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ABACUS3: THE TEACHING ABACUSBACKGROUND AND FIELD OF THE INVENTION

5 This invention relates to a new, 3rd, generation of abacus, specifically designed for teaching foundational math concepts and processes (i.e. numbers and operations) to children from 2 to 12 years old. The earliest abacus (1st Generation) included versions developed and used in Mesopotamia, Egypt, Greece, Rome, India, China (suanpan), Korea (supan) and Japan (soroban). These were all designed for use by adults, merchants, for tallying and counting. They are essentially tokens or beads divided into 2
10 sections by a horizontal central beam or divider; the top section with one to two “5”-beads representing 5s, and the bottom section with four to five “unit”-beads representing 1s. These beads are in turn on vertical groves or rods which indicate place-value: from right to left-units, tens, hundreds etc. So the number 3572 would be represented by **three unit**-beads on the (thousands) rod, **one 5-bead** on the next-moving right-(hundreds) rod, **one 5-bead and two unit-beads** on the next (tens) rod, and **two unit-**
15 **beads** on the next (unit) rod: all these beads being registered (made “active”) by moving them to the central beam.

To add this number to another – say 1338-one starts from the right, and add on the beads (i.e. move to the central beam) one rod (place value) at a time. Thus, starting from the right-most (unit rod) adding 8 to 2 gives 10 beads which is effected by clearing the unit rod (i.e no beads at the divider) and
20 “carrying” 10 by adding one bead to the beam of the next (tens) rod, which would now have 8 beads. Adding 3 beads to this rod causes a carrying of 1 bead to the next (hundreds) rod and remaining 1 at the beam of the ten-rod. And so on till one gets the resulting sum of 4910: represented at the central beam by four unit-beads on the thousands-rod, one 5-bead and four unit-beads on the hundreds-rod, one unit-bead on the tens-rod, and none on the unit-rod. Note that only the first number (3572) is seen on the
25 abacus. The addend (1338) is not depicted at all, and every “addition” step involves only one digit and its

1 corresponding addend digit. Not only is the “big picture” concept of the addition process invisible to the
 user, it reduces addition into a series of mechanical finger operations with at most minimal two single-
 digit additions. Needless to say, such operations involving addition of multiple addends and subtractions
 are even less comprehensible to a child. Multiplication by these abacus-using repetitive additions-is
 5 tedious to say the least. Division would be a stretch, by any means. Except, perhaps, for adults.

In fact, these were used – by adults - until the Arabic/Indian invention of zero leads to the use of
 place value in written form and enable the more efficient paper and pencil approach. So the use of the
 abacus began to decline, even before the advent of electronic calculators.

Eventually and paradoxically, however, starting in Asia, and gradually migrating west, the abacus,
 10 traditionally an adult instrument, began to be used for children. This is the 2nd Generation. There were
 minimal, if any, adaptations to the instrument, but it was appended with innovative instructional
 approaches, in usage and pedagogy.

In spite of these appendages, the fit between an adult instrument and its young students was
 uncertain, from the fact that there has so far not been any qualifying studies of effectiveness, and
 15 according to practitioners, students may need as long as two years to achieve mastery. One reason could
 be that the adult-abacus assumes that the user has developed a fairly large short term memory to hold
 the intermediary results in a multi-step process since the instrument requires the erasure and immediate
 replacement of each intermediary result with the next. Young users have by and large not developed
 such a capacity. Otherwise, the abacus is certainly ideal for teaching young children early numerical and
 20 operational concepts, being visual, concrete and practical.

The next logical evolution of the abacus, the 3rd Generation, would then be a redesign, e.g. doing
 away with the need for a holding memory during the calculation process - to customize the instrument
 specifically for young learners, to be able to teach effectively, beyond just how to use the instrument, but
 essential and foundational mathematics.

25 One of the early attempts in this category would be the Slavonic abacus (schoty) – sometimes

- 1 called the 10 by 10 abacus. It consists of 10 rods, which can be held horizontally or vertically, each with ten beads. The 10 beads on the the top five rods are separated into two sets of fives, distinguished by their respective colours: the 5 on the left-Colour A-and 5 on the right-Colour B. The bottom five rods are similarly strung: each with two sets of 5 beads, in different colours-Colour A for the right group and
- 5 Colour B for the left, respectively. Because of its simplicity, it is ideal for introducing numbers, number bonds, place value, carrying and borrowing in simple addition and subtraction, and early multiplication concepts. It has in fact become the de-facto template for abacus toys. Unfortunately, in spite of its simpler design, it is not much better than 1st Generation adult abacus for teaching complex concepts and processes (e.g. addition of more than two big numbers - without depending on holding memory of
- 10 intermediary solutions between steps, multiplication to 12, division, etc.)

Abacus3 is a 3rd Generation abacus, specially aimed at teaching young and older children essential, comprehensive, foundational mathematical concepts and processes, and enabling feedback and practice where needed.

- It is represented by two versions - Abacus3.1 and Abacus3.2 - which are based on the same
- 15 inventive principles, but are designed specifically for Addition/Subtraction and Multiplication/Division respectively.

- In addition to the simple number concepts and processes covered by the earlier abacus, Abacus3.1 can be used to demonstrate and teach addition of multiple addends (of course, together with place value, decimal places, carrying and borrowing) and subtraction – while showing simultaneously all
- 20 addends, minueands and subtrahends. Abacus3.2 can be used to formulate and understand times tables and division, and also enable practice and feedback.

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REFERENCES TO PRIOR INVENTIONS

As expected, almost all the patents filed on abacus, were on or related to the Generation 1 and Generation 2 abacus.

5 **Patents Related to Generation 1 abacus:**

CN 203870505U 9/10/2014 Conveniently Dismounted Multi-functional Abacus. This invention is based on taking the regular abacus and making the frames, rods, beads and beams detachable and allowing for re-assembly, thus allowing for modification by the user. Thus user could change a 2-bead:5-bead Chinese suanpan into a 1-bead:4-bead Japanese Soroban, and so on. It is indicative of the inventor's realization that the abacus may need improvements which is positive. However, how it's improved for the better will need more than mechanical flexibility.

Patents Related to Generation 2 abacus:

These patents focus on innovative approaches to learning and using the Generation 1 adult abacus in order to make it more accessible to young users. Most of these do not make changes or improvements to the adult abacus, and focus on the learning of the device or on using it for various purposes (such as language) more than the in-depth learning of Math.

WO 2010014464A2 4/2/2010 Computer-based Abacus Training System – This system is meant for training on the Japanese soroban abacus.

20 KR 1020050048552A 24/5/2005 Integrated Learning of Languages and Alphabets using the abacus, mental arithmetic and the teaching and learning of language.

US 20070238078A1 11/10/2007 Method for Teaching Fundamental Abacus Math Skills – This invention combines music and singing with numbers to augment learning, and is based on the Japanese abacus.

US 8672683B2 18/3/2014 Computer-based Abacus Training Systems - this invention utilizes multiple sets of the Japanese Soroban abacus and software for training for application of the soroban.

1 Patents Related to Generation 3 abacus:

These patents are examples of attempts to simplify and modify the abacus for young users, and mostly focus on Math learning.

US 20070048708A1 1/3/2007 Motivational and Educational Enlarged Abacus. This is probably the most

- 5 elementary of abacus, consisting of large beads (presumably 5 or 10) on a single rod, standing horizontally on triangular supports at both ends. It's obviously meant only for display and simple counting.

US 20120251986A1 4/10/2012 Abacus-type Math Teaching Device And Method – This invention is a slightly more complex abacus than the above, with two rods containing 20 beads each, in 2 colours – the

- 10 beads in each rod being of the same colour but different from that on the other rod. It is clearly meant for simple counting and place value concepts.

CN 2898992Y 9/5/2007 Tool for Teaching Children of (sic) Math – This is a very simple abacus with only 2 vertical rods, each with 10 beads. Needless to say, apart from very simple concepts and operations, it is not meant for scaling up for older children.

- 15 CN 2116933U 23/9/1992 Counter – A simplified 3-rod abacus with 10-, 20-, and 20-beads, in colored groups of fives, and no beam. Place value to a hundred, and simple addition and subtraction, and perhaps simple multiplication – using repeated additions. It can deal only with limited numbers of addends and at most one ten-bond in addition.

CN 2101035444Y 12/3/2008 Nine Bead Abacus – and CN 2616930Y 19/5/2004 Nine-bead Digital Abacus

- 20 – This consists of an innovative new abacus design involving no separation-beam and 9 beads in a plurality of rods, arranged in 4-1-4 formation in 3 groups distinguished by different colors. The design is non-intuitive and definitely not meant for young children.

CN 201000587Y 02/1/2008 Ten Beads Ladder Abacus – This invention is a radical design, consisting of 11 vertical rods, no central beam, and 10 beads on each rod in two colours (C1 and C2) so that the first rod

- 25 has 0C1/10C2-i.e. no C1 coloured beads and 10 C2 coloured beads; the second has 1C1/9C2, and so on

- 1 till the 11th which has 10C1/1C2. The visual effect is that of a step ladder – hence the title. Whatever efficacy it has for adult users, it will pose a challenge for children to learn and use.

CN 2611955Y 14/4/2004 Special Abacus for Child (sic) – This is an unusual arrangement, with 9 rods each containing 9 beads separated into 3 parts of 3 beads each, each part being in a different color. With no

- 5 beam, each bead on a rod has the same value, the colours serving only to make counting easier.

US 20080096169A1 24/4/2008 Abacus for Math and English – This invention uses the Japanese abacus, enhancing it for teaching English as well, by connecting arithmetic and alphabets.

- US 20120028229A1-Full 2/2/2012 Augmented Simple Abacus with an Underlying Grid of Numbers of a Blank Sheet – This is an extremely simple abacus with 100 beads in 10 rods of 10 beads each, with a grid
10 of numbers in sequence from 1 to 100 underneath. Thus there's no place value and it's not meant for concepts beyond counting to 100.

- US 20130236864A1 12/9/2013 Use of Colored Beads in An Augmented Simple Abacus – This is an addition to the above invention, with the additional feature of beads being coloured alternatively in 2 colours: thus beads representing 1,2,3, ...100 would be coloured C1,C2,C1,C2....C2. Apart from teaching
15 the concept of even and odd numbers, it is unclear how the augmentation helps.

- US 20070166673A1 19/7/2007 Abacus – This invention offers slight modifications to the existing abacus by: a).ensuring that the number of rods are in a multiple of 3 (R1,R2,R3/R4,R5,R6/...). The inventor's rationale being that the decimal system is divided into place value groups of 3: 1s,10s,100s / 1000s,10000s, 100000s / etc.
20 b) having 3 identifiers [eg colours (C1, C2, C3)] for each group where C1, C2, and C3 are used respectively for each of the rods (R1,R2,R3...) in a group and repeated for the next group/s: thus, for R1, R2, R3 and repeated for R4, R5, R6 and so on.
c) increasing the number of beads per rod to 18 (or more, presumably to multiples of 9) – the colour being the same for all 18 (or more) beads in each rod-the large size of the group does not facilitate
25 subitizing.

- 1 d) eliminating the central (reckoning) beam but introducing a central line to act as a divider and number
rulers/labels on the sides of the frame for use in the counting of beads. Non-participating beads in an
operation are parked above and touching the line, while beads involved in the operation are pushed to
the bottom of the frame-which acts as the “reckoning beam.” The side labels are positioned such as to
5 act as markers for counting the beads above and below the central line. But the constant need to
reference the line and side labels could inhibit with the smooth operation of this abacus.

It is clear that the abacus is essentially one of the best tools for teaching math concept and
processes to pre-school and older children, because of its visual and tactile nature, and ability to
concretize abstract concepts such as place value, carrying and borrowing, multiplication and division.

- 10 But, as the above analysis shows, by its current forms, it is inadequate. Even with innovative instructional
approaches and minor changes (Generation 2) to the adult abacus (Generation 1) there are major short-
comings. It is also clear that the abacus need to be modified and re-invented, specifically for teaching
basic and foundational math concepts and processes to children. There is no better proof of this than the
fact that inventors from east and west are offering a wide range of new products and innovations
15 towards this end, by basically changing the abacus itself (Generation 3.) As the above analysis also shows,
past and current attempts are either too radical, and thus difficult to learn and use, or too limited in their
application, leaving out certain foundational and essential concepts and processes.

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SUMMARY OF THE INVENTION**Abacus3**

This invention addresses the shortfall, and offers a Generation 3 abacus that is not only easy and intuitive for learning and use, but that covers all the essential foundational concepts and processes, from subitizing to multiple addends, construction of multiplication tables, and division. The uniqueness of the invention lies in its use of colour, labels, round and printed flat beads – preferably grouped in 5s, *both* horizontally and vertically - and elastic strings as well as firm rods, albeit held together by a regular frame. Abacus3 is represented in two embodiments below, Abacus3.1 (for Addition & Subtraction) and Abacus3.2 (for Multiplication & Division.)

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Abacus3.1 Addition and Subtraction – See Drawings: Figure 1

This representation is meant for teaching of number concepts and operations from the simplest to the most important and foundational in addition and subtraction. The invention takes young children through number values, number counting, subitizing (counting by recognizing groups of objects), place value, simple addition, addition with multiple addends, “carrying” in addition and “borrowing” in subtraction. Not only does it allow children to be taken through addition/subtraction processes step by step, but it enables children to clearly see and understand how problems are solved, and transit seamlessly to mental math and paper and pencil approaches.

20 Abacus3.2 Multiplication and Division – See Drawings: Figure 2a, 2b, 2c

Abacus3.2 is designed for teaching multiplication and division. Unlike traditional teaching methods which merely provide times tables for children to study and memorize, abacus3.2 enables children to understand and even construct the times tables, and provide feedback and practice. Not only does Abacus3.2 help children to understand the meaning and application of multiplication, it also leads to better understanding and application of division as well.

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DRAWINGS

Abacus embodiments within the scope of the invention are shown in the following drawings. Other variations or embodiments are possible and within the scope of the invention, some of which are described accordingly.

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Figure 1 Abacus3.1

This figure shows an embodiment of Abacus3.1, designed for teaching basic numbers, counting, subitizing, place value, and addition and subtraction. It consists of a square or rectangular frame (1), a multiplicity of rods (or strings)-fifteen in this embodiment (2)-each of which is strung with 10 beads.

10 [Other embodiments can be made by minor modifications: reducing or increasing the rods and beads to make the abacus simpler or more complex.] In this embodiment, the first 5 rods (from the right) represent numbers with place value 1 – i.e. each bead here represents value 1. These rods are bracketed by Labels 3c and 3d, with the latter indicating the place value “1”, and both with Colour C1, say. The top 25 beads in these 5 rods (3a) are in a darker shade of C1, while the bottom 25 beads (3b) are in a lighter shade of C1.

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Similarly, the next 5 rods contain 10 beads each representing place value 10, which is indicated by the “10” printed on label 4d which, with label 4c, bracket the 50 beads. The top 25 beads (4a) are coloured a light shade of C2, and the bottom (4b) are coloured a darker shade of C2, where C2 is the colour of labels 4c and 4d.

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The next 3 rods likewise represent place value 100, bracketed by labels 5c and 5d (printed with “100”), both of Colour C3. The 30 beads here are such that the top 15 beads (5a) are coloured a darker shade of C3, while the bottom 15 (5b) have a lighter shade of C3.

The final 2 rods in this embodiment consist of 10 beads each, with place value 1000, indicated by a label (6d) which brackets these 20 beads with label (6c)-both labels with Colour C4. The top 10 beads (6a) are

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coloured a light shade of C4 and the bottom 10 a darker shade of C4.

1 Note how the colours and shades are designed to identify beads according to their place values (all beads in the same place value sector being of the same colour) as provide horizontal and vertical groupings (by fives) to facilitate subitizing. Also, the beads in this embodiment are spherical with a hole through its centre, for them to be strung on the rods, as shown in 3 views in Figure 1.

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Figure 2a Abacus3.2

This figure shows an embodiment of Abacus3.2, designed for teaching multiplication and division, up to 12. The frame here (13) is taller, with 12 elastic strings strung vertically. 12 labels printed with numbers 1 to 12 are on the top of the frame (9b) indicating columns, and 12 similar labels are on the left-hand side on the top half of the frame (9a) indicating rows of beads.

Each string contains 12 beads, coloured in groups of 5 rows (with remainders) counting from the top. The beads are also coloured in groups of 5 columns, counting the strings of beads, from the left. Thus, the top left group of 25 beads (10a) -in columns 1 to 5, and rows 1 to 5- are coloured c1, the next 25 beads (10b) -in columns 6 to 10, and rows 1 to 5- are coloured c2, and the next 10 beads (10c) -in columns 11 and 12, and rows 1 to 5- are coloured c3. Considering the next set of 5 beads below across the columns, the 25 beads (11a) in columns 1 to 5 and rows 6 to 10 are coloured c4, the next set of 25 (11b) -columns 6 to 10 and rows 6 to 10- are coloured c5, and the 10 beads (11c)-in columns 11 and 12 and rows 6 to 10-are coloured c6. Finally, the last two rows contain 10 beads (12a) in columns 1 to 5 coloured c7, 10 beads (12b) in columns 6 to 10 coloured c8, and 4 beads in columns 11,12 coloured c9.

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Figure2b Abacus3.2

The beads are flat beads as shown in 3 views (Figure 2b), to enable printing of the 144 products of the 12 rows of multipliers X 12 columns of multiplicands. Figure 2b shows the reverse side of the beads—with products printed. The product of 12X5 (multiplier 12 in row12, X multiplicand 5 in col.5) is 60, as shown. Finally, this explains why elastic strings (8) are used: to enable the beads to be flipped over.

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DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS**Abacus3.1 Addition and Subtraction – See Drawings: Figure 1**

This representation is meant for teaching of number concepts and operations from the simplest to the most important and foundational in addition and subtraction. It consists of a plurality of rods (or wires, elastic strings) preferably at least 10 and a multiple of 5, held vertically by a frame (preferably of light, unbreakable material, and of size and dimension that are easily handled by young children.) Each rod is strung with a plurality of beads, preferably at least 10, and a multiple of 5. The beads (which could be spherical or flat circular or rhombus in shape) are coloured, so that groups of 5 beads vertically (in each rod) are of the same colour, to enable *subitizing* – quick counting by seeing number groups- and 5 beads horizontally (in 5 adjacent rods) are of the same colour denoting that they have the same place value.

Thus, every five adjacent rods would represent one place value. For this discussion, we will consider a possible variation, where the first group of 5 rods (starting from the right) represent Unit place value; the second group of 5 rods represent Ten place value; and the third (last) group of 5 rods is divided into two: 3 representing 100 place value, and 2 last rods representing 1000 place value. This variation allows the extension of place value considerations to 1000, without extending the width of the device.

The place value groups are further differentiated visually by labels on the top and bottom surfaces of the frame. Starting from the right, the first pair of labels both coloured C1 is indicated on the top label by the place value 1 represented by the five rods within it ; the second pair, coloured C2, would similarly be indicated by place value 10, represented by the five rods within it; and the third pair. coloured C3, would similarly be indicated by place value 100, represented by the three rods within it; and finally, the fourth pair, coloured C4, would similarly be indicated by place value 1000, represented by the two rods within it.

1 Preferably, the beads within the respective labels are coloured in 2 shades of the labels' colour to identify them clearly to belong to the respective place value sector. Thus, the top 25 beads in the first 5 rods (starting from the right) with place value 1, are coloured a dark shade of C1, and the bottom 25 beads in the same rods are coloured a light shade of C1. Similarly, the top 25 beads in the second 5 rods
5 with place value 10, are coloured a light shade of C2, and the bottom 25 beads in the same rods are coloured a dark shade of C2. Similarly, the top 15 beads in the next 3 rods with place value 100, are coloured a dark shade of C3, and the bottom 15 beads in the same rods are coloured a light shade of C3. Finally, the top 10 beads in the last 2 rods with place value 1000, are coloured a light shade of C4, and the bottom 10 beads in the same rods are coloured a dark shade of C4. This selective and alternating
10 colour scheme is an unique feature of Abacus3.1, allowing for ease of *subitization* and identification of place value, and essential for an abacus that has multiple rods per place value and for depicting operations involving multiple addends.

 Unlike the traditional abacus, Abacus3.1 has no central reckoning beam. Instead, the top of the frame acts as a reckoning beam. Thus, at the beginning of an operation, whether addition or subtraction,
15 all the beads are moved to sit (resting beads) at the bottom of the rods. To add 2457 to 1839 for example, working from the left: 2 beads on rod1 (place value 1000), 4 beads on rod3 (place value 100), 5 beads on rod6 (place value 10) and 7 beads on rod11 (place value 1) are pushed to the top - with the top-most bead touching the frame (active beads). Next, 1 bead (on rod1), 8 beads (6 on rod3 and 2 on rod4), 3 beads (on rod6) and 9 beads (3 on rod11 and 6 on rod12) are moved up. For the younger
20 learners, it may be necessary to leave a gap between the beads representing the two addends so as to enable them to see the process step by step. Note that two separate rods are used for 4 and 8, and for 7 and 9, although the same place values – 100 and 1 respectively – to allow for this step by step process.

 The addition process is then effected, one place value at a time, starting from the right, by pushing the respective active beads to the top together-note the 10-bonds are clearly seen indicated by
25 a rod full of 10 active beads. Thus 7 add to 9 equals 16 beads, leaving 6 beads in place value 1, and

- 1 “carrying 10” : i.e. moving 10 active beads down to the bottom resting position, and moving 1 (resting) bead up at place value 10. Pushing the active beads in place value 10 together to the top, one gets 5 plus 3 plus 1 (carried over from the last addition) equals 9. In place value 100, the total number of active beads combined equals 12 (4 plus 8), leading to carrying of 1000 – i.e. pushing 1 resting bead up on rod
- 5 1- and leaving 2. Finally, at the 1000 place value, total active beads combined equals 2 plus 1 plus 1 (carried from the last addition) equals 4.

The answer is clear to see: with 4 active beads in place value 1000, 2 active beads in place value 100, 9 active beads in place value 10 and 6 active beads in the Unit place: 4,296, Note that not only is the 10-bond clearly visible, but 9 (5+4) and 6 (5+1) are clearly subitized.

- 10 The same process can be applied to 3 to 4 addends, to aid transfer of learning to reinforce the process of adding within a place value, 10-bonds, and “carrying.”

- For subtraction, say $3457 - 1382$, the *minuend*, 3457, is represented by moving up (make active) 3 beads on rod 1, place value 1000, 4 beads up on rod 3, 5 beads up on rod 6, and 7 beads up on rod 11. Starting from the right, and moving left, one place value at a time, the *subtrahend* (1382) is represented
- 15 by taking down (make passive) the respective and relevant beads. Thus, in place value 1, 2 beads from the 7 are moved down. In place value 10, there are only 5 active beads from which 8 beads need to be taken down, so “borrowing” from the next place value is needed: i.e. 1 bead from the 4 that are in place value 100, is moved down (made passive) while at the same time, 10 beads in place value 10 are moved up (made active.) This move increases the beads in place value 10 to 15, from which we can take away 8
- 20 (move down to passive) leaving 7 active beads. In place value 100, we now have 3 active beads, from which we need to take away 3, thus moving all 3 beads down to passive, leaving no active beads. And finally, in place value 1000, we have 3 active beads, from which to take away 1 (move down to make passive) leaving 2 active beads. The answer to the subtraction, the *difference*, is now clearly seen, counting the active beads (easily done with *subitizing*) in each place value sector from left to right: 2075.

- 25 Thereafter, children can be given practice to lead them to be able to visualize abacus addition and

- 1 subtraction operations in mental math, and taught to do addition and subtraction on the abacus side by side with their paper and pencil counter-part, to transition to paper and pencil solutions.

Abacus3.2 Multiplication and Division – See Drawings: Figures 2a, 2b, 2c

- 5 Abacus3.2 is designed for teaching multiplication and division. Abacus3.2 enables children to understand and even construct the times tables – up to 12 in this embodiment.

Like Abacus3.1 this abacus has a frame, and labels. Its labels are on the left-hand side of the frame (indicating rows 1 to 12 -the multipliers) and the top side (indicating columns 1 to 12-the multiplicand) each label with numbers 1 to 12 accordingly. However, this embodiment of Abacus3.2 has
 10 exactly 12 elastic strings (in place of rods) attached vertically to the frame, and each having 12 flat beads strung on it (totaling 144), identified by colour into three groups: 5-5-2 (from the top). [Slight modifications are needed for other embodiments: e.g. for up to 14 times tables, the abacus will have 14 strings, each with 14 beads, in the coloured groups 5-5-4 (from the top)]. The beads are also identified by colours into groups of 5-5-2 horizontally (from the left).

- 15 The 12 rows and 12 columns contain 144 beads: the top 5 rows have 25 beads in columns 1 to 5 coloured c1, 25 beads in columns 6 to 10 coloured c2, and 20 beads in columns 11 to 12 coloured c3. The 5 rows below these have 25 beads in columns 1 to 5 coloured c4, 25 beads in columns 6 to 10 coloured c5, and 20 beads in columns 11 to 12 coloured c6. The last 2 rows below these have 10 beads in col. 1 to 5 coloured c7, 10 beads in col.6 to 10 coloured c8, and 4 beads in col.11 to 12 coloured c9.

- 20 As for Abacus3.1 these selective and alternating groupings in 5s by colours enable subitizing, which in turn facilitates the processes involved in the construction of the times tables.

Abacus3.2 enables the child to construct times tables from the first principle that multiplication is repeated addition or counting in groups. Thus, to construct the 2 times table, beginning from the starting point where all the beads are pushed to the bottom (passive) - the child moves up two rows of beads
 25 (counting from the top row) in 1 to 12 columns (counting from the left) to the top edge of the frame. So,

- 1 $2 \times 4 = 2$ rows of beads in 4 columns = 8 beads. And $2 \times 7 = 2$ rows of beads in 7 columns = 14 beads.

Note that the latter product is clearly seen from the group of 10 (colour c1) and group of 4 (colour c2.) This subitizing advantage is even more prominent when large numbers such as 8×7 are involved.

- Note that the beads used in Abacus3.2 are *flat* beads, strung on elastic strings (of silicon, plastic or metal) so that they can be flipped over, to show the other side, which for each bead is printed (or stickered) with the appropriate product-the result of the multiplication of its row (multiplier) with its column (multiplicand). For example, the bead on the 12th row and 5th column represents 12×5 , and the product, 60, is printed on the flip side. All 144 flat beads are thus likewise printed. This allows the students to get immediate feedback as they construct the times tables step by step. Also, it can be used to turn the abacus into a game-board where students can play solitaire or with one another, guessing the products before they flip the respective beads to check their answers.

Preferably, a background sheet, matched with the 144 beads when moved to the top edge, and printed with a 12×12 table diagram with 144 squares differently colour-coded to indicate the scores associated with each product – the most difficult getting the highest scores etc. (Figure 2c)

- Once multiplication is mastered, Abacus3.2, with no modifications, can be used to teach division. For example, to get $57 \div 6$, student will move 6 rows (the divisor) of beads per string, starting from left to right, up to the top edge (from passive to active) till 57 (the dividend) beads are counted: which would be 9 columns of 6 beads (=54 beads) and another 3 beads on the 10th column. Note that this process is facilitated by subitizing which is enabled by the colour groupings. Note also that the result of the division just jumps out of the abacus: the quotient is 9 (from the columns) with a remainder of 3.

- The layout of the abacus at the end of the process even mirrors the written layout of division: a right-angle with unequal sides, positioned like a long roof and a short side wall at its left edge. The *divisor* is on the left of the “wall” which separates it from the *dividend*, which is directly under the “roof”, and the *quotient* is on the “roof-top” with the *remainder* by the right-side of the *dividend*. This allows the student to easily and “seamlessly” transit from the concrete to the written form.

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CLAIMS**Abacus3.1**

1. An abacus device for teaching foundational number concepts and addition and subtraction operations to young children, which has a multiplicity of rods (or strings) of beads held vertically by a rectangular or square frame, and wherein, instead of one rod of beads representing 1 place value, such as in the traditional abacus, each place value is represented by more than one rod: preferably, but not necessarily, in groups of 5 rods. For example, in the embodiment considered here (Figure 1), there are 5 rods of beads in place value 1 sector, wherein each bead represents the value 1. There are 5 rods in place value 10 sector, wherein each bead represents the value 10, 3 rods in place value 100 sector wherein each bead represents 100, and 2 rods in place value 1000 sector, where each bead has a value of 1000.
2. A device as claimed in Claim 1 wherein the group of rods of beads (a sector) representing each place value is bracketed by labels, one on the top side and the other on the bottom side of the frame, which are differentiated from the labels of other place value sectors by colour.
3. A device as claimed in Claim 2 wherein the top label is printed with a number representing the place value of the respective sector: eg. 1, 10, 100, 1000, etc.
4. A device as claimed in Claim 3 wherein each rod consists of a plurality of beads, in multiples of 5, wherein each group of 5 beads is separated from the others by colour, preferably different shades of the respective sector's colour.
5. A device as claimed in Claim 2 and 4, wherein the beads are thus grouped in 5's by colour both horizontally and vertically, to facilitate subitizing in both directions. For example, in the embodiment that is being used for discussion (Figure 1), there are 5 rods in place value sector 1, each rod containing 10 beads, separated in two groups wherein the top 5 and the bottom 5 are differentiated by being in 2 different shades of colour of the sector. Considering all the 5 rods of beads in this sector, there'll be a group of 25 beads on top and a group of 25 at the bottom,

- 1 each group coloured by a shade of the sector's colour that differentiates it from the other group
where the beads are coloured by a different shade.

Abacus3.2

- 5 6. An abacus device for teaching multiplication and division by helping children understand and
build the times tables from basic principles, which has strings of beads held vertically by a
rectangular or square frame, and wherein, each string has as many beads as there are strings.
For example, in the embodiment shown in Figures 2a, for up to 12 times tables, there are 12
strings and 12 beads in each string.
- 10 7. A device as claimed in Claim 6, wherein the strings are elastic (in metal, silicon or plastic).
8. A device as claimed in Claim 6 wherein there are labels on the left-hand side and the top side of
the frame (converging at the top left-hand corner), with printing from 1 to 12 (for the
embodiment in Figure 2a), indicating rows of beads (numbering from top to bottom) and
columns of strings (numbering from left to right), respectively. The rows represent multipliers
and columns multiplicands, so that 12X5 would be represented by 12 rows and 5 columns of
15 beads; and the product would be represented by the bead at the 12th row and 5th column.
9. A device as claimed by Claim 6 wherein the beads on the strings are flat beads (preferably
round) with printing on one side, showing the product for each set of multiplier and
multiplicand represented by row and column respectively. Thus, referring to the embodiment
20 under consideration (Figure 2b), the total number of 144 beads are printed with the 144
respective products, eg, as shown, the bead at row 12 and column 5 would be labelled 60. The
elasticity of the strings in Claim 7 allows the flat beads to be flipped – hiding and revealing the
printed side as needed.
10. A device as claimed by Claim 6 wherein the beads are coloured, to facilitate subitizing, in groups
25 of 25, counting 5 beads from the left and 5 beads from the top – with remainders where

- 1 applicable. For the embodiment discussed in Figures 2a, the first 5 strings of beads (columns) in
rows 1 to 5 are coloured c1, the next 5 strings of 5 beads each are coloured c2, and the
remaining 5 beads in each of the last two columns are coloured c3. Similarly, considering the
next 5 rows below, the first 5 columns of beads are coloured c4, the next 5 columns c5, and the
5 remaining 2 columns c6. Considering the last 2 rows, the first 5 columns are coloured c7, the
next 5 c8, and the remaining 2 columns (total of 4 beads) are coloured c9.
11. A device as claimed in Claim 6 wherein a sheet of document (preferably detachable and
laminated) in the background, beneath the 144 beads, consisting of a graphic representing a
table (12X12 in this embodiment) with 144 squares, each positioned below one of the beads
10 when all the beads are moved to the top and left (Figure 2c). This may be used as immediate
feedback for students to practice their times tables or in a game where students can use for
practice and revision in a fun way.
12. A device claimed in 1 to 5 and a device claimed in 6 to 11 above, wherein all possible
embodiments in said devices are represented in any form, including but not exclusively: printed,
15 constructed, software, or a projection on a screen.

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