

SELF CALIBRATION FOR ANGULAR RATE ESTIMATION FOR AXISYMMETRIC CORIOLIS VIBRATING GYROSCOPES

TECHNICAL FIELD

The present disclosure, in some embodiments, thereof, relates to an axisymmetric Coriolis vibrating gyroscope and, more particularly, but not exclusively, to determining the bias of an axisymmetric Coriolis vibrating gyroscope.

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BACKGROUND

An axisymmetric Coriolis vibrating gyroscope (CVG) is a sensor used to measure angular velocity based on the Coriolis effect. Axisymmetric CVGs have two well-known modes of operation: The Force To Rebalance (FTR) and Whole Angle (WA) modes.

10 In FTR mode, the vibrating wave is maintained at a specific angular direction and the force applied to hold the wave is proportional to the input angular rate, where the proportional constant is the FTR scale factor. In WA mode, for angular velocities greater than the lock-in rate, the wave rotates at an angular rate that is proportional to the input angular rate, where the proportional constant is the geometric scale factor. For angular
15 rates smaller than the lock-in rate, the wave will precess until it locks at a determined angular position. The FTR mode is capable of overcoming this limitation, but its dynamic range is usually restricted by the rebalance force.

The FTR mode is typically used for low dynamic ranges and for north finding since it delivers the highest precision of both modes. The WA mode is typically used for
20 high dynamics and it has the better scale factor stability of both mechanizations.

SUMMARY OF THE INVENTION

According to some embodiments there is provided an apparatus, a system and a method of calibrating the angular rate estimation of a Coriolis vibrating gyroscope.

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In recent years sensor technologies based on vibrating elements are increasingly entering the field of inertial sensors. This is due to many factors, including their simplicity, high MTBF (Mean Time Between Failures), low power consumption and the low cost of production of these kind of devices. Recently, the Hemispherical Resonator

Gyroscope (HRG) has been used for precise north finding achieving the high precision of the mechanical and optical technologies.

Coriolis Vibrating Gyroscopes (CVGs), such as HRGs, are distinguished from other gyroscopes technologies by the fact that they may be self-calibrated at any moment of their life. This capability is being used in order to improve and preserve the performance during their whole operational life.

Pure inertial navigation without access to external data is highly desired in many applications.

For non-ideal CVGs, the lock-in effect limits the range of angular rates where the WA mode can be used. Particularly, in north finding application, the non-linear behavior of the WA mode for external angular rate close to the lock-in rate limits the WA mechanization in the angle or angular rate estimation.

From the physical point of view the lock-in effect is a force that rotates the vibrating wave until eventually it gets stuck at a determined angle given mainly, by the lock-in value and external angular rate together with the position of the azimuth angle of the axis of maximum damping. When operating inside the lock-in range the dynamics becomes highly non-linear. It is determined by the interaction of lock-in and Coriolis forces, where the lock-in force pushes the wave towards the lock-in angle and the Coriolis force tries to cause the wave to precess according to the external angular rate.

Aspects of the disclosure provide a CVG apparatus and method that allows finding the drift components without need for external data, using an approximate analytical formula for WA operation. Once the drift is found, the output angular rate may be corrected mathematically or by applying physical forces to compensate for the CVG bias during operation. By doing so, it is possible to operate the CVG in the WA mode for all dynamic ranges.

Some embodiments of the disclosure use one or more additional gyroscopes to calibrate the gyroscope even when the external angular rate varies with time.

Advantages of some embodiments of the disclosure may include but are not limited to:

- 1) High accuracy is obtained without relying on the classical calibration processes done during the manufacturing of the Inertial Navigation System.
- 2) The effects of aging and environmental conditions are minimized, since the bias they cause may be calibrated or compensated for during the operation of the gyroscope.

Following is a non-exclusive list of some exemplary aspects and embodiments of the disclosure. The present disclosure also includes examples which include fewer than all the features in an example and examples using features from multiple examples, even
5 if not listed below.

1. An axisymmetric Coriolis vibrating gyroscope, comprising:
 - a resonator configured to vibrate in a standing wave;
 - a plurality of control elements, configured to perform at least one of maintaining vibration in the resonator by applying respective forces to the resonator and sensing vibrations of the resonator around an axis of symmetry of the resonator; and
 - a control circuitry associated with the plurality of control elements, configured to:
 - for each one of a set of initial angles of a vibration on a perimeter of the resonator element:

- control the plurality of control elements so as to position an angular directions of the vibration at the initial angle;
 - allow the vibration to precess for a respective time period;
 - calculate respective angular directions of the vibration at a plurality of sampling times during the time period, based on respective readings of at least two of the control elements at the sampling times; and
 - determine a respective linear coefficient of a polynomial fitting the respective angular directions; and

from the respective linear coefficients, determine Fourier coefficients of an angle-dependent bias of the Coriolis vibrating gyroscope (CVG), such that a bias of a measured angular rate of vibration may be corrected using the determined Fourier coefficients.

2. The axisymmetric Coriolis vibrating gyroscope according to embodiment 1, wherein the control circuitry is further configured to:

- determine a current angular rate of vibration of the CVG from current respective readings of at least two of the control elements;

- estimate a bias at a current angular direction of vibration of the CVG based on the determined Fourier coefficients; and

- correct the determined angular rate using the estimated bias.

3. The axisymmetric Coriolis vibrating gyroscope according to embodiment 2, wherein the angular rate is corrected mathematically by subtracting the estimated bias

from the angular rate of vibration determined from current pickoff readings of at least some of the control elements operating in pickoff mode.

4. The axisymmetric Coriolis vibrating gyroscope according to embodiment 2, wherein the angular rate is corrected by adjusting control signals applied to at least some of the control elements to counteract the estimated bias.

5. The axisymmetric Coriolis vibrating gyroscope according to any one of embodiments 1-4, wherein the Fourier coefficients are obtained from a solution of a system of linear equations defined by the respective linear coefficients and the respective angular directions of the vibration at the initial angles.

6. The axisymmetric Coriolis vibrating gyroscope according to embodiment 5, wherein the set of initial angles are selected such that a determinant of a matrix defining the system of linear equations is non-zero.

7. The axisymmetric Coriolis vibrating gyroscope according to any one of embodiments 1-6, wherein for at least one of the respective time periods an angular velocity of the CVG is at least 1 degree/sec.

8. The axisymmetric Coriolis vibrating gyroscope of any one of embodiments 1-7, wherein the control circuitry is further configured to correct for changes in an angular velocity of the CVG using data obtained from a correction gyroscope having a constant bias.

9. The axisymmetric Coriolis vibrating gyroscope of any one of embodiments 1-8, wherein an angular offset between a first one of the initial angles and a second one of the initial angles is different from an angular offset between the second one of the initial angles and a third one of the initial angles, and wherein the second one of the initial angles is immediately successive to the first initial angle and the third one of the initial angles is immediately successive to the second initial angle.

10. A method of estimating a bias in an axisymmetric Coriolis vibrating gyroscope, comprising:

for each one of a set of initial angles of a vibration on a perimeter of a resonator element of an axisymmetric Coriolis vibrating gyroscope:

positioning an angular direction of vibration of the resonator at the initial angle;

allowing the vibration to precess for a respective time period;

calculating a respective angular direction of the vibration at a plurality of sampling times during the respective time period; and

determining a respective linear coefficient of a polynomial fitting the respective angular directions; and

from the respective linear coefficients, determining Fourier coefficients of an angle-dependent bias of the Coriolis vibrating gyroscope (CVG), such that a bias of a measured angular rate of vibration may be corrected using the determined Fourier coefficients.

11. The method of embodiment 10, further comprising:

determining a current angular rate of vibration of the CVG;

estimating a bias at the current angular direction of vibration of the CVG based on the determined Fourier coefficients; and

correcting the determined angular rate using the estimated bias.

12. The method of embodiment 11, wherein the angular rate is corrected mathematically by subtracting the estimated bias from the angular rate of vibration determined from current pickoff readings of at least some control elements operating in pickoff mode.

13. The method of embodiment 11, wherein the angular rate is corrected by adjusting control signals applied to at least some control elements operating in forcing mode to counteract the estimated bias.

14. The method of any one of embodiments 10-13, wherein the Fourier coefficients are obtained from a solution of a system of linear equations defined by the respective linear coefficients and the respective angular directions of the vibration at the initial angles.

15. The method of embodiment 14, further comprising selecting the set of initial angles such that a determinant of a matrix defining the system of linear equations is non-zero.

16. The method of any one of embodiments 10-15, wherein for at least one of the respective time periods an angular velocity of the CVG is at least 1 degree/sec.

17. The method of any one of embodiments 10-16, further comprising correcting for changes in an angular velocity of the CVG using data obtained from a correction gyroscope having a constant bias.

18. The method of any one of embodiments 10-17, wherein an angular offset between a first one of the initial angles and a second one of the initial angles is different from an angular offset between the second one of the initial angles and a third one of the

initial angles, and wherein the second one of the initial angles is immediately successive to the first initial angle and the third one of the initial angles is immediately successive to the second initial angle.

Unless otherwise defined, all technical and/or scientific terms used within this document have meaning as commonly understood by one of ordinary skill in the art/s to which the present disclosure pertains. Methods and/or materials similar or equivalent to those described herein can be used in the practice and/or testing of embodiments of the present disclosure, and exemplary methods and/or materials are described below. Regarding exemplary embodiments described below, the materials, methods, and examples are illustrative and are not intended to be necessarily limiting.

Some embodiments of the present disclosure may be embodied as a system, method, or computer program product. For example, some embodiments of the present disclosure may take the form of an entirely hardware embodiment or an embodiment combining software and hardware aspects that may all generally be referred to herein as a “circuit,” “circuitry,” “module” and/or “system.”

Implementation of the method and/or system of some embodiments of the present disclosure may involve performing and/or completing selected tasks manually, automatically, or a combination thereof. According to actual instrumentation and/or equipment of some embodiments of the method and/or system of the present disclosure, several selected tasks could be implemented by hardware, by software or by firmware and/or by a combination thereof, e.g., using an operating system.

For example, hardware for performing selected tasks according to some embodiments of the present disclosure could be implemented as a resonator coupled to control elements and/or a circuit. As software, selected tasks according to some embodiments of the present disclosure could be implemented as a plurality of software instructions being executed by a computational device e.g., using any suitable operating system.

In some embodiments, one or more tasks according to some exemplary embodiments of method and/or system as described herein are performed by a data processor, such as a computing platform for executing a plurality of instructions. Optionally, the data processor includes a volatile memory for storing instructions and/or data and/or a non-volatile storage e.g., for storing instructions and/or data. Optionally, a

network connection is provided as well. User interface/s e.g., display/s and/or user input device/s are optionally provided.

Some embodiments of the present disclosure may be described below with reference to flowchart illustrations and/or block diagrams. For example illustrating
5 exemplary methods and/or apparatus (systems) and/or and computer program products according to embodiments of the present disclosure.

The flowchart and block diagrams in the Figures illustrate the architecture, functionality, and operation of possible implementations of systems, methods, and computer program products according to various embodiments of the disclosed subject
10 matter. In this regard, each block in the flowchart or block diagrams may represent a module, segment, or portion of instructions which comprises one or more executable instructions for implementing the specified logical function(s). In some alternative implementations, the functions noted in the block may occur out of the order noted in the figures. For example, two blocks shown in succession may, in fact, be executed
15 substantially concurrently, or the blocks may sometimes be executed in the reverse order, depending upon the functionality involved. It will also be noted that each block of the block diagrams and/or flowchart illustration, and combinations of blocks in the block diagrams and/or flowchart illustration, can be implemented by special purpose hardware-based systems that perform the specified functions or acts or carry out combinations of
20 special purpose hardware and computer instructions.

BRIEF DESCRIPTION OF THE DRAWINGS

In order to understand the invention, embodiments will now be described, by way of non-limiting example only, with reference to the accompanying drawings. Features
25 shown in the drawings are meant to be illustrative of only some embodiments of the present disclosure, unless otherwise indicated. In the drawings like reference numerals are used to indicate corresponding parts.

In block diagrams and flowcharts, optional elements/components and optional stages may be included within dashed boxes.

30 In the figures:

Figs. 1A-1B are simplified side and top sectional views respectively of an exemplary Hemispherical Resonator Gyroscope, according to some examples of the presently disclosed subject matter;

Figs. 2A-2B are simplified views illustrating vibration patterns of an HRG, according to some examples of the presently disclosed subject matter;

Fig. 3 is a simplified block diagram showing basic components of an axisymmetric Coriolis vibrating gyroscope, according to some embodiments of the disclosure;

Fig. 4 is a simplified flowchart of a method of estimating a bias in an axisymmetric Coriolis vibrating gyroscope, according to some aspects of the disclosure;

Fig. 5 is a simplified flowchart of a method of correcting the angle-dependent bias of a CVG, according to embodiments of the disclosure; and

Figs. 6 is a simplified diagram of the time evolution of the angle of vibration at respective starting angular positions and respective angular rates, according to exemplary aspects of the disclosure;

Fig. 7 is a simplified diagram of the time dynamics of the angular position of the vibrating wave, according to an example of the disclosure;

Fig. 8 is a simplified diagram of the angular velocity calculated from the data shown in Fig. 7;

Fig. 9 is a simplified diagram of the calibrated angular velocity using the bias function, according to the example of Fig. 8; and

Figs. 10-11 are simplified diagrams of the time evolution of angular positions of vibrating waves at respective starting angular positions and respective angular rates, according to exemplary aspects of the disclosure

The various embodiments of the present disclosure are described below with reference to the drawings, which are to be considered in all aspects as illustrative only and not restrictive in any manner.

Elements illustrated in the drawings are not necessarily to scale, emphasis instead being placed upon clearly illustrating the principles of embodiments of the present disclosure. Moreover, two different objects in the same figure may be drawn to different scales.

30 DETAILED DESCRIPTION OF EMBODIMENTS

The present disclosure, in some embodiments, thereof, relates to an axisymmetric Coriolis vibrating gyroscope and, more particularly, but not exclusively, to determining the bias of an axisymmetric Coriolis vibrating gyroscope.

The principles, uses and implementations of the teachings herein may be better understood with reference to the accompanying description and figures. Upon perusal of the description and figures present herein, one skilled in the art will be able to implement the teachings herein without undue effort or experimentation.

5 Before explaining at least one embodiment in detail, it is to be understood that the invention is not necessarily limited in its application to the details of construction and the arrangement of the components and/or methods set forth in the following description and/or illustrated in the drawings and/or the Examples. The invention is capable of other embodiments or of being practiced or carried out in various ways.

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I. Coriolis vibrating gyroscope

According to some aspects of the disclosure, a CVG determines an angle-dependent bias of the Coriolis vibrating gyroscope (CVG) based on readings its control elements, without requiring external information. The measured angular rate of vibration may be corrected using the self-determined bias.

According to embodiments of the disclosure, the angular direction of vibration of the CVG resonator is sequentially positioned at multiple initial angles on the CVG resonator. For each initial angle, the vibration is allowed to precess freely for a respective
15 time period. While the vibration is precessing, readings are collected from the control elements at several sampling times during the time period. As described in more detail below, readings collected at multiple initial angles are used to determine Fourier coefficients which may be used to estimate an angle-dependent bias in the angular rate of vibration of the CVG. The estimated bias may be used to correct CVG measurements,
20 thereby cancelling or significantly reducing the effect of the bias on the angular directions calculated from the CVG readings.

As used herein, according to some aspects of the disclosure, the terms “initial angle” and “starting angular position” mean an angular direction that the vibration is forced to before being allowed to precess freely.

25 As used herein, according to some aspects of the disclosure, the term “precess freely” and “allowed to precess” mean that no forces are applied to the resonator in order to influence the angular direction of vibration.

A CVG according to aspects of the disclosure includes a resonator which is driven into oscillation at its natural frequency. When the gyroscope experiences angular rotation,

the Coriolis effect induces secondary vibrations orthogonal to the original oscillation plane.

The CVG also includes at least two control elements which are coupled to the resonator. The control elements are used for two functions, forcing and pickoff. Forcing provides a driving force which controls the direction and amplitude of the resonator vibration. Forcing may be used for functions such as maintaining the resonator's resonant frequency, bias and quadrature correction, preventing amplitude decay and controlling precession of the vibration. Pickoff senses the motion of vibration of the resonator. Pickoff readings from two or more control elements may be used to determine the direction of resonator vibration.

The physical mechanism that couples the control elements and the resonator depends on factors such as their type, the materials they are formed from, and so forth. In one example the coupling between the resonator and the control elements is capacitive. In another example the coupling between the resonator and the control elements is piezo-electric.

Referring now to the drawings, Figs. 1A-1B are simplified side and top sectional views respectively of an exemplary Hemispherical Resonator Gyroscope, according to some examples of the presently disclosed subject matter. Resonator 1 in Figs. 1A-1B is shown without vibration.

For ease of explanation, in some figures according to the disclosure the sizes of some or all of the components of the CVG (e.g. HRG) may be out of proportion and/or exaggerated in size.

Fig. 1A shows a side view of HRG 100. HRG 100 includes resonator 1 (optionally made of coated fused silica), and base 2 (optionally made of coated fused silica) which includes electrodes (not shown) and case 3. Resonator 1 has a stem 1.5 that goes through base 2.

In the exemplary embodiment illustrated in Fig. 1B, base 2 has eight electrodes 4.1-4.8 operating as control elements. Together with resonator 1, electrodes 4.1-4.8 form capacitors which are used to establish a standing wave in resonator 1 and to control it. In the current example, the same capacitors are also used for pick-off (e.g. detection of the position of the standing wave with respect to the gyroscope case).

Other embodiments may include a different number of control elements.

In alternate examples, the control elements are divided into electrodes that are used only for controlling the resonator vibration (i.e. forcing) and other electrodes that are used only for sensing the resonator movement (i.e. pickoff). In additional examples, some electrodes are used only for forcing, some are used only for pickoff and at least one
5 electrode is used for both forcing and pickoff.

Fig. 1B is a top view of HRG 100, showing resonator 1, base 2, case 3 and electrodes 4.1-4.8 in base 2. In order to establish a standing wave in the resonator, appropriate forces are applied to resonator 1 using electrodes 4.1-4.8.

Reference is now made to Figs. 2A-2B which are simplified views illustrating
10 vibration patterns of an HRG, according to some examples of the presently disclosed subject matter.

Fig. 2A depicts a vibration pattern of resonator 1 when no external angular velocity is applied. Fig. 2A shows the second mode of vibration of resonator 1 of a standing wave 5.1. This vibration pattern is characterized by two different position
15 patterns: the first is the position where the standing wave has a maximum amplitude (located at 0, 90, 180 and 270 degrees) and where the standing wave has a minimum amplitude (nodes) (located at 45, 135, 225 and 315 degrees).

Note that the nodes and antinodes are orthogonal in the sense of 45 degrees as shown in Fig. 2A. This defines the x and y axes, and also the $x(t)$ and $y(t)$ gyroscope
20 output signals.

Fig. 2B depicts a vibration pattern 5.2 of resonator 1 when an external angular velocity is applied to HRG 100. Vibration pattern 5.2 responds to the inertial rotation of the resonator 1 by precessing about its z axis (which is 90° orthogonal to the x and y axes) with an angular velocity, where:

$$\eta = \frac{\Omega_{output}}{\Omega_{input}}$$

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is the geometric scale factor (also denoted Bryan's scale factor). Note that when HRG 100 rotates counterclockwise the vibration pattern precesses clockwise. The nodes and antinodes precess, therefore by measuring their position it is possible to know the angular position of the vibration pattern and using the gyroscope scale factor to know how the
30 gyroscope rotated with respect to inertial space.

Reference is now made to Fig. 3, which is a simplified block diagram illustrating basic components of an axisymmetric Coriolis vibrating gyroscope, according to some embodiments of the disclosure. Fig. 3 is presented for illustrative purposes and is not limiting as to the type, shape, size and so forth of the components of the CVG described
5 herein.

CVG 300 includes resonator 310, two or more control elements 321.1-321.n and control circuitry 330.

As used herein, according to some aspects of the disclosure, the term “control element” means a hardware element that converts one form of energy to another. In
10 aspects of this disclosure a control element may serve for forcing and/or pickoff.

As used herein, according to some aspects of the disclosure, the terms “pickoff” and “pickoff mode” mean sensing the motion of the resonator precessing wave so that the resonator’s precessing wave angular direction of vibration may be determined.

As used herein, according to some aspects of the disclosure, the term “forcing”
15 and “forcing mode” means maintaining the vibratory motion of the resonator and controlling the angular direction of the vibratory motion. Optionally, forcing also includes applying suitable forces to the resonator in order to induce resonator vibration and/or maintain amplitude of the resonator vibration and/or to minimize errors caused by mechanical, electronic and/or algorithm defects.

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II. Resonator

Resonator 310 is configured to vibrate in a standing wave. The resonator may be any resonator suitable for use in an axisymmetric CVG. There are many types of resonators having an axisymmetric structure that are known in the art, and others may be developed in the future. Following is a non-limiting list of examples resonator types.

25 In one example, the resonator is a hemisphere resonator.

In a second example, the resonator is a wineglass resonator.

In a third example, the resonator is a disk resonator.

In a fourth example, the resonator is a shell resonator.

In a fifth example, the resonator is cylindrical resonator.

30 In a sixth example, the resonator is honeycomb resonator.

III. Control element

CVG 300 includes multiple control elements 321.1-321.n, where n is at least two. At least two of the control elements are capable of operating in forcing mode and at least two of the control elements are capable of operating in pickoff mode.

For descriptive purposes, Fig. 3 schematically shows control elements 321.1-
5 321.n as aligned with each other. However, the control elements are typically physically distanced from each other and arranged around the resonator.

A control element operating in forcing mode maintains vibration in resonator 310 by applying respective forces to the resonator. Signals from the control elements are also used to control the direction of angular vibration of resonator 310 and to position the
10 angular direction of vibration at a desired angle. The forces applied by the control element are controlled by signals (e.g. voltages) provided by control circuitry 330.

According to optional aspects of the disclosure, the forces applied by the control elements to the resonator may be adapted to perform additional functions, such as quadrature correction and/or or self-calibration.

15 A control element operating in pickoff mode senses vibrations of resonator 310 and outputs signals to control circuitry 330. Control circuitry 330 processes the signals from control elements 321.1-321.n to determine the angular velocity of CVG 300 and to produce control signals for control elements 321.1-321.n.

Control elements may operate:

- 20 a) Only in pickoff mode;
b) Only in forcing mode; or
c) May alternate between forcing mode and pickoff mode.

A control element may be any type of element known in the art which is suitable for forcing and/or pickoff in an axisymmetric CVG.

25 In one example, at least one of the control elements is a piezoelectric element.

In a second example, at least one of the control elements is an electromagnetic element.

In a third example, at least one of the control elements is a capacitive element.

In a fourth example, at least one of the control elements is an optical element.

30 In a fifth example, at least one of the control elements is a piezoresistive element.

In a sixth example, the control elements are of multiple types, such as any combination of the above examples.

According to some aspects of the invention, at least one control element is physically distanced from resonator 310 (e.g. on a base supporting resonator 310). When the control element is not physically coupled to resonator 310, applying a signal to the control element causes it to interact with resonator 310 or with an element physically
5 attached to the resonator, in a way that controls the resonator vibrations. For example, the control unit may be an electrode in the base, where applying a voltage to the electrode creates a capacitance with the coating on resonator 310.

Alternately or additionally, at least one control element is physically coupled to resonator 310 (e.g. glued to the resonator). For example, the control element may be a
10 piezoelectric element which is glued to resonator 310 and is connected to a voltage source. When a voltage is applied to the piezoelectric element, the piezoelectric element vibrates causing a vibration in resonator 310.

IV. Control circuitry

According to embodiments of the disclosure, control circuitry 330 inputs signals
15 from control elements which are operating in pickoff mode and provides control signals to control elements which are operating in forcing mode. Control circuitry 330 performs analog and/or digital processing operations according to any aspects of the disclosure, and optionally to provide additional functionality.

In order to obtain the information used for bias estimation, the angular direction of vibration of resonator 310 is forced in sequence to two or more initial angles. According to some examples of the disclosure, control circuitry 330 controls control elements 321.1-321.n to position the angular direction of the vibration at each of the initial angle in turn. Once the angular direction of the vibration is at a desired initial angle, the vibration is allowed to precess freely for a respective time period. During this time period, control circuitry 330 obtains readings (e.g. pickoff signals) from at least two control elements at multiple sampling times and calculates the respective angular direction at each sampling time.

After readings have been obtained for all the initial angles, control circuitry 330 calculates coefficients which may be used to estimate the CVG bias according to aspects of the disclosure.

For each initial angle, a polynomial equation that fits the calculated angular directions for the given initial angle may be represented as:

$$\theta(t) \approx \theta_0 + p_1(\theta_0, \Omega)t + \sum_{j=1}^M p_{j+1}t^{j+1}$$

where $\theta(t)$ is an azimuth angle, M is an integer, $\theta_0 \equiv \theta(0)$ and t denotes time (where $t = 0$ is the time at which the angular direction begins to precess freely). The linear coefficient of the polynomial is $p_1(\theta_0, \Omega)$.

Respective linear coefficients are determined for each initial angle, by performing polynomial fitting to the angular directions collected during the period of time the angular direction precessed freely from that initial angle.

Equation (10) below presents a more detailed example of polynomial fitting of the calculated angular directions, according to an aspect of the disclosure.

Fourier coefficients of an angle-dependent bias of CVG 300 are determined from the respective linear coefficients, as described in more detail herein. These Fourier coefficients may be used to correct the bias of an angular rate of vibration measured by CVG 300.

According to some aspects of the disclosure, the Fourier coefficients are obtained by solving a system of linear equations defined by the respective linear coefficients and the respective angular directions of the vibration at the initial angles.

According to some aspects of the disclosure, the set of initial angles are selected such that a determinant of a matrix defining the system of linear equations is non-zero.

According to some examples, control circuitry 330 performs bias correction during CVG operation. Control circuitry 330 determines the angular rate of vibration of the CVG from readings from at least two control elements. The bias at the determined angular direction of vibration is estimated based on the Fourier coefficients, and the determined angular rate is corrected using the estimated bias.

In one embodiment of the disclosure, the bias is corrected mathematically by subtracting the bias from the angular rate of vibration determined from the current pickoff readings. For example, the Fourier parameters may be stored in memory after they are calculated. In order to estimate the bias, the Fourier coefficients are retrieved from the memory, and the bias is calculated mathematically based on their values (e.g. using Eqn. (9) below). The estimated bias is used to correct each reading until the Fourier parameters are recalculated.

In an alternate embodiment of the disclosure, the bias is counteracted by adjusting the control signals applied to the control elements operating in forcing mode.

One advantage of the axisymmetric CVG with bias estimation presented herein is its ability to estimate the bias even at relatively high angular velocities. In one example, the angular velocity of the CVG is at least 1 degree/sec for at least one of the time periods the vibration is precessing freely from the respective initial angle.

According to some embodiments of the invention, the initial angles are not distributed evenly around the resonator perimeter. For example, for three successive initial angles, the angular offset between the first initial angle and the second initial angle may be different from the angular offset between the second initial angle and the third initial angle.

According to some examples, control circuitry 330 corrects for changes in an angular velocity of the CVG using data obtained from a separate gyroscope which itself has a constant bias. Examples of suitable gyroscopes include but are not limited to: CVG, ring laser gyroscope (RLG) and fiber-optic gyroscope (FOG).

According to some embodiments of the disclosure, control circuitry 330 includes a demodulator for demodulating signals obtained from the CVG (e.g. signals from control elements operating in pickoff mode).

According to some embodiments of the disclosure, control circuitry 330 includes
5 at least one analog to digital converter (A/D) for converting analog signals from the CVG to digital signals (possibly after analog processing) and/or to convert signals to be supplied to forcing control elements from digital to analog signals.

According to some embodiments of the disclosure, control circuitry 330 includes
10 an oscillator (e.g. a voltage controlled oscillator) which provides a reference frequency signal for use in one or more control loops (e.g. phase-locked loop which synchronizes the resonator's vibration frequency).

According to some embodiments of the disclosure, control circuitry 330 includes
15 at least one processor, configured to perform digital processing operations as required by any example of the disclosure. Non-limiting examples of processor types include: Central Processing Units (CPUs), Digital Signal Processors (DSPs) and Application-Specific Integrated Circuits (ASICs).

According to some embodiments of the disclosure, control circuitry 330 includes non-transitory storage medium storing program instructions which, when executed by the

processor(s), cause the at least one processor to perform the digital processing operations required any example of the disclosure.

V. Feedback loops and additional control circuitry functions

5 In addition to the forcing control loop described above, control circuitry 330 may also implement one or more additional control loops to ensure proper operation of CVG 300.

 In one aspect, control circuitry 330 implements a feedback loop which adjusts the amplitude of the forces applied by the forcing element to maintain a constant resonator
10 vibration amplitude.

 In one aspect, control circuitry 330 implements a phase-locked loop (PLL) which synchronizes the frequency of the resonator's vibration with a reference signal. The reference signal may be provided by an external device and/or may be generated by an internal part of control circuitry 330, such as a voltage controlled oscillator (VCO).

15 In one aspect, control circuitry 330 implements a quadrature control loop which reduces quadrature error.

V. A method of estimating a bias in an axisymmetric CVG

 Reference is now made to Fig. 4, which is a simplified flowchart of a method of
20 estimating a bias in an axisymmetric Coriolis vibrating gyroscope, according to some aspects of the disclosure.

 Optionally, in 410 a vibration is induced in the CVG. Typically, once the vibration is induced it may be maintained by a feedback loop which maintains the resonator at a constant vibration amplitude.

25 As indicated by the arrow from 460 to 420, 420-450 are performed for each initial angle in the calibration cycle. For example, if the bias is being estimated based on measurements at four initial angles, 420-450 are performed four times, once for each initial angle. After the four iterations are performed, a respective linear coefficient value has been determined for each of the four initial angles.

30 In 420, the angular direction of vibration of the resonator is positioned at the initial angle for which a respective linear coefficient is being determined. In 430 the vibration is allowed to precess for the respective time period.

In 440, respective angular directions of the vibration are calculated for multiple sampling times during the time period that the vibration is precessing freely. In 450, a linear coefficient of a polynomial fitting the respective angular directions is determined for the initial angle.

- 5 Based on the decision in 460, 420-450 are repeated for all the initial angles.

When respective linear coefficients have been obtained for all the initial angles, in 470 Fourier coefficients of the angle-dependent bias of the CVG are determined from the linear coefficients. The Fourier coefficients may be used to calculate the angle-dependent bias of the CVG in order to correct the angle-dependent bias of the angular rate of vibration measured by the CVG.

According to some aspects of the disclosure, the Fourier coefficients are obtained by solving a system of linear equations defined by the respective linear coefficients and the respective angular directions of the vibration at the initial angles.

The times at which the Fourier coefficients are calculated may vary. In some embodiments the Fourier coefficients are calculated at one or a combination of:

- i) On turn on of the CVG;
- ii) When there are changes in the operating conditions of the CVG (e.g. temperature);
- iii) Periodically; and
- iv) At system request.

As will be appreciated by the skilled person, the timing for calculating the Fourier coefficients may be selected based on additional or alternate considerations.

According to some aspects of the disclosure, the initial angles are selected such that a determinant of a matrix defining the system of linear equations is non-zero.

According to some aspects of the disclosure, the angular velocity of the CVG is at least 1 degree/sec for at least one of the time periods.

According to some aspects of the disclosure, the method further includes correcting for changes in an angular velocity of the CVG using data obtained from a separate gyroscope having a constant bias.

According to some aspects of the disclosure, the angular offsets between successive initial angles may differ. In this case the initial angles will not be distributed symmetrically around the resonator perimeter.

Reference is now made to Fig. 5 which is a simplified flowchart of a method of correcting the angle-dependent bias of a CVG, according to embodiments of the disclosure.

In 510, the current angular rate of vibration of the CVG is determined. The determination may be performed in any manner known in the art, such as by analysis of pickoff signals from the control elements.

In 520, the bias at the current angular direction of vibration of the CVG is estimated, based on the determined Fourier coefficients.

In 530, the estimated bias is used to correct the current angular rate determined in 510.

As used herein, according to some embodiments of the invention, the term “current angular direction” means the angular direction at the time that the pickoff signals are provided by the control elements.

VI Mathematical model of a non-ideal CVG

A simple model that describes the time evolution of a precessing wave is given by the Lynch's rate equation:

$$5 \quad \frac{d\theta(t)}{dt} = -\eta\Omega + \frac{E\Delta\left(\frac{1}{\tau}\right)}{2n\sqrt{E^2 - Q^2}} \sin 2n[\theta(t) - \theta_\tau] + \frac{Q\Delta\omega}{2n\sqrt{E^2 - Q^2}} \cos 2n[\theta(t) - \theta_\omega] \quad (1)$$

where $\theta(t)$ is the azimuth angle, η is the Bryan's scale factor, Ω is the external angular rate, n the vibration mode, $\Delta\left(\frac{1}{\tau}\right)$ is the difference between the inverse of the damping time constant of x and y axis, θ_τ is azimuth position of the damping principal axis, $\Delta\omega$ is the frequency split, θ_ω is azimuth position of the frequency split principal axis, E is the
10 vibration energy which is kept at a constant value and Q is the quadrature that is usually kept equal to zero.

Equation (1) may be transformed by expanding the trigonometric functions as follows:

$$\begin{aligned} \frac{d\theta(t)}{dt} = & -\eta\Omega + \frac{E\Delta\left(\frac{1}{\tau}\right)}{2n\sqrt{E^2 - Q^2}} [\sin 2n\theta(t) \cos 2n\theta_\tau - \cos 2n\theta(t) \sin 2n\theta_\tau] \\ & + \frac{Q\Delta\omega}{2n\sqrt{E^2 - Q^2}} [\cos 2n\theta(t) \cos 2n\theta_\omega + \sin 2n\theta(t) \sin 2n\theta_\omega] \end{aligned} \quad (2)$$

15 Rearranging terms in Eqn. (2) leads to:

$$\begin{aligned}
\frac{d\theta(t)}{dt} &= -\eta\Omega + \frac{E\Delta\left(\frac{1}{\tau}\right)}{2n\sqrt{E^2 - Q^2}} \sin 2n\theta(t) \cos 2n\theta_\tau \\
&\quad - \frac{E\Delta\left(\frac{1}{\tau}\right)}{2n\sqrt{E^2 - Q^2}} \cos 2n\theta(t) \sin 2n\theta_\tau \\
&\quad + \frac{Q\Delta\omega}{2n\sqrt{E^2 - Q^2}} \cos 2n\theta(t) \cos 2n\theta_\omega \\
&\quad + \frac{Q\Delta\omega}{2n\sqrt{E^2 - Q^2}} \sin 2n\theta(t) \sin 2n\theta_\omega
\end{aligned} \tag{3}$$

Then grouping terms in Eqn. (3) yields:

$$\begin{aligned}
\frac{d\theta(t)}{dt} &= -\eta\Omega + \left[\frac{E\Delta\left(\frac{1}{\tau}\right)}{2n\sqrt{E^2 - Q^2}} \cos 2n\theta_\tau + \frac{Q\Delta\omega}{2n\sqrt{E^2 - Q^2}} \sin 2n\theta_\omega \right] \sin 2n\theta(t) \\
&\quad + \left[\frac{Q\Delta\omega}{2n\sqrt{E^2 - Q^2}} \cos 2n\theta_\omega - \frac{E\Delta\left(\frac{1}{\tau}\right)}{2n\sqrt{E^2 - Q^2}} \sin 2n\theta_\tau \right] \cos 2n\theta(t)
\end{aligned} \tag{4}$$

Defining:

$$\begin{aligned}
\mathcal{S} &\equiv \frac{E\Delta\left(\frac{1}{\tau}\right)}{2n\sqrt{E^2 - Q^2}} \cos 2n\theta_\tau + \frac{Q\Delta\omega}{2n\sqrt{E^2 - Q^2}} \sin 2n\theta_\omega \\
\mathcal{C} &\equiv \frac{Q\Delta\omega}{2n\sqrt{E^2 - Q^2}} \cos 2n\theta_\omega - \frac{E\Delta\left(\frac{1}{\tau}\right)}{2n\sqrt{E^2 - Q^2}} \sin 2n\theta_\tau
\end{aligned} \tag{5}$$

Finally, Eqn. (4) may be written as:

$$\frac{d\theta(t)}{dt} = -\eta\Omega + \underbrace{\mathcal{S} \sin 2n\theta(t) + \mathcal{C} \cos 2n\theta(t)}_{\text{Bias}(\theta(t))} \tag{6}$$

Note that for $Q = 0$ and $E = 1$, Equation (5) reduces to:

$$\begin{aligned}
\mathcal{S} &\equiv \frac{\Delta\left(\frac{1}{\tau}\right)}{2n} \cos 2n\theta_\tau \\
\mathcal{C} &\equiv -\frac{\Delta\left(\frac{1}{\tau}\right)}{2n} \sin 2n\theta_\tau
\end{aligned} \tag{7}$$

10 The term $\text{Bias}(\theta(t)) \equiv \mathcal{S} \sin 2n\theta(t) + \mathcal{C} \cos 2n\theta(t)$ is a bias which depends on the angular position of the vibrating wave i.e. $\theta(t)$. A more general version of Eqn. (6) that accounts for more than one harmonic component may be written as follows:

$$\frac{d\theta(t)}{dt} = -\eta\Omega + \underbrace{\sum_{k=1}^N [\mathcal{S}_k \sin 2kn\theta(t) + \mathcal{C}_k \cos 2kn\theta(t)]}_{B(\theta(t))} \quad (8)$$

Where N is the number of harmonics present in the gyroscope's output.

Approximate Solution of the Generalized Lynch's Rate Equation

- 5 According to some aspects of the disclosure, the bias $B(\theta(t))$ may be canceled by calculating $\mathcal{S}_k$ and $\mathcal{C}_k$, building the term:

$$\sum_{k=1}^N [\mathcal{S}_k \sin 2kn\theta(t) + \mathcal{C}_k \cos 2kn\theta(t)] \quad (9)$$

and cancelling it from the measured angular rate.

- 10 As approximate solution for the angular dynamics (Eqn. (8)) is:

$$\theta(t) \approx \theta_0 + \underbrace{\left[-\eta\Omega + \underbrace{\sum_{k=1}^N (\mathcal{S}_k \sin 2kn\theta_0 + \mathcal{C}_k \cos 2kn\theta_0)}_{B(\theta_0)} \right]}_{p_1(\theta_0, \Omega)} t + \sum_{j=1}^M p_{j+1} t^{j+1} \quad (10)$$

where M is an integer, $\theta_0 \equiv \theta(0)$ and t denotes time.

- Equation (10) describes the time evolution of the angular position of the vibrating wave as a function of time. The term $p_1(\theta_0, \Omega)$, includes the external angular
15 velocity and a bias term i.e. $B(\theta_0)$ which depends on the angular position of the vibrating wave, and therefore leads to drift that depends on the azimuth angle θ_0 . The terms p_j for $j \geq 2$ also depend on the Fourier coefficients, vibration mode and initial angular position. The degree of the polynomial fitting that it is needed to approximate the data may be chosen in order to minimize the residual error.

- 20 According to aspects of the disclosure, the angular time behavior is approximated by a polynomial function of time. As demonstrated by Eqn. (10), measuring the time evolution of the angle $\theta(t)$ when the gyroscope is operated in WA mode enables estimating $p_1(\theta_0, \Omega)$. This estimation may be done by using any method that may fit the data to a polynomial function (for example the Least Square Fitting Method). Then the
25 linear coefficient of the fitting is equal to $p_1(\theta_0, \Omega)$.

Finding $\mathcal{S}_k$ and $\mathcal{C}_k$

In order to cancel N harmonics in Eqn. (8) it is necessary to calculate N terms $\mathcal{S}_k$ and also N terms $\mathcal{C}_k$ which makes a total of $2N$ terms.

5 In one example, when the external angular velocity is not known at the time of the measurement but it is known that the external rate is constant, an additional measurement (i.e. an additional initial angle) may be used to cancel the effects of the constant external rate.

10 In a second example, when the external angular velocity is not known at the time of the measurement and is not constant, the effects of the external rate may be cancelled using information from a second gyroscope as described in more detail below.

A set of measurements $\{1, 2, \dots, 2N + 1\}$ starting at different respective initial angles θ_0^i where $i \in \{1, 2, \dots, 2N + 1\}$ are performed. From these measurements it is possible to estimate a set of values:

$$15 \quad \{p_1(\theta_0^i, \Omega_i)\}_{i=1}^{2N+1} \quad (11)$$

where Ω_i is the external angular velocity during the measurement at position i , Ω_i is known or may be an unknown but constant value. Then:

$$p_1(\theta_0^i, \Omega_i) = -\eta\Omega_i + \sum_{k=1}^N [\mathcal{S}_k \sin 2kn\theta_0^i + \mathcal{C}_k \cos 2kn\theta_0^i] \quad \text{for } 1 \leq i \leq 2N + 1 \quad (12)$$

20 Equation (12) is used to build a system of $2N$ linear equations which are used to calculate $\mathcal{S}_1$ and $\mathcal{C}_1$ as follows.

Without loss of generality, define:

$$\begin{aligned} \mathbb{S}_{j,k} &\equiv \sin 2kn\theta_0^1 - \sin 2kn\theta_0^j \\ \mathbb{C}_{j,k} &\equiv \cos 2kn\theta_0^1 - \cos 2kn\theta_0^j \end{aligned} \quad \text{for } \begin{cases} 1 \leq k \leq N \\ 2 \leq j \leq 2N + 1 \end{cases} \quad (13)$$

$$p_1(\theta_0^1, \Omega_1) - p_1(\theta_0^j, \Omega_j) + \eta(\Omega_1 - \Omega_j) = \sum_{k=1}^N (\mathcal{S}_k \mathbb{S}_{j,k} + \mathcal{C}_k \mathbb{C}_{j,k}) \quad \text{for } 2 \leq j \leq 2N + 1 \quad (14)$$

Equation (14) may be written in matrix form as:

$$\begin{aligned}
 & \begin{pmatrix} \mathbb{S}_{2,1} & \mathbb{C}_{2,1} & \cdots & \mathbb{S}_{2,k} & \mathbb{C}_{12,k} & \cdots & \mathbb{S}_{1,2,N} & \mathbb{C}_{1,2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbb{S}_{j,1} & \mathbb{C}_{j,1} & \cdots & \mathbb{S}_{j,k} & \mathbb{C}_{j,k} & \cdots & \mathbb{S}_{1,j,N} & \mathbb{C}_{1,j,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbb{S}_{2N+1,1} & \mathbb{C}_{2N+1,1} & \cdots & \mathbb{S}_{2N+1,k} & \mathbb{C}_{2N+1,k} & \cdots & \mathbb{S}_{1,2N+1,N} & \mathbb{C}_{1,2N+1,N} \end{pmatrix} \begin{pmatrix} \mathcal{S}_1 \\ \mathcal{C}_1 \\ \vdots \\ \mathcal{S}_k \\ \mathcal{C}_k \\ \vdots \\ \mathcal{S}_N \\ \mathcal{C}_N \end{pmatrix} \\
 &= \begin{pmatrix} p_1(\theta_0^1, \Omega_1) - p_1(\theta_0^2, \Omega_2) + \eta(\Omega_1 - \Omega_2) \\ \vdots \\ p_1(\theta_0^1, \Omega_1) - p_1(\theta_0^j, \Omega_j) + \eta(\Omega_1 - \Omega_j) \\ \vdots \\ p_1(\theta_0^1, \Omega_1) - p_1(\theta_0^{2N+1}, \Omega_{2N+1}) + \eta(\Omega_1 - \Omega_{2N+1}) \end{pmatrix} \quad (15)
 \end{aligned}$$

Defining:

$$M_{\mathbb{S},\mathbb{C}} \equiv \begin{pmatrix} \mathbb{S}_{2,1} & \mathbb{C}_{2,1} & \cdots & \mathbb{S}_{2,k} & \mathbb{C}_{2,k} & \cdots & \mathbb{S}_{1,2,N} & \mathbb{C}_{1,2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbb{S}_{j,1} & \mathbb{C}_{j,1} & \cdots & \mathbb{S}_{j,k} & \mathbb{C}_{j,k} & \cdots & \mathbb{S}_{1,j,N} & \mathbb{C}_{1,j,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbb{S}_{2N+1,1} & \mathbb{C}_{2N+1,1} & \cdots & \mathbb{S}_{2N+1,k} & \mathbb{C}_{2N+1,k} & \cdots & \mathbb{S}_{2N+1,N} & \mathbb{C}_{2N+1,N} \end{pmatrix} \quad (16)$$

5 In order for the system of equations (15) to have a solution, the set of initial angles θ_0^i should be selected such that:

$$\det(M_{\mathbb{S},\mathbb{C}}) \neq 0 \quad (17)$$

10 Note that $\Omega_1 - \Omega_j$ are equal to zero, if they are the same values for all the measurement. If not, they may be determined using an extra gyroscope.

Example 1

In the example presented herein, synthetic data is generated where the harmonic error is known (i.e. $\mathcal{S}_1$ and $\mathcal{C}_1$). To make the example close to real world data, noise is added to the synthetic data.

Synthetic Angle Data

In the instant example, the gyroscope works in mode $n = 2$ with a scale factor $\eta = 0.5$ and has a bias which has one harmonic component i.e. $N = 1$. The bias may be expressed as follows:

$$\text{Bias}(\theta(t)) = \mathcal{S}_1 \sin 4\theta(t) + \mathcal{C}_1 \cos 4\theta(t) \quad (18)$$

Taking the following values for the harmonic components $\mathcal{S}_1 = 6.202440$ deg/sec and $\mathcal{C}_1 = 3.580986$ deg/sec. As described above, three
 5 measurements are needed in order to estimate $\mathcal{S}_1$ and $\mathcal{C}_1$. Without loss of generality, the following initial angles are used:

$$\begin{aligned} \theta_0^1 &= 5 \text{ deg} \\ \theta_0^2 &= 8 \text{ deg} \\ \theta_0^3 &= 17 \text{ deg} \end{aligned} \quad (19)$$

The closer the initial angles are, the less time needed to move the vibrating wave
 10 to the next initial angle. This reduces the time needed to obtain data for bias estimation.

In the example, the gyroscope is exposed to different angular velocities during the simulation, that is $\Omega_1 = -1$ deg/sec, $\Omega_2 = 0.5$ deg/sec, $\Omega_3 = -0.75$ deg/sec, for positions 1, 2 and 3 respectively. Eqn. (8) for the special case of one harmonic component and vibration mode 2 gives:

$$15 \quad \frac{d\theta(t)}{dt} = -\eta\Omega + \mathcal{S}_1 \sin 4\theta(t) + \mathcal{C}_k \cos 4\theta(t) \quad (20)$$

Equation (20) is used to generate synthetic data according to the previous parameters. The data is generated by numerically solving Eqn. (20). The time step, used in the example, is $\Delta t = 0.001$ sec and equal to the sampling time. Three data sets are built
 20 each one corresponding to one of the three starting angular positions.

Fig. 6 illustrates the time evolution of the angle of vibration for $\theta_1(t)$, $\theta_2(t)$ and $\theta_3(t)$:

- i) For $\theta_1(t)$ - The starting angular position is θ_0^1 and the external angular rate is equal to -1 deg/sec ($\Omega_1 = -1$ deg/sec).
- 25 ii) For $\theta_2(t)$ - The starting angular position is θ_0^2 and the external angular rate is equal to 0.5 deg/sec ($\Omega_2 = 0.5$ deg/sec).
- iii) For $\theta_3(t)$ - The starting angular position is θ_0^3 and the external angular rate is equal to -0.75 deg/sec ($\Omega_3 = -0.75$ deg/sec).

To simulate pick-off noise, Gaussian white noise with standard deviation $\sigma = 0.001$ deg was added to the angular positions in all three simulations (i.e. to $\theta_1(t)$, $\theta_2(t)$ and $\theta_3(t)$).

5 Polynomial Fit of the Synthetic Data

A polynomial fit using the Least Square Method was performed on the data shown in Fig. 6:

$$\begin{aligned}\hat{\theta}_1(t) &= 0.00377 t^6 - 0.00948 t^5 - 0.05756 t^4 - 0.05590 t^3 \\ &\quad + 0.96172 t^2 + 5.98642 t + 4.99999 \\ \hat{\theta}_2(t) &= 0.00385 t^6 - 0.00194 t^5 - 0.04975 t^4 - 0.13509 t^3 \\ &\quad + 0.71324 t^2 + 6.07360 t + 8.00000 \\ \hat{\theta}_3(t) &= -0.00238 t^6 + 0.01303 t^5 + 0.03706 t^4 - 0.32049 t^3 \\ &\quad - 0.25838 t^2 + 7.46712 t + 17.00000\end{aligned}\tag{21}$$

10 where the hat denotes the polynomial function and is used to distinguish from the synthetic data.

Eqn. (21) results in:

$$\begin{aligned}p(\theta_0^1, \Omega_1) &= 5.98642 \text{ deg/sec} \\ p(\theta_0^2, \Omega_2) &= 6.07360 \text{ deg/sec} \\ p(\theta_0^3, \Omega_3) &= 7.46712 \text{ deg/sec}\end{aligned}\tag{22}$$

15 Building Functions $\mathbb{S}_{j,1}$ and $\mathbb{C}_{j,1}$

For $k = 1$, Eqn. (13) yields:

$$\begin{aligned}\mathbb{S}_{j,1} &\equiv \sin 4\theta_0^1 - \sin 4\theta_0^j \\ \mathbb{C}_{j,1} &\equiv \cos 4\theta_0^1 - \cos 4\theta_0^j\end{aligned}\quad \text{for } 2 \leq j \leq 3\tag{23}$$

Thus:

$$\begin{aligned}20 \quad \mathbb{S}_{2,1} &\equiv \sin 4\theta_0^1 - \sin 4\theta_0^2 & \mathbb{C}_{2,1} &\equiv \cos 4\theta_0^1 - \cos 4\theta_0^2 \\ \mathbb{S}_{3,1} &\equiv \sin 4\theta_0^1 - \sin 4\theta_0^3 & \mathbb{C}_{3,1} &\equiv \cos 4\theta_0^1 - \cos 4\theta_0^3\end{aligned}\tag{24}$$

Summarizing

$$\begin{aligned}
\mathbb{S}_{2,1} &\equiv \sin 4\theta_0^1 - \sin 4\theta_0^2 = -0.187899 \text{ deg/sec} \\
\mathbb{C}_{2,1} &\equiv \cos 4\theta_0^1 - \cos 4\theta_0^2 = 0.091644 \text{ deg/sec} \\
\mathbb{S}_{3,1} &\equiv \sin 4\theta_0^1 - \sin 4\theta_0^3 = -0.585164 \text{ deg/sec} \\
\mathbb{C}_{3,1} &\equiv \cos 4\theta_0^1 - \cos 4\theta_0^3 = 0.565086 \text{ deg/sec}
\end{aligned} \tag{25}$$

Based on Eqn. (14):

$$\begin{aligned}
p_1(\theta_0^1, \Omega_1) - p_1(\theta_0^j, \Omega_j) + \eta(\Omega_1 - \Omega_j) &= \sum_{k=1}^N (\mathcal{S}_k \mathbb{S}_{j,k} + \mathcal{C}_k \mathbb{C}_{j,k}) \\
&\text{for } 2 \leq j \leq 2N + 1
\end{aligned} \tag{26}$$

Then $N = 1$ yields the following linear system of equations:

$$\begin{aligned}
p_1(\theta_0^1, \Omega_1) - p_1(\theta_0^2, \Omega_2) + \eta(\Omega_1 - \Omega_2) &= \mathcal{S}_1 \mathbb{S}_{2,1} + \mathcal{C}_1 \mathbb{C}_{2,1} \\
p_1(\theta_0^1, \Omega_1) - p_1(\theta_0^3, \Omega_3) + \eta(\Omega_1 - \Omega_3) &= \mathcal{S}_1 \mathbb{S}_{3,1} + \mathcal{C}_1 \mathbb{C}_{3,1}
\end{aligned} \tag{27}$$

10

Expressing Eqn. (27) in matrix form:

$$\begin{bmatrix} p_1(\theta_0^1, \Omega_1) - p_1(\theta_0^2, \Omega_2) + \eta(\Omega_1 - \Omega_2) \\ p_1(\theta_0^1, \Omega_1) - p_1(\theta_0^3, \Omega_3) + \eta(\Omega_1 - \Omega_3) \end{bmatrix} = \begin{pmatrix} \mathbb{S}_{2,1} & \mathbb{C}_{2,1} \\ \mathbb{S}_{3,1} & \mathbb{C}_{3,1} \end{pmatrix} \begin{pmatrix} \mathcal{S}_1 \\ \mathcal{C}_1 \end{pmatrix} \tag{28}$$

15

Solving Eqn. (28):

$$\begin{pmatrix} \mathbb{S}_{2,1} & \mathbb{C}_{2,1} \\ \mathbb{S}_{3,1} & \mathbb{C}_{3,1} \end{pmatrix}^{-1} \begin{bmatrix} p_1(\theta_0^1, \Omega_1) - p_1(\theta_0^2, \Omega_2) + \eta(\Omega_1 - \Omega_2) \\ p_1(\theta_0^1, \Omega_1) - p_1(\theta_0^3, \Omega_3) + \eta(\Omega_1 - \Omega_3) \end{bmatrix} = \begin{pmatrix} \mathcal{S}_1 \\ \mathcal{C}_1 \end{pmatrix} \tag{29}$$

Then:

20

$$\frac{\begin{pmatrix} \mathbb{C}_{3,1} & -\mathbb{C}_{2,1} \\ -\mathbb{S}_{3,1} & \mathbb{S}_{2,1} \end{pmatrix}}{\mathbb{S}_{2,1} \mathbb{C}_{3,1} - \mathbb{S}_{3,1} \mathbb{C}_{2,1}} \begin{bmatrix} p_1(\theta_0^1, \Omega_1) - p_1(\theta_0^2, \Omega_2) + \eta(\Omega_1 - \Omega_2) \\ p_1(\theta_0^1, \Omega_1) - p_1(\theta_0^3, \Omega_3) + \eta(\Omega_1 - \Omega_3) \end{bmatrix} = \begin{pmatrix} \mathcal{S}_1 \\ \mathcal{C}_1 \end{pmatrix} \tag{30}$$

Finally:

$$\begin{pmatrix} -10.7528688 & 1.7438788 \\ -11.1349216 & 3.5754814 \end{pmatrix} \begin{pmatrix} -0.837180 \\ -1.605700 \end{pmatrix} = \begin{pmatrix} 6.201941 \\ 3.580783 \end{pmatrix} \tag{31}$$

25

Table 1 summarizes the estimated values of $\mathcal{S}_1$, $\mathcal{C}_1$ and the theoretical ones. It is seen that the differences between them are very small.

$\mathcal{S}_1$	$\mathcal{S}_1$	$\mathcal{S}_1$	$\mathcal{C}_1$	$\mathcal{C}_1$	$\mathcal{C}_1$
Theoretical value	Estimated value	Difference [%]	Theoretical value	Estimated value	Difference [%]
6.202450	6.201941	0.008	3.580986	3.580783	0.006

Table 1

Calibration

- 5 Equation (32) corresponds to the second term of Eqn. (6), where the Fourier coefficients have been replaced by the estimated values from Table 1:

$$\text{Bias}(\theta(t)) = 6.201941 \sin 4\theta(t) + 3.580783 \cos 4\theta(t) \quad (32)$$

- 10 In order to demonstrate the accuracy of aspects of the disclosure, an additional second simulation using an angular velocity equal to 20 deg/sec is presented.

Fig. 7 illustrates the time dynamics of the angular position of the vibrating wave, clearly showing the harmonic error.

Fig. 8 illustrates the angular velocity calculated from the data of Fig. 9 (i.e.

$$15 \quad \Omega(t) = \frac{d\theta(t)}{dt}.$$

Fig. 9 illustrates the calibrated angular velocity using the bias function of Eqn. (32)

- 20 Using an additional axisymmetric CVG to calculate the difference between two angular rates

According to some aspects of the disclosure, a second gyroscope may be used in order to account for changes in the external angular velocity.

Equation (33) repeats Eqn. (10):

$$25 \quad \theta(t) \approx \theta_0 + \underbrace{\left[-\eta\Omega + \underbrace{\sum_{k=1}^N (\mathcal{S}_k \sin 2kn\theta_0 + \mathcal{C}_k \cos 2kn\theta_0)}_{B(\theta_0)} \right]}_{p_1(\theta_0, \Omega)} t + \sum_{j=1}^M p_{j+1} t^{j+1} \quad (33)$$

where Ω is the external rate.

Assume that for time t_1 the external angular velocity is $\Omega = \Omega_1$ and the initial angle is θ_0 . At time t_2 , the simulation is performed starting again at θ_0 . However at time t_2 the external angular value changed to Ω_2 . By always starting the simulation at the same angular position of the vibrating wave, the difference between two independent
 5 simulations cancel the gyroscope bias of an axisymmetric Coriolis vibrating gyroscope, as be shown below.

$$\begin{aligned}\theta_1(t) &\approx \theta_0 + \underbrace{\left[-\eta\Omega_1 + \sum_{k=1}^N (\mathcal{S}_k \sin 2kn\theta_0 + \mathcal{C}_k \cos 2kn\theta_0) \right]}_{p_1(\theta_0, \Omega_1)} t + \sum_{j=1}^M p_{j+1} t^{j+1} \\ \theta_2(t) &\approx \theta_0 + \underbrace{\left[-\eta\Omega_2 + \sum_{k=1}^N (\mathcal{S}_k \sin 2kn\theta_0 + \mathcal{C}_k \cos 2kn\theta_0) \right]}_{p_1(\theta_0, \Omega_2)} t + \sum_{j=1}^M p_{j+1} t^{j+1}\end{aligned}\quad (34)$$

10 where $\theta_1(t)$ corresponds to the time evolution of the vibrating wave when the external velocity is Ω_1 and $\theta_2(t)$ corresponds to the time evolution of the vibrating wave when the external velocity is Ω_2 .

Estimating $p(\theta_0, \Omega_1)$ and $p(\theta_0, \Omega_2)$ from the best fit to the data gives:

$$\begin{aligned}p(\theta_0, \Omega_1) &= -\eta\Omega_1 + \sum_{k=1}^N (\mathcal{S}_k \sin 2kn\theta_0 + \mathcal{C}_k \cos 2kn\theta_0) \\ p(\theta_0, \Omega_2) &= -\eta\Omega_2 + \sum_{k=1}^N (\mathcal{S}_k \sin 2kn\theta_0 + \mathcal{C}_k \cos 2kn\theta_0)\end{aligned}\quad (35)$$

15 Then:

$$p(\theta_0, \Omega_1) - p(\theta_0, \Omega_2) = -\eta(\Omega_1 - \Omega_2)\quad (36)$$

Finally the difference may be estimated as:

$$\Omega_1 - \Omega_2 = -\frac{p(\theta_0, \Omega_1) - p(\theta_0, \Omega_2)}{\eta}\quad (37)$$

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Example 2

In Example 2, an axisymmetric Coriolis Vibrating Gyroscope with only one harmonic bias component i.e. $N = 1$ is working in the second mode i.e. $n = 2$, with

harmonic values equal to $\mathcal{S}_1 = 5.130302$ deg/sec and $\mathcal{C}_1 = 14.095389$ deg/sec. The simulations are performed starting from the same angular position. By doing so, the bias term will be the same for all simulations. Three simulations are performed in Example 2 in order to calculate the difference between angular velocities as follows:

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$$\begin{aligned}\Omega_1 - \Omega_2 &= -1 - 0.5 &= -1.50 & [\text{deg/sec}] \\ \Omega_1 - \Omega_3 &= -1 - (-0.75) &= -0.25 & [\text{deg/sec}] \\ \Omega_2 - \Omega_3 &= 0.5 - (-0.75) &= 1.25 & [\text{deg/sec}]\end{aligned}$$

Figs. 10-11 show the time evolution of the angular positions of vibrating waves $\theta_1(t)$, $\theta_2(t)$ and $\theta_3(t)$. Fig. 11 is an expanded view of Fig. 10, during the time range of 0.9-1 seconds. Note that angles $\theta_1(t)$ and $\theta_2(t)$ are very close over the entire time range of 0-1 seconds.

For $\theta_1(t)$ the simulation starts at $\theta_0 = 0$ degrees and the external angular velocity is $\Omega_1 = -1$ deg/sec.

Performing a polynomial fit of degree 6 to the data for $\theta_1(t)$ in Fig. 10:

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$$\begin{aligned}\hat{\theta}_1(t) &= -0.240077 t^6 + 1.327372 t^5 - 1.758855 t^4 - 1.884665 t^3 \\ &+ 2.571563 t^2 + 14.598421 t - 0.000053\end{aligned}\tag{38}$$

From the linear term in t in Eqn. (38), $p_1(\theta_0, \Omega_1)$ is estimated as 14.598421 deg/sec.

For $\theta_2(t)$ the simulation starts at $\theta_0 = 0$ degrees and the external angular velocity is $\Omega_2 = -0.5$ deg/sec.

Performing a polynomial fit of degree 6 to the data for $\theta_2(t)$ in Fig. 10:

$$\begin{aligned}\hat{\theta}_2(t) &= -0.179975 t^6 + 1.085866 t^5 - 1.524883 t^4 - 1.694866 t^3 \\ &+ 2.443725 t^2 + 13.847965 t - 0.000046\end{aligned}\tag{39}$$

From linear term t in Eqn. (39), $p_1(\theta_0, \Omega_2)$ is estimate as 13.847965 deg/sec.

For $\theta_3(t)$ the simulation starts at $\theta_0 = 0$ degrees and the external angular velocity is $\Omega_3 = -0.75$ deg/sec.

Performing a polynomial fit of degree 6 to the data for $\theta_3(t)$ in Fig. 10:

$$\begin{aligned}\hat{\theta}_3(t) &= -0.229307 t^6 + 1.284676 t^5 - 1.717762 t^4 - 1.853081 t^3 \\ &+ 2.550620 t^2 + 14.473275 t - 0.000047\end{aligned}\tag{40}$$

From linear term in $\dot{t}$ in Eqn. (40), $p_1(\theta_0, \Omega_3)$ is estimated as 14.473275 deg/sec.

Now calculating:

$$\begin{aligned} -\frac{p_1(\theta_0, \Omega_1) - p_1(\theta_0, \Omega_2)}{\eta} &= -\frac{14.598421 - 13.847965}{\eta} = -1.500912 \quad [\text{deg/sec}] \\ -\frac{p_1(\theta_0, \Omega_1) - p_1(\theta_0, \Omega_3)}{\eta} &= -\frac{14.598421 - 14.473275}{\eta} = -0.250292 \quad [\text{deg/sec}] \\ -\frac{p_1(\theta_0, \Omega_2) - p_1(\theta_0, \Omega_3)}{\eta} &= -\frac{13.847965 - 14.473275}{\eta} = 1.250620 \quad [\text{deg/sec}] \end{aligned}$$

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In summary:

	$\Omega_1 - \Omega_2$ [deg/sec]	$\Omega_1 - \Omega_3$ [deg/sec]	$\Omega_2 - \Omega_3$ [deg/sec]
Theoretical	1.5	-0.25	1.25
Example 2 results	-1.500912	-0.250292	1.250620

	$\Omega_1 - \Omega_2$ Error [%]	$\Omega_1 - \Omega_3$ Error [%]	$\Omega_2 - \Omega_3$ Error [%]
Example 2 results	0.06	0.12	0.05

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General

It is expected that during the life of a patent maturing from this application many relevant Coriolis vibrating gyroscopes, control elements, control circuitry (analog and/or digital) and feedback loops for controlling CVG operation will be developed and the scope of the terms Coriolis vibrating gyroscope, control element, control circuitry and feedback loop are intended to include all such new technologies a priori.

The terms “comprises”, “comprising”, “includes”, “including”, “having” and their conjugates mean “including but not limited to”.

The term “consisting of” means “including and limited to”.

As used herein, singular forms, for example, “a”, “an” and “the” include plural references unless the context clearly dictates otherwise.

Within this application, various quantifications and/or expressions may include use of ranges. Range format should not be construed as an inflexible limitation on the scope of the present disclosure. Accordingly, descriptions including ranges should be considered to have specifically disclosed all the possible subranges as well as individual numerical values within that range. For example, description of a range such as from 1 to

6 should be considered to have specifically disclosed subranges such as from 1 to 3, from 1 to 4, from 1 to 5, from 2 to 4, from 2 to 6, from 3 to 6 etc., as well as individual numbers within the stated range and/or subrange, for example, 1, 2, 3, 4, 5, and 6. Whenever a numerical range is indicated within this document, it is meant to include any cited numeral
5 (fractional or integral) within the indicated range.

It is appreciated that certain features which are (e.g., for clarity) described in the context of separate embodiments, may also be provided in combination in a single embodiment. Where various features of the present disclosure, which are (e.g., for brevity) described in a context of a single embodiment, may also be provided separately
10 or in any suitable sub-combination or may be suitable for use with any other described embodiment. Features described in the context of various embodiments are not to be considered essential features of those embodiments, unless the embodiment is inoperative without those elements.

Although the present disclosure has been described in conjunction with specific
15 embodiments thereof, it is evident that many alternatives, modifications, and variations will be apparent to those skilled in the art. Accordingly, this application intends to embrace all such alternatives, modifications and variations that fall within the spirit and broad scope of the appended claims.

All references (e.g., publications, patents, patent applications) mentioned in this
20 specification are herein incorporated in their entirety by reference into the specification, e.g., as if each individual publication, patent, or patent application was individually indicated to be incorporated herein by reference. Citation or identification of any reference in this application should not be construed as an admission that such reference is available as prior art to the present disclosure. In addition, any priority document(s)
25 and/or document(s) related to this application (e.g., co-filed) are hereby incorporated herein by reference in its/their entirety.

Where section headings are used in this document, they should not be interpreted as necessarily limiting.

CLAIMS:

1. An axisymmetric Coriolis vibrating gyroscope, comprising:
 - a resonator configured to vibrate in a standing wave;
 - a plurality of control elements, configured to perform at least one of maintaining vibration in said resonator by applying respective forces to said resonator and sensing vibrations of said resonator around an axis of symmetry of said resonator; and
 - a control circuitry associated with said plurality of control elements, configured to:
 - for each one of a set of initial angles of a vibration on a perimeter of the resonator element:
 - control said plurality of control elements so as to position an angular directions of said vibration at said initial angle;
 - allow the vibration to precess for a respective time period;
 - calculate respective angular directions of the vibration at a plurality of sampling times during said time period, based on respective readings of at least two of the control elements at said sampling times; and
 - determine a respective linear coefficient of a polynomial fitting said respective angular directions; and
 - from said respective linear coefficients, determine Fourier coefficients of an angle-dependent bias of said Coriolis vibrating gyroscope (CVG), such that a bias of a measured angular rate of vibration may be corrected using said determined Fourier coefficients.
2. The axisymmetric Coriolis vibrating gyroscope of claim 1, wherein said control circuitry is further configured to:
 - determine a current angular rate of vibration of said CVG from current respective readings of at least two of the control elements;
 - estimate a bias at a current angular direction of vibration of said CVG based on said determined Fourier coefficients; and
 - correct said determined angular rate using the estimated bias.
3. The axisymmetric Coriolis vibrating gyroscope of claim 2, wherein said angular rate is corrected mathematically by subtracting the estimated bias from the

angular rate of vibration determined from current pickoff readings of at least some of the control elements operating in pickoff mode.

4. The axisymmetric Coriolis vibrating gyroscope of claim 2, wherein said angular rate is corrected by adjusting control signals applied to at least some of the control elements to counteract the estimated bias.

5. The axisymmetric Coriolis vibrating gyroscope of any one of claims 1-4, wherein said Fourier coefficients are obtained from a solution of a system of linear equations defined by said respective linear coefficients and said respective angular directions of said vibration at said initial angles.

6. The axisymmetric Coriolis vibrating gyroscope of claim 5, wherein said set of initial angles are selected such that a determinant of a matrix defining said system of linear equations is non-zero.

7. The axisymmetric Coriolis vibrating gyroscope of any one of claims 1-6, wherein for at least one of said respective time periods an angular velocity of said CVG is at least 1 degree/sec.

8. The axisymmetric Coriolis vibrating gyroscope of any one of claims 1-7, wherein said control circuitry is further configured to correct for changes in an angular velocity of said CVG using data obtained from a correction gyroscope having a constant bias.

9. The axisymmetric Coriolis vibrating gyroscope of any one of claims 1-8, wherein an angular offset between a first one of said initial angles and a second one of said initial angles is different from an angular offset between said second one of said initial angles and a third one of said initial angles, and wherein said second one of said initial angles is immediately successive to said first initial angle and said third one of said initial angles is immediately successive to said second initial angle.

10. A method of estimating a bias in an axisymmetric Coriolis vibrating gyroscope, comprising:

for each one of a set of initial angles of a vibration on a perimeter of a resonator element of an axisymmetric Coriolis vibrating gyroscope:

positioning an angular direction of vibration of said resonator at said initial angle;

allowing the vibration to precess for a respective time period;

calculating a respective angular direction of the vibration at a plurality of sampling times during said respective time period; and

determining a respective linear coefficient of a polynomial fitting said respective angular directions; and

from said respective linear coefficients, determining Fourier coefficients of an angle-dependent bias of said Coriolis vibrating gyroscope (CVG), such that a bias of a measured angular rate of vibration may be corrected using said determined Fourier coefficients.

11. The method of claim 10, further comprising:

determining a current angular rate of vibration of said CVG;

estimating a bias at said current angular direction of vibration of said CVG based on said determined Fourier coefficients; and

correcting said determined angular rate using the estimated bias.

12. The method of claim 11, wherein said angular rate is corrected mathematically by subtracting the estimated bias from the angular rate of vibration determined from current pickoff readings of at least some control elements operating in pickoff mode.

13. The method of claim 11, wherein said angular rate is corrected by adjusting control signals applied to at least some control elements operating in forcing mode to counteract the estimated bias.

14. The method of any one of claims 10-13, wherein said Fourier coefficients are obtained from a solution of a system of linear equations defined by said respective

linear coefficients and said respective angular directions of said vibration at said initial angles.

15. The method of claim 14, further comprising selecting said set of initial angles such that a determinant of a matrix defining said system of linear equations is non-zero.

16. The method of any one of claims 10-15, wherein for at least one of said respective time periods an angular velocity of said CVG is at least 1 degree/sec.

17. The method of any one of claims 10-16, further comprising correcting for changes in an angular velocity of said CVG using data obtained from a correction gyroscope having a constant bias.

18. The method of any one of claims 10-17, wherein an angular offset between a first one of said initial angles and a second one of said initial angles is different from an angular offset between said second one of said initial angles and a third one of said initial angles, and wherein said second one of said initial angles is immediately successive to said first initial angle and said third one of said initial angles is immediately successive to said second initial angle.