The $\hat{\theta}$ and $\hat{\phi}$ components of the fields are transformed using equations (4) and (5) hereunder:

$$(4)\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix} = E\begin{bmatrix}\sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi\\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi\\ \cos\theta & -\sin\theta & 0\end{bmatrix}\begin{bmatrix}0\\ \sin\tau e^{j\gamma}\\ \cos\tau\end{bmatrix}$$

$$= E\begin{bmatrix}\overbrace{\cos\theta\cos\phi}^{\theta} & \overbrace{-\sin\phi}^{\phi}\\ \cos\theta\sin\phi & \cos\phi\\ -\sin\theta & 0\end{bmatrix}\begin{bmatrix}\sin\tau e^{j\gamma}\\ \cos\tau\end{bmatrix}$$

$$(5)\begin{bmatrix}H_x\\H_y\\H_z\end{bmatrix} = \frac{E}{\eta}\begin{bmatrix}\sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi\\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi\\ \cos\theta & -\sin\theta & 0\end{bmatrix}\begin{bmatrix}0\\ \cos\tau\\ -\sin\tau e^{j\gamma}\end{bmatrix}$$

$$= \frac{E}{\eta}\begin{bmatrix}\overbrace{\cos\theta\cos\phi}^{\theta} & \overbrace{-\sin\phi}^{\phi}\\ \cos\theta\sin\phi & \cos\phi\\ -\sin\theta & 0\end{bmatrix}\begin{bmatrix}\cos\tau\\ -\sin\tau e^{j\gamma}\end{bmatrix}$$

$$= \frac{E}{\eta}\begin{bmatrix}\sin\phi & \cos\theta\cos\phi\\ -\cos\phi & \cos\theta\sin\phi\\ 0 & -\sin\theta\end{bmatrix}\begin{bmatrix}\sin\tau e^{j\gamma}\\ \cos\tau\end{bmatrix}$$

Thus, the Cartesian components of the normalized $\hat{E}$ and $\hat{H}$ fields, $\hat{E} = \frac{\vec{E}}{|E|}$ and $\hat{H} = \frac{\vec{H}}{|H|}$ (normalized by their magnitudes), of the electromagnetic field may be presented in the following generalized vector form $\vec{X}$ as function of the direction (angles) of arrival if the wavefront to the vector sensor (θ, ϕ) and the polarization parameters of the wavefront (γ, τ):

$$(6)\,\vec{X} = \begin{bmatrix}\hat{E}_x\\ \hat{E}_y\\ \hat{E}_z\\ \hat{H}_x\\ \hat{H}_y\\ \hat{H}_z\end{bmatrix} = D\overbrace{\begin{bmatrix}\cos\theta\cos\phi & -\sin\phi\\ \cos\theta\sin\phi & \cos\phi\\ -\sin\theta & 0\\ \sin\phi & \cos\theta\cos\phi\\ -\cos\phi & \cos\theta\sin\phi\\ 0 & -\sin\theta\end{bmatrix}}^{F(\theta,\phi)}\overbrace{\begin{bmatrix}\sin\tau e^{j\gamma}\\ \cos\tau\end{bmatrix}}^{g(\gamma,\tau)}$$

Where here D is conditioning matrix containing parameters like η (the wave impedance). In general, the model shown in (6) describes projection of a single arbitrary polarized (γ, τ) wavefront impinging a vector sensor from a direction(θ, ϕ). In case the