

- 1 -

Method of additively manufacturing a minimal surface structure

FIELD OF THE INVENTION

The present invention relates to a method of additively
5 manufacturing a minimal surface structure of a three-
dimensional article and a minimal surface structure
additively manufactured by said method.

BACKGROUND OF THE INVENTION

10 Additive manufacturing is a manufacturing technology where
articles are made from digital three-dimensional models by
building the article through typically layer-by-layer
addition of material. The additive manufacturing processes
provide a significantly increased freedom of design and
15 allow to produce highly complex shapes and geometries,
compared to conventional e.g. subtractive manufacturing
processes. A prerequisite for producing any three-
dimensional article by additive manufacturing is a digital
three-dimensional model or a computer-aided design file
20 according to which the article can be additively
manufactured by a 3D printer.

Current additive manufacturing processes rely for filling
the interior of hollowed-out objects on tubular scaffolds
or, in elaborate cases, on triply periodic minimal surface
25 (TPMS) infill structures. The generation of triply
periodic minimal surfaces without self-intersections using

- 2 -

a pair of periodic skeleton graphs was described in the NASA technical report NASA TN D-5541 of Alan H. Schoen. A self-intersection-free triply periodic minimal surface generated from a pair of skeleton graphs divides the space into two disjoint labyrinthine regions. According to Schoen, the triply periodic minimal surface can conceptually be described to be generated by simultaneously inflating tubular neighborhoods around the skeletal graphs, where the triply periodic minimal surface emerges when the two inflationary regions collide.

Minimal surfaces, such as TPMS, allow for advantageous force flow and distribution of loads. In case of conventional periodic minimal surfaces, however, the high symmetry due to the periodicity leads to preferred directions in the structure which reduces overall response to physical requirements such as stress or strain due to the preferred directions given by the symmetries. Further, conventional periodic minimal surfaces such as TPMS exhibit poor ability to adapt to bounding geometry or requirements like specific boundary conditions.

SUMMARY OF THE INVENTION

It is therefore an object of the invention to provide a method of additively manufacturing a minimal surface structure of a three-dimensional article and a minimal surface structure additively manufactured by said method which at least partially improve the prior art and avoid at least part of the disadvantages of the prior art.

- 3 -

According to the present invention, this object is achieved by the features of the independent claims. In addition, further advantageous embodiments follow from the dependent claims and the description as well as the
5 figures.

According to an aspect of the invention, the object is particularly achieved by a method of additively manufacturing a minimal surface structure of a three-dimensional article, the method comprising a computer
10 executing the steps of: recording in the computer an envelope of the three-dimensional article; generating a density field across a volume enclosed by the envelope with densities of the density field corresponding to local requirement values of at least one physical parameter at
15 respective positions of the three-dimensional article; generating an adaptive Voronoi tessellation of the volume using the density field; generating a first skeleton graph associated with the adaptive Voronoi tessellation; generating a second skeleton graph associated with the
20 first skeleton graph; generating a digital minimal surface model from the first and second skeleton graphs; wherein the method further comprises a 3D printer additively manufacturing the minimal surface structure according to the digital minimal surface model.

25 The envelope of the three-dimensional article may be an article envelope representing the outer boundary of the article. In order to reduce processing power when generating the digital minimal surface model, a so-called density field envelope representing a simplified article
30 envelope with a simplified geometry may be used as the

- 4 -

envelope of the three-dimensional article. Preferably, the article envelope is fully contained within the density field envelope. For example, a density field envelope in the shape of a polygonal prism and enclosing an article
5 envelope in the shape of a cylinder may be used. In a further example, a density field envelope in the shape of a n -polygonal prism and enclosing an article envelope in the shape of a m -polygonal prism, where $m > n$, may be used.

The adaptive Voronoi tessellation described in the context
10 of the present invention shall be understood as a 3D Voronoi tessellation using three-dimensional Voronoi cells. The term "adaptive" indicates that the adaptive Voronoi tessellation is adaptable to the characteristics of the density field, as described herein.

15 The first and second skeleton graphs are preferably intertwining without intersecting each other. In particular, the second skeleton graph may be based on a dual graph of the first skeleton graph. Further, the second skeleton graph may be based on a dual tessellation
20 of the Voronoi tessellation, such as a 3D-Delaunay triangulation or Delaunay tetrahedralization, respectively, as described further below. The second skeleton graph may therefore be essentially dual to the first skeleton graph. The second skeleton graph, however,
25 may feature one or more corrected segments which deviate from the dual graph of the first skeleton graph in order to adapt to local topological conditions as described further below. Optionally, the first skeleton graph may feature one or more corrected segments which deviate from
30 the dual relationship to the second skeleton graph in

- 5 -

order to adapt to local topological conditions. By using the two skeleton graphs to generate the digital minimal surface model, two disjoint labyrinths, each originating from a skeleton graph, can be generated which are
5 separated by the wall of the minimal surface structure. By the generation of the two labyrinths, two channels of the minimal surface structure can be obtained. The channels may be closed by closures on peripheral openings of the channels or left open.

10 By generating the density field, a spatial mapping of local requirement values of at least one physical parameter can be obtained, as the densities of the density field at respective positions of the article correspond to the local requirement values of the at least one physical
15 parameter at said respective positions. For example, the density field can represent a spatial mapping of load case requirements parametrized by physical parameters such as stress and/or strain across the volume enclosed by the envelope. For the example of mapping the local requirement
20 value of the stress across the three-dimensional article, the densities of the density field may be proportional to the stress.

Using the density field, the adaptive Voronoi tessellation can be generated as a starting point for the generation of
25 the skeleton graphs of the digital minimal surface model, allowing to relate the spatial mapping of local requirement values of the at least one physical parameter to the digital minimal surface model and accordingly, to the additively manufactured minimal surface structure.
30 Therefore, the parametrization of requirements by local

- 6 -

requirement values of the at least one physical parameter can be translated into a parametrization of the digital minimal surface model by using the density field and the adaptive Voronoi tessellation adapted to the density
5 field. In doing so, an additively manufactured minimal surface structure can be obtained which is by design structurally adapted to physical requirements across the three-dimensional article and specific boundary conditions, respectively, owing to the generation of the
10 digital minimal surface model using the density field.

For example, a density field with densities proportional to stress requirements across the three-dimensional article may translate to skeleton graphs being denser at respective positions of the three-dimensional article with
15 increased stress values which in turn results in a minimal surface structure which is structurally denser in said positions in order to be able to withstand the higher stress values prevailing at said positions.

Hence, a so-called Adaptive Density Minimal Surface (ADMS)
20 structure can be obtained by the present method which is inherently locally adapted to input requirement parameters. The present method provides the advantage that said local adaptation to input requirement parameters can be included in a bottom-up fashion while parametrizing the
25 digital minimal surface model itself according to which the minimal surface structure is to be additively manufactured.

The three-dimensional article may comprise a shell, with the minimal surface structure forming an infill structure

- 7 -

within the shell of the three-dimensional article. The shell may coincide with the article envelope.

Alternatively, the minimal surface structure may form the three-dimensional article or part of the three-dimensional article without a shell.

In some embodiments, generating the adaptive Voronoi tessellation comprises: generating a set of scatter points corresponding to a distribution of the densities of the density field; randomly distributing the scatter points across the volume enclosed by the envelope; generating a plurality of Voronoi cells of the adaptive Voronoi tessellation using the randomly distributed scatter points as generating points of the Voronoi cells.

By generating the set of scatter points, the number of Voronoi cells of the adaptive Voronoi tessellation can be adapted to the distribution of densities in order to obtain a minimal surface structure with structural details which correspond to the contrast in the densities of the density field. For the physical parameter of stresses across the three-dimensional article for example, the distribution of stresses can be recorded in a histogram stored in the computer, where the number of scatter points is proportional to the ratio of the sum of all stresses in the bins of the histogram to the product of the largest stress and the number of bins. It is clear to the person skilled in the art that other physical parameter values can accordingly be recorded in a histogram which may be used to calculate the number of scatter points.

- 8 -

In some embodiments, the randomly distributed scatter points are redistributed according to the density field, such that the redistributed distribution of scatter points corresponds to the density field. The redistributed
5 scatter points may then be used as initial generating points for generating the Voronoi cells of the adaptive Voronoi tessellation.

In some embodiments, generating the adaptive Voronoi tessellation using the density field comprises iteratively
10 generating a plurality of Voronoi cells of the Voronoi tessellation by weighted stippling using the density field.

By weighted stippling using the density field, a set of generating points for the plurality of Voronoi cells can
15 be generated, where the position of the generating points is determined by the density values of the density field. In particular, weighted stippling using the density field typically causes regions with higher density values to contain more generating points than regions with lower
20 density values, such that the density field may weight the packing of Voronoi cells of the adaptive Voronoi tessellation. Weighting the packing of Voronoi cells by the density field therefore provides the advantage that the characteristics of the density field can be
25 transferred to structural characteristics of the minimal surface structure originating from the adaptive Voronoi tessellation. Furthermore, iteratively generating the Voronoi cells by weighted stippling allows to start with an initial, for example random, distribution of generating
30 points of a plurality of Voronoi cells and to iteratively

- 9 -

adapt the positioning of the generating points or the packing of the Voronoi cells, respectively, to the density field by repeating the weighted stippling.

In some embodiments, the iterative generation of Voronoi
5 cells by weighted stippling starts with the randomly distributed scatter points or redistributed scatter points as described above. The randomly distributed or redistributed scatter points may therefore serve as initial generating points of the Voronoi cells of the
10 adaptive Voronoi tessellation.

In some embodiments, iteratively generating the plurality of Voronoi cells comprises iterating the steps of: a) calculating the weighted centroid of each Voronoi cell using the density field and shifting generating points of
15 the Voronoi cells to the respective centroids; b) generating new Voronoi cells of the adaptive Voronoi tessellation using the shifted generating points and replacing the Voronoi cells of step a) by the new Voronoi cells; until the calculated centroids conform with the
20 generating points of the Voronoi cells in step a).

By iterating the steps of a) and b) for iteratively generating the plurality of Voronoi cells, a three-dimensional centroidal Voronoi tessellation weighted according to the density field can be achieved as adaptive
25 Voronoi tessellation. The iteration of steps a) and b) preferably ends when the calculated centroids coincide with the generating points of the Voronoi cells in step a). In some embodiments, however, a tolerance is allowed such that the iteration of steps a) and b) ends when the
30 distance between the calculated centroids and the

- 10 -

generating points of the Voronoi cells in step a) is smaller than a predetermined tolerance value. For example, the tolerance value may be 10^{-3} times the minimum wall width of the minimal surface structure.

- 5 In some embodiments, iteratively generating the plurality of Voronoi cells comprises executing the steps of: c) calculating for each Voronoi cell a cell weight by integrating the density field over the respective Voronoi cell; d) recording in the computer a first weight
10 threshold and a second weight threshold, wherein the first weight threshold is larger than the second weight threshold; e) splitting Voronoi cells with a cell weight above the first weight threshold and deleting Voronoi cells with a cell weight below the second weight
15 threshold; after step b) described above.

By executing the steps c)-e), the adaptive Voronoi tessellation can further be adapted to the density field. Furthermore, convergence of the adaptive Voronoi tessellation to a centroidal Voronoi tessellation can be
20 improved. Steps c)-e) advantageously allow to adapt the sizes of the Voronoi cells according to the density field by creating smaller Voronoi cells by splitting and larger Voronoi cells by deleting or merging neighboring Voronoi cells, respectively.

- 25 Splitting of a Voronoi cell may be achieved by randomly generating two generating points within the Voronoi cell and generating two new Voronoi cells from the two generating points.

- 11 -

Usually, Voronoi cells with a cell weight between the first and second weight threshold may be left unmodified.

In some embodiments, the first weight threshold is defined as the ratio of the integral of the density field over the volume enclosed by the envelope to the number of centroids multiplied by a factor $(1+a)$, and the second weight threshold is defined as the ratio of the integral of the density field over the volume enclosed by the envelope to the number of centroids multiplied by a factor $(1-a)$, wherein preferably a being between 0.3 and 0.7, further preferably $a=0.5$.

The steps c)-e) may be executed until the cell weights of the Voronoi cells lie between the first and second weight threshold and no splitting and/or merging of Voronoi cells is required anymore.

In some embodiments, steps c)-e) are executed for the first 5-30%, preferably 10%, of the iterations of steps a)-b).

Executing steps c)-e) after step b) provides the advantage that the convergence to a centroidal Voronoi tessellation can be improved.

In some embodiments, generating the density field comprises: dividing the volume enclosed by the envelope into a plurality of, preferably tetrahedral, primary voxels and generating at least one local requirement value for each primary voxel.

Preferably, the primary voxels are each assigned a local requirement value for a certain physical parameter. For

- 12 -

example, each primary voxel may be assigned a certain stress value. In a further example, each primary voxel may be assigned a certain stress value and a certain strain value.

- 5 The local requirement values for the primary voxels are preferably generated by a FEM simulation. The local requirement values of the primary voxels as generated by the FEM simulation may be output to a histogram stored in the computer.
- 10 In some embodiments, the following pre-processing steps are executed by the computer prior to the generating of the adaptive Voronoi tessellation: dividing the volume enclosed by the envelope into a plurality of, preferably tetrahedral, primary voxels; generating a cuboid envelope
- 15 *BBB* encompassing the envelope; calculating the volume *env_vol* enclosed by the envelope as the sum of the volumes of all primary voxels in the envelope and the volume *bbb_vol* enclosed by the cuboid envelope as the sum of the volumes of all voxels in the cuboid envelope; calculating
- 20 a maximal grid size *max_grid* as 0.5 times the sum of a maximum channel diameter *max_channel* which represents the diameter of a largest channel of the minimal surface structure and a minimum wall width *min_wall* which represents the smallest wall width of the minimal surface
- 25 structure; calculating a number of points *pts_bbb* in the cuboid volume at the maximal grid size as: $pts_bbb = (BBB_width / max_grid) \times (BBB_depth / max_grid) \times (BBB_height / max_grid)$, where *BBB_width*, *BBB_depth*, *BBB_height* denote the width, depth and height of the cuboid envelope *BBB*;
- 30 calculating a number of points *pts_env* in the envelope at

- 13 -

the maximal grid size as: $pts_env = pts_bbb \times (env_vol/bbb_vol)$.

In some embodiments, the number of scatter points pts_use is calculated as: $pts_use = pts_env \times prop_hist$, where
5 $prop_hist$ is the ratio of the sum of all values of a certain physical parameter in bins of a histogram in which the values of the physical parameter are recorded to the product of the largest value of the physical parameter and the number of bins.

10 In some embodiments, the primary voxels generated by the pre-processing steps as described above, are replaced by a plurality of secondary, preferably cubic, voxels. Typically, the number of secondary voxels is, preferably by one or more orders of magnitudes, larger than the
15 number of primary voxels. The secondary voxels are preferably generated by interpolating the density field values of the primary voxels.

In some embodiments, the number of secondary voxels is increased if after the iterations a)-b), optionally
20 including steps c)-e), a Voronoi cell contains less than 10 secondary voxels, and the generation of the adaptive Voronoi tessellation and the iterations a)-b), optionally including steps c)-e), is restarted.

In some embodiments, the first skeleton graph is formed by
25 edges of Voronoi cells of the adaptive Voronoi tessellation.

- 14 -

In some embodiments, the second skeleton graph is generated by Delaunay tetrahedralization of generating points of the adaptive Voronoi tessellation.

Preferably, the first and second skeleton graphs are
5 generated after the adaptive Voronoi tessellation has converged to a centroidal Voronoi tessellation in accordance with the density field, as described above.

In particular, two intertwining skeleton graphs can be achieved in order to generate a minimal surface structure
10 without self-intersections.

Although generated using Delaunay tetrahedralization, the second skeleton graph may comprise one or more corrected segments which do not coincide with the edges of the Delaunay tetrahedralization, in order to adapt to
15 topological conditions. For example, such a topological condition could require segments of the second skeleton graph connecting generating points of neighboring Voronoi cells to only run within said neighboring Voronoi cells. In case such a segment is to run through a third Voronoi
20 cell, an additional point may be inserted at a plane where the neighboring Voronoi cells adjoin, such that the segment may take a detour through said point avoiding crossing of the third Voronoi cell. Such corrected segments may alternatively or additionally be applied to
25 the first skeleton graph.

In some embodiments, from a segment of the first and/or second skeleton graph which traverses the envelope of the three-dimensional article, a first segment part lying outside the volume enclosed by the envelope is removed and

- 15 -

replaced by a segment part which is obtained by mirror-imaging about the envelope at the position where the segment of the first and/or second skeleton graph traverses the envelope a second segment part lying inside
5 the volume and adjoining the first segment part.

In doing so, essentially perpendicular adjoining of the minimal surface structure on the envelope of the three-dimensional article can be achieved. This is particularly advantageous for a three-dimensional article with a shell
10 where the minimal surface structure forms an infill structure, such that the minimal surface structure may meet the shell essentially perpendicularly at places where loads work on the shell.

In some embodiments, endings of the first and/or second
15 skeleton graph are modified to avoid the minimal surface structure to generate overhangs which can not be 3D printed without the use of build supports. This may be achieved by modifying the endings of the first and/or second skeleton graph such that the segments at said
20 endings are inclined towards the center of the three-dimensional article. This is particularly advantageous for three-dimensional articles without a shell where the minimal surface structure is designed to be 3D printed without the use of build supports.

25 Zero mean curvature as a feature of minimal surfaces may not be fulfilled anymore for the minimal surface structure modified at the boundary of the three-dimensional article as described herein. However, zero mean curvature may still be fulfilled for a substantial part of the minimal
30 surface structure and in the context of the present

- 16 -

invention, such a minimal surface structure shall still be considered as being based on a minimal surface.

In some embodiments, Voronoi cells which extend beyond the envelope of the three-dimensional article are trimmed at
5 the envelope and the centroids of the trimmed Voronoi cells are recalculated using the density field.

Alternatively, Voronoi cells which extend beyond the envelope may not be trimmed regardless of their extensions beyond the envelope.

10 The labyrinths or channels, respectively, may be closed by closures on peripheral openings of the channels. In some embodiments, the channels are closed by placing closures on the peripheral openings and applying smoothening by a conformalized mean curvature flow algorithm. By doing so,
15 the internal space smoothness of the channels can advantageously be maximized. In some embodiments, the channels are closed by placing closures on the peripheral openings and applying smoothening by a conformalized mean curvature flow algorithm while retaining the center of the
20 closure. By doing so, the internal volume of the channels can advantageously be maximized. In some embodiments, a V-shaped or rounded V-shaped closure is used to close the peripheral openings of the channels. This may be achieved by increasing the wall width to more than half of the
25 local channel diameter at the peripheral opening of the channel. Rounding may be achieved by a conformalized mean curvature flow algorithm. In some embodiments, the minimal surface structure is joined with the envelope or shell, respectively, at the positions where the minimal surface
30 structure impinges on the envelope or shell, respectively,

- 17 -

without applying a separate closure or modifying the shape of the minimal surface structure, such that the envelope or shell, respectively, closes the respective channels. Different closure schemes may be used for different peripheral openings of the channels. Optionally, the original minimal surface structure (without closures) may additionally be upheld up to the envelope for all closure schemes, in order to enable ideal force transmission and ensure integrability of the minimal surface structure in a higher-level assembly.

Typically, the two labyrinths may be disjoint and do not exhibit interconnections. In some embodiments, however, the wall of the minimal surface structure may exhibit one or more perforations which allow the two labyrinths to be interconnected. The one or more perforations may be achieved by generating additional linking labyrinths between the two labyrinths.

In some embodiments, the two labyrinths may be interconnected through a contact space at the envelope.

In some embodiments, the three-dimensional article may comprise one or more external piping elements arranged beyond the envelope which interconnect one or more peripheral openings of different channels or of the same channel.

In some embodiments, all peripheral openings except two peripheral openings of each channel are closed, such that the non-closed openings may each form a fluid inlet and outlet of a respective channel. In doing so, two separate media may be allowed to flow, especially counterflow,

- 18 -

through the minimal surface structure. This may be particularly advantageous for a heat exchanger or heat equalizer structure.

In some embodiments, the two channels or labyrinths, respectively, are connected by interconnecting one or more peripheral openings of different channels, wherein two peripheral openings are left open and the remaining peripheral openings are closed. The two peripheral openings which are left open are preferably arranged at opposite sides of the three-dimensional article. The two peripheral openings which are left open may serve as an inlet and outlet. Such an embodiment may be used for a crash-resistant fuel tank, for example in a helicopter, or a zero-g capable fuel tank, for example in a spacecraft. The inlet may be used to fill the tank with fuel, wherein while operating the tank, gas may be filled through the inlet to force the fuel to the outlet to exit the tank.

In some embodiments, the two labyrinths are used as storage compartments of a tank to store two components of a fuel, such as for example hydrogen and oxygen, which may be mixed upon exiting the tank. Such an embodiment may be used for example in a rocket. In some embodiments, the minimal surface structure may comprise additional labyrinths embedded inside the wall of the minimal surface structure. For example, a single additional labyrinth may be embedded inside the wall of the minimal surface structure, such that a three-chamber system may be formed. This may be particularly advantageous for heat exchanger structures with an internal coolant loop in order to speed up initial heating and/or cooling. In a further example,

- 19 -

two additional labyrinths may be embedded inside the wall of the minimal surface structure, such that a four-chamber system may be formed. This may be particularly advantageous for heat exchanger structures with two
5 counter-circulating internal coolant loops to speed up initial heating and/or cooling.

In some embodiments with a three or four-chamber system as described above, the two chambers created by the original labyrinths are larger than the one or two chambers which
10 are embedded inside the wall of the minimal surface structure. For cryogenic fuels the larger chambers may comprise one fuel component or two fuel components, and the smaller chambers may comprise a coolant loop. Such an embodiment of a three or four-chamber system may serve as
15 a fuel tank of a rocket, which may be used as a load-bearing structure of the rocket. The load-bearing structure may be enclosed with an ultralightweight envelope or skin which may serve to reduce aerodynamic resistance during a rocket's ascent through the
20 atmosphere.

In some embodiments, generating the digital minimal surface model from the first and second skeleton graphs comprises: generating a minimal surface precursor from the first and second skeleton graphs; generating a minimal
25 surface shape by smoothening the minimal surface precursor; assigning a wall width to the minimal surface shape; generating the digital minimal surface model according to the minimal surface shape and the assigned wall width.

- 20 -

Preferably, the minimal surface precursor is generated as a surface being equidistant to the first and second skeleton graphs. In some embodiments, the minimal surface precursor is generated as a surface with a first distance
5 d to the first skeleton graph and a second distance s to the second skeleton graph. In some embodiments, the first distance d and/or the second distance s vary along the first and/or second skeleton graph.

The wall width may be a global wall width with a global
10 wall width value assigned to the minimal surface shape. Alternatively, the wall width may be a varying wall width which varies while being assigned along the minimal surface shape and accordingly, varies along the minimal surface structure generated according to the digital
15 minimal surface model. Therefore, different local wall width values may be assigned to the minimal surface shape at different positions of the minimal surface shape. By generating the digital minimal surface model according to the minimal surface shape and the assigned wall width, the
20 digital minimal surface model can acquire the geometry as defined by the minimal surface shape, and a wall with the assigned wall width such that the minimal surface structure can be 3D printed according to the digital minimal surface model.

25 A local wall width value may be inferred from a wall width density field. Alternatively, a local wall width value may be inferred from the density field in combination with a set of rules specifying the wall width at different positions of the minimal surface structure or the three-
30 dimensional article, respectively, wherein the set of

- 21 -

rules are defined by requirements of the minimal surface structure or the three-dimensional article, respectively.

For example, a local maximum wall width may be related to a minimum channel diameter of the minimal surface structure and be defined as an upper bound of local wall widths in the set of rules, in order to prevent the walls from closing a channel in the minimal surface structure. In a further example, a rule for the local wall width may be defined by relating the local wall width to a channel diameter in order to close one or more channels of the minimal surface structure. In a further example, a rule for the local wall width may be defined by the requirement of the amount of printed material to be constant at every cross-section of the minimal surface structure.

In order to generate the wall of the digital minimal surface model, a pair of isosurfaces may be generated which is, in some embodiments, equally spaced from the minimal surface shape in both directions facing away from the minimal surface shape. In some embodiments, the pair of isosurfaces are not equally spaced from the minimal surface shape. In some embodiments, the distance or distances of the isosurfaces to the minimal surface shape varies according to the local wall widths assigned to the minimal surface shape, such that the generated wall of the digital minimal surface model may exhibit a finite wall width according to the assigned local wall widths. The pair of isosurfaces may be joined together at the peripheral ends of the isosurfaces by generating an end face which connects the two isosurfaces.

- 22 -

After generating the digital minimal surface model, parts of the digital minimal surface model lying outside the envelope may be removed by projecting said parts back to the envelope, wherein overlapping, self-intersecting
5 and/or zero area surface parts and/or coincident points are removed. Small surface parts with an area below a threshold area may be merged.

In some embodiments, generating the digital minimal surface model from the first and second skeleton graphs
10 comprises: assigning a first electrical charge to the first skeleton graph; assigning a second electrical charge to the second skeleton graph, the second electrical charge being equal in absolute value, but opposite in sign to the first electrical charge; generating the minimal surface
15 precursor as an equipotential surface between the first and second skeleton graph using a Coulomb force field calculated based on the first and second skeleton graphs and their electrical charges.

In doing so, a minimal surface precursor can be achieved
20 which is equidistant to the first and second skeleton graphs. Generating the minimal surface precursor as an equipotential surface between the first and second skeleton graphs using a Coulomb force field provides the advantage of an efficient and processing power-saving
25 scheme to generate a minimal surface precursor being equidistant to the first and second skeleton graphs and separating the two labyrinths defined by the skeleton graphs.

A minimal surface shape may be generated by smoothening
30 for example by using a conformalized mean curvature flow

- 23 -

algorithm, as described e.g. in K. Crane, U. Pinkall, P. Schröder, ACM Transactions on Graphics, July 2013, Article No.: 61, "Robust fairing via conformal curvature flow". The resulting minimal surface shape may be analyzed for
5 the condition of zero mean curvature and the conformalized mean curvature flow algorithm may be reapplied in order to optimize the minimal surface shape with respect to the zero mean curvature condition. Alternatively, the minimal surface shape may be generated from the minimal surface
10 precursor by e.g. minimizing the squared mean curvature of the minimal surface precursor, smoothening using a Laplace operator and/or LS3 Loop subdivision. Further smoothening schemes are described in the application PCT/IB2019/054076 of the present applicant, the description of which is
15 hereby incorporated by reference.

Preferably, the at least one physical parameter is selected from at least one of: mechanical load, stress values and/or distribution, maximal allowable stress, strain, local deformation allowance, stiffness,
20 flexibility, vibration, attenuation across specified frequency range, amount of storable fluid, fluid flow, heat transport, heat transport along the minimal surface structure (within the walls), heat transport across the minimal surface structure (from the labyrinth of the first
25 skeleton graph to the labyrinth of the second skeleton graph), mass budget, mass distribution, momentum distribution, article geometry such as for example increased material density along the perimeter of the article, minimum and/or maximum channel diameter of the
30 minimal surface structure, minimum and/or maximum wall width of the minimal surface structure, enforced cross-

- 24 -

sectional area of the channels of the minimal surface structure or of the wall of the minimal surface structure at a given cross-sectional location (globally equalized across the three-dimensional article or locally
5 specified), 3D printability such as for example increased density adjacent to overhangs or under horizontal outer shells, location of the center of mass of the three-dimensional article, geometry optimization towards bone regrowth for implant applications, material resorption,
10 permeability, volume proportion between the labyrinths (which may be varied by asymmetric wall widths or asymmetric alignment of the minimal surface precursor with respect to the skeleton graphs), minimum and/or maximum voids of the labyrinths.

15 According to a further aspect, the present invention is also directed to a minimal surface structure additively manufactured by the method according to the present invention.

In an embodiment of the minimal surface structure, the
20 minimal surface structure is a quasiperiodic structure.

In an embodiment of the minimal surface structure, the minimal surface structure is an amorphous structure.

According to a further aspect, the present invention is also directed to a non-transitory computer-readable medium
25 having stored thereon computer-executable instructions adapted to cause a 3D printer to additively manufacture a minimal surface structure according to a digital minimal surface model as described herein, the computer-executable instructions comprising a processor executing the steps

- 25 -

of: recording in the computer an envelope of the three-dimensional article; generating a density field across a volume enclosed by the envelope with densities of the density field corresponding to local requirement values of
5 at least one physical parameter at respective positions of the three-dimensional article; generating an adaptive Voronoi tessellation of the volume using the density field; generating a first skeleton graph associated with the adaptive Voronoi tessellation; generating a second
10 skeleton graph associated with the first skeleton graph; generating the digital minimal surface model from the first and second skeleton graphs.

According to a further aspect, the present invention is also directed to a computer-implemented method of
15 generating a digital minimal surface model adapted for additively manufacturing a minimal surface structure according to the digital minimal surface model by a 3D printer as described herein, the computer-implemented method comprising a processor executing the steps of
20 recording in the computer an envelope of the three-dimensional article; generating a density field across a volume enclosed by the envelope with densities of the density field corresponding to local requirement values of at least one physical parameter at respective positions of
25 the three-dimensional article; generating an adaptive Voronoi tessellation of the volume using the density field; generating a first skeleton graph associated with the adaptive Voronoi tessellation; generating a second skeleton graph associated with the first skeleton graph;
30 generating the digital minimal surface model from the

- 26 -

first and second skeleton graphs; storing the digital minimal surface model on a computer-readable medium.

The computer-readable medium may be a non-transitory computer-readable medium or a data signal embodied as a
5 carrier wave.

According to a further aspect, the present invention is also directed to a computer program product comprising computer program code configured to control a computer such that the computer executes the steps of the computer-
10 implemented method according to the present invention.

LIST OF FIGURES

The present invention will be explained in more detail, by way of exemplary embodiments, with reference to the
15 schematic drawings, in which:

- Fig.1 shows a perspective view of an article envelope and a density field envelope;
- Fig.2 shows a perspective view of the density field envelope of Fig.1 and a cuboid envelope;
- 20 Fig.3 shows a perspective view of the density field envelope with an enclosed volume subdivided by a tetrahedral voxel field and the article envelope;
- Fig.4 shows a perspective view of the density field envelope of Fig.3 with a calculated density field within the density field envelope;
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- 27 -

- Fig.5 shows a perspective view of a set of scatter points randomly distributed across the volume enclosed by the article envelope;
- Fig.6 shows a perspective view of an adaptive Voronoi tessellation and a first skeleton graph generated from the adaptive Voronoi tessellation of the volume enclosed by the density field envelope using the scatter points of Fig.5;
- 10 Fig.7 shows a perspective view of a second skeleton graph generated from a Delaunay tetrahedralization of the adaptive Voronoi tessellation of Figure 6;
- Fig.8 shows a perspective view of the first and second skeleton graphs of Fig.6 and Fig.7;
- 15 Fig.9 shows a perspective view of a minimal surface precursor generated from the first and second skeleton graphs and protruding out of the article envelope;
- 20 Fig.10 shows a perspective view of a minimal surface shape obtained from a minimal surface precursor such as the one of Fig.9 and trimmed to the article envelope;
- Fig.11 shows a perspective view of a minimal surface shape such as the one of Fig.10 together with a skeleton graph;
- 25

- 28 -

- Fig.12 shows a perspective view of a digital minimal surface model obtained from the minimal surface shape of Fig.11;
- Fig.13 shows a perspective view of the digital minimal surface of Figure 12 with the second skeleton graph shown in addition;
- Fig.14 shows a perspective view of a satellite chassis with a heat exchanger or heat equalizer as a three-dimensional article comprising an embodiment of a minimal surface structure;
- Fig.15 shows a perspective view of a spinal cage as a three-dimensional article which is formed by an embodiment of a minimal surface structure;
- Figs.16a-c illustrate a sequence of steps where edges of a Delaunay tetrahedralization are corrected in order to adapt to a topological condition;
- Fig.17 shows a flow diagram illustrating an embodiment of the method of additively manufacturing a minimal surface structure of a three-dimensional article;
- Fig.18 shows a block diagram of an embodiment of the method according to the present invention;
- Fig.19 shows an example of treating segments of the skeleton graphs traversing the envelope of the three-dimensional article.

- 29 -

DESCRIPTION OF EXEMPLARY EMBODIMENTS

Figure 1 shows a perspective view of an article envelope 11 of a three-dimensional article to be additively manufactured and a density field envelope 12 enclosing the article envelope 11. The article envelope 11 has a cylindrical shape and represents the boundary of the cylindrical three-dimensional article. The density field envelope 12 is a 7-polygonal prism providing a simplification of the article envelope 11 in order to reduce processing power of the computer generating the digital minimal surface model.

Figure 2 shows a perspective view of the density field envelope 12 of Figure 1 and a cuboid envelope 13 (*BBB*) enclosing the density field envelope 12. Using the cuboid envelope 13 and a subdivision into voxels of the volumes enclosed by the density field envelope 12 and the cuboid envelope 13, a number of points *pts_bbb* in the cuboid volume at *bbb_vol* at the maximal grid size *max_grid* can be calculated as
$$pts_bbb = (BBB_width / max_grid) \times (BBB_depth / max_grid) \times (BBB_height / max_grid),$$
 where *BBB_width*, *BBB_depth*, *BBB_height* denote the width, depth and height of the cuboid envelope *BBB*. The maximal grid size *max_grid* is calculated as 0.5 times the sum of a maximum channel diameter *max_channel* which represents the diameter of a largest channel of the minimal surface structure and a minimum wall width *min_wall* which represents the smallest wall width of the minimal surface structure to be 3D printed. From this, a number of points *pts_env* in the density field envelope at the maximal grid

- 30 -

size can be calculated as: $pts_env = pts_bbb \times (env_vol/bbb_vol)$.

Figure 3 shows a perspective view of the density field envelope 12 with an enclosed volume 121 subdivided by a tetrahedral voxel field with tetrahedral primary voxels 122. The article envelope 11 of Figure 1 is shown in addition. A FEM (Finite Element Method) simulation is performed by a computer in order to generate local requirement values of a physical parameter, such as strain, of the three-dimensional article for each of the primary voxels 122.

Figure 4 shows a density field 2 generated from the local requirement values in the volume 121 enclosed by the density field envelope 12. In the present example of strain, the densities of the density field 2 are proportional of the local strain values in the primary voxels 122. The density field 2 therefore represents a spatial mapping of the local strain within the density field envelope 12. Accordingly, the densities vary across the volume 121. For example, the densities in the lower central region of the volume 121 in Figure 4 (white regions/voxels) are higher than towards the top of the volume 121 in Figure 4 (black regions/voxels), indicating that the stress decreases from the bottom towards the top of the volume 121. It can also be recognized from Figure 4 that the density is higher towards the front side of the volume 121, as indicated by the brighter voxels on the front face of the heptagonal prism of volume 121 compared to the voxels in the two neighboring faces to the left and the right (in the orientation as shown in Figure 4).

- 31 -

Figure 5 shows a perspective view of a set of scatter points 21 redistributed according to the density field 2 of Figure 4 after being randomly distributed across the volume 121 enclosed by the article envelope. The density field characteristics are further included in the calculation of the number of scatter points 21 (*pts_use*) as $pts_use = pts_env \times are_prop$, where $are_prop = area_histo / area_full$. *area_histo* and *area_full* are obtained by a histogram into which all stress values or density values obtained by the FEM simulation are recorded. *area_histo* is the sum of all stresses or densities in the histogram bins and *area_full* is the product of the largest stress in the histogram and the number of bins in the histogram. *pts_env* is the number of points in the density field envelope at the maximal grid size, as described above. The scatter points 21 serve as initial generating points of the Voronoi cells of the adaptive Voronoi tessellation.

Figure 6 shows a perspective view of an adaptive Voronoi tessellation VO and a first skeleton graph A derived from an adaptive Voronoi tessellation VO of the volume 121 enclosed by the density field envelope, wherein the adaptive Voronoi tessellation VO has been generated starting from the scatter points 21 of Figure 5 as initial generating points and by executing thereafter the steps of iteratively generating Voronoi cells of the adaptive Voronoi tessellation VO by weighted stippling as described above, using the density field of Figure 3, resulting in a weighted centroidal Voronoi tessellation VO according to the density field. The skeleton graph A runs along the edges of the Voronoi cells of the adaptive Voronoi tessellation VO. For the skeleton graph A, the iteration

- 32 -

of steps a) and b) for shifting the generating points 21 of the Voronoi cells to the centroids of the Voronoi cells using the density field, as described above, has been executed. Further, the iteration of steps c)-e), as
5 described above, for splitting and/or merging Voronoi cells using the cell weight of the Voronoi cells has also been executed. Therefore, skeleton graph A has been generated from the weighted centroidal Voronoi tessellation VO according to the density field with
10 adapted sizes of Voronoi cells according to the density field.

Figure 7 shows a perspective view of a second skeleton graph B generated from a Delaunay tetrahedralization of the centroidal Voronoi tessellation VO of Figure 6. The
15 centroids C of the Voronoi cells of Figure 6 are also shown. The second skeleton graph B is essentially dual to the first skeleton graph A. The two skeleton graphs A and B are intertwining without intersecting each other.

Figure 8 shows a perspective view of the intertwining
20 first and second skeleton graphs of Figure 6 and Figure 7.

Figure 9 shows a perspective view of a minimal surface precursor 3 generated from the first and second skeleton graphs shown in Figures 6-8 and protruding out of the article envelope 11. The minimal surface precursor 3 is a
25 surface being equidistant to the first and second skeleton graphs. The minimal surface precursor 3 is generated as an equipotential surface using the Coulomb force field calculated using assigned positive and negative electrical

- 33 -

charges to the first and second skeleton graphs, as described above.

Figure 10 shows a perspective view of a minimal surface shape 4 obtained by smoothening from a minimal surface precursor such as the one of Figure 9. The minimal surface shape 4 has been generated by using a conformalized mean curvature flow algorithm where the border of the minimal surface precursor is retained in order to prevent a shrinking of the minimal surface precursor while performing the conformalized mean curvature flow algorithm. The first and second skeleton graphs of Figure 8 and accordingly, the minimal surface precursor 3 of Figure 9 protruding out of the article envelope ensures optimal performance of the conformalized mean curvature flow algorithm also at the location of the article envelope when smoothening the minimal surface precursor 3 in order to obtain the minimal surface shape 4.

Figure 11 shows a perspective view of a minimal surface shape 3 such as the one of Figure 10. The second skeleton graph B of Figure 7 is shown in addition for illustrative purposes. The first skeleton graph has been omitted for improved representation.

Figure 12 shows a perspective view of a digital minimal surface model 5 obtained from the minimal surface shape 3 of Figure 11. The digital minimal surface model 5 comprises a wall 51 which has been generated by a pair of isosurfaces 52.1 and 52.2 which are generated to be equally spaced from the minimal surface shape 3 of Figure 10. The distance between the isosurfaces 52.1 and 52.2 is

- 34 -

equal to the wall width assigned to the minimal surface shape 3 of Figure 10 such that the wall 51 exhibits said wall width. In this example, a constant global wall width has been applied. The isosurfaces 52.1 and 52.2 are
5 connected by end faces 53. It is clear to the person skilled in the art that an illustration of an additively manufactured minimal surface structure according to the digital minimal surface model 5 would essentially look the same as the representation of the digital minimal surface
10 model 5 as shown in Figure 12. The minimal surface structure 3D printed according to the digital minimal surface model 5 may constitute the three-dimensional article. Alternatively, the three-dimensional article may comprise a shell which is arranged at the boundary of the
15 minimal surface structure 3D printed according to the digital minimal surface model 5. The shell may close channels 54. In some embodiments, the channels 54 may be closed by one of the methods described above.

Figure 13 shows a perspective view of the digital minimal
20 surface 5 of Figure 12 with the second skeleton graph B shown in addition for illustrative purposes. The first skeleton graph has been omitted for improved representation.

Figure 14 shows a perspective view of a satellite chassis
25 100 with a heat exchanger or heat equalizer as a three-dimensional article comprising an embodiment of a minimal surface structure 61 which has been additively manufactured according to the present invention. The minimal surface structure 61 of the satellite chassis 100
30 serves as a heat exchanger with a first heat transfer

- 35 -

medium FA flowing through the black pipes and the first labyrinth of the minimal surface structure 61 associated with the first skeleton graph and a second heat transfer medium FB flowing through the white pipes and the second
5 labyrinth of the minimal surface structure 61 associated with the second skeleton graph. The media FA and FB flow in opposite directions, thus providing a temperature equilibrium across the satellite chassis 100 (between sunlit and shaded areas) which allows for the reduction of
10 stress and deformation of the chassis 100 due to temperature differences.

Figure 15 shows a perspective view of a spinal cage 200 as a three-dimensional article which is formed by an embodiment of a minimal surface structure 62 additively
15 manufactured according to the present invention. The scale bar is 1 cm. Small peripheral channels 621 of the minimal surface structure 62 are optimized towards ideal bone ingrowth. Larger channels 622 serve for improving stability. The wall width of the minimal surface structure
20 is 0.4 mm or less, depending on the capability of the 3D printer. For the shown example of the spinal cage 200, the ingrowth characteristics of the bone provides a set of local requirement values for the density field. Bone ingrowth is usually supported by the insertion of own,
25 donor or artificial bone marrow into the spinal cage 200 prior to an implant surgery. For the case of a resorbable implant made of magnesium or a bioceramic like bTCP or HA, the resorption curve in connection with the mechanical load bearing capability provides a set of local
30 requirement values for the density field. For example, the peripheral regions of the minimal surface structure 62

- 36 -

where bone contact arises are required to exhibit channel diameters between 0.8 and 1.2 mm in order to optimize bone ingrowth. Furthermore, for the shown spinal cage 200 which may be made of magnesium or a bioceramic, the walls in a central region of the minimal surface structure 62 are
5 required to exhibit a sufficient wall width in order to ensure sufficient load bearing capability (for the case of bioceramic) and that over the time scale during which the bone ingrowth takes place, bioresorption removes only an
10 amount of material for which the load bearing capability of the minimal surface structure 62 is safeguarded.

Figures 16(a)-(c) illustrate a sequence of steps where edges of a Delaunay tetrahedralization are corrected in order to adapt to a topological condition which requires
15 segments of a second skeleton graph associated with the Delaunay tetrahedralization connecting generating points of neighboring Voronoi cells to only run within said neighboring Voronoi cells. For illustrative purposes, the Figures 16(a)-(c) are shown for a two-dimensional
20 configuration. The person skilled in the art recognizes that the shown correction scheme can be translated to the three-dimensional case accordingly. Figure 16(a) shows an adaptive Voronoi tessellation with Voronoi cells bounded by dotted lines with the generating points and points
25 shown which mark the corners as well as midpoints of the edges of the Voronoi cells. Figure 16(b) shows a Delaunay triangulation by connecting the generating points of the Voronoi cells by solid lines. The edges E denote edges of the Delaunay triangulation which do not satisfy the
30 topological condition and have to be corrected according to the dashed curves. Figure 16(c) shows the corrected

- 37 -

Delaunay triangulation where the edges E of Figure 16(b) have been replaced by edges making a detour and run through midpoints of edges of neighboring Voronoi cells according to the dashed curves of Figure 16(b), such that
5 the topological condition is met. The second skeleton graph obtained from the corrected Delaunay triangulation is essentially dual to the first skeleton graph obtained from the adaptive Voronoi tessellation.

Figure 17 shows a flow diagram illustrating an embodiment
10 of the method of additively manufacturing a minimal surface structure of a three-dimensional article according to the present invention. In step S1, a computer records an envelope of the three-dimensional article in the computer. In step S2, the computer generates a density
15 field across a volume enclosed by the envelope with densities of the density field corresponding to local requirement values of at least one physical parameter at respective positions of the three-dimensional article. In step S3, the computer generates an adaptive Voronoi
20 tessellation of the volume using the density field. In step S4, the computer generates a first skeleton graph associated with the adaptive Voronoi tessellation. In step S5, the computer generates a second skeleton graph associated with the first skeleton graph. In step S6, the
25 computer generates a digital minimal surface model from the first and second skeleton graphs. In step S7, a 3D printer additively manufactures the minimal surface structure according to the digital minimal surface model.

Figure 18 shows a block diagram of an embodiment of the
30 method according to the present invention. First, a

- 38 -

computer 10 with a non-transitory computer-readable medium 101, such as a storage, having stored thereon computer-executable instructions to execute a method as shown in Figure 17, generates a digital minimal surface model 5.

5 The digital minimal surface model 5 is stored as a CAD-file on a non-transitory computer-readable medium 20, such as a storage. Using the CAD-file, a 3D printer 30 prints the minimal surface structure 63 according to the digital minimal surface model 5.

10 Figure 19 shows an example of treating segments of the skeleton graphs traversing the envelope of the three-dimensional article where from a segment of the first and/or second skeleton graph which traverses the envelope of the three-dimensional article, a first segment part
15 lying outside the volume enclosed by the envelope is removed and replaced by a segment part which is obtained by mirror-imaging about the envelope at the position where the segment of the first and/or second skeleton graph traverses the envelope a second segment part lying inside
20 the volume and adjoining the first segment part. As can be seen in Figure 19, open end segments A'''(x), i.e. segments that do not connect to any further segments and end into void, have been removed. Further, from the outermost segment(s), i.e. segments that would traverse
25 the skin 25 of the article, the parts A'''(o) outside of the skin are removed and replaced with a mirror image A'''(m) of the segment part A'''(i) inside the article, whereby the mirror image is a mirror image of the part inside which is mirrored about the skin 25, at the place
30 where it is traversed by the segment. This ascertains that the resulting minimal surface structure touches the

- 39 -

envelope 25 in essentially perpendicular direction, thus providing ideal load conduit.

Claims

1. A method of additively manufacturing a minimal surface structure (61, 62, 63) of a three-dimensional article (100, 200), the method comprising a computer (10) executing the steps of:
 - 5 recording in the computer (10) an envelope (11, 12) of the three-dimensional article (100, 200);
 - generating a density field (2) across a volume (121) enclosed by the envelope (12) with densities of the density field (2) corresponding to local requirement values of at least one physical parameter at
10 respective positions of the three-dimensional article (100, 200);
 - generating an adaptive Voronoi tessellation (V0) of the volume (121) using the density field (2);
15
 - generating a first skeleton graph (A) associated with the adaptive Voronoi tessellation (V0);
 - generating a second skeleton graph (B) associated with the first skeleton graph (A);
 - 20 generating a digital minimal surface model (5) from the first and second skeleton graphs (A; B);
 - wherein the method further comprises a 3D printer (30) additively manufacturing the minimal surface structure (61, 62, 63) according to the digital

minimal surface model (5), wherein generating the adaptive Voronoi tessellation (V0) comprises:

5 generating a set of scatter points (21) corresponding to a distribution of the densities of the density field (2);

randomly distributing the scatter points (21) across the volume (121) enclosed by the envelope (12);

10 generating a plurality of Voronoi cells of the adaptive Voronoi tessellation (V0) using the randomly distributed scatter points (21) as generating points of the Voronoi cells,

wherein the first skeleton graph (A) is formed by edges of Voronoi cells of the adaptive Voronoi tessellation (V0),

15 wherein generating the digital minimal surface model (5) from the first and second skeleton graphs (A; B) comprises:

generating a minimal surface precursor (3) from the first and second skeleton graphs;

20 generating a minimal surface shape (4) by smoothening the minimal surface precursor (3);

assigning a wall width to the minimal surface shape (4);

25 generating the digital minimal surface model (5) according to the minimal surface shape (4) and the assigned wall width.

2. The method according to claim 1, characterized in that generating the adaptive Voronoi tessellation (V0) using the density field (2) comprises
5 iteratively generating a plurality of Voronoi cells of the adaptive Voronoi tessellation (V0) by weighted stippling using the density field (2).
3. The method according to claim 2, characterized in that iteratively generating the plurality of Voronoi
10 cells comprises iterating the steps of:
 - a) calculating the weighted centroid (C) of each Voronoi cell using the density field (2) and shifting generating points of the Voronoi cells to the respective centroids (C);
 - 15 b) generating new Voronoi cells of the adaptive Voronoi tessellation (V0) using the shifted generating points and replacing the Voronoi cells of step a) by the new Voronoi cells;until the calculated centroids (C) conform with the
20 generating points of the Voronoi cells in step a).
4. The method according to claim 3, characterized in that iteratively generating the plurality of Voronoi cells comprises executing the steps of:
 - c) calculating for each Voronoi cell a cell weight by
25 integrating the density field (2) over the respective Voronoi cell;

- d) recording in the computer a first weight threshold and a second weight threshold, wherein the first weight threshold is larger than the second weight threshold;
- 5 e) splitting Voronoi cells with a cell weight above the first weight threshold and deleting Voronoi cells with a cell weight below the second weight threshold;
- after step b) of claim 3.
5. The method according to claim 4, characterized in
10 that the steps c)-e) are executed for the first 10-30%, preferably 20%, of the iterations of steps a)-b) of claim 3.
6. The method according to one of the preceding claims,
15 characterized in that generating the density field (2) comprises:
- dividing the volume (121) enclosed by the envelope (12) into a plurality of, preferably tetrahedral, voxels (122) and generating at least one local requirement value for each voxel (122).
- 20 7. The method according to one of the preceding claims, characterized in that the second skeleton graph (B) is generated by Delaunay tetrahedralization of generating points of the adaptive Voronoi tessellation (V0).
- 25 8. The method according to one of the preceding claims, characterized in that from a segment of the first and/or second skeleton graph (A; B) which traverses

the envelope of the three-dimensional article, a first segment part lying outside the volume (121) enclosed by the envelope (12) is removed and replaced by a segment part which is obtained by mirror-imaging about the envelope (12) at the position where the segment of the first and/or second skeleton graph traverses the envelope (12) a second segment part lying inside the volume and adjoining the first segment part.

9. The method according to one of the preceding claims, characterized in that Voronoi cells which extend beyond the envelope (12) of the three-dimensional article (100, 200) are trimmed at the envelope and the centroids of the trimmed Voronoi cells are recalculated using the density field.

10. The method according to claim 1, characterized in that generating the digital minimal surface model (5) from the first and second skeleton graphs (A; B) comprises:

assigning a first electrical charge to the first skeleton graph (A);

assigning a second electrical charge to the second skeleton graph (B), the second electrical charge being equal in absolute value, but opposite in sign to the first electrical charge;

generating the minimal surface precursor (4) as an equipotential surface between the first and second skeleton graph (A; B) using a Coulomb force field

calculated based on the first and second skeleton graphs (A; B) and their electrical charges.

11. The method according to one of the preceding claims, characterized in that the at least one physical parameter is selected from at least one of: mechanical load, stiffness, amount of storable fluid, fluid flow, heat transport.
12. A non-transitory computer-readable medium (101) having stored thereon computer-executable instructions adapted to cause a 3D printer (30) to additively manufacture a minimal surface structure (61, 62, 63) of a three-dimensional article (100, 200) according to a digital minimal surface model (5), the computer-executable instructions comprising a computer (10) executing the steps of:
 - recording in the computer (10) an envelope (11, 12) of the three-dimensional article (100, 200);
 - generating a density field (2) across a volume (121) enclosed by the envelope (12) with densities of the density field (2) corresponding to local requirement values of at least one physical parameter at respective positions of the three-dimensional article (100, 200);
 - generating an adaptive Voronoi tessellation (V0) of the volume (121) using the density field (2);
 - generating a first skeleton graph (A) associated with the adaptive Voronoi tessellation (V0);

generating a second skeleton graph (B) associated with the first skeleton graph (A);

generating the digital minimal surface model (5) from the first and second skeleton graphs (A; B),

5 wherein generating the adaptive Voronoi tessellation (V0) comprises:

generating a set of scatter points (21) corresponding to a distribution of the densities of the density field (2);

10 randomly distributing the scatter points (21) across the volume (121) enclosed by the envelope (12);

generating a plurality of Voronoi cells of the adaptive Voronoi tessellation (V0) using the randomly distributed scatter points (21) as generating points of the Voronoi cells,

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wherein the first skeleton graph (A) is formed by edges of Voronoi cells of the adaptive Voronoi tessellation (V0),

wherein generating the digital minimal surface model (5) from the first and second skeleton graphs (A; B) comprises:

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generating a minimal surface precursor (3) from the first and second skeleton graphs;

generating a minimal surface shape (4) by smoothening the minimal surface precursor (3);

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assigning a wall width to the minimal surface shape (4);

generating the digital minimal surface model (5) according to the minimal surface shape (4) and the assigned wall width.

13. A computer-implemented method of generating a digital minimal surface model (5) adapted for additively manufacturing a minimal surface structure (61, 62, 63) of a three-dimensional article (100, 200) according to the digital minimal surface model (5) by a 3D printer (30), the computer-implemented method comprising a computer (10) executing the steps of:

recording in the computer (10) an envelope (11, 12) of the three-dimensional article (100, 200);

generating a density field (2) across a volume (121) enclosed by the envelope (12) with densities of the density field (2) corresponding to local requirement values of at least one physical parameter at respective positions of the three-dimensional article (100, 200);

generating an adaptive Voronoi tessellation (V0) of the volume (121) using the density field (2);

generating a first skeleton graph (A) associated with the adaptive Voronoi tessellation (V0);

generating a second skeleton graph (B) associated with the first skeleton graph (A);

generating the digital minimal surface model (5) from the first and second skeleton graphs (A; B);

storing the digital minimal surface model (5) on a computer-readable medium (20),

5 wherein generating the adaptive Voronoi tessellation (V0) comprises:

generating a set of scatter points (21) corresponding to a distribution of the densities of the density field (2);

10 randomly distributing the scatter points (21) across the volume (121) enclosed by the envelope (12);

generating a plurality of Voronoi cells of the adaptive Voronoi tessellation (V0) using the randomly distributed scatter points (21) as generating points of the Voronoi cells,

15

wherein the first skeleton graph (A) is formed by edges of Voronoi cells of the adaptive Voronoi tessellation (V0),

wherein generating the digital minimal surface model (5) from the first and second skeleton graphs (A; B) comprises:

20

generating a minimal surface precursor (3) from the first and second skeleton graphs;

generating a minimal surface shape (4) by smoothening the minimal surface precursor (3);

25

assigning a wall width to the minimal surface shape
(4);

generating the digital minimal surface model (5)
according to the minimal surface shape (4) and the
5 assigned wall width.

14. A computer program product comprising computer
program code configured to control a computer (10)
such that the computer (10) executes the steps of the
method according to claim 13.