

SIMULTANEOUS SOURCE ENCODING AND SOURCE SEPARATION AS A
PRACTICAL SOLUTION FOR FULL WAVEFIELD INVERSION

CROSS REFERENCE TO RELATED APPLICATIONS

[0001] This application claims the benefit of U.S. Provisional Application 61/386,831
5 filed on September 27, 2010, which is incorporated herein by reference in all jurisdictions
that allow it. This application is related to that particular application entitled “Hybrid
Method for Full Waveform Inversion Using Simultaneous and Sequential Source Method”
and claiming the benefit of U.S. Provisional Application 61/836,828 filed on September 27,
2010. This related application is also incorporated by reference herein in all jurisdictions that
10 will allow it.

FIELD OF THE INVENTION

[0002] The invention relates generally to the field of geophysical prospecting, and more
particularly to geophysical data processing. Specifically, the invention is a method for
inversion of data acquired from multiple geophysical sources such as seismic sources,
15 involving geophysical simulation that computes the data from many simultaneously-active
geophysical sources in one execution of the simulation.

BACKGROUND OF THE INVENTION

[0003] Geophysical inversion [1,2] attempts to find a model of subsurface properties that
optimally explains observed data and satisfies geological and geophysical constraints. There
20 are a large number of well known methods of geophysical inversion. These well known
methods fall into one of two categories, iterative inversion and non-iterative inversion. The
following are definitions of what is commonly meant by each of the two categories:

[0004] Non-iterative inversion – inversion that is accomplished by assuming some simple
background model and updating the model based on the input data. This method does not use
25 the updated model as input to another step of inversion. For the case of seismic data these
methods are commonly referred to as imaging, migration, diffraction tomography or Born
inversion.

[0005] Iterative inversion – inversion involving repetitious improvement of the
subsurface properties model such that a model is found that satisfactorily explains the

observed data. If the inversion converges, then the final model will better explain the observed data and will more closely approximate the actual subsurface properties. Iterative inversion usually produces a more accurate model than non-iterative inversion, but is much more expensive to compute.

5 [0006] Iterative inversion is generally preferred over non-iterative inversion, because it yields more accurate subsurface parameter models. Unfortunately, iterative inversion is so computationally expensive that it is impractical to apply it to many problems of interest. This high computational expense is the result of the fact that all inversion techniques require many compute intensive simulations. The compute time of any individual simulation is
10 proportional to the number of sources to be inverted, and typically there are large numbers of sources in geophysical data, where the term *source* as used in the preceding refers to an activation location of a source apparatus. The problem is exacerbated for iterative inversion, because the number of simulations that must be computed is proportional to the number of iterations in the inversion, and the number of iterations required is typically on the order of
15 hundreds to thousands.

[0007] The most commonly employed iterative inversion method employed in geophysics is cost function optimization. Cost function optimization involves iterative minimization or maximization of the value, with respect to the model M , of a cost function $S(M)$ which is a measure of the misfit between the calculated and observed data (this is also
20 sometimes referred to as the objective function), where the calculated data are simulated with a computer using the current geophysical properties model and the physics governing propagation of the source signal in a medium represented by a given geophysical properties model. The simulation computations may be done by any of several numerical methods including but not limited to finite difference, finite element or ray tracing. The simulation
25 computations can be performed in either the frequency or time domain.

[0008] Cost function optimization methods are either local or global [3]. Global methods simply involve computing the cost function $S(M)$ for a population of models $\{M_1, M_2, M_3, \dots\}$ and selecting a set of one or more models from that population that approximately minimize $S(M)$. If further improvement is desired this new selected set of models can then be used as a
30 basis to generate a new population of models that can be again tested relative to the cost function $S(M)$. For global methods each model in the test population can be considered to be

an iteration, or at a higher level each set of populations tested can be considered an iteration. Well known global inversion methods include Monte Carlo, simulated annealing, genetic and evolution algorithms.

[0009] Unfortunately global optimization methods typically converge extremely slowly and therefore most geophysical inversions are based on local cost function optimization. Algorithm 1 summarizes local cost function optimization.

1. selecting a starting model,
2. computing the gradient of the cost function $S(M)$ with respect to the parameters that describe the model,
3. searching for an updated model that is a perturbation of the starting model in the negative gradient direction that better explains the observed data.

Algorithm 1 – Algorithm for performing local cost function optimization.

[0010] This procedure is iterated by using the new updated model as the starting model for another gradient search. The process continues until an updated model is found that satisfactorily explains the observed data. Commonly used local cost function inversion methods include gradient search, conjugate gradients and Newton's method.

[0011] Local cost function optimization of seismic data in the acoustic approximation is a common geophysical inversion task, and is generally illustrative of other types of geophysical inversion. When inverting seismic data in the acoustic approximation the cost function can be written as:

$$S(M) = \sum_{g=1}^{N_g} \sum_{r=1}^{N_r} \sum_{t=1}^{N_t} W(\psi_{calc}(M, r, t, w_g) - \psi_{obs}(r, t, w_g)) \quad (\text{Eqn. 1})$$

where:

S = cost function,

M = vector of N parameters, $(m_1, m_2, \dots, m_N)$ describing the subsurface model,

g = gather index,

- w_g = source function for gather g which is a function of spatial coordinates and time, for a point source this is a delta function of the spatial coordinates,
- N_g = number of gathers,
- r = receiver index within gather,
- 5 N_r = number of receivers in a gather,
- t = time sample index within a trace,
- N_t = number of time samples,
- W = minimization criteria function (we usually choose $W(x)=x^2$ which is the least squares (L2) criteria),
- 10 ψ_{calc} = calculated seismic pressure data from the model M ,
- ψ_{obs} = measured seismic pressure data.

[0012] The gathers can be any type of gather that can be simulated in one run of a seismic forward modeling program. Usually the gathers correspond to a seismic shot, although the shots can be more general than point sources. For point sources the gather index g corresponds to the location of individual point sources. For plane wave sources g would correspond to different plane wave propagation directions. This generalized source data, ψ_{obs} , can either be acquired in the field or can be synthesized from data acquired using point sources. The calculated data ψ_{calc} on the other hand can usually be computed directly by using a generalized source function when forward modeling. For many types of forward modeling, including finite difference modeling, the computation time needed for a generalized source is roughly equal to the computation time needed for a point source.

[0013] Equation(1) can be simplified to:

$$S(M) = \sum_{g=1}^{N_g} W(\delta(M, w_g)) \quad (\text{Eqn. 2})$$

where the sum over receivers and time samples is now implied and,

$$\delta(M, w_g) = \psi_{calc}(M, w_g) - \psi_{obs}(w_g) \quad (\text{Eqn. 3})$$

[0014] Inversion attempts to update the model M such that $S(M)$ is a minimum. This can be accomplished by local cost function optimization which updates the given model $M^{(k)}$ as follows:

$$M^{(k+1)} = M^{(k)} - \alpha^{(k)} \nabla_M S(M) \quad (\text{Eqn. 4})$$

where k is the iteration number, α is the scalar size of the model update, and $\nabla_M S(M)$ is the gradient of the misfit function, taken with respect to the model parameters. The model perturbations, or the values by which the model is updated, are calculated by multiplication of the gradient of the objective function with a step length α , which must be repeatedly
10 calculated.

[0015] From equation (2), the following equation can be derived for the gradient of the cost function:

$$\nabla_M S(M) = \sum_{g=1}^{N_g} \nabla_M W(\delta(M, w_g)) \quad (\text{Eqn. 5})$$

[0016] So to compute the gradient of the cost function one must separately compute the
15 gradient of each gather's contribution to the cost function, then sum those contributions. Therefore, the computational effort required for computing $\nabla_M S(M)$ is N_g times the compute effort required to determine the contribution of a single gather to the gradient. For geophysical problems, N_g usually corresponds to the number of geophysical sources and is on the order of 10,000 to 100,000, greatly magnifying the cost of computing $\nabla_M S(M)$.

20 [0017] Note that computation of $\nabla_M W(\delta)$ requires computation of the derivative of $W(\delta)$ with respect to each of the N model parameters m_i . Since for geophysical problems N is usually very large (usually more than one million), this computation can be extremely time consuming if it had to be performed for each individual model parameter. Fortunately, the adjoint method can be used to efficiently perform this computation for all model parameters

at once [1]. The adjoint method for the least squares objective function and a gridded model parameterization is summarized by the following algorithm:

1. Compute forward simulation of the data using the current model and the gather signature w_g as the source to get $\psi_{calc}(M^{(k)}, w_g)$,
- 5 2. Subtract the observed data from the simulated data giving $\delta(M^{(k)}, w_g)$,
3. Compute the reverse simulation (i.e. backwards in time) using $\delta(M^{(k)}, w_g)$ as the source producing $\psi_{adjoint}(M^{(k)}, w_g)$,
4. Compute the integral over time of the product of $\psi_{calc}(M^{(k)}, w_g)$ and $\psi_{adjoint}(M^{(k)}, w_g)$ to get $\nabla_M W(\delta(M, w_g))$.

10 Algorithm 2 – Algorithm for computing the least-squares cost-function gradient of a gridded model using the adjoint method.

[0018] While computation of the gradients using the adjoint method is efficient relative to other methods, it is still very costly. In particular the adjoint methods requires two simulations, one forward in time and one backward in time, and for geophysical problems
15 these simulations are usually very compute intensive. Also, as discussed above, this adjoint method computation must be performed for each measured data gather individually, increasing the compute cost by a factor of N_g .

[0019] The compute cost of all categories of inversion can be reduced by inverting data from combinations of the sources, rather than inverting the sources individually. This may be
20 called simultaneous source inversion. Several types of source combination are known including: coherently sum closely spaced sources to produce an effective source that produces a wavefront of some desired shape (e.g. a plane wave), sum widely spaces sources, or fully or partially stacking the data before inversion.

[0020] The compute cost reduction gained by inverting combined sources is at least
25 partly offset by the fact that inversion of the combined data usually produces a less accurate inverted model. This loss in accuracy is due to the fact that information is lost when the individual sources are summed, and therefore the summed data does not constrain the inverted model as strongly as the unsummed data. This loss of information during summation

can be minimized by encoding each shot record before summing. Encoding before combination preserves significantly more information in the simultaneous source data, and therefore better constrains the inversion [5]. Encoding also allows combination of closely spaced sources, thus allowing more sources to be combined for a given computational region.

- 5 Various encoding schemes can be used with this technique including time shift encoding and random phase encoding. The remainder of this Background section briefly reviews various published geophysical simultaneous source techniques, both encoded and non-encoded.

[0021] Van Manen [6] suggests using the seismic interferometry method to speed up forward simulation. Seismic interferometry works by placing sources everywhere on the
10 boundary of the region of interest. These sources are modeled individually and the wavefield at all locations for which a Green's function is desired is recorded. The Green's function between any two recorded locations can then be computed by cross-correlating the traces acquired at the two recorded locations and summing over all the boundary sources. If the data to be inverted have a large number of sources and receivers that are within the region of
15 interest (as opposed to having one or the other on the boundary), then this is a very efficient method for computing the desired Green's functions. However, for the seismic data case it is rare that both the source and receiver for the data to be inverted are within the region of interest. Therefore, this improvement has very limited applicability to the seismic inversion problem.

20 [0022] Berkhout [7] and Zhang [8] suggest that inversion in general can be improved by inverting non-encoded simultaneous sources that are summed coherently to produce some desired wave front within some region of the subsurface. For example, point source data could be summed with time shifts that are a linear function of the source location to produce a down-going plane wave at some particular angle with respect to the surface. This technique
25 could be applied to all categories of inversion. A problem with this method is that coherent summation of the source gathers necessarily reduces the amount of information in the data. So for example, summation to produce a plane wave removes all the information in the seismic data related to travel time versus source-receiver offset. This information is critical for updating the slowly varying background velocity model, and therefore Berkhout's method
30 is not well constrained. To overcome this problem many different coherent sums of the data (e.g. many plane waves with different propagation directions) could be inverted, but then efficiency is lost since the cost of inversion is proportional to the number of different sums

inverted. Herein, such coherently summed sources are called generalized sources. Therefore, a generalized source can either be a point source or a sum of point sources that produces a wave front of some desired shape.

5 **[0023]** Van Riel [9] suggests inversion by non-encoded stacking or partial stacking (with respect to source-receiver offset) of the input seismic data, then defining a cost function with respect to this stacked data which will be optimized. Thus, this publication suggests improving cost function based inversion using non-encoded simultaneous sources. As was true of the Berkhout's [7] simultaneous source inversion method, the stacking suggested by this method reduces the amount of information in the data to be inverted and therefore the
10 inversion is less well constrained than it would have been with the original data.

15 **[0024]** Mora [10] proposes inverting data that is the sum of widely spaced sources. Thus, this publication suggests improving the efficiency of inversion using non-encoded simultaneous source simulation. Summing widely spaced sources has the advantage of preserving much more information than the coherent sum proposed by Berkhout. However, summation of widely spaced sources implies that the aperture (model region inverted) that must be used in the inversion must be increased to accommodate all the widely spaced sources. Since the compute time is proportional to the area of this aperture, Mora's method does not produce as much efficiency gain as could be achieved if the summed sources were near each other.

20 **[0025]** Ober [11] suggests speeding up seismic migration, a special case of non-iterative inversion, by using simultaneous encoded sources. After testing various coding methods, Ober found that the resulting migrated images had significantly reduced signal-to-noise ratio due to the fact that broad band encoding functions are necessarily only approximately orthogonal. Thus, when summing more than 16 shots, the quality of the inversion was not
25 satisfactory. Since non-iterative inversion is not very costly to begin with, and since high signal-to-noise ratio inversion is desired, this technique is not widely practiced in the geophysical industry.

30 **[0026]** Ikelle [12] suggests a method for fast forward simulation by simultaneously simulating point sources that are activated (in the simulation) at varying time intervals. A method is also discussed for decoding these time-shifted simultaneous-source simulated data back into the separate simulations that would have been obtained from the individual point

sources. These decoded data could then be used as part of any conventional inversion procedure. A problem with Ikelle's method is that the proposed decoding method will produce separated data having noise levels proportional to the difference between data from adjacent sources. This noise will become significant for subsurface models that are not laterally constant, for example from models containing dipping reflectors. Furthermore, this noise will grow in proportion to the number of simultaneous sources. Due to these difficulties, Ikelle's simultaneous source approach may result in unacceptable levels of noise if used in inverting a subsurface that is not laterally constant.

[0027] Simultaneous inversion of multiple encoded sources is proposed by Krebs et al. in PCT Patent Application Publication No. WO 2008/042081 [5, 18], which is incorporated herein by reference in all jurisdictions that allow it, is a very cost effective method to invert full wave field data. (The same approach of simultaneous inversion of an encoded gather will work for receivers, either via source-receiver reciprocity or by encoding the actual receiver locations in common-source gathers of data.) For fixed receivers, the forward and adjoint computations only need to be performed for a single effective source; see PCT Patent Application Publication No. WO 2009/117174, which is incorporated herein by reference in all jurisdictions that allow it. Given the fact that hundreds of shots are recorded for typical 2D acquisition geometries, and thousands in the case of 3D surveys, computational savings from this method are quite significant. In practice, a fixed receiver assumption is not strictly valid for most common field data acquisition geometries. In the case of marine streamer data, both sources and receivers move for every new shot. Even in surveys where the locations of receivers are fixed, the practice often is that not all receivers are "listening" to every shot, and the receivers that are listening can vary from shot-to-shot. This also violates the "fixed-receiver assumption." In addition, due to logistical problems, it is difficult to record data close to the source, and this means that near-offset data are typically missing. This is true for both marine and land surveys. Both of these factors mean that for a simultaneous source gather, every receiver location will be missing data for some source shots. In summary, in simultaneous encoded-source inversion, for a given simultaneous encoded gather, data are required at all receiver locations for every shot, and this may be referred to as the fixed-receiver assumption of simultaneous encoded-source inversion. In WO 08/042081, some of the disclosed embodiments may work better than others when the fixed-receiver assumption is not satisfied. Therefore, it would be advantageous to have an accommodation or

adjustment to straightforward application of simultaneous encoded sources (and/or receivers) inversion that would enhance its performance when the fixed receiver assumption is compromised. The present invention provides ways of doing this. Haber et al. [25] also describe an approach to the problem of moving receivers in simultaneous encoded source inversion using a stochastic optimization method, and apply it to a direct current resistivity problem.

SUMMARY OF THE INVENTION

[0028] In a broad embodiment, the invention is a computer-implemented method for simultaneous inversion of measured geophysical data from multiple encoded sources to determine a physical properties model for a subsurface region, the measured geophysical data resulting from a survey for which the fixed-receiver assumption of simultaneous encoded-source inversion may not be valid, said method comprising using a computer to perform simultaneous source separation to lessen any effect of the measured geophysical data's not satisfying the fixed-receiver assumption, wherein a data processing step coming after a simultaneous source separation acts to conform model simulated data to the measured geophysical data for source and receiver combinations that are missing in the measured geophysical data.

[0029] In a more specific embodiment, the invention is a computer-implemented method for full-wavefield inversion of measured geophysical data to determine a physical properties model for a subsurface region, said method using a computer to perform steps comprising: (a) constructing an initial physical properties model and using it to simulate synthetic data corresponding to the measured geophysical data, wherein sources and/or receivers in the simulation are encoded and simulated simultaneously, thereby generating simulated encoded data; (b) separating the simulated encoded data according to source, resulting in simulated sequential-source data; (c) processing the simulated sequential-source data to make them conform to the measured geophysical data for source and receiver combinations that are missing in the measured geophysical data; (d) computing differences, called data residuals, between the processed simulated sequential source data and the measured geophysical data for source and receiver combinations that are present in the measured geophysical data; (e) encoding the data residuals, using the same encoding as was used in (a) or different encoding; (f) using the encoded data residuals to compute an update to the initial physical properties

model; and (g) combining the update with the initial model to form an updated physical properties model. Due to linearity, the order of the steps (d) and (e) may be interchanged.

[0030] In a variation on this last embodiment, the following steps may be performed at any time before the computing differences step (c): encoding the measured geophysical data, using the same or different source encoding as was used in generating the simulated encoded data; then separating the encoded measured data according to source, preferably using the same source separation algorithm that was used in separating the simulated encoded data according to source; and using the separated measured data in computing the data residuals.

[0031] In a second more specific embodiment, the present invention is a computer-implemented method for full-wavefield inversion of measured geophysical data to determine a physical properties model for a subsurface region, said method using a computer to perform steps comprising: (a) constructing an initial physical properties model and using it to simulate synthetic data corresponding to the measured geophysical data, wherein sources and/or receivers in the simulation are encoded using selected encoding functions and simulated simultaneously, thereby generating simulated encoded data; (b) separating the simulated encoded data according to source and/or receiver using a source or receiver separation algorithm, resulting in simulated sequential source and/or receiver data; (c) processing the simulated sequential source and/or receiver data to mute those data, referred to as the missing data, corresponding to source and receiver combinations that are missing; (d) encoding the missing data, using encoding functions that are the same or different than said selected encoding functions; (e) encoding the measured geophysical data using encoding functions that are the same as those used in encoding the missing data, and subtracting them, according to source and receiver position, from the simulated encoded data, thereby generating encoded data residuals, which will be incorrect due to the missing source and receiver combinations in the measured geophysical data; (f) subtracting the encoded missing data from the encoded data residuals to generate corrected encoded data residuals; (g) using the corrected encoded data residuals to compute an update to the initial physical properties model; and (h) combining the update with the initial model to form an updated physical properties model.

[0032] In a third more specific embodiment, the present invention is a computer-implemented method for full-wavefield inversion of measured geophysical data to determine a physical properties model for a subsurface region, said method using a computer to perform

steps comprising: (a) constructing an initial physical properties model and using it to simulate synthetic data corresponding to the measured geophysical data, wherein sources and/or receivers in the simulation are encoded, using selected encoding functions, and simulated simultaneously, thereby generating simulated encoded data; (b) using said selected encoding
5 functions to encode the measured geophysical data and subtracting them, according to source and receiver position, from the simulated encoded data, thereby generating encoded data residuals, which will include incorrect values due to the missing source and receiver combinations in the measured geophysical data; (c) separating the encoded data residuals according to source and/or receiver using a source or receiver separation algorithm, resulting
10 in sequential residual data; (d) processing the sequential residual data to estimate simulated data, referred to as missing data, corresponding to source and receiver combinations that are missing in the measured geophysical data; (e) encoding the missing data, using said selected encoding functions; (f) subtracting the encoded missing data from the encoded data residuals to generate corrected encoded data residuals; (g) using the corrected encoded data residuals
15 to compute an update to the initial physical properties model; and (h) combining the update with the initial model to form an updated physical properties model.

BRIEF DESCRIPTION OF THE DRAWINGS

[0033] Due to patent rule restrictions on use of color, the attached drawings are black and white reproductions of color originals. Copies of the U.S. counterpart application or
20 publication with 20 color drawings may be obtained from the U.S. Patent and Trademark Office upon request and payment of the necessary fee.

[0034] The present invention and its advantages will be better understood by referring to the following detailed description and the attached drawings in which:

Fig. 1 is a flowchart showing basic steps in the present inventive method for simultaneous source inversion for a non-fixed receiver geometry problem in an embodiment in which the
25 encoded predicted data are separated using a source separation method;

Fig. 2 is a flowchart showing basic steps in the present inventive method for simultaneous source inversion for a non-fixed receiver geometry problem in an embodiment in which the encoded predicted data as well as encoded measured data are separated using a source
30 separation method to reduce error in the source separation process; and

Figs. 3 and 4 are flowcharts showing basic steps in the present inventive method for simultaneous source inversion for a non-fixed receiver geometry problem in two embodiments in which the incorrect encoded data residuals are corrected by the encoded part of the missing data obtained from a source separation method.

- 5 [0035] The invention will be described in connection with example embodiments. However, to the extent that the following detailed description is specific to a particular embodiment or a particular use of the invention, this is intended to be illustrative only, and is not to be construed as limiting the scope of the invention. On the contrary, it is intended to cover all alternatives, modifications and equivalents that may be included within the scope of
- 10 the invention, as defined by the appended claims.

DETAILED DESCRIPTION OF EXAMPLE EMBODIMENTS

- [0036] The present inventive method uses source encoding and decoding (i.e. source separation) to circumvent the fixed receiver assumption and is applicable for marine streamer as well as land acquisition geometry. In a typical marine streamer and land acquisition, the
- 15 data coverage is often insufficient to satisfy the fixed receiver geometry thus limiting the benefits of simultaneous source full wave inversion (“FWI”) proposed by Krebs et al. [5,18]. In addition to geometry considerations, the field data need to be processed to conform to the physics of the forward simulation used in the inversion. For example, to use acoustic inversion for inverting elastic data, far offsets are typically muted and the data are processed
- 20 to remove other elastic effects. Other practical aspects such as event (reflections, refractions, multiples) based windowing for FWI inversion must assume fixed receiver geometry, an assumption often not satisfied in practice.

- [0037] Most of the work using encoding and decoding has been focused on processing and imaging (migration algorithms) of seismic data (Ikelle [17], Berkhout [16], Beasley et al.
- 25 [26], Stefani et al. [22], Verschuur and Berkhout [23]). Some of the published work on inversion is by Krebs et al. [5,18], Hinkley [4], Ben-Hadj-Ali et al. [15] and Herrmann [24] for inverting full waveform data. A main idea of the present invention is to utilize the power of simultaneous sources (and/or receivers) encoding for the forward modeling, back propagation and inversion process but at the same time be able to handle practical aspects of
- 30 data limitation commonly encountered in field surveys (both land and marine).

[0038] Krebs et al. [5,16] show that the encoded simultaneous source cost function can be computed more efficiently than conventional cost functions while still providing accurate inversions. The simultaneous source cost function is defined here as (compare with equation (2) above):

$$S_{sim}(M) = \sum_{G=1}^{N_G} W \left(\delta \left(M, \sum_{g \in G} c_g \otimes w_g \right) \right) \quad (\text{Eqn. 6})$$

where a summation over receivers and time samples is implied as in Eqn. (2), and:

$\sum_{g=1}^{N_g} = \sum_{G=1}^{N_G} \sum_{g \in G}$ defines a sum over gathers by sub groups of gathers,

S_{sim} = cost function for simultaneous source data,

G = the groups of simultaneous generalized sources, and

10 N_G = the number of groups,

c_g = functions of time that are convolved ($\otimes$) with each gather's source signature to encode the gathers, these encoding functions may be chosen to be approximately orthogonal with respect to some appropriate operation depending on the weighting function W . When W is the L2-Norm the appropriate operation is cross-correlation.

15 [0039] The outer summation in Eqn. (6) is over groups of simultaneous generalized sources corresponding to the gather type (e.g. points sources for common shot gathers). The inner summation, over g , is over the gathers that are grouped for simultaneous computation. For some forward modeling methods, such as finite difference modeling, the computation of the forward model for summed generalized sources (the inner sum over $g \in G$) can be
 20 performed in the same amount of time as the computation for a single source. Therefore, as shown in Krebs et. al. [5] $\delta(M, \sum c_g \otimes w_g)$ can be computed very efficiently using Algorithm 3.

25

1. Simulate $\psi_{calc}(M, \sum c_g \otimes w_g)$ using a single run of the simulator using $\sum c_g \otimes w_g$ as the source,
2. Convolve each measured data gather with the c_g encoding functions, then sum the resulting encoded gathers (i.e. $\sum c_g \otimes \psi_{obs}(w_g)$),

3. Subtract the result of step 2 from the result of step 1.

Algorithm 3 – Algorithm for computing the encoded simultaneous-source data residual. [NOTE: To compute the cost function, the residual needs to be evaluated with the function W ; for example, least squares.]

5 **[0040]** Again as shown in Krebs et al. [5] this algorithm can compute $S_{sim}(M)$ a factor of N_g/N_G times faster than $S(M)$ from Eqn. (2).

[0041] Many types of encoding functions c_g can be used in equation (6) including but not limited to:

- 10 • Linear, random, chirp and modified-chirp frequency-dependent phase encoding as presented in Romero et al. [13];
- The frequency independent phase encoding as presented in Jing et al. [14];
- Random time shift encoding;
- 15 • Frequency division multiplexing (FDMA), time division multiplexing (TDMA) and code division multiplexing (CDMA) used in telecommunications.

[0042] Some of these encoding techniques will work better than others depending upon the application, and some can be combined. In particular, good results have been obtained using frequency dependent random phase encoding and also by combining frequency independent encoding of nearby sources with frequency dependent random phase encoding for more widely separated sources. An indication of the relative merits of different encodings can be obtained by running test inversions with each set of encoding functions to determine which converges faster.

[0043] It should be noted that the simultaneous encoded-source technique can be used for many types of inversion cost function. In particular, it could be used for cost functions based on other norms than L2 discussed above. It could also be used on more sophisticated cost functions than the one presented in Equation 2, including regularized cost functions. Finally, the simultaneous encoded-source method could be used with any type of global or local cost

function inversion method including Monte Carlo, simulated annealing, genetic algorithm, evolution algorithm, gradient line search, conjugate gradients and Newton's method.

[0044] The present inventive method can also be used in conjunction with various types of generalized source techniques, such as those suggested by Berkhout [7]. In this case, rather than encoding different point source gather signatures, one would encode the signatures for different synthesized plane waves.

[0045] Some variations on the embodiment described above include:

- The c_g encoding functions can be changed for each iteration of the inversion. In at least some instances this leads to faster convergence of the inversion.
- 10 • In some cases (e.g., when the source sampling is denser than the receiver sampling) it may be advantageous to use reciprocity to treat the actual receivers as computational sources, and encode the receivers instead of the sources.
- 15 • This invention is not limited to single-component point receivers. For example, the receivers could be receiver arrays or they could be multi-component receivers.
- 20 • The method may be improved by optimizing the encoding to yield the highest quality inversion. For example the encoding functions could be optimized to reduce the number of local minima in the cost function. The encoding functions could be optimized either by manual inspection of tests performed using different encoding functions or using an automated optimization procedure.
- 25 • Acquisition of simultaneous encoded-source data could result in significant geophysical data acquisition cost savings.
- For marine seismic data surveys, it would be very efficient to acquire encoded source data from multiple simultaneously operating marine vibrators that operate continuously while in motion.

- As indicated above, the encoding process in the present invention may be performed in the field acquisition of the data, for example where the pilot signals of multiple simultaneously operating vibrators are encoded with different encoding functions. In the attached claims, steps referring to encoding geophysical data, or to geophysical data from encoded sources, or to obtaining encoded gathers of geophysical data will be understood to include obtaining data already encoded in the field acquisition process, except where the context clearly indicates that encoding is occurring in a data processing step.
- Other definitions for the cost function may be used, including the use of a different norm (e.g. L1 norm (absolute value) instead of L2 norm), and additional terms to regularize and stabilize the inversion (e.g. terms that would penalize models that aren't smooth or models that are not sparse).

[0046] A main idea of the present invention is to use simultaneous source encoding and simultaneous source separation for full waveform inversion in such a way as to lessen the impact when the fixed receiver assumption is compromised, and thereby make simultaneous source FWI more applicable to general data geometries. Figure 1 shows basic steps in one embodiment of the present inventive method. Following the flow chart shown in Figure 1, the steps are given next. The discussion that follows will be in terms of encoded sources; however, alternatively the receivers may be encoded instead of or in addition to the sources.

[0047] 1. For a given physical properties model of the subsurface (100), the survey sources (i.e., source locations) are encoded to form a simultaneous encoded source (101) which is used in the forward modeling, or simulation, (102) to generate simulated encoded data (103). The encoded data are generated assuming fixed receiver geometry with all receivers being live for the encoded source.

[0048] 2. Since the field data typically violate fixed receiver geometry to at least some extent, the encoded data are separated into individual shots using a source separation technique (104) such as that described by Neelamani et al. [19,20,21]. Neelamani's source separation relies on the assumption that the Green's function of the model has sparse representation in a transform domain such as a curvelet or wavelet domain. With the knowledge of the encoding scheme and assuming sparsity of Green's function, the shots can

be separated using for example L1 norm inversion in the transformed domain. This separation problem can be solved for single encoded shot data for the whole model domain, or it can be applied to multiple sets of encoded shot data. Persons who work in the technical field will know of a variety of source separation techniques that can be used as an alternative to that of Neelamani et al. [19,20,21]. For example, Spitz et al. [28] proposed a prediction-subtraction approach which first estimates the primary wavefield of the second source and then subtracts it from the total wavefield via a PEF-based adaptive subtraction. Akerberg et al. [27] used sparse Radon transforms for the source separation. For all source separation algorithms, more than one encoded dataset is preferable.

10 **[0049]** 3. The source separation described in the previous step is crucial to handle practical data geometries. With the separated shots (105), whatever processing steps that may be necessary may be applied (106) so that the simulated data conform to the field data. Thus, with the separated shots, the data can be processed to conform to the field data acquisition geometry such as offset muting, i.e. removing the near and far-offsets, muting
15 based on type of arrivals such as transmission versus reflection component of data, or removing the elastic component of the data if the governing forward engine is based on an acoustic assumption. Therefore, this general step of data processing with the separated shots to make the predicted data (107) similar to the measured data geometry (108) as well as make the data conform to the level of the physics allows production of sequential data residuals
20 (109), i.e. data residuals for selected shots gathers. A data *residual* is a measure of the difference between a measured data value and a corresponding predicted data value.

[0050] 4. These data residuals, which are sequential data residuals (109) by virtue of the source separation step 104, are encoded (110) using the same or different encoding scheme as used in step 101 to produce encoded data residuals (111). This is a
25 significant step since the gaps in the data due to acquisition geometry or due to user choice of specific events in the data are now in these encoded data residuals (111). Next, the encoded data residuals are used to compute the gradient of the objective function (117). For example, the residuals may be back-propagated (115) using the adjoint state equation and cross-correlated (116) with the forward-propagated data (112) from 103 using the simultaneous
30 source to calculate the gradient of the objective function to be minimized as described in PCT Patent Application Publication No. WO 2009/117174. Note that the simulated encoded data 112 may, but do not have to be, the same as the simulated encoded data 103, because they

may be encoded with different encoding functions; however the encoding in 112 must match the encoding in 111.

[0051] 5. In the final step, the model is updated (118) using the computed gradient from the previous step. A variety of well known optimization methods, such as steepest descent, conjugate gradients, or Newton's method can be used to do the model updating to ensure that the value of the objective function decreases. The simultaneous source inversion algorithm represented by the steps within dashed line box 120 is based on WO 2009/117174, but any inversion or forward modeling (manual adjustment) algorithm may be used.

10 **[0052]** For processing and imaging, the accuracy of the source separation is important, since any artifact arising due to source separation process eventually is mapped into the final image. In previous similar work where source separation is used, considerable effort is made to choose the type of encoding to ensure improved accuracy of separation (Ikelle [17]). It is believed that the accuracy requirement will be relaxed for the inversion application of this invention since the small inaccuracies in the data residuals due to source separation can be handled via a line search and model regularization process during the inversion step. For inversion applications, a key is to make the predicted data that are subtracted from measured data conform to the acquisition geometry and adhere to the level of the physics being applied (for example, muting the large offsets when anisotropic effects are present in the data and the forward simulation algorithm assumes an isotropic earth model). Having achieved that via processing the separated shots removes the large errors in the data residuals.

25 **[0053]** Although it is believed that the small inaccuracies in source separation can be handled in the inversion process, large errors can still be a problem. Therefore, an alternative embodiment of the present inventive method is designed to handle the large errors due to source separation, and may be used when this is believed to be a problem. An example of this alternative embodiment is illustrated by the flowchart of Figure 2. To replicate the errors due to separation, the measured data are encoded (108b) using the same encoding scheme and using source separation (108c) to obtain measured separated shot gathers. These measured separated shot gathers can be used to compute the data residuals instead of the original measured shots.

[0054] Another way to view this problem is to determine encoded data residuals that are corrected for the missing far and near offsets traces. So, another alternative can be described as follows, shown in Figure 3. First, the data residual (309) that is a difference between simulated encoded data (103) and the encoded measured data (308) with missing far and near offsets is formed (i.e., simulated encoded minus encoded measured). This data residual (309) is unsuitable for computing the gradient since the simulated encoded data (103) has contribution from the missing part of the data (e.g., near and far offset traces) whereas the measured encoded data (308) do not have a contribution from missing data. In the next step (304), source separation of the simulated encoded data is performed, and the resulting estimate of simulated sequential shots (305) are processed, but instead of removing the missing data part (such as muting out the near and far offsets from the separated shot gathers (306)), the missing part (such as near and far offsets) of the shot gathers (307) is encoded. This encoded missing part of the data (307) can be subtracted (310) from the incorrect data residual (309) to compensate for the missing data information and obtain a corrected data residual (311). (It is subtracted, rather than added, because it needs to offset simulated encoded data which are present in the incorrect data residual with a positive sign, since the residual is defined – see above – as simulated encoded data minus encoded measured data. If that definition, which is arbitrary, were reversed, then the missing data would be *added* to the residual. Moreover, instead of treating the missing-data correction as a subtraction from the simulated data contribution to the data residual, it could be regarded as an addition to the measured data contribution to the data residual. It will be understood that the present invention, including the appended claims, includes all such equivalent approaches.) The key difference of this embodiment compared to that described in Fig. 1 is encoded data residuals are corrected in this embodiment rather than being generated after processing as shown in Fig. 1. If the source separation is perfect, then the two approaches (Fig. 1 and Fig. 3) are mathematically equivalent; however, in the presence of source separation errors, it is possible that the correction approach shown in Fig. 3 may provide a better result.

[0055] Figure 4 describes an alternative to the data residual correction method described Fig. 3. Similar to the method in Fig. 3, first, the encoded data residual (409) that is a difference between simulated encoded data (103) and the encoded measured data (408) with missing far and near offsets is formed. In the next step, source separation (404) is performed

using the encoded data residual, resulting in sequential residual data (405). The source separation estimates the residual between the sequential source simulated data and sequential source measured data. At source and receiver combinations that are not present in the measured data (not acquired), the estimated residual consists of only the simulated data. This
5 missing part (such as near and far offsets) of the shot gathers, determined by processing steps (406), is encoded (407) and then subtracted (410) from the incorrect data residual (409) to obtain a corrected data residual (311). The source separation result can be improved by using multiple encoded data residuals, each computed using a different encoding. In such a case, many of the encoded data residuals can be corrected and then used to update the initial
10 model.

[0056] Other variations to the embodiments of Figs. 1- 4 are possible. An advantage of all embodiments of the present inventive method is that they can handle a variety of field geometries to invert real data and realize speed-up due to the simultaneous encoding. The key step of simultaneous source separation followed by processing steps makes the algorithm
15 practical for field data sets. Neelamani et al. [20,21] show that the compute time for simultaneous source separation is significantly less compared to one forward simulation run, which is one of the primary advantages to this approach. Another embodiment of this invention determines the optimal number of sources so that the separation problem is well-posed. Using fewer, farther spaced sources for encoding helps the source separation
20 problem; however, too much separation may cause spatial aliasing in the model update and eventually degrade the subsurface model. On the other hand, encoding too many closely spaced sources makes the separation problem difficult. Thus, depending on the scale length of the subsurface model to be updated and available sources in a survey, an optimal source spacing (i.e. optimal number of sources) can be chosen to balance the two competing
25 considerations. Thus, to make the source separation problem well-posed may require not using all the sources recorded in a survey. In another embodiment, the encoding is changed every iteration of the inversion, or at least in some iterations, as taught by WO 2008/042081.

[0057] The foregoing application is directed to particular embodiments of the present invention for the purpose of illustrating it. It will be apparent, however, to one skilled in the
30 art, that many modifications and variations to the embodiments described herein are possible. All such modifications and variations are intended to be within the scope of the present invention, as defined in the appended claims. Persons skilled in the art will readily recognize

that in preferred embodiments of the invention, at least some of the steps in the present inventive method are performed on a computer, i.e. the invention is computer implemented. In such cases, the resulting updated physical properties model may be downloaded, displayed, or saved to computer storage.

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CLAIMS

1. A computer-implemented method for simultaneous inversion of measured geophysical data from multiple encoded sources to determine a physical properties model for a subsurface region, the measured geophysical data resulting from a survey for which the
5 fixed-receiver assumption of simultaneous encoded-source inversion may not be valid, said method comprising using a computer to perform simultaneous-source separation to lessen any effect of the measured geophysical data's not satisfying the fixed-receiver assumption, wherein a data processing step coming after a simultaneous source separation acts to conform model-simulated data to the measured geophysical data for source and receiver combinations
10 that are missing in the measured geophysical data.

2. The method of claim 1, wherein the method comprises:

constructing an initial physical properties model and using it to simulate synthetic data corresponding to the measured geophysical data, wherein sources and/or receivers in the simulation are encoded and simulated simultaneously, thereby generating simulated encoded
15 data;

separating the simulated encoded data according to source, resulting in simulated sequential-source data;

processing the simulated sequential-source data to make them conform to the measured geophysical data for source and receiver combinations that are missing in the
20 measured geophysical data;

computing differences, called data residuals, between the processed simulated sequential source data and the measured geophysical data for source and receiver combinations that are present in the measured geophysical data;

encoding the data residuals using the same or different encoding as was used in
25 generating the simulated encoded data;

using the encoded data residuals to compute an update to the initial physical properties model; and

combining the update with the initial model to form an updated physical properties model.

3. The method of claim 2, wherein the encoded data residuals are used to compute a gradient of an objective function, said gradient being with respect to parameters of the physical properties model, which is then used in a selected optimization scheme to update the physical properties model.

4. The method of claim 2, further comprising iterating the method steps at least one time, using the updated physical properties model from one iteration as the initial model for a next iteration.

5. The method of claim 4, wherein the sources are encoded using a selected set of encoding functions, and wherein a different set of encoding functions is selected for at least one of the iterations.

6. The method of claim 2, further comprising at any time before the computing differences step:

encoding the measured geophysical data, using the same or different source encoding as was used in generating the simulated encoded data;

separating the encoded measured data according to source using a source separation algorithm that was also used in separating the simulated encoded data according to source; and

using the separated measured data in computing the data residuals.

7. The method of claim 2, further comprising after the separating step and before the computing differences step, removing source-receiver locations from the simulated sequential-source data to correspond to source-receiver locations with missing data in the measured geophysical data.

8. The method of claim 7, wherein the removing source-receiver locations from the simulated sequential-source data is accomplished by one or more data processing steps from a group consisting of offset muting i.e. removing of near and far offsets; muting based on

type of arrivals including transmission versus reflection component of data, and removing the data's elastic component where forward simulation is based on an acoustic assumption.

9. The method of claim 2, wherein each gather of the simulated encoded data is encoded by temporally convolving all traces from the gather with an encoding signature selected for
5 the gather.

10. The method of claim 1, wherein the method comprises:

constructing an initial physical properties model and using it to simulate synthetic data corresponding to the measured geophysical data, wherein sources and/or receivers in the simulation are encoded using selected encoding functions and simulated simultaneously,
10 thereby generating simulated encoded data;

separating the simulated encoded data according to source and/or receiver using a source or receiver separation algorithm, resulting in simulated sequential source and/or receiver data;

processing the simulated sequential source and/or receiver data to mute those data,
15 referred to as the missing data, corresponding to source and receiver combinations that are missing in the measured geophysical data;

encoding the missing data, using encoding functions that are the same or different than said selected encoding functions;

encoding the measured geophysical data, using encoding functions that are the same
20 as used in the encoding the missing data, and subtracting them, according to source and receiver position, from the simulated encoded data, thereby generating encoded data residuals, which will be incorrect due to the missing source and receiver combinations in the measured geophysical data;

subtracting the encoded missing data from the encoded data residuals to generate
25 corrected encoded data residuals;

using the corrected encoded data residuals to compute an update to the initial physical properties model; and

combining the update with the initial model to form an updated physical properties model.

11. The method of claim 10, further comprising iterating the method steps at least one time, using the updated physical properties model from one iteration as the initial model for a
5 next iteration.

12. The method of claim 11, wherein the encoding functions or signatures are changed for at least one of the iterations.

13. The method of claim 10, wherein fewer than all source and/or receiver locations in the measured geophysical data are used in the generating simulated encoded data in order to
10 improve accuracy of the source or receiver separation algorithm by making the separation well-posed, but not reducing sampling of the measured geophysical data to an extent that causes aliasing.

14. The method of claim 10, wherein the missing data occur at near and far ends of the measured geophysical data's offset range.

15. The method of claim 10, wherein each gather of the simulated encoded data is encoded by temporally convolving all traces from the gather with an encoding signature selected for the gather.

16. The method of claim 10, wherein the encoding the measured geophysical data is accomplished by obtaining gathers of data from a geophysical survey in which data are
20 acquired from a plurality of simultaneously operating, uniquely encoded source devices.

17. The method of claim 1, wherein the method comprises:

constructing an initial physical properties model and using it to simulate synthetic data corresponding to the measured geophysical data, wherein sources and/or receivers in the simulation are encoded, using selected encoding functions, and simulated simultaneously,
25 thereby generating simulated encoded data;

using said selected encoding functions to encode the measured geophysical data and subtracting them, according to source and receiver position, from the simulated encoded data,

thereby generating encoded data residuals, which will include incorrect values due to the missing source and receiver combinations in the measured geophysical data;

separating the encoded data residuals according to source and/or receiver using a source or receiver separation algorithm, resulting in sequential residual data;

- 5 processing the sequential residual data to estimate simulated data, referred to as missing data, corresponding to source and receiver combinations that are missing in the measured geophysical data;

encoding the missing data, using said selected encoding functions;

- subtracting the encoded missing data from the encoded data residuals to generate
10 corrected encoded data residuals;

using the corrected encoded data residuals to compute an update to the initial physical properties model; and

combining the update with the initial model to form an updated physical properties model.

- 15 18. The method of claim 1, wherein the geophysical data are seismic data, of which a full wavefield is inverted.

19. The method of claim 1, wherein a plurality of encoded data sets are used to perform the simultaneous-source separation.

20. A computer program product, comprising a non-transitory computer usable medium
20 having a computer readable program code embodied therein, said computer readable program code adapted to be executed to implement a method for full-wavefield inversion of measured geophysical data to determine a physical properties model for a subsurface region, said method comprising:

- constructing or inputting an initial physical properties model and using it to simulate
25 synthetic data corresponding to the measured geophysical data, wherein sources and/or receivers in the simulation are encoded and simulated simultaneously, thereby generating simulated encoded data;

separating the simulated encoded data according to source, resulting in simulated sequential-source data;

computing differences, called data residuals, between the simulated sequential source data and the corresponding measured geophysical data;

5 encoding the data residuals according to source using the same encoding as was used in generating the simulated encoded data;

 using the encoded data residuals to compute an update to the initial physical properties model; and

 combining the update with the initial model to form an updated physical properties
10 model.

21. A computer program product, comprising a non-transitory computer usable medium having a computer readable program code embodied therein, said computer readable program code adapted to be executed to implement a method for full-wavefield inversion of measured geophysical data to determine a physical properties model for a subsurface region, said
15 method comprising:

 Constructing or inputting an initial physical properties model and using it to simulate synthetic data corresponding to the measured geophysical data, wherein sources and/or receivers in the simulation are encoded using selected encoding functions and simulated simultaneously, thereby generating simulated encoded data;

20 separating the simulated encoded data according to source and/or receiver using a source or receiver separation algorithm, resulting in simulated sequential source and/or receiver data;

 processing the simulated sequential source and/or receiver data to mute those data, referred to as the missing data, corresponding to source and receiver combinations that are
25 missing in the measured geophysical data;

 encoding the missing data, using encoding functions that are the same or different than said selected encoding functions;

encoding the measured geophysical data, using encoding functions that are the same as used in the encoding the missing data, and subtracting them, according to source and receiver position, from the simulated encoded data, thereby generating encoded data residuals;

- 5 subtracting the encoded missing data from the encoded data residuals to generate corrected encoded data residuals;

 using the corrected encoded data residuals to compute an update to the initial physical properties model; and

- combining the update with the initial model to form an updated physical properties
10 model.

22. A computer program product, comprising a non-transitory computer usable medium having a computer readable program code embodied therein, said computer readable program code adapted to be executed to implement a method for full-wavefield inversion of measured geophysical data to determine a physical properties model for a subsurface region, said
15 method comprising:

 Constructing or inputting an initial physical properties model and using it to simulate synthetic data corresponding to the measured geophysical data, wherein sources and/or receivers in the simulation are encoded, using selected encoding functions, and simulated simultaneously, thereby generating simulated encoded data;

- 20 using said selected encoding functions to encode the measured geophysical data and subtracting them, according to source and receiver position, from the simulated encoded data, thereby generating encoded data residuals;

 separating the encoded data residuals according to source and/or receiver using a source or receiver separation algorithm, resulting in sequential residual data;

- 25 processing the sequential residual data to estimate simulated data, referred to as missing data, corresponding to source and receiver combinations that are missing in the measured geophysical data;

 encoding the missing data, using said selected encoding functions;

subtracting the encoded missing data from the encoded data residuals to generate corrected encoded data residuals;

using the corrected encoded data residuals to compute an update to the initial physical properties model; and

5 combining the update with the initial model to form an updated physical properties model.

23. A method for producing hydrocarbons from a subsurface region, comprising:

performing a geophysical survey of the subsurface region resulting in measured geophysical data;

10 processing the measured geophysical data on a computer by a method of claim 1 to generate a physical properties model of the subsurface region;

assessing hydrocarbon potential of the subsurface region using the physical properties model; and

15 drilling a well into the subsurface region based at least in part on the assessment of hydrocarbon potential, and producing hydrocarbons from the well.