

FIG. 1

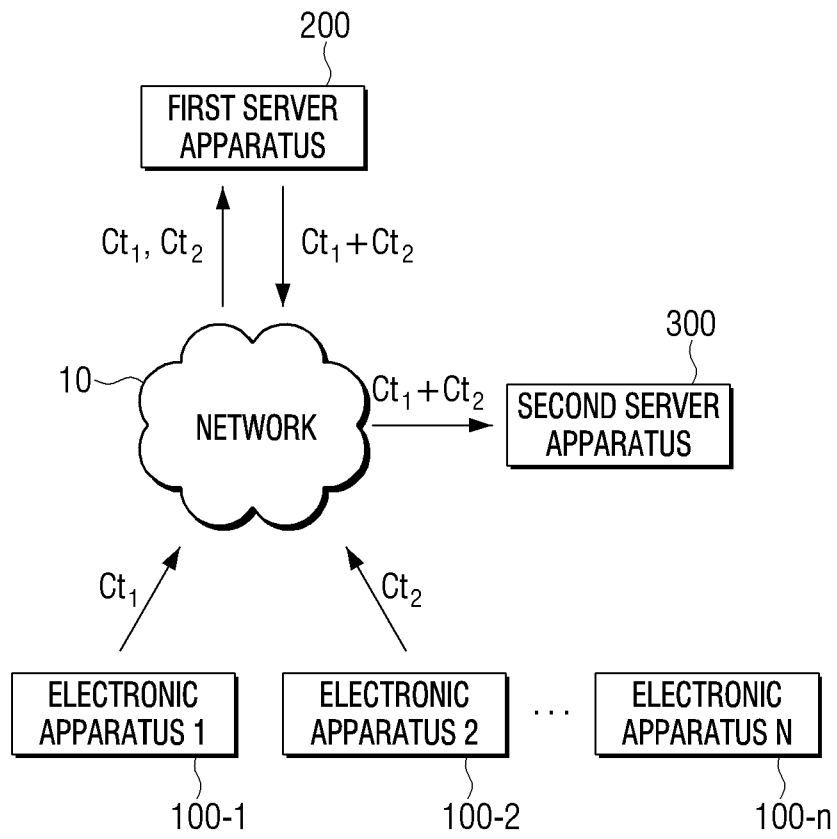


FIG. 2

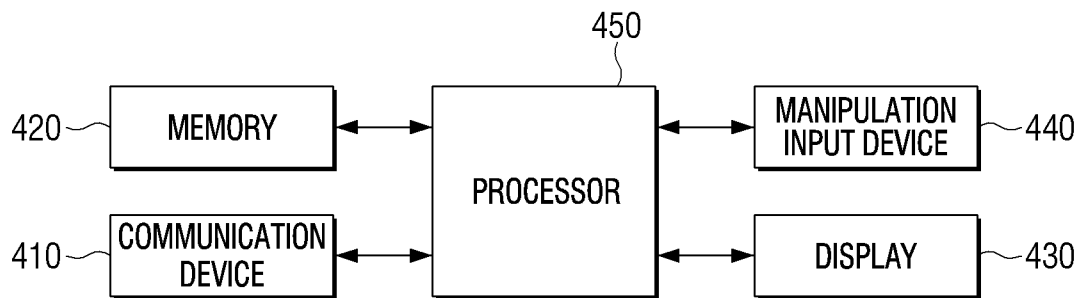
400

FIG. 3

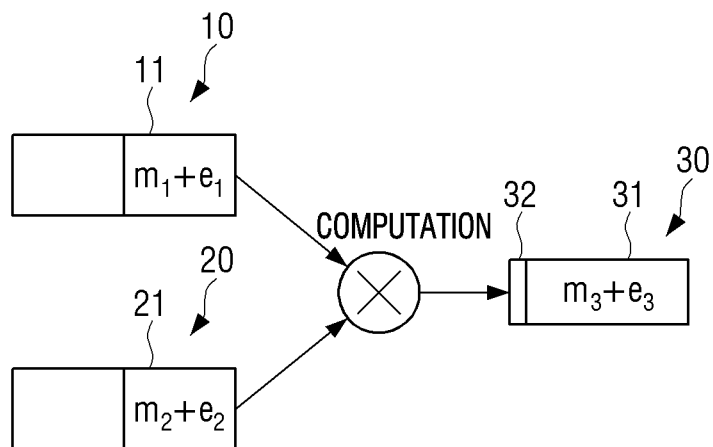


FIG. 4

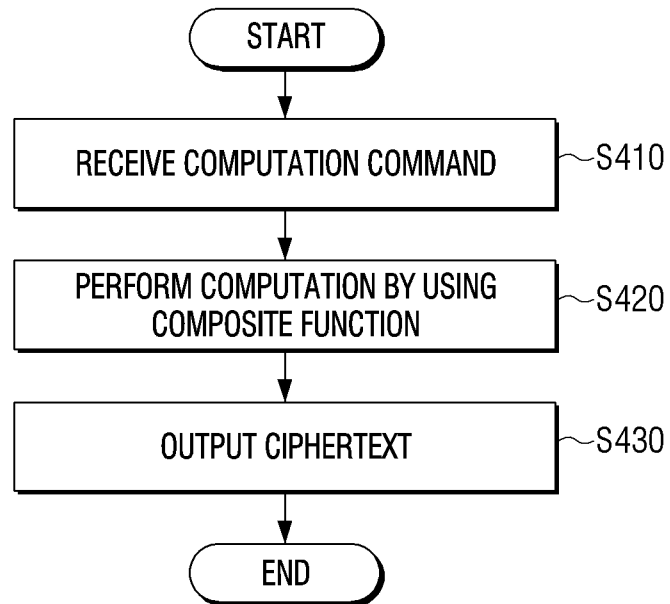


FIG. 5

Algorithm 1 $\text{Inv}(x;d)$

Input: $0 < x < 2, d \in \mathbb{N}$

Output: an approximate value of $1/x$ (refer Lemma 1)

1: $a_0 \leftarrow 2 - x$

2: $b_0 \leftarrow 1 - x$

3: for $n \leftarrow 0$ to $d - 1$ do

4: $b_{n+1} \leftarrow b_n^2$

5: $a_{n+1} \leftarrow a_n \cdot (1 + b_{n+1})$

6: end for

7: return a_d

FIG. 6

Algorithm 2 Sqrt(x;d)

Input: $0 \leq x \leq 1, d \in \mathbb{N}$

Output: an approximate value of $\sqrt{x}$ (refer Lemma 2)

1: $a_0 \leftarrow x$

2: $b_0 \leftarrow x - 1$

3: for $n \leftarrow 0$ to $d - 1$ do

4: $a_{n+1} \leftarrow a_n \left(1 - \frac{b_n}{2}\right)$

5: $b_{n+1} \leftarrow b_n^2 \left(\frac{b_n - 3}{4}\right)$

6: end for

7: return a_d

FIG. 7

Algorithm 3 $\text{Min}(a, b; d), \text{Max}(a, b; d)$

Input: $a, b \in [0, 1), d \in \mathbb{N}$

Output: an approximate value of $\min(a, b)$ and $\max(a, b)$ (refer Theorem 1,2)

1: $x = \frac{a+b}{2}$ and $y = \frac{a-b}{2}$

2: $z \leftarrow \text{Sqrt}(y^2; d)$

3: return $x - z$ for $\text{Min}(a, b; d)$
 $x + z$ for $\text{Max}(a, b; d)$

FIG. 8

Algorithm 4 $\text{Comp}(a, b; d, d', t, m)$

Input: distinct numbers $a, b \in \left[\frac{1}{2}, \frac{3}{2} \right)$, $d, d', t, m \in \mathbb{N}$

Output: an approximate value of $\text{comp}(a, b)$ (refer Theorem 4)

1: $a_0 \leftarrow \frac{a}{2} \cdot \text{Inv} \left(\frac{a+b}{2}; d' \right)$

2: $b_0 \leftarrow 1 - a_0$

3: for $n \leftarrow 0$ to $t - 1$ do

4: $\text{inv} \leftarrow \text{Inv}(a_n^m + b_n^m; d)$

5: $a_{n+1} \leftarrow a_n^m \cdot \text{inv}$

6: $b_{n+1} \leftarrow 1 - a_{n+1}$

7: end for

8: return a_t

FIG. 9

Algorithm 5 NewComp($a, b; n, d$)

Input: $a, b \in [0, 1], n, d \in \mathbb{N}$

Output: An approximate value of 1 if $a > b$, 0 if $a < b$ and $1/2$ otherwise

1: $x \leftarrow a - b$

2: for $i \leftarrow 1$ to d do

3: $x \leftarrow f_n(x)$ // compute $f_n^{(d)}(a - b)$ for input a and b

4: end for

5: return $(x+1)/2$

FIG. 10

Algorithm 6 FindG(n, c)

Input: $n \in \mathbb{N}, 0 < c < 1$

Output: A degree- $(2n+1)$ polynomial $g_{n,c}$ satisfying Prop I & IV with minimal δ of Prop IV.

1: $g_{n,c} \leftarrow x$

2: repeat

3: $\delta_0 \leftarrow$ minimal δ such that $g_{n,c}([\delta, 1]) \subset [1-c, 1]$

4: $g_{\min} \leftarrow$ degree- $(2n+1)$ minimax approx. poly. of $(1 - \frac{c}{2}) \cdot \text{sgn}(x)$ over $[-1, -\delta_0] \cup [\delta_0, 1]$

5: $g_{n,c} \leftarrow g_{\min}$

6: $S \leftarrow \|g_{n,c} - (1 - \frac{c}{2})\|_{\infty, [\delta_0, 1]}$

7: until $S \leq \frac{c}{2}$

8: return $g_{n,c}$

FIG. 11

Algorithm 7 NewCompG($a, b; n, d_f, d_g$)

Input: $a, b \in [0,1], n, d_f, d_g \in \mathbb{N}$

Output: An approximate value of 1 if $a > b$, 0 if $a < b$ and $1/2$ otherwise

1: $x \leftarrow a - b$

2: for $i \leftarrow 1$ to d_g do

3: $x \leftarrow g_n(x)$ // compute $g_n^{(d_g)}(a - b)$ for input a and b

4: end for

5: for $i \leftarrow 1$ to d_f do

6: $x \leftarrow f_n(x)$ // compute $f_n^{(d_f)}(a - b)$ for input a and b

7: end for

8: return $(x+1)/2$

FIG. 12

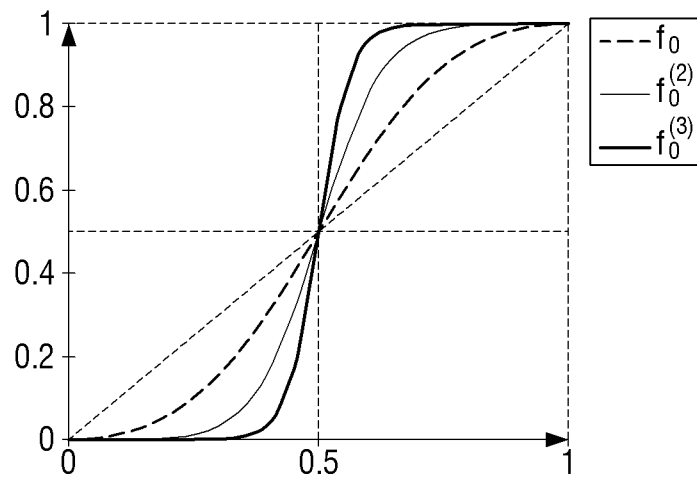


FIG. 13

