signal input to port 211 may be at the resonant frequency ω_a of the resonator formed by resonators 232 and 234.

[0055] The second mode 260, shown in FIG. 2B, may be excited by providing an input pumping signal into port *b* (212) shown in FIG. 2A. The input pumping signal enters the 180° hybrid 221 at the difference port and is therefore output to both resonators 231 and 233 with each of those outputs being 180° out of phase from one another. This phase difference is illustrated in FIG. 2B, which depicts the square shape of loop 250 with the relative phases of inputs to the resonators at corresponding positions around the loop depicted using "+" and "–" symbols. As discussed above, these symbols are intended to convey relative phase, not absolute phase. In order to excite this "b-mode" of the circuit 200, the pumping signal input to port 212 may be at the resonant frequency ω_b of the resonator formed by resonators 231 and 233.

[0056] The third mode 270, shown in FIG. 2B, may be excited by providing an input pumping signal into port *c* (212) shown in FIG. 2A at the common mode frequency of the two other resonant modes (i.e., at a common mode of ω_a and ω_b). This common mode frequency will hereafter be referred to as ω_c. The input pumping signal enters the 180° hybrid 223 at the difference port and is therefore output to both the 180° hybrid 221 and the 180° hybrid 222 at their respective sum ports. The 180° hybrid 221 therefore outputs identical signals to resonators 231 and 233 whereas the 180° hybrid 222 also outputs identical signals to resonators 232 and 234, yet the signal output to resonators 231 and 233 is 180° out of phase with respect to the signal output to resonators 232 and 234 due to the function of 180° hybrid 223. This phase difference is illustrated in FIG. 2B, which shows the relative phase of the four resonators 270.

[0057] The inventors have recognized and appreciated that a circuit, such as circuit 100 shown in FIG. 1 or circuit 200 shown in FIG. 2A, may be off-resonantly pumped in order to couple together modes of the circuit to produce functionality associated with, for example, a circulator or an amplifier.

[0058] As discussed above in relation to the example of FIGs. 2A-2B, each of the three resonant modes of circuit 200 may be excited by providing a pumping input at a particular frequency (ω_a, ω_b or ω_c at ports *a*, *b* or *c*, respectively). The example embodiments discussed below illustrate how pumping of a port at a different frequency

[0066] Similarly, in the gain process shown in FIG. 3B, the interaction Hamiltonian can be written as:

$$\mathcal{H}_{int}^{G} = g_{ab}(a^{\dagger}b^{\dagger}e^{i\phi_p} + abe^{-i\phi_p}),$$

[0067] The gain of the resulting amplification process at zero detuning can then be written as $\sqrt{G} = (1 + |g_{ab}|^2/\kappa_a\kappa_b)/(1 - |g_{ab}|^2/\kappa_a\kappa_b)$. High gain is achieved for $|g_{ab}|^2 \rightarrow \kappa_a\kappa_b$.

[0068] As per the above description, it is clear that each of the three ports in the illustrative circuit 200 shown in FIG. 2A may be pumped in either or both of two ways. That is, each may be pumped according to the conversion process described above and/or pumped according to the gain process described above. Since both processes utilize pumping frequencies that would generally be different at each port, they are generally compatible with one another and could in theory be applied at the same time. FIG. 4 illustrates various ways in which the circuit of FIG. 2A may be pumped to produce combinations of conversion and gain functions, according to some embodiments.

[0069] The inventors have recognized and appreciated that a circuit such as circuit 200 may be operated using a suitable combination of the above-described conversion operations to cause the circuit to function as a circulator. While, as described above, when the circuit is excited as a converter between ports the energy can flow in both directions between those ports (thereby perhaps creating a device that resembles a gyrator with a frequency translation more than a circulator), the inventors have recognized that by appropriately tuning the phases and frequencies of pump inputs to the circuit, the device may nonetheless function as a reversible circulator. This pumping configuration with respect to the illustrative circuit of FIG. 2A will now be described.

[0070] FIGs. 5A-5C illustrate operation of the circuit of FIG. 2A as a reversible circulator, according to some embodiments. The circuit 200 shown in FIG. 2 may be operated as a circulator by pumping each of the three ports in the "conversion mode" described above – that is, at a frequency equal to the difference between the resonant frequencies of the other two ports ($\omega_a^p = |\omega_b - \omega_c|$, $\omega_b^p = |\omega_c - \omega_a|$, and $\omega_c^p = |\omega_b - \omega_a|$), and the three pumping signals at ports a, b and c have phases ϕ_a, ϕ_b and ϕ_c,

respectively. Specifically, the circuit 200 will act as a circulator if $\phi_{tot}^{circ} = \phi_a + \phi_b - \phi_c = \pm\pi/2$ or odd-integer multiples thereof.

[0071] In the ideal case, the circulator 500 uses three conversion processes that would each individually achieve full conversion ($C = 1$). Positive or negative interference occurs as signals travel around the device, their phase being controlled by ϕ_{tot}^{circ}, which is believed to act here like a gauge flux, playing the role of the magnetic field in a conventional circulator. According to some embodiments, the amplitude (power) of the three pumping signals of FIG. 5A are equal, or substantially equal, to one another.

[0072] As shown in Fig. 5B, for $\phi_{tot}^{circ} = +\pi/2$, a matched circulator with clockwise circulation is created. In general, any odd-integer multiple of $\pi/2$ will also produce the same result (i.e., $3\pi/2$, $5\pi/2$, etc.). Similarly, as shown in Fig. 5C, for $\phi_{tot}^{circ} = -\pi/2$, a matched circulator with counter-clockwise circulation is created. In general, any odd-integer multiple of $-\pi/2$ will also produce the same result (i.e., $-3\pi/2$, $-5\pi/2$, etc.).

[0073] It is noted that that this circulator is somewhat different from a ferrite-based circulator in that it translates the frequencies of signals passing through it, but at least in some implementations this may be of little practical consequence since the center frequency of signals can be shifted without degrading their information content.

[0074] As one non-limiting example, ω_a = 9.18 GHz, ω_b = 5.25 GHz, ω_c = 7.17 GHz, and the circulator may be operated by pumping at ω_a^p = 1.92 GHz, ω_b^p = 2.01 GHz, and ω_c^p = 3.93 GHz such that the phases of the three input pumping signals combine such that $\phi_{tot}^{circ} = \phi_a + \phi_b - \phi_c = \pm n\pi/2$ where n is an odd integer. The direction of the circulator may be switched simply by changing the phase of one or more of the pumping signals (e.g., if $\phi_a = \phi_b = \pi/4$ and $\phi_c = 0$, thereby producing a clockwise circulator, ϕ_c may be adjusted to be equal to $-\pi$, thus adjusting the direction of the circulator to counter-clockwise simply by changing the phase of only one of the pumping inputs.

[0075] FIGs. 6A-6D illustrate operation of the circuit of FIG. 2A as a directional amplifier, according to some embodiments. The circuit 200 shown in FIG. 2A may be

CLAIMS

1. A circuit, comprising:

a plurality of Josephson junctions arranged in series in a loop;

at least one magnetic element producing magnetic flux through the loop;

a plurality of superconducting resonators, each resonator coupled to the loop between a different neighboring pair of Josephson junctions of the plurality of Josephson junctions;

a plurality of ports, each port coupled to at least one of the plurality of resonators at ends of the resonators opposite to ends at which the resonators are coupled to the loop; and

at least one controller configured to provide input energy to each of the plurality of ports that causes the circuit to function as a circulator between the plurality of ports.

2. The circuit of claim 1, wherein the plurality of superconducting resonators are each capacitively coupled to one or more of the plurality of ports.

3. The circuit of claim 1, wherein the plurality of Josephson junctions are positioned equidistantly around the loop.

4. The circuit of claim 1, wherein the input energy comprises pumping signals provided to each of the plurality of ports, the pumping signals having different frequencies at each of the plurality of ports.

5. The circuit of claim 1,

wherein the input energy comprises a first pumping signal input to a first port of the plurality of ports, a second pumping signal input to a second port of the plurality of ports, and a third pumping signal input to a third port of the plurality of ports,

wherein the first port is associated with a first resonant mode of the circuit having a first resonance frequency,

wherein the second port is associated with a second resonant mode of the circuit having a second resonance frequency, and

wherein the third port is associated with a third resonant mode of the circuit having a third resonance frequency.

6. The circuit of claim 5, wherein the first pumping signal has a first primary frequency equal to a difference between the second resonance frequency and the third resonance frequency.

7. The circuit of claim 6, wherein the second pumping signal has a second primary frequency equal to a difference between the first resonance frequency and the third resonance frequency, and wherein the third pumping signal has a third primary frequency equal to a difference between the first resonance frequency and the second resonance frequency.

8. The circuit of claim 5, wherein, in a first operating mode, the circulator functions in a first direction between the plurality of ports, and wherein a phase of the first pumping signal plus a phase of the second pumping signal minus a phase of the third pumping signal is equal to an odd integer multiple of $\pi/2$.

9. The circuit of claim 8, wherein, in a second operating mode, the phase of the first pumping signal plus the phase of the second pumping signal minus the phase of the third pumping signal is equal to an odd integer multiple of $-\pi/2$, and wherein the circulator functions in a second direction between the plurality of ports, opposite to the first direction of the first operating mode.

10. The circuit of claim 1, wherein the input energy is first input energy, and wherein the at least one controller is further configured to provide second input energy to each of the plurality of ports, different from the first input energy, that causes the circuit to function as a directional amplifier.

11. The circuit of claim 10, wherein the second input energy comprises a first pumping signal input to a first port of the plurality of ports, a second pumping signal input to a second port of the plurality of ports, and a third pumping signal input to a third port of the plurality of

ports,

wherein the first port is associated with a first resonant mode of the circuit having a first resonance frequency,

wherein the second port is associated with a second resonant mode of the circuit having a second resonance frequency,

wherein the third port is associated with a third resonant mode of the circuit having a third resonance frequency, and

wherein the first pumping signal has a first primary frequency equal to a sum of the second resonance frequency and the third resonance frequency.

12. The circuit of claim 11,

wherein the second pumping signal has a second primary frequency equal to a difference between the first resonance frequency and the third resonance frequency, and

wherein the third pumping signal has a third primary frequency equal to a sum of the first resonance frequency and the second resonance frequency.

13. A method of operating a circuit, the circuit comprising a plurality of Josephson junctions arranged in series in a loop, at least one magnetic element producing magnetic flux through the loop, a plurality of superconducting resonators, each resonator coupled to the loop between a different neighboring pair of Josephson junctions of the plurality of Josephson junctions, a plurality of ports, each port coupled to at least one of the plurality of resonators at ends of the resonators opposite to ends at which the resonators are coupled to the loop, the method comprising:

providing a distinct input pumping signal to each of the plurality of ports to cause the circuit to function as a circulator between the plurality of ports.

14. The method of claim 13, wherein each input pumping signal has a different frequency.

15. The method of claim 13, wherein a first input pumping signal is input to a first port of the plurality of ports, a second input pumping signal is input to a second port of the plurality of ports, and a third input pumping signal is input to a third port of the plurality of ports,

wherein the first port is associated with a first resonant mode of the circuit having a first resonance frequency,

wherein the second port is associated with a second resonant mode of the circuit having a second resonance frequency,

wherein the third port is associated with a third resonant mode of the circuit having a third resonance frequency, and

wherein the first input pumping signal has a first primary frequency equal to a difference between the second resonance frequency and the third resonance frequency.

16. The method of claim 15, wherein the second input pumping signal has a second primary frequency equal to a difference between the first resonance frequency and the third resonance frequency, and wherein the third input pumping signal has a third primary frequency equal to a difference between the first resonance frequency and the second resonance frequency.

17. The method of claim 15, wherein, in a first operating mode, the circulator functions in a first direction between the plurality of ports, and wherein a phase of the first input pumping signal plus a phase of the second input pumping signal minus a phase of the third input pumping signal is equal to an odd integer multiple of $\pi/2$.

18. The method of claim 17, wherein, in a second operating mode, the phase of the first input pumping signal plus the phase of the second input pumping signal minus the phase of the third input pumping signal is equal to an odd integer multiple of $-\pi/2$, and wherein the circulator functions in a second direction between the plurality of ports, opposite to the first direction of the first operating mode.