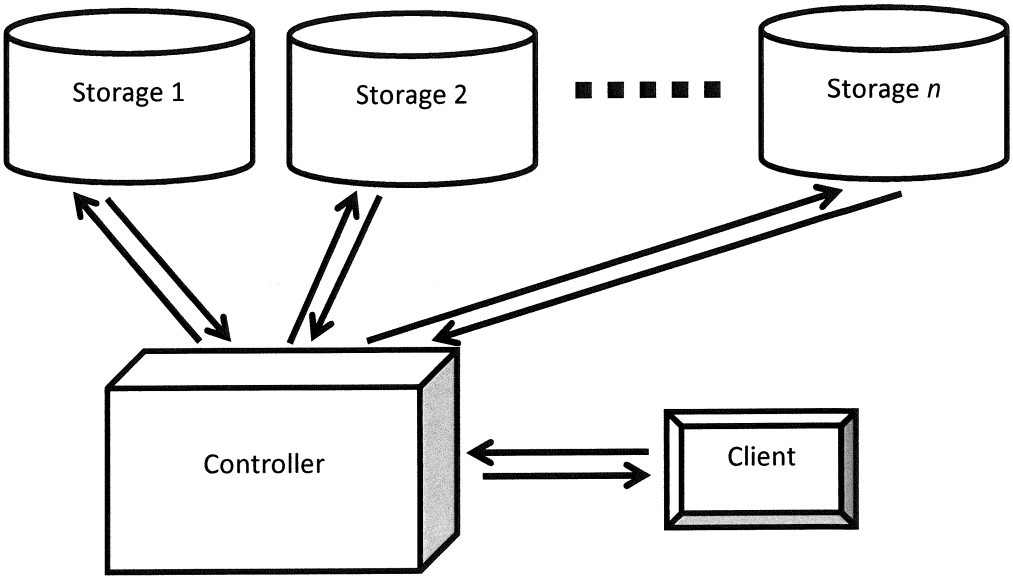
Storage 1
Storage 2
Storage n
Controller
Client

FIG. 1A

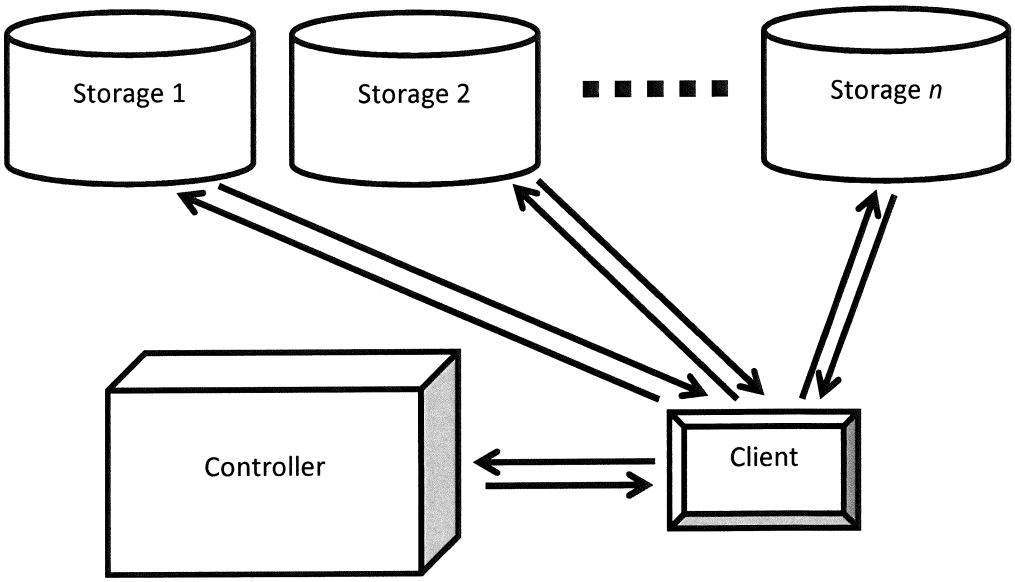
Storage 1
Storage 2
Storage n
Controller
Client

FIG. 1B

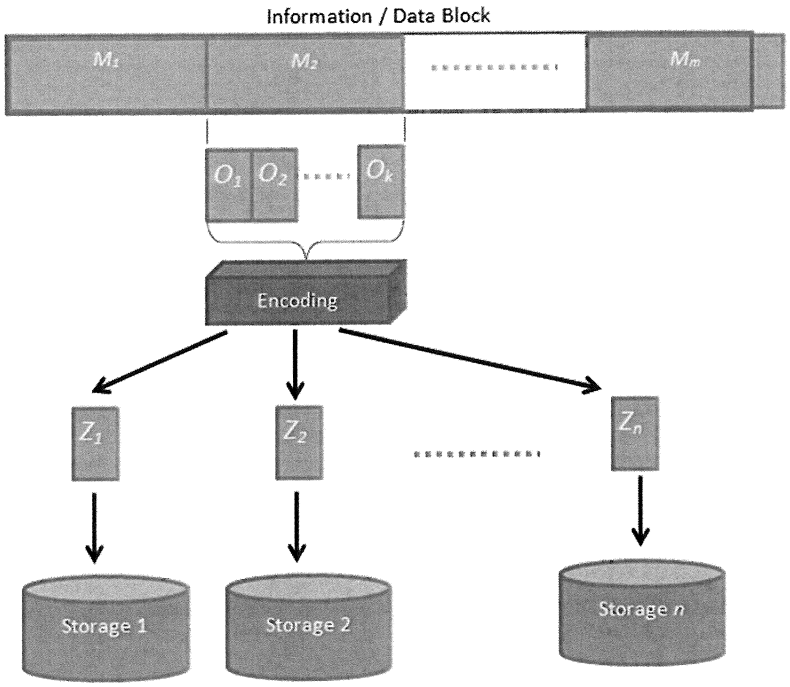
Information / Data Block
M_1
M_2
M_m
O_1 O_2 O_k
Encoding
Z_1
Z_2
Z_n
Storage 1
Storage 2
Storage n

FIG. 2

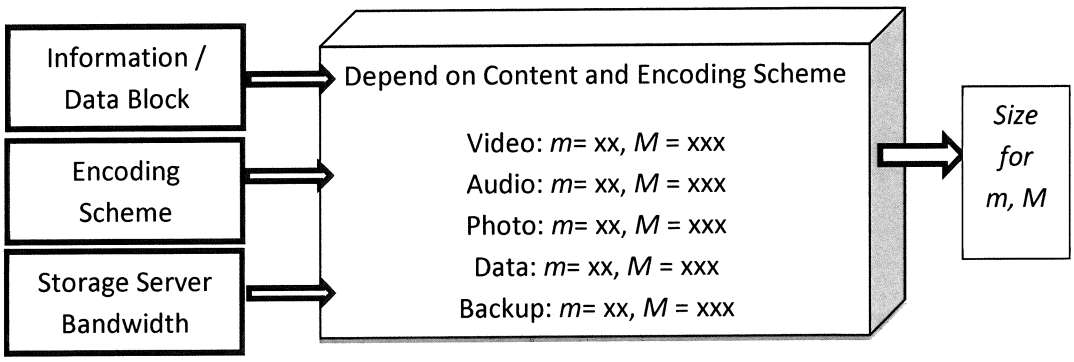
Information / Data Block
Encoding Scheme
Storage Server Bandwidth
Depend on Content and Encoding Scheme
Video: m= xx, M = xxx
Audio: m= xx, M = xxx
Photo: m= xx, M = xxx
Data: m= xx, M = xxx
Backup: m= xx, M = xxx
Size for m, M

FIG. 3

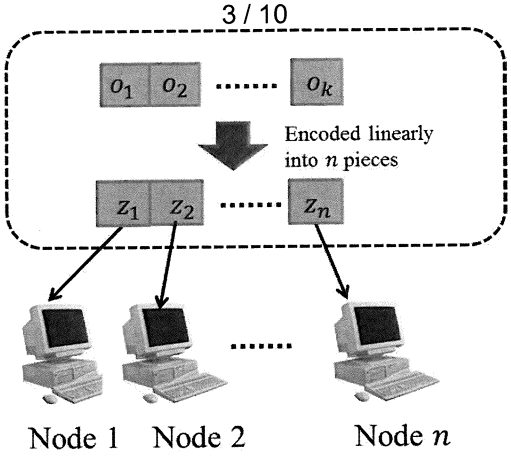
o_1 o_2 ..... o_k
Encoded linearly into n pieces
z_1 z_2 ..... z_n
Node 1
Node 2
Node n

FIG. 4

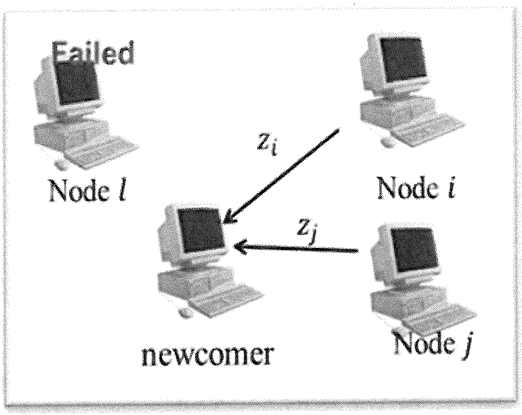
Failed
Node l
z_i
Node i
z_j
newcomer
Node j

FIG. 5

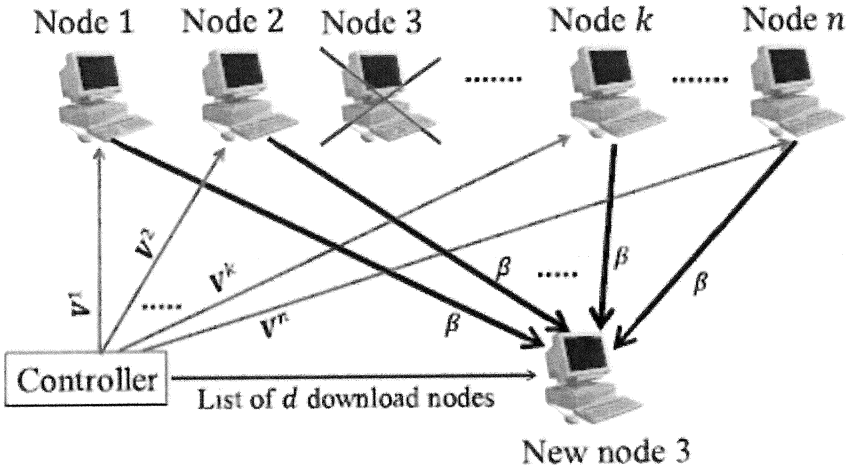
Node 1
Node 2
Node 3
Node k
Node n
V^1
V^2
V^k
V^n
β
Controller
List of d download nodes
New node 3

FIG. 6

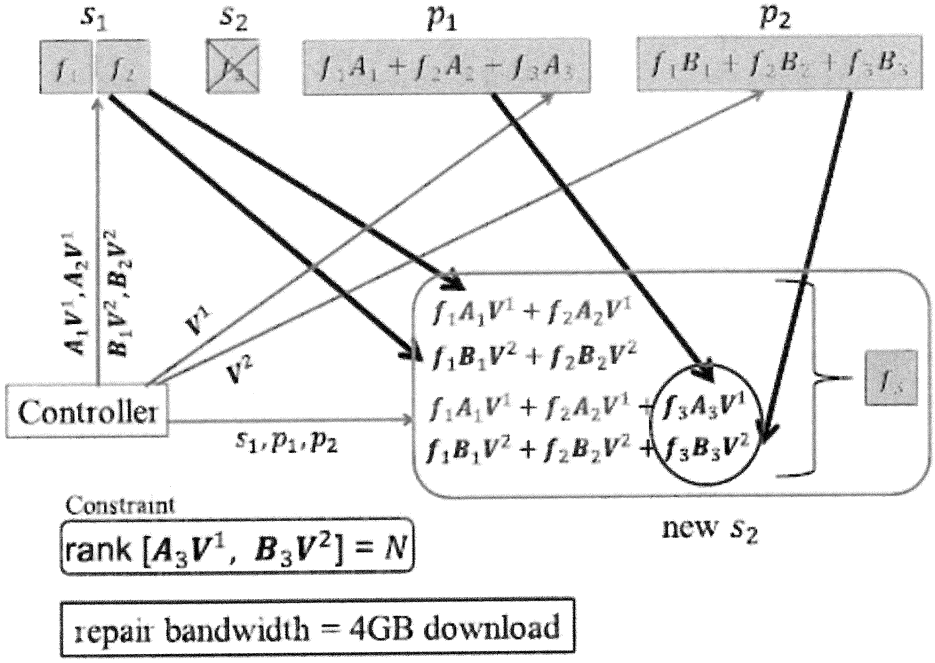
s_1
s_2
p_1
p_2
$f_1A_1 + f_2A_2 - f_3A_3$
$f_1B_1 + f_2B_2 + f_3B_3$
A_1V^1, A_2V^1
B_1V^2, B_2V^2
V^1
V^2
Controller
s_1, p_1, p_2
$f_1A_1V^1 + f_2A_2V^1$
$f_1B_1V^2 + f_2B_2V^2$
$f_1A_1V^1 + f_2A_2V^1 + f_3A_3V^1$
$f_1B_1V^2 + f_2B_2V^2 + f_3B_3V^2$
f_3
new s_2
Constraint
rank $[A_3V^1, B_3V^2] = N$
repair bandwidth = 4GB download

FIG. 7

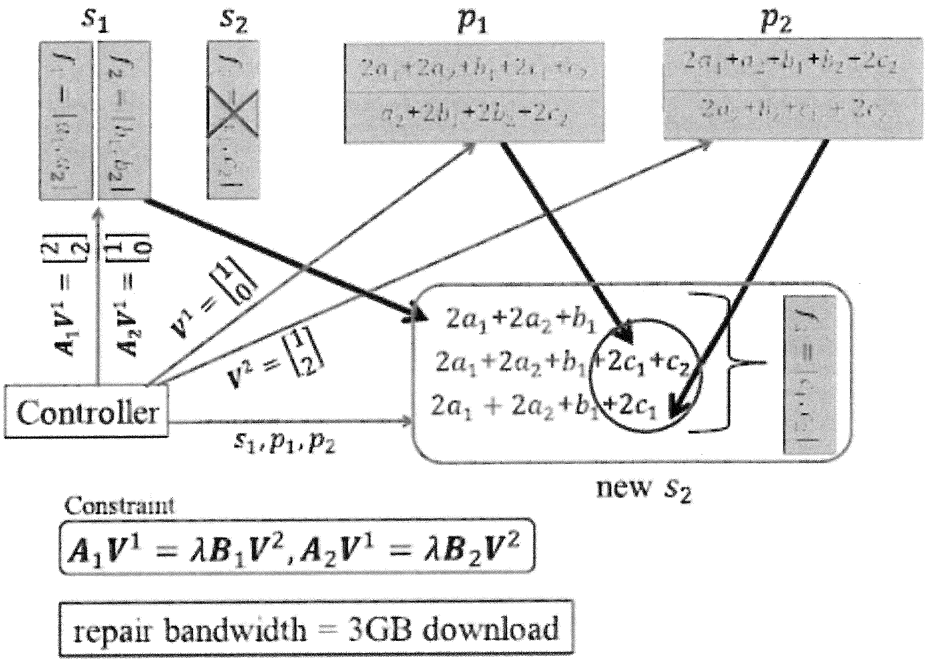
s_1
s_2
p_1
p_2
$A_1V^1 = \begin{bmatrix}2\\2\end{bmatrix}$
$A_2V^1 = \begin{bmatrix}1\\0\end{bmatrix}$
$V^1 = \begin{bmatrix}1\\0\end{bmatrix}$
$V^2 = \begin{bmatrix}1\\2\end{bmatrix}$
$2a_1+2a_2+b_1$
$2a_1+2a_2+b_1+2c_1+c_2$
$2a_1+2a_2+b_1+2c_1$
Controller
s_1, p_1, p_2
new s_2
Constraint
$A_1V^1 = \lambda B_1V^2, A_2V^1 = \lambda B_2V^2$
repair bandwidth = 3GB download

FIG. 8

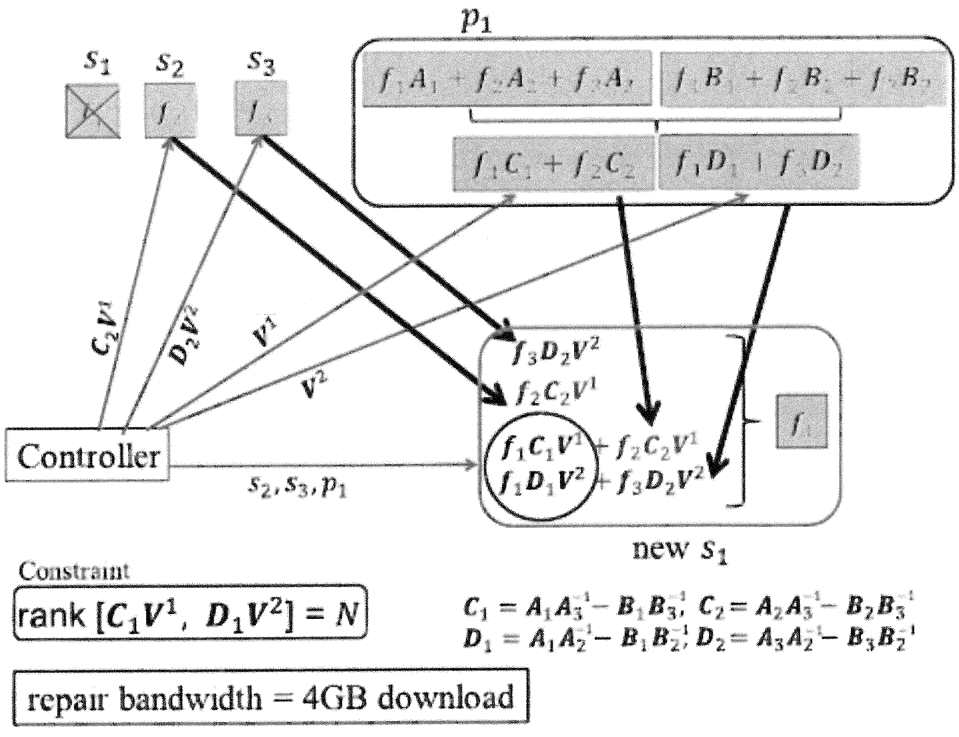
p_1
s_1
s_2
s_3
C_2V^1
D_2V^2
V^1
V^2
$f_3D_2V^2$
$f_2C_2V^1$
$f_1C_1V^1 + f_2C_2V^1$
$f_1D_1V^2 + f_3D_2V^2$
Controller
s_2, s_3, p_1
new s_1
Constraint
rank $[C_1V^1, D_1V^2] = N$
$C_1 = A_1A_3^{-1} - B_1B_3^{-1}$; $C_2 = A_2A_3^{-1} - B_2B_3^{-1}$
$D_1 = A_1A_2^{-1} - B_1B_2^{-1}$, $D_2 = A_3A_2^{-1} - B_3B_2^{-1}$
repair bandwidth = 4GB download

FIG. 9

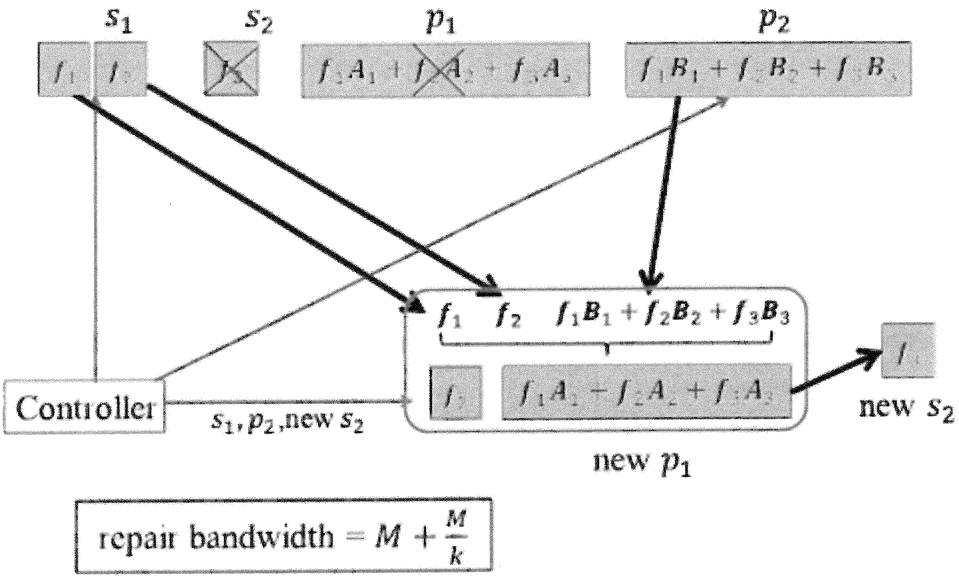

FIG. 10A

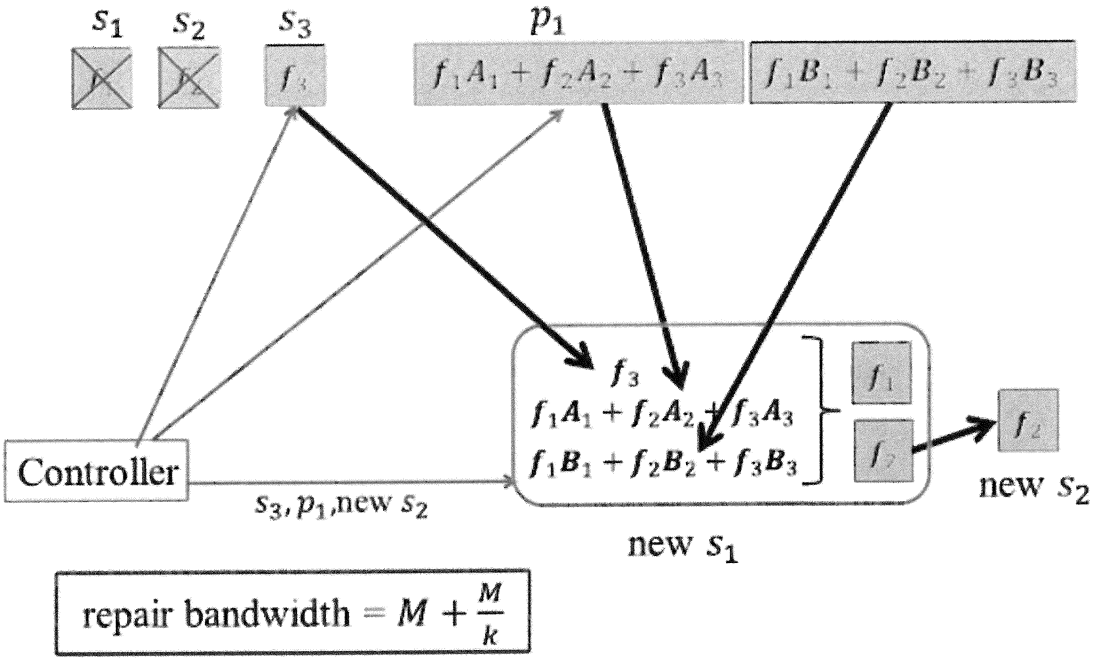

FIG. 10B

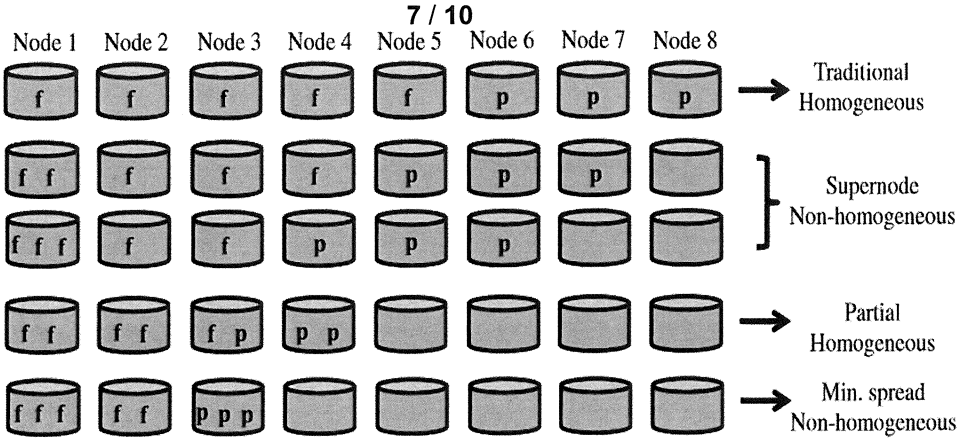
Node 1
Node 2
Node 3
Node 4
Node 5
Node 6
Node 7
Node 8
Traditional
Homogeneous
Supernode
Non-homogeneous
Partial
Homogeneous
Min. spread
Non-homogeneous

FIG. 11

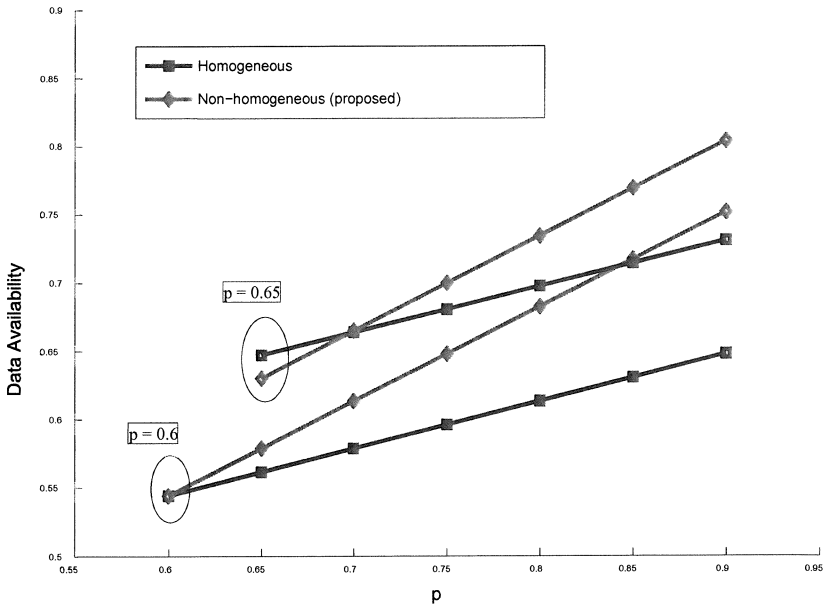
Homogeneous
Non-homogeneous (proposed)
p = 0.65
p = 0.6
Data Availability
p

FIG. 12

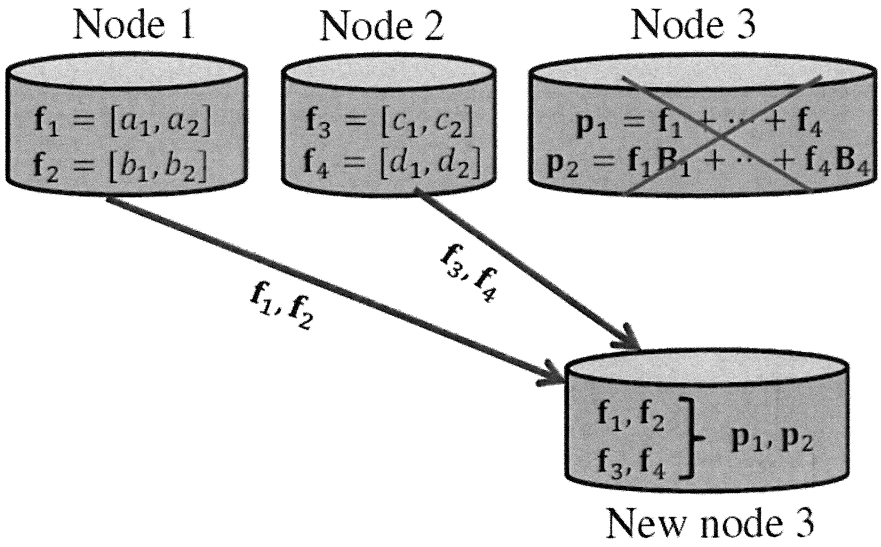
Node 1
Node 2
Node 3
$\mathbf{f}_1 = [a_1, a_2]$
$\mathbf{f}_2 = [b_1, b_2]$
$\mathbf{f}_3 = [c_1, c_2]$
$\mathbf{f}_4 = [d_1, d_2]$
$\mathbf{p}_1 = \mathbf{f}_1 + \cdots + \mathbf{f}_4$
$\mathbf{p}_2 = \mathbf{f}_1\mathbf{B}_1 + \cdots + \mathbf{f}_4\mathbf{B}_4$
$\mathbf{f}_1, \mathbf{f}_2$
$\mathbf{f}_3, \mathbf{f}_4$
$\mathbf{f}_1, \mathbf{f}_2$
$\mathbf{f}_3, \mathbf{f}_4$
$\mathbf{p}_1, \mathbf{p}_2$
New node 3

FIG. 13

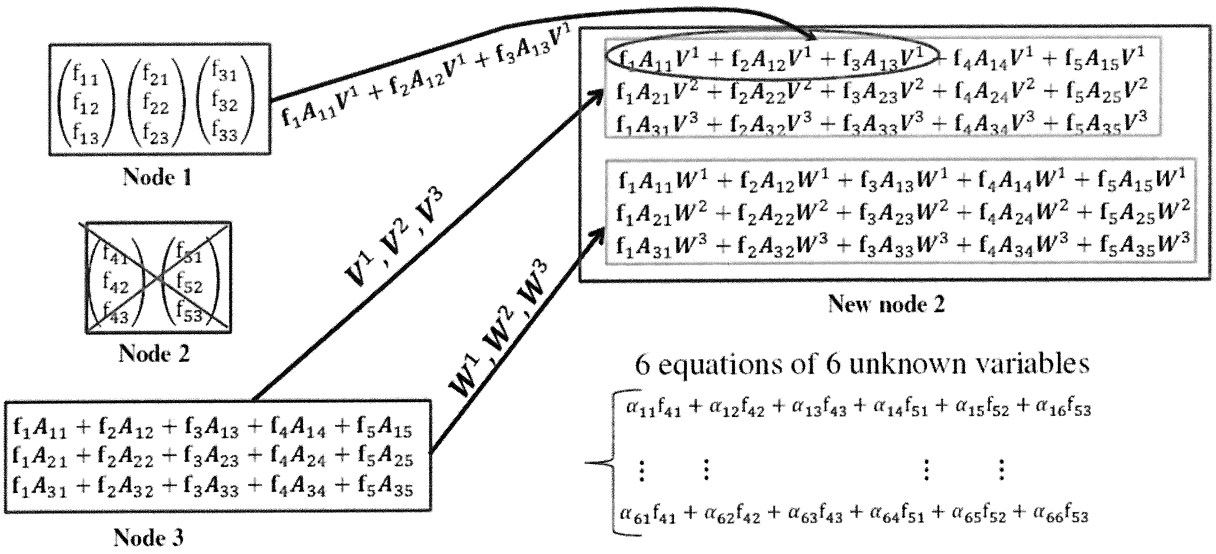
Node 1
Node 2
Node 3
New node 2
$\mathbf{f}_1A_{11}V^1 + \mathbf{f}_2A_{12}V^1 + \mathbf{f}_3A_{13}V^1$
V^1, V^2, V^3
W^1, W^2, W^3
6 equations of 6 unknown variables
$\alpha_{11}\mathbf{f}_{41} + \alpha_{12}\mathbf{f}_{42} + \alpha_{13}\mathbf{f}_{43} + \alpha_{14}\mathbf{f}_{51} + \alpha_{15}\mathbf{f}_{52} + \alpha_{16}\mathbf{f}_{53}$
$\alpha_{61}\mathbf{f}_{41} + \alpha_{62}\mathbf{f}_{42} + \alpha_{63}\mathbf{f}_{43} + \alpha_{64}\mathbf{f}_{51} + \alpha_{65}\mathbf{f}_{52} + \alpha_{66}\mathbf{f}_{53}$

FIG. 14

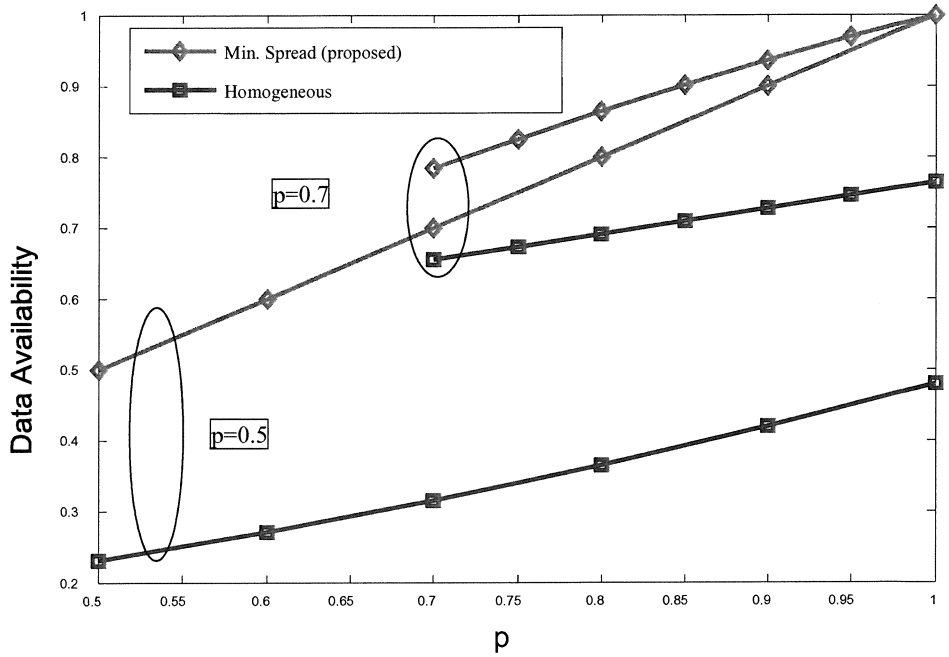
Min. Spread (proposed)
Homogeneous
p=0.7
p=0.5
Data Availability
p
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1

FIG. 15

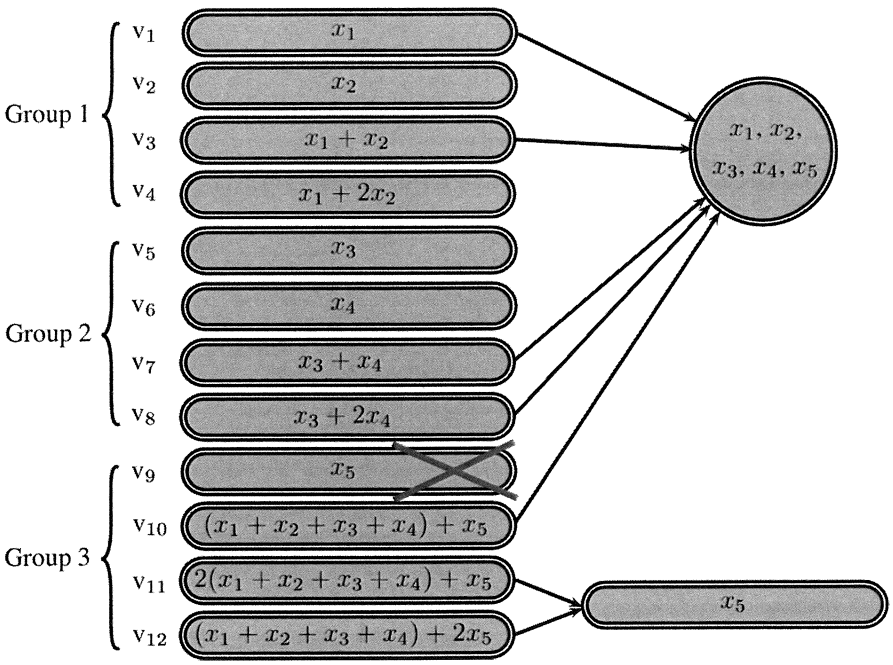
Group 1
Group 2
Group 3
v1
v2
v3
v4
v5
v6
v7
v8
v9
v10
v11
v12
x_1
x_2
$x_1 + x_2$
$x_1 + 2x_2$
x_3
x_4
$x_3 + x_4$
$x_3 + 2x_4$
x_5
$(x_1 + x_2 + x_3 + x_4) + x_5$
$2(x_1 + x_2 + x_3 + x_4) + x_5$
$(x_1 + x_2 + x_3 + x_4) + 2x_5$
x_1, x_2, x_3, x_4, x_5
x_5

FIG. 16

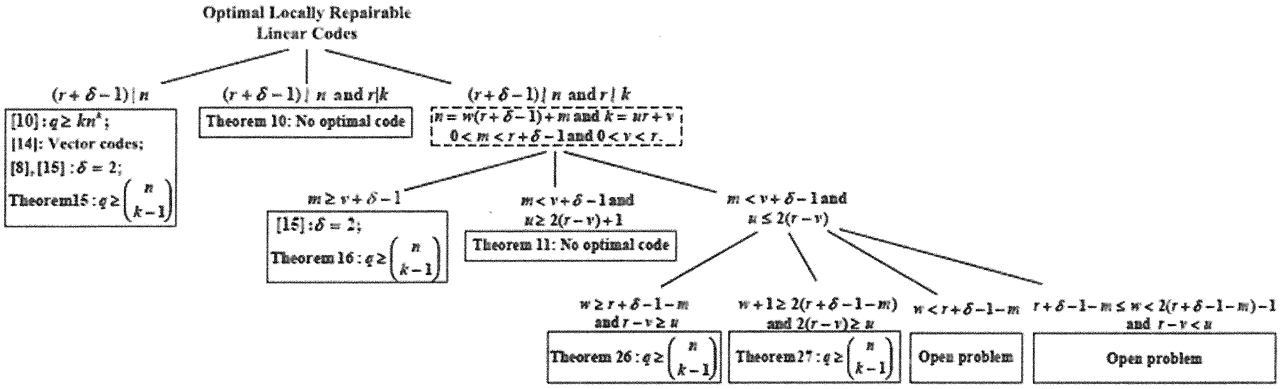
Optimal Locally Repairable Linear Codes
$(r+\delta-1)\mid n$
[10] : $q \geq km^k$;
[14]: Vector codes;
[8], [15] : $\delta = 2$;
Theorem15 : $q \geq \binom{n}{k-1}$
$(r+\delta-1)\mid n$ and $r\mid k$
Theorem 10: No optimal code
$(r+\delta-1)\nmid n$ and $r\nmid k$
$n = w(r+\delta-1)+m$ and $k = ur+v$
$0 < m < r+\delta-1$ and $0 < v < r$.
$m \geq v+\delta-1$
[15] : $\delta = 2$;
Theorem 16 : $q \geq \binom{n}{k-1}$
$m < v+\delta-1$ and $u \geq 2(r-v)+1$
Theorem 11: No optimal code
$m < v+\delta-1$ and $u \leq 2(r-v)$
$w \geq r+\delta-1-m$ and $r-v \geq u$
Theorem 26 : $q \geq \binom{n}{k-1}$
$w+1 \geq 2(r+\delta-1-m)$ and $2(r-v) \geq u$
Theorem 27 : $q \geq \binom{n}{k-1}$
$w < r+\delta-1-m$
Open problem
$r+\delta-1-m \leq w < 2(r+\delta-1-m)-1$ and $r-v < u$
Open problem

FIG. 17

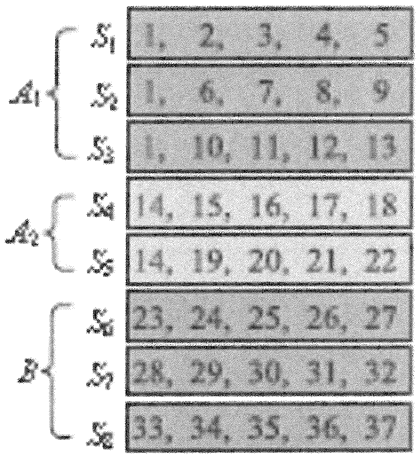
A_1
S_1 1, 2, 3, 4, 5
S_2 1, 6, 7, 8, 9
S_3 1, 10, 11, 12, 13
A_2
S_4 14, 15, 16, 17, 18
S_5 14, 19, 20, 21, 22
B
S_6 23, 24, 25, 26, 27
S_7 28, 29, 30, 31, 32
S_8 33, 34, 35, 36, 37

FIG. 18