



A METHOD OF REDUCING DISTORTION OF AN IMAGE

FIELD OF THE INVENTION

This invention relates to a method of reducing distortion of an image as perceived by a viewer viewing the image. This invention also relates to apparatus for use in carrying out the method.

BACKGROUND TO THE INVENTION

Conventional projectors are commonly used to project images onto display surfaces for use, for example, in presentations. Such projectors are usually digital projectors for projecting an image under the control of a computer on which the image is stored. However, unless the projector is positioned such that its optical axis is normal to a display surface, the resulting image displayed on the display surface will appear geometrically distorted. For example, a rectangular projected image may produce a displayed image that appears trapeziform, that is, with only two parallel sides, one longer than the other. Such distortion is worsened if the image is projected onto a non-planar display surface. Distortion of this kind is sometimes referred to as "keystoning". In order to address this problem, a conventional projector can be moved into a position that results in either no keystoning or an acceptable amount of keystoning. This form of manual, open-loop control can, however, be time consuming and inconvenient. For example, it may only be possible to produce a displayed image that exhibits no keystoning by positioning the projector in a position that blocks the view of one or more viewers intending to view the displayed image.

With some projectors, optical or digital keystone correction is possible under the manual control of an operator of the projector. However, this form of open-loop control can also be time consuming and inconvenient; and, in any event, it does not provide for addressing the problem of distortion of an image displayed on a non-planar surface.

In one proposed method for addressing this problem, it has been proposed to use a camera to observe a projected image displayed on a display screen, with a view to generating mathematical expressions that can be applied to the projected image in an attempt to minimise distortion thereof. This technique is known as "pre-warping", with the expressions being known as "pre-warp values". The pre-warp values compensate for



geometric distortion of the displayed image caused by projector misalignment and are applied prior to the projection of an image that it is desired be viewed, such that a reduction in the distortion of the displayed image can be achieved. By using pre-warp values to compensate for distortion, this closed-loop approach removes the need for manual adjustment of the projector. However, this method is only applicable to circumstances in which the display surface is planar. Distortion resulting from a non-planar display screen is not corrected.

Although methods with the object of compensating for distortion resulting from non-planar surfaces have been proposed, these have in general not proved to be successful. One such method involves dividing a non-planar display surface into several smaller surfaces, each assumed to be planar, before performing a similar pre-warping operation to that described above. Although in theory any non-planar surface can be approximated to a plurality of smaller planar surfaces, for surfaces with complicated topographies, or for continuously curved surfaces such as those of spheres, the number of smaller surfaces required in order to achieve sufficient accuracy for an acceptable result can be impractically high. For example, the necessary computation may be unduly burdensome for a conventional computer.

It is an object of this invention to address these problems.

SUMMARY OF THE INVENTION

According to a first aspect of this invention, there is provided a method of reducing distortion of a desired image that it is desired be viewed at a viewing position, the method including computer processing means carrying out the steps of:

a) receiving information indicative of the desired image;

b) instructing display means to display the desired image such that a first displayed image is displayed at a display position;

c) receiving information from sensing means at a sensing position, including appearance information indicative of a sensed image of the appearance, at the sensing position, of

the first displayed image; and including distance information indicative of the distance of the display position from the sensing position;

- 5 d) operating on the appearance information and the distance information, and the information indicative of the desired image, or other information indicative of the desired image, to produce modified information for use in instructing the display means to display a modified version of the desired image such that another displayed image is displayed at the display position that more closely has the appearance of the desired image at the viewing position than did the first displayed image.
- 10 e) instructing the display means to display the modified desired image.

By sensing and then making use of the distance information in this way, it is possible to modify the desired image to reduce perceived distortion thereof in a manner that is more easily carried out by conventional computer equipment than is the case with methods of the prior art. Thus, conventional computing equipment can be used to carry out the method of the present invention to correct perceived distortion resulting from, for example, a non-planar display surface.

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20 The display means may include an active display screen at the display position such as, for example, a visual display unit (VDU), a television or a large LED display screen of the kind used at outdoor events such as concerts. Where this is the case, the image displayed at the display position at step (b) would usually have the appearance of the desired image. However, a viewer positioned at a viewing position that is to one side of the display screen may still see a distorted image. The method may be used to address this by positioning the sensing means at a sensing position adjacent to the viewing position.

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More preferably, however, the display means includes projection means for projecting images onto a display surface such as a passive screen, which may, for example, simply take the form of a light-coloured surface. It is envisaged, however, that the screen may be any medium capable of reflecting light emitted by the projection means so that that reflected light can be seen by a viewer. In particular, it is envisaged that the screen may

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have a display surface that is non-planar. The surface may, for example, be regularly curved, irregularly curved, jagged or exhibit some other irregular surface topography. It is envisaged that the surface may even change its topography over time. The display surface is positioned at the display position and the projection means is positioned at a projection position. Where the display means includes projection means and a screen, and particularly where the screen is non-planar, the image displayed on the screen in step (b) may well not have the appearance of the desired image, but may instead be a distorted version thereof.

10 Preferably, the sensing position and the viewing position are adjacent one another. Still more preferably, these two positions are coincident.

The projection means preferably includes a digital projector arranged for operation under the control of computer processing means to project an image in response to a projection signal from such means.

In the case of the display means including the projection means, instructing the display means to display the desired image, or the modified desired image, includes instructing the projection means to project the desired image or modified desired image from an image plane of the projection means. The image plane of the projection means is preferably positioned at the projection position. At step (d), the other information indicative of the desired image may be a control signal from the computer processing means to the display means; whereas, the information indicative of the desired image may be data in a format convenient for processing by computer processing means, such as, for example, GIF or JPEG data. The control signal may be in VGA format.

30 Preferably the sensing means is operable to sense both a two-dimensional (2-D) sensed image of the appearance of displayed images as they appear on an image plane of the sensing means, and the distance of the display position from the image plane of the sensing means, the image plane being at the sensing position. The sensing means may include a camera operable to do this. Still more preferably, the sensing means is operable to sense the respective distance of each of a plurality of parts of a surface of the display screen from the sensing position. The sensing means may be arranged to

present the distance information in the form of information indicative of a 2-D range image arranged to correspond with the 2-D sensed image such that parts of the range image each indicate the distance, from the sensing position, of a respective part of the surface of the display screen that is associated with a corresponding part of the sensed image that corresponds with that part of the range image. Each part may be a pixel of the image of which it is a part, or, in the case of the surface of the display screen, each part may be a part of that surface that is substantially covered by a pixel of the displayed image displayed on that surface. Each part may be a point in the respective image, or, in the case of the surface, a point thereon.

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The distance information need not include the actual distance, from the sensing position, of the display surface or parts of that surface. Instead, distance information that includes information indicative of the respective distance, from the sensing position, of each of a plurality of parts of the display surface would suffice, if, for example, each respective distance were not the actual distance, but instead included a constant deviation from the actual distance. Similarly, if each respective distance were not the actual distance, but were scaled up or down from the actual distance, that would suffice.

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20 The sensing means may include two cameras, positioned apart, such that the method makes use of stereo vision to arrive at at least the distance information.

The desired image may be a calibration image including a pattern. The pattern may be such that it contains a number of readily identifiable points. The calibration image may include lines arranged in a grid. The desired image may be a photographic image for viewing by the viewer. The desired image may be a frame from a collection of frames that go to make up a video, film or some such other form of moving picture.

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Step (b) may include the step of sending a control signal to the display means to thereby control those means, the control signal being based on the information received at step (a), to project from the projection means the image of which that information is indicative. Step (e) may similarly include the step of sending a control signal to the display means.

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Step (d) may be such that the modified information produced therein is a modified version of the information received at step (a) and/or a modified version of the control signal sent to the display means in step (b), wherein step (e) would include sending the modified version of the control signal to the display means. Step (d) may include the step of operating on the appearance information and the distance information, and the information or the other information indicative of the desired image, to produce modifying information for use in producing the modified information. The modifying information may include information indicative of a mathematical expression that maps the desired image to the sensed image and/or the sensed image to the desired image. In a preferred embodiment, the mathematical expression is a transform that relies on determining the relative positions of at least six points of the sensed image, determining the relative positions of the six corresponding points in the desired image, such that six pairs of corresponding points are determined; and also determining the distance, from the sensing position, of each of six points of the display surface that are associated with a respective one of the corresponding pairs. Preferably, the distance information includes information indicative of the distance, from the sensing position, of each part of the display surface on which a pixel of the displayed image is shown; and, preferably, the mathematical expression is used to operate on this information. The mathematical expression may be, or at least include, an expression that can be expressed as:

$$\begin{bmatrix} sx_1 \\ sy_1 \\ s \end{bmatrix} = M_{A,3 \times 4} \begin{bmatrix} x_2 \\ y_2 \\ Z_2(x_2, y_2) \\ 1 \end{bmatrix}$$

wherein a point (x_1, y_1) on the image plane of the projection means is projected to an unknown point on the display surface; that point being capture by the sensing means as a point (x_2, y_2) on the image plane thereof; the unknown point on the screen is a 3D point having coordinates of (X_1, Y_1, Z_1) when referenced with respect to a 3-D coordinate system S_1 associated with the projection means and coordinates of (X_2, Y_2, Z_2) when with respect to a coordinate system S_2 associated with the sensing means; wherein the matrix $M_{A,3 \times 4}$ maps points from the image plane of the sensing means to the image plane of the projection means.

This expression may be re-written to be expressed as:

$$\begin{bmatrix} sx_1 \\ sy_1 \\ s \end{bmatrix} = \begin{bmatrix} M_{11}x_2 + M_{12}y_2 + M_{13}Z_2(x_2, y_2) + M_{14} \\ M_{21}x_2 + M_{22}y_2 + M_{23}Z_2(x_2, y_2) + M_{24} \\ M_{31}x_2 + M_{32}y_2 + M_{33}Z_2(x_2, y_2) + M_{34} \end{bmatrix}$$

wherein M_{ij} is the element of $M_{A,3 \times 4}$ at the i-th row and j-th column.

To obtain the inverse mapping from the image plane of the projector means to the image
5 plane of the sensing means, the terms (x_1, y_1) and (x_2, y_2) can be interchanged to give:

$$\begin{bmatrix} sx_2 \\ sy_2 \\ s \end{bmatrix} = M_{B,3 \times 4} \begin{bmatrix} x_1 \\ y_1 \\ Z_2(x_2, y_2) \\ 1 \end{bmatrix}$$

wherein the matrix $M_{B,3 \times 4}$ maps points from the image plane of the projection means to the
image plane of the sensing means. Thus, the mathematical expression may be
10 expressed in this form.

The mathematical expression may further modify the size of the desired image to give a
modified desired image that, when projected, results in an image of a desired size being
viewed by the viewer. The mathematical expression may have the effect of selecting a
15 region within the sensible area at the image plane of the sensing means, that region
corresponding to a region that it is desired be filled by the sensed image and causing
that region to be filled by the sensed image. The mathematical expression may include a
scaling factor that can be applied to the desired image to modify the size and/or position
of that image. Preferably the scaling factor maintains the aspect ratio of the desired
20 image.

The mathematical expression associated with a particular sensing position and/or
projection position may be stored so as to be retrievable for applying to one or more
subsequent desired images. The expression may be stored in a look-up table. The
25 method may include the further steps of storing and/or retrieving the expression.

It will be appreciated that, in the event that the topography of the display surface changes, or the relative position of one or more of the projection means, display surface, sensing means and the viewer changes, the method would need to be repeated to take account of those changes.

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According to a second aspect of this invention, there is provided apparatus including computer processing means programmed to carry out the method of the first aspect defined above.

10 The computer processing means may include one or more microprocessors, which may, for example be contained in one or more personal computers. The computer processing means may additionally, or exclusively, be contained in projection means such as that defined above. The computer processing means may additionally, or exclusively, be contained in sensing means such as that defined above.

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The apparatus may include the display means and/or the sensing means of the first aspect.

According to a third aspect of this invention, there is provided a computer program
20 having code portions which when executed by computer processing means cause those means to carry out the method of the first aspect defined above.

According to a fourth aspect of this invention, there is provided a record carrier, the carrier having recorded thereon information which is readable by computer processing
25 means, or equipment arranged to communicate therewith, to cause those means to carry out the method of the first aspect defined above. The record carrier may be, for example, a CD-ROM, a magnetic disk, a semi-conductor-based memory chip, card or device.

30 According to a fifth aspect of this invention there is provided a signal, the signal conveying information which is readable by computer processing means, or equipment associated therewith, to cause those means to carry out the method of the first aspect defined above. The signal may be an electrical signal provided, such as an electrical

signal provided over a network, such as, for example, the internet. The signal may be a radio signal, such as a radio signal provided over a wireless network.

According to a sixth aspect of this invention, there is provided a method of reducing
5 distortion of a desired image that it is desired be viewed at a viewing position, the method including computer processing means carrying out the steps of:

- a) receiving information indicative of the desired image;
- 10 b) instructing projection means to project the desired image from a projection position such that a first displayed image is displayed on a display surface at a display position;
- c) receiving appearance information from first sensing means at a sensing position, this information being indicative of a sensed image of the appearance, at the sensing
15 position, of the first displayed image;
- d) receiving distance information from second sensing means at the projection position, this information being indicative of the distance of the display position from the projection position;
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- e) operating on the appearance information and the distance information, and the information indicative of the desired image, or other information indicative of the desired image, to produce modified information for use in instructing the projection means to project a modified version of the desired image such that another displayed
25 image is displayed at the display position that more closely has the appearance of the desired image at the viewing position than did the first displayed image.
- f) instructing the projection means to project the modified desired image.

30 One or more of the optional features of the first aspect of this invention may also be features of the other aspects of this invention.

BRIEF DESCRIPTION OF THE DRAWINGS

A specific embodiment of this invention will now be described, by way of example only, with reference to the accompanying drawings, in which:

- 5 Figure 1 is a schematic diagram illustrating the changes in an application image when pre-warping is applied;

Figure 2 is a schematic diagram illustrating the general arrangement of apparatus for performing pre-warping;

- 10 Figure 3 shows co-ordinate systems associated with the projector, display surface and camera;

Figure 4 is a flow chart illustrating calibration steps performed by the apparatus shown in Figure 2;

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Figure 5 is a schematic diagram illustrating a mapping transformation for a calibration pattern between the projector, display surface and camera;

- 20 Figure 6 is a schematic diagram illustrating the mapping transformation for an application image when pre-warping is applied;

Figure 7 is a flow chart illustrating the steps carried out by the apparatus in applying a pre-warp transformation to an application image before it is sent as an input to the projector;

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Figures 8a shows an application image used in a simulation of an embodiment of the present invention;

- 30 Figure 8b shows how the image of Figure 8a appeared when projected onto a non-planar surface;

Figure 9a shows the image captured by a camera notionally positioned at a viewing position;

Figure 9b shows a range image of the non-planar surface captured by the camera;

Figure 10a shows a modified application image sent to a projector; and

Figure 11b shows a substantially distortion-free image as seen at the viewing position.

10 **SPECIFIC DESCRIPTION OF A PREFERRED EMBODIMENT**

In the following description, an embodiment will be described in which geometric distortion of a projected image perceived at a particular viewpoint is reduced by applying a pre-warp transformation to a desired image before that image is sent as an input to a projector. The pre-warp transformation is such the projected image appears to
15 a viewer as the desired image. The desired image may also be termed the “application” image. The pre-warping reduces the image distortion perceived at the particular viewpoint by compensating for the distortions caused by both projector misalignment and the non-planar display surface.

20 The general pre-warping concept is illustrated in Figure 1, which shows the application image is in the form of a rectangular grid pattern. It will be appreciated, however, that a rectangular grid pattern is shown for the sake of simplicity of explanation and that the application image is not limited to rectangular grid features and this assumption is merely for illustrative purposes. As shown in Figure 1, the rectangular application
25 image is skewed before projection by a pre-warp transformation to produce an image 230 with a pre-warped geometry. The pre-warped image is then projected onto the display surface 210 by the projector 200. The values of the pre-warp transformation are chosen so that the geometric distortions introduced by projector misalignment and the non-planar display surface when the pre-warped image 230 is projected onto the display
30 surface and then viewed by the viewer at least partially cancels the pre-warp transformation. The resulting image 240 perceived by the viewer 202 at the selected viewpoint therefore has reduced or no distortion. Accordingly, with the application of

pre-warp transformation in this embodiment, a rectangular image 240 is perceived by the viewer 202.

Figure 2 illustrates apparatus that embodies the present invention. This apparatus is for deriving the pre-warp transformation values and applying these to an application image. The apparatus comprises a projector 200, computer processing means 250, a display surface 210 and a camera 220. The camera 220 is both an optical camera and a range camera, being able to simultaneously capture both an optical image of the display surface (including any image displayed thereon by projection) and range data for each point on the display surface. Range data is a measurement of the distance from a point on the display surface 210 to an image plane of the camera 220. The projector 200, which has a field of projection 201 (demarcated by the solid lines in the Figure), is coupled to the computing means 250 and arranged such that it is able to project digital images in response to suitable control signals from the computer processing means 250. In the present embodiment, the computer processing means 250 is in the form of a personal computer (PC) 250. Alternatively, other portable or fixed computer processing means such as a laptop or workstation are also suitable. During the process of determining the required pre-warp transformation values, the projector projects a calibration image 211 onto the display surface 210. The calibration image includes a calibration pattern that is a structured pattern including one or more calibration feature points arranged in pre-known positions. The calibration feature points for observing changes in the application image may be monochromatic or coloured. In one embodiment, the calibration feature points comprise a plurality of dots of distinguished intensity/colour. Alternatively, the corners of geometrical shapes (e.g. rectangles, polygons) may also be used as calibration feature points.

The optical and range camera 220 is placed at a selected viewpoint 202 and used to sense or, in other words, to observe both the 2-D image of the calibration pattern displayed on the display surface 210 and the range data for the points in the observed 2-D image. Thus every optical image captured by the camera 220 in the present embodiment is supplemented with a corresponding range image which is a composite map of the range data. Information on the profile of the display surface 210 is obtained from the range image. As shown in Figure 2, the camera 220 positioned with its field of

view 221 (demarcated by the broken lines shown in the Figure) intersecting the field of projection 201 of the projector 200 on the display surface 210. In the presently described embodiment, the display surface 210 is curved

5 The camera 220 is coupled to the PC 250 and thus providing a close-loop system where image and range information pertaining to a projected calibration pattern is captured by the camera 220 and fed back to the PC 250 where it is processed to derive the pre-warp transformation values. The pre-warp values are derived using an algorithm based on obtaining the mapping transformation for a given set of features which are projected by
10 the projector 200 and their corresponding locations in the image observed by the camera 220. Details of the algorithm will be described in the succeeding paragraphs. The PC 250 subsequently applies the derived pre-wrap transformation on any application images before they are outputted to the projector, to create a reduced distortion effect at that selected viewpoint 202.

15 Therefore, the pre-warping transformation which compensates for distortion is based on not only observing the 2-D appearance of the displayed projected images, but also the shape of the display surface through the range data captured by a camera placed at a selected view point. Range data and its correspondence with the optical image can be
20 obtained with products that are capable of generating a range image together with the normal optical image. One example is ZCamTM from 3DV Systems, wherein each pixel in the scene captured by the camera is tagged with the distance of the pixel from the camera. Alternatively, the image and range information can also be obtained with normal stereo vision that uses no less than 2 normal cameras. But the system will be
25 bulkier and the generation of range data require heavy computing.

Whereas, in prior art arrangements, the camera only captures the optical image on the display surface and wherein non-planar surfaces are then broken down to multiple planar surfaces and the pre-warp transformation values derived independently for each
30 surface, the present embodiment uses a camera to also capture the range image. In this way, the present embodiment eliminates the problem of having to model a non-planar display surface as multiple planar surfaces encountered in the approach of adopted in the prior art.

The processing of the methods and algorithms described hereinbelow is implemented as a computer program executing on computer processing means. The computer program may be recorded on memory devices located in the processing means or on a portable data storage device or semiconductor memory device coupled to the computing means. Additionally, it is to be appreciated that communication between the projector, computing means and camera may be carried out both wirelessly or through wired communication or the combination of both.

Turning now to Figure 3, the coordinate systems used in an algorithm for deriving the pre-warp transformation are shown. The projector 200 has a projector image plane 202 with an associated 2-D coordinate system. Each application image comprises a series of optical image points represented as co-ordinates (x_1, y_1) on the projector image plane 202. The camera 220 has a separate camera image plane 222 where points on the image captured by the camera are represented as co-ordinates (x_2, y_2) on the associated 2-D coordinate system. . Two additional 3D coordinate systems S_1 and S_2 are also associated with the projector 200 and the camera 220 respectively as shown in Figure 3. The origin of the 2 and 3-D coordinate systems associated with the projector and camera are located at the optical centre (O_1 and O_2) of the projector and camera respectively. The optical centre is the point where the optical axis of the projector or camera intersects the projector or camera image plane.

In the present embodiment, a point (x_1, y_1) on the projector image plane 202 is projected to some unknown point on the display surface 210. The point on the display screen 210 is then captured by the camera 220 as a point (x_2, y_2) on the camera image frame 222. The unknown point on the screen is a 3D point with coordinates of (X_1, Y_1, Z_1) when referenced with respect to 3-D coordinate system S_1 associated with the projector 200 and coordinates of (X_2, Y_2, Z_2) when referenced with respect to coordinate system S_2 associated with the camera 220. Z_1 and Z_2 represent the distance of a point on the display surface 210 from the optical centre of the projector 200 and camera 220 respectively.

The mapping of a point (x_1, y_1) from the projector image frame 202 to the display screen 210 involves a first perspective projection whose parameters are dependent on

factors such as the relative position of the projector to the screen and the projector's optical parameters, such as its focal length. Meanwhile, the mapping of a point from the display screen to the camera image frame (x_2, y_2) involves a second perspective projection whose parameters are dependent on factors such as the relative position of the camera to the screen and the camera's optical parameters. Since the display screen 210 is non-planar, the display screen surface shape also affects both the parameters of first and second perspective projection. Unlike the known methods described earlier, the present invention does not require knowledge of these spatial and optical parameters in order to obtain the $(x_1, y_1) \longrightarrow (x_2, y_2)$ mapping effected by the first and second perspective projections.

In a preferred embodiment, the first and second perspective projections i.e. the relationship between the 3D projected image on the display surface 210 $((X_1, Y_1, Z_1)$ in coordinate system S_1 and (X_2, Y_2, Z_2) in coordinate system S_2) and the image on the projector and camera image planes (x_1, y_1) and (x_2, y_2) is approximated as:

$$\begin{bmatrix} sx_1 \\ sy_1 \\ s \end{bmatrix} = M_{1,3 \times 4} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} \quad (1)$$

and:

$$\begin{bmatrix} s'x_2 \\ s'y_2 \\ s' \end{bmatrix} = M_{2,3 \times 4} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} \quad (2)$$

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where matrices M_1 and M_2 are projection matrices for the projector 200 and the camera 220 respectively.

Equations (1) and (2) are based on the pinhole camera model which is commonly adopted in computer vision area for modelling the perspective projection of the 3D world onto a 2-D image plane. In the pinhole camera model, perspective projection involves an optical ray from a point on a 3D object passing through the optical centre of

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the projector or camera and forming a point on the projector or camera image plane. In equations (1) and (2), this perspective projection is represented by matrices M_1 and M_2 . An introduction to the pinhole camera model transformation can be found in any one of a number of modern computer vision textbooks. One example of such is *Three-*
 5 *Dimensional Computer Vision*, by Olivier Faugeras, MIT Press, 1993. The pinhole camera model has the advantage of using simple, linear equations for modelling the perspective projection thus reducing computational complexity. Alternatively, other models including more complex models which take into account the effects of lens distortion may be used.

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If the relationship between the two 3-D coordinate systems S_1 and S_2 is denoted by a matrix T , then for the same point on the display surface 210, there is the following transformation between its coordinates in S_1 and S_2 :

$$15 \quad \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} = T_{4 \times 4} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} \quad (3)$$

Substituting (3) into (1), we can further have:

$$\begin{aligned} \begin{bmatrix} sx_1 \\ sy_1 \\ s \end{bmatrix} &= M_{1,3 \times 4} \bullet T_{4 \times 4} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} \\ &= M_1' \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} \end{aligned} \quad (4)$$

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With the knowledge of Z_2 which is the range data of a point captured by the camera 220, equations (2) and (4) can be used to formulate the projective transformation between the coordinates in the image planes of the projector 200 and the camera 220, as

a function of the range data Z_2 captured by the camera 220 by the following steps.

Firstly, (4) is re-written in the form of:

$$\begin{bmatrix} sx_1 \\ sy_1 \\ s \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{14} \\ m_{21} & m_{22} & m_{24} \\ m_{31} & m_{32} & m_{34} \end{bmatrix} \begin{bmatrix} X_2 \\ Y_2 \\ 1 \end{bmatrix} + \begin{bmatrix} m_{13}Z_2 \\ m_{23}Z_2 \\ m_{33}Z_2 \end{bmatrix} \quad (5)$$

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where m_{ij} is the element of M_1' at the i-th row and j-th column. In the same way, (2) is re-written as:

$$\begin{bmatrix} s'x_2 \\ s'y_2 \\ s' \end{bmatrix} = \begin{bmatrix} n_{11} & n_{12} & n_{14} \\ n_{21} & n_{22} & n_{24} \\ n_{31} & n_{32} & n_{34} \end{bmatrix} \begin{bmatrix} X_2 \\ Y_2 \\ 1 \end{bmatrix} + \begin{bmatrix} n_{13}Z_2 \\ n_{23}Z_2 \\ n_{33}Z_2 \end{bmatrix} \quad (6)$$

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where n_{ij} is the element of M_2 at the i-th row and j-th column.

From equation (5), the following can be obtained:

$$\begin{bmatrix} X_2 \\ Y_2 \\ 1 \end{bmatrix} = \text{inv} \left(\begin{bmatrix} m_{11} & m_{12} & m_{14} \\ m_{21} & m_{22} & m_{24} \\ m_{31} & m_{32} & m_{34} \end{bmatrix} \right) \left(\begin{bmatrix} sx_1 \\ sy_1 \\ s \end{bmatrix} - \begin{bmatrix} m_{13}Z_2 \\ m_{23}Z_2 \\ m_{33}Z_2 \end{bmatrix} \right) \quad (5A)$$

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Whereas, from equation (6), the following can be obtained:

$$\begin{bmatrix} X_2 \\ Y_2 \\ 1 \end{bmatrix} = \text{inv} \left(\begin{bmatrix} n_{11} & n_{12} & n_{14} \\ n_{21} & n_{22} & n_{24} \\ n_{31} & n_{32} & n_{34} \end{bmatrix} \right) \left(\begin{bmatrix} s'x_2 \\ s'y_2 \\ s' \end{bmatrix} - \begin{bmatrix} n_{13}Z_2 \\ n_{23}Z_2 \\ n_{33}Z_2 \end{bmatrix} \right) \quad (6A)$$

20 By equating these, the projective transformation of coordinates (x_1, y_1) from the projector image plane 202 to coordinates (x_2, y_2) in the camera image plane 222 i.e.

$P: (x_1, y_1) \longrightarrow (x_2, y_2)$ can be fully determined by:

$$\begin{bmatrix} sx_1 - aZ_2(x_2, y_2) \\ sy_1 - bZ_2(x_2, y_2) \\ s - cZ_2(x_2, y_2) \end{bmatrix} = C_{3 \times 3} \begin{bmatrix} x_2 - dZ_2(x_2, y_2) \\ y_2 - eZ_2(x_2, y_2) \\ 1 - fZ_2(x_2, y_2) \end{bmatrix} \quad (7)$$

$\longleftrightarrow$
 Projector image
plane

$\longleftrightarrow$
 Camera image
plane

where a, b, c, d, e, f and $C_{3 \times 3}$ are the parameters obtained from (5) and (6) after some calculations and normalizations. $Z_2(x_2, y_2)$ refers to the range data corresponding to point (x_2, y_2) on the camera image plane 222.

Equation (7) can be further reduced to:

$$\begin{bmatrix} sx_1 \\ sy_1 \\ s \end{bmatrix} = D_{3 \times 3} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} + E_{3 \times 1} Z_2(x_2, y_2) \quad (8)$$

where the matrices D and E are the results of re-arranging the terms of (7).

Equation (8) is very important because it demonstrates the transformation relationship between a point (x_1, y_1) on the projection image plane 202 to a point (x_2, y_2) on the camera image plane 222. For the case where the display screen 210 is planar, only a 2D to 2D projective transformation would be involved and equation (8) would be in the form:

$$\begin{bmatrix} sx_1 \\ sy_1 \\ s \end{bmatrix} = D_{3 \times 3} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}.$$

Thus the effect of the non-planar display screen 210 is taken into account by the term $E_{3 \times 1} Z_2(x_2, y_2)$ in equation (8) which includes the range image associated with the optical image.

Equation (8) can also be written in the form of:

$$\begin{bmatrix} sx_1 \\ sy_1 \\ s \end{bmatrix} = M_{A,3 \times 4} \begin{bmatrix} x_2 \\ y_2 \\ Z_2(x_2, y_2) \\ 1 \end{bmatrix} \quad (9)$$

$$\begin{bmatrix} sx_1 \\ sy_1 \\ s \end{bmatrix} = \begin{bmatrix} M_{11}x_2 + M_{12}y_2 + M_{13}Z_2(x_2, y_2) + M_{14} \\ M_{21}x_2 + M_{22}y_2 + M_{23}Z_2(x_2, y_2) + M_{24} \\ M_{31}x_2 + M_{32}y_2 + M_{33}Z_2(x_2, y_2) + M_{34} \end{bmatrix}$$

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where M_{ij} is the element of $M_{A,3 \times 4}$ at the i -th row and j -th column.

Equation (9) has a form similar to the normal projection from 3D coordinates to the 2D image plane. In equation (9), the matrix $M_{A,3 \times 4}$ maps the points from the camera image plane to the projector image plane. To obtain the inverse mapping from the projector image plane to the camera image plane, the terms (x_1, y_1) and (x_2, y_2) in equation (9) are simply interchanged as shown below.

$$\begin{bmatrix} sx_2 \\ sy_2 \\ s \end{bmatrix} = M_{B,3 \times 4} \begin{bmatrix} x_1 \\ y_1 \\ Z_2(x_1, y_1) \\ 1 \end{bmatrix}$$

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Thus, the first and second projective transformations involved in mapping a point from the projector image plane 202 to the screen and then from the screen to the camera image frame 222 is expressed by a single projective transform that has 11 unknown global parameters (not 12 because there are only 8 degrees of freedom for the matrix D in equation (8)). The single projective transform representing the projective transformation from the projector image plane 202 to the camera image plane 222 will be referred to as P in the succeeding paragraphs. These 11 unknown global parameters can be easily obtained by utilizing the fact that each 2D point (x_1, y_1) on the projector image plane 202 has a corresponding 3D point captured by the camera 220, wherein the

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3D point captured by the camera refers to an optical image point (x_2, y_2) and the range data $Z_2(x_2, y_2)$ corresponding to that point.

With the above in mind, a very simple calibration process can be carried out to estimate the parameters of the projective transform P without having to obtain knowledge of the set-up such as position, optical parameters, surface topography or shape of the display screen, projector or camera.

In the pre-warping method of the present embodiment, the range values do not need to be exact, actual, values. Instead, a bias (such as a constant deviation from the actual value), and/or a scaling of the range values, will cause little or no adverse effect for the processing. This is because any such deviation or scaling can be absorbed by the 11 global parameters obtained in the calibration process.

Figure 4 is a flowchart illustrating the calibration steps 500 for obtaining the 11 global parameters for the projective transformation by using the approach of Figure 2. The camera 220 is positioned with its field of view 221 intersecting the field of projection 201 of the projector 200 on the display surface 210. The calibration steps 500 comprise firstly, at step 510 projecting a digital image comprising a calibration pattern onto the display surface 210. This is done with the PC 250 controlling the projection of images by the projector 200. In the present embodiment shown in Figure 5, the calibration pattern comprises a grid pattern 204 and the calibration feature points are dots 206 with distinguished intensity/colour located at the intersection of the horizontal and vertical grid lines 203. Alternatively, other calibration patterns are also suitable and the calibration feature points needed for calibration can be found on separate frames. For simplicity of illustration, only one calibration feature point is shown in Figure 5. As defined in equation (1), the calibration feature points are mapped from the projector image plane to the display screen 210 by the matrix M_1^{-1} .

The camera 220 captures at step 520 an optical and range image of the calibration pattern 214 displayed on the display surface 210. As defined in equation (2), the mapping from the display screen to the camera image plane is given by M_2 . In the next step 530, software residing in the PC 250 executes a correlation algorithm to identify

the location of calibration feature points 226 in the camera image plane of the optical image captured by the camera 220 corresponding to a given set of known calibration feature points in the projector image plane. The calibration feature points 226 may be identified by means of intensity and/or colour filtering and pattern recognition of the calibration pattern 224 captured by the camera. Since each correspondence between a calibration feature point on the projection image plane and a calibration feature point on the camera image plane provides two independent linear equations, a minimum of 6 pairs of corresponding calibration feature points have to be projected by the projector 200 and captured by the camera 220. In a preferred embodiment, more than 6 pairs of corresponding points are used and equation (8) is solved using least-squares estimation method to correct for noise in the calibration measurements.

Finally, in step 540, software residing on the PC 250 is executed to calculate the 11 global parameters in the mapping transformation P using the optical data associated with the at least 6 pairs of corresponding calibration feature points and the range data associated with the calibration feature points in the camera image plane. Thus the parameters of the mapping transformation P between the image plane of the projector and the image plane of the camera are fully determined by solving the linear equations of equation (8). Therefore, unlike in the prior art methods discussed herein before, there is no need to have knowledge of the projector, camera or display screen set-up. Instead, the mapping transformation for a non-planar display surface is obtained merely by correlating calibration feature points.

Once the parameters of the mapping transformation P have been obtained from the calibration process, the pre-warp transformation required for keystone correction can be determined and applied to any application image before it is outputted from the PC to the projector. As shown in Figure 6, the pre-warp transformation W applied to an application image 258 to produce a pre-warped image on the projector image plane 202. The pre-warp transformation W is given by $P^{-1}S$, where P is the mapping transformation from the projector image plane 202 to the camera image plane 222, and S is a scaling factor. The P^{-1} component corrects for geometric distortion so that a rectangle in the application image 258 will also appear rectangular on the camera image plane, i.e. when perceived by a viewer at a viewpoint located where the camera is

positioned (see Figure 6). Since the mapping transformation P between the projector image plane 202 and camera image plane 222 cancels out the P^{-1} component in the pre-warp, the transformation between the application image 258 and the image “seen” at the camera is just the scaling factor S .

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Figure 7 shows the steps 800 of applying pre-warp transformation to an input application image 258 in accordance with one embodiment of the invention. Firstly, in step 810, the scaling factor S is applied to an application image to adjust the dimensions and placement of the application image so that it fits within a selected target region in the camera image plane. As shown in Figure 6, the target region 225 is a rectangular region within the camera image plane 222 (corresponding to a fraction of the area that can be illuminated by the projector) where the contents of the application image are to be seen by the camera. The selection of the target region is quite flexible with the only restriction being that it has to be within the illuminated area. In a preferred

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embodiment the target region is selected to be as large as possible (limited by image size viewable by the camera) and with the aspect ratio of the application image contents maintained so as to obtain a good viewing effect. In the described embodiment of Figure 6 the scaling factor S is selected so that the aspect ratio of the rectangle in the application image 258 is maintained when mapped to the camera image plane.

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After applying the scaling factor S , the P^{-1} component of the pre-warp transformation which is based on the 11 global parameters obtained from the calibration process is applied to the application image in step 820 before it is output to the projector. After this pre-warping, the image seen by the user at the particular view point will have reduced or substantially no geometric distortion. The required pre-warp transformation

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$P^{-1}S$ for a given viewpoint can be saved in the form of a look-up table and applied to any application image before they are sent to the projector. As a result of the scaling and transformation processes are carried out during the application of pre-warp

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transformation, the number of pixels in the pre-warped image may be larger or smaller than that in the original application image. In order to approximate the value for each

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pixel in the pre-warped image interpolation methods such as bi-linear interpolation and bi-cubic interpolation may be used. In one embodiment, the interpolation algorithms may be arranged into a form of lookup table (where a pixel in the projector image plane is used as an index to obtain the location of the corresponding pixel in the camera image

plane and vice versa). In this way, for a given viewpoint, the required pre-warp transformation $P^{-1}S$ for each pixel can be easily applied to any input application image without going through the same calculations with every different input application image.

5

The steps illustrated in the described embodiment of Figure 4 are required for a fixed set-up of projector 200, display surface 210 and a particular view point 202. Any changes in the set-up may necessitate a new calibration to determine the global parameters of the mapping transformation which are used to calculate the correct pre-warp. In principle, it is impossible to achieve the same amount of reduction in geometric distortion for all viewpoints at the same time. The effects of the pre-warp compensation will clearly be most pronounced for viewpoints located near to the position of the camera. However, for any given view point, this method can be applied to give a correct pre-warp at real-time.

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One or more calibration patterns are displayed alongside the application images and the apparatus and method be configured to continually recalibrate itself by performing steps 520 and 530 at periodic intervals. In this way, it is possible to dynamically adapt to the changes in the set-up (e.g. due to the display surface deforming) by updating the pre-warp transformation in real-time when the mapping transformation P derived from the recalibration differs from the present pre-warp transformation by certain threshold. In order to avoid the interference to the viewer, the calibration pattern may be designed in such a way that it can be embedded into the application images without great influence for the viewing effect. For example, they may be residing only on the edge area of an image.

25

A simulation of the both the calibration and pre-warp processes was carried out in order to illustrate the procedures of the above-described methods and their resulting effect. Actual projectors and cameras were not used in the simulation but the projector and camera images are generated based on the assumptions of the pinhole camera model. An application image 902 (a checkerboard) without pre-warp transformation is provided (see Figure 8a) and its corresponding display image upon projection onto a spherical display surface in 3D space is shown in Figure 8b.

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A simulation based on the pinhole camera model is carried out and a simulated optical image 922a (see Figure 9a) of the checkerboard pattern when captured by an off-centre camera whose optical axis does not overlap with that of the projector is obtained. As

5 evident from the representation of the resulting optical image 922 shown in Figure 9a, geometric distortion of the checkerboard pattern is present when perceived from the viewpoint of the camera. The range value of each pixel in the display image captured by the camera is also simulated and as shown in Figure 9b a range image of the display image is created by grey scaling each pixel according to its range value. Thus every

10 optical image frame captured by the camera has an additional corresponding range image frame containing depth information. As discussed earlier, the depth information is representative of the display surface profile. In one embodiment, the brightness of each pixel in the range image is varied in a manner proportional to its range value.

15 In the present embodiment, the mapping transformation from the projector 202 to the camera image plane 222 P is obtained through a calibration process involving 16 pairs of corresponding projector and camera image points. With the target region chosen as the maximum rectangle within the camera image plane which is illuminated by the projector, pre-warp transformation values are calculated using the parameters of the

20 mapping transformation P obtained from the calibration process and the scaling factor S required to fit the application image into the selected target region. Figure 10a shows the application image 930 which is sent to the projector after pre-warp transformation of the checkerboard pattern while Figure 10b shows the resulting optical image 940 captured by the camera. Geometric distortion present in the optical image 922a of

25 Figure 9a is compensated by the pre-warp transformation thus producing the optical image 940 which is substantially free of geometric distortion. During the application of the pre-warp transformation to an application image, interpolations are applied to determine the value on each pixel. The effect of interpolation is quite obvious as jagged edges are evident in both the pre-warped application image (Figure 10a) and optical

30 image captured by the camera shown in Figure 10b. This poor resolution is due to both the images being quite small (64 by 64) and having highly contrasting patterns. In real applications, the interpolation effect should be unnoticeable. Nevertheless, it is still evident that the geometric distortion seen by the camera is removed.

Once the geometric transformation between the projector and camera image planes has been found, compensation in the colour/intensity/texture can also be done to remove the spatially varying reflectance of the display surface that results from surface imperfections there of due to, for example, a layer of paint on the display surface, or the display surface being formed from tiles or wood. An appropriate method of carrying out this form of compensation would be one such as that disclosed by Shree K. Nayar, Harish Peri, Michael D. Grossberg, and Peter N. Belhumeur in “*A Projection System with Radiometric Compensation for Screen Imperfections*”, Proc. ICCV Workshop on Projector-Camera Systems (PROCAMS), Nice, France, October 2003. By using such an approach, embodiments of the present invention can bring about the projection of images onto non-planar surfaces while preserving their quality.

In an alternative embodiment, the distance measurement is done on the projector side, rather than on the camera side. In this embodiment, the principal difference is that the deduction from equations (1) and (2) to equation (7) would be based on:

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} \quad \text{rather than on:} \quad \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix}$$

The form of equation (8), however, will be the same, but with Z_2 being replaced by Z_1 .

In another alternative embodiment, the sensing position at which the camera senses the displayed image need not be coincident with, or even close to, the viewing position at which images projected onto the display screen are viewed. The reason for this is that an affine transformation between the image captured at the sensing position and the image perceived at the viewing position can be used to take account of the distance between these two position. This transformation can be expressed as:

$$\begin{bmatrix} sx_1 \\ sy_1 \\ 1 \end{bmatrix} = D_{3 \times 3} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$$

- wherein (x_1, y_1) and (x_2, y_2) now represent coordinates of the image sensed at the sensing position and the image sensed at the viewing position. In this embodiment, rather than correcting the projected image to give the desired image at the sensing position, the result of $inv(D_{3 \times 3})$ as applied to the image sensed at the sensing position is
- 5 corrected to give rise to the desired image at the now-displaced viewing position. It is assumed that matrix $D_{3 \times 3}$ is unknown, but it will be appreciated that this can be calculated if the position of the viewing position relative to the sensing position is known, or by way of another calibration process.
- 10 The apparatus and method of the embodiments described hereinabove can be used to project digital images onto display surfaces of almost any form, including non-planar surfaces, such that distortion of the displayed image is minimised when viewed from almost any given view point. The pre-warp needed for correction of the image distortion is calculated based on the normal image and range image provided by a special camera
- 15 put at the view point. Compared to other known methods, the present embodiment provides several advantages. Firstly, it is suitable for projection onto non-planar surfaces; and, secondly, it provides a more accurate (pixel by pixel) result and involves a much easier calibration procedure with much lower computation requirements than those associated with prior art arrangements.

CLAIMS

1. A method of reducing distortion of a desired image that it is desired be viewed at a viewing position, the method including computer processing means carrying out the steps of:
- 5
- a) receiving information indicative of the desired image;
 - b) instructing display means to display the desired image such that a first displayed
10 image is displayed at a display position;
 - c) receiving information from sensing means at a sensing position, including appearance information indicative of a sensed image of the appearance, at the sensing position, of the first displayed image; and including distance information indicative of the distance
15 of the display position from the sensing position;
 - d) operating on the appearance information and the distance information, and the information indicative of the desired image, or other information indicative of the desired image, to produce modified information for use in instructing the display means
20 to display a modified version of the desired image such that another displayed image is displayed at the display position that more closely has the appearance of the desired image at the viewing position than did the first displayed image.
 - e) instructing the display means to display the modified desired image.
25
2. A method according to claim 1, wherein the display means includes projection means for projecting images onto a display surface, the display surface being positioned at the display position and the projection means being positioned at a projection position.
- 30 3. A method according to claim 1 or claim 2, wherein the sensing position and the viewing position are adjacent one another.

4. A method according to any preceding claims, wherein the sensing position and the viewing position are coincident.
5. A method according to any one of claims 2 to 4, wherein the projection means preferably includes a digital projector arranged for operation under the control of the computer processing means to project an image in response to a projection signal from those means.
- 10 6. A method according to any one of claims 2 to 5, wherein instructing the display means to display the desired image, or the modified desired image, includes instructing the projection means to project the desired image or modified desired image from an image plane of the projection means, the image plane of the projection means being positioned at the projection position.
- 15 7. A method according to any preceding claim, wherein, at step (d), the other information indicative of the desired image is a control signal from the computer processing means to the display means.
- 20 8. A method according to any preceding claim, wherein, at step (d), the information indicative of the desired image is data in a format convenient for processing by computer processing means, such as, for example, GIF or JPEG data.
- 25 9. A method according to any preceding claim, wherein the sensing means is operable to sense both a two-dimensional (2-D) sensed image of the appearance of displayed images as they appear on an image plane of the sensing means, and the distance of the display position from the image plane of the sensing means, the image plane of the sensing means being at the sensing position.
- 30 10. A method according to any preceding claim, wherein the sensing means is operable to sense the respective distance of each of a plurality of parts of a surface of the display screen from the sensing position.

11. A method according to claim 9 or 10 when dependent on claim 9, wherein the sensing means is arranged to present the distance information in the form of information indicative of a 2-D range image arranged to correspond with the 2-D sensed image such that parts of the range image each indicate the distance, from the sensing position, of a
5 respective part of the surface of the display screen that is associated with a respective part of the sensed image that corresponds with that part of the range image.

12. A method according to claim 11, wherein each part is a pixel of the image of which it is a part, or, in the case of the surface of the display screen, each part is a part of that
10 surface that is substantially covered by a pixel of the first displayed image displayed on that surface.

13. A method according to any one of the preceding claims, wherein the sensing means includes a single camera operable to sense both the appearance information and the
15 distance information.

14. A method according to any one of claims 1 to 12, wherein the sensing means includes two cameras, positioned apart, such that the method makes use of stereo vision to arrive at at least the distance information.
20

15. A method according to any preceding claim, wherein the desired image is a calibration image including a pattern.

16. A method according to any preceding claim, wherein the desired image is a
25 photographic image for viewing by the viewer.

17. A method according to any preceding claim, where step (b) includes the step of sending to the display means a control signal based on the information received at step (a) to thereby control those means to project from the projection means the image of
30 which that information is indicative; and wherein step (e) similarly includes the step of sending a control signal to the display means.

18. A method according to any preceding claim, wherein step (d) is such that the modified information produced therein is a modified version of the information received at step (a); and/or, when according to claim 17, a modified version of the control signal sent to the display means in step (b), wherein step (e) would include sending the
5 modified version of the control signal to the display means.

19. A method according to any preceding claim, wherein step (d) includes the step of operating on the appearance information and the distance information, and the information indicative of the desired image or the other information indicative of the
10 desired image, to produce modifying information for use in producing the modified information.

20. A method according to claim 19, wherein the modifying information includes information indicative of a mathematical expression that maps the desired image to the sensed image and/or the sensed image to the desired image.
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21. A method according to claim 20, wherein the mathematical expression is a transform that relies on determining the relative positions of at least six points of the sensed image, determining the relative positions of the six corresponding points in the
20 desired image, such that six pairs of corresponding points are determined; and also determining the distance, from the sensing position, of each of six points of the display surface that are associated with a respective one of the corresponding pairs.

22. A method according to claim 20 or claim 21, wherein the transform further relies on
25 determining the distance, from the sensing position, of each part of the display surface on which a pixel of the first displayed image is displayed.

23. A method according to any one of claims 20 to 22, wherein the mathematical expression at least includes an expression that can be expressed as:

$$\begin{bmatrix} sx_1 \\ sy_1 \\ s \end{bmatrix} = M_{A,3 \times 4} \begin{bmatrix} x_2 \\ y_2 \\ Z_2(x_2, y_2) \\ 1 \end{bmatrix}$$

wherein a point (x_1, y_1) on the image plane of the projection means is projected to an unknown point on the display surface; that point being captured by the sensing means as a point (x_2, y_2) on the image plane thereof; the unknown point on the screen is a 3D point having coordinates of (X_1, Y_1, Z_1) when referenced with respect to a 3-D coordinate system S_1 associated with the projection means and coordinates of (X_2, Y_2, Z_2) when with respect to a coordinate system S_2 associated with the sensing means; wherein the matrix $M_{A,3 \times 4}$ maps points from the image plane of the sensing means to the image plane of the projection means;

10 this expression being re-writable to be expressed as:

$$\begin{bmatrix} sx_1 \\ sy_1 \\ s \end{bmatrix} = \begin{bmatrix} M_{11}x_2 + M_{12}y_2 + M_{13}Z_2(x_2, y_2) + M_{14} \\ M_{21}x_2 + M_{22}y_2 + M_{23}Z_2(x_2, y_2) + M_{24} \\ M_{31}x_2 + M_{32}y_2 + M_{33}Z_2(x_2, y_2) + M_{34} \end{bmatrix}$$

wherein M_{ij} is the element of $M_{A,3 \times 4}$ at the i-th row and j-th column.

24. A method according to any one of claims 20 to 22, wherein the mathematical expression at least includes an expression that can be expressed as:

$$\begin{bmatrix} sx_2 \\ sy_2 \\ s \end{bmatrix} = M_{B,3 \times 4} \begin{bmatrix} x_1 \\ y_1 \\ Z_2(x_2, y_2) \\ 1 \end{bmatrix}$$

wherein a point (x_1, y_1) on the image plane of the projection means is projected to an unknown point on the display surface; that point being capture by the sensing means as a point (x_2, y_2) on the image plane thereof; the unknown point on the screen is a 3D point having coordinates of (X_1, Y_1, Z_1) when referenced with respect to a 3-D coordinate system S_1 associated with the projection means and coordinates of (X_2, Y_2, Z_2) when with respect to a coordinate system S_2 associated with the sensing means;

Z₂) when with respect to a coordinate system S₂ associated with the sensing means; wherein the matrix $M_{B,3 \times 4}$ maps points from the image plane of the projection means to the image plane of the sensing means.

- 5 25. A method according to any one of claims 20 to 24, wherein the mathematical expression is arranged to modify the size of the desired image to give a modified desired image that, when projected, results in an image of a desired size being viewed by the viewer.
- 10 26. A method according to claim 25, wherein the mathematical expression is such that it has the effect of selecting a region within the sensible area at the image plane of the sensing means, that region corresponding to a region that it is desired be filled by the sensed image and causing that region to be filled by the sensed image.
- 15 27. A method according to claim 25 or 26, wherein the mathematical expression may include a scaling factor that can be applied to the desired image to modify the size and/or position of that image.
- 20 28. A method according to any one of claims 25 to 27, wherein the mathematical expression associated with a particular sensing position and/or projection position is stored so as to be retrievable for applying to one or more subsequent desired images.
- 25 29. A method according to claim 28, wherein the expression is stored in a look-up table and the method includes the further steps of storing and/or retrieving the expression.
- 30 30. A method of reducing distortion of a desired image that it is desired be viewed at a viewing position, the method including computer processing means carrying out the steps of:
 - a) receiving information indicative of the desired image;
 - b) instructing projection means to project the desired image from a projection position such that a first displayed image is displayed on a display surface at a display position;

- c) receiving appearance information from first sensing means at a sensing position, this information being indicative of a sensed image of the appearance, at the sensing position, of the first displayed image;
- 5 d) receiving distance information from second sensing means at the projection position, this information being indicative of the distance of the display position from the projection position;
- 10 e) operating on the appearance information and the distance information, and the information indicative of the desired image, or other information indicative of the desired image, to produce modified information for use in instructing the projection means to project a modified version of the desired image such that another displayed image is displayed at the display position that more closely has the appearance of the
- 15 desired image at the viewing position than did the first displayed image.
- f) instructing the projection means to project the modified desired image.
31. A method according claim 30, wherein the first sensing means is operable to sense a
- 20 two-dimensional (2-D) sensed image of the appearance of displayed images as they appear on an image plane of the first sensing means, the image plane of the first sensing means being at the sensing position; and wherein the second sensing means is operable to sense the distance of the display position from an image plane of the second sensing means, the image plane of the second sensing means being at the projection position.
- 25 32. A method according to claim 30 or 31, wherein the second sensing means is operable to sense the respective distance of each of a plurality of parts of a surface of the display screen from the projection position.
- 30 33. A method according to claim 30, 31 or 32, wherein the second sensing means is arranged to present the distance information in the form of information indicative of a 2-D range image arranged to correspond with the 2-D sensed image such that parts of the range image each indicate the distance, from the projection position, of a respective part

of the surface of the display screen that is associated with a corresponding part of the sensed image that corresponds with that part of the range image.

34. A method according to claim 33, wherein each part is a pixel of the image of which
5 it is a part, or, in the case of the surface of the display screen, each part is a part of that surface that is substantially covered by a pixel of the first displayed image displayed on that surface.

35. A method according to any one of claims 30 to 34, wherein step (e) includes the step
10 of operating on the appearance information and the distance information, and the information indicative of the desired image, or the other information indicative of the desired image, to produce modifying information for use in producing the modified information.

15 36. A method according to claim 35, wherein the modifying information includes information indicative of a mathematical expression that maps the desired image to the sensed image and/or the sensed image to the desired image.

37. A method according to claim 36, wherein the mathematical expression is a
20 transform that relies on determining the relative positions of at least six points of the sensed image, determining the relative positions of the six corresponding points in the desired image, such that six pairs of corresponding points are determined; and also determining the distance, from the projection position, of each of six points of the display surface that are associated with a respective one of the corresponding pairs.

25 38. A method according to claim 36 or claim 37, wherein the transform further relies on determining the distance, from the projection position, of each part of the display surface on which a pixel of the first displayed image is displayed.

30 39. A method according to any one of claims 36 to 38, wherein the mathematical expression at least includes an expression that can be expressed as:

$$\begin{bmatrix} sx_1 \\ sy_1 \\ s \end{bmatrix} = M_{A,3 \times 4} \begin{bmatrix} x_2 \\ y_2 \\ Z_1(x_2, y_2) \\ 1 \end{bmatrix}$$

wherein a point (x_1, y_1) on the image plane of the projection means is projected to an unknown point on the display surface; that point being capture by the sensing means as a point (x_2, y_2) on the image plane thereof; the unknown point on the screen is a 3D point having coordinates of (X_1, Y_1, Z_1) when referenced with respect to a 3-D coordinate system S_1 associated with the projection means and coordinates of (X_2, Y_2, Z_2) when with respect to a coordinate system S_2 associated with the sensing means; wherein the matrix $M_{A,3 \times 4}$ maps points from the image plane of the sensing means to the image plane of the projection means.

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40. Apparatus including computer processing means programmed to carry out the method of any one of the preceding claims.

15 41. Apparatus according to claim 40 and further including the display means and/or the sensing means defined in any one of claims 1 to 39.

42. Apparatus according to claim 41, wherein the display means and/or the sensing means includes the computer processing means therein.

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43. A computer program having code portions which when executed by computer processing means cause those means to carry out a method according to any one of claims 1 to 39.

25 44. A record carrier, the carrier having recorded thereon information which is readable by computer processing means, or equipment arranged to communicate therewith, to cause those means to carry out a method according to any one of claims 1 to 39.

45. A signal, the signal conveying information which is readable by computer processing means, or equipment associated therewith, to cause those means to carry out a method according to any one of claims 1 to 39.