

LOGARITHM/INVERSE-LOGARITHM CONVERTER UTILIZING
SECOND-ORDER TERM AND METHOD OF USING SAME

Related Inventions

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The present invention is related to the following inventions which are assigned to the same assignee as the present invention:

(1) "Logarithm/Inverse-Logarithm Converter and Method
10 of Using Same", having Serial No. 08/381,368, filed on 31 January 1995.

(2) "Logarithm/Inverse-Logarithm Converter Utilizing a Truncated Taylor Series and Method of Use Thereof", having Serial No. 08/381,167, filed on 31 January 1995.

15 (3) "Logarithm/Inverse-Logarithm Converter Utilizing Linear Interpolation and Method of Using Same", having Serial No. 08/391,880, filed on 22 February 1995.

(4) "Neural Network Utilizing a Logarithmic Function and Method of Using Same", having Serial No. 08/176,601,
20 filed on 03 January 1994.

(5) "Computer Processor Utilizing Logarithmic Conversion and Method of Use Thereof", having Serial No. 08/403,158, filed on 13 March 1995.

25 The subject matter of the above-identified related inventions is hereby incorporated by reference into the disclosure of this invention.

Technical Field

30 The present invention relates generally to computing circuits and, in particular, to a computing circuit which may be used to perform either a logarithmic or inverse-logarithmic conversion on an input value.

Background of the Invention

Logarithmic converters are devices used to implement logarithmic functions and are most commonly found in items
5 such as hand-held calculators and spread sheet software programs. Logarithmic functions, or log functions, belong to a class of mathematical functions called transcendental functions which are important in wide variety applications such as data communications, control systems, chemical
10 processes, and computer simulation. A log function is abbreviated with the following mathematical equation: $y = \log_b(x)$. In this equation, x represents an input value which may be any number greater than or equal to zero; b represents a base number system; and y represents a
15 logarithm value, or log value, which corresponds to the input value x .

Inverse-log converters are devices used to implement inverse-logarithmic, or inverse-log, functions. Essentially, an inverse-log function is the reverse of a
20 log function. What is meant by "reverse" is explained in the following discussion. A log function converts an input value, x , which is in a domain of input value into a definite log value which is in a range of log values. Basically, the log function establishes a one-to-one
25 correspondence between input values in the domain and log values in the range. This correspondence is represented symbolically by $x \rightarrow y$. An inverse-log function establishes a reverse correspondence between log values and input values which is represented by $y \rightarrow x$. An
30 inverse-log function is abbreviated with the following mathematical equation: $y = \log_b^{-1}(x)$. In this equation, x represents an input value; b represents a base number system; and y represents an inverse-log value which corresponds to the input value x and may be any number

greater than or equal to zero. Like log functions, inverse log functions are important in a wide variety applications.

Two techniques of computing log and inverse-log values are commonly used today. Both techniques are analogously
5 used to compute either log or inverse-log values; thus, for the sake of brevity, the following discussion will focus on using the techniques to compute log values, with the understanding that the techniques may be used in a like manner to compute inverse-log values.

10 The first technique involves storing a corresponding log value in a look-up table for every possible input value. This approach allows a log value to be computed relatively quickly and is practical for applications requiring limited precision and having input values within
15 a small domain. However, in many applications this technique is impractical because it requires too great a memory space to store the look-up table. For example, in a digital system using an IEEE standard 32-bit floating point number, which has a 23-bit mantissa, such a look-up table
20 would be required to store 2^{23} log values -- one for every possible mantissa value. A computer memory for storing this number of log values would be prohibitively expensive, if not impossible, to build.

The second technique of computing log values involves
25 computing a power series to approximate a log function. An example of a power series which approximates a log function is given as:

$$y = \text{Log}(1 + x) = x - x^2/2 + x^3/3 - x^4/4 + \dots \quad (1)$$

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In this example, the log function is a specific type of log function known as a natural logarithm, which is widely used in science and engineering applications. The variables in Equation 1 are defined as follows: y represents a log
35 value, and x represents an input value in the domain $-1 \leq x$

< 1. Although the technique of using a power series to approximate a log function allows a log value to be computed with a high degree of precision, it requires a large number of computer operations and therefore requires
5 a relatively long period of time to execute. In other words, this technique is generally slow and negatively affects the throughput of a computer.

In summary, there is a need for a converter which can perform either a log or an inverse-log function quickly,
10 thus allowing a computer to operate with greater throughput. Such a converter should also reduce the amount of memory space required to perform the conversions, and it should produce log or inverse-log values which have a high degree of precision.

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Brief Description of the Drawings

The invention is pointed out with particularity in the appended claims. However, other features of the invention
20 will become more apparent and the invention will be best understood by referring to the following detailed description in conjunction with the accompanying drawings in which:

FIG. 1 illustrates a logarithm/inverse-logarithm
25 converter in accordance with a preferred embodiment of the present invention.

FIG. 2 shows a graph of a logarithmic function.

FIG. 3 shows a graph of an inverse-logarithmic function.

30 FIG. 4 illustrates a flow chart diagram of a method of using the logarithm/inverse-logarithm converter shown in FIG. 1.

FIG. 5 illustrates a flow chart diagram of a method of computing and storing parameters and second-order terms
35 used in conjunction with the converter of FIG. 1

FIG. 6 illustrates one version of a computing device which incorporates an embodiment of the present invention.

FIG. 7 illustrates another version of a computing device which incorporates an embodiment of the present invention.

Detailed Description of a Preferred Embodiment

It is an advantage of the present invention to provide
10 a converter which can be used to perform either a logarithmic or inverse-logarithmic conversion, in a computer, processor or circuit, without altering the design of the converter. A further advantage of the present invention is that it provides a converter which computes
15 log and inverse-log values with a high degree of precision. In fact, a specific advantage of the present invention is that it provides a converter which can compute log and inverse-log values of an IEEE standard 32-bit floating point number, wherein the IEEE 32-bit floating point
20 representations of the log and inverse-log values have less than one LSB (least significant bit) of error in the mantissa. Yet another advantage of the present invention is that it provides a converter which requires a significantly reduced amount of memory space and may be
25 efficiently implemented in an integrated circuit which consumes little power. An additional advantage of the present invention is to provide a method of using the converter to produce either log or inverse-log values.

Generally, the present invention provides a converter
30 for computing either a log or an inverse-log functions. The converter comprises a memory and an arithmetic means. The memory stores a plurality of parameters and second-order terms, wherein the parameters and terms are calculated using a least squares method to estimate either
35 a log or an inverse-log function over a domain of input

values. The arithmetic means generates a log or an inverse-log value, depending on the parameters and terms stored in the memory, by performing arithmetic operations on an input value and ones of the parameters and second-order terms.

The converter may be easily re-configured to perform either log or inverse-log functions, or functions having a different base number system or domain of input values, by loading the memory with a different set of parameter pairs and second-order terms.

FIG. 1 illustrates a logarithm/inverse-logarithm converter in accordance with a preferred embodiment of the present invention. The converter comprises a memory 20, a multiplier 22, a first adder 24, and a second adder 26. The memory 20 stores a plurality of second-order terms in a second-order look-up table 32 and a plurality of parameter pairs. The parameter pairs and second-order terms are calculated using a least squares method to estimate either a log or an inverse-log function over an interval of input values. Each parameter pair has a zero-order parameter, which is stored in a zero-order look-up table 28, and a first-order parameter which is stored in a first-order look-up table 30.

The converter operates as follows. An input value 34 provides an address 36 which is used to retrieve a corresponding parameter pair from the zero-order and first-order look-up tables 28,30. In a preferred embodiment, the input value 34 is a binary value having a length of 23-bits, denoted by [22:0], wherein the most significant nine bits, [22:14], of the input value 24 are provided to the memory 20 as the address 36. Additionally, the zero-order and first-order parameters 42, 44 are located in the memory 20 at a common address which corresponds to the input value 34.

A term address 38 is also derived from the input value 34. The term address is used to retrieve the second-order term 46 from the second-order look-up table 32. In one embodiment of the present invention, the input value 34 is
5 a binary value having an upper portion and a lower portion, and the second-order term 46 is retrievable from the memory 20 using a first bit slice from the upper portion concatenated with a second bit slice from the lower portion.

10 Preferably, the input value 34 has a length of 23 bits, denoted by [22:0], where the upper portion includes the most significant nine bits, [22:14], and the lower portion includes the remaining 14 bits, [13:0]. The term address 38 is a nine-bit value, which comprises the most
15 significant four bits of the upper portion concatenated with the most significant five bits of the lower portion, i.e., bits [22:19][13:9] of input value 34.

The memory 20 provides as output the zero-order parameter 42, the first-order parameter 44, and the second-
20 order term 26 which correspond to the input value 34. In a preferred embodiment, the zero-order parameter 42 is a 24-bit binary value, the first-order parameter 44 is a 15-bit value, and the second-order term 46 is a 9-bit value. A total of 512 parameter pairs and 512 second-order terms are
25 stored in the memory 20.

The multiplier 22 multiplies the first-order parameter 44 by the bit slice 40 to produce a proportional term. Although the bit slice 40 may include the entire input value 34, in a preferred embodiment it includes the
30 fourteen least significant bits, [13:0], of the input value 34. The first adder 24 sums the proportional term and the second-order term 46 to produce a first sum. In turn, the second adder 26 sums the first sum and the zero-order parameter 42 to produce an output value 48. The output
35 value 48 is preferably 23 bits in length.

In one embodiment of the present invention, the converter is implemented with an integrated circuit. However, one of ordinary skill will recognize that a programmable logic array, application specific integrated circuit (ASIC), or other digital logic device, as well as software running on a processor such as a microprocessor could also be used to implement the converter.

FIG. 2 shows a graph of a logarithmic function 50 over a domain of input values. The input values are given along the x-axis and the log values are given along the y-axis. The domain of the input values is 1 to 2, and the range of the log values is 0 to 1. Several intervals within the domain are indicated with vertical dotted lines at input values 1.1, 1.2, 1.3, 1.4, and 1.5. Each interval contains data points between the dotted lines. The log function 50 uses a base number system of two.

FIG. 3 shows a graph of an inverse-logarithmic function 52 over a domain of input values. The input values are given along the x-axis and the inverse-log values are given along the y-axis. The domain of the input values is 0 to 1, and the range of the inverse-log values is 1 to 2. Intervals within the domain are indicated with vertical dotted lines at input values 0.1, 0.2, 0.3, and 0.4. In the example shown, each interval contains data points between the dotted lines. The inverse-log function 52 also uses a base two number system.

One of ordinary skill in the art will realize that the converter used for computing any log or inverse-log function; however, in a preferred embodiment of the present invention, the converter computes the log function depicted in FIG. 2 and the inverse-log function depicted in FIG. 3.

An IEEE standard 32-bit floating point number may be easily converted to either a log or an inverse-log value using the converter of FIG. 1. Values represented by IEEE floating point numbers have one sign bit, a 23-bit

mantissa, and an 8-bit exponent. Only the mantissa is provided to the converter as the input value 34. Both the sign bit and exponent bypass the converter and are later combined with the output value 48. Essentially, the converter computes either $y = \log_2(1.M)$ or $\log_2^{-1}(1.M)$, where y represents the output value and M represents the mantissa. The exponent of the floating point number is a base-two value, 2^e , where e represents the exponent. A log or an inverse-log value corresponding to the floating point number is generated by summing of the output value 48 and the exponent following conversion of the mantissa. This summation results in a value which is represented in a fixed point number format having a j -bit fractional portion and a k -bit integer portion. The integers j and k can vary, depending on the desired precision of the converter.

FIG. 4 illustrates a flow chart diagram of a method of using the logarithm/inverse-logarithm converter shown in FIG. 1. The method may be used for generating either a log value or an inverse-log value. In box 60, a plurality of parameters and a plurality of second-order terms are stored in the memory 20. The parameters are calculated using a least squares method to estimate either a log or an inverse-log function. In box 62, an input value 34 is received. Next, in box 64, parameters and a second-order term 46 which correspond to the input value 34 are retrieve from the memory 20.

In box 66, arithmetic operations are performed on the retrieved parameters, second-order term 46, and the input value 34 to produce the output value 48. The retrieved parameters include the zero-order parameter 42 and the first-order parameter 44, and the arithmetic operations are performed as follows. First, the first-order parameter 44 is multiplied by the input value 34 to produce a proportional term. In a prefer embodiment of the present invention, the first-order parameter 44 is multiplied by a

bit slice of the input value 34. Next, the proportional term and the second-order term 46 are summed to produce a first sum. Finally, the first sum and the zero-order parameter are summed to produce the output value 48.

5 FIG. 5 illustrates a flow chart diagram of a method of computing and storing parameters and second-order terms used in conjunction with the converter of FIG. 1. This method may be used for either a log or inverse-log function. In box 68, a domain of input values is selected
10 for the function. In box 70, the domain is partitioned into a plurality of intervals. In box 71, the function is approximated, over the domain of input values, with a second-order polynomial which has a plurality of parameters and a second-order term. Next, in box 72, the parameters
15 for an interval are computed using a least squares method. In box 74, a second-order term for each possible input value within the interval is computed, whereby producing one or more second-order terms for the interval. Next, in
20 box 76, the parameters and second-order term for the interval are stored in the memory 20. In decision box 76 a check is made to determine whether parameters and second-order terms are to be calculated for any additional intervals. If so, the procedure returns to box 72. If not, the procedure terminates.

25 In a preferred embodiment of the present invention, log and inverse-log functions are approximated with a second order polynomial having the form:

$$y = b_0 + b_1x + b_2x^2 \quad (2)$$

30 where y represents the log or inverse-log value, b_0 represents the zero-order parameter, b_1 represents the first-order parameter, b_2 represents a second-order parameter, x represents an input value, and b_2x^2 represents
35 the second-order term.

The domain of input values is partitioned into 512 intervals which each contain 2^{14} data points. For each interval, the parameters b_0 , b_1 , and b_2 are calculated using a second order regression on the data points. Rather
5 than storing the second-order parameter along with the zero-order and first-order parameter, a selected number of second-order terms are stored instead. The second-orders terms which are stored are empirically determined by observing the variance of the second-term over the domain
10 of input values. This eliminates the need for an addition multiplier and a squaring means to produce the second-order term. In addition, this allows second-order terms having a same value at different input values to reside at a common address in the memory.

15 FIG. 6 illustrates one version of a computing device which incorporates an embodiment of the present invention. The computing device comprises a processing unit 102 for executing a computer operation which utilizes a log/inverse-log value 106; a computer memory 100 for
20 storing a computer program which includes the computer operation; a bus 101 connecting the processing unit 102 and the computer memory 100; and a converter 108 which receives an input value 104.

The converter 108 may include the converter shown in
25 FIG. 1. In the example shown, the converter 108 includes a memory 112 and an arithmetic means 110 which is operatively coupled to the memory 112. The arithmetic means receives the input value 104 from the processing unit 102 and, in turn, generates either a log value or an inverse-log value
30 depending on the parameters stored in the memory 112.

FIG. 7 illustrates another version of a computing device which incorporates an embodiment of the present invention. The computing device comprises a processing unit 102 for executing a computer operation which utilizes
35 a log/inverse-log value 106; a computer memory 100 for

storing a computer program which includes the computer operation; a bus 101 connecting the processing unit 102 and the computer memory 100; and a converter 120 which receives an input value 104.

5 The converter 120 may include the converter shown in FIG. 1. In the example shown, the converter 120 includes a memory 124 and an arithmetic means 122 which is operatively coupled to the memory 124. The arithmetic means 122 receives the input value 104 and, in turn, generates either
10 a log value or an inverse-log value depending on the parameters stored in the memory 124.

 Thus there has been described herein a concept, as well as a preferred embodiment, of a converter which may be used to compute either log or inverse-log function.
15 Because the embodiment of the converter as herein-described utilizes a second-order polynomial to approximate a function it can perform either a log or inverse-log conversion using two adders, a single multiplier, and a look-up table which allows a significant reduction in
20 memory size. Such a converter allows both improved data throughput and computation precision with a vast reduction in the cost and complexity of implementation, whether on a semiconductor chip or in a computer program.

 While the various embodiments of the present invention
25 have been described in terms of the processing of an input value into an output value, the present invention should be construed to include circuits and methods implemented by these circuits for processing an input signal representative of the input value into an output signal
30 representative of the output value.

 While specific embodiments of the present invention have been shown and described, it will be apparent to those skilled in the art that the disclosed invention may be modified in numerous ways and may assume many embodiments

other than the preferred form specifically set out and described above.

Accordingly, it is intended by the appended claims to cover all modifications of the invention which fall within
5 the true spirit and scope of the invention.

What is claimed is:

CLAIMS

- 5 1. A logarithm converter, which comprises:
 a memory for storing a plurality of parameters and a
 plurality of second-order terms, wherein the parameters are
 calculated using a least squares method to estimate a
 logarithmic function over a domain of input values, the
10 memory providing as output ones of the parameters and a
 second-order term which correspond to an input value; and
 arithmetic means, operatively coupled to the memory,
 for generating a log value by performing arithmetic
 operations on the input value, the ones of the parameters,
15 and the second-order term.



2. The logarithm converter of claim 1, wherein the logarithmic function is estimated with a second-order polynomial.

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3. The logarithm converter of claim 2, wherein the ones of the plurality of parameters include a zero-order parameter and a first-order parameter, and the arithmetic means includes:

- 10 a multiplier for multiplying the first-order parameter by the input value to produce a proportional term;
a first adder for summing the proportional term and the second-order term to produce a first sum; and
a second adder for summing the first sum and the zero-
15 order parameter to produce the log value.

4. The logarithm converter of claim 2, wherein the ones of the plurality of parameters include a zero-order parameter and a first-order parameter, and the arithmetic
20 means includes:

- a multiplier for multiplying the first-order parameter by a bit slice of the input value to produce a proportional term;
a first adder for summing the proportional term and
25 the second-order term to produce a first sum; and
a second adder for summing the first sum and the zero-order parameter to produce the log value.

5. The logarithm converter of claim 1, wherein the
30 input value is a binary value having a plurality of bits used to retrieve from the memory the ones of the plurality of parameters and the second-order term.



6. The logarithm converter of claim 1, wherein the input value is a binary value having an upper portion and a lower portion, and the second-order term is retrievable
- 5 from the memory using a first bit slice from the upper portion concatenated with a second bit slice from the lower portion.



7. A method for generating a log value, which method comprises the following steps:

receiving an input value;

5 retrieving from a memory a plurality of parameters and a second-order term which corresponds to the input value, wherein the plurality of parameters are calculated using a least squares method to estimate a logarithmic function; and

10 performing arithmetic operations on the plurality of parameters, the second-order term, and the input value to produce the log value.



8. A logarithm converter for generating a log value corresponding to an input value, the logarithm converter comprising:

- 5 a memory for storing a plurality of second-order terms and a plurality of parameter pairs, wherein each of the parameter pairs is calculated using a least squares method to estimate a logarithmic function over an interval of input values, the memory providing as output a parameter
- 10 pair and a second-order term which correspond to the input value, wherein the parameter pair has a zero-order parameter and a first-order parameter;
- a multiplier for multiplying the first-order parameter by a bit slice of the input value to produce a proportional
- 15 term;
- a first adder for summing the proportional term and the second-order term to produce a first sum; and
- a second adder for summing the first sum and the zero-order parameter to produce the log value.

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9. A computing device, which comprises:
a processing unit for executing a computer operation
which utilizes a log value;
5 a computer memory for storing a computer program which
includes the computer operation;
a bus for connecting the processing unit and the
computer memory; and
a logarithm converter which receives an input value
10 and includes:
a memory for storing a plurality of parameters and a
plurality of second-order terms, wherein the
parameters are calculated using a least squares
method to estimate a logarithmic function over a
15 domain of input values, the memory providing as
output ones of the parameters and a second-order
term which correspond to the input value; and
arithmetic means, operatively coupled to the memory,
for generating the log value by performing
20 arithmetic operations on the input value, the
ones of the parameters, and the second-order
term.



10. An inverse-logarithm converter for generating an inverse-log value corresponding to an input value, the inverse-logarithm converter comprising:

- 5 a memory for storing a plurality of second-order terms and a plurality of parameter pairs, wherein each of the parameter pairs is calculated using a least squares method to estimate an inverse-logarithmic function over an interval of input values, the memory providing as output a
10 parameter pair and a second-order term which correspond to the input value, wherein the parameter pair has a zero-order parameter and a first-order parameter;
a multiplier for multiplying the first-order parameter by a bit slice of the input value to produce a proportional
15 term;
a first adder for summing the proportional term and the second-order term to produce a first sum; and
a second adder for summing the first sum and the zero-order parameter to produce the inverse-log value.

