

**Method of determining a noise characterization, quantum
computer, classical computer, and system**

Technical field

The present disclosure relates to determining a noise characterization of a quantum processing unit configurable to execute at least a part of a quantum circuit comprising at least two consecutive layers of quantum gates. It moreover relates to executing a quantum circuit on a quantum processing unit, wherein executing the quantum circuit includes steps of error mitigation and/or error correction using the determined noise characterization as well as a quantum computer having a quantum processing unit configured to implement the determination of the noise characterization, a classical computer having a classical processing unit configured to implement the determination of the noise characterization and a system of the quantum computer and the classical computer.

Background

In the technical field of quantum computing, one of the more prominent, if not the most prominent model, is the quantum circuit (QC) model. This model can be seen as a quantum version of the classical circuit model used in classical computing in which classical bits are replaced by qubits (short for quantum bits) and quantum operations replace classical operations.

In a QC, the quantum operations may be represented by quantum gates. In general, a quantum gate (or simply gate) refers to a fundamental physical operation that manipulates the quantum state of one or more qubits. The skilled person understands that quantum gates may be represented by matrices and qubits (or more

precisely: the quantum state of the qubit) may be represented by vectors or matrices, and the application to qubits involves matrix multiplications. When a quantum gate is applied to one or more qubits, it transforms the quantum state of those qubits. Performing a gate on one or more qubits may thus be considered as involving a manipulation of the corresponding quantum state(s) of the qubit(s) according to the specific operations defined by the gate.

However, quantum devices, in particular in the current era of Noisy Intermediate-Scale Quantum (NISQ) devices, are often subject to substantial noise leading to faulty quantum operations. While conceptually, quantum error correction by means of so-called error correction codes, is possible, at the present stage of development, the technical requirements for it to be feasible are not met yet. This is in particular due to the no-cloning theorem of quantum mechanics which requires quantum error correction to use more advanced techniques than usable in classical computation. Quantum error mitigation, on the other hand, aims at mitigating, not outright removing the error. For this to be feasible, accurate knowledge about the source of the errors, that is, the noise, is necessary such that the mitigation techniques can function. While an accurate noise characterization can lead to meaningful quantum error mitigation, an imprecise or even incorrect noise characterization may lead to quantum error mitigation being less useful as a tool or even detrimental. Indeed, an imperfect error characterization of the noise has direct detrimental effects on the implementation of techniques like probabilistic error cancellation and/or probabilistic Zero Noise Extrapolation (ZNE), where it can lead to over or under compensation of the noise. Moreover, a precise characterization can help in improving calibration by providing detailed feedback, in identifying good and stable patches with best performing qubits and, in general, develop a better understanding of how the quantum processing unit (QPU) works. Accordingly, an accurate noise characterization is important for quantum computing, specifically for implementing the above-mentioned noise-aware error mitigation techniques as well as to boost error correction

codes such that the quantum advantage of quantum computing outperforming (in certain tasks) classical computing can be achieved.

A full noise characterization, for example using quantum process tomography, of N qubits requires 16^N parameters and this exponential scaling makes this not feasible. Consequently, effective models scaling polynomial with the number of qubits N have been developed, thus paving the way to a scalable noise characterization. One such a model is the Sparse Pauli-Lindblad (SPL) model, in which the noise is assumed to be Markovian, limited spatially and represented by Pauli operators ubiquitous in quantum computing. Within this model, the noise characterization can be obtained when so-called "Pauli fidelities", further defined in Eq. (2) below, related to the noise model are known.

Based on this model, further discussed in detail below, techniques have been developed to provide a scalable noise characterization of individual layers of a quantum circuit, specifically to infer the Pauli fidelities.

Providing such a noise characterization entails various difficulties. First, noise is not to be understood as merely a state-related or element-related phenomenon but is in particular a process-related phenomenon. In other words, it is not only the qubit or the quantum state that is noisy, but in particular the quantum gate. Hence, a noise event is commonly understood as an additional, unwanted quantum operation taking place together with the desired quantum gate but remains dependent at least on the specific qubit. Consequently, the same quantum gate performed on different qubits may have different noise and different quantum gates performed on the same qubit may have different noise as well.

Second, noise includes (spatial, but possibly also temporal) cross talk and thus a noise characterization of two qubits subject to a two-qubit gate has to take into account the remaining qubits as well. This pertains not only to other qubits

also subject to gates at the same time, but also to idling qubits (qubits not subject to any gate). Hence, accurate noise characterization has to take into account the entirety of qubits, specifically all qubits and the gates performed on the same in parallel.

Third, the execution of the gate is not the only source of noise since every quantum circuit includes a state preparation step as well as a measurement step. Both steps are subject to noise as well and separating this noise, also called SPAM (State Preparation And Measurement) noise, from the noise genuinely originating from the gate(s) to be characterized is a difficult task.

One of these techniques proven to be powerful is cycle benchmarking" originally proposed in A. Erhard et al.: "Characterizing large-scale quantum computers via cycle benchmarking", Nature Communications, volume 10, Article number: 5347 (2019). In this technique, a single layer of the quantum circuit to be characterized is repeatedly executed, which allows to deduce information related to the noise associated with said layer with high accuracy, in particular it allows to separate the SPAM noise from the noise directly related to the gate(s) to be characterized such that the latter can be obtained. This technique has been successfully implemented as detailed in Y. Kim et al.: "Evidence for the utility of quantum computing before fault tolerance", Nature, volume 618, pages 500-505 (2023) and will be referred to in the present disclosure as "single-layer cycle benchmarking.

However, the amount of information deducible by this technique is limited as some Pauli fidelities remain "unlearnable" by means of cycle benchmarking. As demonstrated in S. Chen et al.: "The learnability of Pauli noise", Nature Communications, volume 14, Article number: 52 (2023), only a subset of all Pauli fidelities relevant for the noise characterization can be inferred using cycle benchmarking. Accordingly, a scalable noise characterization requires additional techniques with lower

accuracy, leading to an overall less accurate scalable noise characterization.

A further approach is "randomized benchmarking" (RB), originally proposed by J. Emerson, et al. in "Scalable noise estimation with random unitary operators," J. Opt. B Quantum Semiclassical Opt. 7, S347 (2005). Here, randomized gates are performed on the quantum processing unit such that an average error can be inferred. While this technique as well as related techniques such as, for example, interleaved RB, mirror RB, unitarity RB, binary RB, simultaneous RB, can avoid SPAM noise, it is not tied to the quantum circuit to be executed but only to the quantum processing unit and hence is limited when a noise characterization taking into account both the quantum processing unit and the quantum circuit. In other words, randomized benchmarking provides information about average properties of the gates. This can be useful to assess the quality of the quantum processing unit, however, in general, does not provide a quantitative description which specific errors determine the average quality. In this sense, randomized benchmarking is not detailed enough to be used as a characterization technique to implement error mitigation or any other usage that requires details over the specifics of the errors/noise events.

In view of these existing approaches, there is a need for techniques providing a more accurate scalable noise characterization, in particular for techniques that provide additional information about the noise with high accuracy, overcoming the existing limiting factors of a scalable noise characterization, while maintaining the computational effort substantially unchanged, in particular maintained the same scaling with respect to the system parameters.

Summary

The present disclosure has been made in view of the above technical limitations of currently existing methods of providing a scalable noise characterization and thus provides a method for

providing a scalable noise characterization that increases the accuracy of said scalable noise characterization by pushing the boundaries of what information can be deduced with high accuracy without increasing the computational effort substantially.

According to an aspect of the present disclosure, a method of determining a noise characterization of a quantum processing unit configurable to execute at least a part of a quantum circuit comprising at least two consecutive layers of quantum gates, wherein the noise characterization is based on a predetermined number of parameters is provided, the method comprising the steps of: performing a first process based on a single layer of the part of the quantum circuit for obtaining a first set of fidelities and/or products of fidelities, performing a second process based on at least two consecutive layers of the part of the quantum circuit for obtaining a second set of fidelities and/or products of fidelities, and determining the predetermined number of parameters based on at least the first set and the second set.

According to another aspect of the present disclosure, a method of executing a quantum circuit on a quantum processing unit is provided, wherein executing the quantum circuit includes steps of error mitigation and/or error correction using a noise characterization of the quantum processing unit obtained by the above method.

According to a further aspect of the present invention, a quantum computer having a quantum processing unit is provided, wherein the quantum computer is configured to implement the above method of determining a noise characterization.

According to a further aspect of the present invention, a classical computer having a classical processing unit is provided, wherein the classical computer is configured to implement the above method of determining a noise characterization.

According to a further aspect of the present invention, a system of the above quantum computer and the above classical computer is provided.

In addition, preferred aspects of the present disclosure are defined in the dependent claims.

According to the invention, a more accurate scalable noise characterization can be provided, in particular without increasing the required computational resources in a substantial manner.

Brief description of the drawings

Embodiments of the present disclosure, which are presented for better understanding the inventive concepts, but which are not to be seen as limiting the disclosure, will be described with reference to the figures in which:

- Fig. 1 shows a flow chart for a method of determining a noise characterization of a quantum processing unit;
- Fig. 2 shows an illustration of a noise characterization of a quantum processing unit;
- Fig. 3 shows an illustration of a quantum circuit;
- Fig. 4 shows a further flow chart of a noise characterization of a quantum processing unit;
- Fig. 5 shows a flow chart of further steps of performing a first process of determining a noise characterization of a quantum processing unit;
- Fig. 6 shows an illustration of single-layer cycle benchmarking;

- Fig. 7 shows a flow chart of further steps of performing a second process of determining a noise characterization of a quantum processing unit;
- Fig. 8 shows an illustration of multi-layer cycle benchmarking;
- Fig. 9 shows a flow chart of further steps of performing a second process of determining a noise characterization of a quantum processing unit related to minimal structures contributing to the noise characterization;
- Fig. 10 shows a flow chart of further steps of performing a second process of determining a noise characterization of a quantum processing unit related to chains of quantum gates;
- Fig. 11 shows a comparison of error model mismatch and a mismatch reduction factor of a method of determining a noise characterization of a quantum processing unit according to the present disclosure with conventional approaches;
- Fig. 12 shows a visualization of two layer consisting of parallel CZ gates involving different qubits on a square topology;
- Fig. 13 shows minimal blocks to perform multi-layer cycle benchmarking on open chains;
- Fig. 14 shows minimal blocks to perform multi-layer cycle benchmarking on closed chains;
- Fig. 15 shows a quantum processing unit with square topology and various structures obtained by considering pairs of layers of a quantum circuit;

Fig. 16 shows random generated model parameters for four layers; and

Fig. 17 shows differences between correct model parameters and reconstructed ones using a conventional method and a method according to the present invention.

Detailed description

As the present disclosure relates to the technical field of quantum computing, in particular to details of how to characterize the noise present when executing a quantum gate/a quantum circuit, the following paragraphs will provide further details regarding the technology referred to and the terms used within this disclosure to facilitate the understanding of the present disclosure and the inventive concepts disclosed herein.

Quantum computing can generally be understood as technically implemented computing based on or exploiting quantum mechanical phenomena. Under certain conditions, in particular at small scales, classical theories of physical matter have to be replaced by quantum theories. One core element of these theories is that physical matter exhibits properties of both particles and waves. Quantum computing is built on the fact that leveraging this behavior can lead to what is called a "quantum advantage": For some calculations, there exist quantum algorithms that outperform classical algorithms, i.e., algorithms performed on a classical computer, by a substantial margin, in some cases even exponentially faster.

However, quantum computing as understood at present may not replace classical computing in general and for every type of calculation but only for specific technical applications for which a quantum algorithm outperforming known classical algorithms is known. Typical examples thereof include Shor's algorithm for finding prime factors of an integer showing an exponential speedup compared to known classical algorithms and

Grover's algorithm for an unstructured search showing a quadratic speedup compared to known classical algorithms, both having a wide range of possible applications. Further fields where quantum computing is expected to outperform classical computing is the field of quantum simulation, i.e., simulating a quantum system by using another quantum system governed by equivalent equations, originally proposed by Richard Feynman as well as specific optimization problems, in particular hybrid algorithms combining quantum computing aspects with classical optimization techniques. These two examples are followed by various industries as they could improve the performance and feasibility of many computationally very demanding tasks such as drug discovery and drug development, logistics as well as engineering.

At the same time, the fragile nature of quantum states leads to the possible computational advantage from quantum computing to be closely tied to a demanding engineering challenge as the quantum behavior of these states has to be preserved for a sufficient amount of time. Due to the presence of noise disrupting the quantum behavior, the number of operations that can be performed on a quantum computer are limited and as a consequence, large-scale algorithms cannot be realized on the currently available Noisy Intermediate-Scale Quantum (NISQ) devices, i.e., devices with non-negligible noise and for which scaling the number of qubits remains a challenge.

Further, detailed knowledge about the noise present during quantum computing may enhance the computational capabilities of quantum computing via techniques such as quantum error correction and/or quantum error mitigation. These techniques may allow to adjust the quantum computing process in view of the noise to counteract/mitigate the same and/or perform processing on the final result to correct for errors that occurred during the quantum computing process. In this manner, the capabilities of quantum computing can be enhanced and/or it can be ensured that said capabilities can be used to their fullest.

As quantum computing originates in quantum physics, but relates to the field of computer technology, there is a need for a model or representation to bring quantum physics and computer technology together. The model currently most established is the QC model based on the classical circuit model. In the (classical) circuit model, a (classical) circuit is comprised of bits, having either the value 0 or the value 1, to which gates are applied. In the QC model, each of these elements is replaced by its "quantum version". An example of a part of such a quantum circuit is shown in Fig. 3, which will be described in more detail later.

The quantum version of the bit is the qubit (also referred to as quantum bit). It is, similar to a classical bit, a two-level (or two-state) system, however, a quantum-mechanical two-level system, possibly an effective two-level system. As a consequence of quantum physics, a qubit may be in any coherent superposition of both states 0 and 1 simultaneously.

A qubit or quantum bit may be considered as the basic unit of quantum information technology as well as the two-level quantum-mechanical system. It may refer to a physical qubit (that is, physically implemented qubit) and/or a logical qubit. As the present disclosure relates to noise characterization of a quantum processing unit (QPU), the focus will be on the physical qubit.

A quantum gate (or simply gate) may be considered as the basic quantum circuit operating on one or more qubits. Depending on whether one refers to the logical qubits or the physical qubits, the quantum gate may thus either refer to an operation on logical qubits, or to an operation in the context of the physical quantum-mechanical two-level system, i.e., a quantum gate operating on the physical qubits (as implemented by hardware, the physical quantum system).

Quantum gates may operate on a various number of qubits. If it operates only on one qubit, the gate is also called a "single-qubit gate". Accordingly, "two-qubit gates" operate on two qubits. While also gates operating on three or more qubits are

possible, for most applications, only single- and two-qubit gates are used.

A quantum circuit may be considered as a set of quantum gates, in particular a set of quantum gates spanning one or more layers.

A layer of quantum gates, as part of the quantum circuit, may be considered as a set of quantum gates of the quantum circuit that can be executed in parallel. Here, executing gates may mean that the on the physical quantum systems operating are carried out that correspond to the execution of the quantum gates on the physical qubits.

A noise characterization of a quantum processing unit may be considered as the properties of quantum processing unit related to the noise the quantum processing unit is subjected to. While the noise characterization may be considered to mainly relate to the quantum processing unit, it cannot be considered separately from the gates to be executed. Moreover, while such a noise characterization relies mostly on the individual (two qubit) gates, it may also depend on potential (spatial) cross talk and other phenomena. That is, the noise characterization may not necessary depend only on the two qubits but may depend on the whole quantum circuit and the whole quantum processing unit.

At present, there are various possible platforms for quantum computing. One of the more prominent ones are super conducting circuits featuring one or more non-linear Josephson junctions used as the physical system of the (physical) qubits.

Moreover, even though manufacture of such superconducting circuits is done by high precision machines, at the scales of quantum computing, two circuits are rarely if ever equal and miniscule differences in manufacture but also within the quantum processing unit or the quantum computer can influence the noise each superconducting circuit, and thus each qubit, is subject to. This illustrates why a detailed noise characterization needs to take into account the individual qubit and the individual gate.

In addition, it is noted that drift, for example, changes of the internal parameters of the quantum computer and/or quantum processing unit, within the quantum computer and/or processing unit may make it necessary or at least appropriate to re-determine a noise configuration in given intervals, such as, for example, daily or weekly.

A full noise characterization of a quantum processing unit having N qubits requires 16^N parameters, that is, scales exponentially with the number of qubits and is thus not feasible. Instead, an effective model of the noise is required such that a polynomial scaling of the required number of parameters is achieved.

One such effective model is the Sparse Pauli-Lindblad (SPL) model, which is a particular model of the established Lindblad model, see also E. van den Berg et al.: "Probabilistic error cancellation with sparse Pauli-Lindblad models on noisy quantum processors", Nature Physics, volume 19, pages 1116-1121 (2023). Specifically, it is assumed that jump operators determining the Lindbladian, which generates the noise, are Pauli operators, hence the name "Pauli-Lindblad", and that the noise acts either on a single qubit (weight-1) or on two adjacent qubits (weight-2), wherein here "adjacent" refers to the physical qubits of the quantum processing unit being spatially next to each other. This assumption motivates the "Sparse" part of the model.

In more details, this model assumes that for each layer l of the quantum circuit, an effective Lindblad operator (Lindbladian)

$$\mathcal{L}_l(\rho) = \sum_{k \in \mathcal{K}} \lambda_k \left(P_k \rho P_k - \frac{1}{2} \{P_k P_k, \rho\} \right) = \sum_{k \in \mathcal{K}} \lambda_k (P_k \rho P_k - \rho) \quad (1)$$

generates the noise map Λ , wherein it is assumed that the jump operators P_k are Pauli strings, created from local Pauli operators I, X, Y, Z , belonging to the set $\mathcal{K}$, this set consisting of all weight-1 and weight-2 operators as discussed above. Here, ρ is the density matrix representing the quantum state and λ_k are weights associated with the jump operators P_k . Further,

$\{A, B\} = AB + BA$ denotes the anticommutator. Further, noise map may be considered as a mapping from one quantum state to another quantum state that represents the noise present in the system.

Importantly, the number of parameters λ_k of the SPL model has a polynomial scaling in the number of qubits N and in the case of a quantum processing unit having a square grid, the number of parameters scales linearly with the number of qubits. Thus, using the SPL model, a scalable noise characterization becomes feasible.

It is to be noted that the limitation to weight-1 and weight-2 is not necessary for the model to be applicable, but be extended, for example to weight-3 or weight-4 operators, see also E. van den Berg and P. Wocjan: "Techniques for learning sparse Pauli-Lindblad noise models", arXiv:2311.15408. Even in this case, possibly further improving the accuracy of the model and hence the accuracy of the resulting noise characterization, the scaling in terms of the number of qubits N remains polynomial.

The noise map Λ per layer (dropping now the index l) can then be obtained by exponentiating $\mathcal{L}$: $\Lambda = \exp(\mathcal{L})$. It can then be shown (for details, we refer to the detailed discussion below) that the set of parameters λ_k are equivalent to the set of Pauli fidelities

$$f_\alpha = \frac{1}{2^m} \text{Tr}[P_\alpha \Lambda P_\alpha] \quad (2)$$

where 2^m is the size of the Hilbert space to which the Pauli operator P_α belongs. Hence, obtaining the Pauli fidelities by means of experiments allows to infer the parameters λ_k and thus leads to a scalable noise characterization.

Determining these Pauli fidelities through experiments is challenging in so far as the Pauli fidelities as such cannot be measured, since every quantum experiment includes a state preparation as well as a measurement. Since both of these

introduce noise as well (the so-called SPAM noise), it is necessary to devise strategies how the effect of said SPAM noise can be separated from the Pauli fidelities. One such strategy is the above-mentioned single-layer cycle benchmarking. The core idea of cycle benchmarking, while referring to the details below, is that if the gates corresponding to a Pauli fidelity f_α are performed d times, the resulting measurement scales with f_α^d , but since state preparation and measurement are performed only once, the SPAM error remains the same for any value of d (and manifests itself as a constant prefactor and, possibly, with a fixed offset). Performing several experiments with different values of d thus allows to infer the value of the Pauli fidelity f_α without the influence of the SPAM noise.

In addition, it has been shown by S. Chen et al. in "The learnability of Pauli noise", Nature Communications, volume 14, Article number: 52 (2023) that for some Pauli fidelities f_α it is only possible to determine products such as $f_\alpha f_\beta$, where $\alpha \neq \beta$, and hence additional algebraic manipulations are necessary. In this paper, it has been shown that not all of the Pauli fidelities can be extracted by this technique because, due to a gauge freedom, some Pauli fidelities will remain unobtainable by this technique. The combination of this limitation of cycle benchmarking together with the above-mentioned SPAM noise is what currently limits the accuracy of scalable noise characterization.

Using the above, the following describes embodiments of the present disclosure in detail.

Fig. 1 shows a flow chart for a method of determining a noise characterization, more in detail a flow chart for a computer-implemented method of determining a noise characterization of a quantum processing unit comprising at least two consecutive layers of quantum gates, wherein the noise characterization is based on a predetermined number of parameters, the method comprising the steps of: performing a first process based on a single layer of the part of the quantum circuit for obtaining a first set of fidelities and/or products of fidelities (S100),

performing a second process based on at least two consecutive layers of the part of the quantum circuit for obtaining a second set of fidelities and/or products of fidelities (S200), and determining the predetermined number of parameters based on at least the first set and the second set (S400).

Here, fidelity may be considered to mean a quantity from which information about the noise characterization can be inferred. Further, process may be considered to mean any type of manipulation on the quantum processing unit.

The step of performing the first process (S100) is based on a single layer of the part of the quantum circuit and thus the obtained first set of fidelities and/or products of fidelities may be considered to provide information related to the noise characterization of this single layer. This step may, as also discussed in further details below, be implemented by techniques such as single-layer cycle benchmarking (SL CB).

The step of performing the second process (S200) is based on at least two consecutive layers of the part of the quantum circuit and thus the obtained second set of fidelities and/or products of fidelities may be considered to provide information related to the noise characterization of these at least two consecutive layers. These at least two consecutive layers may be different from each other. In particular, at least two of the entirety of these at least two consecutive layers may be different. This step may, as also discussed in further details below, be implemented by techniques such as a generalization of single-layer cycle benchmarking, which - due to it being applied to multiple layers - will also be referred to multi-layer cycle benchmarking (ML CB).

It is noted that the first process (S100) and second process (S200) are not to be performed in any particular order, indeed, as these two processes can be performed independently, any order such as first S100, then S200; or first S200, then S200, or S100 and S200 in parallel is possible.

Further, the step of determining (S400) the predetermined number of parameters, on which the noise characterization is based, uses the first and the second set. The second set may be different, preferably disjoint and/or independent, from the first set. This may already follow from the fact that the second set is based on at least two layers and hence takes into account more of the quantum circuit than is used to obtain the first set.

That is, while both sets comprise fidelities and/or products of fidelities, a striking difference between the two sets is that the second set is obtained by a process involving at least two consecutive layers of the quantum circuit. As a result of the second process, additional fidelities beyond those that can be obtained using the first process can be obtained. This additional accurate information obtained by going beyond the single layer of the quantum circuit allows to determine a more accurate noise characterization. In other words, the present invention provides a generalization of existing techniques for noise characterization, significantly increasing the number of fidelities (parameters) that can be obtained accurately, thereby increasing the accuracy of the scalable noise characterization.

Herein, performing a process, including the first and the second process, but not limited thereto, may also include "sending instructions to a quantum processing unit to carry out the necessary steps". That is, the method of determining a noise characterization may relate to a method in which the steps are performed on a quantum computer comprising the quantum processing unit, or a method in which the steps are performed partially on a quantum computer comprising the quantum processing and partially on a classical computer, or a method in which all steps are performed on a classical computer, these steps being limited to instructing a quantum computer comprising the quantum processing unit to perform experiments to provide result and evaluating these results to obtain the noise characterization.

In other words, since the noise characterization of a quantum processing unit can be seen as a result of data processing measurements results obtained by performing experiments on the quantum processing unit, the present invention does not necessarily involve the quantum processing unit but may also be implemented through interacting with the same by sending instructions/input to the quantum processing unit and receiving results/output of, for example, the first/second process, from the quantum processing unit.

Herein, "fidelities" may also refer to the values of the fidelities. That is, the result of the first and the second process is not the set of fidelities and/or products of fidelities as the fidelities are given by the model underlying the noise characterization but rather values attributed to the same.

It is further noted that since the noise characterization is based on the predetermined number of parameters, determining said predetermined number of parameters (that is, the values of said predetermined number of parameters, the predetermined number of parameters are given by the underlying model) can be seen as equivalent to determining the noise characterization.

In fact, as discussed above in connection with the SPL model, the noise characterization may be given by the set of parameters λ_k defined by the SPL model, which are equivalent to the set of Pauli fidelities f_α and hence step S400 of determining the predetermined number of parameters based on at least the first set and the second set may be understood as taking the Pauli fidelities f_α and/or products of said Pauli fidelities f_α and determining the parameters λ_k , which is equivalent to determining the noise characterization within the chosen SPL model.

Fig. 2 shows an illustration of a noise characterization of a quantum processing unit, specifically a noise characterization obtained using the SPL model. This figure illustrates in particular that a detailed noise characterization depends not

only on the individual qubits, but also on the individual single qubit or two-qubit gate to be performed.

Fig. 2 shows four numbered qubits represented by circles, wherein each circle is split into three equally big pieces, making the circle look like a pie chart. As indicated in the legend, each piece of each pie chart represents a numerical value of the model coefficients λ_k belonging of the SPL model, i.e., the coefficients of the corresponding Pauli jump operators in the Lindbladian which generate the noise. The magnitude of each of these single qubit Pauli fidelities is indicated, for simplicity, as one of "low", "medium" or "high" by the hatching of the piece. Alternatively, as this magnitude is a numerical value, it may be indicated by a gradual color scheme.

Fig. 2 shows further the Pauli fidelities associated to nine different weight-2 Pauli operators arising from the combination of the above three Pauli operators. Also here, the orientation is counterclockwise, that is, the first letter "A" of a pair "AB" corresponds to a first qubit and the second letter "B" of said pair corresponds to a second qubit following the first qubit in a counterclockwise sense. Similar to the weight-1 Pauli operators, different hatchings are used to indicate the magnitude of these errors as one of "low", "medium" or "high", and may alternatively, as these magnitudes are numerical values, be indicated by a gradual color scheme.

Fig. 3 shows an illustration of a quantum circuit. Here, the horizontal lines represent qubits, squares on a single horizontal line represent single-qubit gates, and symbols (circles) on two horizontal lines linked by vertical line represent two-qubit gates. Specifically, Fig. 3 shows four layers of a quantum circuit comprising two-qubit gates. As can be seen from the symbols and the qubits, the first and fourth of these layers are identical, while the second and third of these layers are different. Further, the single-qubit gates before each of these four layers and after the fourth of these layers, while all illustrated by squares may not be the same single-qubit gates. It is noted that Fig. 3 is related to and in parts

repeated in Fig. 6 and Fig. 8 explaining details of particular implementation of the first and the second process, respectively.

In a preferred embodiment of the above method, the first process may be performed for at least two of the at least two consecutive layers, thereby obtaining two first sets of fidelities and/or products of fidelities, and wherein the predetermined number of parameters is determined based on at least the two first sets and the second set. In other words, in this embodiment, the first process is performed twice, once for a first layer of the at least two consecutive layers and once for a second layer of the at least two consecutive layers. Additionally, the second process is performed. The determination of the predetermined number of parameters is then based on the output of these at least three process: The two first processes and the second process.

In other words, since the second process involves at least two consecutive layers and consequently a noise characterization related to these at least two consecutive layer is determined, it may be preferred that also the first process based on a single layer is performed for the at least two of the layers used for the second process. As an example, in relation to Fig. 3, this may be understood in that the first process (S100) is performed for the second layer of the shown quantum circuit and for the third layer of the shown quantum circuit, each resulting in a first set of fidelities and/or products of fidelities, and the second process (S200) is performed for the second and the third layer, resulting in second set of fidelities and/or products of fidelities. Then, determining the predetermined number of parameters (S400) uses at least the first set obtained from the first process on the second layer, the first set obtained from the first process on the third layer and second set obtained from the second process on the second and the third layer.

In the following, where no distinction between the different "first sets" is required, these may collectively be referred to as "first set", that is, the singular form is used to refer to

the entirety of first sets obtained by executing (multiple times) the first process for improved readability.

In a preferred embodiment, the predetermined number of parameters may be polynomial in the number of qubits of the quantum processing unit. As explained above, while a full noise characterization scales exponentially with the number of qubits and it thus not feasible for any practical application, and in particular would be detrimental to any quantum advantage, an effective model for the noise characterization having a polynomial, preferably even a linear, scaling in the number of qubits can be used to determine a scalable noise characterization.

As also discussed above, in a preferred embodiment, the noise characterization may be based on an effective noise model, preferably a sparse Lindblad-Pauli model.

It is noted that the effective model is not particular limited as long as it allows to arrive at a scalable noise characterization, which may be considered to correspond to a noise characterization that has a polynomial scaling with the number of qubits of the quantum processing unit.

To this end, various assumption underlying the effective model may be made. A first assumption can be that the noise is Markovian, that is, is modeled by a Markov process. In other words, the noise depends only on the present state of system, but not on its history. This assumption would break down if the system would have some form of memory regarding the state evolution and the noise, and hence this assumption is typically considered a reasonable assumption. This assumption may simplify modelling the noise substantially as handling Markov processes is well studied. This assumption can be considered as equivalent to using a Lindblad equation for modelling the noise as for example done above in Eq. (1).

A second assumption may be that the noise is spatially limited, that is, that noise is limited to interaction between a few

qubits. A common assumption is that noise is limited to single qubits and two qubits. In view of the architectures currently commonly used to realize quantum processing unit and in view of gates being limited to single-qubit gates and two-qubit gates (which corresponds to weight-1 and weight-2 operators representing the noise), this assumption is oftentimes found to reproduce experimental result quite well. An extension to spatially larger noise to model noise more accurately, such as weight-3 and weight-4 operators, is possible and will likely increase the accuracy of the noise characterization at the expense of increased computational costs and/or more experiments being necessary to obtain the relevant data.

A third assumption is that the noise operators can be represented by Pauli operators. In view of Pauli Twirling, originally introduced by C. Bennett et al.: "Mixed-state entanglement and quantum error correction", Physical Review A 54, 3824 (1996), this assumption is understood to be a not particularly limiting assumption as Pauli Twirling allows to recast noise into effective Pauli noise.

In this context, it is noted that the second assumption, that is, using a sparse effective model or in other words that the noise characterization is based on a sparse effective noise model, may be an important assumption to arrive at a polynomial scaling. In fact, assuming Markovian noise as well as that the noise can be represented by Pauli operators, there remain 4^N parameters, with N being the number of qubits of the quantum processing unit.

In a preferred embodiment, the fidelities are Pauli fidelities. This can be understood as equivalent to using the SPL model as the effective model underlying the scalable noise characterization. Reference is also made to Eq. (2) above.

In a preferred embodiment, the two-qubit gates of the quantum circuit may be Clifford gates. It is noted that while the two-qubit gates being Clifford gates may be advantageous in that it makes the mathematical description of the process of determining

the noise characterization simpler, this is not required as, for example stated in the already above cited paper by A. Erhard et al. (see page 3, left column, beginning of second paragraph of the paper).

Nevertheless, the limitation to Clifford gates as two-qubit gates may be advantageous if randomized compiling (see also J. Wallman and J Emerson: "Noise tailoring for scalable quantum computation via randomized compiling", Phys. Rev. A 94, 052325 (2016)) is performed, that is, may be advantageous within the context of cycle benchmarking, either single-layer benchmarking or multi-layer benchmarking.

Clifford gates are understood to be the elements of the Clifford group. While Quantum circuits that consist of only Clifford gates can be efficiently simulated with a classical computer due to the Gottesman-Knill theorem, and thus cannot achieve a quantum advantage, the Clifford group together with, for example, the phase shift gate with $\varphi = \pi/8$ form a universal quantum gate set for quantum computation. Clifford gates may be particular advantageous when quantum error correction is included in the quantum computing process.

Fig. 4 shows a further flow chart of a noise characterization of a quantum processing unit. This flow chart shows a preferred embodiment of the method described in connection with Fig. 1. In particular, Fig. 4 shows a flow chart in which in addition to steps S100, S200 and S400 a step of estimating (S300) a third set of fidelities that comprises fidelities that cannot be determined from the first set and the second set is performed, wherein determining the predetermined number of parameters is based on at least the first set, the second set and the third set.

As the steps S100, S200 and S400 are the same as in Fig. 1, a detail discussion of these steps is omitted.

As the third set of fidelities correspond to fidelities that cannot be determined from the first and second process, these

may, for example, be Pauli fidelities that cannot be determined using cycle benchmarking, that is, cannot be determined using single-layer cycle benchmarking and/or multi-layer cycle benchmarking. This can be understood in view of the above-mentioned observation that (single-layer) cycle benchmarking is not able to infer all fidelities and due to the fact that similar observations have been made not only with respect to single-layer cycle benchmarking but also multi-layer cycle benchmarking.

As for this third set of fidelities, cycle benchmarking or other accurate strategies for determining these fidelities and/or products of fidelities may not be available, less accurate strategies may be used. This, in turn, may reduce the overall accuracy of the noise characterization. This further underlines that the second process, providing a second set of fidelities and/or products of fidelities in addition to the first set of fidelities obtainable by conventional methods, reduces the amount of fidelities that have to be obtained using such less accurate strategies used to obtain the third set of fidelities and thus increases the overall accuracy of the noise characterization.

Further, it is noted that one way of characterizing whether a process of determining fidelities and/or products of fidelities has a low accuracy or has a high accuracy is their relationship to SPAM noise discussed above. Any process of determining fidelities and/or products of fidelities in a way that is robust with respect to SPAM noise, that is, can separate the SPAM noise from the remaining noise related to the quantum gates/layer/quantum circuit analyzed, may be considered to have a high accuracy, while any process of determining fidelities and/or products of fidelities that is not robust with respect to SPAM noise, that is, cannot separate the SPAM noise from the remaining noise may be considered to have low accuracy.

Hence, the processes S100 and S200 can be seen, with respect to process S300, to be characterized by being SPAM robust process while process S300 is not SPAM robust. In view of the differences

between S100 and S200, the three process may overall be characterized as follows: The process S100 of determining the first set is a SPAM robust process based on a single layer of a quantum circuit, the process S200 of determining the second set is a SPAM robust process based on at least two consecutive layer of the quantum circuit and the process S300 of determining the third set of a SPAM in a non-robust process. It is noted that this characterization also applies for embodiment without process S300.

It is further noted that SPAM noise may increase with the number of qubits and can be larger than gate noise, thus making SPAM noise robustness particular important when considering larger quantum processing units.

In a preferred embodiment, estimating the third set of fidelities may include applying symmetries among the fidelities of the first and/or second set and/or performing a process using a unit depth quantum circuit. Here, "applying a symmetry" may mean "assuming a symmetry", that is, it may be assumed that two fidelities only obtainable as their product are equal to each other.

A unit depth quantum circuit may be considered as a quantum circuit consisting of one layer. As such in such a unit depth quantum circuit, the relevant quantum gate(s) is(are) performed only once, it is difficult to separate the SPAM noise from the noise associated with the quantum gate(s) and hence the accuracy of such process based on unit depth quantum circuit can be expected to be low.

Fig. 5 shows a flow chart of further steps of performing a first process of determining a noise characterization of a quantum processing unit. Specifically, Fig. 5 shows a further embodiment of the first process S100 in which said first process comprises single-layer cycle benchmarking, herein now referred to as "S100a". That is, in a preferred embodiment, the first process may comprise single-layer cycle benchmarking.

In general, cycle benchmarking (either single-layer benchmarking or multi-layer benchmarking) may be considered as a process of repeatedly performing set of quantum gates on a quantum processing unit for obtaining information related to the noise associated with the set of quantum gates on the quantum processing unit. Cycle benchmarking may in particular be characterized by decoupling the noise associated with the quantum gate from noise associated with state preparation and measurement (SPAM), as also discussed above. Indeed, it can be understood that if a quantum gate is repeated several times, the noise associated with this quantum gate is generated equally often, while the SPAM noise is generated only once. Comparing the resulting measurements results for different numbers of repetitions allows to deduce which part of the measured noise is to be associated to the SPAM noise and which part to the quantum gate noise.

Further, different from randomized benchmarking and related techniques, the present invention provides a noise characterization that is not only directed at the quantum processing unit but directed at the quantum circuit to be executed on the quantum processing unit. In other words, it may be said that randomized benchmarking does not provide a noise characterization in the sense of the present invention as randomized benchmarking only provides an average error associated with the quantum processing unit. Thus, with respect to randomized benchmarking, the present invention may be understood to include the quantum circuit to be executed into the considerations to arrive at a noise characterization rather performing merely randomized quantum gates to acquire an average error. In this context, it is further noted that the predetermined number of parameters may be two or more and/or may scale polynomial in the number of qubits of the quantum processing unit.

Discussing now in more detail Fig. 5, therein it is shown that the step of performing the first process, comprising the single-layer cycle benchmarking, includes: preparing (S111) qubits of the quantum processing unit to be in first initial states,

repeatedly executing (S112) the gates of the single layer, executing (S113) randomized single qubit Pauli gates on the qubits before each execution of the gates of the single layer, executing (S114) randomized single qubit Pauli gates on the qubits after a last execution of the gates of the single layer, and measuring (S115) the qubits.

The initial states may, in particular when the SPL model is used, be eigenstates of the Pauli operators corresponding to the Pauli fidelities to be determined with the cycle benchmarking.

Similarly, the measurement may be a measurement in the basis of the Pauli operators corresponding to the Pauli fidelities to be determined with the cycle benchmarking, that is, the expectation value of said Pauli operator may be measured.

In a preferred embodiment, single qubit Clifford gates are interleaved into the randomized single qubit Pauli gates. Specifically, the first process S100a may include two subprocesses, one in which single qubit Clifford gates are interleaved and one in which they are not interleaved. Both subprocesses contribute to the first set of fidelities and/or products of fidelities. It is further noted that, as stated in the paper by Ewout van den Berg and Pawel Wocjan cited above, this can be avoided if one implements a generalized version of randomized compiling, which uses Clifford gates.

Fig. 6 shows an illustration of single-layer cycle benchmarking. This illustration is closely related to Fig. 5 describing details of the single-layer cycle benchmarking and also related to the elements introduced in Fig. 3. Specifically, Fig. 6 shows the state preparation illustrated by the squares on grey background on the left, then the m times repeated execution of the single layer of the quantum circuit also shown in Fig. 3, wherein before each execution and after the last execution randomized single-qubit gates, illustrated by the squares, are executed, and the measurement, also illustrated by squares on grey background. It is noted that state preparation and measurement are different processes that are depicted in a common manner here only to

symbolize that they are the sources of the SPAM noise. The randomized single-qubit gates may serve the purpose of performing Pauli Twirling and/or readout twirling, see Ewout van den Berg et al.: "Model-free readout-error mitigation for quantum expectation values", Phys. Rev. A 105, 032620 (2022). The single-qubit gates may be Pauli gates and/or may be Clifford gates.

It is noted that within the context of these methods implemented by the single-qubit gates, while the individual single-qubit layer is randomized, the overall set of single-qubit layers is to be chosen such that, if no noise were present, the single-qubit layers as a whole do not modify the quantum circuit on a logical level, that is, the randomized single-qubit gates should have no effect on the result of the quantum circuit.

Further, it is noted that gates for state preparation/measurement and their respective adjacent single-qubit gates may be compiled into a single layer of single-qubit gates. In other words, a single layer of single-qubit gates at the beginning and the end may be sufficient.

Fig. 7 shows a flow chart of further steps of performing a second process of determining a noise characterization of a quantum processing unit. Specifically, Fig. 7 shows a further embodiment of the second process S200 in which said second process comprises multi-layer cycle benchmarking, herein now referred to as "S200a". That is, in a preferred embodiment, the second process may comprise multi-layer cycle benchmarking.

The above considerations regarding cycle benchmarking and single-layer cycle may apply to multi-layer cycle benchmarking as well, with the distinction that - as indicated by the name - multi-layer cycle benchmarking is not directed to a single layer of the quantum circuit but instead is directed to at least two consecutive layers of the quantum circuit. In other words, multi-layer cycle benchmarking also makes use of the conceptual idea that by repeating the layers, information regarding the noise associated with this layer can be deduced in a way that separates

it from SPAM noise since the SPAM noise is not affected by the repeated execution of the layer and thus multi-layer cycle benchmarking is SPAM noise robust. It is noted, for the sake of completeness, that here "layer" refers to a layer of two-qubit gates.

Discussing now in more detail Fig. 7, therein it is shown that the step of performing the second process, comprising the multi-layer cycle benchmarking, includes: preparing (S211) qubits of the quantum processing unit to be in second initial states, repeatedly (S212) executing the gates of the at least two consecutive layers, wherein the gates of the at least two consecutive layers are alternated, executing (S213) randomized single qubit Pauli gates on the qubits before each execution of the gates of one of the at least two consecutive layers, executing (S214) randomized single qubit Pauli gates on the qubits after a last execution of the gates of the at least two consecutive layers, and measuring (S215) the qubits.

Here, considerations analogous to the discussion of Fig. 5 apply regarding the second initial states and the measurements. Further, single qubit Clifford gates may be interleaved as well.

Fig. 8 shows an illustration of multi-layer cycle benchmarking. This illustration is closely related to Fig. 7 describing details of the single-layer cycle benchmarking and also related to the elements introduced in Fig. 3. Moreover, it also illustrates the similarities and differences compared to the single-layer cycle benchmarking shown in Fig. 6. Specifically, Fig. 8 shows the state preparation illustrated by the squares on grey background on the left, then the m times repeated execution of, in this case, two layers of the quantum circuit also shown in Fig. 3, wherein before each execution of each layer and after the last execution randomized single-qubit gates, illustrated by the squares, are executed, and the measurement, also illustrated by squares on grey background.

From Fig. 8 one can thus understand that multi-layer cycle benchmarking achieves SPAM robustness in a similar manner as

single-layer cycle benchmarking, namely, by repeating the layers. Comparing experimental results with different numbers of repetition thus allows to separate also here the effects, that is, the noise, of the multiple layers from the SPAM noise, achieving SPAM robustness.

Different from single-layer cycle benchmarking, multi-layer benchmarking allows to infer further information about the fidelities, in particular is less limited by the above discussed "unlearnability" of some Pauli fidelities of single-layer benchmarking. Hence, multi-layer benchmarking may be understood as a generalization of single-layer cycle benchmarking that allows to determine more Pauli fidelities (and/or products thereof) in a SPAM robust manner, i.e., determining these quantities more accurately, thus increasing the accuracy of the noise characterization.

In a preferred embodiment, determining the noise characterization may include: reducing the third set of fidelities based on the first set of fidelities and/or the second set of fidelities, and determining the predetermined number of parameters using the reduced third set of fidelities, the first set of Pauli fidelities and the second set of Pauli fidelities.

This illustrates how the second process determining the second set improves the accuracy of the noise characterization compared to conventional methods: By reducing the third set using the first and/or second set, the influence of the low accuracy fidelities of the third set on the noise characterization can be reduced, hence improving the noise characterization.

In a further preferred embodiment determining the noise characterization may include: refining the third set of fidelities based on the first set of fidelities and/or the second set of fidelities, and determining the predetermined number of parameters using the refined third set of fidelities and the first set of Pauli fidelities and/or products of Paul fidelities.

Here, refining may be considered to mean that the values of the third set are manipulating/corrected. In other words, in this embodiment, the second set improves the accuracy by improving the contribution the third set makes to the noise characterization, thus overall improving the noise characterization.

A further possibility is to use the redundancy in data provided by the second set. That is, since a noise characterization can be provided using the first set and the third set, the second set can be understood as "redundant information". This redundant information can then be used to provide a more accurate determination of the predetermined number of parameters and hence an improved noise characterization, in particular when using methods such as the (non-negative) least-square fitting method.

Fig. 9 shows a flow chart of further steps of performing a second process of determining a noise characterization of a quantum processing unit related to minimal structures contributing to the noise characterization.

According to a preferred embodiment illustrated by Fig. 9, performing the second process (S200a) may further include: identifying (S221) minimal structures, that are part of the at least consecutive two layers, that contribute to the characterization beyond the single-layer cycle benchmarking, and limiting (S222) the multi-layer cycle benchmarking to the identified minimal structures.

Minimal structures may here refer to subsets of the at least two consecutive layers, for example, subsets of qubits and gates. Specifically, due to the local nature of the noise model, for example when employing the SPL model, it can be shown that the Pauli fidelities relevant for the noise characterization are influenced only by spatially localized interactions. This may be further exploited when limiting the second process to information that cannot be obtained by means of the first process. Hence, determining a specific Pauli fidelity does not

necessarily require performing multi-layer cycle benchmarking over the entire range of the quantum processing circuit, i.e., all qubits, but can be limited to some qubits and thus also limited to some gates.

This may further improve the second process, in particular make it computationally less demanding as the second process itself is modified to contain less gates to be performed and as a consequence also less data may have to be processed in the step of determining the predetermined number of parameters.

In summary, the separation into minimal structures/chains allows to improve the second process as i) it requires only a finite, rather small, number of rules of how to construct and analyze these minimal structures to cover the most generic scenario, ii) the analysis of these minimal structures can be carried out in parallel in the sense the one can determine all the necessary information to study them individually in post processing with a single instance of multi-layer cycle benchmarking, for example per selected pair of consecutive layers - this is due to the fact that the analysis of the different minimal structures shares the same state preparation and measurement stages, and iii) as for the first process, even if the generators of the noise are local with the model chosen, the measurement made take into account potential cross talk.

Fig. 10 shows a further flow chart of further steps of performing a second process of determining a noise characterization of a quantum processing unit related to chains of quantum gates.

According to a preferred embodiment illustrated by Fig. 10, performing the second process (S200a) may further include: separating (S231) the at least two consecutive layers into one or more chains of quantum gates, and performing (S232) the second process for each chain of quantum gates. It is noted that the step of separating of the layers into one or more chains may be understood as conceptually and not necessarily experimentally. In other words, since the one or more chains can be analyzed independently from each other, the postprocessing of the

experimental results can be carried out independently to obtain the relevant information. Nevertheless, the experimental aspect, that is, executing the quantum gates corresponding to the one or more chains, can be done in parallel.

Moreover, performing the second process for each chain in parallel may be advantageous as the noise in the system may not be confined to individual qubits, but may reach farther. By performing the second process for each chain parallel, also this additional part of the noise can be assessed.

In this context it is further noted that since the second process is performed in addition to the first process, it is only necessary to obtain additional information, i.e., information that cannot be obtained from the first process, and this can be taken into account when performing the second process.

Here, a chain of quantum gates may be considered to be a set of quantum gates in which each quantum gate "connects" the two qubits on which it acts, thereby building a chain. This may require that the at least two consecutive layers are exactly two consecutive layers.

Separating the layers into chains may be possible due to the underlying model used for determining the noise characterization being spatially limited. As a consequence of this, the noise to be considered in the second process cannot, when considering the fidelities, propagate through the set of qubits but only to a finite number of qubits. This finite number of qubits is in particular for quantum processing unit within many qubits smaller than the total number of qubits, thus simplifying the second process. It is noted that if the noise is limited to weight-1 and weight-2 Pauli operators within the above discussed SPL model, chains no longer than five qubits may have to be considered.

In line with the above, in a preferred embodiment, chains with a size exceeding a predetermined threshold may be processed,

when performing the second process, as open chains with a size corresponding to the predetermined threshold.

In a further preferred embodiment, the second process may be performed in parallel for at least two of the one or more chains of quantum gates. In other words, since chains can be considered as independent from each other, they can be performed in parallel, providing a further simplification of the second process, while maintaining a more accurate determination of the noise since by means of the parallel execution of the gates corresponding to the quantum circuit the potential cross talk across the quantum processing unit can be taken into account.

Further, the one or more chains of quantum gates may comprise one or more open chains of quantum gates and/or one or more closed chains of quantum gates. Open chains may be considered to be chains in which there are two qubits that are connected only via one quantum gates with another qubit. These qubits may be considered to be the beginning and the end of the chain. Closed chains may be considered to be chains in which every qubit is connected with at least two quantum gates with other qubits. Thus, there is no beginning/end of the chain.

Fig. 11 shows a comparison of error model mismatch and a mismatch reduction factor of a method of determining a noise characterization of a quantum processing unit according to the present disclosure with conventional approaches. This comparison is based on numerical simulations focusing on a quantum processing unit with 16 qubits, arranged in a 4x4 square topology, wherein connectivity between qubits is limited to adjacent qubits. Further, the quantum circuit is comprised of executing all possible CZ two-qubit gates allowed by the topology with the highest parallelization, that is, the quantum circuit is comprised of four two-qubit gate layers.

When considering only single-layer cycle benchmarking, a total of 44 experiments (11 per layer) can be carried out and of the total number noise parameters (which are 264) 48 unlearnable

quantities remain and have to be determined in a non-SPAM robust manner.

When also considering multi-layer cycle benchmarking, specifically, two-layer cycle benchmarking, 32 additional quantities can be determined. In other words, the amount of "unlearnable", that is, non-SPAM robust manner determined quantities, can be reduced by 2/3. It is noted that if a larger quantum processing unit would be considered, this number would approach 3/4 as in this case the overwhelming majority of the qubits would be in the bulk of the quantum processing unit and could thus be analyzed using multi-layer cycle benchmarking.

Four additional, in the specific case even only three additional, experiments are necessary to obtain the 32 additional quantities in a SPAM robust. Hence, the overhead produced by multi-layer cycle benchmarking compared to conventional methods is below 7% ($=3/44$). It is evident from these numbers that the additional information comes with a relatively small additional effort, in particular when comparing the gain (up to 75% less unlearnable quantities) with the additional experiments being less than 7%.

Further, Fig. 11 shows the extents to which the overall accuracy of the noise characterization can be improved. The underlying simulations are based on the same quantum processing unit discussed above and includes the following steps. First, the 264 parameters are randomly generated aiming at mimicking behavior of real devices. Based on these parameters, calculating the exact Pauli fidelities (and their products) is straightforward. In other words, in this step, the exact noise characterization is provided.

Second, the results of single-layer cycle benchmarking are simulated by adding to the exact values obtained a statistical uncertainty σ_{CB} . Unlearnable quantities are simulated in the same way, however, with a bigger uncertainty $\sigma_{Un} > \sigma_{CB}$, further also allowing a non-zero offset $|o_{Un}| \geq 0$ for these unlearnable

quantities. This is to mimic the fact that the unlearnable quantities cannot be determined in a SPAM robust manner.

Third, the additional quantities obtained using multi-layer cycle benchmarking may, depending on the concrete example and implementation, require further processing of single-layer cycle benchmarking, thus their statistical uncertainty σ_{ML} should be bigger than the one of single-layer cycle benchmarking.

The results shown in Fig. 11 are obtained by sampling over 50 different noise models, considering different accuracy levels of σ_{CB} and σ_{Un} , while choosing $\sigma_{ML} = 8\sigma_{CB}$. Further, the first row shows data with offset $\sigma_{Un} = 0$ and the second row shows data with offset $\sigma_{Un} = \sigma_{Un}/2$.

The two left plots in Fig. 11 show the error model mismatch in percentage calculated using the L1 norm, wherein solid lines show multi-layer cycle benchmarking according to the present invention and dashed lines show the conventional approach using only single-layer cycle benchmarking. In each plot, different value of σ_{Un} are indicated by different line colors and different symbols as indicated in the legend. The two right plots in Fig. 11 show the mismatch reduction factor, that is, the ratio between the error model mismatch of the conventional approach and the error model mismatch of multi-layer cycle benchmarking according to the present invention. A factor of less than 1 indicates that multi-layer cycle benchmarking according to the present invention provides a more accurate noise characterization. When considering that in these simulations, it is assumed that $\sigma_{ML} = 8\sigma_{CB}$, one can understand from these plots that once $\sigma_{ML} < \sigma_{Un}$, the noise characterization provided by multi-layer cycle benchmarking according to the present invention has an accuracy increased up to 40-50%. That is, the additional information obtained in a SPAM-robust manner as discussed above leads to a significant improvement in the overall scalable noise characterization. This, in turn, means that the performance of, for example, noise-aware error mitigation techniques may be improved through the use of this improved noise

characterization, thus improving the overall performance of the quantum computing process.

In line with the above, in a preferred embodiment of the present invention, there is provided a method of performing a quantum circuit on a quantum processing unit, wherein executing the quantum circuit includes steps of error mitigation and/or error correction using a noise characterization of the quantum processing unit obtained by a method according to an embodiment of the present invention.

Herein, the error mitigation and/or error correction is not particular limited and may, for example, be based on the above cited paper titled "Probabilistic error cancellation with sparse Pauli-Lindblad models on noisy quantum processors". In fact, any error mitigation and/or error correction that can be used on the basis of a conventional approach, such as single-layer cycle benchmarking on its own, may equally be used in combination with the techniques presented herein.

In a further preferred embodiment of the presented invention, there is provided a quantum computer having a quantum processing unit, the quantum computer configured to implement a method according to an embodiment of the present invention.

This quantum computer may, for example, be able to receive, as an input, a quantum circuit to be executed by the quantum computer and then perform, based on appropriately defined routines in line with the above, a method for determining the noise characterization of said quantum circuit. In other words, the quantum computer can perform, given the quantum circuit, the noise characterization of said quantum circuit with respect to the quantum computer, that is, one may say that the quantum computer performs the noise characterization autonomously.

There are no particular limitations regarding the possible hardware platforms on which the present invention may be implemented. Quantum processing unit based on superconducting circuits/superconducting qubits are a clear use case, in

particular as the above cited papers have demonstrated that conventional cycle benchmarking can be realized on this platform. Nevertheless, an adaption to any platform such spin qubits, ultra-cold atoms and the like is possible as the above discussion is platform independent.

In a further embodiment according to the present invention, there is provided a classical computer having a classical processing unit, the classical computer configured to implement a method for determining a noise characterization according to an embodiment of the present invention.

This corresponds to the above discussion that the present invention may be implemented by a classical computer controlling a quantum computer, providing input to the quantum computer and receiving out from it. In this scenario, the quantum computer could be seen as an external (technical) entity controlled by a classical computer.

In a still further embodiment according to the present invention, there is provided a system of a quantum computer according to an embodiment of the present invention and a classical computer according to an embodiment of the present invention. In other words, this system comprises the quantum computer as well as the classical computer. In such a system, the various steps involved in determining the noise characterization may be shared and/or distributed among the classical computer and the quantum computer. They may in particular be distributed in a manner that allows efficient performance of said steps.

In summary, the present invention provides methods for determining a noise characterization by considering at least two consecutive layers of a quantum circuit instead of, as done in conventional methods, a single layer of the quantum circuit. In this manner, additional fidelities relevant for the determination of noise characterization can be determined in an accurate manner, that is, in a manner that is robust with respect to noise resulting from state preparation and measurement.

Consequently, the noise characterization provided by the present invention reduces the number of fidelities that have to be determined in a low accuracy manner by a substantial margin. In specific, relevant examples by up to 75%. It is noted that this advantage is related to the fact that for a noise characterization not all mathematically possible fidelities are relevant. In fact, in realistic scenario, it is only a small portion that is relevant for the noise characterization. The present invention is able to target this small portion of the fidelities.

Advantageously, the implementation of this additional step in the noise characterization does not affect the scalability of the overall noise characterization as the runtime overhead is fixed and typically below 10%, possibly even less depending on the specific scenario.

Further, the present invention is not particular limited to a quantum processing unit topology, a quantum processing unit platform/technology or the like, and thus can find application across the whole spectrum of quantum computing technologies, in particular can implemented whenever conventional approaches such as single-layer cycle benchmarking can be implemented.

Further technical details

In the following, further technical details relating to the above discussed methods are presented. These details serve in particular to provide further details on some aspects of the present invention and to highlight some of the complexities involved when generalized the conventional approach limited to a single layer of a quantum circuit to at let two consecutive layers of the quantum circuit.

First, the sparse Pauli-Lindblad (SPL) model will be discussed in detail. This model has been discussed by the above cited paper by E. van den Berg et al. in 2022. Therein, it is assumed that the noise map Λ_l , associated with a layer l of two-qubit gates, is generated by a Lindbladian whose jump operators are low-

weight (specifically, but not limited to weight-1 and weight-2) Pauli operators, which are local in terms of the topology of the QPU. Such a model is scalable and has been successfully implemented on IBM hardware (albeit without tunable couplers) up to 127 qubits, see the above cited paper by Y. Kim et al.

The effective Lindbladian that generate the noise map Λ_l associated with the execution of a layer l of (dressed) two-qubit gates reads (this equation corresponds to Eq. (1)) above.

$$\mathcal{L}_\ell(\rho) = \sum_{k \in \mathcal{K}} \lambda_k \left(P_k \rho P_k - \frac{1}{2} \{P_k P_k, \rho\} \right) = \sum_{k \in \mathcal{K}} \lambda_k (P_k \rho P_k - \rho). \quad (3)$$

Note that the second formulation of this equation arises from the fact that the square of all Pauli operators is equal to the identity: $P_k^2 = I$. Further, $\{A, B\} = AB + BA$ denotes anticommutator, and (used later) $[A, B] = AB - BA$ denotes the commutator. Here, the jump operators P_k are Pauli operators belonging to the set $\mathcal{K}$. The latter consist of all weight-1 Pauli operators (i.e., acting on only one qubit) and all weight-2 Pauli operators with support on two adjacent qubits (in terms of the QPU topology). The number of model coefficients $\lambda_k \geq 0$ is given by

$$K(n, c) = |\mathcal{K}| = 3n + 9c. \quad (4)$$

where c is the number of connections between the qubits. For a square grid, $c \simeq 2n - 2\sqrt{n}$. By considering unit time, the noise map is simply obtained by exponentiating $\mathcal{L}$ (from hereon, we drop the layer index l for simplicity). Since Pauli channels $\mathcal{P}(\rho) = P\rho P$ commute, one simply has

$$\Lambda = \exp(\mathcal{L}) = \exp\left(\sum_{k \in \mathcal{K}} \lambda_k (\mathcal{P}_k - \mathcal{I})\right) = \prod_{k \in \mathcal{K}} e^{-\lambda_k \mathcal{I}} e^{\lambda_k \mathcal{P}_k} = \prod_{k \in \mathcal{K}} \omega_k \mathcal{I} + (1 - \omega_k) \mathcal{P}_k, \quad (5)$$

where we introduced the non-negative coefficients

$$\omega_k = \frac{1+e^{-2\lambda_k}}{2}, \quad (6)$$

and $\mathcal{I}$ is the identity superoperator. The SPL model may be seen to have several advantages. Using only a linear number of parameters, the resulting noise map, which contains also high-weight Paulis, can capture (at least some degree of) spatial cross-talks associated with the executed of the layer of single-qubit gates. Moreover, it is extremely straightforward to (probabilistically) modify the noise map, which may have applications for error mitigation, such as probabilistic error cancellation (PEC) and probabilistic ZNE. Indeed, to effectively multiply the noise level by a factor β it is sufficient to probabilistically implement the map

$$\Lambda^{(\beta)} = \prod_{k \in \mathcal{K}} \omega_k^{(\beta)} \mathcal{I} + (1 - \omega_k^{(\beta)}) \mathcal{P}_k \quad (7)$$

with

$$\omega_k^{(\beta)} = \frac{1 + e^{-2(\beta-1)\lambda_k}}{2} \quad (8)$$

since

$$\Lambda^{(\beta)}(\Lambda) = e^{\beta \mathcal{L}}. \quad (9)$$

For example, the choice $\beta = 0$ allows to completely cancel the noise (in that case $\Lambda^{(\beta)}$ is not physical and can be implemented using quasi-probability).

Having introduced an important effective model, we will now discuss details how a noise characterization of this model can be obtained using conventional approaches. This illustrates not only concepts relevant for the extension provided by the present invention but also illustrates the shortcomings of conventional approaches.

To characterize a layer of two-qubit gates according to the SPL model, all coefficients λ_k of the model need to be determined. To this end, we can exploit their direct connection with the Pauli fidelities f_a associated with the noise map. Indeed, we find, as also discussed above, (here m is the size of the Hilbert space)

$$\begin{aligned} f_a &= \frac{1}{2^m} \text{Tr}[P_a \Lambda(P_a)] = \prod_{k \text{ s.t. } \{P_a, P_k\}=0} 2\omega_k - 1 \prod_{k \text{ s.t. } [P_a, P_k]=0} 1 = \prod_{k \text{ s.t. } \{P_a, P_k\}=0} e^{-2\lambda_k} \\ &= \exp\left(-2 \sum_k \langle a, k \rangle_{sp} \lambda_k\right). \end{aligned} \tag{10}$$

Here, we make use of the fact that two Pauli operators P_a and P_k either commute or anticommute, a fact that can be captured by the symplectic inner product

$$\langle a, k \rangle_{sp} = \begin{cases} 0 & \text{if } [P_a, P_k] = 0 \\ 1 & \text{if } \{P_a, P_k\} = 0 \end{cases} \tag{11}$$

Moreover, by introducing introduced the binary matrix $M_{ak} = \langle a, k \rangle_{sp}$, the vectors of model parameters $\vec{\lambda} = (\lambda_1, \lambda_2, \dots)$ and fidelities $\vec{f} = (f_1, f_2, \dots)$, we obtain the simple expression (in which the logarithm is applied elementwise)

$$\vec{f} = \exp(-2M\vec{\lambda}) \Leftrightarrow -\log(\vec{f})/2 = M\vec{\lambda}. \tag{12}$$

If we find a way to measure enough Pauli fidelities so that the matrix M has full rank (which equivalent to the number of model parameters $|\mathcal{K}|$), then Eq. (12) can be inverted and the model parameters determined. It is convenient to perform such an operation by implementing a non-negative least square fit, that is, solving the optimization problem

$$\vec{\lambda}(\vec{f}) = \operatorname{argmin}_{\lambda \geq 0} \|M\vec{\lambda} + \log(\vec{f})/2\|_2^2. \quad (13)$$

We will now turn to how to obtain, that is, measure Pauli fidelities using conventional (single-layer) cycle benchmarking.

Pauli fidelities can be determined in a SPAM robust way via cycle benchmarking (CB). To extract the fidelity of the Pauli operator P_α , the basic idea is to apply d times the Pauli error map $\Lambda(\rho) = \sum_k p_k P_k \rho P_k$ that we want to characterize to one eigenstate $|\alpha\rangle$ of P_α and then measure the expectation value of P_α . Indeed, for $d = 1$, this would correspond to

$$\begin{aligned} \operatorname{Tr}[P_\alpha \Lambda(|\alpha\rangle\langle\alpha|)] &= \sum_k \operatorname{Tr}[p_k P_k P_\alpha P_k |\alpha\rangle\langle\alpha|] = \sum_k (-1)^{\langle\alpha, k\rangle_{sp}} p_k = \sum_k p_k \operatorname{Tr}[P_\alpha P_k P_\alpha P_k]/2^m \\ &= \operatorname{Tr}[P_\alpha \Lambda(P_\alpha)] = f_\alpha. \end{aligned} \quad (14)$$

In presence of SPAM errors and for generic d , the expectation value of P_α would read, under the assumption of doing Pauli twirling and readout twirling

$$\langle P_\alpha \rangle^d = A f_\alpha^d, \quad (15)$$

where the depth-independent factor A is due to SPAM errors/SPAM noise. The fidelity can be then extracted in a SPAM-robust way via an exponential fit.

The above assumes that one can apply the noise map Λ without the layer of two-qubit gates associated to it. This assumption does not hold and hence some complications for the computation of isolated Pauli fidelities are introduced.

We assume, without any particular loss of generality, that the layer of two-qubit gates is Clifford and is described by the unitary C . While this guarantees that $C P_\alpha C^\dagger$ is still a Pauli operator P_β , in general, one can have $P_\alpha \neq P_\beta$. Therefore, in general, the repeated action of the noisy layer will not lead to

an expectation value proportional to the power of a single Pauli fidelity, but rather to a product of different Pauli fidelities.

Let us clarify this point with a simple example, which will be particularly relevant in the following, and focus on a two-qubit layer consisting of a single CZ (Controlled-Z) gate. In this case, the conjugation table reads:

$$\begin{array}{cccccccccccccccc} P_\alpha & IX & IY & IZ & XI & XX & XY & XZ & YI & YX & YY & YZ & ZI & ZX & ZY & ZZ \\ CP_\alpha C^\dagger & ZX & ZY & IZ & XZ & YY & YX & XI & YZ & XY & XX & YI & ZI & IX & IY & ZZ \end{array}$$

We immediately see that $CP_\alpha C^\dagger = P_\alpha$ only in few cases ($P_\alpha = IZ, ZI, ZZ$). In all the other cases, the CZ gate changes the Pauli operator and the original Pauli operator is recovered by applying the CZ twice (as it should be, since two CZs correspond to the identity). By using CB in such a scenario, we can then directly access the value of three individual Pauli fidelities (IZ , ZI , and ZZ) as well as of six pairs of fidelities.

Fortunately, we can easily extract more information by interleaving the layer we want to characterize with specific single-qubit Clifford gates. Indeed, as long as the layer unitary C does not change the support of a Pauli, it is always possible to apply a combination of H (Hadamard) and S single-qubit gates to restore the original Pauli after every application of C . These extra single-qubit gates can be then compiled together with the single-qubit Pauli gates used for randomized compiling. Effectively, they can thus be implemented without changing the depth of the circuit. In the simple example of the single CZ gate discussed before, it is particularly convenient to implement one extra S gate on each of the two qubits, after the CZ gate. This corresponds to a unitary operation $\tilde{C} = (S \otimes S)C$, whose conjugation table now reads

$$\begin{array}{cccccccccccccccc} P_\alpha & IX & IY & IZ & XI & XX & XY & XZ & YI & YX & YY & YZ & ZI & ZX & ZY & ZZ \\ \tilde{C}P_\alpha\tilde{C}^\dagger & ZY & ZX & IZ & YZ & XX & XY & YI & XZ & YX & YY & XI & ZI & IY & IX & ZZ \end{array}$$

Now, $\tilde{C}P_\alpha\tilde{C}^\dagger = P_\alpha$ for seven Pauli operators: ZI , IZ , ZZ , XX , YY , XY , and YX . Moreover, we can measure the values of four new pairs of fidelities. Unfortunately, even with these results, it is not

possible to determine all the 15 Pauli fidelities associated with a CZ gate. Indeed, out of the 4 conditions on the pairs of Pauli fidelities involving the operators IX, ZX, IY, and ZY, only three are independent:

$$f_{XI}f_{XZ} = \frac{(f_{XI}f_{YZ})(f_{YI}f_{XZ})}{(f_{YZ}f_{YI})}. \quad (16)$$

An analogous relation holds for the other four Pauli fidelities XI, XZ, YI, and YZ. As a result, by combining “normal” and “interleaved” CB, it is only possible to directly determine 7 individual fidelities and 6 extra independent conditions on fidelity pairs, leaving two degrees of freedom that cannot be determined in a SPAM robust way. Importantly, this particular result can be generalized to arbitrary Clifford layers, demonstrating that single-layer cycle benchmarking cannot provide a complete scalable noise characterization. In the general case, the total number of Pauli fidelities is obviously $4n - 1$ (excluding the identity, which is trivial) and the number of unlearnable degrees of freedom is $2n - \hat{c}$, where $\hat{c}$ is the number of connected components of the pattern transfer graph associated with the layer. This general result can be understood in terms of the presence of a gauge transformation, which affects state preparation, gate layer, and measurement errors in such a way that some (unlearnable) fidelities of the layer can be modified arbitrarily without affecting the values of the observables that can be measured. Note that the fraction of unlearnable fidelities is exponentially small in the number of qubits.

We will now discuss the effect of this limited learnability. The impossibility to determine all Pauli fidelities clearly poses an issue for the noise characterization using the SPL model. Indeed, in general, instead of the vector of single fidelities $\vec{f}$, one has only access to the elementwise product of two vectors $\vec{f}_1$ and $\vec{f}_2$. By means of Eq. (12), we can relate each one of them to $\vec{\lambda}$ via binary matrices M_1 and M_2 as follows

$$-\log(\vec{f}_1 \vec{f}_2) / 2 = -\log(\vec{f}_1) / 2 - \log(\vec{f}_2) / 2 = (M_1 + M_2) \vec{\lambda}. \quad (17)$$

As before, we can then derive the model parameters λ only when the rank of the matrix $M = M_1 + M_2$ corresponds to the number of parameters $|\mathcal{K}|$.

Focusing on the case of a layer consisting of non-overlapping CZ gates, it has been shown that a sufficient condition for determining all the model parameters is that

- for every qubit, all the fidelities of weight-one Paulis are known,
- for each weight-two Pauli operator P_k with $k \in \mathcal{K}$, the product of its fidelity times the fidelity of its conjugate under the Clifford layer is known.

The last condition on the weight-two Pauli operators is straightforward to meet, since it is exactly what CB (or, equivalently, interleaved CB) can provide out-of-the-box. The first condition on the weight-one Pauli operators is more complicated. If a qubit does not belong to the support of the CZ gates in the layer, it is easy to measure all its three individual fidelities with weight one. If, on the other hand, the qubit is “active” and undergoes a CZ gate, only the Z fidelity can be measured directly. It is possible to find a constraint on the two remaining fidelities, X and Y, by combining the results from CB and interleaved CB. Indeed, focusing only on the pairs of qubits the CZ acts on, one has

$$f_{YI} = f_{XI} \frac{f_{YI} f_{XZ}}{f_{XI} f_{XZ}}. \quad (18)$$

Despite this, one is left with a single degree of freedom per “active” qubit that cannot be determined using CB and has to be measured in a different, that is, non-SPAM robust manner.

Further, it is important to determine which circuits are actually needed to perform all the required CB instances. In general, for a given eigenstate and for a given measurement basis, several fidelities can be extracted from the same circuit. For example, if one wants to measure the fidelity of the Pauli operator $P_\alpha = XZZ$, one needs to create an eigenstate $|\alpha\rangle$ of P_α , implement the Clifford layer a certain number of times, and, to determine the expectation value of P_α , measure in the basis XZZ . With the very same circuit, however, I can determine the fidelities of all the Pauli operators that can be derived from P_α by replacing with identities one or more entry of the Pauli string (e.g., IZZ , XIZ , XII , ...). This can be exploited to dramatically reduce the number of circuits that need to be run on the quantum processing unit.

In particular, for quantum processing unit with square topology and, more generally, for topologies that allows to order the qubits such that no qubit is preceded by more than two of its neighbors, it is possible to select only nine Pauli strings such that, for each pair of neighbor qubits, the corresponding substrings cover all the possibilities, i.e., $\{X, Y, Z\}^{\otimes 2}$.

In view of the above, (noise) characterizing a layer of m non-overlapping CZs on a quantum processing unit with square topology using CB can be realized with the following procedure.

1. Perform standard CB on the nine Pauli bases, that is, 9 CB instances;
2. Perform interleaved CB, that is, with S gates following each active qubit, using two different bases, so that XX and XY are measured for each pair of active qubits, that is, 2 further CB instances;
3. Measure/infer the individual fidelity of the weight-one X Pauli operator for every active qubit, that is, two for each CZ gate, that is, two or zero extra circuits evaluation.

The first step provides a list of pair fidelities that can be processed using Eq. (17), however, this will not lead to a full rank matrix M . In particular, for each CZ, there are 6 individual

fidelities that are still not determined and the rank of M is thus $K(n, c) - 6m$. The input from the interleaved CB allows to further increase the rank up to $K(n, c) - 2m$, since it relates X and Y Pauli operators. In the last step, we have to measure/infer the remaining $2m$ individual weight-one fidelities, which cannot be done in a SPAM-robust way.

The individual weight-1 X fidelity of active qubits can be determined in several ways. A relevant one is to implement the CB procedure with $d = 1$, i.e., applying the layer only once (a unit depth circuit). In this case one can directly extract the desired fidelity, but it is not possible to separate it from SPAM noise. For such a scenario, two extra circuits have to be executed: one considering a Pauli string (for eigenstate preparation and measurement basis) with X on half of the active qubits and Z on their partners, and another one with X and Z exchanged.

Another option to infer those unlearnable fidelities is to assume some extra relations/symmetries. For example, if $f_{X_I} \sim f_{X_Z}$, then one can simply infer $f_{X_I} \sim \sqrt{f_{X_I} f_{X_Z}}$. Other different methods (or combinations of methods) can be of course implemented. In any case, the estimations of the weight-1 X fidelities are very likely to be affected by a much larger uncertainty with respect to the values that can be extracted from the SPAM-robust CB procedure and such an uncertainty is detrimental for the precise determination of the model coefficients λ_k .

To sum up, to characterize a layer consisting of m non-overlapping CZ gates, it is necessary to implement a total of 11 CB experiments (each one using preferably at least three different values of d), regardless of the total number of qubits. This leaves $2m$ unlearnable quantities that have to be determined in a different, non SPAM-robust way.

Having introduced the technical details of the conventional approach, thereby setting the ground for what is to follow, we now turn to the details of how considering more than one layer may increase the accuracy of the conventional methods.

Typically, a quantum circuit consists of L distinct layers of two-qubit gates, each one consisting of m_l ($l = 1, \dots, L$) gates. From the above, it is understood that conventional methods require a minimum of $11L$ CB experiments, resulting in $2\sum_l m_l$ unlearnable fidelities that have to be determined using non-SPAM robust technique having larger uncertainties, for example by using extra $2L$ circuits or assuming the existence of extra symmetries as discussed above.

However, if additional experiments similar to conventional CB experiments, but with respect more than one layer of the quantum circuit, are executed, they allow us to identify extra constraints, which can be measured in a SPAM-robust way, on products of unlearnable fidelities. As also discussed above, this leads to a significant and immediately quantifiable reduction of the total number of unlearnable quantities while being associated with an overhead due to execution of the additional multi-layer CB experiments that is limited and under control, in particular it does not affect the polynomial scaling and thus the feasibility of the noise characterization. In the following, technical details of how this can be implemented in a concrete and reasonable use case will be discussed.

As done above, the following will focus on two layers of parallel CZ gates. As shown in Fig. 12, the two layers can be visualized by highlighting with different line styles, i.e., black solid lines and gray dashed lines, the connections between the qubit (illustrated by circles) associated with the execution of a CZ. Since a single qubit cannot be connected to more than one line of the same color (the CZ within a layer have to be executed in parallel) the resulting patterns of grey and black lines are relatively simple. In particular, the only structures allowed are open chains (left hand side of Fig. 12) and closed chains (right hand side of Fig. 12) of qubits, linked via lines of alternating line styles. Those chains cannot intersect, as this would imply having non-parallel CZ within a single layer. Further, also two qubits may be linked via a single CZ gate and are not affected by another layer (see the middle of Fig. 12). Moreover, the closed chains necessarily involve an even number

of qubits. While Fig. 12 shows a square topology, these considerations apply to any QPU topology.

It is stressed that during a multi-layer CB experiment, qubits belonging to the same (open or closed) chain can become entangled, however, different chains remain completely independent from each other. As a result, when studying multi-layer CB, one can focus on each chain of qubit separately. This will greatly simplify our analysis and guarantee the scalability of our approach.

In view of the additional complications which fidelities can be determined by the additional multi-layer CB experiments, the focus is now on a qubit belonging to the bulk of one of the chains, that is, not at the ends of the chains. Given the above discussion of conventional approaches, it is clear that such a qubit is associated with two unlearnable fidelities, one for each layer.

In the following, it will be shown how multi-layer CB can provide us with a SPAM-robust constraint on the product of these two unlearnable fidelities.

Importantly, in the CB experiment, it is sufficient to consider one single initial state $|+\rangle^{\otimes N} = H|0\rangle^{\otimes N}$, which is an eigenstate of all Pauli operators that are tensor products of single-qubit X and I operators.

At first, we consider, with reference to Fig. 13, open chains of qubits. The shortest non-trivial open chain consists of three qubits, see also Fig. 13(a). We label the central one with 0 and assume it is coupled to qubit 1 via the black layer and to qubit 2 via the grey one. The two unlearnable fidelities associated with qubit 0 can be identified with f_{IXI}^R and f_{ZIX}^B . In this section, the fidelities are labelled such that the upper index indicates the layer, wherein here R is chosen for the black and B for grey one, and the lower index indicates the Pauli string. The latter is ordered starting from qubit 0 on the leftmost position.

Fig. 13(a) graphically shows that, if one prepares one eigenstate of the Pauli operator IXX , after the noiseless application of the black and grey layer, we obtain again one eigenstate of the same operator. If one repeats this minimal structure m times in presence of noise (and interleaving the two-qubit gate layers with single-qubit gates to perform RC and twirl the noise into Pauli, a step not shown in the Figure) and then measures the expectation value of operator IXX , one obtain $A(f_{IXX}^R f_{ZXX}^B)^m$. In complete analogy with standard CB, by considering different values of m and performing an exponential fit, one can thus determine the product $f_{IXX}^R f_{ZXX}^B$ in a way that is robust against SPAM noise/errors. Despite the similarity, this product is not identical to the product of the unlearnable fidelities that is to be determined, namely, $f_{IXI}^R f_{ZIX}^B$. However, as detailed further below, within the SPL model the Pauli fidelities are not all independent from each other. In the present case, it turns out that, assuming to know all learnable fidelities, those two products are indeed completely equivalent. In particular, one can show that $f_{IXI}^R = f_{IXX}^R / f_{IIX}^R$ and $f_{ZIX}^B = f_{ZXX}^B f_{ZII}^B / f_{ZXI}^B$ where the fidelities $f_{ZXI}^B, f_{ZII}^B, f_{IIX}^R$ are all learnable. As a result, it has been shown that multi-layer CB on a length-three open chains allows to determine the product of two unlearnable quantities involving the central qubit.

The above approach and result can be generalized without particular difficulties to longer open chains, i.e., when qubits 1 and 2 are connected to other qubits as well as shown in Fig. 13(b-d). The main difference is that, to obtain a minimal block that can be repeated identically m times, one needs to apply the layers four times (RBRB). This is necessary to take care of the extra Z operators that appear at next-to-nearest neighbors of qubit 0. Importantly, the presence of additional qubits further away from qubit 0 has no effect, see the gray shaded areas in Fig. 13(b-d). One can thus analyze in detail which products of (unlearnable) fidelities can be determined via multi-layer CB by focusing on chains of four and five qubits. In particular, as discussed in further detail below:

- for the four-qubit chains depicted in Fig. 13(c-d), we can measure the product $f_{IXXI}^R f_{ZXXZ}^B f_{IXXZ}^R f_{ZXXI}^B$, which is equivalent to $f_{IXII}^R f_{ZIXI}^B$;
- for the five-qubit chain depicted in Fig. 13(b), we can measure the product $f_{IXXII}^R f_{ZXXIZ}^B f_{IXXZZ}^R f_{ZXXZI}^B$ which is equivalent to $f_{IXIII}^R f_{ZIXII}^B$.

With this, the analysis can turn to the closed chains. Importantly, closed chains featuring six or more qubits can be readily studied using the result for open chains with five or more qubits. This is because, as discussed above, the effects of weight-2 Pauli operators are spatially limited. One is thus left with the closed chains of shorter length depicted in Fig. 14.

Closed chains involving only two qubits are straightforward to analyze since multi-layer CB provides us directly with the product of unlearnable fidelities $f_{IX}^R f_{ZX}^B$, see Fig. 14(a).

The situation is more complicated for closed chains consisting of four qubits. In this case, one has again to apply the layers four times (RBRB) to obtain a minimal block that can be repeated, see Fig. 14(b). This allows us to measure the product $f_{IXXX}^R f_{ZXYX}^B f_{IYXX}^R f_{ZYXY}^B$, which is equivalent to the simple product of unlearnable fidelities $f_{IXII}^R f_{ZIXI}^B$, see further below for details. While the above analysis is sufficient to cover all the possibilities and allows to assess the general performance of multi-layer CB with two layers, it can be useful to discuss some slightly different approaches to the analysis of closed chains of length four.

In particular, the same product $f_{IXII}^R f_{ZIXI}^B$ can also be determined by interleaving the layers with S gates, as depicted in Fig. 14(c), thus working with minimal blocks consisting only of two layers. This alternative approach offers potential advantages, namely shallower circuits, and potential disadvantages, namely the position of the S gates is not universal but depends on the one of qubit 0, meaning that different instances of multi-layer CB are needed. Finally, the compact scheme depicted in Fig. 14(d) allows the measurement of

the product $f_{IXXI}^R f_{ZXXZ}^B$, which is equivalent to $f_{IXII}^R f_{ZIXI}^B f_{IIIXI}^R f_{IXIZ}^B$. While this constraint may be seen less strict compared to the one obtained with the approaches in Fig. 14(b), it can still be useful.

Based on the detailed analysis above, it can be concluded that multi-layer CB provides additional SPAM-robust constraints on unlearnable fidelities. In particular, one of these extra constraints can be obtained for each qubit that is active in both layers. Importantly, this comes with small and fixed run time increase, since it is required only one instance of multi-layer CB (with state preparation and measurement in the $X^{\otimes N}$ basis) in addition to the $11L=22$ standard (and interleaved) CB instances. The overhead in this example is thus less than 5%.

For circuits featuring the highest possible density of CZ, in the asymptotic limit $N \rightarrow \infty$ basically all the qubits belong to both layers. This means that the total number of unlearnable quantities with state-of-the-art approach, which is $2N$, can be reduced by 50% down to N .

The above considerations have been made for two consecutive layers, however, as explained below the concepts of the present invention are not limited thereto but can readily be expanded to multiple layers.

If one defines S_i as the set of layers for which the i -th qubit is active, the size of such a set is typically $t_i > 0$, otherwise that qubit would be always unentangled with the rest of the quantum processing unit and therefore useless. Further, $t_i \leq L$, with L being the total number of layers. Since two different layers can actually share a CZ gate, t_i can be larger than the connectivity of the i -th qubit.

Using the conventional approach, the i -th qubit is associated with t_i unlearnable fidelities. These can be labelled as $f_{\chi(l,j)}^l$, where $l \in S^i$ are the involved layers and the Pauli strings $\chi(l,j)$ consists of identities everywhere apart from an X operator

associated with the qubit which is connected to the i -th one via layer l .

Note that those unlearnable quantities can also be related to other fidelities, namely $f_{\vec{\chi}(l,j)}^l$, where the Pauli strings $\vec{\chi}(l,j)$ consist of identities everywhere apart from an X operator associated with the qubit which is connected to the j -th one via layer l and to a Z operator on the j -th qubit. For each pair (A, B) of layers in L^i , one can apply the results from the previous discussion, that is we can learn a product of two unlearnable fidelities, one associated with layer A and one with layer B.

Further, by using the equivalence relation detailed further below, one can write this product in terms of the unlearnable fidelities identified before, that is, as $f_{\vec{\chi}(A,j)}^A f_{\vec{\chi}(B,j)}^B$. Therefore, by performing multi-layer CB on $t_i - 1$ pairs of layers, one can find $t_i - 1$ extra SPAM-robust constraints, thus effectively reducing the number of unlearnable quantities all the way to 1. In general, the number of unlearnable quantities with the state-of-the-art approach is $\sum_{i=1}^N t_i$, which can be reduced to N using multi-layer CB. The overhead in terms of extra CB instances depends on how many different layer pairs are needed.

Following the above discussion how a multi-layer process may provide a more accurate noise characterization, now further details of a concrete example will be discussed, illustrating this process. This example corresponds to the example discussed in connection with Fig. 11 and will be discussed in connection with Fig. 15.

In this example, a quantum processing unit with 16 qubits, arranged in a 4×4 grid will be considered, see also Fig. 15(a). The quantum circuit executes all possible CZ gates allowed by the topology with the highest parallelization possible. In this scenario, the number of different two-qubit layers is then equal to the maximum connectivity of the qubits, i.e., four. The four different layers of parallel CZ gates are shown in black, dark grey, light grey and dashed grey.

Applying the general results from the previous discussion leads to the following. The total number parameters in the SPL model is given by $K(n = 16, c = 24) = 264$. The black and grey layer feature $m = 8$ CZ gates, while light grey and dashed grey layers feature only $m = 4$ CZ gates. In these cases, after performing conventional approach, i.e., standard and interleaved CB, for a total of 11 CB instances, we are left with 16 resp. 8 unlearnable quantities and overall, a grand total of 44 CB experiments needs to be performed and 48 unlearnable quantities needs to be determined in a non-SPAM-robust alternative way if one is restricted to conventional approaches.

If, on the other hand, multi-layer processes, e.g., multi-layer CB, is used, the following improvements can be achieved.

The four qubits in the bulk belong to the four layers, while the four qubits on the vertexes belong only two layers. Further, the remaining eight qubits on the edges belong to three layers. Thus, by implementing multi-layer CB, we can thus determine a total of $4 \cdot 3 + 4 \cdot 1 + 8 \cdot 2 = 32$ additional constraints in a SPAM-robust way, which corresponds to a 67% reduction of the number of unlearnable quantities. This figure approaches 75% for larger numbers of qubits, when the majority of them lay in the bulk and thus the majority of qubits belongs to all four layers.

Further, there are several different ways to determine these constrains. A straightforward way is to recognize the presence of square structures determined by pairs of layers, e.g., black-dark grey, black-light grey, grey-dashed grey and dashed grey-light grey, see Fig. 15 (b-e) respectively.

By performing multi-layer CB focused on these square structures, see also Fig. 14(b) or Fig. 14(c) for the details, it is thus possible to measure all the required constraints with only 4 different CB experiments. Moreover, in the present case, due to the structure of the overall problem considered here, the analysis of the dashed grey-light grey pair is redundant and can be skipped, reducing the number of addition CB instances down to three.

Alternatively, one could focus on horizontal and vertical lines defined by black-dashed grey and dark grey-light grey pairs as shown in Fig. 15(f) and Fig. 15(g), respectively, and only on squared defined by the black-dark grey layers. Also in this case, with only three additional CB experiments, all the 32 additional SPAM-robust constraints can be obtained. The overhead in run-time is thus $3/44$, that is, below 7%.

Having shown that multi-layer CB allows to extract more SPAM-robust information out of the quantum processing unit to be characterized, the discussion will now focus on the improvement on the noise characterization by means of numerical simulations structured as follows.

1. For each layer, the values of the 264 model parameters are randomly generated according to rules that aim at mimicking the behavior of a real device. Specifically, the model parameters are generated according to normal distributions centered around realistic values based on characterizations of IQM's QPUs and what is available from, for example, IBM literature, taking into account the weight of the Pauli jump operator as well as the status (active/idling) of the involved qubit(s).
2. It is noted that while this approach is understood to produce reasonable values, it is limited in so far as it does not reproduce potential correlations between the different parameters present in the real device. Once the noise model is established, it is straightforward to compute the associated Pauli fidelities (and their products) in line with the above discussion.
3. The results of standard/interleaved CB experiments are simulated by adding, to the ideal values of the (product of) fidelities, a statistical uncertainty with standard deviation σ_{CB} . The unlearnable fidelities are simulated in a similar manner, however, because they can only be accessed with low-accuracy procedures, a larger statistical error $\sigma_{Un} > \sigma_{CB}$ is assumed and the presence of an offset ($|o_{Un}| \geq 0$) is considered as well.

4. As discussed above, there are generally different ways to implement multi-layer CB by using different pairs of layers and therefore different structures. Thus, an effective outcome of the multi-layer CB procedure was considered. Ideally, one can determine products of unlearnable fidelities of the form $f_{\chi(Aj)}^A f_{\chi(Bj)}^B$ but, to actually determine the numerical values of these products, it is generally necessary to combine different individual CB measurements, thus these values are associated with a higher uncertainty than normal CB measurement. We take this fact into account by considering a larger measurement uncertainty σ_{ML} on these products. Also here, it is no be noted that this simple approach is reasonable, but it may not capture possible correlations between the different terms which might be present in a real scenario.
5. Knowing all the outcomes of standard/interleaved CB, multi-layer CB and unlearnable fidelities measurements, all with finite accuracy, the noise characterization, that is, the SPL model, can be reconstructed and compared to the correct one, i.e., the "original". The L1 norm is used to quantify the disagreement between the two models.

A distribution of the 264 randomly generated parameter is shown in Fig. 16, while Fig. 17 shows the difference between correct model parameters and reconstructed ones using the conventional approach (individual bars) and the approach according to the present invention (connected solid line). Parameters used for these simulations are $\sigma_{CB} = 5 \cdot 10^{-5}$, $\sigma_{ML} = 4 \cdot 10^{-4}$, $\sigma_{Un} = 1 \cdot 10^{-3}$, and $\sigma_{Un} = 0$. From Fig. 17 it can be seen that the difference between the correct model parameters and the ones reconstructed using the present invention is overall smaller, indicating that the addition information provided by the multi-layer process indeed increases the accuracy of the resulting noise characterization. Indeed, by summing the errors up, one obtains a L1 mismatch of $3.84 \cdot 10^{-2}$ using the conventional art approach and a L1 mismatch of $2.34 \cdot 10^{-2}$ using the increased learnability due to multi-layer CB. This specific scenario thus corresponds to a 40% increase in accuracy.

It is important to stress that, in the reconstruction of the noise model, the multi-layer CB results can be used in different ways. One approach is to feed these additional constraints to the non-negative least-square (nnLS) fit used to derive noise parameters, either leaving or removing the entries associated with now unnecessary unlearnable quantities.

Another approach is to first use the high-accuracy constraints from multi-layer CB to refine/reduce the set of unlearnable quantities, via standard (possibly analytical) least-square optimization. This allows us to effectively increase the accuracy of the unlearnable quantities. This is then followed by a standard approach based on nnLS, using these refined/reduce quantities as input. This approach scales better with the number of layers.

As discussed in more detail above in connection with Fig. 11, these simulations are repeated for 50 different noise models, also considering different accuracy levels for σ_{CB} and σ_{Un} while keeping $\sigma_{ML} = 8\sigma_{CB}$ and further considering two values of the offset, namely $\sigma_{Un} = 0$, and $\sigma_{Un} = \sigma_{Un}/2$. For each instance, the L1 mismatch with the correct error model is computed and the conventional solution is compared the solution provided by the present invention. As discussed above, it can be observed that multi-layer CB can increase the accuracy of the characterization up to 40 - 50%, provided that $\sigma_{ML} < \sigma_{Un}$. This inequality can be understood as roughly meaning that the accuracy of additional measurements is better than the accuracy of the non-SPAM-robust measurements; it is reasonable to assume that this holds in most, if not all relevant realistic experimental scenarios, in particular for bigger quantum processing units.

In the following, a summary of the complete noise characterization procedure will be given, providing also further details which steps are carried out by which elements of the system.

A. Hardware measurements

1. Standard CB

- Identifying the layers of parallel gates whose execution have to be characterized. Those layers can be either for a specific quantum circuit for implementation of noise-aware error mitigation techniques is desired or they can be part of a quantum circuit used for characterization/benchmarking of the quantum processing unit.
- For each layer, perform at least 11 CB experiments to measure products of Pauli fidelities with high accuracy. Out of these experiments, 9 are standard executions of the CB protocol while (at least) 2 requires interleaving the layer with extra S gates.
- Given the limitations of CB, a subset of (unlearnable) fidelities needs to be measured/determined with alternative low-accuracy protocols. For example, by direct measurement or by assuming symmetries among the different fidelities.

2. Multi-layer CB

The following steps are added/are part of multi-layer CB and added on top of single-layer CB.

- Determining the achievable performance gain associated with multi-layer CB. The maximum number of extra constraints that can be measured is given by $\sum_j (t_j - 1)$ as discussed above.
- Analyzing the structures (open and closed chains) that emerges when pairs of layers are considered. Selecting a minimal number of these structure that allows to determine the desired constraints.
- Performing one multi-layer CB experiment (initializing and measuring the qubits in the $X^{\otimes N}$ basis) for each relevant pairs of layers. Within each individual experiment, the procedures discussed above in Fig. 13 and 14 for each type of open/closed chain are followed, wherein it is noted that multiple chains can be studied in parallel.

B. Data analysis and model fitting

- Using the high-fidelity data from multi-layer CB to improve the quality of the fidelities associated with unlearnable quantities, which can be executed different ways and may be problem dependent.
- Combine the high-fidelity data obtained from standard/interleaved and multi-layer CB with the (refined/reduced) unlearnable fidelities and fit the noise model.

Now, we discuss the equivalent between various fidelities within the SPL model. For the most generic noise model consisting of 4^N parameters, assuming Markovian noise and Pauli operators as the Lindblad jump operators, all the Pauli fidelities are clearly independent from each other.

However, if an effective noise model like the SPL model, which features only a small polynomial number of parameters, is considered, not all the Pauli fidelities are independent from each other. Hence, it can happen that the value of a specific Pauli fidelity is completely determined by the knowledge of a set of other fidelities. The same applies to product of fidelities. Moreover, there is a strategy to determine whether this is the case.

Given a product of fidelities $F^{(i)} = \prod_{j \in S_i} f_j$, where S_i is a set of Pauli strings, the SPL model allows to derive its value from the model parameters $\vec{\lambda} = (\lambda_1, \lambda_2, \dots)$ via Eq. (12), that is

$$-\log(F^{(i)})/2 = -\log\left(\prod_{j \in S_i} f_j\right)/2 = M^{(i)}(\vec{\lambda}) \quad (19)$$

Here, $M^{(i)} = \sum_{j \in S_i} M_j$ and each M_j is a single-row binary array based on the symplectic inner product between Pauli operators $M_j = (\langle j, 1 \rangle_{sp}, \dots, \langle j, k \rangle_{sp})$, defined in Eq. (11). Given a set of products of fidelities $\{F^{(i)}\}$, we can then determine whether they are all

independent or not by computing the rank of the matrix obtained by stacking together all the single-row matrices $M(i)$. Only if such a matrix is full rank, then all the products of fidelities are independent. This simple and general procedure is useful in order to establish the possible equivalence between different unlearnable quantities. Once it is known that two (products) of fidelities are equivalent, it can be useful to derive an explicit relation between them. In general, such a relation is not unique.

Claims

1. A method of determining a noise characterization of a quantum processing unit configurable to execute at least a part of a quantum circuit comprising at least two consecutive layers of quantum gates, wherein the noise characterization is based on a predetermined number of parameters, the method comprising the steps of:
 - performing a first process based on a single layer of the part of the quantum circuit for obtaining a first set of fidelities and/or products of fidelities,
 - performing a second process based on at least two consecutive layers of the part of the quantum circuit for obtaining a second set of fidelities and/or products of fidelities, and
 - determining the predetermined number of parameters based on at least the first set and the second set.
2. The method according to claim 1, wherein the first process is performed for at least two of the at least two consecutive layers, thereby obtaining two first sets of fidelities and/or products of fidelities, and wherein the predetermined number of parameters is determined based on at least the two first sets and the second set,

and/or

wherein the predetermined number of parameters is polynomial in the number of qubits of the quantum processing unit.
3. The method according to claim 1 or 2, wherein the noise characterization is based on an effective noise model, preferably a sparse Lindblad-Pauli model,

and/or

wherein the fidelities are Pauli fidelities.

4. The method according to any one of claims 1 to 3, further comprising a step of estimating a third set of fidelities that comprises fidelities that cannot be determined from the first set and the second set, wherein determining the predetermined number of parameters is based on at least the first set, the second set and the third set,

wherein preferably estimating the third set of fidelities includes applying symmetries among the fidelities of the first and/or second set and/or performing a process using a unit depth quantum circuit.

5. The method according to any one of claims 1 to 4, wherein the first process comprises performing single-layer cycle benchmarking,

wherein preferably the step of performing the single-layer cycle benchmarking includes

- preparing qubits of the quantum processing unit to be in first initial states,
- repeatedly executing the gates of the single layer,
- executing randomized single qubit Pauli gates on the qubits before each execution of the gates of the single layer,
- executing randomized single qubit Pauli gates on the qubits after a last execution of the gates of the single layer, and
- measuring the qubits.

6. The method according to any one of claims 1 to 5, wherein the second process comprises performing multi-layer cycle benchmarking,

wherein preferably the step of performing the multi-layer cycle benchmarking includes

- preparing qubits of the quantum processing unit to be in second initial states,
 - repeatedly executing the gates of the at least two consecutive layers, wherein the gates of the at least two consecutive layers are alternated,
 - executing randomized single qubit Pauli gates on the qubits before each execution of the gates of one of the at least two consecutive layers,
 - executing randomized single qubit Pauli gates on the qubits after a last execution of the gates of the at least two consecutive layers, and
 - measuring the qubits.
7. The method according to claim 5 or 6, when dependent on claim 5, wherein single qubit Clifford gates are interleaved into the randomized single qubit Pauli gates.
8. The method according to any one of claims 1 to 7, when dependent on claims 5 and 6, wherein performing the second process further includes
- identifying minimal structures, that are part of the at least consecutive two layers, that contribute to the noise characterization beyond the single-layer cycle benchmarking, and
 - limiting the multi-layer cycle benchmarking to the identified minimal structures.
9. The method according to any one of claims 1 to 8, wherein performing the second process further includes

- separating the at least two consecutive layers into one or more chains of quantum gates, and
- performing the second process for each chain of quantum gates,

and/or

wherein the second process is performed in parallel for at least two of the one or more chains of quantum gates,

and/or

wherein the one or more chains of quantum gates comprise one or more open chains of quantum gates and/or one or more closed chains of quantum gates,

and/or

wherein chains with a size exceeding a predetermined threshold are processed, when performing the second process, as open chains with a size corresponding to the predetermined threshold.

10. The method according to any one of claims 1 to 9, wherein two-qubit gates of the quantum circuit are Clifford gates.
11. The method according to any one of claims 1 to 10, when dependent on claim 5, wherein determining the noise characterization includes
 - reducing the third set of fidelities based on the first set of fidelities and/or the second set of fidelities, and
 - determining the predetermined number of parameters using the reduced third set of fidelities, the first set of Pauli fidelities and the second set of Pauli fidelities.

or

- refining the third set of fidelities based on the first set of fidelities and/or the second set of fidelities, and
 - determining the predetermined number of parameters using the refined third set of fidelities and the first set of Pauli fidelities and/or products of Pauli fidelities.
12. A method of executing a quantum circuit on a quantum processing unit, wherein executing the quantum circuit includes steps of error mitigation and/or error correction using a noise characterization of the quantum processing unit obtained by a method according to any one of claims 1 to 11.
13. A quantum computer having a quantum processing unit, the quantum computer configured to implement the method according to any one of claims 1 to 12.
14. A classical computer having a classical processing unit, the classical computer configured to implement the method according to any one of claims 1 to 12.
15. A system of a quantum computer according to claims 13 and a classical computer according to claim 14.