

SENSOR ARRAY, SYSTEMS AND METHODS FOR BUILDING A MAGNETIC
MAP, AND FOR LOCALIZING A MOBILE DEVICE BASED ON A MAGNETIC
MAP

5

Cross-Reference To Related Application

[0001] This application claims the benefit of priority of Singapore patent application No. 10202110176U, filed 15 September 2021, the content of it being hereby incorporated by reference in its entirety for all purposes.

10

Technical Field

[0002] Various embodiments relate to a sensor array, a magnetic map, systems and methods for building a magnetic map and for localizing a mobile device, as well as a method for determining a rotation-invariant ambient magnetic field vector to build a magnetic map.

15

Background

[0003] Existing approaches for mobile devices localization in GNSS-challenged environments (e.g. hotels, hospitals, airports, industrial warehouses, container seaports, amongst others) depend heavily on the pre-installed infrastructures (e.g. Wi-Fi routers, RFID, UWB, QR Codes, magnetic guiding tapes, amongst others), which are inflexible, and demand huge costs for deployment and maintenance. The magnetic field around us in our daily lives (i.e. the ambient magnetic field, AMF) exhibits high distinctiveness and ubiquity at different location, that proves to be a feasible infrastructure-free localization solution and has been gradually utilized for a wide range of applications such as navigation, mapping, assistance in intelligent operation, and so on.

25

[0004] Existing approaches on AMF-based mapping utilize a single magnetometer sensor / inertial measurement unit to measure the 3-D orientation-dependent AMF data and build

30

the magnetic map, where the efficiency is significantly low thus not applicable to large-scale environments.

[0005] Existing approaches on AMF-based localization mainly utilize the 3-D orientation-dependent AMF data for single fingerprint-based matching in 2-D space, where they are sensitive to orientation errors and the localization accuracy is not good.

[0006] Thus, there is a need for novel sensors, systems and methods for building a magnetic map and for localizing a mobile device that address at least the problems mentioned above.

Summary

[0007] According to an embodiment, a sensor array is provided. The sensor array may include a plurality of magnetic field sensors configured to operate substantially simultaneously for building a magnetic map representative of an environment. Each magnetic field sensor may be configured to measure an ambient magnetic field at a position of the magnetic field sensor, the position being within the environment or within a working distance of the magnetic field sensor from an edge boundary of the environment. Each magnetic field sensor may be arranged spaced apart from a neighbouring magnetic field sensor at a distance such that the respective ambient magnetic fields measured by the magnetic field sensor and the neighbouring magnetic field sensor are disambiguated.

[0008] According to an embodiment, a system for building a magnetic map representative of an environment and localizing a mobile device is provided. The system may include a sensor array configured to provide measurements of ambient magnetic fields in an environment; a mapping module configured to receive the ambient magnetic fields and determine rotation-invariant ambient magnetic field vectors, based on the received ambient magnetic fields, to build a magnetic map representative of the environment; and a localization module. The localization module may be configured to: based on ambient magnetic fields measured over a period of time by a plurality of magnetic field sensors releasably coupled with a mobile device, infer candidate initial poses of the mobile device from candidate locations in the magnetic map; based on the candidate initial poses and variations in the measured ambient magnetic fields over the period of time, infer a fine

pose of the mobile device; and evaluate a joint probabilistic distribution based on a probability distribution of the candidate initial poses and a probability distribution of the fine pose to estimate a specific pose of the mobile device.

[0009] According to an embodiment, a system for building a magnetic map representative of an environment is provided. The system may include a sensor array configured to provide measurements of ambient magnetic fields in an environment; and a mapping module configured to receive the ambient magnetic fields and determine rotation-invariant ambient magnetic field vectors, based on the received ambient magnetic fields, to build a magnetic map representative of the environment.

10 [0010] According to an embodiment, a system for localizing a mobile device is provided. The system may include a plurality of magnetic field sensors releasably coupled with a mobile device, wherein the plurality of magnetic field sensors may be configured to measure ambient magnetic fields over a period of time; and a localization module. The localization module may be configured to: based on the measured ambient magnetic fields, infer candidate initial poses of the mobile device from candidate locations in a magnetic map; based on the candidate initial poses and variations in the measured ambient magnetic fields over the period of time, to infer a fine pose of the mobile device; and evaluate a joint probabilistic distribution based on a probability distribution of the candidate initial poses and a probability distribution of the fine pose to estimate a specific pose of the mobile device.

[0011] According to an embodiment, a magnetic map representative of an environment is provided. The magnetic map may include coordinates established for each of a plurality of locations in the environment; and a rotation-invariant ambient magnetic field fingerprint at each location.

25 [0012] According to an embodiment, a method for determining a rotation-invariant ambient magnetic field vector to build a magnetic map of an environment is provided. The method may include adapting an ambient magnetic field measured by a magnetic field sensor at a position within the environment or within a working distance of the magnetic field sensor from an edge boundary of the environment into an ambient magnetic field vector; inferring, based on the ambient magnetic field vector, an orientation of the magnetic field sensor; and aligning, based on the inferred orientation, the ambient magnetic field

vector in a local coordinate system to a global coordinate system to generate the rotation-invariant ambient magnetic field vector. The rotation-invariant ambient magnetic field vector may include two orthogonal components and an integrated intensity of the measured ambient magnetic field. The magnetic field sensor may be releasably coupled with a mobile device moved to the position.

5 [0013] According to an embodiment, a method for building a magnetic map representative of an environment is provided. The method may include establishing coordinates for each of a plurality of locations in the environment; collecting rotation-invariant ambient magnetic field vectors at each location, the rotation-invariant ambient magnetic field
10 vectors being determined from ambient magnetic fields measured at the location; determining, based on the collected rotation-invariant ambient magnetic field vectors, an ambient magnetic field fingerprint for each location; and for more than one of the plurality of locations, selecting the respective ambient magnetic field fingerprints and associating each of the respective ambient magnetic field fingerprints with the corresponding
15 coordinates.

[0014] According to an embodiment, a method for localizing a mobile device is provided. The method may include performing coarse localization, based on ambient magnetic fields, to infer candidate initial poses of the mobile device from candidate locations in a magnetic map, wherein the ambient magnetic fields may be measured over a period of time by a
20 plurality of magnetic field sensors releasably coupled with the mobile device; performing fine localization, based on the candidate initial poses and variations in the measured ambient magnetic fields over the period of time, to infer a fine pose of the mobile device; and evaluating a joint probabilistic distribution based on a probability distribution of the candidate initial poses and a probability distribution of the fine pose to estimate a specific
25 pose of the mobile device.

[0015] According to an embodiment, there is provided a computer readable storage medium including computer readable instructions operable when executed by a computer to perform at least one of the following: determine a rotation-invariant ambient magnetic field vector to build a magnetic map of an environment, build a magnetic map
30 representative of an environment, or localize a mobile device.

Brief Description of the Drawings

[0016] In the drawings, like reference characters generally refer to like parts throughout the different views. The drawings are not necessarily to scale, emphasis instead generally being placed upon illustrating the principles of the invention. In the following description, various embodiments of the invention are described with reference to the following drawings, in which:

[0017] FIG. 1A shows a schematic view of a sensor array according to various embodiments.

10 [0018] FIG. 1B shows a schematic view of a system for building a magnetic map and localizing a mobile device, according to various embodiments.

[0019] FIG. 1C shows a schematic view of a system for building a magnetic map, according to various embodiments.

15 [0020] FIG. 1D shows a schematic view of a system for localizing a mobile device, according to various embodiments.

[0021] FIG. 1E shows a schematic view of a magnetic map, according to various embodiments.

[0022] FIG. 2A shows a flow chart illustrating a method for determining a rotation-invariant ambient magnetic field vector to build a magnetic map of an environment, according to various embodiments.

20 [0023] FIG. 2B shows a flow chart illustrating a method for building a magnetic map representative of an environment, according to various embodiments.

[0024] FIG. 2C shows a flow chart illustrating a method for localizing a mobile device, according to various embodiments.

25 [0025] FIG. 3A shows a schematic representation of multiple magnetometer sensors array, according to an exemplary embodiment.

[0026] FIG. 3B shows a schematic perspective view of FIG. 3A.

[0027] FIG. 4 shows a flow diagram depicting a magnetic map building strategy involving the features of a mapping module working with the multiple magnetometer sensors array of FIG. 3A, according to one example.

30 [0028] FIG. 5 shows a flow diagram illustrating an exemplary localization framework.

[0029] FIG. 6 shows an exemplary embodiment of the magnetic map.

[0030] FIG. 7A shows a graphical representation of ambient magnetic field in the global coordinate system, according to an example.

5 [0031] FIG. 7B shows a graphical representation of measured x/y-axes ambient magnetic field data with different orientations of a magnetometer sensor coupled to a mobile device at a same location of a magnetic field interference-free environment, according to an example.

[0032] FIG. 8A shows a schematic diagram illustrating magnetometer sensors coordinates and different orientation candidates of the mobile device, according to various examples.

10 [0033] FIG. 8B shows a schematic diagram illustrating coordinates transformation from the local coordinate system to the global coordinate system, according to an example.

[0034] FIG. 9A shows a perspective view illustrating a multiple magnetometer sensors array installed on a simulated mobile device having magnetometers in single-line shape, according to an example.

15 [0035] FIG. 9B shows a photograph of a multiple magnetometer sensors array installed on a real mobile device having magnetometers in single-line shape, according to an example.

[0036] FIGS. 10A and 10B respectively show interpolation (IP)-based magnetic maps of a simulated industrial warehouse environment and a real office corridor environment built by the magnetic map building method of FIG. 2B or based on the magnetic map building strategy of FIG. 4.

[0037] FIGS. 11A, 11B, and 11C show front perspective views illustrating configurations of the multiple magnetometer sensors arrays with bar lengths of 0.5m, 1.0m, and 1.5m, respectively, having three, five, and seven magnetometer sensors in single-line shape, according to various examples.

25 [0038] FIGS. 11D, 11E, and 11F show front perspective views illustrating configurations of the multiple magnetometer sensors arrays with bar length of 1.0m having nine, fifteen, and twenty-one magnetometer sensors in multiple-lines shape, respectively, according to various examples.

30 [0039] FIG. 12A shows a perspective view illustrating a simulated mobile device in Gazebo simulator with single magnetometer and 2-D LiDAR configuration for mobile device online localization, according to one example.

[0040] FIG. 12B shows a perspective view illustrating a real mobile device with single magnetometer and 2-D LiDAR configuration for mobile device online localization, according to one example.

[0041] FIGS. 13A and 13B show the fused interpolation (IP)-based magnetic maps and 2-D occupancy grid maps (OGMs) (left side of FIG. 13A, bottom part of FIG. 13B) of the simulated industrial warehouse environment (right side of FIG. 13A) and the real office corridor environment (top part of FIG. 13B), according to various examples.

[0042] FIGS. 14A and 14B show histograms illustrating the localization robustness for the simulated industrial warehouse environment and the real office corridor environment, respectively, assessed by the Correct Localization Rate (CLR), according to various examples.

[0043] FIGS. 14C and 14D show Cumulative Distribution Function (CDF) plots of the overall localization error for the simulated industrial warehouse environment and the real office corridor environment, respectively, according to various examples.

[0044] FIGS. 15A, 15B, and 15C show graphs depicting the root-mean-square (RMS) of the positional error in the fine localization (*i.e.*, pose tracking) phase for three randomly-selected waypoints (WPs) in the simulated warehouse environment, respectively, according to various examples.

[0045] FIGS. 15D, 15E, and 15F show graphs depicting the root-mean-square (RMS) of the positional error in the fine localization (*i.e.*, pose tracking) phase for three randomly-selected WPs in the real corridor environment, respectively, according to various examples.

Detailed Description

[0046] The following detailed description refers to the accompanying drawings that show, by way of illustration, specific details and embodiments in which the invention may be practiced. These embodiments are described in sufficient detail to enable those skilled in the art to practice the invention. Other embodiments may be utilized and structural, logical, and electrical changes may be made without departing from the scope of the invention. The various embodiments are not necessarily mutually exclusive, as some embodiments can be combined with one or more other embodiments to form new embodiments.

[0047] Embodiments described in the context of one of the methods or devices are analogously valid for the other methods or devices. Similarly, embodiments described in the context of a method are analogously valid for a device, and vice versa.

[0048] Features that are described in the context of an embodiment may correspondingly be applicable to the same or similar features in the other embodiments. Features that are described in the context of an embodiment may correspondingly be applicable to the other embodiments, even if not explicitly described in these other embodiments. Furthermore, additions and/or combinations and/or alternatives as described for a feature in the context of an embodiment may correspondingly be applicable to the same or similar feature in the other embodiments.

[0049] In the context of various embodiments, the articles “a”, “an” and “the” as used with regard to a feature or element include a reference to one or more of the features or elements.

[0050] In the context of various embodiments, the term “substantially” may include “exactly” and a reasonable variance.

[0051] In the context of various embodiments, the term “about” as applied to a numeric value encompasses the exact value and a reasonable variance.

[0052] As used herein, the term “and/or” includes any and all combinations of one or more of the associated listed items.

[0053] As used herein, the phrase of the form of “at least one of A or B” may include A or B or both A and B. Correspondingly, the phrase of the form of “at least one of A or B or C”, or including further listed items, may include any and all combinations of one or more of the associated listed items.

[0054] As used herein, the expression “configured to” may mean “constructed to” or “arranged to”. The expression “system” may be interchangeably referred to as apparatus.

[0055] Various embodiments may be described for ambient magnetic field-based mapping and localization.

[0056] More specifically, various embodiments may provide systems and methods for ambient magnetic field (AMF)-based mapping and localization for mobile devices (e.g. autonomous robots, automated guided vehicles, unmanned ground vehicles, etc.) equipped with magnetometer sensors (or any other sensors with magnetic field sensing functionality, such as the inertial measurement unit (IMU), digital compass, amongst others). The

expression “ambient magnetic field” and its abbreviated form “AMF” may be interchangeably used throughout the context here. For example, a system for AMF-based mapping and localization may include a mapping module/submodule configured to measure the AMF with a multiple-magnetometer-sensors array to build the magnetic map with certain attributes, and a localization submodule configured to determine a unique location or position of the mobile devices in two-dimensional (2-D) or three-dimensional (3-D) space.

[0057] FIG. 1A shows a schematic view of a sensor array 100 in accordance with various embodiments. The sensor array 100 may include a plurality of magnetic field sensors 102 configured to operate substantially simultaneously for building a magnetic map (e.g. 180 of FIG. 1E) representative of an environment. Each magnetic field sensor 102 may be configured to measure an ambient magnetic field at a position of the magnetic field sensor 102, the position being within the environment or within a working distance of the magnetic field sensor 102 from an edge boundary of the environment. When the position is within the working distance of the magnetic field sensor 102 from the edge boundary of the environment, the magnetic field sensor 102 may be located just outside of or sufficiently close to the environment so that the magnetic field sensor 102 may still be able to measure the ambient magnetic field propagating within the environment, e.g. near the edge boundary. Each magnetic field sensor 102 may be arranged spaced apart from a neighbouring magnetic field sensor 102 at a distance such that the respective ambient magnetic fields measured by the magnetic field sensor 102 and the neighbouring magnetic field sensor 102 are disambiguated.

[0058] In the context of various embodiments, the term “disambiguated” may mean distinct in a manner where the respective magnetic fields do not cause interference to another. The term “environment” may refer to a physical area of interest, for example, a large-scale environment, with hundreds-of-meters size, facing problems of unavailable or severely challenged global navigation satellite system or global positioning system signals. The environment may include both indoor and outdoor environments for 2-dimensional or 3-dimensional localization problems.

[0059] The sensor array 100 may be a q-dimensional sensor array, where q is 1, 2 or 3. The sensor array 100 may be releasably couplable with a mobile device, the mobile device being movable to the position.

5 [0060] In various embodiments, the plurality of magnetic field sensors 102 may be arranged at substantially regular spatial intervals.

[0061] The plurality of magnetic field sensors 102 may be calibrated to obtain calibration parameters to facilitate building the magnetic map. The plurality of magnetic field sensors 102 may be calibrated individually and as an entire array to obtain intra-sensor and inter-sensor parameters. The intra-sensor and inter-sensor parameters may then be used for
10 adjustments or compensations to build the magnetic map with higher accuracy.

[0062] The plurality of magnetic field sensors 102 may include at least one of the followings: a plurality of magnetometer sensors, a plurality of inertial measurement units, a plurality of digital compasses, a plurality of magnetoresistive sensors, a plurality of magnetoinductive sensors, a plurality of Hall effect sensors, or a plurality of
15 superconducting quantum interference devices.

[0063] It should be appreciated that the sensor array 100, in accordance with various embodiments differs from the existing arrays, for example, having a hierarchical localization framework with a single (not multiple) IMU and a 2-D LiDAR sensor. The sensor array 100 is designed in a manner to build 3-D magnetic maps of large-scale
20 environments in a much faster and efficient way.

[0064] For example, the sensor array 100 may include a 3-D multiple magnetometer sensors array to measure the AMF data with high efficiency and accuracy. The efficiency of the AMF data collection by the multiple magnetometer sensors array is significantly improved comparing with the existing single magnetometer sensor / IMU based methods.

25 [0065] FIG. 3A and FIG. 3B respectively show a schematic representation and a schematic perspective view of the multiple magnetometer sensors array 300, according to exemplary embodiments. The multiple magnetometer sensors array 300 may be any form, such as cubic, circular, L-shaped, amongst others. In a cubic example embodiment, the array 300 may include multiple magnetometer sensors 302 at different heights and the exact number
30 of magnetometer sensors 302 on each height may be changeable based on the different requirements in different scenarios. The vertical height interval, each of $h_1 \dots h_{N-1}$, of

different magnetometer sensors 302 and the lateral interval, each of $d_1, d_2 \dots d_{N-1}$, between any two magnetometer sensors 302 on the same height may vary from different implementations but may be sufficiently long to disambiguate the AMF.

[0066] As shown in FIG. 3B, the multiple magnetometer sensors array 300 may be three-dimensional, and may include a plurality of sensor layers 303, the layers 303 being spaced apart by a predetermined height; and at least one magnetometer sensor 302 disposed in each layer 303. The multiple magnetometer sensors 302 in the array 300 may be configured to simultaneously measure the AMF and may be calibrated by the standard calibration procedures. The calibration parameters obtained may then be used in the processing of the measured rotation-invariant AMF vectors before transmitting to the mobile device and fusing with the corresponding coordinates data. In operation, each magnetometer sensor 302 measures the AMF at its corresponding location and all of the magnetometer sensors 302 in the array measure simultaneously. Other configurations/layouts of the multiple magnetometer sensors array, not shown in the figures, may be applicable.

[0067] FIG. 1B shows a schematic view of a system 120 for building a magnetic map and localizing a mobile device, in accordance with various embodiments. The system 120 may include the sensor array 100 configured to provide measurements of ambient magnetic fields in an environment; a mapping module 124 configured to receive the ambient magnetic fields and determine rotation-invariant ambient magnetic field vectors, based on the received ambient magnetic fields, to build a magnetic map representative of the environment; and a localization module 126. Details on how the rotation-invariant ambient magnetic field vectors may be determined will be described at the relevant section below. It should be appreciated that with the rotation-invariant ambient magnetic field vectors having good distinguishability, the system 120 may be configured to operate without extra radio source infrastructures (e.g., Wi-Fi, Bluetooth, active RFID) to accurately localize the mobile device.

[0068] The localization module 126 may be configured to: based on ambient magnetic fields measured over a period of time by a plurality of magnetic field sensors (not shown in FIG. 1B) releasably coupled with a mobile device (not shown in FIG. 1B), infer candidate initial poses of the mobile device from candidate locations in the magnetic map; based on the candidate initial poses and variations in the measured ambient magnetic fields

over the period of time, infer a fine pose of the mobile device; and evaluate a joint probabilistic distribution based on a probability distribution of the candidate initial poses and a probability distribution of the fine pose to estimate a specific pose of the mobile device. The specific pose may be of low or negligible error.

- 5 [0069] In some embodiments, the sensor array 100 may include the plurality of magnetic field sensors used by the localization module 126. The sensor array 100 may be in communication with the mapping module 124 and optionally, the localization module 126 as denoted by lines 128, 130. The mapping module 124 may be in communication with the localization module 126 as denoted by line 130.

10

Mapping Module

- [0070] In various embodiments, the mapping module 124 may be further configured to estimate coordinates for each of a plurality of locations in the environment; collect the rotation-invariant ambient magnetic field vectors at each location; determine, based on the
15 collected rotation-invariant ambient magnetic field vectors, an ambient magnetic field fingerprint for each location; and for more than one of the plurality of locations, select the respective ambient magnetic field fingerprints and associate each of the respective ambient magnetic field fingerprints with the corresponding coordinates.

- [0071] In the context of various embodiments, the phrase “rotation-invariant” may mean
20 being free from orientation errors.

- [0072] The system 120 may further include an auxiliary sensor in communication with the mapping module 124. The auxiliary sensor may be configured to provide measurements to establish the coordinates. For example, the auxiliary sensor may include a Global navigation satellite system (GNSS) / Global Positioning System (GPS) receiver, a Light
25 Detection and Ranging (LiDAR) system, or a camera.

- [0073] In various embodiments, the ambient magnetic field fingerprint may include a rotation-invariant ambient magnetic field fingerprint.

- [0074] The mapping module 124 may be further configured to determine the ambient magnetic field fingerprint for each location by averaging the collected rotation-invariant
30 ambient magnetic field vectors within the location.

[0075] In one embodiment, the mapping module 124 may be further configured to fill at least one vacancy between one of the collected rotation-invariant ambient magnetic field vectors and an adjacent collected rotation-invariant ambient magnetic field vector. The mapping module 124 may be further configured to fill the at least one vacancy by performing interpolation on the collected rotation-invariant ambient magnetic field vectors.

[0076] In another embodiment, the mapping module 124 may be further configured to fill at least one vacancy between one of the measured ambient magnetic fields and an adjacent measured ambient magnetic field. The mapping module 124 may be further configured to fill the at least one vacancy by performing interpolation on the measured ambient magnetic fields.

[0077] The filling of vacancies may be an interpolation process performed by the mapping module 124 to enhance the data of interest, e.g. the collected rotation-invariant ambient magnetic field vectors or the measured ambient magnetic fields.

[0078] In other words, the system 120 may be configured to systematically associate rotation-invariant AMF data collected by the sensor array 100 (e.g. the multiple magnetometer sensors array 300) with the corresponding coordinates data obtained by the auxiliary sensor, build the magnetic map, and to localize the mobile device or mobile devices. The magnetic map may be switched between two modes, namely, fast fingerprints-based magnetic map and accurate interpolation-based magnetic map.

[0079] FIG. 4 shows a flow diagram depicting a magnetic map building strategy 400 involving the features of a mapping module 424 working with the multiple magnetometer sensors array 300, according to one example. The mapping module 424 may be described in similar context to the mapping module 124 of FIG. 1B. The mapping module 424 and the multiple magnetometer sensors array 300 may include the same or like elements or components as those of the mapping module 124 of FIG. 1B and the sensor array 100 of FIGS. 1A, 1B, respectively, and as such, the similar ending numerals are assigned and the like elements may be as described in the context of the mapping module 124 of FIG. 1B and the sensor array 100 of FIGS. 1A, 1B, and therefore the corresponding descriptions may be omitted here.

[0080] Since the measurement range of the magnetometer sensor 302 may be narrow and it may not be feasible for the mobile device to survey every single location in the

environment, the rotation-invariant AMF vectors (that is, at least part of the AMF features 450) may be collected with the mobile device teleoperated on different desired survey routes. According to the speed of the mobile device platform and the capabilities of the magnetometer sensor(s) 302, the rotation-invariant AMF vectors 450 may be collected at a predetermined (e.g. every l -cm) interval along the desired mobile device survey routes. For one designated grid/cubic location, there may only be one AMF fingerprint 452 which may be derived by averaging over multiple measured AMF vectors 450 within this grid/cubic formed by the target environment. The 2-D grid or 3-D cubic coordinates data may be provided by the auxiliary sensor 454. After the collection of all the AMF fingerprints 452, a process may be implemented to select the AMF fingerprints 452 at different grid/cubic locations, thus building magnetic maps with different grid/ cubic sizes.

[0081] The aforementioned rotation-invariant AMF fingerprints 452 be directly utilized to constitute the magnetic map, which is called the fingerprints (FP)-based magnetic map 458a. Apart from the FP-based approach 456a, interpolation (IP)-based methods 456b (e.g. bilinear interpolation, Gaussian Process Regression, Kriging interpolation, amongst others) may also be implemented to fill the measurement vacancies between the desired mobile device survey routes to build a more complete and accurate IP-based magnetic map 458b. The FP-based approaches 456a may be the fastest and most intuitive way which takes the least time for building the magnetic map 458a, while the IP-based methods 456b may provide a more accurate magnetic map 458b with higher computational burden. The magnetic map building strategy 400 may be switched between the aforementioned two approaches 456a, 456b based on the actual requirements.

Localization Module

[0082] The magnetic map may be pre-built for localization purpose. In other words, the pre-built magnetic map may be built by either the mapping module 124 (or even by a different mapping process not described herein) and stored for future usage by the localization module 126. The localization module 126 may perform coarse-to-fine localization based on the pre-built magnetic map with the measured rotation-invariant AMF sequential data to localize the mobile device accurately and robustly. At the initial stage of localization, every single location in the magnetic map may be considered as a

candidate location (e.g. candidate locations may be any possible random locations in the magnetic map). The ambient magnetic field data measured by the plurality of magnetic field sensors constitutes a magnetic field sequence, which may be aligned and matched with the pre-built magnetic map. The measured magnetic field sequence and candidate magnetic field sequences stored in the magnetic map have substantially similar patterns. Euclidean distances may be calculated between the magnetic field values of the measured magnetic field sequence and the magnetic field values of the candidate magnetic field sequences stored in the magnetic map. A criterion may be set for this Euclidean distance to identify the candidate magnetic field sequences stored in the magnetic map, thus determining the eligible candidate initial poses/locations accordingly.

[0083] In one embodiment, the plurality of magnetic field sensors used by the localization module 126 may be the plurality of magnetic field sensors 102 of the sensor array 100. In another embodiment, the plurality of magnetic field sensors used by the localization module 126 may be independent/different from the plurality of magnetic field sensors 102 of the sensor array 100, in which case, the plurality of magnetic field sensors used by the localization module 126 may nevertheless be described in similar context to the plurality of magnetic field sensors 102.

[0084] In various embodiments, the localization module 162 may be configured to infer the candidate initial poses by calculating the probability distribution of the candidate initial poses based on a normalization constant for coarse localization, a prior distribution of the candidate initial poses and an observation model for coarse localization. The prior distribution of the candidate initial poses may be a uniform distribution.

[0085] The observation model for coarse localization may be denoted by $p(\mathbf{z}_0 | \mathbf{x}_0, \mathbf{M}^B)$, where

$$p(\mathbf{z}_0 | \mathbf{x}_0, \mathbf{M}^B) \propto \prod_{i=1}^N \frac{\exp\left(-\frac{1}{2}(\mathbf{B}_{\mathbf{O}_i}^0 - \mathbf{B}_{\mathbf{M}_{k,i}}^0)^T (\boldsymbol{\Sigma}_{\mathbf{x}_i^k}^0)^{-1} (\mathbf{B}_{\mathbf{O}_i}^0 - \mathbf{B}_{\mathbf{M}_{k,i}}^0)\right)}{(\sqrt{2\pi})^3 \sqrt{|\boldsymbol{\Sigma}_{\mathbf{x}_i^k}^0|}}$$

denoting Equation (1),

$\mathbf{z}_0$ denotes ambient magnetic field observations of the plurality of magnetic field sensors at initial timestep,

$\mathbf{x}_0$ denotes the candidate initial poses,

$\mathbf{M}^B$ denotes the magnetic map,

N denotes the number of magnetic field sensors utilized for localization,

$\mathbf{B}_{O_i}^0 = [B_{O_i}^{h,0}, B_{O_i}^{v,0}, F_{O_i}^0]^T$ and denotes measured ambient magnetic field data of the i^{th} magnetic field sensor,

$\mathbf{B}_{M_{k,i}}^0 = [B_{M_{k,i}}^{h,0}, B_{M_{k,i}}^{v,0}, F_{M_{k,i}}^0]^T$ and denotes corresponding ambient magnetic field data relative to the i^{th} magnetic field sensor at the k^{th} candidate location in the magnetic map, and

$\Sigma_{\mathbf{x}_i^0}^0$ is a diagonal covariance matrix of the measured ambient magnetic field data of the i^{th} magnetic field sensor at the k^{th} candidate location at the initial timestep, diagonal elements

of the diagonal covariance matrix being shown as variances $\sigma_{\mathbf{x}_i^k}^{h,0}$, $\sigma_{\mathbf{x}_i^k}^{v,0}$, and $\sigma_{\mathbf{x}_i^k}^{F,0}$.

[0086] In various embodiments, the system 120 may further include an assistant localization apparatus in communication with the localization module 126, wherein the assistant localization apparatus may be configured to expedite the inference of the candidate initial poses of the mobile device from the candidate locations in the magnetic map. For example, the assistant localization apparatus may include a laser-based localization apparatus, a vision-based localization apparatus, or a GNSS/GPS-based localization apparatus. The assistant localization apparatus may supplement measurement data, for example, to reduce a target area on the magnetic map that may have a higher probability where the specific pose may be, and thereby reduce the number of candidate initial poses required, and/or to estimate the orientation of the mobile device so as to eliminate or at least minimize the contribution of orientation errors.

[0087] The localization module 126 may be configured to infer the fine pose of the mobile device by calculating the probability distribution of the fine pose based on a normalization constant for fine localization, an observation model for fine localization, and a motion model for fine localization.

[0088] The observation model for fine localization may be derived from a multidimensional ambient magnetic field sequential matching module for aligning two time-series ambient magnetic field sequences with substantially similar patterns and amplitudes. The multidimensional ambient magnetic field sequential matching module may be configured to estimate an orientation of the mobile device; take rotation-invariant

ambient magnetic field vectors, determined from the measured ambient magnetic fields at predetermined time intervals over the period of time and the estimated orientation of the mobile device, as one of the two time-series ambient magnetic field sequences; derive, by the motion model for fine localization, the other of the two time-series ambient magnetic field sequences at the predetermined time intervals and at one of the candidate locations in the magnetic map; and derive the observation model for fine localization based on a multivariate multi-dimensional Gaussian distribution by utilizing the two time-series ambient magnetic field sequences and positive covariances of the two time-series ambient magnetic field sequences at the respective candidate location. The multidimensional ambient magnetic field sequential matching module may be configured to estimate the orientation of the mobile device by averaging yaw angles of the plurality of magnetic field sensors, wherein for each magnetic field sensor, the yaw angle represents an orientation of the magnetic field sensor that is inferred based on an ambient magnetic field vector. The ambient magnetic field vector may be adapted from the ambient magnetic field measured by the magnetic field sensor.

[0089] The motion model for fine localization may be based on a probability distribution utilizing the inferred fine pose, the candidate initial poses and a control input for fine localization.

[0090] The observation model for fine localization is denoted by $p(\mathbf{z}_t | \mathbf{x}_t, \mathbf{M}^B)$, where

$$p(\mathbf{z}_t | \mathbf{x}_t, \mathbf{M}^B) \propto \prod_{i=1}^N \frac{\exp\left(-\frac{1}{2}(\mathbf{B}_{\mathbf{O}_i} - \mathbf{B}_{\mathbf{M}_{k,i}})^T \Sigma_{\mathbf{x}_t}^{-1} (\mathbf{B}_{\mathbf{O}_i} - \mathbf{B}_{\mathbf{M}_{k,i}})\right)}{(\sqrt{2\pi})^3 \sqrt{|\Sigma_{\mathbf{x}_t}|}}$$

denoting Equation (2),

$\mathbf{z}_t$ denotes ambient magnetic field observations of the plurality of magnetic field sensors at timestep t ,

$\mathbf{x}_t$ denotes the specific pose of the mobile device at timestep t ,

$\mathbf{M}^B$ denotes the magnetic map,

N denotes the number of magnetic field sensors utilized for localization,

$\mathbf{B}_{0_i} = \{\mathbf{B}_{0_i}^{t-n}, \mathbf{B}_{0_i}^{t-n+1}, \dots, \mathbf{B}_{0_i}^t\}$ and denotes measured ambient magnetic field sequence of the i^{th} magnetic field sensor,

$\mathbf{B}_{0_i}^t = [B_{0_i}^{h,t}, B_{0_i}^{v,t}, F_{0_i}^t]^T$ and denotes rotation-invariant 3-D ambient magnetic field vector measured at time t ,

- 5 $\mathbf{B}_{\mathbf{M}_{k,i}} = \{\mathbf{B}_{\mathbf{M}_{k,i}}^{t-n}, \mathbf{B}_{\mathbf{M}_{k,i}}^{t-n+1}, \dots, \mathbf{B}_{\mathbf{M}_{k,i}}^t\}$ and denotes corresponding ambient magnetic field sequence derivated from the motion model of the i^{th} magnetic field sensor at the k^{th} candidate location for fine localization,

$\mathbf{B}_{\mathbf{M}_{k,i}}^t = [B_{\mathbf{M}_{k,i}}^{h,t}, B_{\mathbf{M}_{k,i}}^{v,t}, F_{\mathbf{M}_{k,i}}^t]^T$ and denotes corresponding rotation-invariant 3-D ambient magnetic field vector at time t ,

- 10 $\Sigma_{\mathbf{x}_i^k} = \{\Sigma_{\mathbf{x}_i^k}^{t-n}, \Sigma_{\mathbf{x}_i^k}^{t-n+1}, \dots, \Sigma_{\mathbf{x}_i^k}^t\}$ and is a sequence of diagonal covariance matrix of the measured ambient magnetic field of the i^{th} magnetic field sensor at the k^{th} candidate location, diagonal elements of the diagonal covariance matrix at time t being shown as variances $\sigma_{\mathbf{x}_i^k}^{h,t}$, $\sigma_{\mathbf{x}_i^k}^{v,t}$, and $\sigma_{\mathbf{x}_i^k}^{F,t}$.

[0091] The specific pose of the mobile device at timestep t may be estimated by:

$$\frac{\sum_{k=1}^R \left(\mathbf{x}_t^k \cdot p(\mathbf{x}_0^k | \mathbf{z}_0, \mathbf{M}^B) \cdot p(\mathbf{x}_t^k | \mathbf{x}_0^k, \mathbf{z}_{1:t}, \mathbf{u}_{1:t}, \mathbf{M}^B) \right)}{\sum_{k=1}^R \left(p(\mathbf{x}_0^k | \mathbf{z}_0, \mathbf{M}^B) \cdot p(\mathbf{x}_t^k | \mathbf{x}_0^k, \mathbf{z}_{1:t}, \mathbf{u}_{1:t}, \mathbf{M}^B) \right)}$$

15

denoting Equation (3),

where R includes selected candidate locations,

k corresponds to the k^{th} candidate location,

- 20 $\mathbf{z}_0$ denotes ambient magnetic field observations of the plurality of magnetic field sensors at initial timestep,

$\mathbf{z}_{1:t}$ denotes multidimensional ambient magnetic field sequential observations from timestep 1 to t ,

$\mathbf{x}_0^k$ denotes a candidate pose based on the k^{th} candidate location at the initial timestep,

- 25 $\mathbf{x}_t^k$ denotes a candidate pose based on the k^{th} candidate location at timestep t ,

$\mathbf{u}_{1:t}$ is a control input from timestep 1 to t , and

$\mathbf{M}^B$ is the magnetic map.

[0092] The localization module 126 may be further configured to calculate an estimated pose of the mobile device at timestep $t+1$ based on the specific pose at timestep t and data provided by the motion model for fine localization during the timestep t to the timestep $t+1$; select an area in the magnetic map based on the estimated pose; match, in terms of substantially similar patterns and amplitudes, a first time-series ambient magnetic field sequences corresponding to locations within the selected area and a second time-series ambient magnetic field sequences associated with rotation-invariant ambient magnetic field vectors determined from the measured ambient magnetic fields to identify candidate subsequent locations; and infer a subsequent pose based on the identified candidate subsequent locations, thereby enabling substantially continuous tracking of poses of the mobile device.

[0093] The subsequent pose of the mobile device at timestep $t+1$ may be estimated by:

$$\frac{\sum_{k=1}^{R'} \left(x_{t+1}^k \cdot p(x_t^k | z_t, M^B) \cdot p(x_{t+1}^k | x_t^k, z_{1:t+1}, u_{1:t+1}, M^B) \right)}{\sum_{k=1}^{R'} \left(p(x_t^k | z_t, M^B) \cdot p(x_{t+1}^k | x_t^k, z_{1:t+1}, u_{1:t+1}, M^B) \right)}$$

denoting Equation (4),

where R' includes the identified candidate subsequent locations,
 k corresponds to the k^{th} candidate subsequent location,
 z_t denotes ambient magnetic field observations of the plurality of magnetic field sensors at timestep t ,
 $z_{1:t+1}$ denotes multidimensional ambient magnetic field sequential observations from timestep 1 to $t+1$,
 x_t^k denotes a candidate pose based on the k^{th} candidate subsequent location at timestep t ,
 x_{t+1}^k denotes a candidate pose based on the k^{th} candidate subsequent location at timestep $t+1$,
 $u_{1:t+1}$ is a control input from timestep 1 to $t+1$, and
 M^B is the magnetic map.

[0094] Details relating to the localization module 126 will be described in more detail below, with reference to an exemplary localization framework 500 shown in a flow diagram of FIG. 5. The exemplary localization framework 500 may include the same or

like elements or components as those of the localization module 126 of FIG. 1B, and as such, the same ending numerals are assigned and the like elements may be as described in the context of the localization module 126 of FIG. 1B, and therefore the corresponding descriptions may be omitted here.

5 [0095] Commonly used AMF-based localization algorithm may have problems such as mismatches to similar AMF fingerprints and high computational overhead. Moreover, orientation-dependent characteristic of the 3-D AMF vector used in such AMF-based localization algorithm may be a troublesome limitation which significantly hinders the 3-D AMF vector-based localization applications. Therefore, considering the low
10 computational complexity and high accuracy characteristics of the rotation-invariant AMF sequential matching, a novel probabilistic localization framework (e.g. 500) is designed to solve or at least address the problem of mobile devices global localization. In this framework 500, assistant localization methods 572 (e.g. laser-based localization, vision-based localization, GNSS-based localization, amongst others) may be utilized to expedite
15 the localization process, thus improving the localization efficiency and accuracy.

[0096] The initial mobile device pose $\mathbf{x}_0$ may be introduced as the hidden state to enhance mobile device localization performance, which may be derived by the coarse localization
560 by leveraging the multiple AMF data-based initialization. Thus, the challenge of coarse-to-fine probabilistic localization may be formulated as the estimation of the joint
20 probabilistic distribution, given by:

$$p(\mathbf{x}_t, \mathbf{x}_0 | \mathbf{z}_0, \mathbf{z}_{1:t}, \mathbf{u}_{1:t}, \mathbf{M}^B) = \underbrace{p(\mathbf{x}_0 | \mathbf{z}_0, \mathbf{M}^B)}_{\text{Coarse}} \underbrace{p(\mathbf{x}_t | \mathbf{x}_0, \mathbf{z}_{1:t}, \mathbf{u}_{1:t}, \mathbf{M}^B)}_{\text{Fine}}$$

denoting Equation (5).

[0097] Assuming that each sensor observation is independent, the sensor observation may
25 be depicted as $\mathbf{z} = [\mathbf{z}_0, \mathbf{z}_{1:t}]^T$, where $\mathbf{z}_0$ and $\mathbf{z}_{1:t}$ denote the AMF observations of multiple magnetometer sensors of the coarse localization at timestep 0 and multi-dimensional AMF sequential observations of the fine localization from timestep 1 to t , respectively. $\mathbf{x}_0 \in \{\langle \mathbf{x}_0^1, \mathbf{x}_0^2, \dots, \mathbf{x}_0^K \rangle \mid k=1, \dots, K\}$ denotes the candidate initial poses and K denotes the total numbers of the selected candidates. $\mathbf{x}_t \in \{\langle \mathbf{x}_t^1, \mathbf{x}_t^2, \dots, \mathbf{x}_t^K \rangle \mid k=1, \dots, K\}$ denotes the fine pose
30 corresponds with $\mathbf{x}_0$. $\mathbf{u}$ is the control input of the fine localization submodule. $\mathbf{M}^B$ denotes

the pre-built magnetic map. The advantage of introducing x_0 comes at providing an accurate and efficient initialization for the localization.

Coarse Localization

- 5 [0098] The first term on the right-hand side of Equation (5) is the initial pose probability $p(x_0 | z_0, M^B)$. The objective of the coarse localization 560 is to infer the candidate initial pose x_0 , which may be derivated by following the Bayesian theorem as:

$$p(x_0 | z_0, M^B) = \eta_0 p(x_0) p(z_0 | x_0, M^B)$$

denoting Equation (6),

- 10 where η_0 is a normalization constant, term $p(x_0)$ and $p(z_0 | x_0, M^B)$ are the prior distribution and observation model of the coarse localization 560 submodule, respectively. [0099] The prior distribution of the candidate initial pose $p(x_0)$ may be assumed to be a uniform distribution. It may be assumed that N magnetometer sensors 502 are utilized for the localization tasks. Then, the rotation-invariant AMF data of multiple magnetometer
- 15 sensors 502 may be utilized in the observation model at timestep 0 for AMF sequence based initialization 564, where the measured AMF data of the i^{th} magnetometer sensor 502 is $\mathbf{B}_{O_i}^0 = [B_{O_i}^{h,0}, B_{O_i}^{v,0}, F_{O_i}^0]^T$ and the corresponding AMF data relative to the i^{th} magnetometer sensor 502 at the k^{th} candidate location in the magnetic map 458a, 458b is $\mathbf{B}_{M_{k,i}}^0 = [B_{M_{k,i}}^{h,0}, B_{M_{k,i}}^{v,0}, F_{M_{k,i}}^0]^T$. The observation model may be provided as in Equation (1) discussed
- 20 above. The obtained initial candidate poses may then be sent as input for the fine localization 562 submodule.

Fine Localization

- [0100] The second term on the right-hand side of Equation (5) is the fine pose inference. 25 The focus of the fine localization 562 is to estimate the accurate mobile device pose x_t by applying Bayesian theorem as:

$$p(x_t | x_0, z_{1:t}, u_{1:t}, M^B) = \eta_t p(z_t | x_t, x_0, z_{1:t-1}, u_{1:t}, M^B) p(x_t | x_0, z_{1:t-1}, u_{1:t}, M^B)$$

denoting Equation (7),

where η_t is another normalization constant. As Markov property may be applied, the 30 following equation may be obtained:

$$\eta_t p(z_t | x_t, x_0, z_{1:t-1}, u_{1:t}, M^B) p(x_t | x_0, z_{1:t-1}, u_{1:t}, M^B) = \eta_t p(z_t | x_t, M^B) p(x_t | x_0, u_{1:t})$$

denoting Equation (8),

where $p(z_t | x_t, M^B)$ and $p(x_t | x_0, u_{1:t})$ are the observation model and motion model 566 of the fine localization 562 submodule, respectively. A corresponding motion model may
 5 be employed for the specific mobile device. As for the observation model, it may be derivated with multi-dimensional AMF sequential matching as provided in the multi-dimensional AMF sequence based localization 568. Detail of the multi-dimensional AMF sequential matching will be discussed in the relevant section below.

10 (a) Orientation Estimation

[0101] Due to the magnetic field interferences and measurement noises of each single magnetometer sensor 502, it may not be sufficiently accurate to determine the orientation of the mobile device based on one single magnetometer sensor 502. Thus, the average value of the aforementioned yaw angle γ of each magnetometer sensor may be used to initially
 15 estimate the orientation of the mobile device. Based on this initial estimation, the subsequent multi-dimensional AMF sequential matching may be implemented with higher efficiency and the final matching results in return reduce the orientation estimation errors. Moreover, the orientation of the mobile device may also be measurable by auxiliary sensors (e.g., IMU, LiDAR, Camera), not shown in FIG. 5, to further reduce the errors and improve
 20 the estimation accuracy.

(b) Multi-dimensional AMF Sequential Matching

[0102] The rotational-invariant 3-D AMF data collected at certain time intervals may be regarded as time series data. It may be observed that when the mobile device visits similar
 25 places in the environment, the AMF data follow a similar pattern but with variations in AMF intensity due to slight changes in the coordinates. Moreover, since the mobile device may not follow a similar movement pattern with the previous magnetic map building phase (e.g. the mobile device is moving straightly when building the magnetic map, while the mobile device is moving in a zigzag/curve route when doing the localization), the
 30 traditional dynamic time warping (DTW)-based AMF sequential route matching will fail. Thus, the multi-dimensional AMF sequential matching is designed to align two time series

AMF sequences with substantially similar patterns and amplitude. It may also be assumed that N magnetometer sensors 502 are utilized for the localization tasks. In general, considering two multi-dimensional AMF sequences from timestep $t - n$ to t (i.e. $n + 1$ dimension): the measured AMF sequence of the i^{th} magnetometer sensor is $\mathbf{B}_{\mathbf{O}_i} =$
 5 $\{\mathbf{B}_{\mathbf{O}_i}^{t-n}, \mathbf{B}_{\mathbf{O}_i}^{t-n+1}, \dots, \mathbf{B}_{\mathbf{O}_i}^t\}$, where $\mathbf{B}_{\mathbf{O}_i}^t = [B_{\mathbf{O}_i}^{h,t}, B_{\mathbf{O}_i}^{v,t}, F_{\mathbf{O}_i}^t]^T$ denotes the rotation-invariant 3-D AMF vector measured at time t , and the sequence relative to the i^{th} magnetometer sensor 502 at the k^{th} candidate location derivated by the motion model 566 in the magnetic map is $\mathbf{B}_{\mathbf{M}_{k,i}} = \{\mathbf{B}_{\mathbf{M}_{k,i}}^{t-n}, \mathbf{B}_{\mathbf{M}_{k,i}}^{t-n+1}, \dots, \mathbf{B}_{\mathbf{M}_{k,i}}^t\}$, where $\mathbf{B}_{\mathbf{M}_{k,i}}^t = [B_{\mathbf{M}_{k,i}}^{h,t}, B_{\mathbf{M}_{k,i}}^{v,t}, F_{\mathbf{M}_{k,i}}^t]^T$. Thereafter, the observation model of the fine localization submodule may be derivated based on the
 10 multivariate multi-dimensional Gaussian distribution as provided in Equation (2) discussed above.

[0103] After performing the multi-dimensional AMF sequential matching, empirical criterions are set to eliminate false matches with large errors. Then, eligible matches are selected between the measured AMF sequence $\mathbf{B}_0 = \{\mathbf{B}_{\mathbf{O}_1}, \mathbf{B}_{\mathbf{O}_2}, \dots, \mathbf{B}_{\mathbf{O}_i} | i = 1, 2, \dots, N\}$
 15 and the chosen sequence at the k^{th} candidate location in the magnetic map 458a, 458b being $\mathbf{B}_{\mathbf{M}_k} = \{\mathbf{B}_{\mathbf{M}_{k,1}}, \mathbf{B}_{\mathbf{M}_{k,2}}, \dots, \mathbf{B}_{\mathbf{M}_{k,i}} | i = 1, 2, \dots, N\}$. Eventually, by combining the coarse localization (Equation (6)) and fine localization (Equation (7)), the accurate mobile device pose 570 defined in Equation (5) may be estimated by the R selected candidate locations as $\varepsilon[x_t]$ being provided in Equation (3) discussed above.

20 [0104] In a different aspect, various embodiments may provide a system 140 for building a magnetic map as shown in FIG. 1C. The system 140 may include the sensor array 100 configured to provide measurements of ambient magnetic fields in an environment; and the mapping module 124 configured to receive the ambient magnetic fields and determine rotation-invariant ambient magnetic field vectors, based on the received ambient magnetic
 25 fields, to build a magnetic map representative of the environment. The system 140 shares like features of the system 120 (FIG. 1B) with respect to the mapping module 124 and the corresponding descriptions are omitted here. It should be appreciated that the system 140 does not include the localization module 126 of the system 120.

[0105] In yet another different aspect, various embodiments may provide a system 160 for
 30 localizing a mobile device as shown in FIG. 1D. The system 160 may include a plurality

of magnetic field sensors 162 releasably coupled with a mobile device, wherein the plurality of magnetic field sensors 162 may be configured to measure ambient magnetic fields over a period of time; and a localization module 126. The localization module 126 may be configured to: based on the measured ambient magnetic fields, infer candidate
5 initial poses of the mobile device from candidate locations in a magnetic map; based on the candidate initial poses and variations in the measured ambient magnetic fields over the period of time, to infer a fine pose of the mobile device; and evaluate a joint probabilistic distribution based on a probability distribution of the candidate initial poses and a probability distribution of the fine pose to estimate a specific pose of the mobile device.
10 The system 160 shares like features of the system 120 (FIG. 1B) with respect to the localization module 126 and the corresponding descriptions are omitted here. It should be appreciated that the system 160 does not include the mapping module 124 of the system 120. In one example, the plurality of magnetic field sensors 162 used by the localization module 126 may be independent from, but described in similar context to, the plurality of
15 magnetic field sensors 102 of the sensor array 100 (FIG. 1A). In another example, the plurality of magnetic field sensors 162 used by the localization module 126 may be the plurality of magnetic field sensors 102. The plurality of magnetic field sensors 162 may be in communication with the localization module 126 as denoted by line 170.

20 ***Magnetic Map***

[0106] FIG. 1E shows a schematic view of a magnetic map 180, in accordance with various embodiments. The magnetic map 180 may be pre-built for localization purposes. The magnetic map 180 may include coordinates 181 established for each of a plurality of locations in the environment; and a rotation-invariant ambient magnetic field fingerprint
25 183 at each location. Each coordinate 181 and each rotation-invariant ambient magnetic field fingerprint 183 may be related to each other as denoted by line 185.

[0107] In one embodiment, the coordinates 181 may be 2-dimensional grid coordinates, and the magnetic map 180 may be a 2-dimensional magnetic map. In another embodiment, the coordinates 181 may be 3-dimensional cubic coordinates, and the magnetic map 180
30 may be a 3-dimensional magnetic map.

[0108] The magnetic map 180 may be built or pre-built using a method 220 (FIG. 2B) described below. The rotation-invariant ambient magnetic field fingerprint 183 may be determined based on rotation-invariant ambient magnetic field vectors being determined from ambient magnetic fields measured at the location.

5 [0109] In the context of various embodiments discussed herein, the rotation-invariant ambient magnetic field vectors may be determined using a method 200 (FIG. 2A) described below. The ambient magnetic fields may be measured by the sensor array 100 (FIG. 1A).

[0110] The magnetic map may be free from geometric limitations and may be implemented in almost any environment.

10 [0111] FIG. 6 shows an exemplary embodiment of the magnetic map 600. The magnetic map 600 may be described in similar context to the magnetic map 180 of FIG. 1E. The magnetic map 600, represented by $M^B = \{ f p_j = (\mathbf{p}_j, \mathbf{q}_j) \mid j=1, \dots, J \}$, is defined as a set of rotation-invariant AMF fingerprints (e.g. 183) $\mathbf{p}_j = [B_m^{j,h}, B_m^{j,v}, F_m^j]^T$ at each designated square grid/cubic location, i.e. 2-D grid coordinates $\mathbf{q}_j = [x_j, y_j]^T$ or 3-D cubic coordinates

15 $\mathbf{q}_j = [x_j, y_j, z_j]^T$ (e.g. 181) with specific size. Firstly, the target environment may be divided into a grid/cubic of side length k -centimeter(cm) and the rotation-invariant AMF vectors within each grid/cubic may be collected.

Rotation-invariant Ambient Magnetic Field Vector

20 [0112] FIG. 2A shows a flow chart illustrating a method 200 for determining a rotation-invariant ambient magnetic field vector to build a magnetic map (e.g. 180 of FIG. 1E) of an environment, according to various embodiments. In Step 201, an ambient magnetic field measured by a magnetic field sensor at a position within the environment or within a working distance of the magnetic field sensor from an edge boundary of the environment

25 may be adapted into an ambient magnetic field vector. The magnetic field sensor may be one or each of the magnetic field sensors 102 of the sensor array 100 (FIGS. 1A, 1B, 1C). In Step 203, based on the ambient magnetic field vector, an orientation of the magnetic field sensor may be inferred. In Step 205, based on the inferred orientation, the ambient magnetic field vector in a local coordinate system may be aligned to a global coordinate

30 system to generate the rotation-invariant ambient magnetic field vector. The rotation-

invariant ambient magnetic field vector may include two orthogonal components and an integrated intensity of the measured ambient magnetic field.

[0113] The rotation-invariant AMF data (vector) may be advantageously free from orientation errors and may take significantly less time to be collected than the 3-D orientation-dependent AMF data collected by the existing methods, for example, with the magnetic field database being defined with respect to the local coordinate frame with 3-D orientation-dependent MF vectors.

[0114] In general, the 3-D AMF $\mathbf{B}_e \in \mathbf{R}^3$ at the same location may be modeled as $\mathbf{B}_e = [B_e^x, B_e^y, B_e^z]^T$ within the global coordinate system (GCS), where the orthogonal components B_e^x 710, B_e^y 716, and B_e^z 712 represent magnetic field intensities in Geographic North, Geographic East and vertical downward directions (*i.e.*, North-East-Down (NED) coordinate system), respectively. The vertical component is in parallel with the unit vector $\mathbf{n}$ of the Earth gravity field, which may be detectable by the accelerometer of the mobile devices. If without strong magnetic field interference, the vector sum of B_e^x 710 and B_e^y 716 points to the *Magnetic North* B_{North} 718. The AMF integrated intensity, F_e 714, may be represented as $F_e = \sqrt{(B_e^x)^2 + (B_e^y)^2 + (B_e^z)^2}$, which is demonstrated in a graphical representation 700 of FIG. 7A.

[0115] In various embodiments, the measured ambient magnetic field may be dependent on the orientation of the magnetic sensor, and represented by:

$$W R_z(\gamma) R_y(\theta) R_x(\phi) \mathbf{B}_e + V$$

denoting Equation (9),

where $R_z(\gamma)$, $R_y(\theta)$, and $R_x(\phi)$ are corresponding rotation matrices in yaw, pitch, and roll, respectively;

W and V are a soft-iron effect matrix and a hard-iron effect offset vector of the magnetic field sensor, respectively; and

$\mathbf{B}_e = [B_e^x, B_e^y, B_e^z]^T$ within the global coordinate system, B_e^x , B_e^y , and B_e^z being orthogonal components representing magnetic field intensities in Geographic North, Geographic East and vertical downward directions, respectively, and B_e^z being parallel with a unit vector of the Earth's gravity field.

[0116] The measured ambient magnetic field may be denoted as B_m and interchangeably referred to as the calibrated AMF measured in the sensor/local coordinate system (*LCS*).

[0117] The AMF is directional, thus the measured x/y-axes AMF components B_m^x and B_m^y are highly dependent on the orientation of the magnetometer sensors (i.e. the orientation-dependent characteristic). In order to show this characteristic, AMF data may be measured at the same location of an environment free from magnetic field interference but with different orientations of the magnetometer sensor, where the results are shown in a graphical representation 730 of FIG. 7B. It may be observed that the AMF data measured at different orientations exhibit dramatic variations in the x/y-axes, while the magnetic field horizontal component B_m^h (i.e. radius 734 of the circle 732) remains stable.

[0118] Due to the aforementioned orientation-dependent characteristic of the AMF, the direct usage of 3-D AMF vector may be a challenging and intractable problem. Only after the heading direction is determined, the measured AMF in a magnetometer's *LCS* may be generalized to describe a vector measurement. However, in practical tasks, the magnetometer sensors (e.g. 302) of the magnetometer sensor array (e.g. 300) mounted on the mobile device 801 may head towards any direction with the yaw angle $\psi \in [0, 2\pi]$ as shown in a schematic diagram 800 of FIG. 8A, where the 0 degree is defined with respect to the direction of *Magnetic North*. The AMF data B_m is measured in the *LCS* and may be aligned in the unified *GCS*, which is shown in a schematic diagram 830 of FIG. 8B.

[0119] The ambient magnetic field vector may include three orthogonal components in the local coordinate system and the integrated intensity of the measured ambient magnetic

$$\begin{bmatrix} B_m^x \\ B_m^y \\ B_m^z \\ F_m \end{bmatrix}$$

field, represented by:

where B_m^x is a x-direction component; B_m^y is a y-direction component; B_m^z is a z-direction component; and $F_m = \sqrt{(B_m^x)^2 + (B_m^y)^2 + (B_m^z)^2}$.

[0120] Since the height is settled for both the mapping and localization phases, the $R_y(\theta)$ and $R_x(\phi)$ are both set to 1. Empirically, the values of B_m^x and B_m^y may be utilized to infer the mobile device orientation ψ in the environments free from strong MF interference.

[0121] The inferred orientation of the magnetic field sensor (e.g. 102) or a magnetometer sensor (e.g. 302) may be represented by a yaw angle, ψ , where

$$\psi = \begin{cases} 2\pi - \alpha, & \text{if } B_m^x > 0, B_m^y > 0 \\ -\alpha, & \text{if } B_m^x > 0, B_m^y < 0 \\ \pi - \alpha, & \text{if } B_m^x < 0, B_m^y > 0 \\ \pi + \alpha, & \text{if } B_m^x < 0, B_m^y < 0 \end{cases}$$

denoting Equation (10),

$\alpha = \arctan(|B_m^y| / |B_m^x|)$ denoting Equation (11), and ψ ranges between 0 radian and 2π radian, the 0 radian being defined with respect to a direction of Magnetic North.

[0122] Since the calibration has been done for every magnetometer sensor, transformed rotation-invariant AMF vector $\mathbf{B}_m^e$ in the *GCS* may be depicted respectively as:

$$\begin{aligned} \mathbf{B}_m^e &= R_z^\top(\psi) \mathbf{B}_m \\ &= \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 & 0 \\ -\sin(\psi) & \cos(\psi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} B_m^x \\ B_m^y \\ B_m^z \\ F_m \end{bmatrix} = \begin{bmatrix} B_m^h \\ 0 \\ B_m^v \\ F_m \end{bmatrix} \end{aligned}$$

denoting Equation (12).

[0123] Eventually, the transformed AMF data in the *GCS* may be denoted as: $\mathbf{B}_m^e = [B_m^h, B_m^v, F_m]^T$ denoting Equation (13). In other words, one of the two orthogonal components of the rotation-invariant ambient magnetic field vector may be a vertical component parallel with a unit vector $\mathbf{n}$ of the Earth's gravity field, and the other may be a rotation-invariant horizontal component orthogonal to the vertical component. The rotation-invariant horizontal component, B_m^h may be $\sqrt{(B_m^x)^2 + (B_m^y)^2}$ denoting Equation (13), and the vertical component, B_m^v may be B_m^z . It should be noted that the rotation-invariant AMF vector incorporates the crucial rotation-invariant horizontal component B_m^h to provide a high level of uniqueness disambiguating the similar magnetic field features.

[0124] The AMF integrated intensity F_m may be included to further improve the spatial locality. The AMF vector represented in the *GCS* may be stationary when the mobile device rotates without any translational movement.

5 ***Method for Building a Magnetic Map***

[0125] Various embodiments may further provide a method 220 for building a magnetic map (e.g. 180) representative of an environment, as shown in a flow diagram of FIG. 2B. This flexible magnetic map building method 220 is introduced, where the method 220 may be switched between two modes, i.e. fingerprints-based extremely fast mode and
10 interpolation-based highly accurate mode. In Step 221, coordinates (e.g. 181) for each of a plurality of locations in the environment may be established. In Step 223, rotation-invariant ambient magnetic field vectors at each location may be collected. The rotation-invariant ambient magnetic field vectors may be determined from ambient magnetic fields measured at the location. In one example, each location may be a grid location, and the coordinates
15 may be 2-D grid coordinates. In another example, each location may be a cubic location, and the coordinates may be 3-D cubic coordinates. In Step 225, based on the collected rotation-invariant ambient magnetic field vectors, an ambient magnetic field fingerprint for each location may be determined. In Step 227, for more than one of the plurality of locations, the respective ambient magnetic field fingerprints may be selected and each of
20 the respective ambient magnetic field fingerprints may be associated with the corresponding coordinates.

[0126] The systems 120, 140 with respect to the mapping module 124 may perform the method 220. The method 220 may include the same or like elements or components as those of the systems 120, 140 of FIGS. 1B and 1C, respectively, and as such, the same
25 ending numerals are assigned and the like elements may be as described in the context of the systems 120, 140 of FIGS. 1B and 1C, and therefore the corresponding descriptions are omitted here.

Method for localizing a Mobile Device

30 [0127] Various embodiments may also provide a method 240 for localizing a mobile device, as shown in a flow diagram of FIG. 2C. This localization method 240 that matches

the measured rotation-invariant AMF sequential data with the pre-built magnetic map (e.g. 180) demonstrates significantly improved localization performances as compared to the existing methods. It should be appreciated that only a magnetic map (and no other types of maps) is required for the localization process. In Step 241, coarse localization may be performed, based on ambient magnetic fields, to infer candidate initial poses of the mobile device from candidate locations in a magnetic map. The ambient magnetic fields may be measured over a period of time by a plurality of magnetic field sensors (e.g. 162 of FIG. 1D) releasably coupled with the mobile device. In Step 243, fine localization may be performed, based on the candidate initial poses and variations in the measured ambient magnetic fields over the period of time, to infer a fine pose of the mobile device. In Step 245, a joint probabilistic distribution may be evaluated based on a probability distribution of the candidate initial poses and a probability distribution of the fine pose to estimate a specific pose of the mobile device.

[0128] In various embodiments, the method 240 may further include applying an assistant localization method to the step of performing coarse localization to expedite the inference of the candidate initial poses of the mobile device from the candidate locations in the magnetic map. The assistant localization method may include, for example, a laser-based localization method, a vision-based localization method, or a GNSS/GPS-based localization method.

[0129] The method 240 may further include performing substantially continuous tracking of poses of the mobile device by recursively inferring subsequent poses based on candidate subsequent locations. One candidate subsequent location may be identified by calculating an estimated pose of the mobile device at timestep $t+1$ based on the specific pose at timestep t and data provided by a motion model for fine localization during the timestep t to the timestep $t+1$; selecting an area in the magnetic map based on the estimated pose; and matching, in terms of substantially similar patterns and amplitudes, a first time-series ambient magnetic field sequences corresponding to locations within the selected area and a second time-series ambient magnetic field sequences associated with the rotation-invariant ambient magnetic field vectors determined from the measured ambient magnetic fields to identify the candidate subsequent location.

[0130] The systems 120, 160 with respect to the localization module 126 may perform the method 240. The method 240 may include the same or like elements or components as those of the systems 120, 160 of FIGS. 1B and 1D, respectively, and as such, the same ending numerals are assigned and the like elements may be as described in the context of the systems 120, 160 of FIGS. 1B and 1D, and therefore the corresponding descriptions are omitted here.

[0131] While the method described above is illustrated and described as a series of steps or events, it will be appreciated that any ordering of such steps or events are not to be interpreted in a limiting sense. For example, some steps may occur in different orders and/or concurrently with other steps or events apart from those illustrated and/or described herein. In addition, not all illustrated steps may be required to implement one or more aspects or embodiments described herein. Also, one or more of the steps depicted herein may be carried out in one or more separate acts and/or phases.

[0132] Various embodiments may additionally provide a computer readable storage medium including computer readable instructions operable when executed by a computer to perform at least one of the following: determine a rotation-invariant ambient magnetic field vector to build a magnetic map of an environment, the computer readable instructions configured to perform the method 200; to build a magnetic map representative of an environment, the computer readable instructions configured to perform the method 220; or to localize a mobile device, the computer readable instructions configured to perform the method 240.

Simulations and Experiments

[0133] Examples of simulations and real-world experiments to showcase the feasibility and utility of the array of multiple magnetometer sensors to measure the ambient magnetic field (AMF) data and build magnetic map will be described below.

[0134] FIG. 9A shows a perspective view illustrating the multiple magnetometer sensors array 900a installed on a simulated mobile device 905a having seven magnetometers in single-line shape. FIG. 9B shows a photograph of the multiple magnetometer sensors array 900b installed on a real mobile device 905b having three magnetometers in single-line shape. A laptop 903 is provided to the real mobile device 905b to at least receive and/or

manipulate data from the multiple magnetometer sensors array 900b. The lateral distance between any two neighboring magnetometers was empirically set at 0.5m to balance between the magnetic mapping accuracy and efficiency.

[0135] FIGS. 10A and 10B respectively show the interpolation (IP)-based magnetic map 1001 of a simulated industrial warehouse environment (area being about 85m x 75m) and the interpolation (IP)-based magnetic map 1003 of a real office corridor environment (area being about 112m x 21m) built by the magnetic map building method (e.g. as described based on FIG. 2B). The AMF data was collected by the multiple magnetometer sensors array 300 (FIG. 3A). Within each fused map, the shade depicts the magnetic field integrated intensity based on the legend provided in FIGS. 10A and 10B.

[0136] Other exemplary embodiments of the multiple magnetometer sensors array may be as shown in FIGS. 11A to 11F. More specifically, FIGS. 11A, 11B, and 11C show perspective views illustrating configurations of the multiple magnetometer sensors arrays with bar lengths of 0.5m, 1.0m, and 1.5m, respectively, having three, five, and seven magnetometer sensors in single-line shape to respectively give a 1x3 array, a 1x5 array, a 1x7 array at each of the abovementioned bar lengths. FIGS. 11D, 11E, and 11F show top views illustrating configurations of the multiple magnetometer sensors arrays with bar length of 1.0m having nine, fifteen, and twenty-one magnetometer sensors in multiple-lines shape to respectively give a 3x3 array, a 3x5 array, a 3x7 array.

[0137] Examples of extensive simulations and real-world experiments to showcase the robustness and accuracy of the AMF-based localization will be described below.

[0138] FIG. 12A shows a perspective view illustrating a simulated mobile device 1205a in Gazebo simulator with single magnetometer 1202a and 2-D LiDAR 1207a configuration for mobile device online localization. FIG. 12B shows a perspective view illustrating a real mobile device 1205b with single magnetometer 1202b (VN-100T IMU) and Hokuyo 2-D LiDAR 1207b configuration for mobile device online localization. A laptop 1203 is provided to the real mobile device 1205b to at least receive and/or manipulate data from the single magnetometer 1202b. The AMF data collected by the single magnetometer sensor 1202a, 1202b with a designed sliding mechanism (e.g. 1209) simulates the AMF data collected by multiple magnetometer sensors.

[0139] FIGS. 13A and 13B respectively show the fused interpolation (IP)-based magnetic maps and 2-D occupancy grid maps (OGMs) (left side of FIG. 13A) of the simulated industrial warehouse environment (area being about 85m x 75m) (right side of FIG. 13A), and the fused interpolation (IP)-based magnetic maps and 2-D occupancy grid maps (OGMs) (bottom part of FIG. 13B) of the real office corridor environment (area being about 112m x 21m) (top part of FIG. 13B). The real office corridor may have similar shapes with the paths in the container seaports or industrial warehouses, which may serve as an adequate test site for developing the localization technology described herein. In each of the fused maps, the interpolation-based magnetic maps are overlapped with the traversable area of the 2-D occupancy grid maps (OGMs). These traversable areas are the areas of interest to localize the mobile device(s) where the mobile device(s) may operate. Within each fused map, the shade depicts the magnetic field integrated intensity based on the respective legends provided in FIGS. 13A and 13B. Black dots denote randomly-selected waypoints (WPs, i.e. global localization starting locations) and black lines near the WPs denote the ground-truth trajectories of the mobile device.

[0140] As shown in FIGS. 13A and 13B, extensive simulations and real-world experiments were carried out. Twenty waypoints (WPs) were randomly selected in each environment to do the global localization performance evaluation. For each evaluated approach of each WP, ten individual random tests were conducted.

[0141] The localization system 160 (or the localization module 126) or the localization method (e.g. 240) were quantitatively analyzed by comparing with six state-of-the-art magnetic field-based localization and tracking algorithms, namely i) MSTSL 1421 being multi-sensor based two-step localization; ii) k-NN 1423 being the well-known k-NN classification method; iii) LocateMe 1425 being the original dynamic time warping (DTW) algorithm with 1-D magnetic field magnitude; iv) MagSequence 1427 being the DTW algorithm with 3-D magnetic field vectors; v) 1-D PF 1431 being particle filtering (PF)-based 1-D magnetic field magnitude localization; vi) 3-D PF 1433 being PF-based 3-D magnetic field vector localization.

[0142] The global localization performance of each comparative method was evaluated by the robustness and accuracy criteria. First, the robustness was assessed by the Correct Localization Rate (CLR), which denotes the ratio of successful cases over the total number

of localization cases. A localization error threshold of 3.0m was empirically defined to determine whether the localization in each WP is successful or failed. In particular, the localization case is considered successful if its mean squared error is less than 3.0m, and localization case is considered failed if its mean squared error is equal to or more than 3.0m. Besides, the Cumulative Distribution Function (CDF) of the mean squared translational error was used to evaluate the overall localization accuracy of the mobile device. Specifically, root-mean-square (RMS) of the translational error was adopted to evaluate the detailed mobile device fine localization (i.e., pose tracking) accuracy.

[0143] FIGS. 14A and 14B are the histograms 1401, 1403 illustrating the localization robustness for the simulated industrial warehouse environment and the real office corridor environment, respectively, assessed by the Correct Localization Rate (CLR). The method (e.g. 240) performed by the localization system (e.g. 120, 160) has achieved excellent localization success rates of 100% for the simulated industrial warehouse environment and 95% for the real office corridor, thereby proving the robustness of the method. FIGS. 14C and 14D illustrate the CDF plots 1405, 1407 of the overall localization error for the simulated industrial warehouse environment and the real office corridor environment, respectively. For the localization error that is within 2.0m, the method (e.g. 240) performed by the localization system (e.g. 160) or the localization module 126 yields a more than 90 percentile CDF reflecting high accuracy, and is relatively advantageous over the aforementioned comparative state-of-the-art methods i) to vi).

[0144] The fine localization (i.e., pose tracking) accuracy of the method (e.g. 240) were specifically evaluated. FIGS. 15A to 15C show graphs 1501, 1503, 1505 depicting the root-mean-square (RMS) of the positional error in the fine localization (i.e., pose tracking) phase for three randomly-selected WPs in the warehouse environment, respectively. FIGS. 15D to 15F show graphs 1507, 1509, 1511 depicting the root-mean-square (RMS) of the positional error in the fine localization (i.e., pose tracking) phase for three randomly-selected WPs in the corridor environment, respectively. For simplicity and better visualization of the graphs, the method having RMS error of every single tracking point being more than the 5-metre pre-defined threshold is omitted and marked with a × in the legend of each of FIGS. 15A to 15F. As shown in FIGS. 15A to 15F, it is easily observed that the localization system 160, in accordance with various embodiments, displays the

most accurate and robust pose tracking performance, where the average positional error is less than 0.1m in the warehouse environment and less than 0.3m in the corridor environment, respectively.

[0145] In summary, the simulations and real-world experiments, that were conducted, have exhibited the feasibility and utility of the multiple magnetometer sensors array (e.g. 300) to measure the ambient magnetic field (AMF) data and build magnetic map. Moreover, extensive simulations and real-world experiments were carried out to prove the robustness and accuracy of the AMF-based localization (e.g. 240).

10 ***Commercial Applications***

[0146] As mentioned herein, the ambient magnetic field (AMF) has been gradually utilized in a wide range of applications due to its infrastructure-free, low cost, and ease of deployment characteristics. The AMF-based mapping and localization systems according to various embodiments as described herein are of high practical value and may provide significant help in various related commercial applications. Such commercial applications may include, but is not limited to:

- warehouse autonomous robots / automated guided vehicle (AGV) / unmanned ground vehicle (UGV) localization and navigation;
- container seaports automation;
- 20 • intelligent service robots / security robots operating in the hotels / hospitals / transportation centers / airports
- multi-robots / multi-AGVs / multi-UGVs collaboration tasks

[0147] While the invention has been particularly shown and described with reference to specific embodiments, it should be understood by those skilled in the art that various changes in form and detail may be made therein without departing from the spirit and scope of the invention as defined by the appended claims. The scope of the invention is thus indicated by the appended claims and all changes which come within the meaning and range of equivalency of the claims are therefore intended to be embraced.

CLAIMS

1. A sensor array comprising:
a plurality of magnetic field sensors configured to operate substantially
5 simultaneously for building a magnetic map representative of an environment, wherein
each magnetic field sensor is configured to measure an ambient magnetic
field at a position of the magnetic field sensor, the position being within the
environment or within a working distance of the magnetic field sensor from an edge
boundary of the environment; and
10 each magnetic field sensor is arranged spaced apart from a neighbouring
magnetic field sensor at a distance such that the respective ambient magnetic fields
measured by the magnetic field sensor and the neighbouring magnetic field sensor
are disambiguated.
- 15 2. The sensor array as claimed in claim 1, wherein the plurality of magnetic field
sensors comprises at least one of the followings:
a plurality of magnetometer sensors,
a plurality of inertial measurement units,
a plurality of digital compasses,
20 a plurality of magnetoresistive sensors,
a plurality of magnetoinductive sensors,
a plurality of Hall effect sensors, or
a plurality of superconducting quantum interference devices.
- 25 3. The sensor array as claimed in claim 1 or 2, wherein the plurality of magnetic field
sensors is arranged at substantially regular spatial intervals.
4. The sensor array as claimed in any one of claims 1 to 3, wherein the sensor array is
a q-dimensional sensor array, where q is 1, 2 or 3.

30

5. The sensor array as claimed in any one of claims 1 to 4, wherein the plurality of magnetic field sensors is calibrated to obtain calibration parameters to facilitate building the magnetic map.
- 5 6. The sensor array as claimed in any one of claims 1 to 5, being releasably couplable with a mobile device, the mobile device being movable to the position.
7. A system comprising:
a sensor array configured to provide measurements of ambient magnetic fields in
10 an environment;
a mapping module configured to receive the ambient magnetic fields and determine rotation-invariant ambient magnetic field vectors, based on the received ambient magnetic fields, to build a magnetic map representative of the environment; and
a localization module configured to
15 based on ambient magnetic fields measured over a period of time by a plurality of magnetic field sensors releasably coupled with a mobile device, infer candidate initial poses of the mobile device from candidate locations in the magnetic map,
based on the candidate initial poses and variations in the measured ambient
20 magnetic fields over the period of time, infer a fine pose of the mobile device, and
evaluate a joint probabilistic distribution based on a probability distribution of the candidate initial poses and a probability distribution of the fine pose to estimate a specific pose of the mobile device.
- 25 8. The system as claimed in claim 7, wherein the sensor array comprises the plurality of magnetic field sensors.
9. The system as claimed in claim 7 or 8, wherein the mapping module is further configured to:
30 estimate coordinates for each of a plurality of locations in the environment;
collect the rotation-invariant ambient magnetic field vectors at each location;

determine, based on the collected rotation-invariant ambient magnetic field vectors, an ambient magnetic field fingerprint for each location; and

for more than one of the plurality of locations, select the respective ambient magnetic field fingerprints and associate each of the respective ambient magnetic field fingerprints with the corresponding coordinates.

10. The system as claimed in claim 9, wherein each location is either:
a grid location, and the coordinates are 2-dimensional grid coordinates; or
a cubic location, and the coordinates are 3-dimensional cubic coordinates.

11. The system as claimed in claim 9 or 10, further comprising an auxiliary sensor in communication with the mapping module, the auxiliary sensor configured to provide measurements to establish the coordinates.

12. The system as claimed in claim 11, wherein the auxiliary sensor comprises a GNSS/GPS receiver, a LiDAR system, or a camera.

13. The system as claimed in any one of claims 9 to 12, wherein the ambient magnetic field fingerprint comprises a rotation-invariant ambient magnetic field fingerprint.

14. The system as claimed in any one of claims 9 to 13, wherein the mapping module is further configured to determine the ambient magnetic field fingerprint for each location by averaging the collected rotation-invariant ambient magnetic field vectors within the location.

15. The system as claimed in any one of claims 9 to 14, wherein the mapping module is further configured to fill at least one vacancy between one of the collected rotation-invariant ambient magnetic field vectors and an adjacent collected rotation-invariant ambient magnetic field vector.

16. The system as claimed in claim 15, wherein the mapping module is further configured to fill the at least one vacancy by performing interpolation on the collected rotation-invariant ambient magnetic field vectors.

5 17. The system as claimed in any one of claims 9 to 14, wherein the mapping module is further configured to fill at least one vacancy between one of the measured ambient magnetic fields and an adjacent measured ambient magnetic field.

18. The system as claimed in claim 17, wherein the mapping module is further
10 configured to fill the at least one vacancy by performing interpolation on the measured ambient magnetic fields.

19. The system as claimed in any one of claims 7 to 18, wherein the localization module is configured to infer the candidate initial poses by calculating the probability distribution
15 of the candidate initial poses based on a normalization constant for coarse localization, a prior distribution of the candidate initial poses and an observation model for coarse localization.

20. The system as claimed in claim 19, wherein the prior distribution of the candidate
20 initial poses is a uniform distribution.

21. The system as claimed in claim 19 or 20, wherein the observation model for coarse

localization is denoted by $p(\mathbf{z}_0|\mathbf{x}_0, \mathbf{M}^B)$, where

$$p(\mathbf{z}_0|\mathbf{x}_0, \mathbf{M}^B) \propto \prod_{i=1}^N \frac{\exp\left(-\frac{1}{2}(\mathbf{B}_{\mathbf{O}_i}^0 - \mathbf{B}_{\mathbf{M}_{k,i}}^0)^T (\boldsymbol{\Sigma}_{\mathbf{x}_i^k}^0)^{-1} (\mathbf{B}_{\mathbf{O}_i}^0 - \mathbf{B}_{\mathbf{M}_{k,i}}^0)\right)}{(\sqrt{2\pi})^3 \sqrt{|\boldsymbol{\Sigma}_{\mathbf{x}_i^k}^0|}},$$

25 $\mathbf{z}_0$ denotes ambient magnetic field observations of the plurality of magnetic field sensors at initial timestep,

$\mathbf{x}_0$ denotes the candidate initial poses,

$\mathbf{M}^B$ denotes the magnetic map,

N denotes the number of magnetic field sensors utilized for localization,

$\mathbf{B}_{O_i}^0 = [B_{O_i}^{h,0}, B_{O_i}^{v,0}, F_{O_i}^0]^T$ and denotes measured ambient magnetic field data of the i^{th} magnetic field sensor,

5 $\mathbf{B}_{M_{k,i}}^0 = [B_{M_{k,i}}^{h,0}, B_{M_{k,i}}^{v,0}, F_{M_{k,i}}^0]^T$ and denotes corresponding ambient magnetic field data relative to the i^{th} magnetic field sensor at the k^{th} candidate location in the magnetic map, and $\Sigma_{\mathbf{x}_i^k}^0$ is a diagonal covariance matrix of the measured ambient magnetic field data of the i^{th} magnetic field sensor at the k^{th} candidate location at the initial timestep, diagonal elements of the diagonal covariance matrix being shown as variances $\sigma_{\mathbf{x}_i^k}^{h,0}$, $\sigma_{\mathbf{x}_i^k}^{v,0}$, and $\sigma_{\mathbf{x}_i^k}^{F,0}$.

10 22. The system as claimed in any one of claims 7 to 21, further comprising an assistant localization apparatus in communication with the localization module, wherein the assistant localization apparatus is configured to expedite the inference of the candidate initial poses of the mobile device from the candidate locations in the magnetic map.

15 23. The system as claimed in claim 22, wherein the assistant localization apparatus comprises a laser-based localization apparatus, a vision-based localization apparatus, or a GNSS/GPS-based localization apparatus.

20 24. The system as claimed in any one of claims 7 to 23, wherein the localization module is configured to infer the fine pose of the mobile device by calculating the probability distribution of the fine pose based on a normalization constant for fine localization, an observation model for fine localization, and a motion model for fine localization.

25 25. The system as claimed in claim 24, wherein the observation model for fine localization is derived from a multidimensional ambient magnetic field sequential matching module for aligning two time-series ambient magnetic field sequences with substantially similar patterns and amplitudes.

30 26. The system as claimed in claim 25, wherein the multidimensional ambient magnetic field sequential matching module is configured to:

estimate an orientation of the mobile device,

take rotation-invariant ambient magnetic field vectors, determined from the measured ambient magnetic fields at predetermined time intervals over the period of time and the estimated orientation of the mobile device, as one of the two time-series ambient magnetic field sequences;

derive, by the motion model for fine localization, the other of the two time-series ambient magnetic field sequences at the predetermined time intervals and at one of the candidate locations in the magnetic map, and

derive the observation model for fine localization based on a multivariate multi-dimensional Gaussian distribution by utilizing the two time-series ambient magnetic field sequences and positive covariances of the two time-series ambient magnetic field sequences at the respective candidate location.

27. The system as claimed in claim 26, wherein the multidimensional ambient magnetic field sequential matching module is configured to estimate the orientation of the mobile device by averaging yaw angles of the plurality of magnetic field sensors,

wherein for each magnetic field sensor, the yaw angle represents an orientation of the magnetic field sensor that is inferred based on an ambient magnetic field vector, the ambient magnetic field vector being adapted from the ambient magnetic field measured by the magnetic field sensor.

28. The system as claimed in any one of claims 24 to 27, wherein the motion model for fine localization is based on a probability distribution utilizing the inferred fine pose, the candidate initial poses and a control input for fine localization.

29. The system as claimed in any one of claims 24 to 28, wherein the observation model for fine localization is denoted by

$$p(\mathbf{z}_t | \mathbf{x}_t, \mathbf{M}^B)$$

, where

$$p(\mathbf{z}_t | \mathbf{x}_t, \mathbf{M}^B) \propto \prod_{i=1}^N \frac{\exp\left(-\frac{1}{2}(\mathbf{B}_{\mathbf{O}_i} - \mathbf{B}_{\mathbf{M}_{k,i}})^T \Sigma_{\mathbf{x}_i^k}^{-1} (\mathbf{B}_{\mathbf{O}_i} - \mathbf{B}_{\mathbf{M}_{k,i}})\right)}{(\sqrt{2\pi})^3 \sqrt{|\Sigma_{\mathbf{x}_i^k}|}}$$

$\mathbf{z}_t$ denotes ambient magnetic field observations of the plurality of magnetic field sensors at timestep t ,

$\mathbf{x}_t$ denotes the specific pose of the mobile device at timestep t ,

5 $\mathbf{M}^B$ denotes the magnetic map,

N denotes the number of magnetic field sensors utilized for localization,

$\mathbf{B}_{\mathbf{O}_i} = \{\mathbf{B}_{\mathbf{O}_i}^{t-n}, \mathbf{B}_{\mathbf{O}_i}^{t-n+1}, \dots, \mathbf{B}_{\mathbf{O}_i}^t\}$ and denotes measured ambient magnetic field sequence of the i^{th} magnetic field sensor,

$\mathbf{B}_{\mathbf{O}_i}^t = [B_{\mathbf{O}_i}^{h,t}, B_{\mathbf{O}_i}^{v,t}, F_{\mathbf{O}_i}^t]^T$ and denotes rotation-invariant 3-D ambient magnetic field vector
10 measured at time t ,

$\mathbf{B}_{\mathbf{M}_{k,i}} = \{\mathbf{B}_{\mathbf{M}_{k,i}}^{t-n}, \mathbf{B}_{\mathbf{M}_{k,i}}^{t-n+1}, \dots, \mathbf{B}_{\mathbf{M}_{k,i}}^t\}$ and denotes corresponding ambient magnetic field sequence derivated from the motion model of the i^{th} magnetic field sensor at the k^{th} candidate location for fine localization,

$\mathbf{B}_{\mathbf{M}_{k,i}}^t = [B_{\mathbf{M}_{k,i}}^{h,t}, B_{\mathbf{M}_{k,i}}^{v,t}, F_{\mathbf{M}_{k,i}}^t]^T$ and denotes corresponding rotation-invariant 3-D ambient
15 magnetic field vector at time t ,

$\Sigma_{\mathbf{x}_i^k} = \{\Sigma_{\mathbf{x}_i^k}^{t-n}, \Sigma_{\mathbf{x}_i^k}^{t-n+1}, \dots, \Sigma_{\mathbf{x}_i^k}^t\}$ and is a sequence of diagonal covariance matrix of the measured ambient magnetic field of the i^{th} magnetic field sensor at the k^{th} candidate location, diagonal elements of the diagonal covariance matrix at time t being shown as variances $\sigma_{\mathbf{x}_i^k}^{h,t}$, $\sigma_{\mathbf{x}_i^k}^{v,t}$, and $\sigma_{\mathbf{x}_i^k}^{F,t}$.

20

30. The system as claimed in any one of claims 24 to 29, wherein the localization module is further configured to:

calculate an estimated pose of the mobile device at timestep $t+1$ based on the specific pose at timestep t and data provided by the motion model for fine localization
25 during the timestep t to the timestep $t+1$;

select an area in the magnetic map based on the estimated pose;

match, in terms of substantially similar patterns and amplitudes, a first time-series ambient magnetic field sequences corresponding to locations within the selected area and

a second time-series ambient magnetic field sequences associated with rotation-invariant ambient magnetic field vectors determined from the measured ambient magnetic fields to identify candidate subsequent locations; and

- infer a subsequent pose based on the identified candidate subsequent locations,
 5 thereby enabling substantially continuous tracking of poses of the mobile device.

31. The system as claimed in claim 30, wherein the subsequent pose of the mobile device at timestep $t+1$ is estimated by:

$$\frac{\sum_{k=1}^{R'} \left(x_{t+1}^k \cdot p(x_t^k | z_t, M^B) \cdot p(x_{t+1}^k | x_t^k, z_{1:t+1}, u_{1:t+1}, M^B) \right)}{\sum_{k=1}^{R'} \left(p(x_t^k | z_t, M^B) \cdot p(x_{t+1}^k | x_t^k, z_{1:t+1}, u_{1:t+1}, M^B) \right)}$$

- 10 where R' comprises the identified candidate subsequent locations,

k corresponds to the k^{th} candidate subsequent location,

z_t denotes ambient magnetic field observations of the plurality of magnetic field sensors at timestep t ,

- $z_{1:t+1}$ denotes multidimensional ambient magnetic field sequential observations
 15 from timestep 1 to $t+1$,

x_t^k denotes a candidate pose based on the k^{th} candidate subsequent location at timestep t ,

x_{t+1}^k denotes a candidate pose based on the k^{th} candidate subsequent location at timestep $t+1$,

- 20 $u_{1:t+1}$ is a control input from timestep 1 to $t+1$, and

M^B is the magnetic map.

32. The system as claimed in any one of claims 7 to 31, wherein the specific pose of the mobile device at timestep t is estimated by:

$$\frac{\sum_{k=1}^R \left(x_t^k \cdot p(x_0^k | z_0, M^B) \cdot p(x_t^k | x_0^k, z_{1:t}, u_{1:t}, M^B) \right)}{\sum_{k=1}^R \left(p(x_0^k | z_0, M^B) \cdot p(x_t^k | x_0^k, z_{1:t}, u_{1:t}, M^B) \right)}$$

25

where R comprises selected candidate locations,

k corresponds to the k^{th} candidate location,

$\mathbf{z}_0$ denotes ambient magnetic field observations of the plurality of magnetic field sensors at initial timestep,

$\mathbf{z}_{1:t}$ denotes multidimensional ambient magnetic field sequential observations from timestep 1 to t ,

5 $\mathbf{x}_0^k$ denotes a candidate pose based on the k^{th} candidate location at the initial timestep,

$\mathbf{x}_t^k$ denotes a candidate pose based on the k^{th} candidate location at timestep t ,

$\mathbf{u}_{1:t}$ is a control input from timestep 1 to t , and

M^B is the magnetic map.

10

33. A system comprising:

a sensor array configured to provide measurements of ambient magnetic fields in an environment; and

15 a mapping module configured to receive the ambient magnetic fields and determine rotation-invariant ambient magnetic field vectors, based on the received ambient magnetic fields, to build a magnetic map representative of the environment.

34. A system comprising:

20 a plurality of magnetic field sensors releasably coupled with a mobile device, wherein the plurality of magnetic field sensors is configured to measure ambient magnetic fields over a period of time; and

a localization module configured to

based on the measured ambient magnetic fields, infer candidate initial poses of the mobile device from candidate locations in a magnetic map,

25 based on the candidate initial poses and variations in the measured ambient magnetic fields over the period of time, to infer a fine pose of the mobile device, and

30 evaluate a joint probabilistic distribution based on a probability distribution of the candidate initial poses and a probability distribution of the fine pose to estimate a specific pose of the mobile device.

35. A magnetic map representative of an environment, the magnetic map comprising:
coordinates established for each of a plurality of locations in the environment; and
a rotation-invariant ambient magnetic field fingerprint at each location.

5 36. The magnetic map as claimed in claim 35, wherein the coordinates are either:
2-dimensional grid coordinates, and the magnetic map is a 2-dimensional magnetic
map; or
3-dimensional cubic coordinates, and the magnetic map is a 3-dimensional
magnetic map.

10

37. The magnetic map as claimed in claim 35 or 36, wherein the rotation-invariant
ambient magnetic field fingerprint is determined based on rotation-invariant ambient
magnetic field vectors being determined from ambient magnetic fields measured at the
location.

15

38. A method for determining a rotation-invariant ambient magnetic field vector to
build a magnetic map of an environment, the method comprising:

adapting an ambient magnetic field measured by a magnetic field sensor at a
position within the environment or within a working distance of the magnetic field sensor
20 from an edge boundary of the environment into an ambient magnetic field vector, wherein
the magnetic field sensor is releasably coupled with a mobile device moved to the position;
inferring, based on the ambient magnetic field vector, an orientation of the magnetic
field sensor; and

aligning, based on the inferred orientation, the ambient magnetic field vector in a
25 local coordinate system to a global coordinate system to generate the rotation-invariant
ambient magnetic field vector, the rotation-invariant ambient magnetic field vector
comprising two orthogonal components and an integrated intensity of the measured
ambient magnetic field.

30 39. The method as claimed in claim 38, wherein the measured ambient magnetic field
is dependent on the orientation of the magnetic sensor, and represented by:

$$W R_z(\gamma) R_y(\theta) R_x(\phi) B_e + V$$

where $R_z(\gamma)$, $R_y(\theta)$, and $R_x(\phi)$ are corresponding rotation matrices in yaw, pitch, and roll, respectively;

5 W and V are a soft-iron effect matrix and a hard-iron effect offset vector of the magnetic field sensor, respectively; and

$B_e = [B_e^x, B_e^y, B_e^z]^T$ within the global coordinate system, B_e^x , B_e^y , and B_e^z being orthogonal components representing magnetic field intensities in Geographic North, Geographic East and vertical downward directions, respectively, and B_e^z being parallel with a unit vector of the Earth's gravity field.

10

40. The method as claimed in claim 38 or 39, wherein the ambient magnetic field vector comprises three orthogonal components in the local coordinate system and the integrated

$$\begin{bmatrix} B_m^x \\ B_m^y \\ B_m^z \\ F_m \end{bmatrix}$$

intensity of the measured ambient magnetic field, represented by:

where B_m^x is a x-direction component;

15 B_m^y is a y-direction component;

B_m^z is a z-direction component; and

$$F_m = \sqrt{(B_m^x)^2 + (B_m^y)^2 + (B_m^z)^2}.$$

41. The method as claimed in claim 40, wherein the inferred orientation of the magnetic
20 field sensor is represented by a yaw angle, ψ , where

$$\psi = \begin{cases} 2\pi - \alpha, & \text{if } B_m^x > 0, B_m^y > 0 \\ -\alpha, & \text{if } B_m^x > 0, B_m^y < 0 \\ \pi - \alpha, & \text{if } B_m^x < 0, B_m^y > 0 \\ \pi + \alpha, & \text{if } B_m^x < 0, B_m^y < 0 \end{cases},$$

$\alpha = \arctan (|B_m^y| / |B_m^x|)$, and

ψ ranges between 0 radian and 2π radian, the 0 radian being defined with respect to a direction of Magnetic North.

5 42. The method as claimed in claim 40 or 41, wherein one of the two orthogonal components of the rotation-invariant ambient magnetic field vector is a vertical component parallel with a unit vector of the Earth's gravity field, and the other is a rotation-invariant horizontal component orthogonal to the vertical component.

10 43. The method as claimed in claim 42, wherein the rotation-invariant horizontal component is $\sqrt{(B_m^x)^2 + (B_m^y)^2}$, and the vertical component is B_m^z .

44. A method for building a magnetic map representative of an environment, the method comprising:

15 establishing coordinates for each of a plurality of locations in the environment;
collecting rotation-invariant ambient magnetic field vectors at each location, the rotation-invariant ambient magnetic field vectors being determined from ambient magnetic fields measured at the location;
determining, based on the collected rotation-invariant ambient magnetic field
20 vectors, an ambient magnetic field fingerprint for each location; and
for more than one of the plurality of locations, selecting the respective ambient magnetic field fingerprints and associating each of the respective ambient magnetic field fingerprints with the corresponding coordinates.

25 45. A method for localizing a mobile device, the method comprising:
performing coarse localization, based on ambient magnetic fields, to infer candidate initial poses of the mobile device from candidate locations in a magnetic map, wherein the ambient magnetic fields are measured over a period of time by a plurality of magnetic field sensors releasably coupled with the mobile device;

performing fine localization, based on the candidate initial poses and variations in the measured ambient magnetic fields over the period of time, to infer a fine pose of the mobile device; and

5 evaluating a joint probabilistic distribution based on a probability distribution of the candidate initial poses and a probability distribution of the fine pose to estimate a specific pose of the mobile device.

46. The method as claimed in claim 45, further comprising applying an assistant localization method to the step of performing coarse localization to expedite the inference
10 of the candidate initial poses of the mobile device from the candidate locations in the magnetic map.

47. The method as claimed in claim 46, wherein the assistant localization method comprises a laser-based localization method, a vision-based localization method, or a
15 GNSS/GPS-based localization method.

48. The method as claimed in any one of claims 45 to 47, further comprising performing substantially continuous tracking of poses of the mobile device by recursively inferring subsequent poses based on candidate subsequent locations, wherein one candidate
20 subsequent location is identified by

calculating an estimated pose of the mobile device at timestep $t+1$ based on the specific pose at timestep t and data provided by a motion model for fine localization during the timestep t to the timestep $t+1$;

selecting an area in the magnetic map based on the estimated pose; and

25 matching, in terms of substantially similar patterns and amplitudes, a first time-series ambient magnetic field sequences corresponding to locations within the selected area and a second time-series ambient magnetic field sequences associated with the rotation-invariant ambient magnetic field vectors determined from the measured ambient magnetic fields to identify the candidate subsequent location.

30

49. A computer readable storage medium comprising computer readable instructions operable when executed by a computer to perform at least one of the following:

determine a rotation-invariant ambient magnetic field vector to build a magnetic map of an environment, the computer readable instructions configured to perform a method

5 as claimed in any one of claims 38 to 43;

build a magnetic map representative of an environment, the computer readable instructions configured to perform a method as claimed in claim 44; or

localize a mobile device, the computer readable instructions configured to perform a method as claimed in any one of claims 45 to 48.

10