



**METHOD FOR DETERMINING MAGNETISATION AND THE FIELD
RADIATED BY A FERROMAGNETIC
PLATE**

The invention concerns the field of arrangements and devices for measuring the intensity and the direction of the magnetic field emitted by a plate or structure, such as for example the hull of a ship, as well as the field of facilities for the degaussing of ships.

It is known that the presence of magnetic materials on board a ship makes it detectable and identifiable by detection means, commonly known as magnetic anomaly detectors, which detect the ship's magnetic signature. These detection means can for example be embarked on board a vessel, mine, torpedo, missile or aircraft.

To limit the magnetic signature of a ship, the use of antimagnetic materials is known. However, there are still ferromagnetic materials on board. In order to limit the cost of ships, it seems preferable to produce a ferromagnetic metal hull and, by means of field sensors (magnetometers) installed inside the ship and degaussing coils, to generate magnetic fields capable of compensating for those fields linked to the hull itself.

Nevertheless, to achieve this compensation, it is necessary to determine beforehand the magnetisation of the hull. Knowledge of this magnetisation then allows determination of the currents required for this compensation in each of the degaussing coils.

One of the objectives of the invention is to provide a method which enables the



magnetisation of a plate (ferromagnetic steel plate of small thickness) to be estimated by using field sensors (magnetometers) and deducing from that the field radiated by the plate everywhere outside of the latter.

The magnetisation of a ferromagnetic material is the resultant of the sum of an induced magnetisation, which corresponds to the magnetisation acquired by the material under the influence of an induction field, like the terrestrial field for example, and a permanent magnetisation which depends on the history of the material. The induced magnetisation is completely deterministic and depends only on the induction field, the geometry of the plate (the hull) and the relative permeability of the material; on the other hand the permanent magnetisation develops slowly over time as a function of the induction field, thermal stress, pressure stresses or even manufacturing conditions. There are a great number of these parameters and they are difficult to control. Therefore the permanent magnetisation cannot be modelled, it can be estimated only by measurement of the field which induced it. As a result of the last-mentioned fact and in order to determine the total magnetisation of a plate, it is necessary to construct a model linking the measurement to the magnetisation, then inverting the model to deduce the magnetisation. Consequently all the difficulties connected with solving inverted problems are encountered.

To circumvent this problem, FR2679514 can be cited. That patent describes a portable station for measuring and adjusting the magnetic signature of a naval vessel, the said station enabling determination and modification of the characteristics of currents circulating in the magnetic degaussing coils with which the vessel is provided.

This station includes several magnetic sensors forming a deformable chain and capable of

being laid on the sea-bed and linked to the vessel through signal transmission/reception means which transmit/receive signals generated by the vessel

The method for determining the magnetic signature of the vessel consists in

- evaluating the shape of the chain of sensors so as to determine their relative position,
- determining the position of the vessel relative to the said sensor chain,
- determining a track and collecting the data from the sensors during at least one transit of the vessel along the said track.

With a chain of sensors necessarily placed outside the vessel, this method permits the magnetisation perceptible in the far field to be estimated, but it does not permit estimation of the magnetisation from an anomaly with the aid of sensors placed inside the vessel.

At present, one of the known methods for determining the signature of a metal-hulled ship based on internal measurements consists in determining by inversion of a linear system, not the magnetisation, but the distribution of mathematical quantity called magnetic charge at the nodes of a mesh on the surface of the hull then deducing from that the field at the defined reference distance. This method seeks to estimate a charge at each node of the surface mesh by considering it as being an independent element uncorrelated to its neighbours. This solution necessitates making use of a considerable number of sensors, given that there are as many parameters to be estimated as there are nodes defining the mesh. Furthermore, the obtained distribution of charges is not necessarily physical and there is the risk of the result being unstable in signature terms

The aim of the invention is to resolve the above-mentioned deficiencies by proposing a method which allows a physical estimation of the total induced and permanent magnetisation of a plate to be carried out. This magnetisation can then be exploited to estimate the field at every point and allows the reference signature of metal-hulled ship to be estimated. In addition, the sensors can be placed inside or outside the magnetic anomaly, that is to say the plate or the hull.

Moreover, the estimated magnetisation can be exploited by a device for direct compensation of the magnetisation, based on the calculation of degaussing coil currents minimising not the field estimated on the reference points, but minimising directly the magnetisation of the hull. Such a method of compensation has the advantage of minimising the field everywhere and, consequently, both above the hull with regard to aerial hazards and below the hull with regard to the hazards connected with mines and torpedoes. This allows the two sets of adjustments generally found in the operational procedures of ships, depending on whether the risk of detection is airborne (detection by aircraft) or linked to mines, to be avoided.

The invention consists of a method of determining the magnetisation and the field radiated by a ferromagnetic plate using magnetic field sensors distributed along the plate and at known positions relative to the latter, characterised in that it consists in:

- producing a mesh of the said plate by dividing it into elementary surfaces
- producing a first modelling of the magnetic field associated with the measurements and a function of the surface mesh produced on the plate, its thickness, the measurements, the coordinates of the measurement points and the induction field $B_0 = \mu_0 H_0$
- producing a second modelling of the physical behaviour of the material forming the plate, based on the mesh, its thickness, the induction field and the reversible permeability of the material.

- calculating the permanent and induced magnetisation of each of the elementary surfaces using as a basis the said first and second modellings and measurements performed by the said magnetic field sensors,
- calculating the magnetic signature of the plate at every point based on the said permanent and induced magnetisation of each of the elementary surfaces, on the surface mesh produced on the plate, on its thickness, on the measurements, on the coordinates of the calculation points and on the induction field $B_0 \approx \mu_0 H_0$.

“Plate” is understood to mean either a single plate, or a set of plates delimiting in this way a volume.

According to a particular characteristic, characterised in that the first modelling consists in modelling the magnetic field by either of the following two formulas:

$$B = B_0 + \frac{e\mu_0}{4\pi} \sum_{N_i} \int_{L_i} M_i \cdot n_i \frac{\vec{r}}{r^3} dL_i$$

$$B = B_0 - \frac{e\mu_0}{4\pi} \sum_{N_i} \text{grad} \int_{S_i} M_i \cdot \frac{\vec{r}}{r^3} dS_i$$

where,

M_i is the total magnetisation, that is to say the sum of the permanent magnetisation M_i^{per} and the induced magnetisation M_i^{ind} of the element i ,

N_i is the number of elementary surfaces,

L_i the edges of the elementary surfaces,

S_i the surface areas of the elements

r is the distance between the current point of the edge and the measurement point,

n_i is the normal to the edge i of the current surface element, tangential to the element,

e is the thickness of the hull,

that is to say, a matrix system taking the following matrix form.

$$B - B_0 = FM = FM^{ind} + FM^{per}$$

According to another particular characteristic guaranteeing high stability with regard to errors and measurement noise which permits consideration of its exploitation in a realistic environment, the second modelling consists in modelling the magnetic field at the level of each of the surface elements by either of the following two formulas:

$$\frac{M_i^{ind}}{(\mu_r - 1)} = H_0 - \frac{e}{4\pi} \sum_{N_i} \int_{L_j} M_j^{ind} \cdot n_j \frac{\vec{r}_{ij}}{r_{ij}^3} dL_j - \frac{e}{4\pi} \sum_{N_i} \int_{L_j} M_j^{per} \cdot n_j \frac{\vec{r}_{ij}}{r_{ij}^3} dL_j$$

$$\frac{M_i^{ind}}{(\mu_r - 1)} = H_0 + \frac{e}{4\pi} \sum_{N_i} \text{grad} \int_{S_j} M_j^{ind} \cdot \frac{\vec{r}_{ij}}{r_{ij}^3} dS_j + \frac{e}{4\pi} \sum_{N_i} \text{grad} \int_{S_j} M_j^{per} \cdot \frac{\vec{r}_{ij}}{r_{ij}^3} dS_j$$

where,

M_j^{ind} is the induced magnetisation of the elementary surface j,

M_j^{per} is the permanent magnetisation of the elementary surface j,

N_i is the number of elementary surfaces

L_j the edges of the elementary surfaces,

S_j the surface areas of the elements,

r_{ij} is the distance between the barycentres of the surface elements i and j,

n_j is the normal to the edge i of the current surface element, tangential to the element,

e is the thickness of the hull.

that is to say globally a matrix system taking the following matrix form:

$$(C + Id)M^{ind} + CM^{per} = f(H_0)$$

According to a particular characteristic, the calculation of the permanent and induced magnetisation of each of the elementary surfaces is carried out by solving the following matrix

system, its solution being $\tilde{M}$:

$$\begin{bmatrix} F & F \\ C + Id & C \end{bmatrix} \begin{bmatrix} M_{ind} \\ M_{per} \end{bmatrix} = \begin{bmatrix} B - B_0 \\ D \end{bmatrix}$$

According to an additional characteristic, the calculation of the magnetic signature S of the plate at a point I is carried out by any one of the following formulas:

$$S = B_0 + \frac{e\mu_0}{4\pi} \sum_{N_i} \int_{L_i} \tilde{M}_i \cdot n_i \frac{\vec{r}}{r^3} dL_i$$

$$S = B_0 - \frac{e\mu_0}{4\pi} \sum_{N_i} \text{grad} \int_{S_i} \tilde{M}_i \cdot \frac{\vec{r}}{r^3} dS_i$$

where.

$\tilde{M}$ represents the estimated magnetisation

N_i is the number of surface elements (triangular or square facet).

L_i the edges of the elements

S_i the surface area of the elements

r is the distance between the barycentre of the current surface element and the point of calculation of the field

n_i is the normal to the edge i of the current surface element, tangential to the element

e is the thickness of the hull.

According to another characteristic, a method in accordance with the invention includes a step which consists in compensating for the magnetisation of the plate or its magnetic signature by using a system of degaussing coils arranged close to the plate and in which a current of adjustable strength I can be generated, this step being able to consist in establishing a linear system G linking the currents I to the magnetisation M' which they generate on the plate and a function of the geometries of the plate and coils, and then to minimise the function $\tilde{M} - M'$, this

minimisation of the function $\tilde{M} - M'$ capable of being performed by the method of least squares

A method according to the invention may be particularly applied to the determination of the magnetic signature of the hull of a ship and even to the determination of the magnetisation and the magnetic signature of a ship as a whole.

It can be applied to the determination of the magnetisation and the magnetic signature of a ferromagnetic structure.

Other characteristics and advantages of the invention will become apparent to the reader of the following description of a preferred mode of implementation of the invention, given by way of a simple illustrative and non-limiting example, and annexed drawings, in which.

- Figure 1 shows the means of implementing a method according to the invention,
- Figure 2 shows a diagram of the various steps of a method according to the preferred mode of implementation,
- Figure 3 shows the modelling of a hull and its surface mesh,
- Figure 4 shows the spectrum of singular values,
- Figure 5 shows the magnetisation of each elementary surface of the surface mesh,
- Figure 6 shows the predicted magnetic signature of the hull,
- Figures 7, 8 and 9 show, in an orthonormal reference Ox, y, z , the component of the predicted and measured magnetic signature of the hull along the x, y and z axes respectively

Figure 1 shows the means of implementing a method according to the implementation mode of the invention. These means include n magnetic sensors $C1$ to Cn , for example magnetometers, each being capable of transmitting a signal to the processing means 1 of these signals, in this case a microcomputer.

The microcomputer and the said sensors are arranged inside the hull 2 of a ship

The method according to the described preferred mode of implementation consists in a first step in producing a model, for example by computer-aided design, of the hull 2 of the ship while respecting the proportions of the various surfaces, then a surface mesh is created over the whole surface of the hull, each elementary surface possibly being formed for example by a triangle or square as shown in Figure 2 [sic], the position of each of the elementary surfaces being defined relative to a predetermined triaxial reference point.

The next step consists in establishing a model associated with measurements produced by the n sensors C1 to Cn.

The model, denoted F, associated with the measurements denoted B, uses the knowledge of the geometric model composed of the surface mesh of the hull, its thickness, the measurements, the coordinates of the measurement points and the induction field $B_0 = \mu_0 H_0$ (μ_0 is the permeability of a vacuum or air):

$$B = B_0 + \frac{e\mu_0}{4\pi} \sum_{N_1} \int_{L_i} M_i \cdot n_i \frac{\vec{r}}{r^3} dL_i$$

$$B = B_0 - \frac{e\mu_0}{4\pi} \sum_{N_1} \text{grad} \int \int_{S_i} M_i \cdot \frac{\vec{r}}{r^3} dS_i$$

where,

M_i is the total magnetisation, that is to say the sum of the permanent magnetisation M_i^{per} and the induced magnetisation M_i^{ind} of the element i,

N_1 is the number of elementary surfaces,

L_i the edges of the elementary surfaces,

S_i the surface areas of the elements

r is the distance between the current point of the edge and the measurement point,

n_i is the normal to the edge i of the current surface element, tangential to the element,

e is the thickness of the hull,

This equation reduces numerically to solve a problem which takes the following matrix form:

$$B - B_0 = FM = FM^{ind} + FM^{per}$$

This system possesses the following characteristics:

- It is representative of the measurements produced by the sensors.
- It is a priori under-dimensioned, that is to say that there are less equations than unknowns.
- It is unconditional, that is to say that it is difficult to invert, this difficulty increasing with the number of sensors.

To make good this shortcoming, a model of the physical behaviour of the magnetisation is created.

The overall formulation of the physical behaviour exploits the knowledge of the geometric model forming the surface mesh of the hull and its thickness, of the induction field denoted H_0 and of the relative permeability of the material denoted μ_r . This formulation is described in the form of the following very general equation:

$$B = \mu_0 (H + M) \quad (1)$$

to which is added the following equation intrinsic to the material:

$$B = \mu H + B_p \quad (2)$$

By combining (1) and (2), we get:

$$H = \mu_r H + M - M_p \quad (3)$$

With M_p being the permanent magnetisation, M_i the induced magnetisation and M the total magnetisation.

Since $M - M_p = M_i$, we obtain

$$M_i = (\mu_r - 1)H$$

At the same time, M_i is created by M_i and M_p in the whole of the plate. We then have:

$$\frac{M_i^{ind}}{(\mu_r - 1)} = H_0 - \frac{e}{4\pi} \sum_{N_i} \int_{L_j} M_j^{ind} \cdot n_j \frac{\vec{r}_{ij}}{r_{ij}^3} dL_j - \frac{e}{4\pi} \sum_{N_i} \int_{L_j} M_j^{per} \cdot n_j \frac{\vec{r}_{ij}}{r_{ij}^3} dL_j \quad (4)$$

$$\frac{M_i^{ind}}{(\mu_r - 1)} = H_0 + \frac{e}{4\pi} \sum_{N_i} \text{grad} \int_{S_j} M_j^{ind} \cdot \frac{\vec{r}_{ij}}{r_{ij}^3} dS_j + \frac{e}{4\pi} \sum_{N_i} \text{grad} \int_{S_j} M_j^{per} \cdot \frac{\vec{r}_{ij}}{r_{ij}^3} dS_j$$

where,

M_j^{ind} is the induced magnetisation of the elementary surface j,

M_j^{per} is the permanent magnetisation of the elementary surface j,

N_i is the number of elementary surfaces

L_j the edges of the elementary surfaces,

S_j the surface areas of the elements,

r_{ij} is the distance between the barycentres of the surface elements i and j,

n_j is the normal to the edge i of the current surface element, tangential to the element,

e is the thickness of the hull.

By writing this equation for each element, numerically the following matrix system is obtained

$$(C + Id)M^{ind} + CM^{per} = f(H_0)$$

where C is a matrix which is dependent on the geometry of the mesh, the thickness of the plate and the relative permeability μ_r , Id is the identity matrix, M_i and M_p are respectively the induced and permanent magnetisation such that $M = M_i + M_p$.

This system has the following characteristics:

- It is representative of the physical behaviour of the plate,
- It is under-dimensioned, that is to say that it has more equations than unknowns,
- It is highly conditional, that is to say it is easy to invert.

The 2 models have to be verified simultaneously, which leads to constructing then solving the

following global model:

$$\begin{bmatrix} F & F \\ C + Id & C \end{bmatrix} \begin{bmatrix} M_i \\ M_p \end{bmatrix} = \begin{bmatrix} B - B_0 \\ D \end{bmatrix}$$

that is:

$$AM = Y$$

with:

$$A = \begin{bmatrix} F & F \\ C + Id & C \end{bmatrix} \quad M = \begin{bmatrix} M_i \\ M_p \end{bmatrix} \quad Y = \begin{bmatrix} B - B_0 \\ D \end{bmatrix}$$

The resultant matrix to be inverted, denoted A, is then naturally highly conditional, therefore easy to invert; despite the fact that the system is under-dimensioned, it requires neither empirical estimation of weighting coefficients nor use of a specific normalisation technique; a generalised inversion by resolving into singular values is sufficient for finding the magnetisation

The inversion allows the estimation of **M**, denoted $\tilde{M}$ to be obtained.

The matrix $A_{(X,N)}$ with $N > X$ may be resolved in the following way.

$$A_{(X,N)} = P_{(X,N)} L_{(N,N)} Q_{(N,N)}^T$$

where the matrices P and Q are orthogonal, that is to say:

$$\sum_{i=1}^X P_{ik} P_{in} = \delta_{kn}$$

$$\sum_{j=1}^X Q_{jk} Q_{jn} = \delta_{kn}$$

for

$$1 \leq k, n \leq N$$

L is a diagonal matrix whose diagonal elements are the eigenvalues of A. L has the following form

$$L = \begin{bmatrix} l_1 & 0 & 0 & 0 & 0 \\ 0 & . & 0 & 0 & 0 \\ 0 & 0 & l_i & 0 & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & l_N \end{bmatrix}$$

We then have

$$A^{-1} = QL^{-1}P^T$$

with L^{-1} , which is equivalent to

$$L = \begin{bmatrix} 1/l_1 & 0 & 0 & 0 & 0 \\ 0 & . & 0 & 0 & 0 \\ 0 & 0 & 1/l_i & 0 & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & 1/l_N \end{bmatrix}$$

When an eigenvalue l_i is zero, it is replaced by 0 in L^{-1}

The estimation of M , that is to say , $\tilde{M}$ is obtained quite simply by.

$$\tilde{M} = A^{-1}Y \quad (5)$$

The model, denoted A , associated with the calculation points of the signature denoted S , is theoretically written in the same way as the model associated with the measurement points described by 0. It uses the knowledge of the geometric model forming the surface mesh of the hull, its thickness, the measurements, the coordinates of the calculation points and the induction field $B_0 = \mu_0 H_0$ (μ_0 is the permeability of a vacuum or air):

$$S = B_0 + \frac{e\mu_0}{4\pi} \sum_{N_1} \int_{L_i} \tilde{M} \cdot n_i \frac{\vec{r}}{r^3} dL_i \quad (6)$$

$\tilde{M}$ represents the estimated magnetisation

N_1 is the number of surface elements (triangular or square facet).

L_i the edges of the elements

r is the distance between the barycentre of the current surface element and the point of calculation of the field.

n_i is the normal to the edge of the current surface element, tangential to the element.

e is the thickness of the hull

This equation reduces numerically to solve a problem which takes the following matrix form

$$S = A\tilde{M}$$

thus a simple matrix product.

A complementary step consists in compensating for the magnetisation of the plate or its magnetic signature by using a known system of degaussing coils arranged close to plate and in which currents of adjustable strength I can be generated. To this end a linear system G is set up which is generally considerably over-dimensioned, linking the currents I to the magnetisation on a hull having regard to the geometries of the hull and degaussing coils. We can then simply write that.

$$\tilde{M} = GI$$

The compensation currents I are then estimated, for example by generalised inversion by expansion into singular values of the matrix G , in the following manner:

$$I = (G^T G)^{-1} G^T \tilde{M}$$

The following step consists in minimising the function $\tilde{M} - M'$, this minimisation of the

function $\tilde{M} - M'$ being performed by the method of least squares

The method according to the mode of implementation described previously was implemented on a steel hull which was modelled on a computer and on which a surface mesh was created, as shown in Figure 3. This model comprises 271 elementary surfaces which have square forms 4 or triangular forms 5. As shown in Figure 3, 32 triaxial mini-Crouzet sensors 6 were arranged inside the hull.

The hull was placed on a northerly heading so as to not require simulation of the terrestrial field. It is thus placed in a highly uniform induction field. The values of this field are:

$$B_{ox} = -553 \text{ nT}$$

$$B_{oy} = -22549 \text{ nT}$$

$$B_{oz} = -40662 \text{ nT}$$

The internal sensors 6 were mounted on a structure isolated from the hull. It is thus possible to produce two types of measurements: measurements of the field inside the mock-up and measurements with no hull, after extraction of the latter and putting back in place the said structure. This approach permits, by simple subtraction, measurement directly of the reaction of the material in relation to the induction field while removing the constraint of the problems inherent in the offsets of the sensors. This approach is referred to as differential.

Since the hull was created in a mesh of 271 surface elements, there are therefore 542 unknowns to be identified, starting with 99 pieces of information, that is to say 96 measurements coming from 32 triaxial sensors 6 plus the three components of the induction field.

Then the three following steps were performed

- performing a first modelling of the magnetic field associated with the measurements

and a function of the surface mesh created on the plate, its thickness, the measurements, the coordinates of the measurement points and the induction field $B_0 = \mu_0 H_0$.

- performing a second modelling of the physical behaviour of the material constituting the plate taking as a basis the mesh, the plate's thickness, the induction field and the reversible permeability of the material.

- calculating the permanent and induced magnetisation of each of the elementary surfaces based on the said first and second modellings and measurements of the magnetic field carried out by the said sensors

This last step requires the inversion of a matrix. Before the inversion, it is necessary to examine its singular value expansion. On that subject, the smaller the singular values, the more difficult the system is to invert. In particular, if these values are very close to 0, it is then necessary to bring normalisation methods into play. These methods are relatively cumbersome for 3D problems.

This expansion into singular values, shown in Figure 4, clearly makes apparent two groups of values. The first group is composed of significant values, related to the singular values stemming from the first modelling. This system effectively shows a relatively low conditioning. The second group shows values a great deal closer to zero, it being directly the image of the system of measurements which has a very poor conditioning. The latter thus contributes towards the overall degradation but to a lesser extent since the number of equations connected with it is not very large. The overall characteristics of the system to be inverted are:

smallest singular value: 0.0753 conditioning: 400

The system can thus be inverted by the method of least squares generalised with no particular precaution. After inversion, we thus obtain the residual and induced magnetisation, the

sum being the total magnetisation.

The magnetisation of each of the elementary surfaces is shown in Figure 5, firstly by strength: the greater the strength, the darker is the colour of the surface, and secondly by direction which is represented by an arrow

By applying Equation 6, we now calculate the magnetic signature of the plate at every point taking as a basis the said permanent and induced magnetisation of each of the elementary surfaces, the surface mesh created on the plate, its thickness, the measurements, the coordinates of the calculation points and the induction field $B_0 = \mu_0 H_0$

The modulus of this magnetic signature, thus the magnetic anomaly calculated in this way, is shown in Figure 6

In order to show the agreement between the calculated signature values and the actual values, 7 sensors of the Ultra type were placed in a horizontal plane situated at 30 cm above the hull, and perpendicular to the axis of the hull were evenly distributed over a total distance of about 1m.

The results are shown in Figures 7, 8 and 9 representing in a orthonormal reference Ox, y, z, the component of the predicted magnetic signature 10 and measured magnetic signature 20 along the x, y and z axes respectively.

Each component is shown by ordinates in units of nanotesla while the abscissa is the distance in relation to the plane of symmetry of the hull.

It will be noted that there is very good agreement between the predicted and measured signature values. Therefore since the invention allows the magnetisation and magnetic signature of the hull to be determined precisely, it is then possible to compensate almost completely for the magnetisation of the hull by using a system of degaussing coils arranged close to the plate and in

which a current of adjustable strength I can be generated, and by performing a first step consisting in establishing a linear system G linking the currents I to the magnetisation M' which they generate on the hull and a function of the geometries of the plate and coils, and a second step consisting in minimising the function $\tilde{M} - M'$ by the method of least squares

CLAIMS

1) A method of determining the magnetisation and the field radiated by a ferromagnetic plate using magnetic field sensors distributed along the plate and at known positions relative to the latter, characterised in that it consists in

- producing a mesh of the said plate by dividing it into elementary surfaces.
- producing a first modelling of the magnetic field associated with the measurements and a function of the surface mesh produced on the plate, its thickness, the measurements, the coordinates of the measurement points and the induction field $B_0 = \mu_0 H_0$
- producing a second modelling of the physical behaviour of the material forming the plate, based on the mesh, its thickness, the induction field and the reversible permeability of the material.
- calculating the permanent and induced magnetisation of each of the elementary surfaces using as a basis the said first and second modellings and measurements performed by the said magnetic field sensors,
- calculating the magnetic signature of the plate at every point based on the said permanent and induced magnetisation of each of the elementary surfaces, the surface mesh produced on the plate, on its thickness, the measurements, the coordinates of the calculation points and the induction field $B_0 = \mu_0 H_0$

2) A method according to Claim 1, characterised in that the first modelling consists in modelling the magnetic field by a formula linking the magnetic field measured by the sensors, the magnetisation, the induction field and the geometry, such as one of the following formulas

$$B = B_0 + \frac{e\mu_0}{4\pi} \sum_{N_i} \int_{L_i} M_i \cdot n_i \frac{\vec{r}}{r^3} dL_i$$

$$B = B_0 - \frac{e\mu_0}{4\pi} \sum_{N_i} \text{grad} \int_{S_i} M_i \cdot \frac{\vec{r}}{r^3} dS_i$$

where,

M_i is the total magnetisation, that is to say the sum of the permanent magnetisation M_i^{per} and the induced magnetisation M_i^{ind} of the element i ,

N_i is the number of elementary surfaces,

L_i the edges of the elementary surfaces,

S_i the surface areas of the elements

r is the distance between the current point of the edge and the measurement point,

n_i is the normal to the edge i of the current surface element, tangential to the element,

e is the thickness of the hull,

that is to say, a matrix system taking the following matrix form:

$$B - B_0 = FM = FM^{\text{ind}} + FM^{\text{per}}$$

3) A method according to either one of Claims 1 and 2, characterised in that the second modelling consists in modelling the magnetic behaviour of the plate by a formula linking the magnetisation, the induction field, the reversible relative permeability of the material and the geometry, such as one of the following formulas

$$\frac{M_i^{\text{ind}}}{(\mu_r - 1)} = H_0 - \frac{e}{4\pi} \sum_{N_j} \int_{L_j} M_j^{\text{ind}} \cdot n_j \frac{\vec{r}_{ij}}{r_{ij}^3} dL_j - \frac{e}{4\pi} \sum_{N_j} \int_{L_j} M_j^{\text{per}} \cdot n_j \frac{\vec{r}_{ij}}{r_{ij}^3} dL_j$$

$$\frac{M_i^{ind}}{(\mu_r - 1)} = H_0 + \frac{e}{4\pi} \sum_{N_i} grad \iint_{S_j} M_j^{ind} \cdot \frac{\vec{r}_{ij}}{r_{ij}^3} dS_j + \frac{e}{4\pi} \sum_{N_i} grad \iint_{S_j} M_j^{per} \cdot \frac{\vec{r}_{ij}}{r_{ij}^3} dS_j,$$

where,

M_j^{ind} is the induced magnetisation of the elementary surface j,

M_j^{per} is the permanent magnetisation of the elementary surface j,

N_i is the number of elementary surfaces

L_j the edges of the elementary surfaces,

S_j the surface areas of the elements,

r_{ij} is the distance between the barycentres of the surface elements i and j,

n_j is the normal to the edge i of the current surface element, tangential to the element,

e is the thickness of the hull.

that is to say globally a matrix system taking the following matrix form:

$$(C + Id)M^{ind} + CM^{per} = f(H_0)$$

4) A method according to Claims 2 and 3, characterised in that the calculation of the permanent and induced magnetisation of each of the elementary surfaces is carried out by solving the following

matrix system, its solution being $\tilde{M}$

$$\begin{bmatrix} F & F \\ C + Id & C \end{bmatrix} \begin{bmatrix} M^{ind} \\ M^{per} \end{bmatrix} = \begin{bmatrix} B - B_0 \\ D \end{bmatrix}$$

5) A method according to Claim 4, characterised in that the calculation of the magnetic signature S of the plate at a point I is carried out by any one of the following formulas:

$$S = B_0 + \frac{e\mu_0}{4\pi} \sum_{N_i} \int_{L_i} \tilde{M}_i \cdot n_i \frac{\vec{r}}{r^3} dL_i$$

$$S = B_0 - \frac{e\mu_0}{4\pi} \sum_{N_i} \text{grad} \int_{S_i} \tilde{M}_i \cdot \frac{\vec{r}}{r^3} dS_i$$

$\tilde{M}$ represents the estimated magnetisation

N_i is the number of surface elements (triangular or square)

L_i the edges of the elements

S_i the surface area of the elements

r is the distance between the barycentre of the current surface element and the point of calculation of the field.

n_i is the normal to the edge i of the current surface element, tangential to the element.

e is the thickness of the hull

6) A method according to any one of Claims 1 to 4, characterised in that it includes a step which consists in compensating for the magnetisation of the plate or its magnetic signature by using a system of degaussing coils arranged close to the plate and in which a current of adjustable strength I can be generated.

7) A method according to Claim 5, characterised in that it includes a first step consisting in establishing a linear system G linking the currents I to the magnetisation M' which they generate on the plate and a function of the geometries of the plate and coils, and a second step consisting in minimisation of the function $\tilde{M} - M'$.

8) A method according to Claim 6, characterised in that the step of minimising the function

$\tilde{M} - M'$ is performed by the method of least squares

9) Application of a method according to any one of the preceding claims to the determination of the magnetisation and the magnetic signature of the hull of a ship

10) Application of a method according to any one of the preceding claims to the determination of the magnetisation and the magnetic signature of a ship as a whole

11) Application of a method according to any one of the preceding claims to the determination of the magnetisation and the magnetic signature of a ferromagnetic structure