in which multiple reflections have an apparent velocity which differs from the apparent velocity of primary reflections in dependence upon the geometries



designed, and stacked for multiple attenuation as shown in FIGURE 3.

FIGURE 8 illustrates one system for evaluation of the impulse response of the filters Y1, Y2, and Y3 by carrying out the operations as indicated in Equation 3.

Variable voltage sources 100, 101 and 102 provide variable unidirectional voltages which may be adjusted in magnitude in accordance with the desired values of the functions S_{10} , S_{20} , and S_{30} respectively. Sources 105, 106 and 107 similarly are adjustable voltage sources which will provide voltages representative of the values M_{11} , M_{21} and M_{31} respectively, the latter being represented mathematically in Equation 3.

Sources 110, 111 and 112 are provided for representing the values M_{12} , M_{22} , M_{32} . Sources 115, 116 and 117 are provided to supply voltages representative of the elements M_{13} , M_{23} and M_{33} .

Multipliers 120, 121 and 122 are also provided. The source 100 is connected to one input terminal of each of the multipliers 120—122. The source 105 is connected to the second input terminal of multiplier 120. The source 106 is connected to the second input terminal of multiplier 121. The source 107 is connected to the second input of multiplier 122. The outputs of the multipliers 120—122 are summed in a network 123 with the sum being applied to a meter-recorder unit 124 to produce a record 125. For each setting of the sources 100, 105, 106 and 107, one point on the curve 126 will be plotted. The point will represent the particular frequency at which the values of the Equation 3 are determined. Thus, an amplitude frequency plot is formed on record 125. This plot characterizes the filter Y1. The Y1 filter information is then applied to a unit 127 to provide a Fourier transform on chart 128. The function recorded on chart 128 represents the impulse response of the filter Y1 at the record time selected for the data for the values to be used in the exponential of Equation 9. In a similar manner, charts 129 and 130 are produced which represent respectively the impulse response of the filter Y2 and filter Y3 respectively.

The impulse responses of the filters Y1, Y2 and Y3 are illustrated in detail in FIGURE 4. The impulse response as plotted in FIGURE 4 is then employed to set or adjust the values of the resistors 40a—84a of the unit of FIGURE 3. More particularly for channel X1 at Time=0, the magnitude of the resistor 42a is made proportional to the ordinate 42a'. The value of resistor 72a is proportional to the ordinate 72'. The value of resistor 82a is proportional to the ordinate 82a'.

Similarly, the resistor 41a is proportional to the ordinate 41a'; resistor 71a to ordinate 71a' (with provision being made though not shown for polarity reversal); resistor 81a to ordinate 81a'; resistor 43a to ordinate 43a'; resistor 73a to ordinate 73a'; and resistor 83a to ordinate 83a'. In a similar manner, each of the resistors in the filters will be adjusted to magnitudes and polarities which are dependent upon the respective impulse response curves of FIGURE 4. The filters will then serve to eliminate multiples so long as the data, such as represented in the plot 5A of FIGURE 5 is representative of the multiple excess moveout.

In FIGURE 6 it will be noted that the curves are segmented. Consistent therewith, the filters will be changed as to their respective impulse responses as a function of record time. This may be accomplished in FIGURE 3 by actuating linkage 89 at 2.5 seconds, 4 seconds, and 7 seconds record time as indicated in Figure 6. The filter switches successively couple one set of resistors and then another in each of the filter banks. The values of the selected resistors are selected in dependence upon functions such as in FIGURE 6 to provide for multiple removal in each of the record intervals.

It will be appreciated that the foregoing description of an analog type of apparatus for carrying out the invention may be substituted by a digital computer which is programmed to carry out the operations mathematically and graphically described above. The input data will comprise the seismic reflection data and the design data for the optimum filters. Three filters for each of eight different sets will be employed.

It is to be understood that the invention is not limited to the specific field technique illustrated in FIGURE 1. For example, it may be of the type described in the above-identified article from *Geophysics* dealing with expanding spreads. In such case, the expanding spread data could be rendered free from multiples. Other field procedures may also be employed where more than but single coverage of the subsurface is accomplished.

WHAT WE CLAIM IS:—

1. A method for reducing multiple reflections in seismic signals including the steps of generating a plurality of signals representative of seismic waves detected after travel to and from the same subsurface reflecting point over a like plurality of paths all of different lengths and in which multiple reflections have an apparent velocity which differs from the apparent velocity of primary reflections in dependence upon the geometries

of and velocity profiles along said paths, applying normal moveout correction to align the primary reflections in each of said plurality of signals, applying separate filters to each of said plurality of signals to alter the time distribution of the multiple reflections of each of said signals relative to the primary reflections therein in dependence upon the geometries of the respective paths of the associated seismic waves to and from said point and upon the velocity profile along said path, and combining the aligned and filtered signals to produce a composite signal in which said primary reflections are enhanced relative to said multiple reflections.

2. The method as defined in claim 1, in which the composite signal is recorded and the degree of modification of the time distribution of the multiple reflections in each signal is itself modified as a function of record time.

3. The method as defined in claim 1 or claim 2, in which the seismic waves are generated successively at shot points spaced at selected distances from a reference point and are detected at points located at like distances from the reference point on the side thereof opposite the respective shot points.

4. The method of reducing multiple reflections in seismic signals which includes the steps of

(a) generating in signal channels a plurality of signals representative of seismic waves generated at shot points and detected after travel to and from the same subsurface reflecting point over a like plurality of paths all of different lengths and in which multiple reflections have an apparent velocity which differs from the apparent velocity of primary reflections in dependence upon the geometries of and velocity profiles along said paths, and applying normal moveout corrections to align the primary reflections in each of the said plurality of signals,

(b) modifying the time distribution of the multiple reflections in each signal relative to the primary reflections therein by filtering in accordance with the filter operator $Y_i(f)$ given by the relationship

$$[S_{ij}^*(f) + N_{ij}^*(f)] [Y_i(f)] = [S_{io}^*(f)]$$

where:

$$i = 1, 2, 3, \dots, n;$$

$$j = 1, 2, 3, \dots, n;$$

matrix elements $S_{ij}^*(f)$ and $N_{ij}^*(f)$ are the complex conjugates of $S_{ij}(f)$ and $N_{ij}(f)$ which are the cross spectral densities between signal channels i and j for signal and noise respectively; and

elements $S_{io}^*(f)$ are the complex conjugates of $S_{io}(f)$ which are the cross spectral densities between the desired output signal and the signal in signal channels $i = 1, 2, \dots, n$ respectively; and

(c) combining the aligned and modified signals to produce a composite signal in which said primary reflections are enhanced relative to said multiple reflections.

5. The method as defined in claim 4, wherein the step of generating a plurality of signals includes the successive generation of seismic waves at each of a plurality of laterally spaced near-surface shot points located at different distances from a reference point and where said combined and modified signal is recorded as a time function.

6. The method as defined in claim 4, which includes the steps of:

(a) storing the functions to be performed by the filters in reproducible form, the filter functions being dependent upon the velocity profiles and the geometries of seismic wave paths to and from a given subsurface reflecting point for a plurality of different shot point-detector distances,

(b) convolving the stored filter functions for each seismic wave path with the signal from the corresponding detector corrected for normal moveout, and

(c) combining the resultant functions.

7. Apparatus for carrying out the method of claim 3, including means operable to generate the said seismic waves at the said shot points, seismic wave detector means operable to generate seismic signals representative of the said seismic waves, means for applying the said normal moveout corrections to align the primary reflections in each of the said plurality of signals, for each seismic signal thus generated, an individual filter having a time-response characteristic dependent upon the seismic wave path from the respective shot point via the common subsurface reflection point to the respective detector and the velocity profile along the said seismic wave path, the filter being operable to modify the time distribution of the multiple reflections relative to the primary reflections in the seismic signal, and signal combining means interconnecting the outputs of the filters, the combining means being operable to produce a composite



signal in which primary reflections are enhanced relative to the multiple reflections.

8. Apparatus as defined in claim 7, wherein the filter for each signal has a characteristic $Y_i(f)$ dependent upon the said seismic wave paths (channels) from the shot points to the detectors and the velocity profiles along the seismic wave paths, defined by the expression:—

$$[S_{ij}^*(f) + N_{ij}^*(f)] [Y_i(f)] = [S_{io}^*(f)]$$

where:

$i = 1, 2, 3, \dots, n;$

$j = 1, 2, 3, \dots, n;$

10 matrix elements $S_{ij}(f)$ and $N_{ij}(f)$ are the cross spectral densities between channels i and j for signal and noise respectively; and

elements $S_{io}(f)$ are the cross spectral densities between the desired output signal and signal in channels $i = 1, 2, \dots, n$ respectively.

9. The method for attenuation of multiple reflections in seismic signals according to claim 1, substantially as described herein with reference to the accompanying drawings.

ABEL & IMRAY,
Chartered Patent Agents,

Quality House, Quality Court, Chancery Lane, London, W.C.2.

Leamington Spa: Printed for Her Majesty's Stationery Office, by the Courier Press.

—1968. Published by The Patent Office, 25 Southampton Buildings, London, W.C.2,
from which copies may be obtained.

