

SYSTEM, DEVICE AND METHOD FOR IMPROVING A DEFINED PROPERTY OF TRANSFORM-DOMAIN SIGNALS

Related Application

5 This application is a continuation-in-part of US Application No. 09/058,671 filed April 10, 1998, which is hereby incorporated by reference in its entirety.

Field Of Invention

10 This invention relates to a system, device and method for improving a defined property of transform-domain signals, and more particularly to a system, device and method for reducing the peak-to-average energy ratio (PAR) of time-domain signals.

Background Of Invention

15 The large time-domain peak-to-average energy ratio (PAR) of discrete multitone (DMT) signals is often cited as a major disadvantage of DMT systems. This problem exists in systems using other modulation schemes as well, such as in systems using orthogonal frequency division
20 multiplexing (OFDM) and orthogonal quadrature amplitude modulation (OQAM).

 A large PAR requires implementation of a high-precision digital-to-analog converter (DAC), or else requires the system to be tolerant of signal distortion (clipping) introduced when input signals exceed the DAC range. For
25 a fixed DAC precision, scaling the input signal so that signal values are always within range may result in excessive quantization noise; on the other hand, insufficient signal scaling may result in excessive clipping noise.

 A number of approaches to reduce the time-domain peak amplitude of DMT and OFDM symbols have been proposed. These
30 techniques can be divided into three classes. In the first class, multiple symbols are used to represent the same data and side information, transmitted on reserved tones, is used to tell the receiver which of the symbols was transmitted. For example in J.S. Chow, J.A.C. Bingham, and M.S.
 Flowers, "Mitigating clipping noise in multicarrier systems," *Proc. 1997 Int.*
35 *Conf. Commun. (ICC'97)*, pp. 715-719, June 1997, if the peak of the DMT symbol is too high, the DMT symbol is scaled and reserved tones are used to relay the scaling factor to the receiver. This technique reduces the signal-to-

noise ratio (SNR) of the transmitted symbol and thus results in an increased bit error rate. In Djokovic, "PAR reduction without noise enhancement", submission T1E1.4/97 270 to ADSL Standard Issue 2, Sept. 1997., the transmitter chooses between the original DMT symbol and its conjugate
5 formed by scrambling the original symbol. In D.J. Mestdagh and P.M. Spruyt, "A method to reduce the probability of clipping in DMT-based transceivers," *IEEE Trans. On Commun.*, vol. 44, 1234-1238, Oct. 1996, a pseudo-random phase sequence is added to the original DMT symbol. The most significant drawback with this class of techniques is that the transmitter must relay side
10 information about the transmitted symbol to the receiver. In addition to incurring a data rate loss or an increase in bandwidth, if the side information is corrupted, the entire DMT symbol will be destroyed.

The second class of PAR reduction techniques is based on determining sequences which have good PAR properties. See for example, S. Shepherd,
15 J. Orriss, and S. Barton, "Asymptotic limits in peak envelope power reduction by redundant coding in orthogonal frequency-division multiplex modulation," *IEEE Trans. on Commun.*, vol. 46, pp. 5-10, Jan. 1998. These methods generally involve removing "bad" time-domain sequences from the set of possible transmitted symbols and thus result in a data rate loss. Furthermore,
20 these methods require mapping the data to the "good" symbols. This map is generally accomplished via a lookup table. The size of the required lookup table becomes impractical in a DMT system with many tones and large constellation sizes.

In the third class of schemes, PAR reduction is achieved via a
25 redundant signal representation, in which a given data block can be represented by any of a number of possible transmitted signals from some equivalence class, with the "most desirable" class representative—in this case, a representative with small time-domain peak value—chosen for transmission. In such schemes, the receiver
30 is designed to operate "modulo equivalence classes" producing the data block associated with an equivalence class whenever it detects an element of that class. In this way, the receiver requires no knowledge of the precise algorithm used to select a class representative at the transmitter. One way to operate "modulo
35 equivalence classes" in the DMT case is to have the receiver simply ignore the contents of various frequency bins. See A. Gatherer and M. Polley, "Controlling clipping probability in DMT transmission," *Conf. Record 31st Asilomar Conf. On Sign. Sys. And Comp.*, Nov. 1997; A. Gatherer and M.

Polley, "Proposed PAR Reduction Techniques for G.lite," Universal ADSL Technical Group Contribution TG/98-025, Feb. 4, 1998; J. Tellado and J.M. Cioffi, "PAR reduction in multicarrier transmission systems," contribution 97-367 to T1E1.4 standards committee, Dec. 1997. For any given data block, the transmitter can place values in these unused bins to minimize (as far as possible) the peak value of the transmitted time-domain signal. These techniques incur a significant data rate loss since several bins are not used to transmit data.

Therefore, there exists a need for a PAR reduction technique which utilizes the data carrying or complex frequency bins in DMT modulation schemes to reduce PAR without affecting the data rate. There is also a need for such a technique which can be applied generally to other modulation schemes and to improve other properties of the time-domain, or, in general, the transform-domain signal.

Brief Description of the Drawings

FIG. 1A is a schematic block diagram of a DMT transmitter configured according to this invention;

FIG. 1B is a schematic block diagram of an alternative DMT transmitter configuration according to this invention;

FIG. 2 is an expanded signal point constellation in accordance with this invention;

FIG. 3 is a schematic block diagram of a receiver configured according to this invention;

FIG. 4 is a schematic block diagram of the offset coset representative generator of FIGS. 1A and 1B;

FIG. 5 is a schematic block diagram of the perturbation device of FIG. 1A;

FIG. 6 is a schematic block diagram of the valid perturbation generator in the perturbation device of FIG. 5;

FIG. 7 is a schematic block diagram of the perturbation selector in the perturbation device of FIG. 5;

FIG. 8 is a schematic block diagram of the offset decoder of the receiver of FIG. 3;

FIG. 9 is a schematic block diagram of an alternative, frame based configuration of the offset coset representative generator of FIGS. 1A and 1B;

FIG. 10A is a schematic block diagram of an alternative, frame based configuration of the perturbation device of FIG. 1A;

FIG. 10B is a schematic block diagram of the perturbation device of FIG. 10A incorporating look-ahead;

5 FIG. 11 is a schematic block diagram of a valid perturbation generator depicted in FIG. 10A;

FIG. 12 is a schematic block diagram of a perturbation selector depicted in FIG. 10;

10 FIG. 13 is a schematic block diagram of an alternative, frame based configuration of the offset decoder of FIG. 3;

FIG. 14 is a schematic block diagram of an alternative configuration of the perturbation selector of FIG. 5 for use in a DSL splitterless application; and

FIG. 15 is an alternative, rotated expanded signal point constellation in accordance with this invention.

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Description of a Preferred Embodiment

The present invention is generally directed to a system and method for improving a defined property of a signal after block transformation, hereinafter referred to as a transform-domain signal. In order to provide a more readily
20 understandable description of the invention we describe herein an actual application of the invention to reduce the peak-to-average energy ratio (PAR) of the time-domain signal (more generally referred to herein as the transform-domain signal) in discrete multitone (DMT) modulation schemes. It will be apparent to those skilled in the art that the invention is generally applicable to
25 other modulation schemes, such as orthogonal frequency division multiplexing (OFDM), and orthogonal quadrature amplitude modulation (OQAM). Moreover, it will be apparent to those skilled in the art that the invention may be used to improve other defined properties in the transform-domain signal in addition to PAR. For example, it may also be used to shape the spectrum of the
30 transform-domain signal after it goes through a non-linearity, as for example in the splitterless operation of a DSL system where it is desirable to reduce the voiceband (0-4kHz) interference generated by the non-linearity in the plain old telephone service (POTS) phone.

The transmitting scheme of the DMT system is based on blocks of N
35 symbols. Each symbol in a block corresponds to a different frequency bin. Thus, each symbol block X consists of frequency domain symbols X_0 - X_{N-1} . In asymmetrical digital subscriber line (ADSL) systems using DMT (assuming N is even), there are no symbols transmitted in the zero (X_0) and Nyquist ($X_{N/2}$)

frequency bins. There are symbols transmitted in the lower $N/2-1$ complex frequency bins ($X_1-X_{N/2-1}$) and the upper $N/2-1$ complex frequency bins ($X_{N/2+1}-X_{N-1}$) are selected as the complex conjugate images of the lower $N/2-1$ bins so that the resulting frequency-domain signal possesses the Hermitian symmetry
 5 needed to make the time-domain signal real-valued. Therefore, there are actually n (where $n=N/2-1$) complex frequency bin symbols per block.

As shown in FIG. 1A the DMT transmitter 10 receives a serial digital bit stream over line 12 from data terminal equipment (not shown), such as a personal computer. The serial bit stream is converted to parallel format by a
 10 serial to parallel converter 14. For each block, serial to parallel converter outputs $(kn-r)+m$ information bits, v and u , where r is a number of redundancy bits and k is a number of bits needed to represent the equivalence classes in the expanded constellation. The variables r and k as well as the terms equivalence class and expanded constellation are described below.

15 As noted above, n is the number of complex frequency bin symbols generated per block. The m base information bits, u , correspond to $m = m_1 + m_2 + \dots + m_n$, where m_i represents the number of base information bits transmitted in complex frequency bin i . For each of the n complex frequency bins, base constellation mapper maps m_i base information bits into a symbol
 20 from a base constellation. The i th base constellation contains 2^{m_i} points.

In the base constellation mapper 16, a base constellation for each frequency bin is determined by the channel quality for the frequency bin and n base symbols, g , for each block are generated. The channel quality is typically determined by probing the channel during a training sequence. The
 25 size of the constellation, and hence the number of input data bits that can be represented by the symbol chosen from the constellation, is dependent upon the quality of the channel in the frequency range of the bin. A channel having better quality can use a denser constellation with more closely spaced points and therefore more bits can be transmitted with each symbol. Thus, the
 30 number of input data bits represented by a block of symbols is dependent upon the quality of the channel.

To illustrate the general idea of a base constellation in accordance with this invention with a specific example, consider a DMT system with base constellation capable of transmitting two bits per symbol. In FIG. 2, there is
 35 shown base constellation 30 (which is assumed to reside in one quadrant) containing points A, B, C, and D, from which the base constellation symbols are selected by the base constellation mapper 16. Also in accordance with this invention, at least some of the base constellations are expanded to

include additional points beyond the 2^m points necessary to transmit m base information bits per symbol. These expanded constellations are partitioned into equivalence classes.

The base signal constellations are expanded to support the
5 transmission of up to k additional bits per symbol. Some of these kn bits are used to send additional information bits while others can be used to provide the transmitter with some flexibility in choosing the transmitted symbols. This extra degree of freedom can be used to optimize some objective function of the resulting signal – for example the peak time-domain amplitude or the
10 spectral shape of the transmitted signal after a non-linear transformation. We refer to these kn additional bits as "offset bits."

In the example depicted in FIG. 2, base constellation 30 is expanded by a factor of 4 to form 16 point expanded constellation 32. Therefore, $k=2$ offset bits per symbol are needed to determine which of the equivalent points in
15 expanded constellation 32 is transmitted. Expanded constellation 32 includes base constellation 30 and expansion areas 34, 36, and 38 each containing four points labeled A-D. All the points with the same label belong to the same equivalence class.

In the example of FIG. 2, the expanded constellation is formed from the
20 base constellation by repeating the base constellation in each of the four quadrants. The minimum distance between neighboring points in the constellation for the i th symbol is defined as d_i . This distance is dependent upon channel quality. This type of constellation expansion will be referred to as an additive expansion because the equivalent points in the expanded
25 constellation are generated by adding a value (0 or $-2d_i$ in each dimension in this example) to points in the base constellation.

In this example, we have expanded the base constellation by a factor of 4. Of course, the base constellation can also be expanded by a factor larger than 4. For example, by adding 0 or integer multiples of $\pm 2d_i$ in each
30 dimension to points in the base constellation, we can generate additional equivalence class points. As will be seen below, both the performance capability and system complexity increase with larger constellation expansions. For a general square constellation with c points, expanded constellation points may be generated by adding integer multiples of $cd_i/2$ in
35 each dimension. Other methods of generating expanded constellations containing several constellation point equivalence classes will be clear to those skilled in the art. Below an expanded constellation generated via rotation of the base constellation is described.

Referring again to FIG. 1A, $kn-r$ bits, v , are provided to offset coset representative generator 18 which produces kn offset (or more particularly coset representative offset) bits. The n base symbols g and the kn coset representative offset bits t are combined by expanded constellation mapper 20 to form n expansion symbols, h , from the expanded constellations. In the example above, with $k=2$, the base symbols g correspond to the base constellation points and the k offset bits select the quadrant of the corresponding symbols. The n expansion symbols are mapped to a block of N conjugate symmetric symbols X (X_0 - X_{N-1}) by Hermitian symmetry block generator 22. In Hermitian symmetry block generator 22, the n symbols are mapped to symbols X_1 - $X_{N/2-1}$ and symbols $X_{N/2+1}$ - X_{N-1} are the complex conjugates of X_1 - $X_{N/2-1}$. The block of N symbols X is provided to an invertible transform device, such as inverse discrete Fourier transform (IDFT) device 24 which transforms the N frequency domain symbols into N time-domain symbols x (x_0 - x_{N-1}). Perturbation device 26 modifies the N time-domain symbols x to produce perturbed time-domain blocks y by modifying the coset representative offset bits t to improve a defined property of the N time domain symbols, in this example, minimizing the peak value, as described below. The perturbed time-domain blocks y are provided to parallel to serial converter 28 which converts the perturbed time-domain blocks y from parallel form to serial form and transmits them over the channel.

Base constellation mapper 16, offset coset representative generator 18, expanded constellation mapper 20 and Hermitian symmetry block generator 22 collectively form signal mapper 23 which maps the input data from serial to parallel converter 14 to blocks X of frequency domain symbols. And, IDFT device 24 and perturbation device 26 collectively form perturbation/transform device 27.

In FIG. 3 there is shown a schematic block diagram of receiver 40 according to this invention. The perturbed time domain symbols y , after going through the channel, are received as symbols w at serial to parallel converter 44 which receives the time-domain symbols w in serial form and converts them to blocks of received time-domain symbols w , w_0 - w_{N-1} . The blocks of received time-domain symbols w , w_0 - w_{N-1} , are provided to discrete Fourier transform (DFT) device 46 which converts the blocks of time domain symbols into blocks of received frequency-domain symbols W , W_0 - W_{N-1} . The blocks of received frequency-domain symbols W , W_0 - W_{N-1} , are provided to frequency domain equalizer device 48 which takes into account the effect of the channel on the transmitted perturbed frequency domain blocks Y , Y_0 - Y_{N-1} , and scales

the received symbols W, W_0-W_{N-1} , to produce symbols $Y', Y'_0-Y'_{N-1}$ which are estimates of the transmitted blocks Y, Y_0-Y_{N-1} .

The estimates of the transmitted perturbed frequency domain blocks Y' are provided to base symbol and offset extractor 50 which extracts base symbols g and valid perturbation offset bits s . These bits do not correspond exactly to coset representative offset bits t because the offset bits were modified in perturbation device 26, FIG. 1A. The base symbol and offset extractor 50 first decodes each of the n symbols g transmitted in the block of N symbols, Y , to a point in the corresponding expanded constellation. The offset bits s of these n symbols are provided to offset decoder 52 to recover the offset information bits v' , which are equivalent to offset information bits v , as described below. The n base symbols g are provided to base constellation demapper 54, to recover the m base information bits u . The base symbols g correspond to points in the base constellations.

The decoded information bits, u and v , may then be further processed and provided to data terminal equipment, such as a personal computer.

Offset Coset Representative Generator

Offset coset representative generator 18 is depicted in more detail in FIG. 4. The offset information bits v , considered to be a $1 \times (kn-r)$ row vector, are post-multiplied (modulo 2) (i.e., filtered) in matrix block 60 by matrix H^T , having $kn-r$ rows and kn columns to produce the $1 \times kn$ row vector of coset representative offset bits t which are provided to expanded constellation mapper 20 and perturbation device 26, FIG. 1.

An example of matrix H^T when there are $n=3$ symbols transmitted per block, $k=2$ and there is one redundancy bit per symbol ($r=3$) is as follows:

$$H^T = \begin{matrix} & 1 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 1 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 1 & 0 \end{matrix} \quad (1)$$

In this example, if the offset information bits are given by $v = [v_0 \ v_1 \ v_2]$, the output bits are simply the input bits padded with zeros, i.e. $t = [v_0 \ 0 \ v_1 \ 0 \ v_2 \ 0]$.

Perturbation Device

Perturbation device 26 is depicted in more detail in FIG. 5. Perturbation device 26 operates in the time-domain to perturb the blocks of symbols; however, it can be readily modified to operate in the frequency domain as with

perturbation device 26' in perturbation/transform device 27', FIG. 1B. The kn coset representative offset bits t are provided to valid perturbation generator 70. The valid perturbation generator 70, generates 2^r valid perturbation vectors (or some subset thereof to reduce complexity), where r is the number of redundancy bits. In general, the "perturbations" are not additive, but can be considered as such according to the following scheme. Let y_i be the perturbed time domain block corresponding to perturbation i , $i=0,1, \dots, 2^r-1$. Let $p_i = y_i - x$, where $x=y_0$. The vectors $\{p_i : i=0,1, \dots, 2^r-1\}$ are referred to as "valid perturbation vectors" and x is referred to as the "nominal time-domain block."

The perturbation selector 72 receives these 2^r valid perturbation vectors (or some subset thereof), p_i ($p_{i,0}-p_{i,N-1}$), and adds each of these perturbation vectors to the nominal time-domain block x (x_0-x_{N-1}) to generate $y_i = x + p_i$. The vector y with smallest time-domain peak (or that which achieves some other objective function) is selected for transmission.

If the expanded constellation is formed by expanding the base constellation for each of the n symbols by a factor of 2^k , then in addition to the m base bits, kn offset bits t can be transmitted over the channel in each block of symbols. Of these kn offset bits, $kn-r$ coset representative offset bits are used to send additional information bits and the flexibility afforded by r redundancy bits is used to improve a desired property of the transform-domain signal. Larger values of r provide greater flexibility in improving the defined property of the transform-domain signal, but result in lower bit rates for information transmission.

25 Valid perturbation generator

Valid perturbation generator 70 is described herein based on binary linear codes, though it will be apparent to those skilled in the art that this structure can be extended to non-binary group codes.

Valid perturbation generator 70, FIG. 6, generates valid perturbation vectors that correspond to modifications of the offset for each symbol. The kn coset representative offset bits t provided to valid perturbation generator 70 define a coset representative for a defined linear code C generated by perturbation codeword generator 80 using a matrix G having r rows and kn columns.

Matrix G and matrix block H^T 60, FIG. 4, are selected such that $GH^T=0$ where H is a matrix having $kn-r$ rows and kn columns, itself satisfying the property that $H^T H^T = I_{kn-r}$ where I_{kn-r} is the $(kn-r) \times (kn-r)$ identity matrix. In other words, H^T is a right inverse of H^T . It is required that G have row rank r

and that H have row rank $kn-r$. An example of matrix G when $n=3$, $r=3$ and $k=2$ is shown as follows:

$$5 \quad G = \begin{matrix} & 0 & 1 & 0 & 1 & 0 & 1 \\ & 0 & 1 & 0 & 1 & 0 & 0 \\ & 0 & 1 & 0 & 0 & 0 & 0 \end{matrix} \quad (2)$$

Here G can be chosen as the generator matrix for any well-known binary linear code, or it could correspond to the generator matrix for a truncated or terminated convolutional code, optimized for Hamming distance properties or according to some other criterion.

Using the kn coset representative offset bits t , valid perturbation generator 70 modifies offset bits t by EXCLUSIVE OR'ing, i.e., adding modulo 2, the bits with valid codewords c , defined by matrix G . These codewords, $c_r = r, G$, are generated by all 2^r possible choices for the r redundancy bits denoted r , producing 2^r valid codewords c_r . The codewords generated by perturbation codeword generator 80 have the property that $cH^T=0$. The result of EXCLUSIVE OR'ing the valid codewords c with the coset representative offset bits t is to produce a set of valid perturbation offset bits $s_r = t \oplus c_r$. the valid perturbation offset bits s_r are mapped to N -symbol block P , via perturbation mapper 82 and Hermitian symmetry block generator 84, as described below. It should be noted that with this selection process any of the valid perturbation offset bits s_r may be used and will be decoded, as described below, to the offset information bits v .

Each set of kn valid perturbation offset bits s_r corresponds to k offset bits per symbol. Continuing with the earlier example where it is assumed $k=2$ and $r=n$, the offset bits s_r define the quadrants of the n symbols. Equivalently, the $k=2$ bits per symbol define the displacement of the point in the base constellation required to generate the appropriate point in the equivalence class containing the original base constellation point. In the nominal time-domain block x , the offset bits are defined by t . Each s_r corresponds to a modification of these offset bits. Equivalently, in the example, the valid perturbation offset bits s_r correspond to changing the quadrants of the transmitted symbols. Recall that t was generated from the offset information bits v . Therefore the valid perturbation offset bits s_r which are formed from t are information dependent.

If H^T is defined as in the example above, and $r=n$ (1 redundancy bit per symbol), then the unmodified coset representative offset bits t consist of n

pairs of bits, where the second bit in each pair is zero. Therefore the coset representative offset bits t only choose between one of two quadrants, those represented by 00 and by 10. In this example, the valid codewords c_i will consist of n pairs of bits whose first bit in each pair is 0. If the second bit in a pair is non-zero, c_i modifies the quadrant from 00 to 01 or from 10 to 11. Let d be the distance between neighboring points in the base constellation. In this example, if quadrant 00 is defined to denote the quadrant containing the base constellation, quadrant 10 is defined to be the quadrant below the base constellation, quadrant 01 is defined to be the quadrant to the left of the base constellation, and quadrant 11 is defined to be the remaining quadrant, then the valid perturbation offset bits modify the coset chosen by the information dependent coset representative offset bits t by a perturbation of 0 or $-2d$ for each symbol.

The perturbation mapper 82 maps each set of valid perturbation offset bits s_i into n symbol perturbations. These n symbol perturbations represent the resulting perturbation from changing the offset bits from t to s_i . In other words, recall that each of the n expansion symbols in h was determined from base symbols g and offset bits t . Denote by h_i' the n expansion symbols corresponding to base symbols g and offset bits s_i . The n perturbation symbols q_i are the difference between h_i' and h , i.e., $q_i = h_i' - h$. In the example above, q_i contains perturbations of 0 or $-2d$ in each symbol.

For each set of valid perturbation offset bits s_i , the n perturbation symbols (0 or $-2d$ for each symbol in example above) are mapped by the Hermitian symmetry block generator 84 into an N -symbol frequency domain symbol P_i with complex conjugate symmetry. The operation of the Hermitian symmetry block generator 84 was described above. The frequency domain symbols P_i are provided to IDFT device 86 to generate $2'$ time-domain perturbation vectors p_i .

It will be apparent to those skilled in the art that different values for k , r , H , and G , will lead to different perturbation vectors. Also, different methods of mapping the bits to symbols will lead to different perturbation vectors. The valid perturbation vectors generated are dependent on offset information bits v .

In general, for a fixed k , H , G , and r , and for a fixed mapping scheme, the set of $2'$ valid perturbation vectors will come from a set of 2^{kn} possible time-domain perturbation vectors. Instead of generating these valid perturbation vectors for each incoming t as described above, valid perturbation vector generator 70 could store all 2^{kn} possible time-domain perturbation vectors in

memory and use the coset representative offset bits t to determine which 2^r of these perturbation vectors (or some subset thereof) are valid for the given t . Note that if the expanded constellation is not an additive expansion as described above, the perturbation symbols may depend on not only the coset representative offset bits t , but also the base symbols g . In this case there may be more than 2^{kn} possible time-domain perturbation vectors.

Perturbation Selector

Perturbation selector 72 is shown in more detail in FIG. 7. For each of the 2^r valid time-domain perturbation vectors p_i , perturbed time-domain block y_i is computed by block 90, where $y_i = x + p_i$. Then all of the y_i perturbed time domain blocks computed are evaluated in block 92 and the y_i with the smallest peak value is selected as the perturbed time-domain block of symbols to be transmitted.

Base Symbol and Offset Extractor

Base symbol and offset extractor 50, FIG. 3, maps the frequency domain equalized blocks Y' to n symbol points in the expanded constellations. Each point in the expanded constellation is equivalent to a point in the base constellation (equivalence class representative). The offset signifies which of the 2^k equivalent points was actually transmitted. The 2^k equivalence class points are represented by k offset bits per symbol. The equivalence class point transmitted is represented by kn offset bits, s . These offset bits are provided to the offset decoder 52 which determines the information bits encoded in the offset bits, as described below. The n equivalence class representatives in the base constellations are the estimates of the transmitted base symbols g and are provided to base constellation de-mapper 54 which de-maps these points to estimates of the transmitted base information bits u .

Offset Decoder

Offset decoder 52, shown in greater detail in FIG 8, includes matrix block 100. In matrix block 100 the $1 \times kn$ row vector of offset bits s is post-multiplied (modulo 2) (i.e., filtered) by matrix H^T having kn rows and $kn-r$ columns to recover the $1 \times (kn-r)$ row vector of offset information bits v' .

In order to demonstrate how the each of the candidates of valid perturbation offset bits s_i are decoded to the same offset information bits, the encoding and decoding processes must be expressed mathematically. The information bits recovered, v' , (decoding) can be expressed mathematically as follows:

$$\mathbf{v}' = \mathbf{sH}^T \quad (3)$$

and the valid perturbation offset bits $\mathbf{s}$ (encoding) can be expressed
 5 mathematically as follows:

$$\mathbf{s} = \mathbf{vH}^T + \mathbf{rG}, \quad (4)$$

where $\mathbf{c} = \mathbf{rG}$ is a valid codeword generated by perturbation codeword
 generator 80, FIG. 6. If the right hand side of equation (4) is substituted into
 10 equation (3) for $\mathbf{s}$, then the following equation is derived:

$$\mathbf{v}' = \mathbf{vH}^T\mathbf{H}^T + \mathbf{rGH}^T \quad (5)$$

By selecting $\mathbf{G}$, $\mathbf{H}^T$ and $\mathbf{H}^T$ so that the following conditions are satisfied: (1) $\mathbf{H}^T$
 15 $\mathbf{H}^T = \mathbf{I}$ (where $\mathbf{I}$ is the identity matrix); and (2) $\mathbf{GH}^T = 0$, then $\mathbf{v}' = \mathbf{v}$ regardless
 of the value of $\mathbf{r}$.

Frame Based Perturbations

20 In a block based system, such as a DMT based system, bits are
 mapped to blocks of N symbols. In the invention described above, we assume
 that the offset bits are modified on a block by block basis. In other words all
 $n=N/2-1$ symbols transmitted on the block of N symbols are perturbed jointly.
 It may be useful in some cases to divide the blocks into frames having a size
 25 less than n symbols. For example if n and r are large, a large set of valid
 perturbation vectors must be generated and/or stored and/or tested. If smaller
 frame sizes are used, and the perturbations performed on a frame by frame
 basis, the number of valid perturbation vectors that must be tested and/or
 stored and/or generated will be reduced. The cost of this approach is some
 30 loss in performance since the perturbations are selected to optimize the
 desired property on a frame by frame basis. Some of this performance can be
 recovered by using look-ahead, as described below. This of course again
 increases the system complexity.

With frame based perturbation, the transmitter of this invention differs in the
 35 following two ways: 1)The offset coset representative generator operates on f
 frames of kn/f bits as described below; and 2)The perturbation device divides
 its input and operates on f frames of kn/f bits as described below. And, the
 receiver of this invention differs in one way; namely, the offset decoder divides
 its input and operates on f frames of kn/f bits as described below.

Note that to simplify the explanation, it is easiest to assume n/f is an integer, otherwise the offset coset representative generator and perturbation device and offset decoder would need to operate on frames of different sizes. Nevertheless, generalization to the case where n/f is not an integer is straightforward.

Frame Based Offset Coset Representative Generator

Offset coset representative generator 18a, FIG. 9, includes frame divider 110 which receives $kn-r$ information bits v and divides the $kn-r$ bits v into frames of size $kn/f - r/f$. These frames are denoted as v_j and $n'=n/f$ and $r'=r/f$. Thus, for each of the f frames, $kn'-r'$ information bits are transmitted via v_j , and r' redundancy bits are used to improve a desired property of the transform-domain symbol. These $1 \times (kn'-r')$ frames of offset information bits are post-multiplied (modulo 2) (i.e., filtered) in matrix blocks 112_0-112_{f-1} by matrix H^T having $kn'-r'$ rows and kn' columns to produce a $1 \times kn'$ frame of kn' coset representative offset bits t_j (t_0-t_{f-1}). The f frames (t_0-t_{f-1}) are concatenated in frame concatenator 114 to form kn coset representative offset bits t which are provided to expanded constellation mapper 20, FIG. 1, and perturbation device 26a, FIG. 10.

Frame Based Perturbation Device

Perturbation device 26a, FIG. 10A, includes frame divider 120 which receives kn coset representative offset bits t and divides the bits into f frames of size kn' , denoted by t_0-t_{f-1} . Alternatively, the frames of size kn' can be provided directly from the offset coset representative generator 18a, FIG. 9. Each frame of coset representative offset bits t_j is provided to a valid perturbation generator 112_j (112_0-112_{f-1}) which generates 2^r valid perturbation vectors (or some subset thereof) and provides these to the j th perturbation selector 124_j (124_0-124_{f-1}) corresponding to the j th frame.

In general, the perturbations are not additive, but can be considered as such according to the following scheme. Let $y_{j,i}$ be the time domain signal corresponding to perturbation i , $i=0,1, \dots, 2^r-1$. Let $p_{j,i} = y_{j,i} - y_{j,0}$, and let $y_{j,0} = y_{j-1}$ which will be defined subsequently. The $\{p_{j,i} : i=0,1, \dots, 2^r-1\}$ are referred to as the 'valid perturbation vectors' corresponding to the j th frame of coset representative offset bits t_j .

The j th perturbation selector is provided 2^r valid perturbation (or some subset thereof) vectors $p_{j,i}$ corresponding to the j th frame of coset representative offset bits t_j . It is also provided with the output of Perturbation

selector 124_{j-1} , which will be denoted y_{j-1} . The first perturbation selector, perturbation selector 124_0 is provided with nominal time-domain block x , which we will denote by y_{-1} . Perturbation selector 124_j computes $y_{j,i} = y_{j-1} + p_{j,i}$, for each of the valid perturbation vectors provided to perturbation selector 124_j . It provides the y_j with the smallest time domain peak to perturbation selector 124_{j+1} . The last perturbation selector, perturbation selector 124_{f-1} , outputs $y = y_{f-1}$ to the channel.

In perturbation device 26a, the perturbations are selected sequentially on a frame by frame basis. The performance of this device can be improved by incorporating look-ahead. That is, instead of selecting the valid perturbation offset bits s_j and corresponding perturbed output vector y_j based solely on the current frame, perturbation selector 124_j may use the valid perturbation offset bits s for the current frame and for future frames to decide which perturbed output vector achieves the lowest peak time-domain power.

To illustrate this idea, consider first a look-ahead depth of 1. Perturbation device 26b, FIG. 10B, includes perturbation selector 124_j which looks at the perturbation vectors entering perturbation selector 124_{j+1} to determine which valid perturbation offset bits s_j combined with valid perturbation offset bits s_{j+1} reduces the peak power of $y_{j+1} = y_{j-1} + s_j + s_{j+1}$ the most. Then vector $y_j = y_{j-1} + s_j$ is output to perturbation selector 124_{j+1} . Similarly, perturbation selector 124_{j+1} looks ahead to perturbation selector 124_{j+2} .

If the look-ahead depth is Δ , then perturbation selector 124_j looks ahead to the valid perturbation vectors entering perturbation selectors 124_{j+1} to $124_{j+\Delta}$ to determine which valid perturbation offset bits s_j combined with valid perturbation offset bits s_{j+1} to $s_{j+\Delta}$ reduces the peak power of $y_{j+\Delta} = y_{j-1} + s_j + s_{j+1} + \dots + s_{j+\Delta}$. Note that it is not possible in this implementation of the device to look beyond block boundaries. Therefore the last $\Delta-1$ perturbation selectors will have look-ahead depth less than Δ . Furthermore, the last $\Delta-1$ perturbation vectors are fully determined at perturbation selector $f-\Delta-1$. Perturbation device 26b has a look-ahead depth $\Delta=1$. When we are trying to improve the peak power of a time-domain block of symbols, looking beyond block boundaries is not helpful. For other objective functions, look-ahead beyond block boundaries may be useful. It will be clear to those skilled in the art that this invention can be modified to look-ahead beyond block boundaries. Note that if the look-ahead depth Δ is equal to the number of frames in one block f , this

scheme reduces to the first perturbation selector described above, i.e., it is equivalent to assuming 1 frame of size n symbols.

Frame Based Valid Perturbation Generator

5 The configuration of valid perturbation generators 122₀-122 _{$t-1$} is depicted in Fig 11. The valid perturbation generators are provided with their respective frames of kn' bits corresponding to frames of n' symbols and generates valid perturbation vectors of N -symbols that are used to modify the time-domain symbol x in order to minimize its peak power.

10 A valid perturbation generator generates valid perturbation vectors that correspond to modifications of the offset bits for n' symbols in each frame. The kn' coset representative offset bits t_j provided to a valid perturbation generator define a coset representative for a defined linear code C generated by perturbation codeword generator 126 using a matrix G having r' rows and
15 kn' columns. Matrix G and matrix block H^T (in offset coset representative generator 18, FIG. 1) are selected such that $GH^T=0$ where H is a matrix having $kn'-r'$ rows and kn' columns, itself satisfying the property that $H^TH^T = I_{kn'-r'}$, where $I_{kn'-r'}$ is the $(kn'-r') \times (kn'-r')$ identity matrix. In other words, H^T is a right inverse of H . We require that G have row rank r' and that H have row rank
20 $kn'-r'$.

In perturbation codeword generator 126, $2^{r'}$ codewords, (or some subset thereof) $c_i = r_i G$, are generated by post-multiplying all $2^{r'}$ possible choices r_i (or some subset thereof) for the r' redundancy bits denoted r_i by G . Codewords generated by perturbation codeword generator 126 have the property that
25 $c_i H^T = 0$. There are $2^{r'}$ valid codewords c_i . Using the kn' coset representative bits t_j , the valid perturbation generator modifies coset representative sign bits t_j by EXCLUSIVE OR'ing, i.e., adding modulo 2, the bits with valid codewords c_i defined by perturbation codeword generator 126. The resulting valid perturbation bits $s_{j,i} = t_j \oplus c_i$ are mapped to N -symbol block $P_{j,i}$ via
30 perturbation mapper 128 and Hermitian symmetry block generator 130. It should be noted that with this selection process any of the valid perturbation offset bits $s_{j,i}$ may be used and will be decoded, as described below, to the information bits v_j .

Perturbation mapper 128 maps each set of valid perturbation offset bits
35 $s_{j,i}$ into n' symbol perturbations, $q_{j,i}'$. These n' symbol perturbations represent the resulting perturbation from changing the offset bits of frame j from t_j to $s_{j,i}$. Let h_j denote the j th frame of n' expansion symbols in h . These expansion symbols were determined from base symbols g and offset bits t_j . Denote by

$h_{j,i}'$ the n' expansion symbols corresponding to base symbols g and offset bits $s_{j,i}$. The n' perturbation symbols $q_{j,i}'$ are the difference between $h_{j,i}'$ and $h_{j,i}$, i.e., $q_{j,i}' = h_{j,i}' - h_{j,i}$. These n' symbol perturbations $q_{j,i}'$ are mapped to $q_{j,i}$ a set of n symbol perturbations, where only the j th frame of n' symbols in the set of n symbol perturbations $q_{j,i}$ is non-zero. For each set of valid perturbation offset bits $s_{j,i}$, the n perturbation symbols are mapped by the Hermitian symmetry block generator 130 into an N -symbol frequency domain symbol $P_{j,i}$ with complex conjugate symmetry. The operation of the Hermitian symmetry block generator 130 is described above. The frequency domain symbols $P_{j,i}$ are provided to IDFT device 132 to generate 2^r time-domain perturbation vectors $p_{j,i}$.

It will be apparent to those skilled in the art that different values for k , r , H , and G , will lead to different perturbation vectors. Also, different methods of mapping the bits to symbols will lead to different perturbation vectors. The valid perturbation vectors generated are dependent on information bits v_j . In general, for a fixed k , H , G , and r , and for a fixed mapping scheme, the set of 2^r valid perturbation vectors will come from a set of $2^{kn'}$ possible time-domain perturbation vectors. Instead of generating these valid perturbation vectors for each incoming t_j as described above, the valid perturbation vector generator could store all $2^{kn'}$ possible time-domain perturbation vectors in memory and use the coset representative offset bits t_j to determine which 2^r of these perturbation vectors, (or some subset thereof) are valid for the given t_j . Note that if the expanded constellation is not an additive expansion as described above, the perturbation symbols may depend on not only the coset representative offset bits t_j , but also the base symbols g . In this case there may be more than $2^{kn'}$ possible time-domain perturbation vectors.

Frame Based Perturbation Selector

Perturbation selector 124_j (124₀ – 124_{t-1}) is shown in more detail in FIG.

12. For each of the 2^r valid time-domain perturbation vectors $p_{j,i}$, (or some subset thereof), perturbed time-domain block $y_{j,i}$ is computed by block 140, where $y_{j,i} = y_{j-1} + p_{j,i}$. (Note: the input to Perturbation selector 124₀ is $y_{-1} = x$.) Then all of the $y_{j,i}$ perturbed time domain blocks computed are evaluated in block 142 and the $y_j'' = y_{j,i}$ with the smallest peak value is selected as the perturbed time-domain block of symbols to be provided to Perturbation selector 124_{j+1}. (Note: the output of Perturbation selector 124_{t-1} is output to the channel $y_{t-1} = y$.)

Frame Based Offset Decoder

Offset decoder 52a, FIG. 13, includes frame divider 150 which divides the kn valid perturbation offset bits s into f frames of $kn'=kn/f$ bits each. Each frame s_0-s_{f-1} is provided to a matrix block (152₀-152_{f-1}). In the j th matrix block, the j th frame of $1 \times kn'$ valid perturbation offset bits is multiplied (modulo 2) (i.e., filtered) by matrix H^T having kn' rows and $kn'-r$ columns to recover the j th frame of $1 \times (kn' - r)$ offset information bits v'_j . The f frames of offset information bits $v'_0-v'_{f-1}$ are passed to frame concatenator 154 which concatenates the f frames to form an estimate of the $kn-r$ offset information bits v' .

In order to demonstrate how each of the candidates of valid perturbation offset bits s_j are decoded to the same offset information bits v_j , the encoding and decoding processes must be expressed mathematically. The offset information bits recovered, v'_j , (decoding) can be expressed mathematically as follows:

$$v'_j = s_j H^T \quad (6)$$

and the valid perturbation offset bits s_j (encoding) can be expressed mathematically as follows:

$$s_j = v_j H^{-T} + rG, \quad (7)$$

where $c = rG$ is a valid codeword generated by perturbation codeword generator 126, FIG. 11. If the right hand side of equation (7) is substituted into equation (6) for s , then the following equation is derived:

$$v'_j = v_j H^{-T} H^T + rGH^T \quad (8)$$

By selecting G , H^T and H^{-T} so that the following conditions are satisfied: (1) $H^{-T} H^T = I$ (where I is the identity matrix); and (2) $GH^T = 0$, then $v'_j = v_j$ regardless of the value of r

Splitterless Operation

In an Asymmetric Digital Subscriber Line (ADSL) modem operating without a splitter, the transmitted ADSL signal results in interference in the voice band (0-4kHz) at the POTS phone. This interference is the result of inter-modulation effects due to the non-linear devices in the POTS phone. This interference can be reduced by using the present invention above to improve an appropriate objective function of the transmitted signal. One possible

objective function is to create a notch in the spectrum of the transmitted ADSL signal after it goes through the non-linearity at 2kHz.

Let the subscript k denote time. Then $X(k)$ denotes the k th unperturbed time-domain DMT symbol block and $x(k+1)$ denotes the $k+1^{\text{st}}$ block etc.

5 Similarly, y_k denotes the k th transmitted perturbed time-domain DMT symbol block and $y(k+1)$ denotes the $k+1^{\text{st}}$ symbol block etc. Let $Z(k)$ denote the output of the spectrum calculator 164 Fig. 14, i.e. the spectrum of the transmitted signals y transmitted up to time k , after the POTS non-linearity, evaluated at 2kHz.

10 This objective function can be improved using the above described inventions. In fact, this can be achieved by incorporating the new objective function in the selection criterion of the perturbation selector. As shown in Fig 14, perturbation selector 72a of this embodiment consists of a perturber 160, a non-linear device 162, a spectrum calculator 164, and a selector 166. The
15 perturber 160 modifies the nominal time domain block x with each of the valid perturbation vectors to produce candidate transmit blocks y_i . These blocks are provided to the non-linear device 162 which mimics the POTS non-linearity. Spectrum calculator 164 computes the power of the non-linear distorted signals around 2 kHz and selector 166 chooses the candidate perturbed time-
20 domain block y_i that minimizes the output of the spectrum calculator 164.

In our examples above we assumed the "additive" constellation expansion as described in conjunction with Fig 2. For this particular objective function, further benefit may be obtained by using an alternative constellation expansion as described below. In particular, as will become clear,
25 implementation complexity can be significantly reduced.

A rotated expanded constellation 170, FIG. 15, is formed by rotating the symbols in base constellation 172, rather than by shifting the base constellation (which was referred to as an additive constellation expansion earlier) as shown in Fig. 2.

30 In the specific example shown in Fig 15, we consider a DMT system with base constellation 172 capable of transmitting two bits per symbol. Base constellation 172 contains points A, B, C, and D, from which the base constellation symbols are selected by the base constellation mapper 16, FIG. 1. The base constellation 172 is expanded by a factor of 4 to form a 16 point
35 constellation. Thus, $k=2$ bits per symbol are needed to determine which of the equivalent points in the expanded constellation is transmitted. Expanded constellation 170 includes base constellation 172 and expansion areas 174, 176, and 178 each containing four points labeled A-D. Expanded constellation

170 is formed from the base constellation 172 by rotating each of the points in the base constellation by 0° , 90° , 180° , and 270° .

For each of the n base symbols, g , base constellation mapper 16 chooses a point in base constellation 172. The expanded constellation mapper 20, FIG. 1, uses the kn or $2n$ (two bits per symbol) coset representative offset bits t to rotate the n symbols by 0° , 90° , 180° , or 270° . One way to define the mapping of the two bits per symbol is: 00 corresponds to a 0° rotation, 01 corresponds to a 90° rotation, 11 corresponds to a 180° rotation, and 10 corresponds to a 270° rotation,

To reduce the implementation complexity, this scheme uses perturbation codewords that do not require re-computing the IDFT, i.e., the rows of the matrix G of perturbation codeword generator 80, FIG. 1, having r rows and kn or $2n$ columns are chosen such that the codewords c_i generated from this matrix lead to perturbed time-domain blocks y_i that can easily be obtained from the nominal time-domain block x . Recall, G , H^T , and H^{-T} must be chosen such that $GH^T=0$, and $H^{-T}H^T=I_{2n \times 2n}$, where $I_{2n \times 2n}$ is the $2n \times 2n$ identity matrix.

This reduced complexity scheme can be achieved by allowing three rows for matrix G , which also corresponds to $r=3$ redundancy bits. Assuming $n=8$, the three rows are:

1. 11 11 11 11 11 11 11 11, which corresponds to a sign inversion of x .
2. 00 11 00 11 00 11 00 11, which corresponds to a circular rotation of x by $N/2$ samples.
3. 00 01 11 10 00 01 11 10, which corresponds to a circular rotation of x by $N/4$ samples.

In this specific example, there are $r=3$ redundancy bits which corresponds to $2^3=8$ possible perturbations y_i . The selector 166 selects the best of these 8 perturbations to minimize the output of the spectrum calculator 164 i.e., to create a null at 2kHz in the spectrum of the transmitted blocks y after they are distorted by the POTS non-linearity. Note that although these perturbations do not change the peak of the transmitted symbols, they can be used to shape the non-linearly distorted spectrum of the transmitted symbols.

The perturbation selector 72a, FIG. 14, can improve its performance by incorporating look-ahead. With a one-step look-ahead, the perturbation selector would select the perturbed time-domain block $y(k)$ such that $y(k)$ in combination with the best choice for $y(k+1)$ creates the deepest null in the spectrum Z_{k+1} . With a Δ -step look-ahead, the perturbation selector operating on the k th block would select the perturbed time-domain block $y(k)$ such that

$y(k)$ in combination with the best choice for $y(k+1) - y(k+\Delta)$ that minimizes the output of the spectrum calculator.

Although we have described an uncoded system in our examples, this invention can be easily extended to the case of a coded system, for example a trellis-coded system, in a manner that will be apparent to those skilled in the art.

It should be noted that this invention may be embodied in software and/or firmware, which may be stored on a computer useable medium, such as a computer disk or memory chip. The invention may also take the form of a computer data signal embodied in a carrier wave, such as when the invention is embodied in software/firmware, which is electrically transmitted, for example, over the Internet.

The present invention may be embodied in other specific forms without departing from the spirit or essential characteristics. The described embodiments are to be considered in all respects only as illustrative and not restrictive. The scope of the invention is, therefore, indicated by the appended claims rather than by the foregoing description. All changes, which come within the meaning and range within the equivalency of the claims, are to be embraced within their scope.

What is claimed is:

Claims

1. A device for improving a defined property of transform-domain symbols, the device, comprising:
 - 5 a signal mapper which maps input data into blocks of symbols in a first domain and which generates offset bits corresponding to each block of symbols; and
 - a perturbation/transform device, responsive to the blocks of symbols in the first domain and the corresponding offset bits, which produces blocks of perturbed transform domain symbols in order to improve the defined property of the transformed symbols.
2. The device of claim 1 wherein the perturbation/transform device includes:
 - 15 an invertible transform device, responsive to the signal mapper, which transforms each block of symbols in the first domain into a block of transform domain symbols; and
 - a perturbation device, responsive to the invertible transform device and the corresponding offset bits, which selectively perturbs each block of transform domain symbols to form the blocks of perturbed transform domain symbols.
3. The device of claim 1 wherein the perturbation/transform device includes:
 - 25 a perturbation device, responsive to the signal mapper and the corresponding offset bits, which selectively perturbs each block of symbols to form blocks of perturbed symbols; and
 - an invertible transform device, responsive to the perturbation device, which transforms the blocks of perturbed symbols in the first domain into the blocks of perturbed transform domain.
4. A receiver for receiving and decoding symbols created by the perturbation/transform device of claim 1.
5. The device of claim 1 further characterized by: a selector for selecting
35 a block of perturbed transform domain symbols that meets a predetermined criteria.



6. The device of claim 5 wherein the predetermined criteria is a the smallest Peak to Average power Ratio.
7. The device of claim 5 wherein the predetermined criteria is the least amount of nonlinear interference in a voice band.
8. The device of claim 5 further characterized by a transmitter for transmitting the selected block of perturbed transform domain symbols.
9. A receiver for receiving blocks of perturbed symbols characterized by: a transforming device to transform the received blocks of perturbed symbols into frequency domain symbols; an offset extractor for extracting base symbols and perturbation offset bits and transforming said base symbols into base information bits; and an offset decoder for transforming the perturbation offset bits into offset information bits.

