



DOPPLER TRACKING METHOD AND DEVICE FOR A WIDEBAND MODEM

The invention relates to a method and device allowing Doppler tracking for a broadband modem.

More generally, it also relates to a system for estimating and tracking the frequency offset for a transmission system whose bandwidth is not "insignificant" with respect to the central frequency.

The problem posed is the transmission of data of any type between two correspondents submerged in the water and at a rate of several hundred bits per second while using a band-pass as small as possible and a computing power that is reduced to the minimum, in particular for standalone equipment running on batteries.

The problem is also that of transmitting data between two correspondents submerged in a given environment.

The electromagnetic waves conventionally used (from a few hundred kHz to several GHz being propagated extremely poorly in the water, the modem (modulator - demodulator) therefore uses acoustic waves to transport the messages transmitted.

For information, the submarine environment has an extremely complex behavior for acoustic waves. Because of its inhomogeneities (salinity, temperature, currents, etc.) mean that the path followed by the waves is in a straight line only for a very short distance and this route varies over time.

The waves that arrive at the receiver may follow several routes with very different delays. These routes may be either "direct" (several different journeys converging on the receiver), or "indirect" (reflections on the surface of the water, possibly disturbed by waves, and on the bottom which may have all kinds of types of relief).

In practice, this results in the arrival at the receiver of many repetitions of the emitted signal, each having, at a given moment, its own amplitude, phase, delay and frequency offset.

To this must be added, where necessary, a frequency offset due to the relative radial speed of the correspondents (Doppler effect) and a pseudo-periodic frequency offset due to the swell if at least one of the correspondents is on the surface.



In some cases, the reliability of the transmission must be virtually perfect. If it is a text, it is possible to tolerate several isolated errors, because they will be automatically corrected by the reader. When it is a binary file (for example a JPEG image), the required error rate is approximately 0, otherwise the image will be
5 very greatly degraded.

In addition, the system is "broadband". In practical applications, the bandwidth of the signal transmitted is not inconsiderable compared with the central transmission frequency. The Doppler offset being proportional to the frequency, the low frequencies have a lower frequency offset than the high frequencies.

10 What is more, since the relative speed of the transmitter and of the receiver is not always negligible compared with the speed of sound in water (approximately 1500 m/s), an effect of expansion or contraction of time is observed depending on the sign of the relative speed.

For example, a message transmitted over a period of 10 s will be
15 received in 10.13 seconds if the speed of separation is +10 m/s (36 km/h or approximately 20 knots) which poses problems of synchronization tracking.

To the knowledge of the applicant, there are systems of the "spread spectrum" type in which the payload rate is very low compared with the bandwidth used and in which discretion is appropriate, which prohibits the use of fixed carriers
20 that are easily identifiable.

Patent application WO 2005/055442 discloses a "broadband" system that estimates only once and for the whole duration of the message a Doppler offset value without carrying out tracking.

The subject of the present invention relates notably to a method making
25 it possible to continuously estimate the frequency offset (Doppler) for a modem operating in a submarine environment.

The invention relates notably to a system of estimating and tracking the frequency offset for a transmission system whose bandwidth is not insignificant with respect to the central frequency, the system emitting a waveform including fixed
30 carriers whose level is substantially greater than that of the carriers transporting the information, the carriers having frequencies $F_0 \dots F_{N-1}$, and the payload signal carrying the information, characterized in that it comprises at least the following elements:

- N banks of band-pass filters, each bank comprising P filters, suitable for

covering all of the values that the frequencies $F_0 \dots F_{N-1}$ may take after Doppler offset,

- A filter output is connected to a quadratic envelope detector,
- P adders suitable for adding the corresponding outputs of the N banks of filters,
- A system for detecting the position of the two largest maxima from the P sums and for validating the position of the largest maximum,
- A system for storing the positions of the two largest maxima and their "weight" and for predicting the current value,
- A system for validating/replacing the position found,
- A system converting the positions into estimated Doppler and filtering the latter through a low-pass filter.

The invention also relates to a method making it possible to estimate and track the Doppler effect for a signal consisting of a waveform including fixed carriers of a level substantially greater than the level of the carriers transporting the information, the carriers having frequencies $F_0 \dots F_{N-1}$, and the payload signal carrying the information in a transmission system whose bandwidth is not insignificant with respect to the central frequency, characterized in that it comprises at least the following steps:

1. Transmitting the composite signal comprising the payload signal carrying information and the reference signal to a filtering step using the frequencies of the carriers,
2. Determining, for each filter, the envelope of the signal, $e_{n,p}$,
3. Adding the various outputs $e_{n,p}$,
4. Determining the values of the two largest maxima, M and m, for several moments, t,
5. Determining the probable value of the position at the moment t, considering the maxima found that define uniquely the associated Doppler offset value,
6. Carrying out a low-pass filtering on the Doppler offset values in order to obtain a smoothed value.

The method comprises for example a step in which the smoothed value is interpolated at intermediate moments by means of low-pass filters derived from the filter defined above.

The filtering step 1 comprises, for example, at least the following steps:

- The input signal is windowed by a Hanning window (simple raised cosine) for a period of 160 ms, with overlap of 80 ms between two successive windows,
- to the 1.024 points obtained, 3.072 null points are added, then:
- 5 • the 4096-point fast Fourier transform FFT is carried out on this signal whose output is marked $Z_0, Z_1 \dots Z_{4095}$,
- for a desired central frequency $F_{n,p}$ the rank n of the "bin" Z_n of the fast Fourier transform FFT that is just below is computed,
- the actual number x lying between 0 and 1 which is $(F_{n,p} - 1.5625 n) / 1.5625$
- 10 is computed and two complex numbers $z0(x)$ and $z1(x)$ that are functions of x are selected,
- the filter output is determined by $z0(x) Z_n + z1(x) Z_{n+1}$.

The method and the system according to the invention notably have the following advantages. They make it possible to carry out submarine communications of long duration, with variable Doppler, between a transmitter and a receiver traveling on any trajectories.

- Figure 1 shows, in a time-frequency space, the effect of the offset on the carriers,
- Figure 2 shows a typical structure of the modules and steps used by the system according to the invention,
- 20 • Figure 3 shows a step for filtering the signal by windowing the input signal,
- Figure 4 shows an example of outputs of the sensors,
- Figure 5 shows an example of the outputs of the adders,
- Figure 6, shows an example of parabolic regression,
- 25 • Figures 7 to 9 show the results of the final filtering, and
- Figure 10 shows an example of a frequency response obtained.

In order to better explain the principle used in the invention, the following example is given for a modem operating in a submarine environment, characterized by a considerable and rapidly variable time spread and a not inconsiderable bandwidth with respect to the bandwidth which makes the frequency offset due to the Doppler effect variable in the very bandwidth of the signal ("broadband" effect).

The waveform emitted by the modem in question includes fixed carriers (unmodulated) whose level is substantially greater than that of the carriers transporting the information. It is based on these carriers that the estimate of the average Doppler offset of the signal is made, an offset that is capable of varying rapidly.

The reference carriers are equal to N in number and have frequencies marked $F_0, F_1, \dots, F_{N-1}$.

The maximum Doppler offset is given by a dimensionless number, marked δ , obtained by the equation:

$$\delta = \frac{v_r(R) - v_r(E)}{C - v_r(R)}$$

C: speed of sound in the water

$v_r(E)$: radial speed of the transmitter

$v_r(R)$: radial speed of the receiver

Frame of reference: water, considered to be immobile

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In the presence of such a Doppler offset, each frequency carrier F_i sees its frequency modified to $(1 + \delta) F_i$.

Figure 1 describes in a time-frequency space the effect of the Doppler offset on the carriers, for $\delta = -\delta_{\max}$, $d = 0$, and $d = +\delta_{\max}$, $\delta_{\max}$ being a design element of the system.

Any frequencies may be chosen, but, for δ to be able to be estimated unambiguously, the small rectangles 1 of figure 1 have a positive height, which results in the following simple condition:

$$(1 - \delta_{\max}) F_n > (1 + \delta_{\max}) F_{n-1}$$

or

$$F_n - F_{n-1} > \delta_{\max} (F_n + F_{n-1})$$

$$n = 1 \dots N-1$$

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Figure 2 represents the various steps used in the invention and the elements constituting the device.

This system requires the presence, throughout the whole message and even a little longer, of a set of N unmodulated fixed carriers in addition to the payload signal.

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The complex signal (reference carriers and useful signal) is received on

a processing subsystem comprising, in this order:

- a set, 100, of N banks of P band-pass filters covering (slightly more than) at least all of the values that the frequencies $F_0 \dots F_{N-1}$ that can be taken after Doppler offset.
- 5 Each band-pass filter is followed by a quadratic envelope sensor not shown separately from the filters in the figure for reasons of simplification.
- a set, 200, of P adders adding the outputs of the quadratic envelope sensors of the same rank for each filter,
- a system, 300, for detecting the position of the two largest maxima from the P sums and for validating their positions at the output of the series of adders,
- 10 - a system, 400, for storing the positions and their "weights" and for predicting the current value,
- a system, 500, for validating/replacing the position found,
- 15 - a system, 600, converting the positions into estimated Doppler and filtering the latter through a low-pass filter which gives an estimate of the Doppler offset (with a fixed delay).

The estimation and tracking mechanism may also include an interpolation mechanism making it possible to have Doppler offset samples more frequently.

The detail of the various elements forming part of the system according to the invention is given below.

Banks of filters 100

A bank of filters is suitable notably for covering the frequencies lying between $(1 - \delta_{\max}) F_n$ and $(1 + \delta_{\max}) F_n$ where F_n is the frequency after Doppler offset for the bank of filters of rank n.

Each bank of filters comprises, for example, the same number P of band-pass filters of evenly spaced central frequencies.

The p^{th} filter of the n^{th} bank will be centered on a frequency equal to:

$$F_{n,p} = F_n \left(1 + \delta_{\max} \left(\frac{2p - P + 1}{P} \right) \right)$$

$n = 0 \dots N-1$
 $p = 0 \dots P-1$

F_n : the frequency after Doppler offset for the bank of filters of rank n.

A simple way of producing the bank of filters is described below.

As an example, it is assumed that the carriers are spaced at multiples of 25 Hz and that the sampling frequency is 6.400 Hz.

It is assumed that, during the demodulation phase, a new symbol
5 appears every 80 ms and then a new frequency offset value is needed.

Assuming that the differential phase demodulation may have up to four phase states, that is at least 90° , if the error of estimation of the frequency offset is dF , the additional phase rotation that it introduces is $360^\circ \times 80 \text{ ms} \times dF$.

So that it does not exceed a quarter of the minimum rotation (empirical
10 rule from experience), the error of estimation of the frequency offset dF must be at most $df_{\text{MAX}} = 0.78125 \text{ Hz}$ (in absolute value), which means that the fast Fourier transform, FFT, to be used must be able to separate two frequencies separated by $2 df_{\text{MAX}} = 1.5625 \text{ Hz}$. Its number of points must therefore be at least $6.400/1.5625 = 4.096$.

15 For this, the process is as follows:

- The input signal is windowed (weighting) by a Hanning window (simple raised cosine) for a period of 160 ms, with overlap of 80 ms between two successive windows. This step is illustrated in figure 3.
- added to the $0.16 \times 6.400 = 1.024$ points obtained are $4.096 - 1.024 = 3.072$
20 null points, then:

1. the 4096-point fast Fourier transform FFT is carried out on this signal whose output is marked $Z_0, Z_1 \dots Z_{4095}$,
2. for a desired central frequency $F_{n,p}$, a computation is made of the rank n of the "bin" Z_n of the Fourier transform FFT that is just below, that is to
25 say that for which F_n lies between n times 1.5625 Hz and $(n + 1)$ times 1.5625 Hz,
3. the real number x lying between 0 and 1 is computed which is $(F_{n,p} - 1.5625 n) / 1.5625$ and two complex numbers $z0(x)$ and $z1(x)$ that are functions of x (and computed in advance for convenience) are selected,
- 30 4. the output of the filter is simply given by $z0(x) Z_n + z1(x) Z_{n+1}$.

The coefficients $z0$ and $z1$ are optimized so that the result is as close as possible to the one that a filter really centered on $F_{n,p}$ would give.

One of the two coefficients $z0$ or $z1$ may be chosen to be REAL (and the other corrected as a consequence) since the phase of the output signal is of no

importance.

Each filter is followed by a quadratic envelope detector giving the square of the modulus of its output (the process is usually carried out on a complex signal, in baseband, i.e. centered on the frequency 0).

- 5 The output of the p^{th} detector of the n^{th} bank will be marked $e_{n,p}$.

Figure 4 gives an example of outputs of the quadratic envelope sensors, for $P = 100$, and $N = 7$ frequencies, with an input signal offset toward the high frequencies.

Adders 200

- 10 The output of the p^{th} adder is given by:

$$Sp = \sum_{n=0}^{N-1} e_{n,p}$$

$$p = 0 \dots P - 1$$

where $e_{n,p}$ is the output of the p^{th} sensor of the n^{th} bank of filters.

- Figure 5 represents an example of the outputs of the adders for the case represented by figure 4, where the smoothing effect provided by the adding (due to the propagation, the gain may vary a great deal in the vicinity of each of the N carriers) is clearly seen.
- 15

Detection of the two largest maxima out of P 300

The signal originating from the adders 200 is transmitted to a maxima detection system 300 having notably the role of finding two maxima.

- 20 Specifically, if there are two maxima of adjacent amplitudes, they are usually of low amplitude and correspond to nothing precise: there is probably only noise at the input of the receiver.

To find these two maxima, M (the largest) and m (the second largest), the method may use the following algorithm, written in simplified C language.

- 25 /* Initialization */
 $M = 0$
 position_M = 0
 $m = 0$
 position_m = 0
- 30 /* Loop to find a maximum in positions 1 ... P-2 */
 for(i = 1 to P - 2) {
 if ($S(i) \geq S(i - 1)$ and $S(i) \geq S(i + 1)$) {

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/* S(i) is a local maximum */
if ( S(i) > M ) {
/* S(i) is the new largest maximum */
    m = M
5    position_m = position_M
    M = S(i)
    position_M = i
} else {
    if ( S(i) > m ) {
10    /* S(i) is the second new maximum */
        m = S(i)
        position_m = i
    }
}
15 }
}

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If position_M is not null (0 is an illegal value) and if m is less than a given proportion of M (for example 50%), M and its position (position_M) may be considered valid. Two maxima have been found.

20 Otherwise they are ignored subsequently (it is sufficient to put M at zero).

The value of the position of the two maxima is then refined for example by quadratic regression, that is to say that the amplitudes of the sums adjacent to the maximal sum M are approximated by a parabola using three or four neighbors
25 depending on the number of neighbors available, a value indicated subsequently as the corrected value.

For the drafting of the search algorithm, any language other than C language may be used.

Figure 6 gives an example of parabolic regression, still on the same data
30 of figures 4 and 5.

The position of the maximum of the parabola is the modified value of position_M which, an integer, becomes a real number.

For position_M = 1, the correction is equal to:

$$\frac{11 S_0 - 3 S_1 - 7 S_2 - S_3}{10 (S_0 - S_1 - S_2 + S_3)}$$

For position_M = 2...P-3, the correction is equal to:

$$\frac{7(2S_{p-2} + S_{p-1} + S_{p+1} - 2S_{p+2})}{10(2S_{p-2} - S_{p-1} - 2S_p - S_{p+1} + 2S_{p+2})}$$

For position_M=P-2 the correction is equal to:

$$5 \frac{(S_{p-4} + 7S_{p-3} + 3S_{p-2} - 11S_{p-1})}{10(S_{p-4} - S_{p-3} - S_{p-2} + S_{p-1})}$$

At the output of the maxima detector, the user has two values of maxima and their position.

Position predictor 400

10 The prediction/replacement system 400 uses the value of the position corrected at the moment t , and at the moments $t - 1, t - 2, \dots, t - T$ (t being taken to be an integer for the convenience of the computations and at least equal to 4). The value of the corrected position corresponds to the value of the position deducted from the maxima and refined by quadratic regression.

15 Its function is notably to evaluate, based on the past, the probable value of the position at the moment t , first considering the maxima of largest amplitude ($M_{t-1}, M_{t-2}, \dots, M_{t-T}$), because they are the smallest. It is constituted notably of a delay line containing the last $T+1$ M/position_M pairs.

20 Since, over a limited period, the Doppler variation may be assimilated locally to a parabola, the value of the position at the moment t is predicted by means of a weighted quadratic regression.

For this, the coefficients a, b, c of a parabola are computed minimizing the following weighted quadratic error (in which the variable pos_i indicates the value of position_M after correction):

$$25 \quad E = \sum_{i=t-T}^t M_i ((ai^2 + bi + c) - pos_i)^2$$

The predicted value is simply

$$\widehat{pos}_t = a t^2 + b t + c$$

Its computation is simple and given by the following series of equations:

$$U_k = \sum_{i=t-T}^t M_i (t-i)^k, k = 0, \dots, 4$$

$$W_k = \sum_{i=t-T}^t M_i \text{pos}_i (t-i)^k, k = 0, \dots, 2$$

$$\widehat{\text{pos}}_t = \frac{W_0(U_2 U_4 - U_3^2) + W_1(U_2 U_3 - U_1 U_4) + W_2(U_1 U_3 - U_2^2)}{U_0(U_2 U_4 - U_3^2) + U_1(U_2 U_3 - U_1 U_4) + U_2(U_1 U_3 - U_2^2)}$$

5 Choice of the position to use, 500

The value predicted for the position is transmitted to a device 500 of final choice of the position to be used. This latter also receives the information originating from the step for validating/refining the position value.

- If the corrected position is not valid, the computation is made with $M_t = 0$,
 10 and the predicted value (pos_t) is replaced by its value estimated by means of the formula given above and M_t by the average of the preceding values.

- If the corrected position is valid, its value estimated with the measured value of M_t is computed. If the difference between the estimated position and the corrected position is sufficiently small (in absolute value), the value of pos_t is
 15 retained; otherwise, the procedure is as explained above.

This mechanism makes it possible to eliminate the aberrant values.

Initialization is carried out with $T + 1$ values M_i equal and small and values pos_i corresponding to a null frequency offset, that is $(P - 1) / 2$.

- The approximation of the Doppler variation by a sinusoid would perhaps
 20 be more appropriate to the submarine acoustic channel (because of the periodic character of the swell). The computations are however more complex.

The conversion of the position to "Doppler" can be carried out after the correction of the position, simply by replacing pos_t with:

$$\delta_i = \delta_{\max} \left(\frac{2 \text{pos}_i - P + 1}{P} \right)$$

- 25 The computations will be the same, simply replacing pos_i with δ_i .

Final filtering, 600

This module receives the value originating from the conversion of the

position to Doppler. It comprises for example several low-pass filters and a commutator.

- The last step of the process consists in carrying out a low-pass filtering consequent on the values of the Doppler offsets in order to obtain as faithful as possible a smoothed value δ while the instantaneous frequency of δ remains lying

between two frequencies $-F_0$ and $+F_0$.
For example, a linear-phase low-pass filter will be used with an odd number of coefficients, namely:

$$\delta = \sum_{i=t-2K}^{i=t} h_{t-i} \delta_i$$

where

$$(1) \sum_{i=0}^{i=2K} h_i = 1$$

and

$$(2) h_i = h_{2K-i}, i = 0, \dots, K-1$$

- The condition (1) guarantees that a constant offset at the input will be unchanged at the output.

- The condition (2) expresses the linear phase, which guarantees a constant delay irrespective of the instantaneous speed of variation of δ . This delay is equal to K times the sampling period. The delay due to the foregoing processes must be added to this, notably the filtering step for which all the $N \times P$ filters must have the same delay, which is the case with the exemplary embodiment proposed.

For the exact design of the filter, which must be as "flat" as possible between $-F_0$ and $+F_0$, it is possible to use the following method:

- 1 - obtain an ideal filter whose frequency response is given by:

$$h(f) = 1 + \alpha \left(\frac{f}{F_0} \right)^2$$

$$-F_0 \leq f \leq +F_0$$

$$0 \text{ elsewhere}$$

an example is given in figure 7 for $F_0 = 1$ Hz and $\alpha = 0.2$.

- 2 - calculate its impulse response (modified to be 1 for $t = 0$)

$$\theta = 2 \pi F_0 t$$

$$h(t) = 3 \frac{2 \alpha \theta \cos(\theta) + (\theta^2 (1 + \alpha) - 2 \alpha) \sin(\theta)}{(3 + \alpha) \theta^3}$$

(where: $h(0) = 1$)

given by:

an example of impulse response is given in figure 8.

- 3 - compute its impulse response sampled at intervals equal to T, and weight it
5 by a Hamming window, for example:

$$h_i = h((i - K) T) \left(0.54 + 0.46 \cos \left(\pi \frac{i - K}{K + 0.5} \right) \right)$$

$$i = 0 \dots 2K$$

For T = 80 ms, and K = 16, for example, a response is represented in
figure 9.

- 4 - adjust the coefficient α to have a frequency response as flat as possible in
10 the band,
5 - normalize so that the sum of the coefficients is 1.

Figure 10 represents an example of frequency response obtained (the
value of α is intentionally a little too high).

- If it is desired to have Doppler values at a more rapid rate than every
15 80 ms, a technique of interpolation should be used to obtain the intermediate values.

The technique well known to those skilled in the art and called
"polyphase filters" makes it possible to obtain the intermediate samples.

In an exemplary embodiment, the coefficients h'_i of a filter giving
samples offset in time can be calculated simply by the following generic formula:

20

$$h'_p = \frac{(-1)^{p+1} \sin(\pi\theta)}{\pi} \left(\frac{h_0}{p - \theta} + \sum_{i=1}^K (-1)^i h_i \left(\frac{1}{p + i - \theta} + \frac{1}{p - i - \theta} \right) \right)$$

where θ depends on the desired delay.

The value $\theta = 0$ corresponds to the source filter.

$$h'_p = \frac{(-1)^{p+1} \sin(\pi\theta)}{\pi} \left(\frac{h_0}{p - \theta} + \sum_{i=1}^K (-1)^i h_i \left(\frac{1}{p + i - \theta} + \frac{1}{p - i - \theta} \right) \right)$$

CLAIMS

1 - A system of estimating and tracking the frequency offset for a transmission system whose bandwidth is not insignificant with respect to the central frequency, the system transmitting a waveform including fixed carriers whose level is substantially greater than that of the carriers transporting the information, the carriers having frequencies $F_0 \dots F_{N-1}$, and the payload signal carrying the information, characterized in that it comprises at least the following elements:

- N banks (100) of band-pass filters, each bank comprising P filters, suitable for covering all of the values that the frequencies $F_0 \dots F_{N-1}$ may take after Doppler offset,
- A filter output is connected to a quadratic envelope detector,
- P adders (200) suitable for adding the corresponding outputs of the N banks of filters,
- A system (300) for detecting the position of the two largest maxima from the P sums and for validating the position of the largest maximum,
- A system (400) for storing the positions of the two largest maxima and their "weight" and for predicting the current value,
- A system (500) for validating/replacing the position found,
- A system (600) converting the positions into estimated Doppler and filtering the latter through a low-pass filter.

2 - A method making it possible to estimate and track the Doppler effect for a signal consisting of a waveform including fixed carriers of a level substantially greater than the level of the carriers transporting the information, the carriers having frequencies $F_0 \dots F_{N-1}$, and the payload signal carrying the information in a transmission system whose bandwidth is not insignificant with respect to the central frequency, characterized in that it comprises at least the following steps:

1. Transmitting the composite signal comprising the useful signal carrying information and the reference signal to a filtering step using the frequencies of the carriers, (100),
2. Determining, for each filter, the envelope of the signal, $e_{n,p}$,
3. Adding the various outputs $e_{n,p}$, (200),

4. Determining the values of the two largest maxima, M and m , for several moments, t , (300),
5. Determining the probable value of the position at the moment t , considering the maxima found that define uniquely the associated Doppler offset value, (400, 500),
6. Carrying out a low-pass filtering on the Doppler offset values in order to obtain a smoothed value (600).

3 - The method as claimed in claim 2, characterized in that it comprises a step in which the smoothed value is interpolated at intermediate moments by means of low-pass filters derived from the filter of the preceding claim.

4 - The method as claimed in claim 2, characterized in that the filtering step 1 comprises at least the following steps:

- The input signal is windowed by a Hanning window (simple raised cosine) for a period of 160 ms, with overlap of 80 ms between two successive windows,
- to the 1.024 points obtained, 3.072 null points are added, then:
- the 4096-point fast Fourier transform FFT is carried out on this signal whose output is marked $Z_0, Z_1 \dots Z_{4095}$,
- for a desired central frequency $F_{n,p}$ the rank n of the "bin" Z_n of the fast Fourier transform FFT that is just below is computed,
- the real number x lying between 0 and 1 which is $(F_{n,p} - 1.5625 n) / 1.5625$ is computed and two complex numbers $z0(x)$ and $z1(x)$ that are functions of x are selected,
- the filter output is determined by $z0(x) Z_n + z1(x) Z_{n+1}$.

5 - The method as claimed in claim 2, characterized in that the value of the two maxima found is refined by quadratic regression.

6 - The method as claimed in claim 2, characterized in that the step 6 of low-pass filter is carried out with a linear-phase low-pass filter with an odd number of coefficients.