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Multiplier and squaring circuit

to be used for the same

This invention relates to a multiplier and a squaring circuit to be used for the same and more particularly, to a multiplier including a plurality of squaring circuits having differential input terminal pairs and adapted to be arranged on a bipolar integrated circuit and a squaring circuit to be used for the same.

Conventional multipliers are a Gilbert multiplier in general. The Gilbert multiplier has such a structure that transistor pairs are provided in a two-stage stack manner and a constant electric current source I_0 as shown in Fig.1. The operation thereof will be explained below.

In Fig. 1, an electric current (emitter current) I_E of a junction diode forming a transistor can be expressed by the following equation (1), where I_s is saturation current, k is Boltzmann's constant, q is a unit electron charge, V_{BE} is voltage between base and emitter and T is absolute

temperature.

$$I_E = I_s \cdot \exp(q \cdot V_{BE}/kT) - 1 \dots\dots\dots (1)$$

Here, if $V_T = kT/q$, as $V_{BE} \gg V_T$, when $\exp(V_{BE}/V_T) \gg 1$ in Eq. (1), the emitter current I_E can be approximated as follows;

$$I_E \approx I_s \cdot \exp(V_{BE}/V_T) \dots\dots\dots (2)$$

As a result, collector currents I_{C43} , I_{C44} , I_{C45} , I_{C46} , I_{C41} and I_{C42} of the transistors $Q43$, $Q44$, $Q45$, $Q46$, $Q41$ and $Q42$ can be expressed by the following equations (3), (4), (5), (6), (7) and (8), respectively;

$$I_{C43} = \frac{\alpha_F \cdot I_{C41}}{1 + \exp(-V_{41}/V_T)} \dots\dots\dots (3)$$

$$I_{C44} = \frac{\alpha_F \cdot I_{C41}}{1 + \exp(V_{41}/V_T)} \dots\dots\dots (4)$$

$$IC45 = \frac{\alpha F \cdot IC42}{1 + \exp(V41/VT)} \quad \dots\dots\dots (5)$$

$$IC46 = \frac{\alpha F \cdot IC42}{1 + \exp(-V41/VT)} \quad \dots\dots\dots (6)$$

$$IC41 = \frac{\alpha F \cdot I0}{1 + \exp(-V42/VT)} \quad \dots\dots\dots (7)$$

$$IC42 = \frac{\alpha F \cdot I0}{1 + \exp(-V42/VT)} \quad \dots\dots\dots (8)$$

In the above equations, V41 is an input voltage of the transistors Q43, Q44, Q45 and Q46, V42 is an input voltage of the transistors Q41 and Q42, αF is current amplification factor thereof designated by the large signal forward gain for the common base configuration.

Hence, the collector currents IC43, IC44, IC45 and IC46 of the transistors Q43, Q44, Q45 and Q46 can be expressed by the following equations (9), (10), (11) and (12), respectively;

$$IC_{43} = \frac{\alpha F^2 \cdot I_0}{\{1 + \exp(-V_{41}/V_T)\} \{1 + \exp(-V_{42}/V_T)\}} \dots\dots\dots (9)$$

$$IC_{44} = \frac{\alpha F^2 \cdot I_0}{\{1 + \exp(V_{41}/V_T)\} \{1 + \exp(-V_{42}/V_T)\}} \dots\dots\dots (10)$$

$$IC_{45} = \frac{\alpha F^2 \cdot I_0}{\{1 + \exp(V_{41}/V_T)\} \{1 + \exp(V_{42}/V_T)\}} \dots\dots\dots (11)$$

$$IC_{46} = \frac{\alpha F^2 \cdot I_0}{\{1 + \exp(-V_{41}/V_T)\} \{1 + \exp(V_{42}/V_T)\}} \dots\dots\dots (12)$$

As a result, the differential current ΔI between an output current IC_{43-45} and an output current IC_{44-46} can be expressed by the following equation (13);

$$\begin{aligned} I &= IC_{43-45} - IC_{44-46} \\ &= (IC_{43} + IC_{45}) - (IC_{44} + IC_{46}) \\ &= (IC_{43} - IC_{46}) - (IC_{44} - IC_{45}) \\ &= \alpha F^2 \cdot I_0 \cdot \{\tanh(V_{41}/2V_T)\} \cdot \{\tanh(V_{42}/2V_T)\} \dots\dots (13) \end{aligned}$$

Here, $\tanh x$ can be expanded in series as shown by the

following equation (14) as;

$$\tanh x = x - (x^3/3) \dots\dots\dots (14),$$

so that if $x \ll 1$, it can be approximated as $\tanh x = x$.

Accordingly, if $V_{41} \ll 2V_T$ and $V_{42} \ll 2V_T$, the differential current ΔI can be approximated by the following equation (15); From Eq. (15), it can be found that the circuit shown in Fig. 1 becomes a multiplier for the input voltages V_{41} and V_{42} as a small signal.

$$I \approx (1/4) (\alpha F/V_T)^2 \cdot V_{41} \cdot V_{42} \dots\dots\dots (15)$$

In this case, however, the conventional Gilbert multiplier as explained above has transistor pairs stacked in two stages, so that there arises such a problem that the source voltage cannot be decreased.

Next, a conventional squaring circuit formed on a C-MOS integrated circuit obtains a squaring characteristic by using a MOS transistor at the source follower as shown in Fig. 2. The drain current I_d thereof can be expressed by the following equation (16) in the saturation region, where

W is gate width, L is gate length, VGS is voltage between gate and source, Vt is threshold voltage, μn is mobility of electron, and COX is unit gate oxide film capacity;

$$I_d = \mu n \cdot (COX/2) (W/L) (V_{GS} - V_t)^2 \quad \dots\dots (16)$$

According to Eq. (16), the drain current I_d changes with the threshold voltage V_t . The threshold voltage V_t has a variation on a production basis. This means that with the conventional squaring circuit using MOS transistor at the source follower, the drain current I_d cannot be made constant even by applying the same gate voltage V_{GS} . As a result, there arises such a problem that the conventional squaring circuit is difficult to be integrated on a large-scale basis.

In consideration of the above-mentioned problems, an object of this invention is to provide a multiplier capable of reducing a source voltage.

Another object of this invention is to provide a squaring circuit which is easy to be integrated on a large-scale basis and which is adapted to be used for a multiplier.

SUMMARY OF THE INVENTION

(1) In a first aspect of this invention, a multiplier is provided which comprises a first and second squaring circuits each having a differential input terminal pair and whose outputs are connected in common. A first input terminal of the first squaring circuit is applied with a first input voltage and a second input terminal thereof is applied with a second input voltage which is opposite in phase to the first input voltage. A first input terminal of the second squaring circuit is applied with the second input voltage and a second input terminal thereof is applied with the first input voltage. The first and second squaring circuits each includes two sets of unbalanced differential transistor pairs which are arranged so that their inputs are opposite in phase and their outputs are connected in common. Said unbalanced differential transistor pairs have different emitter sizes from each other.

In the preferred embodiments of this aspect, two squaring circuits are provided whose input signals are opposite in phase from each other and applied to respective differential input terminal pairs. These two squaring circuits are formed of two sets of differential transistor

pairs whose emitters to be connected in common are with an emitter size ratio of $K:1$ ($K>1$). The two sets of differential transistor pairs are arranged so that the bases of the transistors which are respectively unequal in emitter size are connected in common for making a differential input terminal pair. The four sets of differential transistor pairs are arranged so that the collectors of four transistors which are respectively equal in emitter size are connected in common for making respective differential outputs.

Two transistors having different emitter sizes constituting each differential transistor pair may be connected with an emitter resistor with a resistant value inversely proportional to the emitter size ratio to the both or one of them.

Two transistors constituting each differential transistor pair may be made equal in emitter size to each other. In this case, only one transistor thereof has an emitter resistor to be connected. Also, in case of being equal in emitter size, one transistor thereof may have a Darlington connection.

(2) In a second aspect of this invention, similar to the

first aspect, a multiplier is provided which comprises a first and second squaring circuits. That is, it comprises the first squaring circuit including a first and second unbalanced differential transistor pairs whose outputs are connected in common and the second squaring circuit including a third and fourth unbalanced differential transistor pairs whose outputs are connected in common, and the outputs of the both squaring circuits are connected in common. A first input voltage is applied between one input terminal of said first unbalanced differential transistor pair and one input terminal of said second unbalanced differential transistor pair, and a second input voltage is applied between the other input terminal of the first unbalanced differential transistor pair and the other input terminal of the second unbalanced differential transistor pair. The second input voltage is applied between one input terminal of said third unbalanced differential transistor pair and one input terminal of said fourth unbalanced differential transistor pair, and the first input voltage is applied between the other input terminal of said unbalanced differential transistor pair and the other input terminal of said fourth unbalanced differential transistor pair. Two

transistors including each unbalanced differential transistor pair have different emitter sizes from each other as in the first aspect.

In the preferred embodiments of this aspect, a first and second differential input terminal pairs whose input signals are opposite in phase to each other and four sets of differential transistor pairs whose emitters to be connected in common are with an emitter size ratio of $K:1$ ($K>1$). In the four sets of differential transistor pairs, the base of the transistor having an emitter size ratio of K of the first differential transistor pair and that of the transistor having an emitter size ratio of 1 of the third differential transistor pair are connected in common to one input terminal (one polarity) of said first differential input terminal pair. Also, the base of the transistor having an emitter size ratio of 1 of the first differential transistor pair and that of the transistor having an emitter size ratio of K of the fourth differential transistor pair are connected in common to one input terminal (one polarity) of said second input terminal pair. The base of the transistor having an emitter size ratio of K of the second differential transistor pair and that of the transistor

having an emitter size ratio of 1 of said fourth differential transistor pair are connected in common to the other input terminal (the other polarity) of said first input terminal pair. The base of the transistor having an emitter size ratio of 1 of the second differential transistor pair and that of the transistor having an emitter size ratio of K of the third differential transistor pair are connected in common to the other input terminal (the other polarity) of said second input terminal pair. In addition, the collectors of four transistors which are respectively equal in emitter size are connected in common for making respective differential outputs.

As in the first aspect, two transistors different in emitter size from each other, which constitutes each differential transistor pair, may be connected respectively with emitter resistors having a resistant value inversely proportional to the emitter size ratio, or only one of them may be connected with an emitter resistor having a resistant value as above. In addition, two transistors constituting each differential transistor pair may be made equal in emitter size, but, only one transistor thereof is connected with an emitter resistor in this case. In case of being

equal in emitter size, one of two transistors constituting each differential transistor pair may have a Darlington connection.

(3) In a third aspect of this invention, a multiplier is provided which comprises a first, second and third squaring circuits each having a differential input terminal pair and which is arranged so that the output of said first squaring circuit is opposite in phase to those of said second and third squaring circuits. In this multiplier, a first input voltage is applied to one input terminal of said first squaring circuit and a second input voltage is applied to the other input terminal thereof. The first input voltage is applied across an input terminal pair of said second squaring circuit and the second input voltage is applied across an input terminal pair of said third squaring circuit. The two transistors constituting each differential transistor pair have different emitter sizes from each other as in the first and second aspects.

In the preferred embodiments of this aspect, the multiplier comprises a first and second input terminal pairs whose input signals are equal in phase to each other and whose one input terminals are made as a common input



terminal and three squaring circuits, first, second and third, which are arranged between said first and second input terminal pairs. The three squaring circuits each includes two sets of unbalanced differential transistor pairs whose emitters to be connected in common are with an emitter size ratio of $K:1$ ($K>1$), and in which the collectors of the transistors which are respectively equal in emitter size are connected in common and the bases of the transistors which are respectively unequal in emitter size are connected in common. In addition, one bases of the first and second squaring circuits are connected in common to the other input terminal of said first input terminal pair, and the other bases of the first and third squaring circuits are connected in common to the other input terminal of said second input terminal pair, and the other bases of the second squaring circuit and one bases of said third squaring circuit are connected in common to the common input terminal. In addition, the collectors of the transistors which are respectively equal in emitter size of said second and third squaring circuits are connected in common to be connected respectively to the collectors which are respectively unequal in emitter size of said first squaring



circuit.

In this multiplier, as in the first aspect, two transistors having different emitter sizes from each other, which constitute each differential transistor pair, may be connected respectively with emitter resistors having a resistant value inversely proportional to the emitter size ratio, or only one of them may be connected with an emitter resistor having a resistant value as above. In addition, two transistors constituting each differential transistor pair may be made equal in emitter size, but only one transistor thereof is connected with an emitter resistor in this case. In case of being equal in emitter size, one of the transistors of each differential transistor pair may have a Darlington connection.

(4) In a fourth aspect of this invention, additionally to the multiplier of the third aspect, a multiplier is provided which is obtained by addingly provided one squaring circuit to the multiplier of the third aspect. This multiplier comprises a first, second, third and fourth squaring circuits each having a differential input terminal pair, in which the output of the first squaring circuit is opposite in phase to and connected with the outputs of the second,



third and fourth squaring circuits. As in the third aspect, a first input voltage is applied to one input terminal of said first squaring circuit, and a second input voltage is applied to the other input terminal thereof. The first input voltage is applied across an input terminal pair of said second squaring circuit, and the second input voltage is applied across an input terminal pair of said third squaring circuit. Across an input terminal pair of said fourth squaring circuit, the first or second input voltage is applied. The two transistors constituting each differential transistor pair have different emitter sizes from each other as in the first, second and third aspects.

In the preferred embodiments of this aspect, the multiplier comprises a first and second input terminal pairs whose input signals are equal in phase to each other and whose one input terminals are made as a common input terminal, and four squaring circuits, first, second, third and fourth, which are arranged between said first and second input terminal pairs. The four squaring circuits each includes two sets of unbalanced differential transistor pairs (driven by respective constant current sources) whose emitters to be connected in common are with an emitter size



ratio of $K:1$ ($K>1$), and in which the collectors of the transistors which are respectively equal in emitter size are connected in common and the bases of the transistors which are respectively unequal in emitter size are connected in common. In addition, one bases of the first and second squaring circuits are connected in common to the other input terminal of said first input terminal pair, and the other bases of the first and fourth squaring circuits are connected in common to the other input terminal of said second input terminal pair, the other bases of said second squaring circuit and one bases of said third squaring bases are connected in common to said common input terminal, and the other bases of said third squaring circuit and one bases of said fourth squaring circuit are connected in common. In addition, between the first and third squaring circuits and between the second and fourth squaring circuits, the collectors of the transistors which are respectively equal in emitter size are connected in common, and the collectors of the transistors which are respectively unequal in emitter size are connected in common.

As in the first aspect, two transistors having different emitter sizes from each other, which constitute



each differential transistor pair, may be connected respectively with emitter resistors having a resistant value inversely proportional to the emitter size ratio, or only one of them may be connected with an emitter resistor having a resistant value as above. Two transistors constituting each differential transistor pair may be made equal in emitter size, but only one of them is connected with an emitter resistor in this case. In case of being equal in emitter size, one of such two transistors may have a Darlington connection.

Each of the multipliers shown in the first to fourth aspects as above does not have a plurality of differential transistor pairs arranged in a stack manner as of the prior art, but has them arranged so-called in a line transversally to be driven by a constant voltage source. As a result, it can be operated at a lower source voltage than that in the prior art.

(5) In a fifth aspect of this invention, a squaring circuit is provided which is adapted to be used for each multiplier shown above. This squaring circuit comprises a first differential transistor pair including a first MOS transistor having a gate width (W) and gate length (L) ratio



(W/L) of one (1) and a second MOS transistor having a ratio (W/L) of H ($H \neq 1$), which are driven by a constant current source I_0 , and a second differential transistor pair including a third and fourth MOS transistors having such a ratio (W/L) as:

$$\{4H \cdot H^{1/2} / (H+1)^2\} ,$$

which is driven by a constant current source of

$$\{2 \cdot H^{1/2} / (H+1)\} \cdot I_0 .$$

The drains of the first and third transistors are connected in common, and the drains of the second and fourth transistors are connected in common, and the gates of the first and fourth transistors are connected in common and the gates of the second and third transistors are connected in common.

This squaring circuit comprises two sets of differential transistor pairs including MOS transistors each having a gate width and gate length ratio (W/L) appropriately selected for making a differential input. This



means that such a squaring circuit that is completely independent of a variation in threshold voltage due to manufacturing dispersion of transistors and adapted to be integrated on a large - scale basis can be realized. Consequently, this squaring circuit can be preferably used instead of those used in these multipliers shown in the first to fourth aspects as above.

BRIEF DESCRIPTION OF THE DRAWINGS

Fig. 1 is a circuit diagram of a conventional multiplier.

Fig. 2 is a circuit diagram of a conventional squaring circuit using a MOS transistor.

Fig. 3 is a block diagram of a multiplier according to first to sixth embodiments of this invention.

Fig. 4 is a circuit diagram of a multiplier according to a first embodiment of this invention.

Fig. 5 is an output characteristic diagram of a squaring circuit to be used for the multiplier shown in Fig. 4.

Fig. 6 is an output characteristic diagram of the multiplier shown in Fig. 4.

Fig. 7 is a diagram of an output transformer conductance characteristic of the multiplier shown in Fig. 4.

Fig. 8 is an output characteristic diagram of the multiplier shown in Fig. 4.

Fig. 9 is a circuit diagram of a squaring circuit to be used for a multiplier according to a second embodiment of this invention.

Fig. 10 is an output characteristic diagram of the squaring circuit shown in Fig. 9.

Fig. 11 is an output characteristic diagram of the multiplier according to the second embodiment of this invention.

Fig. 12 is a circuit diagram of a squaring circuit to be used for a multiplier according to a third embodiment of this invention.

Fig. 13 is an output characteristic diagram of the squaring circuit shown in Fig. 12.

Fig. 14 is an output characteristic diagram of the multiplier according to the third embodiment of this invention.

Fig. 15 is a circuit diagram of a squaring circuit

to be used for a multiplier according to a fourth embodiment of this invention.

Fig. 16 is an output characteristic diagram of the squaring circuit shown in Fig. 15.

Fig. 17 is an output characteristic diagram of the multiplier according to the fourth embodiment of this invention.

Fig. 18 is a circuit diagram of a squaring circuit to be used for a multiplier according to a fifth embodiment of this invention.

Fig. 19 is an output characteristic diagram of the squaring circuit shown in Fig. 18.

Fig. 20 is an output characteristic diagram of the multiplier according to the fifth embodiment of this invention.

Fig. 21 is a circuit diagram of a multiplier according to a sixth embodiment of this invention.

Fig. 22 is a block diagram of a multiplier according to a seventh and eighth embodiments of this invention.

Fig. 23 is an output characteristic diagram of a multiplier according to a seventh embodiment of this invention.

Fig. 24 is a circuit diagram of a multiplier according to an eighth embodiment of this invention.

Fig. 25 is an output characteristic diagram of the multiplier shown in Fig. 24.

Fig. 26 is a circuit diagram of a squaring circuit to be used for a multiplier according to a ninth embodiment of this invention.

DESCRIPTION OF THE PREFERRED EMBODIMENTS

The preferred embodiments of this invention will be described below while referring to Figs. 3 to 26.

Fig. 3 schematically shows a multiplier according to first to sixth embodiments of this invention. In Fig. 3, as each squaring circuit has a differential input terminal pair, a differential input voltage of a first squaring circuit becomes $(V1+V2)$, and that of a second squaring circuit becomes $(V2-V1)$. As a result, the outputs of these two squaring circuits are subtracted to generate an output voltage V_{OUT} , which can be expressed as follows;

$$\begin{aligned} V_{OUT} &= (V1+V2)^2 - (V2-V1)^2 \\ &= 4V1 \cdot V2 \end{aligned} \quad \dots\dots\dots (21)$$

That is, the output voltage V_{OUT} can be expressed by the product $(V_1 \cdot V_2)$ of the first input voltage V_1 and the second input voltage V_2 , which means that such a circuit that comprises two squaring circuits as shown in Fig. 3 has a multiplier characteristic.

[First Embodiment]

Fig. 4 shows a multiplier according to a first embodiment of this invention. This multiplier basically comprises four sets of differential transistor pairs respectively consisting of differential transistor pairs (Q1 and Q2), (Q3 and Q4), (Q5 and Q6), and (Q7 and Q8) whose emitters are connected in common. In this case, if the emitter size of each of one transistors Q2, Q3, Q6 and Q7 of respective four sets of them is made one (1), that of the other transistors Q1, Q4, Q5 and Q8 is made K times ($K > 1$). Also, two sets of differential transistor pairs consisting of the transistors Q1 and Q2 and the transistors Q3 and Q4, and the two sets of differential transistor pairs consisting of the transistors Q5 and Q6 and the transistors Q7 and Q8 form squaring circuits, respectively. These squaring

circuits are supplied with respective electric currents in parallel, and an input signal (voltage VA) to be applied to one differential input terminal pair (1 and 2) is opposite in phase to an input signal (voltage VB) to be applied to the other differential input pair (3 and 4).

In the two squaring circuits which consist respectively of the two sets of transistor pairs (Q1 and Q2) and (Q3 and Q4) and two sets of transistor pairs (Q5 and Q6) and (Q7 and Q8), the bases of the transistors whose emitter sizes are different from each other, that is, of the transistors Q1 and Q3, Q2 and Q4, Q6 and Q8, and Q5 and Q7 are connected in common, and the bases of the transistors Q1 and Q3 are connected to one input terminal 1 of the differential input terminal pair (1 and 2), and the bases of the transistors Q2 and Q4 are connected to the other input terminal 2 thereof. In addition, the bases of the transistors Q5 and Q7 are connected to one input terminal 3 of the differential input terminal pair (3 and 4), and the bases of the transistors Q6 and Q8 are connected to the other input terminal 4 thereof. Also, the collectors of the transistors whose emitter sizes are equal to each other, that is, of the four transistors Q1 and Q4, Q6 and Q7 and of the four

transistors Q2 and Q3, and Q5 and Q8 are connected in common to form differential output signals I_p and I_q , respectively. The transistor pairs are connected to respective constant electric current sources I_0 .

In the multiplier thus obtained, the collector currents I_{C1} and I_{C2} of the differential transistor pair Q1 and Q2 can be expressed as follows;

$$I_{C1} = \frac{\alpha F \cdot I_0}{1 + (1/k) \cdot \exp(-V_A/V_T)} \dots\dots\dots (22)$$

$$I_{C2} = \frac{\alpha F \cdot I_0}{1 + k \cdot \exp(V_A/V_T)} \dots\dots\dots (23)$$

where, $\alpha F \cdot I_0$ can be expressed as follows;

$$\alpha F \cdot I_0 = I_{C1} + I_{C2} \dots\dots\dots (24)$$

Hence, the difference between the collector currents, $(I_{C1} - I_{C2})$, can be expressed as follows;

$$I_{C1} - I_{C2}$$

$$= \alpha F \cdot I_0 \frac{k^{1/2} \cdot \exp \cdot (VA/2VT) - (1/k) \cdot \exp \cdot (-VA/2VT)}{k^{1/2} \cdot \exp \cdot (VA/2VT) + (1/k) \cdot \exp \cdot (-VA/2VT)} \dots\dots\dots (25)$$

Here, supposing that VK is expressed as;

$$VK = VT \cdot \ln(K) \dots\dots\dots (26),$$

K can be obtained as follows;

$$K = \exp \cdot (VK/VT) \dots\dots\dots (27)$$

Thus, Eq. (25) showing the difference between the collector currents IC1 and IC2 can be expressed by the following equation (28);

$$\begin{aligned} & IC1 - IC2 \\ &= \alpha F \cdot I_0 \frac{\exp \cdot \{(VA+VK)/2VT\} - \exp \cdot \{(-VA+VK)/2VT\}}{\exp \cdot \{(VA+VK)/2VT\} + \exp \cdot \{(-VA+VK)/2VT\}} \\ &= \alpha F \cdot I_0 \cdot \tanh \{(VA+VK)/2VT\} \dots\dots\dots (28) \end{aligned}$$

Next, the difference between the collector currents IC3 and IC4 of respective differential transistor pair Q3 and Q4 can be obtained in the same way as shown above, that is,

$$\begin{aligned} IC4 - IC3 &= \alpha F \cdot I_0 \cdot \tanh\{(-VA+VK)/2VT\} \\ &= -\alpha F \cdot I_0 \cdot \tanh\{(VA-VK)/2VT\} \dots\dots\dots (29) \end{aligned}$$

Here, if the sum of Eq. (28) and Eq. (29) is IA, it can be expressed as follows;

$$\begin{aligned} IA &= IK - IU = (IC1-IC2) + (IC4-IC3) \\ &= \alpha F \cdot I_0 \cdot [\tanh\{(VA+VK)/2VT\} - \tanh\{(VA-VK)/2VT\}] \\ &\dots\dots\dots (30) \end{aligned}$$

Then, $\tanh x$ can be expanded as shown in Eq. (14) when $|x| \ll 1$, so that when $|VA+VK| \ll 2VT$ and $|VA-VK| \ll 2VT$, Eq. (30) becomes as shown by the following equation(31), resulting in being obtainable a differential electric current proportional to the square of the input voltage VA. Accordingly, it can be found that a squaring circuit can be obtained by combiningly using two sets of unbalanced differential transistor pairs having an emitter size ratio

of K:1.

$$\begin{aligned}
 \Delta I_A &= (I_{C1} - I_{C2}) + (I_{C4} - I_{C3}) \\
 &= \alpha F \cdot I_0 \cdot \left[\left\{ \frac{(V_A + V_K)}{2V_T} - \frac{1}{3} \right\} \left\{ \frac{(V_A + V_K)}{2V_T} \right\}^3 \right] \dots \\
 &\quad - \left\{ \frac{(V_A - V_K)}{2V_T} - \frac{1}{3} \right\} \left\{ \frac{(V_A + V_K)}{2V_T} \right\}^3 \right] \dots \\
 &= \alpha F \cdot I_0 \cdot \left\{ \left(\frac{V_K}{V_T} \right) - \left(\frac{V_K}{8V_T^3} \right) V_A^2 - \frac{1}{3} \left(\frac{V_K}{2V_T} \right)^3 \right\} \dots \\
 &\dots\dots\dots (31)
 \end{aligned}$$

Fig. 5 is an output characteristic diagram of the squaring circuit shown in Fig. 4, in which SPICE simulation values are graphically shown with the K as a parameter. From Fig. 5, it can be found that good squaring characteristic is provided.

Similar to the above explanations, for the transistor pairs (Q5 and Q6) and (Q7 and Q8), the following equations (32), (33) and (34) can be established, and the differential electric current ΔI_B between both the differential transistor pairs can be found to be proportional to the square of the input voltage V_B as;

$$I_{C5} - I_{C6} = -\alpha F \cdot I_0 \cdot \tanh\left\{\frac{(V_B - V_K)}{2V_T}\right\} \dots\dots\dots (32)$$

$$I_{C8} - I_{C7} = \alpha F \cdot I_0 \cdot \tanh\left\{\frac{(V_B + V_K)}{2V_T}\right\} \dots\dots\dots (33)$$

$$\begin{aligned}\Delta I_B &= (I_{C5} - I_{C6}) + (I_{C8} - I_{C7}) \\ &= -\alpha F \cdot I_0 \cdot \left\{ (V_K/V_T) - (V_K/8V_T^3) \cdot V_B^2 - (1/3)(V_K/2V_T)^3 \dots \right\} \\ &\dots\dots\dots (34)\end{aligned}$$

As a result, if the sum of the differential electric currents, $\Delta I_A + \Delta I_B$, is expressed ΔI , the following equation can be established as;

$$\begin{aligned}\Delta I &= \Delta I_A + \Delta I_B \\ &\approx \left\{ -(\alpha F \cdot I_0 \cdot V_K)/8V_T \right\} \cdot (V_A^2 - V_B^2) \dots\dots\dots (35)\end{aligned}$$

And, if the input voltages V_A and V_B are expressed as;

$$V_A = V_1 - V_2 \dots\dots\dots (36)$$

$$V_B = V_1 + V_2 \dots\dots\dots (37)$$

Eq. (35) can be expressed by the following equation (38), which means that a differential current I proportional to the product of the voltages V_1 and V_2 can be obtained, thus being obtainable a multiplier.

$$\Delta I \approx \alpha F \cdot I_0 \cdot (VK/2VT) V_1 \cdot V_2 \dots\dots\dots (38)$$

Fig. 6 is a characteristic diagram of a differential output current I using a hyperbolic tangent function. From this, it can be found that good multiplier characteristic is obtainable in the range of an input voltage smaller than V_K .

Fig. 7 is a gain characteristic diagram of the multiplier which is obtained by differentiating the differential output current ΔI using a hyperbolic tangent function with respect to the first input voltage V_1 . From this, it can be found that good multiplier characteristic is obtainable in the range of an input voltage smaller than V_K .

Fig. 8 shows the result obtained from a multiplier whose individual components were produced as $K = 7$. The transistor used was of 2SC2785 produced by NEC. From this, it can be found that though an offset is appeared in the output because these components are realized on an individual basis, good multiplier characteristic is obtainable. In addition, this diagram was prepared in such a manner that V_2 was changed as a parameter from zero (0) to 100 mV in a step manner at an interval of 20 mV, and

converted into voltage as follows:

$$VM1 = VCC - RL \cdot I_p$$

$$VM2 = VCC - RL \cdot I_q$$

[Second Embodiment]

Fig. 9 shows a squaring circuit to be used for a multiplier according to a second embodiment of this invention. This multiplier comprises two squaring circuits as shown in Fig. 4. The squaring circuit to be used for this embodiment is substantially equal in structure to that in the first embodiment shown in Fig. 4. What is different from the first embodiment is that respective transistors (Q1 and Q2) and (Q3 and Q4) forming two sets of differential transistor pairs have emitter resistors. The transistors Q2 and Q3 with an emitter size of 1 have emitter resistors with a resistant value of R, and the transistors Q1 and Q4 with an emitter size of K have emitter resistors with a resistant value of (R/K) which is inversely proportional to be the emitter size ratio.

The operational characteristic of this squaring circuit

cannot be analytically resolved because of including emitter resistors into differential transistor pairs. As a result, SPICE simulation values were obtained using the product $(R \cdot I_0)$ of the resistant value R of the emitter resistance and the current value I_0 of the driving current source as a parameter, which is shown in Fig. 10. From Fig. 10, it can be found that the range of input voltage can be expanded and yet good squaring characteristic can be obtained by appropriately selecting the value of the product $(R \cdot I_0)$.

Next, with $K = 3$, and $R \cdot I_0 = 8.6VT$, experiments were carried out using individual components, the result of which is shown in Fig. 11. The transistors used was of 2SC2785. From Fig. 11, it can be found that though an offset is appeared in the output because these components are realized on an individual basis, good multiplier characteristic is obtainable. In addition, this diagram was prepared in such a manner that V_2 was changed as a parameter from zero (0) to 400 mV in a step manner at an interval of 100 mV. Compared with the result shown in Fig. 8, it can be found that the input voltage range in Fig. 11 is expanded approximately three times. As a result, a multiplier using the squaring circuit having emitter resistors as shown in Fig. 9 makes

obtainable good characteristic and yet advantageously expanded input voltage range.

[Third Embodiment]

Fig.12 is a circuit diagram of a squaring circuit to be used for a multiplier according to a third embodiment of this invention, which comprises two squaring circuits combinedly arranged as shown in Fig. 4. This squaring circuit is substantially equal in structure to that in the first embodiment shown in Fig. 4 excepting that respective transistors (Q1 and Q2) and (Q3 and Q4) forming two sets of differential transistor pairs have emitter resistors on their one transistors. That is, the transistors Q2 and Q3 with an emitter size of 1 each has an emitter resistor with a resistant value of R and the transistors Q1 and Q4 with an emitter size of K each does not have an emitter resistor.

The operational characteristic of this squaring circuit cannot be analytically resolved because of including emitter resistors into differential transistor pairs. As a result, SPICE simulation values were obtained using product $(R \cdot I_0)$ of the resistant value R of the emitter resistor and the current value I_0 of the driving current source as a

parameter, which is shown in Fig. 13. From Fig. 13, it can be found that the range of the input voltage can be expanded and yet good squaring characteristic can be obtained by appropriately selecting the value of the product ($R \cdot I_0$).

Next, with $K = 3$ and $(R \cdot I_0) = 8.6 \text{ VT}$, experiments were carried out using individual components, the result of which is shown in Fig. 14. What was used for this purpose was 2SC2785 transistor. From Fig. 14, it can be found that though an offset is appeared in the output because these components are individually realized, good multiplier characteristic is obtainable. In addition, this diagram was prepared in such a manner that V_2 was changed as a parameter from zero (0) to 400 mV in a step manner at an interval of 100 mV. Compared with the result shown in Fig. 8, it can be found that the input voltage range in Fig. 14 is expanded approximately four times. As a result, a multiplier using the squaring circuit having emitter resistors as shown Fig. 12 makes obtainable good characteristic and yet advantageously expanded input voltage range.

[Fourth Embodiment]

Fig. 15 shows a squaring circuit to be used for a

multiplier according to a fourth embodiment of this invention, which comprises two squaring circuit combinedly arranged as shown in Fig. 4, and substantially equal in structure to that in the first embodiment shown in Fig. 4 excepting that respective transistors (Q1 and Q2) and (Q3 and Q4) forming two sets of differential transistor pairs have the same emitter size and yet only the transistors Q2 and Q4 have emitter resistors, respectively.

The operational characteristic of this squaring circuit cannot be analytically resolved because including emitter resistors into differential transistor pairs. As a result, SPICE simulation values were obtained using the product ($R \cdot I_0$) of the resistant value R of the emitter resistor and the current value I_0 of the driving current source as a parameter, which is shown in Fig. 16. From Fig. 16, it can be found that the input voltage range can be expanded and yet good squaring characteristic can be obtained by approximately selecting the value of the product ($R \cdot I_0$).

Next, with $K = 3$ and $(R \cdot I_0) = 8.6VT$, experiments were carried out using individual components, the result of which is shown in Fig. 17. The transistor used in the experiments was of 2SC2785. From Fig. 17, it can be found that though

there appears an offset in the output because these components were individually realized, good multiplier characteristic is obtainable. In addition, this diagram was prepared in such a manner that V_2 was changed as a parameter from zero (0) to 400 mV in a step manner at an interval of 100 mV. Compared with the result shown in Fig. 8, it can be found that the input voltage range is expanded approximately three times. As a result, a multiplier using the squaring circuit having emitter resistors as shown in Fig. 15 makes obtainable good multiplier characteristic and advantageously expanded input voltage range.

[Fifth Embodiment]

Fig. 18 shows a squaring circuit to be used for a multiplier according to a fifth embodiment of this invention, which comprises two squaring circuits combinedly arranged as shown in Fig. 4, and substantially equal in structure to that in the first embodiment shown in Fig. 4 excepting that two sets of differential transistor pairs respectively have transistors (Q1a and Q1b) and (Q4a and Q4b) having a Darlington connection. The transistors Q1a, Q1b, Q2, Q3, Q4a and Q4b are equal in emitter size and the

transistors Q2 and Q3 each has an emitter resistor with a resistant value of R.

The operational characteristic cannot be analytically resolved because of including emitter resistances into differential transistor pairs. As a result, SPICE simulation values were obtained using the product ($R \cdot I_0$) of the resistant value R of the emitter resistor and the current value I_0 of the driving current source as a parameter, which is shown in Fig. 19. From Fig. 19, it can be found that the input voltage range can be expanded and yet good squaring characteristic can be obtained by appropriately selecting the value of the product ($R \cdot I_0$).

Next, with $K = 3$ and $(R \cdot I_0) = 8.6VT$, experiments were carried out using individual components, the result of which is shown in Fig. 20. The transistor used for the experiments was of 2SC2785. From Fig. 20, it can be found that though there appears an offset in the output because these components were individually realized, good multiplier characteristic is obtainable. In addition, this diagram was prepared in such a manner that V_2 was changed as a parameter from zero (0) to 400 mV in a step manner at an interval of 100 mV. Compared with the result shown in Fig. 8, it can be

found that the input voltage range is expanded approximately five times. As a result, a multiplier using the squaring circuit having emitter resistors as shown in Fig. 18 makes obtainable good multiplier characteristic and yet advantageously expanded input voltage range.

[Sixth Embodiment]

Fig. 21 shows a multiplier according to a sixth embodiment of this invention, which is structured basically in the same manner as in the first embodiment in that four sets of differential transistor pairs (Q21 and Q22), (Q23 and Q24), (Q25 and Q26) and (Q27 and Q28) having emitters connected in common are combinedly structured. In this embodiment, the differential transistor pairs are respectively supplied with electric current in parallel, and if the emitter size of each of one transistors Q22, Q23, Q26 and Q27 is made one (1), that of each of the other transistors Q21, Q24, Q25 and Q28 is made K ($K > 1$).

In addition, in this embodiment, the differential input terminal pair (1 and 2), and differential input terminal pair (3 and 4) are applied with input signals (voltages V_{21} and V_{22}), respectively, which are equal in phase.

The four sets of differential transistor pairs as shown above are combinedly arranged in such a manner that the bases of the transistors (Q21 and Q27), (Q22 and Q25), (Q23 and Q28) and (Q24 and Q26), which are respectively unequal in emitter size to each other, are respectively connected in common, and the base of the transistor Q21 and that of the transistor Q27 are connected to the input terminal 1 of the differential input terminal pair (1 and 2) and the base of the transistor Q24 and that of the transistor Q26 are connected to the input terminal 2 of the differential input terminal pair (1 and 2). In addition, the base of the transistor Q24 and that of the transistor Q25 are connected to the input terminal 3 of the differential input terminal pair (3 and 4), and the base of the transistor Q23 and that of the transistor Q28 are connected to the input terminal 4 of the differential input terminal pair (3 and 4). On the other hand, the collectors of the four transistors Q21, Q24, Q26 and Q27 and those of the transistors Q22, Q23, Q25 and Q28 are connected in common to form differential outputs I_p and I_q , respectively. In addition, each differential transistor pair is connected to the constant current source I_0 .

Here, if the reference voltage is expressed as V_R , respective base voltages V_{B21} , V_{B22} , V_{B23} , V_{B24} , V_{B25} , V_{B26} , V_{B27} and V_{B28} of the transistors of a first differential transistor pair Q_{21} and Q_{22} , a second differential transistor pair Q_{23} and Q_{24} , a third differential transistor pair Q_{25} and Q_{26} , and a fourth differential transistor pair Q_{27} and Q_{28} can be expressed as follows;

$$V_{B21} = V_{B27} = V_R + (1/2)V_{21} \quad \dots\dots\dots (39)$$

$$V_{B22} = V_{B25} = V_R + (1/2)V_{22} \quad \dots\dots\dots (40)$$

$$V_{B23} = V_{B28} = V_R - (1/2)V_{22} \quad \dots\dots\dots (41)$$

$$V_{B24} = V_{B26} = V_R - (1/2)V_{21} \quad \dots\dots\dots (42)$$

Here, the inter-base voltage of the first differential transistor pair Q_{21} and Q_{22} , and the inter-base voltage of the second differential transistor pair Q_{23} and Q_{24} can be expressed by the following equations (43) and (44), and the both are equal to each other as shown by the following equation (45), which is defined as V_A for matching the first

embodiment;

$$VB21 - VB22 = (1/2) (V21 - V22) \dots\dots\dots (43)$$

$$VB23 - VB24 = (1/2) (V21 - V22) \dots\dots\dots (44)$$

$$VB21 - VB22 = VB23 - VB24 = VA = (1/2) (V21 - V22) \dots\dots\dots (45)$$

In addition, the inter-base voltage of the third differential transistor pair (Q27 and Q28), and that of the fourth differential transistor pair (Q25 and Q26) can be expressed by the following equations (46) and (47), and the both are equal to each other as shown by the following equation (48), which is defined as VB for matching the first embodiment;

$$VB26 - VB25 = (-1/2) (V21 + V22) \dots\dots\dots (46)$$

$$VB28 - VB27 = (-1/2) (V21 + V22) \dots\dots\dots (47)$$

$$VB26 - VB25 = VB28 - VB27 = VB = (-1/2) (V21 + V22) \dots\dots\dots (48)$$

Then, substituting V_A and V_B into Eq. (35), the following equation (49) can be obtained, which means that a differential current proportional to the product of the input voltages V_{12} and V_{22} , thus being obtainable a multiplier circuit;

$$\begin{aligned}
 I &\approx (-\alpha F \cdot I_0 \cdot V_K / 8V_T) \times \\
 &\quad [\{ (1/2)(V_{21} - V_{22}) \}^2 - \{ (-1/2)(V_{21} - V_{22}) \}^2] \\
 &= \alpha F \cdot I_0 \cdot (V_K / 4V_T) \cdot V_{21} \cdot V_{22} \quad \dots\dots\dots (49)
 \end{aligned}$$

In addition, the differential current ΔI can be expressed as $\Delta I = I_p - I_q$ in Figs. 4 and 21. In this case, however, due to the fact that the currents I_p and I_q are opposite in phase to each other, each of them includes such a current component as the product of the voltages V_1 (V_{21}) and V_2 (V_{22}). However, the magnitude thereof will become only half the differential current ΔI .

Even in this embodiment, such squaring circuits as shown in the second through fifth embodiments (see Figs. 9, 12, 15 and 18) can be used instead of each squaring circuit

shown in Fig. 21. As a result, the input voltage range can be expanded.

As explained above, according to the first through sixth embodiments, four sets of differential transistor pairs are not so arranged in a stack manner as in the prior art, but arranged so-called in a line transversally thereby allowing them to be operated at the same source voltage, so that the multipliers shown above can be effectively operated at lower source voltage than those in the prior art.

[Seventh Embodiment]

Fig. 22 schematically shows a multiplier according to a seventh embodiment of this invention. In Fig. 22, three squaring circuits each has a differential input terminal pair, and a differential input voltage of a first squaring circuit becomes $(V1-V2)$, a differential input voltage of a second squaring circuit becomes $V1$ and a differential input voltage of a third squaring circuit becomes $V2$. As a result, an output voltage $VOUT$ of the three squaring circuits can be expressed as follows;

$$VOUT = -(V1-V2)^2 + V1^2 + V2^2$$

$$= 2V_1 \cdot V_2 \quad \dots\dots\dots (50)$$

This means that the output V_{OUT} can be expressed in terms of the product $(V_1 \cdot V_2)$ of respective output voltages V_1 and V_2 of the first and second squaring circuits, and it can be found that the circuit shown in Fig. 22 has a multiplier characteristic as the case of the two squaring circuits shown in Fig. 3.

Fig. 23 is a circuit diagram of the multiplier of this embodiment. This multiplier basically comprises six unbalanced differential transistor pairs $(Q_1 \text{ and } Q_2)$, $(Q_3 \text{ and } Q_4)$, $(Q_5 \text{ and } Q_6)$, $(Q_7 \text{ and } Q_8)$, $(Q_9 \text{ and } Q_{10})$ and $(Q_{11} \text{ and } Q_{12})$, whose emitters are connected in common, respectively. Here, if the emitter size of each of one transistors Q_2 , Q_3 , Q_6 , Q_7 , Q_{10} and Q_{11} is made one (1), that of each of the other transistors Q_1 , Q_4 , Q_5 , Q_8 , Q_9 and Q_{12} is made K ($K > 1$). In addition, two sets of the transistor pairs $(Q_1 \text{ and } Q_2)$ and $(Q_3 \text{ and } Q_4)$, two sets of the transistor pairs $(Q_5 \text{ and } Q_6)$ and $(Q_7 \text{ and } Q_8)$ and two sets of the transistor pairs $(Q_9 \text{ and } Q_{10})$ and $(Q_{11} \text{ and } Q_{12})$ respectively constitute squaring circuits and supplied with electric current in parallel to be driven by a constant current source I_0 .

In the three squaring circuits shown above, two sets of unbalanced differential transistor pairs of each squaring circuit are structured so that the collectors of the transistors (Q1 and Q4), (Q2 and Q3), (Q5 and Q8), (Q6 and Q7), (Q9 and Q12) and (Q10 and Q11), which are respectively equal in emitter size to each other, are connected in common, and the bases of the transistors (Q1 and Q3), (Q2 and Q4), (Q5 and Q7), (Q6 and Q8), (Q8 and Q11) and (Q10 and Q12), which are respectively unequal in emitter size to each other, are connected in common.

In addition, referring to the inter-relation between the three squaring circuits, the bases of the transistors Q1 and Q3 of the two sets of unbalanced differential transistor pairs (Q1 and Q2) and (Q3 and Q4) as the first squaring circuit and the those of the transistors Q5 and Q7 of the two sets of unbalanced differential transistor pairs (Q5 and Q6) and (Q7 and Q8) as the second squaring circuit are connected in common to the first input terminal 1, the bases of the transistors Q2 and Q4 of the first squaring circuit and those of the transistors Q9 and Q11 of the two sets of the unbalanced differential transistor pairs (Q9 and Q10) and (Q11 and Q12) are connected in common to the input

terminal 2, and the bases of the transistors Q6 and Q8 of the second squaring circuit and those of the transistors Q10 and Q12 of the third squaring circuit are connected in common to the common input terminal 3.

In addition, the collectors of the transistors (Q5, Q8, Q9 and Q12) and (Q6, Q7, Q10 and Q11), which are equal in emitter size to each other in respective second and third squaring circuits, are connected in common, which are connected to the collectors of the transistors not equal in emitter size to each other of the first squaring circuit, respectively, thereby making the differential output currents I_p' and I_q' .

Also, the input terminal 1 and the common input terminal 3 makes a first input terminal pair to be applied with one input signal voltage V_1 and the input terminal 2 and the common input terminal 3 makes a second input terminal pair to be applied with the other input signal voltage V_2 , and as shown in Fig. 23, to the input terminals 1 and 2, the polarity of one of two input signals is applied, and to the common input terminal 3, the polarity of the other thereof is applied.

With the structure as shown above, the differential

currents I_A and I_B of the unbalanced differential transistor pairs (Q1 and Q2), (Q3 and Q4), (Q5 and Q6) and (Q7 and Q8) can be obtained in the same way as in the first embodiment (see Eqs. (30) and (34)). Next, those of the unbalanced differential transistor pairs (Q9 and Q10) and (Q11 and Q12) can be obtained similarly by the following equations (51) and (52), so that the differential current I_C of the both pairs can be expressed by the following equation (53), showing that it is proportional to the square of the input voltage V_2 .

$$I_{C9} - I_{C10} = \alpha F \cdot I_0 \cdot \tanh \{(V_2 + V_K)/2V_T\} \dots\dots (51)$$

$$I_{C12} - I_{C11} = - \alpha F \cdot I_0 \cdot \tanh \{(V_2 - V_K)/2V_T\} \dots\dots (52)$$

$$\begin{aligned} \Delta I_C &= (I_{C9} + I_{C12}) - (I_{C10} + I_{C11}) \\ &= (I_{C9} - I_{C10}) + (I_{C12} - I_{C11}) \\ &= \alpha F \cdot I_0 \cdot [\tanh \{(V_2 + V_K)/2V_T\} - \tanh \{(V_2 - V_K)/2V_T\}] \\ &= \alpha F \cdot I_0 \cdot \{(V_K/V_T) - (V_K/4V_T^3)V_2^2 - (2/3)(V_K/2V_T)^3 \dots\dots\} \\ &\dots\dots (53) \end{aligned}$$

As a result, in Fig. 23, if the difference ($I_{p'} - I_{q'}$) of

the differential output currents I_p' and I_q' is expressed as $\Delta I'$, the following equation will be obtained;

$$\begin{aligned}
 \Delta I' &= I_p' - I_q' \\
 &= -\Delta I_A + \Delta I_B + \Delta I_C \\
 &= \alpha F \cdot I_0 \cdot X \\
 &\quad [(VK/VT) - (2/3)(VK/2VT)^3 - (VK/4VT^3) \{V_1^2 + V_2^2 - (V_1 - V_2)^2\} \dots] \\
 &= -\alpha F \cdot I_0 \cdot X \\
 &\quad [(VK/VT) - (2/3)(VK/2VT)^3 - (VK/2VT^3) \cdot V_1 \cdot V_2 \dots]
 \end{aligned}
 \tag{54}$$

Here, as $V_A = V_1 - V_2$, $V_B = V_1$ and $V_C = V_2$, the following equation (55) can be obtained;

$$\Delta I' \approx \alpha F \cdot I_0 \cdot [(VK/2VT^3) \cdot V_1 \cdot V_2 - \{(VK/VT) - (2/3)(VK/2VT)^2\}]
 \tag{55}$$

This means that the differential current ΔI proportional to the product $(V_1 \cdot V_2)$ of the input voltages V_1 and V_2 , resulting in obtaining a multiplier circuit.

[Eighth Embodiment]

Fig. 24 is a multiplier according to an eighth embodiment of this invention, which comprises squaring circuits having one squaring circuit added to the multiplier of the seventh embodiment, and for the sake of convenience of explanations, the transistors are indicated by the sequential reference numerals.

The multiplier of this embodiment basically comprises eight unbalanced differential transistor pairs (Q1 and Q2), (Q3 and Q4), (Q5 and Q6), (Q7 and Q8), (Q9 and Q10), (Q11 and Q12), (Q13 and Q14) and (Q15 and Q16) respectively having the emitters connected in common. Here, if the emitter size of each of one transistors Q2, Q3, Q6, Q7, Q10, Q11, Q14 and Q15 of the eight pairs is made one (1), the emitter size of each of the other transistors Q1, Q4, Q5, Q8, Q9, Q12, Q13 and Q16 is made K ($K > 1$). In addition, two sets of the pairs (Q1 and Q2) and (Q3 and Q4), two sets of the pairs (Q5 and Q6) and (Q7 and Q8), two sets of the pairs (Q9 and Q10) and (Q11 and Q12), and two sets of the pairs (Q13 and Q14) and (Q15 and Q16) respectively form squaring circuits and supplied with source currents in parallel to be driven by the constant current source I_0 .

In the four squaring circuit shown above, two sets of

unbalanced differential transistor pairs of each squaring circuit are structured so that the collectors of the transistors (Q1 and Q4), (Q2 and Q3), (Q5 and Q8), (Q6 and Q7), (Q9 and Q12), (Q10 and Q11), (Q13 and Q16) and (Q14 and Q15), which are respectively equal in emitter size to each other, are connected in common, and the bases of the transistors (Q1 and Q3), (Q2 and Q4), (Q5 and Q7), (Q6 and Q8), (Q9 and Q11), (Q10 and Q12), (Q13 and Q15) and (Q14 and Q16), which are not equal in emitter size to each other, are connected in common.

In addition, referring to the inter-relation of the four squaring circuits shown above, the bases of the transistors Q1 and Q3 of the two sets of unbalanced differential transistor pairs (Q1 and Q2) and (Q3 and Q4) as the first squaring circuit and those of the transistors Q5 and Q7 of the two sets of unbalanced differential transistor pairs (Q5 and Q6) and (Q7 and Q8) as the second squaring circuit are connected in common to the input terminal 1, the bases of the transistors Q2 and Q4 of the first squaring circuit and those of the transistors Q9 and Q11 of the two sets of unbalanced differential transistor pairs (Q9 and Q10) and (Q11 and Q12) are connected in common to the input

terminal 2, the bases of the transistors Q6 and Q8 of the second squaring circuit and those of the transistors Q14 and Q16 of the third squaring circuit are connected in common to the common input terminal 3, and the bases of the transistors Q13 and Q15 of the third squaring circuit and those of the transistors Q12 and Q10 of the fourth squaring circuit are connected in common to each other. The bases of the transistors Q13 and Q14 are connected in common to each other.

Further, the collectors of the transistors (Q1 and Q4), (Q13 and Q16), (Q3 and Q2), (Q14 and Q15), (Q5 and Q8), (Q12 and Q9), (Q6 and Q7), and (Q10 and Q11), which are respectively equal in emitter size to each other, are connected in common, and the collectors of the transistors (Q1, Q4, Q13 and Q16), (Q6, Q7, Q10 and Q11), (Q3, Q2, Q14 and Q15) and (Q12, Q8, Q5 and Q9), which are respectively not equal in emitter size to each other, are connected in common, thereby forming the differential output currents I_p and I_q .

Also, similar to the case of the seventh embodiment, the input terminal 1 and the common input terminal 3 makes a first input terminal pair to be applied with one input

signal (voltage V_1) and the input terminal 2 and the common input terminal 3 makes a second input terminal pair to be applied with the other input signal (voltage V_2), and as shown in Fig. 24, to the input terminals 1 and 2, the polarity of one of two input signals is applied, and to the common input terminal 3, the polarity of the other thereof is applied.

With the structure as shown above, in the fourth squaring circuit additionally provided, that is, the two sets of unbalanced differential transistor pairs (Q_{13} and Q_{14}) and (Q_{15} and Q_{16}), the collector currents (I_{C13} and I_{C14}) and (I_{C15} and I_{C16}) and their differential currents ($I_{C13}-I_{C14}$) and ($I_{C16}-I_{C15}$) can be obtained as follows and the differential current ΔI_D between the both can be expressed as follows;

$$I_{C13} - I_{C14} = \alpha F \cdot I_0 \cdot \tanh (V_K/2V_T) \quad \dots\dots (56)$$

$$I_{C16} - I_{C15} = \alpha F \cdot I_0 \cdot \tanh (V_K/2V_T) \quad \dots\dots (57)$$

$$\begin{aligned} \Delta I_D &= 2 \alpha F \cdot I_0 \cdot \tanh (V_K/2V_T) \\ &= 2 \alpha F \cdot I_0 \cdot \{ (V_K/2V_T) - (1/3)(V_K/2V_T)^3 \dots\dots \} \end{aligned}$$

$$= \alpha F \cdot I_0 \cdot \{ (VK/VT) - (2/3)(VK/2VT)^3 \} \dots\dots\dots (58)$$

As a result, in Fig. 24, if the difference ($I_p'' - I_q''$) of the differential output currents I_p'' and I_q'' is expressed as $\Delta I''$, it can be expressed by the following equation (59);

$$\begin{aligned} \Delta I'' &= I_p'' - I_q'' \\ &= -\Delta I_A + \Delta I_B + \Delta I_C - \Delta I_D \\ &= \alpha F \cdot I_0 \cdot (VK/2VT^3) \cdot V_1 \cdot V_2 \dots\dots\dots (59) \end{aligned}$$

As a result, the direct current term of Eq. (55) that is, $-\alpha F \cdot I_0 \cdot [(VK/VT) - (2/3)(VK/2VT)^2]$, can be cancelled, thus being capable of being approximated by the following equation (60);

$$\Delta I' \approx \alpha F \cdot I_0 \cdot (VK/2VT^3) \cdot V_1 \cdot V_2 \dots\dots\dots (60)$$

Therefore, in the same way as in the first embodiment, the differential current $\Delta I''$ proportional to the product ($V_1 \cdot V_2$) of the input voltages V_1 and V_2 can be obtained, which means that a multiplier circuit can be obtained. In

addition, the multiplier characteristic of this embodiment was analyzed in terms of hyperbolic tangent function, the result of which is shown in Fig. 25.

Even in the seventh and eighth embodiments of this invention, the squaring circuits described in the second through fifth embodiments can be used instead of those shown in Figs. 23 and 24 (see Figs. 9, 12, 15 and 18). As a result, the input voltage range can be advantageously expanded.

As explained above, in case of the multipliers shown in the seventh and eighth embodiments, six or eight unbalanced differential transistor pairs are not arranged in a stuck manner as in the prior art, but arranged so-called in a line transversally, thereby allowing them to be operated at the same source voltage, so that the multipliers shown above can be effectively operated at lower source voltage than those in the prior art.

[Ninth Embodiment]

Fig. 26 shows a squaring circuit to be used for a multiplier according to a ninth embodiment of this invention, which comprises four MOS transistors. In Fig. 26,

MOS transistors M1 and M2 form a first differential transistor pair to be driven by a constant current source I0, and MOS transistors M3 and M4 form a second differential transistor pair to be driven by a constant current source in conformity with the following equation (61);

$$\{2 \cdot H^{1/2} / (H+1)\} \cdot I_0 \quad \dots\dots\dots (61)$$

Referring to the inter-relation between the both differential transistor pairs, the drains of the transistors M1 and M3 and those of the transistors M2 and M4 are connected in common, and the gates of the transistors M1 and M4 and those of the transistors M2 and M3 are connected in common respectively.

Here, in the first transistor pair, the transistor M1 has a ratio of a gate width W1 and gate length L1, or W1/L1, of one (1), and the transistor M2 has a ratio of gate width W2 and gate length L2, or W2/L2, of H. Namely, H can be expressed as follows;

$$(W_2/L_2)/(W_1/L_1) = H \quad (H \neq 1) \quad \dots\dots\dots (62)$$

On the other hand, in the second differential transistor pair, the transistor M3 has a ratio of gate width and gate length, or $W3/L3$, and the transistor M4 has a ratio of gate width and gate length, or $W4/L4$, which are equal to each other as shown below;

$$(W3/L3) = (W4/L4) = 4H \cdot H^{1/2}/(H+1)^2 \quad \dots\dots\dots (63)$$

Thus, respective drain currents I_{d1} and I_{d2} of the transistors M1 and M2 of the first differential transistor pair can be expressed as follows;

$$I_{d1} = \mu_n \cdot (COX/2) (W1/L1) (V_{GS1} - V_T)^2 \quad \dots\dots\dots (64)$$

$$I_{d2} = \mu_n \cdot (COX/2) \cdot H \cdot (W1/L1) (V_{GS2} - V_T)^2 \quad \dots\dots\dots (65)$$

In addition, the constant current source I_0 and the input voltage V_{IN} can be respectively expressed as follows;

$$I_{d1} + I_{d2} = I_0 \quad \dots\dots\dots (66)$$

$$V_{GS1} - V_{GS2} = V_{IN} \quad \dots\dots\dots (67)$$

Here, if ΔI_{dp} is expressed by the following equation (68);

$$\Delta I_{dp} = I_{d1} - I_{d2} \quad \dots\dots\dots (68),$$

it can be obtained as follows;

$$\Delta I_{dp} = \frac{-(1-1/H) \{ (1+1/H) I_0 - 2 \beta_1 \cdot V_{IN}^2 \} + 4 \beta_1 \{ (1+1/H) (I_0 / \beta_1) - V_{IN}^2 \}^{1/2}}{(1+1/H)^2} \quad \dots\dots\dots (69)$$

where,

$$\beta_1 = \mu_n (C_{OX}/2) (W_1/L_1) \quad \dots\dots\dots (70)$$

Similarly, in the second differential transistor pair, respective drain currents I_{d3} and I_{d4} of the transistors M_3 and M_4 can be expressed as follows;

$$I_{d3} = \{4H \cdot H^{1/2}/(H+1)^2\} \cdot \beta_1 (V_{GS3} - V_T)^2 \dots\dots\dots (71)$$

$$I_{d4} = \{4H \cdot H^{1/2}/(H+1)^2\} \cdot \beta_1 (V_{GS4} - V_T)^2 \dots\dots\dots (72)$$

In addition, the constant current source and the input voltage V_{IN} can be respectively expressed as follows:

$$I_{d3} + I_{d4} = \{2 \cdot H^{1/2}/(H+1)\} \cdot I_0 \dots\dots\dots (73)$$

$$V_{GS4} - V_{GS3} = V_{IN} \dots\dots\dots (74)$$

Here, if

$$I_{dQ} = I_{d3} - I_{d4} \dots\dots\dots (75),$$

it can be obtained by the following equation (76):

$$\begin{aligned} I_{dQ} &= \{-4H \cdot H^{1/2}/(H+1)^2\} \cdot \beta_1 \cdot V_{IN} \times \\ &\quad [4 \{H^{1/2} \cdot I_0/(H+1)\} \cdot \{(H+1)^2/4H \cdot H^{1/2} \cdot \beta_1\} - V_{IN}^2]^{1/2} \\ &= -\{4 \cdot \beta_1 \cdot (1/H^{1/2})/(H+1)^2\} \cdot V_{IN} \times \\ &\quad [\{4 H^{1/2} \cdot I_0/(H+1)\} \cdot \{(H+1)^2/4H \cdot H^{1/2} \cdot \beta_1\} - V_{IN}^2]^{1/2} \\ &\dots\dots\dots (76) \end{aligned}$$

As a result, the differential output current I can be calculated by the following equation (77);

$$\begin{aligned}
 \Delta I &= I_1 - I_2 = \Delta I_{dP} - \Delta I_{dQ} \\
 &= \frac{(1-1/H) \cdot \{2\beta_1 \cdot V_{IN}^2 - (1+1/H) \cdot I_0\}}{(1+1/H)^2} \\
 &= \{2H \cdot (H-1) \cdot \beta_1 / (H+1)^2\} \cdot V_{IN}^2 - \{(H-1)/(H+1)\} \cdot I_0
 \end{aligned}
 \tag{77}$$

That is, the differential output current proportional to the square of the input voltage V_{IN} can be obtained, thus being obtainable a multiplier circuit.

As explained above, according to this embodiment, a squaring circuit comprises two sets of differential transistor pairs having gate width and gate length ratios appropriately selected for making a differential input, so that such a squaring circuit can be realized that is completely independent of variation in threshold voltage due to manufacturing dispersion of transistors. Consequently,

a squaring circuit adapted to be integrated on a large-scale basis as well as to be preferably used for a multiplier can be effectively provided.



CLAIMS:

1. A multiplier comprising a first squaring circuit and a second squaring circuit each having a differential input terminal pair, in which an output of said first squaring circuit and an output of said second squaring circuit are connected in common to each other, a first input terminal of said first squaring circuit is applied with a first input voltage, a second input terminal thereof is applied with a second input voltage opposite in phase to said first input voltage, a first input terminal of said second squaring circuit is applied with said second input voltage, and a second input terminal thereof is applied with said first input voltage, wherein

each of said first and second squaring circuits comprises two sets of unbalanced differential transistor pairs each being arranged so that the inputs thereof are opposite in phase and the outputs thereof are connected in common, and the transistors thereof have different emitter sizes from each other.

2. A multiplier as claimed in claim 1, wherein said each





differential transistor pair has two transistor emitters respectively connected to resistors, and a ratio of a resistant value of a resistor connected to a transistor having a large emitter size and that of a resistor connected to a transistor having a small emitter size is inversely proportional to an emitter size ratio of said each differential transistor pair.

3. A multiplier as claimed in claim 1, wherein only one transistor of said each differential transistor pair has an emitter connected to a resistor.

4. A multiplier comprising a first squaring circuit and a second squaring circuit each having a differential transistor pair, in which an output of said first squaring circuit and an output of said second squaring circuit are connected in common to each other, a first input terminal of said first squaring circuit is applied with a first input voltage, a second input terminal thereof is applied with second input voltage opposite in phase to said first input voltage, a first input terminal of said second squaring circuit is applied with said second input voltage, and a





second input terminal thereof is applied with said first input voltage, wherein

each of said first and second squaring circuits comprises two sets of unbalanced differential transistor pairs whose transistors have the same emitter size and only one transistor of each pair has a resistor.

A multiplier as claimed in claim 4, wherein said unbalanced differential transistor pairs each includes two transistors having a Darlington connection.

A multiplier comprising two squaring circuits having two input signals applied to respective differential input terminal pairs thereof so as to be opposite in phase to each other, in which said two squaring circuits comprises two sets of differential transistor pairs whose emitters to be connected in common have an emitter ratio of $K:1$ ($K>1$),

wherein said two sets of differential transistor pairs are mutually arranged so that the bases of the transistors which are unequal in emitter size to each other are connected in common to form a differential input terminal pair and yet the collectors of the four transistors having





the same emitter size forming said four sets of differential transistor pairs are connected in common for making respective differential outputs.

7. A multiplier comprising a first squaring circuit including a first differential transistor pair and a second differential transistor pair whose outputs are connected in common and a second squaring circuit including a third differential transistor pair and a fourth differential transistor pair whose outputs are connected in common, outputs of said first and second squaring circuits being connected in common, wherein

a first input voltage is applied between one input terminal of said first differential transistor pair and one input terminal of said second differential transistor pair, a second input voltage is applied between the other input terminal of said first differential transistor pair and the other input terminal of said second differential transistor pair, said second input voltage being applied between one input terminal of said third differential transistor pair and one input terminal of said fourth differential transistor pair, and said first input voltage being applied





between the other input terminal of said third differential transistor pair and the other input terminal of said fourth differential transistor pair, and two transistors constituting said each differential transistor pair having different emitter sizes from each other.

8. A multiplier as claimed in claim 7, wherein said each differential transistor pair has two transistor emitters respectively connected to resistor, and a ratio of the resistant value of the resistor connected to the emitter of said transistor with a large emitter size and that of the resistor connected to the emitter of said transistor with a small emitter size is inversely proportional to an emitter size ratio of said each differential transistor pair.

9. A multiplier as claimed in claim 7, wherein only one transistor of said each differential transistor pair has an emitter connected to a resistor.

10. A multiplier comprising a first squaring circuit including a first differential transistor pair and a second differential transistor pair whose outputs are connected in





common, and a second squaring circuit including a third differential transistor pair and a fourth differential transistor pair whose outputs are connected in common, outputs of said first and second squaring circuits being connected in common, wherein

a first input voltage is applied between one input terminal of said first differential transistor pair and one input terminal of said second differential transistor pair, a second input voltage is applied between the other input terminal of said first differential transistor pair and the other input terminal of said second differential transistor pair, said second input voltage being applied between one input terminal of said third differential transistor pair and one input terminal of said fourth differential transistor pair, and said first input voltage being applied between the other input terminal of said third differential transistor pair and the other input terminal of said fourth differential transistor pair, and two transistors of each differential transistor pair respectively having the same emitter size and emitters only one of which has a resistor.

11. A multiplier as claimed in claim 10, wherein said





differential transistor pairs respectively include two transistors having a Darlington connection.

12. A multiplier comprising a first input terminal pair and second input terminal pair whose input signals are opposite in phase, and four sets of differential transistor pairs whose emitters to be connected in common are with an emitter size ratio of $K:1$ ($K>1$), wherein

said four sets of differential transistor pairs are arranged such that the base of a transistor with an emitter size of K of a first differential transistor pair and that of a transistor with an emitter size of 1 of a third differential transistor pair are connected in common to one input terminal of said first differential input terminal pair, the base of the transistor with an emitter size of 1 of the first differential transistor pair and that of the transistor with an emitter size of K of a fourth differential transistor pair are connected in common to one input terminal of said second differential input terminal pair, the base of a transistor with an emitter size of K of a second differential transistor pair and that of the transistor with an emitter size of 1 of the fourth





differential transistor pair are connected in common to the other input terminal of said first input terminal pair, and the base of the transistor with an emitter size of 1 of the second differential transistor pair and that of the transistor with an emitter size of K of the third differential transistor pair are connected in common to the other terminal of said second differential input terminal pair, and the collectors of the four transistors having the same emitter size of said four sets of differential transistor pairs are connected in common for forming respective differential outputs.

13. A multiplier comprising a first squaring circuit, a second squaring circuit and a third squaring circuit each having a differential input terminal pair, in which said first squaring circuit has an output connected so as to be opposite in phase to said second and third squaring circuits, wherein

a first input voltage is applied to one input terminal of said first squaring circuit, a second input voltage is applied to the other input terminal of said first squaring circuit, the first input voltage being applied across the





input terminal pair of said second squaring circuit, the second input voltage being applied across the input terminal pair of said third squaring circuit, and the two transistors constituting said each differential pair have different emitter sizes from each other.

14. A multiplier as claimed in claim 13, wherein said each differential transistor pair has two transistor emitters respectively connected to resistors, and a ratio of a resistant value of the resistor connected to the emitter of said transistor with a large emitter size and that of the resistor connected to the emitter of said transistor with a small emitter size is inversely proportional to an emitter size ratio of said each differential transistor pair.

15. A multiplier as claimed in claim 13, wherein only one transistor of said each differential transistor pair has an emitter connected to a resistor.

16. A multiplier comprising a first squaring circuit, a second squaring circuit and a third squaring circuit each having a differential input terminal pair, wherein an output





of said first squaring circuit is connected so as to be opposite in phase respectively to said second and third squaring circuits, wherein

a first input voltage is applied to one input terminal of said first squaring circuit, a second input voltage is applied to the other input terminal of said first squaring circuit, the first input voltage being applied across the input terminal pair of said second squaring circuit, and the second input voltage being applied across the input terminal pair of said third squaring circuit, and two transistors constituting the differential transistor pair of said each squaring circuit have the same emitter size and only one of which has an emitter connected to a resistor.

17. A multiplier as claimed in claim 16, wherein said differential transistor pairs respectively include two transistors having a Darlington connection.

18. A multiplier comprising a first input terminal pair and a second input terminal pair whose input signals are equal in phase and whose one input terminals are made as a common input terminal, and three squaring circuits including first,





second and third squaring circuits provided between said first and second input terminal pairs, in which said three squaring circuits each includes two sets of unbalanced differential transistor pairs whose emitters to be connected in common are with an emitter size ratio of $K:1$ ($K>1$), and the collectors of the transistors which are equal in emitter size and the collectors of the transistors which are unequal in emitter size are respectively connected in common, wherein

one bases of the first and second squaring circuits are connected in common to the other input terminal of said first input terminal pair, the other bases of the first and third squaring circuits are connected in common to the other input terminal of said second input terminal pair, the other bases of the second squaring circuit and one bases of the third squaring circuit are respectively connected in common to said common input terminal, and the collectors of the transistors which are equal in emitter size of the second and third squaring circuits are connected in common thereby to be connected respectively to the collectors of the transistors which are unequal in emitter size of the first squaring circuit.

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19. A multiplier comprising a first squaring circuit, a second squaring circuit, a third squaring circuit and a fourth squaring circuit each having a differential input terminal pair, in which an output of said first squaring circuit is connected so as to be opposite in phase to said second, third and fourth squaring circuits, wherein

a first input voltage is applied to one input terminal of said first squaring circuit, a second input voltage is applied to the other input terminal of said first squaring circuit, the first input voltage being applied across an input terminal pair of said second squaring circuit, the second input voltage being applied across an input terminal pair of said third squaring circuit, the first input voltage or the second input voltage being applied across an input terminal pair of said fourth squaring circuit, and two transistors constituting each of the differential transistor pairs of said each squaring circuit have different emitter sizes from each other.

20. A multiplier as claimed in claim 19, wherein said each differential transistor pair has two transistor emitters





respectively connected to resistors, and a ratio of a resistant value of the resistor connected to the emitter of said transistor with a large emitter size and that of the resistor connected to the emitter of said transistor with a small emitter size is inversely proportional to an emitter size ratio of said each differential transistor pair.

21. A multiplier as claimed in claim 19, wherein only one transistor of said each differential transistor pair has an emitter connected to a resistor.

22. A multiplier comprising a first squaring circuit, a second squaring circuit, a third squaring circuit and a fourth squaring circuit each having a differential input terminal pair, in which an output of said first squaring circuit is connected so as to be opposite in phase respectively to said second, third and fourth squaring circuits, wherein

a first input voltage is applied to one input terminal of said first squaring circuit, a second input voltage is applied to the other input terminal of said first squaring circuit, the first input voltage being applied across an





input terminal pair of said second squaring circuit, the second input voltage being applied across an input terminal pair of said third squaring circuit, the first input voltage or the second input voltage being applied across an input terminal pair of said fourth squaring circuit, and two transistors constituting each of the differential transistor pairs of said each squaring circuit have different emitter sizes from each other and only one transistor of said each differential transistor pair has an emitter connected to a resistor.

23. A multiplier as claimed in claim 22, wherein said differential transistor pairs respectively include two transistors having a Darlington connection.

24. A multiplier comprising a first input terminal pair and a second input terminal pair whose input signals are equal in phase and whose one input terminals are made as a common input terminal, and four squaring circuits comprising first, second, third and fourth squaring circuits provided between said first and second input terminal pairs, wherein

said four squaring circuits each includes two sets of





unbalanced differential transistor pairs whose emitters to be connected in common are with an emitter size ratio $K:1$ ($K>1$), the collectors of the transistors which are equal in emitter size and the collectors of the transistors which are unequal in emitter size are respectively connected in common, one bases of the first and second squaring circuits are connected in common to the other input terminal of said first input terminal pair, the other bases of the first and fourth squaring circuits are connected in common to the other input terminal of said second input terminal pair, the other bases of the second squaring circuit and one bases of the third squaring circuit are connected in common to said common input terminal, the other bases of the third squaring circuit and one bases of the fourth squaring circuit are connected in common, between the first and third squaring circuits and between the second and fourth squaring circuits, the collectors of the transistors which are equal in emitter size are connected in common, and the collectors of the transistors which are unequal in emitter size are connected in common.

25. A squaring circuit comprising a first differential





transistor pair and a second differential transistor pair which are driven by a constant electric current source, wherein

said first differential transistor pair is driven by a constant electric current source I_0 and said second differential transistor pair is driven by such a constant electric current source as;

$$\{2 \cdot H^{1/2} / (H+1)\} \cdot I_0$$

said first differential transistor pair including a first MOS transistor whose gate width (W) and gate length (L) ratio (W/L) is one (1), and a second MOS transistor whose gate width (W) and gate length (L) ratio (W/L) is H ($H \neq 1$), and said second differential transistor pair including a third MOS transistor and a fourth MOS transistor whose gate width (W) and gate length (L) ratios (W/L) are as follows;

$$\{4H \cdot H^{1/2} / (H+1)^2\}$$

and the drains of said first and third transistors, and





the drains of said second and fourth transistors are respectively connected in common, the gates of said first and fourth transistors and the gates of said second and third transistors are respectively connected in common.

26. A multiplier having at least two squaring circuits as claimed in claim 25.

