

FAST VISCOACOUSTIC AND VISCOELASTIC FULL-WAVEFIELD INVERSIONCROSS-REFERENCE TO RELATED APPLICATION

[0001] This application claims the benefit of U.S. Provisional Patent Application 62/013,455, filed June 17, 2014, entitled FAST VISCOACOUSTIC AND VISCOELASTIC
5 FULL-WAVEFIELD INVERSION, the entirety of which is incorporated by reference herein.

TECHNOLOGICAL FIELD

[0002] Exemplary embodiments described herein pertain to geophysical prospecting and, more particularly, to seismic data processing that includes recovering viscoacoustic and/or viscoelastic parameters from seismic data using the full-wavefield inversion (FWI)
10 technique, and that also includes imaging subsurface interfaces using reverse time migration (RTM).

BACKGROUND

[0003] This section is intended to introduce various aspects of the art, which may be associated with exemplary embodiments of the present invention. This discussion is believed
15 to assist in providing a framework to facilitate a better understanding of particular aspects of the present invention. Accordingly, it should be understood that this section should be read in this light, and not necessarily as admissions of prior art.

VISCOACOUSTIC AND VISCOELASTIC WAVEFIELD MODELING

[0004] In the time-domain, intrinsic attenuation (absorption and dispersion) is
20 formulated with convolution operators between strain and relaxation functions. Such convolutions are computationally impractical for large-scale wave propagation simulations using time-marching methods. Incorporation of realistic attenuation into time-domain computations was first achieved using Padé approximants by Day and Minster (*Day and Minster, 1984*). Later, Padé approximants evolved into sophisticated viscoacoustic and
25 viscoelastic rheological models in rational forms, such as the generalized-Maxwell (GMB) and Standard-Linear-Solid (SLS) models. Attenuation modeling methods in the time domain are based on either the GMB or SLS formulations given by Emmerich and Korn (1987) and Carcione et al. (1988), respectively, and use rheological models based on relaxation mechanisms.

30 [0005] A relaxation mechanism is the unit of the time-domain attenuation model representing viscous effects in a narrow frequency band. Multiple relaxation mechanisms are combined to model attenuation over a desired frequency band, introducing additional state

variables and partial differential equations (PDE) to the acoustic and elastic forward wave equations. A considerable amount of computational time and memory are consumed by these additional variables and equations during forward wave simulations and to an even greater degree during adjoint simulations.

5 **[0006]** Only simplified versions of approximate SLS formulations have been incorporated into FWI to infer a frequency-invariant attenuation at a reduced computational cost, such as *Robertsson et al. 1994*, *Charara et al. 2000*, *Hestholm et al. 2006*, and *Royle 2011*. Not only does the accuracy of such approximations deteriorate with decreasing quality factor values (stronger attenuation), but also they cannot be used to model quality factors that
10 vary with frequency. Another approach is to limit the number of relaxation mechanisms to one or two in order to minimize the complexity of the SLS model, which results in poor representation of the targeted quality factor over the seismic frequency band of interest. For examples, Charara (*Charara 2006*) suggests using only two relaxation mechanisms for the SLS model to reduce the computational cost in computing gradients. Their formulation
15 requires computing spatial derivatives of memory variables in the adjoint simulations, which limits them in the total number of relaxation mechanisms that can be used in their approximate SLS model. Also, Bai (*Bai et al., 2012*) presented a method for compensating attenuation effects for FWI using a single mechanism SLS model with the assumption of a frequency-invariant quality-factor.

20 **[0007]** While only SLS attenuation models have been incorporated into time-domain viscoacoustic and viscoelastic FWI techniques, both SLS and GMB models have been used for forward viscoacoustic and viscoelastic modeling of waves. For example, Käser et al (*Käser et al., 2007*) applied GMB for modeling viscoelastic waves with a discontinuous Galerkin method.

25 **[0008]** A common measure of attenuation is the quality factor, a dimensionless quantity that defines the frequency dependence of the acoustic or elastic moduli. The quality factor itself can be frequency dependent, especially for fluid-bearing rocks such as hydrocarbons, and is typically assumed to be frequency invariant for dry rocks (*Muller et al., 2010; Quintal, 2012*). Therefore, it is useful to infer the frequency dependence of the quality
30 factor to identify fluid bearing rocks such as reservoirs.

[0009] The SLS attenuation model represents the numerical inverse of the quality factor Q , also called the loss factor, by the following relation (*Carcione et al., 1988*):

$$Q^{-1}(x, \omega) = \frac{\sum_{l=1}^L \frac{\omega(\tau_{\epsilon l}(x) - \tau_{\sigma l}(x))}{1 + \omega^2(\tau_{\sigma l}(x))^2}}{\sum_{l=1}^L \frac{1 + \omega^2 \tau_{\epsilon l}(x) \tau_{\sigma l}(x)}{1 + \omega^2(\tau_{\sigma l}(x))^2}} \quad (1)$$

where

Q = quality factor,

$\tau_{\epsilon l}$ = strain relaxation time of mechanism l in SLS model,

$\tau_{\sigma l}$ = stress relaxation time of mechanism l in SLS model,

5 x = spatial coordinate,

ω = frequency,

L = number of relaxation mechanisms used in the SLS model.

Conceptually, the quality factor Q represents the ratio of stored to dissipated energy in a medium. The strain and stress relaxation times are determined to best fit the desired quality factor distribution over the frequency band.

[0010] A conventional first-order form of the linear viscoacoustic wave equations for simulating waves in attenuating acoustic media (*JafarGandomi et al. 2007*) is:

$$\begin{aligned} \frac{\partial p}{\partial t} + \kappa \nabla \cdot \mathbf{v} + \sum_{l=1}^L \phi_l m_l &= s_p, \\ \frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\rho} \nabla p &= \mathbf{s}_v, \\ \frac{\partial m_l}{\partial t} + \kappa \alpha_l \nabla \cdot \mathbf{v} + \phi_l m_l &= 0, \end{aligned} \quad (2)$$

with appropriate initial and boundary conditions for pressure p , velocity $\mathbf{v}$, and memory variables m_l . Note that

15 ∇ = divergence operator,

κ = unrelaxed bulk modulus ($\lim_{\omega \rightarrow \infty} \kappa(\omega) \rightarrow \kappa$),

ρ = mass density,

$\mathbf{v}$ = velocity ($\mathbf{v} = \{v_x \ v_y \ v_z\}^T$ in 3D space),

p = pressure,

20 m_l = memory variable for mechanism l ,

s_p = pressure source,

$\mathbf{s}_v$ = velocity source,

$\phi_l = \frac{1}{\tau_{\sigma l}}$ and $\alpha_l = \left(1 - \frac{\tau_{\sigma l}}{\tau_{\epsilon l}}\right)$ where relaxation parameters $\tau_{\epsilon l}$ and $\tau_{\sigma l}$ may be determined

by Equation (1) for a given quality factor profile.

Note that continuous scalar variables are denoted by italicized characters and vector and matrices are denoted by bold non-italicized characters throughout this document.

[0011] FWI methods based on computing gradients of an objective function with respect to the parameters are often efficiently implemented by using adjoint methods, which have been proved to outperform other relevant methods, such as direct sensitivity analyses, finite differences or complex variable methods. One can find a detailed discussion about adjoint methods in *Thevenin et al., 2008*.

[0012] The continuous adjoint of the conventional viscoacoustic system (Equations (2)) is

$$\begin{aligned}\frac{\partial \bar{p}}{\partial t} + \nabla \cdot \left(\frac{1}{\rho} \bar{\mathbf{v}} \right) &= \frac{\partial \mathcal{F}}{\partial p}, \\ \frac{\partial \bar{\mathbf{v}}}{\partial t} + \nabla(\kappa \bar{p}) + \sum_{l=1}^L \nabla(\kappa \alpha_l \bar{m}_l) &= \frac{\partial \mathcal{F}}{\partial \mathbf{v}}, \\ \frac{\partial \bar{m}_l}{\partial t} + \phi_l \bar{p} + \phi_l \bar{m}_l &= 0,\end{aligned}\tag{3}$$

10 where

$\bar{p}$ = adjoint pressure,

$\bar{\mathbf{v}}$ = adjoint velocity,

$\bar{m}_l$ = adjoint memory variable for mechanism l , and

$\partial \mathcal{F} / \partial p$ and $\partial \mathcal{F} / \partial \mathbf{v}$ are derivatives of the objective function $\mathcal{F}$ with respect to the pressure and velocity respectively. Although the objective function and its derivative with respect to the pressure and velocity will be explicitly defined in the Viscoacoustic and Viscoelastic Full Wavefield Inversion Section, $\partial \mathcal{F} / \partial p$ and $\partial \mathcal{F} / \partial \mathbf{v}$ can simply be considered as sources to the adjoint equations.

[0013] Note that the derivation of the adjoint equations involves using integration by parts which introduces both spatial and temporal boundary terms into the adjoint equations (which are evaluated at the spatial and temporal boundaries respectively). These adjoint spatial boundary terms are not included in Equations (3), but need to be included (i.e. in code) in order to correctly compute the unique solutions for the adjoint variables, and the temporal boundary terms are zero and thus drop out.

[0014] The spatial derivatives of memory variables $\nabla \kappa \alpha_l \bar{m}_l$ in the adjoint equations (Equations (3)) lead to a significant amount of computational cost. Figure 5 shows the cost of adjoint computations relative to forward computations for second through twelfth order (spatially) accurate finite-difference (FD) time-domain methods. The relative cost is based on comparing the number of flops. For three relaxation mechanisms and an eighth-order

finite-difference method, the cost of solving the adjoint equations is 2.25 times more expensive than the cost of solving the forward equations (Equations (2)). Also note that as the spatial order of the finite-difference method and the number of relaxation mechanisms increase, the relative cost of solving the adjoint equations increases. Furthermore, it is expected that for large-scale distributed-memory parallel processing computations, the cost of solving the adjoint will increase due to the additional communication of the memory variables.

[0015] The linear viscoacoustic equations can be extended to the following linear viscoelastic equations:

$$\begin{aligned} \frac{\partial \boldsymbol{\sigma}}{\partial t} + \mathbf{C} \boldsymbol{\kappa}^T \mathbf{v} + \sum_{l=1}^L \phi_l \mathbf{m}_l &= \mathbf{s}_{\boldsymbol{\sigma}}, \\ \frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\rho} \boldsymbol{\kappa} \boldsymbol{\sigma} &= \mathbf{s}_{\mathbf{v}}, \\ \frac{\partial \mathbf{m}_l}{\partial t} + \mathbf{D}_l \boldsymbol{\kappa}^T \mathbf{v} + \phi_l \mathbf{m}_l &= \mathbf{0}, \end{aligned} \quad (4)$$

10 where in the 3D Cartesian coordinate system,

$$\boldsymbol{\sigma} = \text{stress}, \boldsymbol{\sigma} = \{\sigma_{xx} \ \sigma_{yy} \ \sigma_{zz} \ \sigma_{xz} \ \sigma_{yz} \ \sigma_{xy}\}^T,$$

$\mathbf{m}_l$ = memory variable for mechanism l ,

$$\mathbf{m}_l = \{m_{l,xx} \ m_{l,yy} \ m_{l,zz} \ m_{l,xz} \ m_{l,yz} \ m_{l,xy}\}^T,$$

$$\mathbf{C} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}, \text{ which is the elastic constitutive relationship}$$

15 for the isotropic unrelaxed system in terms of Lamé constants λ and μ ,

$$\mathbf{D}_l = \begin{bmatrix} \lambda \alpha_l^\lambda + 2\mu \alpha_l^\mu & \lambda \alpha_l^\lambda & \lambda \alpha_l^\lambda & 0 & 0 & 0 \\ \lambda \alpha_l^\lambda & \lambda \alpha_l^\lambda + 2\mu \alpha_l^\mu & \lambda \alpha_l^\lambda & 0 & 0 & 0 \\ \lambda \alpha_l^\lambda & \lambda \alpha_l^\lambda & \lambda \alpha_l^\lambda + 2\mu \alpha_l^\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu \alpha_l^\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu \alpha_l^\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \alpha_l^\mu \end{bmatrix}, \text{ which is the}$$

constitutive relationship for the memory system (for the SLS formulation $\alpha_l^{\lambda,\mu} =$

$$\left(1 - \frac{\tau_{\sigma l}}{\tau_{\epsilon l}^{\lambda,\mu}}\right),$$

and $\tau_{\epsilon l}^\lambda$ and $\tau_{\epsilon l}^\mu$ are computed from the compressional and shear wave quality factors Q^P

20 and Q^S using Equation (1)),

$\aleph^T$ = strain operator,

$$\aleph^T = \begin{bmatrix} \partial_x & 0 & 0 & \partial_z & 0 & \partial_y \\ 0 & \partial_y & 0 & 0 & \partial_z & \partial_x \\ 0 & 0 & \partial_z & \partial_x & \partial_y & 0 \end{bmatrix}^T.$$

The continuous adjoint of the conventional viscoelastic system (Equations (4)) is

$$\begin{aligned} \frac{\partial \bar{\sigma}}{\partial t} + \aleph^T \left(\frac{1}{\rho} \bar{\mathbf{v}} \right) &= \frac{\partial \mathcal{F}}{\partial \sigma}, \\ \frac{\partial \bar{\mathbf{v}}}{\partial t} + \aleph(\mathbf{C}\bar{\sigma}) + \sum_{l=1}^L \aleph(\mathbf{D}_l \bar{\mathbf{m}}_l) &= \frac{\partial \mathcal{F}}{\partial \mathbf{v}}, \\ \frac{\partial \bar{\mathbf{m}}_l}{\partial t} + \phi_l \bar{\sigma} + \phi_l \bar{\mathbf{m}}_l &= \mathbf{0}, \end{aligned} \quad (5)$$

where in the 3D Cartesian coordinate system,

$$\bar{\sigma} = \text{adjoint stress, } \bar{\sigma} = \{\bar{\sigma}_{xx} \quad \bar{\sigma}_{yy} \quad \bar{\sigma}_{zz} \quad \bar{\sigma}_{xz} \quad \bar{\sigma}_{yz} \quad \bar{\sigma}_{xy}\}^T,$$

$\bar{\mathbf{m}}_l$ = adjoint memory field for mechanism l ,

$\bar{\mathbf{m}}_l = \{\bar{m}_{l,xx} \quad \bar{m}_{l,yy} \quad \bar{m}_{l,zz} \quad \bar{m}_{l,xz} \quad \bar{m}_{l,yz} \quad \bar{m}_{l,xy}\}^T$, and $\partial \mathcal{F} / \partial \sigma$ and $\partial \mathcal{F} / \partial \mathbf{v}$ are derivatives of the objective function $\mathcal{F}$ with respect to the stress and velocity respectively.

[0016] Note that the spatial boundary terms are not included in Equations (4). The temporal boundary terms are zero and thus drop out.

[0017] As for the viscoacoustic case, the adjoint equations for the viscoelastic case also contain spatial derivatives of memory variables, which will add a large computational cost to viscoelastic FWI.

VISCOACOUSTIC AND VISCOELASTIC FULL WAVEFIELD INVERSION

[0018] FWI is a partial-differential-equation-constrained optimization method which iteratively minimizes a norm of the misfit between measured and computed wavefields. Seismic FWI involves multiple iterations, and a single iteration can involve the following computations: (1) solution of the forward equations, (2) solution of the adjoint equations, and (3) convolutions of these forward and adjoint solutions to yield a gradient of the cost function. Note that for second-order optimization methods, such as Gauss-Newton, the (4) solution of the perturbed forward equations is also required. A more robust mathematical justification for this case can be found, for example, in U.S. Patent Publication 20130238246, the entire content of which is hereby incorporated by reference.

[0019] For viscoacoustic and viscoelastic seismic FWI, the cost of each simulation is exacerbated due to the fact that the relaxation mechanisms needed to accurately model the quality factor's frequency-dependent profile introduce even more variables and equations. Furthermore, the number of simulations that must be computed is proportional to the total

number of iterations in the inversion, which is typically on the order of hundreds to thousands. Nevertheless, the benefit of inferring the attenuation properties of the subsurface using this method is expected to outweigh the cost, and development of algorithms and workflows that lead to faster turnaround times is a key step towards making this technology
 5 feasible for field-scale data, allowing users to solve larger scale problems faster.

[0020] A common iterative inversion method used in geophysics is cost function optimization. Cost function optimization involves iterative minimization or maximization of the value of a cost function $\mathcal{F}(\theta)$ with respect to the model θ . The cost function, also referred to as the objective function, is a measure of the misfit between the simulated and
 10 observed data. The simulations (simulated data) are conducted by first discretizing the physics governing propagation of the source signal in a medium with an appropriate numerical method, such as the finite difference or finite element method, and computing the numerical solutions on a computer using the current geophysical properties model.

[0021] The following summarizes a local cost function optimization procedure for
 15 FWI: (1) select a starting model; (2) compute a search direction $S(\theta)$; and (3) search for an updated model that is a perturbation of the model in the search direction.

[0022] The cost function optimization procedure is iterated by using the new updated model as the starting model for finding another search direction, which will then be used to perturb the model in order to better explain the observed data. The process continues until an
 20 updated model is found that satisfactorily explains the observed data. Commonly used local cost function optimization methods include gradient search, conjugate gradients, quasi-Newton, Gauss-Newton and Newton's method.

[0023] Local cost function optimization of seismic data in the acoustic approximation is a common geophysical inversion task, and is generally illustrative of other types of
 25 geophysical inversion. When inverting seismic data in the acoustic approximation, the cost function can be written as:

$$\mathcal{F}(\theta) = \frac{1}{2} \sum_{g=1}^{N_g} \sum_{r=1}^{N_r} \sum_{t=1}^{N_t} W \left(\psi_{calc}(\theta, r, t, w_g) - \psi_{obs}(r, t, w_g) \right), \quad (6)$$

where

$\mathcal{F}(\theta)$ = cost function,

θ = vector of N parameters, $(\theta_1, \theta_2, \dots, \theta_N)$ describing the subsurface model,

30 g = gather index,

w_g = source function for gather g which is a function of spatial coordinates and time, for a point source this is a delta function of the spatial coordinates,

- N_g = number of gathers,
 r = receiver index within gather,
 N_r = number of receivers in a gather,
 t = time sample index within a trace,
5 N_t = number of time samples,
 W = norm function (minimization function, e.g. for least squares function $(x) = x^2$),
 ψ_{calc} = calculated seismic data from the model θ ,
 ψ_{obs} = measured seismic data (pressure, stress, velocities and/or acceleration).

[0024] The gathers, data from a number of sensors that share a common geometry,
10 can be any type of gather (common midpoint, common source, common offset, common receiver, etc.) that can be simulated in one run of a seismic forward modeling program. Usually the gathers correspond to a seismic shot, although the shots can be more general than point sources. For point sources, the gather index g corresponds to the location of individual point sources. This generalized source data, ψ_{obs} , can either be acquired in the field or can
15 be synthesized from data acquired using point sources. The calculated data ψ_{calc} on the other hand can usually be computed directly by using a generalized source function when forward modeling.

[0025] FWI attempts to update the discretized model θ such that $\mathcal{F}(\theta)$ is a minimum. This can be accomplished by local cost function optimization which updates the given model
20 $\theta^{(k)}$ as follows:

$$\theta^{(i+1)} = \theta^{(i)} + \gamma^{(i)} \mathbf{S}(\theta^{(i)}), \quad (7)$$

where i is the iteration number, γ is the scalar step size of the model update, and $\mathbf{S}(\theta)$ is the search direction. For steepest descent, $\mathbf{S}(\theta) = -\nabla_{\theta} \mathcal{F}(\theta)$, which is the negative of the gradient of the misfit function taken with respect to the model parameters. In this case, the model perturbations, or the values by which the model is updated, are calculated by multiplication of
25 the gradient of the objective function with a step length γ , which must be repeatedly calculated. For second-order optimization techniques, the gradient is scaled by the Hessian (second-order derivatives of objective function with respect to the model parameters). The computation of $\nabla_{\theta} \mathcal{F}(\theta)$ requires computation of the derivative of $\mathcal{F}(\theta)$ with respect to each of the N model parameters. N is usually very large in geophysical problems (more than one
30 million), and this computation can be extremely time consuming if it has to be performed for each individual model parameter. Fortunately, the adjoint method can be used to efficiently perform this computation for all model parameters at once (*Tarantola, 1984*). While

computation of the gradients using the adjoint method is efficient relative to other methods, it is still very costly for viscoacoustic and viscoelastic FWI.

[0026] Conventional viscoacoustic and viscoelastic full-waveform inversion methods in the time domain compute the gradient of the memory variables to time-march the adjoint equations. Earth models taking attenuation into account have been presented both in the frequency and time domains (*Ursin and Toverud, 2002*). The main focus of frequency domain methods has been establishing a relationship between attenuation and medium velocities using complex number properties under causality principals, which make them straightforward to directly apply for FWI (*Hak and Mulder, 2010*).

SUMMARY

[0027] A method, including: obtaining an initial geophysical model; modeling a forward wavefield with viscoacoustic or viscoelastic wave equations; modeling an adjoint wavefield with adjoint viscoacoustic or adjoint viscoelastic wave equations, wherein the adjoint viscoacoustic wave equations are based on an auxiliary variable that is a function of pressure and a memory variable or the adjoint viscoelastic wave equations are based on a combination of stress and a memory variable, respectively; obtaining a gradient of a cost function based on a combination of a model of the forward wavefield and a model of the adjoint wavefield; and using the gradient of the cost function to update the initial geophysical model and obtain an updated geophysical model.

BRIEF DESCRIPTION OF THE DRAWINGS

[0028] While the present disclosure is susceptible to various modifications and alternative forms, specific example embodiments thereof have been shown in the drawings and are herein described in detail. It should be understood, however, that the description herein of specific example embodiments is not intended to limit the disclosure to the particular forms disclosed herein, but on the contrary, this disclosure is to cover all modifications and equivalents as defined by the appended claims. It should also be understood that the drawings are not necessarily to scale, emphasis instead being placed upon clearly illustrating principles of exemplary embodiments of the present invention. Moreover, certain dimensions may be exaggerated to help visually convey such principles.

[0029] Figure 1 illustrates exemplary quality-factor models based on 1, 2, 3 and 5 relaxation mechanisms for the Generalized-Maxwell Body model optimized for a frequency-invariant $Q = 30$ (frequency band from 2.5 to 40 Hz).

[0030] Figure 2 illustrates exemplary velocity dispersion curves computed with 1, 2, 3 and 5 relaxation mechanisms for the Generalized-Maxwell Body model of Figure 1.

[0031] Figure 3 illustrates exemplary quality-factor models based on 1, 2, 3 and 5 relaxation mechanisms for the Generalized-Maxwell Body model optimized for a linearly-varying quality factor over the band from 2.5 to 40 Hz.

[0032] Figure 4 illustrates exemplary velocity dispersion curves computed with 1, 2, 3 and 5 relaxation mechanisms for the Generalized-Maxwell Body model of Figure 3.

[0033] Figure 5 illustrates a non-limiting example of the cost (in terms of floating-point operations – flops) of adjoint relative to the forward for conventional 3D viscoacoustic for $L = 3$ and $L = 5$ relaxation mechanisms.

[0034] Figure 6 illustrates an exemplary method for viscoacoustic full-wavefield inversion.

[0035] Figure 7 illustrates an exemplary method for viscoelastic full-wavefield inversion.

[0036] Figure 8 is a non-limiting example of the cost of solving 3-D viscoacoustic forward and adjoint equations according to the present technological advancement relative to cost of solving conventional acoustic forward equations with a three relaxation mechanism attenuation model.

[0037] Figure 9 is a non-limiting example of the speedup for 3-D viscoacoustic adjoint according to the present technological advancement relative to conventional adjoint for $L = 2$, $L = 3$ and $L = 5$ relaxation mechanism attenuation models.

[0038] Figure 10 is an example of a Marmousi velocity model (scale unit is in km/s).

[0039] Figure 11 is an example of a true quality-factor model (scale unit is for inverse of quality factor).

[0040] Figure 12 is an example of an inverted velocity model (scale unit is in km/s).

[0041] Figure 13 is an example of an inverted quality-factor model (scale unit is for inverse of quality factor).

[0042] Figure 14 is an example of a change of objective function (least-square norm of data misfit) with number of iterations.

[0043] Figure 15 is an example of a computer that can execute methods that embody the present technological advancement.

DETAILED DESCRIPTION

[0044] Exemplary embodiments are described herein. However, to the extent that the following description is specific to a particular embodiment, this is intended to be for exemplary purposes only and simply provides a description of the exemplary embodiments. Accordingly, the invention is not limited to the specific embodiments described below, but rather, it includes all alternatives, modifications, and equivalents falling within the true spirit and scope of the appended claims.

[0045] An exemplary embodiment of the present technological advancement is a method for performing full-wavefield inversion (FWI) or reverse-time migration (RTM) requiring viscoacoustic or viscoelastic wave propagation. The method includes attenuation in the time domain using a multi-relaxation rheological model which optimally represents attenuation for a given number of relaxation mechanisms. This attenuation model is flexible for representing the constant or varying frequency dependence of the quality factor over a given seismic frequency bandwidth. The method formulates the viscoacoustic and/or viscoelastic wave equations using new auxiliary state variables which are a combination of pressure and memory variables for the viscoacoustic case or a combination of stress and memory variables for the viscoelastic case. These formulations lead to new wave equation systems with reduced computational complexity.

[0046] An exemplary embodiment of the viscoacoustic and viscoelastic FWI framework relies on the GMB rheological model to account for attenuation, which defines the loss factor in the frequency domain ω as

$$Q^{-1}(x, \omega) = \sum_{l=1}^L \alpha_l(x) \frac{\phi_l \omega + \phi_l^2 Q^{-1}}{\phi_l^2 + \omega^2}, \quad (8)$$

where

ϕ_l = Relaxation frequency for mechanism l ($\Phi = \{\phi_1, \phi_2, \dots, \phi_L\}$),

α_l = Relaxation moduli ratio for mechanism ($\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_L\}$).

The relaxation frequencies are typically chosen to vary exponentially over the frequency band and are spatially invariant (*Robertsson et al., 1994*). Relaxation moduli are determined by the following approach. An over-determined system of equations is formed by discretizing Equation (8) over the frequency band $\omega = \{\omega_1, \omega_2, \dots, \omega_n\}$ of the source, where n is the number of frequencies discretized over the band and larger than the number of memory variables L , and is typically $2L + 1$ for nearly frequency-invariant quality factor and larger for frequency-varying quality factor. The frequencies are typically discretized exponentially over the frequency band for computational effectiveness. This over-determined system of

equations is for discretized quality factor $\mathbf{Q} = \{Q(\omega_1, x), Q(\omega_2, x), \dots, Q(\omega_n, x)\}$ at coordinate x :

$$\mathbf{A}(\omega, \phi, \mathbf{Q}^{-1}(\omega))\alpha = \mathbf{Q}^{-1}(\omega), \quad (9)$$

where

$$\mathbf{A}_{l,i} = \frac{\phi_l \omega_i + \phi_l^2 Q(\omega_i)^{-1}}{\phi_l^2 + \omega_i^2} \text{ discretized quality-factor relationship (7) and } \mathbf{A} \text{ has}$$

5 dimensions of L by n .

α can be the inversion parameter. However, if the quality factor Q is constant over the frequency band, it can directly be considered as an inversion parameter instead of α . In such cases, the derivative of the objective functional with respect to quality factor Q can be obtained using the chain rule

$$\frac{\partial \mathcal{F}}{\partial Q} = \sum_l \frac{\partial \alpha_l}{\partial Q} \frac{\partial \mathcal{F}}{\partial \alpha_l}, \quad (10)$$

10 where the derivate of relaxation moduli with respect to Q , $\partial \alpha / \partial Q$ is obtained by solving

$$\mathbf{A} \frac{\partial \alpha}{\partial Q} = -\frac{\partial \mathbf{A}}{\partial Q} \alpha - \mathbf{Q}^{-2}. \quad (11)$$

[0047] Figures 1 and 2 illustrate examples of the quality-factor and velocity dispersion curves obtained by 1, 2, 3 and 5 relaxation mechanisms of GMB models which are designed to represent a frequency-invariant quality factor of 30 (target Q) over a frequency band from 2.5 to 40 Hz. The parameters of these attenuation models are computed using
15 Equation (9). These figures show that the accuracy of the modeled attenuation and dispersion increases with the number of relaxation mechanisms. Figures 3 and 4 illustrate an example of the quality-factor and dispersion responses of these models for a linearly-varying quality factor (over the frequency band), and once again show that the accuracy of the modeled attenuation and dispersion increases with the number of relaxation mechanisms.

20 [0048] We derive the viscoacoustic and viscoelastic adjoint equations in continuous form (often referred to as a continuous adjoint method) in order to demonstrate the improved efficiency of the present technological advancement. However, one may first discretize the forward equations and then derive the adjoints of the discretized equations (Betts et al., 2005), which is referred to as a discrete adjoint method. The present technological
25 advancement described in this application is applicable for both types of derivations of the FWI equations.

[0049] One embodiment of the present technological advancement provides an improved viscoacoustic FWI method which is based on a new form of viscoacoustic equations. A new auxiliary variable is defined as

$$\zeta_l = p - \frac{1}{\alpha_l} m_l, \quad (12)$$

and Equation (12) is used to carry out a linear transformation of Equations (2). The resulting viscoacoustic wave equations are

$$\begin{aligned} \frac{\partial p}{\partial t} + \kappa \nabla \cdot \mathbf{v} + r &= 0, \\ \frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\rho} \nabla p &= \mathbf{s}_v, \\ \frac{\partial \zeta_l}{\partial t} - \phi_l(p - \zeta_l) + r &= 0, \\ r &= \sum_{l=1}^L \alpha_l \phi_l(p - \zeta_l) - s_p, \end{aligned} \quad (13)$$

where r is a new variable which is defined for clarity. The relaxation moduli ratios used in the memory equations are obtained from Equation (9).

5 **[0050]** The adjoint of Equations (13) is

$$\begin{aligned} \frac{\partial \bar{p}}{\partial t} + \nabla \cdot \left(\frac{1}{\rho} \bar{\mathbf{v}} \right) + \sum_{l=1}^L \phi_l(-\bar{\zeta}_l + \bar{r}_l) &= \frac{\partial \mathcal{F}}{\partial p}, \\ \frac{\partial \bar{\mathbf{v}}}{\partial t} + \nabla(\kappa \bar{p}) &= \frac{\partial \mathcal{F}}{\partial \mathbf{v}}, \\ \frac{\partial \bar{\zeta}_l}{\partial t} + \phi_l \bar{\zeta}_l - \phi_l \bar{r}_l &= 0, \\ \bar{r}_l &= \alpha_l(\bar{p} + \sum_{k=1}^L \bar{\zeta}_k). \end{aligned} \quad (14)$$

Note that the spatial boundary terms are not included in Equations (14). The temporal boundary terms are zero and thus drop out.

[0051] The auxiliary variable introduced (Equation (12)), which is a combination of pressure and memory variables, replaces the memory variable in order to lower the cost of solving the adjoint wavefield equations compared to alternative equations that do not employ the auxiliary variable.

[0052] In contrast to the conventional viscoacoustic adjoint (Equations (3)), spatial derivatives of the memory variables are not present in the adjoint Equations (14), which reduces the computational cost for solving the adjoint. In the exemplary method embodying the present technological advancement, the cost of solving the adjoint (Equations (14)) is approximately the same as the cost of solving the forward equations (Equations (13)). Figure 8 illustrates the cost (based on comparing the number of flops) in solving the conventional and new forward and adjoint viscoacoustic equations relative to the forward acoustic equations. In Figure 8, solid lines depict the cost of solving conventional equations and dashed lines depict the cost of solving equations embodying the present technological advancement. Even though the cost of the new forward solver actually increases by a small

margin (versus the conventional forward solver), the cost of the new adjoint solver significantly decreases versus the state-of-the-art adjoint solver, and the overall cost therefore decreases. For example, for an eighth-order FD time-domain method and rheological model using three relaxation mechanisms, the relative adjoint cost is 2.68 for the state-of-the-art method and reduces to 1.24 for the new method. Furthermore, the adjoint speed-up of the present advancement relative to the state-of-the-art is shown in Figure 9 for 2, 3 and 5 relaxation mechanisms.

[0053] The continuous gradients of the objective function with respect to the inversion parameters are

$$\begin{aligned}\frac{\partial \mathcal{F}}{\partial \kappa} &= \int_0^T \bar{p} \nabla \cdot \mathbf{v} dt, \\ \frac{\partial \mathcal{F}}{\partial \rho} &= - \int_0^T \bar{\mathbf{v}} \cdot \frac{1}{\rho^2} \nabla p dt, \\ \frac{\partial \mathcal{F}}{\partial \alpha_l} &= \int_0^T \bar{r} \phi_l(p - \zeta_l) dt \text{ or} \\ \frac{\partial \mathcal{F}}{\partial Q} &= \int_0^T \bar{r} \sum_{l=1}^L \frac{\partial \alpha_l}{\partial Q} \phi_l(p - \zeta_l) dt \text{ if } Q \text{ is the inversion parameter,}\end{aligned}\tag{15}$$

10 Thus, $\nabla_{\boldsymbol{\theta}} \mathcal{F}(\boldsymbol{\theta}) = \left\{ \frac{\partial \mathcal{F}}{\partial \kappa} \quad \frac{\partial \mathcal{F}}{\partial \rho} \quad \frac{\partial \mathcal{F}}{\partial \alpha_l} \right\}^T$ or $\left\{ \frac{\partial \mathcal{F}}{\partial \kappa} \quad \frac{\partial \mathcal{F}}{\partial \rho} \quad \frac{\partial \mathcal{F}}{\partial Q} \right\}^T$.

[0054] The perturbed forward equations required for a second-order optimization method are

$$\begin{aligned}\frac{\partial \hat{p}}{\partial t} + \kappa \nabla \cdot \hat{\mathbf{v}} + r &= -\delta \kappa \nabla \cdot \mathbf{v}, \\ \frac{\partial \hat{\mathbf{v}}}{\partial t} + \frac{1}{\rho} \nabla \hat{p} &= \frac{\delta \rho}{\rho^2} \nabla p, \\ \frac{\partial \hat{\zeta}_l}{\partial t} - \phi_l(\hat{p} - \hat{\zeta}_l) + r &= 0, \\ r - \sum_{l=1}^L \alpha_l \phi_l(\hat{p} - \hat{\zeta}_l) &= \begin{cases} \sum_{l=1}^L \delta \alpha_l \phi_l(p - \zeta_l) & \text{if } \alpha_l \text{ is the inversion parameter} \\ \delta Q \sum_{l=1}^L \frac{\partial \alpha_l}{\partial Q} \phi_l(p - \zeta_l) & \text{if } Q \text{ is the parameter} \end{cases},\end{aligned}\tag{16}$$

where

$\hat{p}$ = perturbed pressure,

15 $\hat{\mathbf{v}}$ = perturbed velocity,

$\hat{m}_l$ = perturbed memory variable for mechanism l .

[0055] The same approach can be extended to the viscoelastic wave equations. The computational savings will be larger for a given number of relaxation mechanisms because the total number of memory variables for the 3D viscoelastic equations will be six times larger than that for the viscoacoustic case. We define new auxiliary variables

20

$$\boldsymbol{\zeta}_l = \boldsymbol{\sigma} - \mathbf{C}\mathbf{D}_l^{-1}\mathbf{m}_l, \quad (17)$$

and carry out a linear transformation of Equations (4) with the new auxiliary variables. This transformation eliminates spatial derivatives of the memory variables in the adjoint, and reduces the computational cost for solving the adjoint system. The resulting viscoelastic wave equations are

$$\begin{aligned} \frac{\partial \boldsymbol{\sigma}}{\partial t} + \mathbf{C}\boldsymbol{\kappa}^T \mathbf{v} + \mathbf{r} &= \mathbf{0}, \\ \frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\rho} \boldsymbol{\kappa} \boldsymbol{\sigma} &= \mathbf{s}_v, \\ \frac{\partial \boldsymbol{\zeta}_l}{\partial t} - \phi_l(\boldsymbol{\sigma} - \boldsymbol{\zeta}_l) + \mathbf{r} &= \mathbf{0}, \\ \mathbf{r} &= \sum_{l=1}^L \phi_l \mathbf{D}_l \mathbf{C}^{-1}(\boldsymbol{\sigma} - \boldsymbol{\zeta}_l) - \mathbf{s}_\sigma, \end{aligned} \quad (18)$$

5 where $\mathbf{r}$ is a new variable which is defined for clarity. The relaxation moduli ratios used in the memory equations are obtained from Equation (9).

[0056] The adjoint of Equations (18) is

$$\begin{aligned} \frac{\partial \bar{\boldsymbol{\sigma}}}{\partial t} + \boldsymbol{\kappa}^T \left(\frac{1}{\rho} \bar{\mathbf{v}} \right) + \sum_{l=1}^L \phi_l (-\bar{\boldsymbol{\zeta}}_l + \bar{\mathbf{r}}_l) &= \frac{\partial \mathcal{F}}{\partial \boldsymbol{\sigma}}, \\ \frac{\partial \bar{\mathbf{v}}}{\partial t} + \boldsymbol{\kappa}(\mathbf{C}\bar{\boldsymbol{\sigma}}) &= \frac{\partial \mathcal{F}}{\partial \mathbf{v}}, \\ \frac{\partial \bar{\boldsymbol{\zeta}}_l}{\partial t} + \phi_l \bar{\boldsymbol{\zeta}}_l - \phi_l \bar{\mathbf{r}}_l &= \mathbf{0}, \\ \bar{\mathbf{r}}_l &= \mathbf{D}_l \mathbf{C}^{-1}(\bar{\boldsymbol{\sigma}} + \sum_{k=1}^L \bar{\boldsymbol{\zeta}}_k), \end{aligned} \quad (19)$$

where

$\bar{\boldsymbol{\zeta}}_l$ = adjoint memory field for mechanism l ,

$$10 \quad \bar{\boldsymbol{\zeta}}_l = \{\bar{\zeta}_{l,xx} \quad \bar{\zeta}_{l,yy} \quad \bar{\zeta}_{l,zz} \quad \bar{\zeta}_{l,xz} \quad \bar{\zeta}_{l,yz} \quad \bar{\zeta}_{l,xy}\}^T,$$

$\bar{\mathbf{r}}_l$ = variable defined for clarity for mechanism l ,

$$\bar{\mathbf{r}}_l = \{\bar{r}_{l,xx} \quad \bar{r}_{l,yy} \quad \bar{r}_{l,zz} \quad \bar{r}_{l,xz} \quad \bar{r}_{l,yz} \quad \bar{r}_{l,xy}\}^T.$$

Note that the spatial boundary terms are not included in Equations (19). The temporal boundary terms are zero and thus drop out.

15 [0057] It is straightforward to derive alternative computationally efficient second-order time derivative formulations of the acoustic or elastic wave equations as opposed to first-order time derivative formulations as shown in this application. For the acoustic case, one may derive the second-order form of the acoustic equations by taking a time-derivative of both the first (pressure) and third (memory) equations in (13) and eliminating the $\partial \mathbf{v} / \partial t$ term

20 using the second (velocity) equation. For the elastic case, one may derive the second-order form of the elastic equations by taking a time-derivative of both the first (stress) and third (memory) equations in (18) and eliminating the $\partial \mathbf{v} / \partial t$ term using the second (velocity) equation. Spatial derivatives of the memory variables will not be present in the adjoint

equations of such second-order systems as demonstrated for first-order systems (equations 14 and 19).

EXAMPLE

[0058] Figure 6 illustrates an exemplary method for a viscoacoustic FWI method using Equations (13) to (16). In step 601, a geophysical model is assumed. A geophysical model gives one or more subsurface properties as a function of location in a region. For this example, a Marmousi velocity model (Figure 10) and a quality-factor model that has several low-Q anomalies, layers of intermediate Q values, and a smooth background (Figure 11) are used. The velocity and quality-factor models are structurally uncorrelated.

[0059] In step 603, attenuation parameters α are computed using Equations (9). The GMB model can be used for an assumed Q frequency dependence.

[0060] In steps 605a and 605b, the forward wavefield model Equations (13) and the adjoint model Equations (14) are solved. Step 605b can utilize measured data in the modeling of the adjoint wavefield.

[0061] In step 607, the gradient of the cost function is obtained from a convolution of Equations (13) and (14) in order to arrive at the gradients of the objective function with respect to the inversion parameters (14). Step 607 can utilize measured data in obtaining the gradient of the cost function.

[0062] In step 609, the gradient of the cost function (which provides the rate of the change of the cost function in a given direction) is then used to update the geophysical model in order to minimize the cost function. Step 609 can include searching for an updated geophysical property model that is a perturbation of the initial geophysical property model in the gradient direction that better explains the observed data. The iterative process of Figure 6 can be repeated until predetermined convergence between measured data and the updated model is obtained. Figure 14 shows how the data misfit of the objective function (least-square norm of data misfit) decreases with number of iterations.

[0063] Assuming a 16km maximum offset and 10 seconds of recorded (synthetic) data, 40 sources, 100 receivers, and a 10Hz Ricker wavelet source, the inverted or updated results for velocity and Q models are shown in Figures 12 and 13 respectively, demonstrating that the present technological advancement is able to invert for both parameters (velocity and Q) at a greatly reduced computational cost.

[0064] When the updated assumed model converges, the process proceeds to step 611. In step 611, an updated subsurface model is used to manage hydrocarbons. As used

herein, hydrocarbon management includes hydrocarbon extraction, hydrocarbon production, hydrocarbon exploration, identifying potential hydrocarbon resources, identifying well locations, determining well injection and/or extraction rates, identifying reservoir connectivity, acquiring, disposing of and/or abandoning hydrocarbon resources, reviewing
5 prior hydrocarbon management decisions, and any other hydrocarbon-related acts or activities.

[0065] Figure 7 illustrates an exemplary method for a viscoelastic FWI method using Equations (18) to (19). The method of Figure 7 (steps 701-711) is analogous to that of Figure 6 (steps 601-611), except that the equations have changed.

10 [0066] Figure 15 is a block diagram of a computer system 2400 that can be used to execute the present geophysical inversion techniques. A central processing unit (CPU) 2402 is coupled to system bus 2404. The CPU 2402 may be any general- purpose CPU, although other types of architectures of CPU 2402 (or other components of exemplary system 2400) may be used as long as CPU 2402 (and other components of system 2400) supports the
15 operations as described herein. Those of ordinary skill in the art will appreciate that, while only a single CPU 2402 is shown in Fig. 15, additional CPUs may be present. Moreover, the computer system 2400 may comprise a networked, multi-processor computer system that may include a hybrid parallel CPU/GPU system. The CPU 402 may execute the various logical instructions according to various teachings disclosed herein. For example, the CPU
20 2402 may execute machine-level instructions for performing processing according to the operational flow described.

[0067] The computer system 2400 may also include computer components such as non- transitory, computer -readable media. Examples of computer -readable media include a random access memory (RAM) 2406, which may be SRAM, DRAM, SDRAM, or the like.
25 The computer system 2400 may also include additional non-transitory, computer -readable media such as a read-only memory (ROM) 2408, which may be PROM, EPROM, EEPROM, or the like. RAM 2406 and ROM 2408 hold user and system data and programs, as is known in the art. The computer system 2400 may also include an input/output (I/O) adapter 2410, a communications adapter 2422, a user interface adapter 2424, and a display adapter 2418.

30 [0068] The I/O adapter 2410 may connect additional non-transitory, computer -readable media such as a storage device(s) 2412, including, for example, a hard drive, a compact disc (CD) drive, a floppy disk drive, a tape drive, and the like to computer system 2400. The storage device(s) may be used when RAM 2406 is insufficient for the memory requirements associated with storing data for operations of the present techniques. The data

storage of the computer system 2400 may be used for storing information and/or other data used or generated as disclosed herein. For example, storage device(s) 2412 may be used to store configuration information or additional plug-ins in accordance with the present techniques. Further, user interface adapter 2424 couples user input devices, such as a keyboard 2428, a pointing device 2426 and/or output devices to the computer system 400. The display adapter 2418 is driven by the CPU 2402 to control the display on a display device 2420 to, for example, present information to the user regarding available plug-ins.

[0069] The architecture of system 2400 may be varied as desired. For example, any suitable processor-based device may be used, including without limitation personal computers, laptop computers, computer workstations, and multi-processor servers. Moreover, the present technological advancement may be implemented on application specific integrated circuits (ASICs) or very large scale integrated (VLSI) circuits. In fact, persons of ordinary skill in the art may use any number of suitable hardware structures capable of executing logical operations according to the present technological advancement. The term "processing circuit" encompasses a hardware processor (such as those found in the hardware devices noted above), ASICs, and VLSI circuits. Input data to the computer system 2400 may include various plug-ins and library files. Input data may additionally include configuration information.

[0070] The present techniques may be susceptible to various modifications and alternative forms, and the examples discussed above have been shown only by way of example. However, the present techniques are not intended to be limited to the particular examples disclosed herein. Indeed, the present techniques include all alternatives, modifications, and equivalents falling within the spirit and scope of the appended claims.

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CLAIMS

1. A method of managing hydrocarbons, comprising:
 - obtaining, with a computer, an initial geophysical model;
 - modeling, with the computer, a forward wavefield with viscoacoustic or viscoelastic wave equations, wherein the forward wavefield with viscoacoustic wave equations is based on a first auxiliary variable that is a function of both pressure and a memory variable or the forward wavefield with viscoelastic wave equations is based on a second auxiliary variable that is a function of both stress and a memory variable, respectively, and the first or second auxiliary variable is chosen to eliminate a velocity term from the forward wavefield with viscoacoustic or viscoelastic wave equations;
 - modeling, with the computer, an adjoint wavefield with adjoint viscoacoustic or adjoint viscoelastic wave equations, which are derived from the forward viscoacoustic or the forward viscoelastic wave equations, respectively, and are based on an adjoint of the first or second auxiliary variable, respectively,
 - wherein the modeling includes causing a spatial derivative of the memory variable to not be present in the adjoint viscoacoustic or viscoelastic wave equations by a change of variables comprising replacing the memory variable with the first or second auxiliary variable in the viscoacoustic or viscoelastic wave equations, respectively, thereby causing the adjoint viscoacoustic or viscoelastic wave equations to include spatially-pointwise operations of the first or second auxiliary variable, respectively, instead of the spatial derivative operations of the memory variable;
 - obtaining, with the computer, a gradient of a cost function based on a combination of a model of the forward wavefield and a model of the adjoint wavefield;
 - updating, with the computer, the initial geophysical model with the gradient of the cost function, and obtaining and displaying an updated geophysical model; and
 - managing for hydrocarbons using the updated geophysical model.
2. The method of claim 1, wherein obtaining the initial geophysical model includes obtaining relaxation parameters for a given quality factor using an initial rheological model and acoustic or elastic medium parameters.
3. The method of any one of claims 1-2, wherein the adjoint wavefield is modeled using the adjoint viscoacoustic equations.

4. The method of any one of claims 1-3, wherein the first auxiliary variable is $\zeta_l = p - \frac{1}{\alpha_l} m_l$, where p is pressure, m_l is the memory variable for mechanism l , where l is an integer greater than or equal to 1, and α_l is inversion parameter for mechanism l .

5. The method of any one of claims 1-4, wherein the adjoint viscoacoustic equations are

$$\frac{\partial \bar{p}}{\partial t} + \nabla \cdot \left(\frac{1}{\rho} \bar{\mathbf{v}} \right) + \sum_{l=1}^L \phi_l (-\bar{\zeta}_l + \bar{r}_l) = \frac{\partial \mathcal{F}}{\partial p},$$

$$\frac{\partial \bar{\mathbf{v}}}{\partial t} + \nabla(\kappa \bar{p}) = \frac{\partial \mathcal{F}}{\partial \mathbf{v}},$$

$$\frac{\partial \bar{\zeta}_l}{\partial t} + \phi_l \bar{\zeta}_l - \phi_l \bar{r}_l = 0,$$

$$\bar{r}_l = \alpha_l (\bar{p} + \sum_{k=1}^L \bar{\zeta}_k),$$

where

∇ is a divergence operator,

t is time,

κ is an unrelaxed bulk modulus ($\lim_{\omega \rightarrow \infty} \kappa(\omega) \rightarrow \kappa$),

ρ is a mass density,

$\mathbf{v}$ is a velocity ($\mathbf{v} = \{v_x \ v_y \ v_z\}^T$ in 3D space),

$\bar{p}$ is an adjoint pressure,

$\mathcal{F}(\boldsymbol{\theta})$ is the cost function, where $\boldsymbol{\theta}$ is the model parameters (i.e. some combination of Q or α_l , κ and ρ),

$\bar{\mathbf{v}}$ is an adjoint velocity, and

ϕ_l is a relaxation frequency for mechanism l ,

$\partial \mathcal{F} / \partial p$ and $\partial \mathcal{F} / \partial \mathbf{v}$ are derivatives of the objective function $\mathcal{F}$ with respect to pressure and velocity respectively.

6. The method of claim 1, wherein the adjoint wavefield is modeled using the adjoint viscoelastic equations.

7. The method of claim 6, wherein the second auxiliary variable is $\bar{\zeta}_l = \sigma - \mathbf{C}\mathbf{D}_l^{-1}\mathbf{m}_l$, where σ is stress, $\mathbf{m}_l$ is a memory variable for mechanism l , where l is an integer greater than or equal to 1, $\mathbf{C}$ is an elastic constitutive relationship for an isotropic unrelaxed system and $\mathbf{D}_l$ is a constitutive relationship for a memory system.

8. The method of claim 7, wherein the adjoint viscoelastic equations are

$$\frac{\partial \bar{\sigma}}{\partial t} + \kappa^T \left(\frac{1}{\rho} \bar{\mathbf{v}} \right) + \sum_{l=1}^L \phi_l (-\bar{\zeta}_l + \bar{\mathbf{r}}_l) = \frac{\partial \mathcal{F}}{\partial \sigma},$$

$$\frac{\partial \bar{\mathbf{v}}}{\partial t} + \kappa(\mathbf{C}\bar{\sigma}) = \frac{\partial \mathcal{F}}{\partial \mathbf{v}},$$

$$\frac{\partial \bar{\zeta}_l}{\partial t} + \phi_l \bar{\zeta}_l - \phi_l \bar{\mathbf{r}}_l = 0,$$

$$\bar{\mathbf{r}}_l = \mathbf{D}_l \mathbf{C}^{-1} (\bar{\sigma} + \sum_k \bar{\zeta}_k),$$

where

t is time,

$\bar{\sigma}$ is an adjoint stress,

$$\bar{\sigma} = \{\bar{\sigma}_{xx} \quad \bar{\sigma}_{yy} \quad \bar{\sigma}_{zz} \quad \bar{\sigma}_{xz} \quad \bar{\sigma}_{yz} \quad \bar{\sigma}_{xy}\}^T,$$

$\bar{\mathbf{v}}$ is an adjoint velocity,

$\bar{\zeta}_l$ is an adjoint memory field for mechanism l ,

$$\bar{\zeta}_l = \{\bar{\zeta}_{l,xx} \quad \bar{\zeta}_{l,yy} \quad \bar{\zeta}_{l,zz} \quad \bar{\zeta}_{l,xz} \quad \bar{\zeta}_{l,yz} \quad \bar{\zeta}_{l,xy}\}^T,$$

$\bar{\mathbf{r}}_l$ is a variable defined for clarity for mechanism l ,

$$\bar{\mathbf{r}}_l = \{\bar{r}_{l,xx} \quad \bar{r}_{l,yy} \quad \bar{r}_{l,zz} \quad \bar{r}_{l,xz} \quad \bar{r}_{l,yz} \quad \bar{r}_{l,xy}\}^T,$$

ρ is a mass density,

$\partial \mathcal{F} / \partial \sigma$ and $\partial \mathcal{F} / \partial \mathbf{v}$, are derivatives of the objective function $\mathcal{F}$ with respect to the stress and velocity respectively.

$$\mathbf{C} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}, \text{ which is the elastic constitutive}$$

relationship for the isotropic unrelaxed system in terms of Lamé constants λ and μ ,

$$\mathbf{D}_i = \begin{bmatrix} \lambda\alpha_i^\lambda + 2\mu\alpha_i^\mu & \lambda\alpha_i^\lambda & \lambda\alpha_i^\lambda & 0 & 0 & 0 \\ \lambda\alpha_i^\lambda & \lambda\alpha_i^\lambda + 2\mu\alpha_i^\mu & \lambda\alpha_i^\lambda & 0 & 0 & 0 \\ \lambda\alpha_i^\lambda & \lambda\alpha_i^\lambda & \lambda\alpha_i^\lambda + 2\mu\alpha_i^\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu\alpha_i^\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu\alpha_i^\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu\alpha_i^\mu \end{bmatrix}, \text{ which is the}$$

constitutive relationship for the memory system ($\alpha_i^{\lambda,\mu}$ are computed from the compressional and shear wave quality factor using Equation (9)), and $\mathbf{\aleph}^T$ = strain

$$\text{operator, } \mathbf{\aleph}^T = \begin{bmatrix} \partial_x & 0 & 0 & \partial_z & 0 & \partial_y \\ 0 & \partial_y & 0 & 0 & \partial_z & \partial_x \\ 0 & 0 & \partial_z & \partial_x & \partial_y & 0 \end{bmatrix}^T.$$

$\mathcal{F}(\boldsymbol{\theta})$ is the cost function, where $\boldsymbol{\theta}$ is the model parameters (i.e. some combination of Q^P , Q^S or $\alpha_i^{\lambda,\mu}$, λ , μ and ρ).

9. The method of any one of claims 1-8, wherein the initial geophysical model includes a velocity model.
10. The method of any one of claims 1-9, wherein the initial geophysical model includes a quality factor model.
11. The method of any one of claims 1-10, wherein the first or second auxiliary variable is based on a plurality of relaxation mechanisms.
12. The method of any one of claims 1-11, wherein the obtaining the initial geophysical model includes obtaining relaxation parameters for a given quality factor using a Generalized-Maxwell rheological model and acoustic or elastic medium parameters.
13. The method of any one of claims 1-12, further comprising causing a well to be drilled at a location derived from the updated geophysical model.
14. The method of any one of claims 1-13, wherein the updated geophysical model resulting from the updating identifies location of structure in earth's subsurface that returned seismic wave to receivers that recorded.