

CRYPTOGRAPHY BY PARAMETERIZING ON AN ELLIPTIC CURVE

The present invention relates to message cryptography based on the use
 5 of points on an elliptic curve, and more particularly said use in the field of
 algorithms of the EKE (Encrypted Key Exchange) type.

Methods of authentication by password are based on the use of an
 algorithm of the EKE type. This type of algorithm is illustrated in Figure 1.

A device D 10 wishes to be authenticated on the basis of a password π
 10 at a controller C 11. Each of these entities knows the password π . Moreover,
 let us consider an elliptic curve $E_{a,b}$, and a generator G of the set of points on
 the elliptic curve, as public parameters. The elliptic curve satisfies the following
 equation:

$$E_{a,b}(x, y) : x^3 + ax + b = y^2 \quad (1)$$

15 At a step 12, the device D 10 generates a random number r_1 . Then, it
 transmits this random number in the form of a point on the elliptic curve to the
 controller 11. For this purpose, it determines a value V to be transmitted
 satisfying:

$$V = r_1 \cdot G$$

20 This result is then encrypted with the password in the form $E_\pi(r_1 \cdot G)$, E_π
 being an encryption function by password.

Then, the device sends a message 13 to the controller stating the value
 $E_\pi(r_1 \cdot G)$.

On receiving the message 13, in its turn the controller 11 generates a
 25 random value r_2 at a step 14. Then it transmits this value in the form of a point
 on the elliptic curve, and transmits a message 15 to the device 10 stating the
 result:

$$E_\pi(r_2 \cdot G)$$

Following this exchange of random values r_1 and r_2 in encrypted form, the
 30 device 10 recovers at a step 16 the random value r_2 generated by the
 controller by deciphering, using a decryption function with password D_π , the
 information contained in the message 15:

$$r_2.G = D_\pi E_\pi(r_2.G)$$

and the controller 11 recovers at a step 17 the random value r_1 generated by the device 10 by deciphering the information contained in the message 13:

$$r_1.G = D_\pi E_\pi(r_1.G)$$

5 Thus, following a protected exchange, each entity is able to calculate a common key K:

$$K = r_1.r_2.G$$

This type of algorithm aims to exchange values in a form encrypted with a password or a key derived from a password. However, it should be noted that
 10 according to a classical representation of an elliptic curve satisfying equation (1) on a finite field F_q , the exchanges described with reference to Figure 1 can allow a potential attacker to deduce information relating to the password π . In fact, exchanging random values in an encrypted form as described above, via the two messages 13 and 15, supplies a redundancy of information which may
 15 allow the secret of the password to be broken. More precisely, at each listening-in, an attacker could test a password to decipher the information exchanged in messages 13 and 15. He is then presented with two cases. In the first case, the decrypted information corresponds to a point on the curve and accordingly the password is correct. In the second case, the decrypted
 20 information does not correspond to a point on the elliptic curve and the password is not discovered. By multiplying the listenings-in and the separate passwords, it is thus possible to find the password belonging to a finite set of elements.

The present invention aims to improve the situation.

25 A first aspect of the present invention proposes a method of controlling a device by a controller on the basis of a password;
 said method comprising the following steps, at the device or at the controller:

/1/ on the basis of a random value r_1 , determine a point $P(X,Y)$ on an elliptic curve, in a finite field F_q , q being an integer, of equation:

30
$$E_{a,b}(x,y) : x^3 + ax + b = y^2 \quad (1)$$

/2/ obtain first and second parameters k and k' , such that

$$P=F(k,k')$$

where F is a surjective function of $F_q \times F_q$ in F_q

/ 3/ obtain the first and second parameters in encrypted form by encryption as a function of the password; and

5 / 4/ transmit said first and second encrypted parameters to the controller;

in which the function F is such that, regardless of z and z' input elements of F_q , $F(z, z')$ is a point on the elliptic curve, and the input elements do not satisfy equation (1).

Owing to these arrangements, it is possible to avoid the attacks such as
 10 described above with respect to the prior art. In fact, under these conditions it is no longer possible to test a password by determining whether the result obtained corresponds to a point (X, Y) on the elliptic curve $E_{a,b}$ since the function F always supplies a point on the elliptic curve as output regardless of the parameters that were supplied as input, as the input parameters do not
 15 satisfy the classical equation of an elliptic curve. Ingeniously, it therefore envisages representing a point on the elliptic curve in a different way, using this surjective function F , so as not to supply any information to a potential attacker that can enable him to deduce the password used.

In order to determine said function F allowing a point $P(X, Y)$ on an elliptic
 20 curve to be represented according to some other parameterization than that of equation (1), it is possible to use as a basis an invertible function $f_{a,b}$, the inverse function $f_{a,b}^{-1}$ of which makes it possible to recover a point on the curve, according to two parameters that do not satisfy equation (1), from an input parameter.

25 The function F can be written:

$$F(k, k') = f'(k') + f_{a,b}(k)$$

where $f_{a,b}$ is an invertible function, based on the coefficients a and b of the elliptic curve, taking an input parameter and supplying a point on the elliptic curve and

30 where f' is a function generating a point on the elliptic curve as a function of a parameter;

and

where, at step /2/, the parameters k and k' are obtained by the following steps:

- randomly generate a value of the parameter k' ;
- calculate a value of $f'(k')$;
- determine a value of the parameter k from the following equation:

$$5 \quad k = f_{a,b}^{-1}(P(X,Y) - f'(k'))$$

Advantageously, by combining a function f' that makes it possible to generate a point on the elliptic curve on the basis of a parameter k' of random value and an invertible function $f_{a,b}$ based on the coefficients of the elliptic curve a and b , it is then possible to obtain a function F that allows
 10 parameterization of a point on the curve while avoiding the attacks described.

The function f' can in particular be written:

$$f'(k') = k'.G$$

where G is the generator of the set of points on the elliptic curve; and a value of the parameter k is determined from the following equation:

$$15 \quad k = f_{a,b}^{-1}(P(X,Y) - k'.G)$$

Under these conditions, the function F can be written:

$$F(k, k') = f_{a,b}(k) + k'.G$$

Alternatively, the function f' can be written:

$$f'(k') = f_{a,b}(k')$$

20 and

a value of the parameter k is determined from the following equation:

$$k = f_{a,b}^{-1}(P(X,Y) - f_{a,b}(k'))$$

Under these conditions, the function F can be written as follows:

$$F(k, k') = f_{a,b}(k) + f_{a,b}(k')$$

25 Thus, generally, for parameterizing a point $P(X,Y)$ on the curve according to the present invention, a first step envisages obtaining a point on the curve for a value of the parameter k' . Then a point on the curve is determined which corresponds to the subtraction of the point $P(X, Y)$ to be represented and of the point obtained at the first step. Then another point on the elliptic curve is
 30 obtained. Next, the inverse function of $f_{a,b}$ is applied to this point and a value of parameter k is obtained. The point P is then represented in the form of the pair

of parameters k and k' that does not satisfy equation (1).

In one embodiment of the present invention, the function F comprises at least one invertible function $f_{a,b}$, such that

$$\forall P = (X, Y) \in E_{a,b}, |f_{a,b}^{-1}(X, Y)| < L$$

5 where L is an integer having a relatively small value in relation to the number of points on the elliptic curve (1).

The fact that function F comprises a function $f_{a,b}$ means that application of the function F to first and second parameters corresponds to applying this function $f_{a,b}$ to at least one of the first and second parameters. We can thus
10 envisage, on the one hand, generating a point on the curve on the basis of the first parameter and, on the other hand, applying the inverse function of $f_{a,b}$ to obtain the second parameter. By following this procedure, a point on the elliptic curve can be represented according to these two parameters.

Thus, application of the function F to first and second parameters can
15 correspond to application, to at least one of the two parameters, of a function $f_{a,b}$ such that

$$\forall P(X, Y) \in E_{a,b}, |f_{a,b}^{-1}(X, Y)| < L \quad (4)$$

where L is an integer having a relatively small value in relation to the number of points on the elliptic curve (1).

20 In other words, the function $f_{a,b}$ considered here has a pre-image corresponding to a set of points P on the elliptic curve $E_{a,b}$ bounded by a quite small maximum value relative to the number of points on the elliptic curve. In fact, if this is not so, it is then possible to invert the function $f_{a,b}$ simply by taking a number at random. In this context, it can for example be considered that L is
25 smaller than $1/2^{80}$ times the number of points on the curve.

By using an invertible function satisfying conditions (4), it is easy to obtain a function F for representing a point on the curve via parameters k and k' which do not permit an attack on the password used for encrypting them, such as that described above.

30 In one embodiment of the present invention, the function $f_{a,b}$ makes a pair of parameters (x, y) correspond to a parameter u such that x is the only solution of the following equation:

$$x^3 + ax + b - (ux + Q(u, a, b))^2 = 0 \quad (5)$$

and y satisfies:

$$y = ux + Q(u, a, b)$$

This equation (5) is obtained by replacing y in equation (1) by the term
 5 $ux + Q(u, a, b)$. The term $Q(u, a, b)$ denotes a rational fraction in the variables u, a and b.

The fact that this equation (5) only allows a single root is equivalent to the fact that the discriminant denoted $\Delta(u, a, b)$ of the following term is not a square whatever the parameter u:

$$10 \quad x^3 + ax + b - (ux + Q(u, a, b))^2$$

For $q \equiv 2 \pmod{3}$, we can write:

$$\Delta(u, a, b) = -3R(u, a, b)^2$$

where R is a rational fraction.

In fact, for $p \equiv 2 \pmod{3}$, -3 is not a square and therefore $-3R(u, a, b)^2$ is never
 15 a squared term.

Since equation (5) only allows a single root then the following term is a polynomial of degree 1:

$$\gcd(x^3 + ax + b - (ux + Q(u, a, b))^2, X^p - X)$$

where gcd is a large common denominator.

20 The root of this polynomial, denoted X_P , is an abscissa of a point on the elliptic curve. Finally, the point with abscissa X_P and ordinate $u.X_P + Q(u, a, b)$ is a point on the curve.

In one example, a rational fraction $Q(u, a, b)$ that satisfies the following equation can be taken into consideration:

$$25 \quad Q(u, a, b) = \frac{236196u^2ab + 405u^8a - 20412u^6b - 45927u^4a^2 - u^{12} + 19683a^3}{54u(-162au^4 + 2187a^2 - u^8 + 2916bu^2)}$$

In one embodiment, the function $f_{a,b}$ can advantageously be defined as follows in F_q , q being equal to 2 mod 3. It makes a pair of parameters (x, y) correspond to a parameter u, such that:

$$x = (v^2 - b - u^6/27)^{1/3} + u^2/3$$

30 and

$$y = u.x + v$$

$$\text{with } v = \frac{3a - u^4}{6u} = Q(u, a, b)$$

and with $f_{a,b}(0)=0$.

Such a function is invertible. Thus, for a $P(X,Y)$ on the elliptic curve, a
5 pre-image of the point by the function $f_{a,b}$ is a solution of the following equation:

$$u^4 - 6u^2x + 6uy - 3a = 0$$

Now, a polynomial equation of this kind can easily be inverted.

Moreover, the set of pre-image points P of this function $f_{a,b}$ is bounded
by L that is small relative to the number of points on the elliptic curve.

10 Thus, this function satisfies the characteristics defined previously.

We can also envisage basing the function F on the use of a function $f_{a,b}$
defined on the basis of polynomials satisfying Skalba's equation.

It should be noted that polynomials satisfying Skalba's equality are
defined in the document 'Rational points on certain hyperelliptic curves over
15 finite fields' by Maciej Ulas, published 11 June 2007. These functions are
invertible. In fact, given polynomials $X_1(k)$, $X_2(k)$, $X_3(k)$ and $U(k)$ satisfying
Skalba's equality, i.e. satisfying:

$$f(X_1(k)).f(X_2(k)).f(X_3(k))=U(k)^2$$

where f is the polynomial that defines the elliptic curve

20 $E_{a,b}$.

More precisely, f satisfies the equation:

$$f(x)=y^2$$

where x and y are elements of F_q^2 , and the pair (x,y) represents a
point on $E_{a,b}$.

25 $f_{a,b}(k)$ is defined as being the point $P=(X_i(k), f(X_i(k))^{1/2})$, where i is such
that $f(X_i(k))$ is a square in F_q . Thus, to invert this function $f_{a,b}$, given a point
 $P=(X,Y)$, we calculate the solutions k_s of the three polynomial equations:

$$X_1(k_s)-X=0$$

$$X_2(k_s)-X=0$$

30 $X_3(k_s)-X=0$

Each of these solutions is a pre-image of P by $f_{a,b}$.

In the case where $F(k,k')$ is written:

$$F(k, k') = f_{a,b}(k) + f_{a,b}(k')$$

We can advantageously obtain a uniform distribution of the points, output values of $f_{a,b}$, for a uniform distribution of input values. In this case, if the input values are randomly determined, then the output values also have a random distribution.

By following this procedure, it is possible to avoid attacks based on a statistical analysis of values used for the parameter, which could supply information. In fact, in the case where the inverse of this function $f_{a,b}$ on a point P on the elliptic curve corresponds to a plurality of values of the input parameter of this inverse function, it is possible that some of these values recur more often than others according to a statistical law. Thus, if the attacker knows this statistical law, by testing a password on the information exchanged, he can decide whether or not the values of the parameter exchanged follow this law. He can thus deduce from this whether the password tested is correct or not.

In order to avoid this type of attack, it can be advantageous to employ an algorithm such as that described below.

D_i denotes the set of points on the elliptic curve $E_{a,b}$ which has exactly a number i of pre-images by the function $f_{a,b}$. In the example described below, five sets are defined: D_1 , D_2 , D_3 , D_4 and D_5 .

GEN denotes a function that generates, randomly and uniformly distributed, points on the elliptic curve. Let $\delta_i = \frac{i}{L}$, a probability associated with the set D_i .

For $t=0$ at $T-1$, where T is an integer

- generate a point P_t with the function GEN;
- if P_t is an element of D_0 , then go to the start;
- If P_t is an element of D_i
 - o Select at random a value b between 0 and 1 satisfying:
 $\Pr(b = 0) = \delta_i$
 - o If b is equal to 1, then go to step (1);
 - o Otherwise

- select at random an element u in the set $f_{a,b}^{-1}(P_t)$
- return u

It should be noted that, to obtain a $1-1/2^k$ probability of success, it is sufficient to have T equal to a polynomial evaluated in k of degree 1, in other words T is a multiple of k .

Moreover, the following steps can be employed:

- receive a message stating first and second parameters in a form encrypted with the password;
- decipher the first and second parameters and obtain parameters p and p' ; and
- calculate a common secret value K with the controller according to the following equation:

$$K = r_1.F(p, p')$$

where r_1 is the random value used at step /1/.

Thus, the device and the controller can finally have a common key at their disposal without exchanging information that puts the secrecy of the password in jeopardy.

A second aspect of the present invention proposes an electronic entity comprising means suitable for implementing a method of control according to the first aspect of the present invention.

A third aspect of the present invention proposes a system of control by password comprising a first electronic entity according to the second aspect as controller and a second electronic entity according to the second aspect as device to be controlled.

Other aspects, aims and advantages of the invention will become clear on reading the description of one of its embodiments.

The invention will also be better understood with the aid of the following figures:

- Figure 1, already described, illustrates the main steps of a method of control according to the prior art;
- Figure 2 shows the main steps of a method of control according to one embodiment of the present invention;

- Figure 3 illustrates an implementation of a method of control between a device and a controller according to one embodiment of the present invention; and
- Figure 4 illustrates a device and a controller according to one embodiment of the present invention.

Figure 2 illustrates the main steps of a method of control according to one embodiment of the present invention.

At a step 21, a point $P=(X,Y)$ on an elliptic curve in a finite field F_q , with q an integer, is determined on the basis of a random value r_1 . Then, at a step 22, first and second parameters k and k' are obtained, such that

$$P=F(k,k')$$

where F is a surjective function such as of $F_q \times F_q$ in F_q .

The parameters thus obtained make it possible to represent the point P ingeniously so as to protect the secrecy of the password during exchanges between the device and the controller.

At a step 23, the first and second parameters are obtained in encrypted form by encryption as a function of the password π $E_\pi(k,k')$.

In this ingenious form, the random value r_1 can advantageously be transmitted, at a step 24, from the device to be controlled to the controller, or from the controller to the device to be controlled so as to generate a secret common key.

The function F is such that, whatever z and z' element of F_q , $F(z,z')$ is a point on the elliptical curve and the elements z and z' do not satisfy equation (1).

Figure 3 illustrates an exchange of messages between a device to be controlled 10 and a controller 11 according to one embodiment of the present invention.

The device to be controlled D 10 generates firstly at a step 31 a random value r_1 . When the order of the set of points on the elliptical curve is an integer N , then r_1 can be taken at random by the set of values: $[0, N-1]$.

Then, starting from this random value, a point $P=(X,Y)$ on the elliptic curve $E_{a,b}$ is generated using a generator G . Said generator is such that, for

any point P on the elliptic curve, there is a value h such that:

$$P=h.G$$

Thus, a point P_1 is obtained such that:

$$P_1=r_1.G$$

5 Then, values of the parameters k_1 and k_1' are determined, at a step 33, for representing the point P_1 according to parameters different from its coordinates X and Y , which, for their part, satisfy equation (2).

In order to determine these parameters k_1 and k_1' , firstly a random value can be generated for the parameter k_1' . Then, a point on the elliptic curve P_{k_1} corresponds to this random value k_1' , either by applying a function $f_{a,b}$ that makes coordinates x, y of a point on the curve correspond to a parameter, or by using a generator G of the points on the elliptic curve.

Then, a value of the parameter k_1 is obtained, by applying the inverse application of $f_{a,b}$ to the point corresponding to a subtraction in the group of points on the elliptic curve: $P(X,Y)-P_{k_1}$.

The parameters k_1 and k_1' represent the point $P(X,Y)$. Next, the first and second parameters obtained previously are encrypted by an encryption function E_π based on the password π , to transmit them to the controller via a message 34.

20 In a symmetrical fashion, the controller generates a random value r_2 at a step 35. Then it transforms this value into a point P_2 on the elliptic curve, at a step 36. Then, as described previously at step 33, the controller obtains respective values of the first and second parameters k_2 and k_2' , at a step 37. Then, it encrypts these values before transmitting them to the device to be controlled 10, via a message 38.

The device to be controlled 10 therefore has at its disposal the random value r_1 , the values k_2 and k_2' and the function F .

Thus, at a step 39, it recovers the point P_2 :

$$P_2=F(k_2,k_2')$$

30 It therefore obtains a secret key K shared with the controller but without having supplied any information about the password, which satisfies:

$$K=r_1.r_2.G$$

The controller recovers the secret key K in a symmetric fashion at a

step 301.

The function F as defined above can be written in a different form. The following sections describe different representations of this point P which advantageously makes it possible not to supply redundancy of information during encrypted exchanges of the random values.

The function F is surjective of $F_q \times F_q$ in F_q , such that:

$$P = F(k, k') \quad (3)$$

and such that, for any pair of values (k, k') , P is a point on the elliptic curve, without equation (1) being satisfied by k and k' .

In such a context, advantageously, no information can be deduced about the password, on the basis of exchanges of the random values exchanged, in contrast to the prior art. In fact, the pair of values (k, k') is then exchanged in encrypted fashion between the device and the controller. But, whatever the password tested by a potential attacker to recover this pair of values, it is impossible to determine whether this password is correct or not, since any result obtained corresponds to a point on the elliptic curve, without equation (1) being satisfied by k and k' .

Such a representation can be based on advantageous characteristics of a function $f_{a,b}(U)$ that has the following characteristic:

$$\forall P \in E_{a,b}, |f_{a,b}^{-1}(P)| < L$$

with L an integer.

In other words, the size of the pre-image set of this function is bounded by a small number L relative to the number of points on the elliptic curve.

By considering the following function $f_{a,b}(u)$, advantageously a function F can be determined according to one embodiment of the present invention, which makes a pair of parameters $(x, u.x+v)$ in F_q^2 correspond to a parameter u in F_q , such that:

$$x = (v^2 - b - u^6 / 27)^{1/3} + u^2 / 3$$

and

$$v = \frac{3a - u^4}{6u}$$

putting $f_{a,b}(0)=0$.

The function F can then be written:

$$F(k,k')=f_{a,b}(k)+f_{a,b}(k')$$

or

$$F(k,k')=f_{a,b}(k)+k'.G$$

- 5 It can also be considered that $f_{a,b}$ is a function obtained from polynomials satisfying Skalba's equality:

$$f(X_1(k)).f(X_2(k)).f(X_3(k))=U(k)^2$$

- in which $X_1(k)$, $X_2(k)$, $X_3(k)$ and $U(k)$ are polynomials satisfying Skalba's equality, as defined for example in the document 'Rational points on certain hyperelliptic curves over finite fields' by Maciej Ulas, published 11 June 2007. $F_{a,b}(k)$ is defined here as $(X_i(k), f(X_i(k))^{1/2})$ where i is such that $f(X_i(k))$ is a square in F_q .

Figure 4 illustrates a control system comprising a device to be controlled and a controller according to one embodiment of the present invention.

- 15 Such a control system comprises at least one electronic entity as device to be controlled 10, and a control entity or controller 11 according to one embodiment of the present invention. Such an electronic entity, whether it is used as device to be controlled or as controller, can comprise:

- a determination unit 41 suitable for determining a point $P(X,Y)$ on an elliptic curve in a finite field F_q , q being an integer, on the basis of a random value r_1 , the elliptic curve satisfying the equation:

$$E_{a,b}(x,y): x^3 + ax + b = y^2 \quad (1)$$

- an obtaining unit 42 suitable for obtaining first and second parameters k and k' , such that

25
$$P=F(k,k')$$

where F is a surjective function of $F_q \times F_q$ in F_q ,

- an encryption unit 43 suitable for obtaining first and second parameters in encrypted form by encryption as a function of the password; and
- an interface unit 44 suitable for transmitting said first and second encrypted parameters to the controller;

- 30 the function F being such that, whatever z and z' input elements of F_q , $F(z,z')$ is a point on the elliptic curve and the input elements do not satisfy equation (1).

The encryption function by password E_π can be defined differently. It takes a bit string as input parameter and returns a bit string as output. Nevertheless it is necessary, to ensure password confidentiality, that the value returned does not give information about the password. Thus, two cases can be distinguished:

- either the password is used as encryption key of a classical encryption function,
- or the password is an index which, in a database, points to the value of the public parameters of the elliptic curve.

In both cases, the encryption algorithm can proceed as follows, for transforming a value of a parameter k or k' into a bit string, and the following method can be applied:

Select a random value r . Then transform a value v of a parameter into a bit string by calculating:

$$v' = v + r \cdot q'$$

with q' corresponding either to q or to N , q being the number of elements of the basic body F_q , N being the number of points on the elliptic curve. It is then possible to represent v' by a bit string.

In the case when the password π is an index, v' is the encrypted value of v under the password, as only the person knowing the public parameters (q or N) can find the value of v .

In the case when the password is used as encryption key of a classical encryption function, it is sufficient to encrypt v' by using the encryption function E_π .

Thus, on the receiving side, in the first case, v can be recovered by calculating $v' \bmod q'$. In the second case, it is necessary to decipher the bit string sent by the decryption function, in order to find v' and finally be able to calculate v by calculating $v' \bmod N$.

The present invention can be applied advantageously in any type of cryptographic calculation using elliptic curves. It can in particular be advantageous in protocols for authentication by password, such as PACE (Password Authenticated Connection Establishment). In this case, it allows an improvement in calculation performance, while not allowing any attack linked

with the execution time of the cryptographic calculation.

The present invention can also be applied advantageously in the context of privacy protocols, such as those used for checking electronic identity documents, such as electronic passports. In fact, listening in on the
5 protocol presented does not make it possible to find the public parameters of the elliptic curve used, in contrast to the prior art.

CLAIMS

1. Method of control of a device (10) by a controller (11) on the basis of a
5 password (π);

said method comprising the following steps, at the device or at the controller:

/1/ on the basis of a random value r_1 , determine a point $P(X,Y)$ on an
elliptic curve, in a finite field F_q , q being an integer, of equation:

$$E_{a,b}(x,y) : x^3 + ax + b = y^2 \quad (1)$$

10 /2/ obtain first and second parameters k and k' , such that

$$P(X,Y)=F(k,k')$$

where F is a surjective function of $F_q \times F_q$ in F_q ,

/3/ obtain first and second parameters in encrypted form by
encryption as a function of the password; and

15 /4/ transmit said first and second encrypted parameters to the
controller;

in which the function F is such that, regardless of z and z' input elements of F_q ,
 $F(z,z')$ is a point on the elliptic curve, and the input elements do not satisfy
equation (1).

20

2. Method of control according to claim 1, in which the function F is
written:

$$F(k, k') = f'(k') + f_{a,b}(k)$$

25 where $f_{a,b}$ is an invertible function, based on the coefficients a and b of
the elliptic curve, taking an input parameter and supplying a point on the elliptic
curve and

where f' is a function generating a point on the elliptic curve as a
function of a parameter;

and

30 in which, at step /2/, the parameters k and k' are obtained according to the
following steps:

- randomly generate a value of the parameter k' ;

- calculate a value of $f'(k')$;
- determine a value of the parameter k according to the following equation:

$$k = f_{a,b}^{-1}(P(X,Y) - f'(k'))$$

5

3. Method of control according to claim 2, in which the function f' is written:

$$f'(k') = k'.G$$

with G the generator of the set of points on the elliptic curve; and

10 in which a value of the parameter k is determined according to the following equation:

$$k = f_{a,b}^{-1}(P(X,Y) - k'.G)$$

15 4. Method of control according to claim 2, in which the function f' is written:

$$f'(k') = f_{a,b}(k')$$

and

in which a value of the parameter k is determined according to the following equation:

20
$$k = f_{a,b}^{-1}(P(X,Y) - f_{a,b}(k'))$$

5. Method of control according to any one of the preceding claims, in which the function F comprises at least one invertible function $f_{a,b}$, obtained by means of polynomials $X_1(k)$, $X_2(k)$, $X_3(k)$ and $U(k)$ satisfying Skalba's equation:

25

$$f(X_1(k)).f(X_2(k)).f(X_3(k)) = U(k)^2$$

where f is the polynomial that defines the elliptic curve $E_{a,b}$.

6. Method of control according to claim 5, in which the function $f_{a,b}$ makes
30 a pair of parameters (x,y) in F_q^2 correspond to a parameter u in F_q , where q is equal to 2 mod 3, such that:

$$x = (v^2 - b - u^6 / 27)^{1/3} + u^2 / 3$$

and $y = u.x + v$

with $v = \frac{3a - u^4}{6u}$

and $f_{a,b}(0)=0$.

5

7. Method of control according to any one of the preceding claims, further comprising the following steps:

- receive a message indicating the first and second parameters in a form encrypted with the password;

10 - decrypt the first and second parameters and obtain parameters p and p' ; and

- calculate a common secret value K with the controller according to the following equation:

$$K = r_1.F(p, p')$$

15 where r_1 is the random value used at step /1/.

8. Electronic entity in a system for control by password comprising:

- a determination unit (41) suitable for determining a point $P(X,Y)$ on an elliptic curve in a finite field F_q , q being an integer, on the basis of a random value r_1 ,
20 the elliptic curve being of equation:

$$E_{a,b}(x,y) : x^3 + ax + b = y^2 \quad (1)$$

- an obtaining unit (42) suitable for obtaining first and second parameters k and k' , such that

$$P=F(k,k')$$

25 where F is a surjective function of $F_q \times F_q$ in F_q ,

- an encryption unit (43) suitable for obtaining the first and second parameters in encrypted form by encryption as a function of the password; and
- an interface unit (44) suitable for transmitting said first and second encrypted parameters to the controller;

30 the function F being such that, whatever z and z' input elements of F_q , $F(z,z')$ is a point on the elliptic curve, and the input elements do not satisfy equation (1).

9. Electronic entity according to claim 8, comprising means suitable for implementing a method of control according to any one of claims 1 to 7.
- 5 10. System for control by password comprising at least one first electronic entity according to claim 8 or 9 as controller and a second electronic entity according to claim 8 or 9 as device to be controlled.