

HOMOGENEOUS PROCESS OPTIMAL CONTROL (HOPOC)

BACKGROUND OF THE INVENTION

This patent relates to a method of nonlinear process control.

We encounter process control everyday. Temperature of a house in summer time is maintained near a desired value or range of values by air-conditioners. The temperature in the house is determined by a thermostat, which compares the measured value of room temperature to a desired range, say 25 °C to 27°C. If temperature is over 27°C, air-conditioners are turned on, more cooling air is pumped into the room to reduce the heat inside the house, so temperature drops down. When temperature is decreased to below 25°C, the air-conditioners are turned off, no more cooling air is introduced into the room. Heat will accumulate inside the house till temperature increases to above 27°C, that air-conditioners are turned on and the control cycle is repeated. In industries, process control is widely used in most industrial processes such as aerospace, oil & gas, iron & steel, petrochemical & refinery, food & beverage, pulp & paper, pharmaceuticals, semiconductor, water treatment, coal mining etc.

The quest confronting process control is: processes should be manipulated in acceptable control behaviors and within acceptable control time. If house temperature rises to above 40°C sometimes and drops to below 20°C in other times, we call this unacceptable control behavior. If it takes many hours to bring room temperature into the desired range, we call this unacceptable control time. Either of these happenings will make people staying in the house very uncomfortable.

Process control theory addresses this quest. Process dynamics is studied and mathematical model is obtained to describe its behaviors. Based on the studied dynamics, control scheme is designed to find appropriate actions which will manipulate process to desired performance. Remarkable successes have been achieved in the field of linear system control.

But real world is mostly nonlinear, that many of the achievements in linear system control cannot be applied. Nonlinear processes have their unique features, making the study and application of nonlinear process control rather difficult. Existing control methods are constrained by dynamics of processes. Each method works well only for some type or types of process dynamics. For example, linear control methods are applied ideally in linear processes.

There are three major nonlinear control methods. Nonlinear model predictive control (NMPC) is an approximate algorithm based on solving a constrained optimization problem with finite receding control horizon. Generalized feedback linearization (GPL) is generally applied to bilinear systems, to transform nonlinear models into linear ones, and uses linear control methods to design proper controllers. Backstepping is recursive algorithm of finding stable controllers satisfying Lyapunov function in each computing step, generally applied in bilinear processes as well.

So far there are no control methods which provide robust controllers for general processes.

SUMMARY OF THE INVENTION

An object of the invention is to propose HOPOC scheme to design robust controllers for nonlinear processes, which can be applied by computer control systems (such as Honeywell Experion PKS) in industrial plants.

Another object of the invention is to define a QRH process.

Still another object is to provide optimal control scheme for the QRH processes.

Still a further object is to derive mapping equations to obtain mapping functions which convert non-QRH processes into QRH.

A further object is to provide solution procedure to solve mapping equations to find the mapping functions.

There are other advantages of the patent, as could be seen from the following descriptions.

Process dynamics or behaviors can be described by differential or difference equations called first principle models. The models are derived from fundamental theories and laws, such as conservations of mass and energy, Newton's laws, electromagnetism, thermodynamics, fluid mechanics etc. Controlled variables, manipulated variables and sometimes disturbance variables are identified for the studied processes or plants. Controlled variables are often the features or performances we seek to achieve, such as room temperature, which can be measured in quantity. Manipulated variables are means we seek to manipulate, such as valve opening. Adjustments of manipulated variables shall result in changes of controlled variables. Disturbance variables are those that affect the controlled variables and manipulated variables, such as changes of ambient temperature, which can be measured but are difficult to control. For an enclosed room where room temperature is to be controlled by air conditioner, theory of thermodynamics can be applied to derive a first principle mathematical model.

Process dynamics or behaviors can also be described by empirical models. These models illustrate dynamic relationship between controlled variables and manipulated variables, and disturbance variables. By adjusting manipulated variables about the nominal conditions, the effects on controlled variables can be measured and recorded. These testing data can be used to develop mathematical functions reflecting relationship between process variables. There are two typical types of empirical modeling methods. Graphical method can be used to develop empirical models so that model outputs fit the actual measured curves of real processes. Statistics methods such as least square, maximum likelihood, neural network, group method data handling and other tools, can be used to determine the most appropriate empirical models which satisfy certain criteria.

There is a special type of process models, which is discovered by this patent as quadratic regulator homogeneous (QRH) model. QRH model consists of one controlled variable and one manipulated variable. Its dynamics can be described by a mathematical function of the ratio of controlled variable to manipulated variable, the function can be of any mathematical form, be it polynomials of any order, or exponential, or trigonometric, or a compound function of the abovementioned functions. For example, it can be Taylor's polynomial functions, which is often employed to approximate any smooth analytical functions. It can also be Fourier's series, which can approximate any functions including discontinuous ones.

It is found in this patent that optimal controllers can be solved explicitly for QRH processes. Optimization of the process is based upon the quadratic regulator cost function of minimizing the controlled variable errors and manipulated variable amplitudes. Minimization of controlled variable errors ensures process closely following predetermined setpoints; minimization of manipulated variable amplitudes maintains process adjustment to acceptable level, thus achieving smooth operation and energy saving. The quadratic regulator optimal control problem for QRH processes can be solved from the Hamilton-Jacob-Bellman equation. Such optimal controllers are robust.

When process models are non-QRH, mapping functions are found to convert these models into QRH. By applying Euler's Homogeneous Theorem, partial differential equation is derived which is called mapping equation by this patent. Appropriate mapping functions can be solved from the mapping equation, which shall convert the non-QRH process into QRH.

This patent introduces a solution procedure to solve the mapping equation, and proposes selection criteria to choose the appropriate mapping functions. Applying mapping functions to the non-QRH model, that is, projecting the controlled variable and manipulated variable by mapping functions, it is yielded a new process model consisting of projected controlled variable and manipulated variable, which is QRH. Optimal controller can be solved for the converted QRH process. Applying mapping functions to the optimal controller, robust controller is obtained for the original non-QRH process.

Such is the HOPOC scheme. It can be implemented conveniently by real-time computer control systems, such as Honeywell Experion PKS systems. Controlled variables and manipulated variables are measured by field sensors and are sampled into computer systems; process dynamics are modeled by mathematical functions which can be programmed in computers; HOPOC scheme is applied to the process models yielding robust controllers; controllers are programmed in computers to calculate adjustments of manipulated variables; validated adjustments are applied in each control interval by computer control systems to drive field actuators to operate motors and control vales etc, thus maintaining process temperature, pressure, level and flowrate to predetermined setpoints for industrial plants.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a functional block diagram showing a QRH process with one controlled variable and one manipulated variable, where HOPOC scheme is applied to design an optimal controller for the process.

FIG. 2 is a functional block diagram showing a non-QRH process with one controlled variable and one manipulated variable, where HOPOC scheme is applied to design a robust controller for the process.

DESCRIPTION OF THE PREFERRED EMBODIMENT

(a) HOPOC for QRH Process

To achieve better understanding, mathematical development of HOPOC scheme for QRH process is illustrated hereby together with drawing FIG.1. A detailed definition of QRH process can be found in Appendix 1.

Referring FIG. 1, there is shown an optimal controller **10** and a QRH process **20**. Controller **10** has one controlled variable $y(t)$ as input variable, such as bottom level of a distillation column in a refinery crude distillation unit; one manipulated variable $u(t)$ as output variable, such as the opening of a bottom level control valve of a distillation column; process **20** has one manipulated variable $u(t)$ as input variable and one controlled variable $y(t)$ as output variable.

The QRH process can be described by differential equation (1):

$$\frac{dy(t)}{dt} = y(t)F\left(\frac{u(t)}{y(t)}\right) \quad (1)$$

Where $y(t)$ is controlled variable at time t , $u(t)$ is manipulated variable at time t , $F(\cdot)$ is any mathematical function.

There are generally two types of methods for obtaining differential equation (1), commonly referred to as industrial process model. The first principle method develops differential equation based upon fundamental principles and laws of nature, such as Newton's laws or mass conservation law; the Empirical method obtains the dynamics of a process based upon the perturbation effects of manipulated variables affecting upon the controlled variables. In a distillation column, when feed flow is perturbed by a 5% changes, tray levels, temperatures and pressures of the column will be disturbed and fluctuating over certain period of time, until a new equilibrium is reached. Column tray levels, temperatures and pressures are sampled and recorded over the transient period. These recorded data can be used to develop process model for describing dynamics of column behaviors over feed flow changes. Sometimes it is necessary to combine both methods to obtain a satisfactory model: first principle model is derived to define system architectures with certain unknown parameters; and empirical method is applied to find actual values of these unknown parameters.

For ease of applying the invention in industrial process control computers, difference equation is used throughout this paper. The QRH process can be expressed in difference equation (2)

$$y(k+1) = y(k)F\left(\frac{u(k)}{y(k)}\right) \quad (2)$$

Where $y(k)$ is controlled variable deviation at sampling interval k , $u(k)$ is manipulated variable at sampling interval k , $y(k+1)$ is controlled variable deviation at sampling interval $k+1$, $F(\cdot)$ is any mathematical function.

Quadratic regulator cost function is defined to penalize controlled variable deviation $y(k)$ and manipulated variable amplitudes $u(k)$ shown as (3):

$$J = \sum_{k=1}^{N-1} [\alpha y^2(k) + \beta u^2(k)] + \gamma y^2(N) \quad (3)$$

Where N is end time of overall process, α , β , γ are weighting factors. Larger weighting factor indicates more penalties applied to its related values. For example, we can set a relatively larger value α to keep controlled variable deviation to smaller value thus ensuring smooth process behaviors. In plants where drastic actions are discouraged, we may assign larger β to satisfy such constraints. By solving the Hamilton-Jacob-Bellman equation (see Appendix 1), optimal controller is obtained which is represented in the linear feedback form of controlled variable $y(k)$ shown as (4)

$$u(k) = l y(k) \quad (4)$$

Note equation $F(\cdot)$ can be function of any complexity, be it polynomial, exponential, trigonometric, or a compound function which is combination of mathematical functions.

Such is the HOPOC scheme for QRH processes, which provides method for finding optimal controllers that are stable and robust. Stable systems indicate that under disturbances of controlled variable or manipulated variable, systems are able to return to its original state eventually. Robust systems imply that under the disturbances of process dynamics, also referred as model mismatches, the systems are still stable. In industrial plants where process dynamics often changes, existing process models may no more be accurate or are mismatched. It is desirable that the control systems obtained from existing models still maintain stability under change of process dynamics or model mismatches. As quadratic regulator optimal controller is both stable and robust, the HOPOC scheme provides an appropriate solution.

(b) HOPOC for non-QRH Process

The invention is about applying HOPOC scheme to not only QRH processes but to other nonlinear processes as well. To achieve this there are barricades to overcome. So far HOPOC scheme has provided solution for QRH processes but there are many nonlinear processes that are non-QRH, where HOPOC scheme is not yet applicable. In order to apply HOPOC to these non-QRH processes, a possible solution is to project non-QRH processes into QRH. But we don't know such projections (mapping functions) indeed exist and how to obtain them. And even if such mapping functions are found, we don't know if the solution ensures stability and robustness of the processes.

It is introduced hereby a scheme to address above difficulties, which is the HOPOC for non-QRH processes. Referring FIG. 2, there is shown controller **30** and non-QRH process **40**. The controller **30** has one controlled variable $y(k)$ and one manipulated variable $u(k)$. The non-QRH process **40** has one manipulated variable $u(k)$ and one controlled variable $y(k)$. Mapping functions (Φ, Γ) **50** are obtained to project controlled variable $y(k)$ into $z(k)$, and to project the pair of controlled variable $y(k)$ and manipulated variable $u(k)$ into $w(k)$. Upon such projections, new process model is obtained which is QRH process **60**. It is now able to derive a quadratic regulator optimal controller **70** in linear form of the new controlled variable $z(k)$. After applying the mapping functions (Φ, Γ) **80**, it is yielded a robust controller **30**.

Let's define the given non-QRH process model as (5):

$$y(k+1) = y(k) f(y(k), u(k)) \quad (5)$$

Assumed there exist mapping functions projecting process (5) from coordinates system (y, u) into new coordinates system (z, w) such that new process is QRH. Validity of this assumption is further explored in Appendix 2 and Appendix 3, where it is shown mapping functions do exist and can be obtained. As for now let's satisfied with the assumption and note these mapping functions as $\Phi()$ and $\Gamma()$ shown as (6):

$$\begin{aligned} z(k) &= \Phi(y(k)) \\ w(k) &= \Gamma(y(k), u(k)) \end{aligned} \quad (6)$$

Under this projection we have new process inside coordinate system (z, w) which is QRH, as shown in (7).

$$z(k+1) = \Phi(y(k)) F\left(\frac{\Gamma(u(k))}{\Phi(y(k))}\right) = z(k) F\left(\frac{w(k)}{z(k)}\right) \quad (7)$$

Where $z(k)$ is projected controlled variable at sampling interval k , $w(k)$ is projected manipulated variable at sampling interval k , $z(k+1)$ is projected controlled variable at sampling interval $k+1$, $F()$ is yielded from $f()$ of (5) under the projections of $\Phi()$ and $\Gamma()$.

Process (7) being QRH which is same as (2), it is able to follow the same arguments and procedure to find quadratic regulator optimal controller, as illustrated in previous section: that quadratic regulator cost function is defined to penalize controlled variable deviation $z(k)$ and manipulated variable amplitudes $w(k)$, resulting in cost function that is same as (3). By solving the Hamilton-Jacob-Bellman equation, optimal controller is obtained which is represented in the linear feedback form shown as (8)

$$w(k) = l z(k) \quad (8)$$

Applying mapping functions (6) by replace $z(k)$ and $w(k)$ into (8), we have found the function of which the robust controller must satisfy.

$$\Gamma(y(k), u(k)) = l \Phi(y(k)) \quad (9)$$

Above discussion reveals a solution procedure of applying HOPOC in non-QRH processes which comprises of three steps.

Step-1) Verification

By applying Euler's Homogeneous Theorem to any given process, it is able to verify if a given process is QRH. If the process is found to be QRH, we can obtain the quadratic regulator optimal controller (4) which is robust.

Euler's Homogeneous Theorem states that a given function $f(y(k), u(k))$ satisfies equation (10) is homogeneous:

$$y(k) \frac{\partial f(y, u)}{\partial y} + u(k) \frac{\partial f(y, u)}{\partial u} = 0 \quad (10)$$

From Appendix 1, we know a given function $f(y(k), u(k))$ of process (5) satisfying equation (10) is QRH, whereas a given function $f(y(k), u(k))$ of process (5) not satisfying (10) is non-QRH.

Step-2) Projection

If a given process is verified by (10) to be non-QRH, it is shown in Appendix 2, (A2-2) that mapping functions Φ, Γ satisfying the mapping equation (11) will convert it into QRH:

$$[\Phi \Phi_f f_y - \Phi_y \Phi^f] \Gamma_u - \Phi \Phi_f f_u \Gamma_y + \Phi_f \Phi_y f_u \Gamma = 0 \quad (11)$$

Where mapping functions are defined as:

$$z = \Phi(y) \quad w = \Gamma(y, u)$$

And the converted QRH function is $F(z(k), w(k))$. The solution of mapping equation (11) can be derived as shown in (12), illustrated in Appendix 3, (A3-4):

$$\Gamma = \xi \left(\frac{\Phi^f}{\Phi} \right) \exp \left(\int \frac{\Phi_y \Phi^f - \Phi \Phi_f f_y}{\Phi_f \Phi_y f_u} dp \right) \quad (12)$$

Mapping functions shall be selected carefully, following the criterion stated in Appendix 4. Such mapping functions are suitable to ensure the controller to be robust.

Step-3) Control

For converted QRH function $F(z(k), w(k))$, optimal controller is obtained as (13)

$$w(k) = l z(k) \quad (13)$$

We can apply mapping functions on (13) to obtain a nonlinear process controller $u(k)$ (14)

$$\Gamma(y(k), u(k)) - l \Phi(y(k)) = 0 \quad (14)$$

Since any nonlinear process is either QRH or non-QRH, and HOPOC can be applied in both cases to yield a robust controller, we have indeed obtained the solution for designing a suitable nonlinear process controller.

APPENDIX 1. QRH AND OPTIMAL CONTROL

There are four elements to define Quadratic Regulator Homogeneous (QRH) process and its optimal control scheme: (i) dynamics of QRH process, (ii) quantities that are observable at a given time, (iii) optimization criterion, (iv) optimal controller.

(i) Quadratic Regulator Homogeneous (QRH process): If continuous process consisting of controlled variable $y(\cdot)$ and manipulated variable $u(\cdot)$, which can be expressed eventually in the form of (A1-1)

$$\frac{dy(t)}{dt} = y(t) f\left(\frac{u(t)}{y(t)}\right) \quad (A1-1)$$

Or a discrete process consisting of controlled variable $y(\cdot)$ and manipulated variable $u(\cdot)$, which can be expressed eventually in the form of (A1-2)

$$y(k+1) = y(k) f\left(\frac{u(k)}{y(k)}\right) \quad (A1-2)$$

Whereby t is continuous time, k is discrete interval, $f(\cdot)$ is any mathematical function. Such process (A1-1) or (A1-2) is defined as Quadratic Regulator Homogeneous (QRH) process. Any continuous or discrete process that can not be expressed by (A1-1) or (A1-2) is defined as non-QRH process.

(ii) Quadratic Regulator Optimal Controller for QRH process: Suppose

(a) controlled variable $y(\cdot)$ and manipulated variable $u(\cdot)$ are quantifiable and observable at any sampling time

(b) optimization criterion is defined by the cost function (A1-3)

$$C = \sum_{i=0}^{\infty} (\alpha y(i)^2 + \beta u(i)^2) \quad (A1-3)$$

Where weighting factors α, β are positive real numbers. The one-step cost is

$$c(k) = \alpha y(k)^2 + \beta u(k)^2 \quad (A1-4)$$

Then optimal controller $u^*(\cdot)$ to minimize cost function (A1-3) can be found as (A1-5):

$$u^*(k) = ly(k) \quad (A1-5)$$

Where l is real number.

To find quadratic regulator optimal control scheme, let's consider the infinitive optimization problem. Rewriting (A1-3) as:

$$C = \sum_{i=0}^{\infty} (\alpha y(i)^2 + \beta u(i)^2) \quad (A1-6)$$

The Hamilton-Jacobi-Bellman equation becomes:

$$C^*(y, k) = \min_u \{ \alpha y(k)^2 + \beta u(k)^2 + C^*(y, u, k+1) \} \quad (A1-7)$$

Assuming that optimized stage cost function has the following form (p is unknown):

$$C^*(k) = py(k)^2$$

We obtain the recursive equation of stage cost function (A1-7) as:

$$C^*(k) = \min_u \{ \alpha y(k)^2 + \beta u(k)^2 + py(k+1)^2 \}$$

Solving above Hamilton-Jacobi-Bellman equation yields the following set of equations:

$$\begin{cases} \alpha y(k)^2 + \beta u(k)^2 + py(k+1)^2 = py(k)^2 \\ \frac{\partial}{\partial u} \{ \alpha y(k)^2 + \beta u(k)^2 + py(k+1)^2 \} = 0 \end{cases} \quad (A1-8)$$

The optimal controller $u^*(\cdot)$ is found to be:

$$u^*(k) = ly(k) \quad (A1-9)$$

Where variables l, p are obtained from the following set of equations:

$$\begin{cases} \beta l + pf_v(l) = 0 \\ \alpha + \beta l^2 + pf_v(l)^2 = p \end{cases} \quad (A1-10)$$

Note that $f_v(\cdot)$ is defined as:

$$f_v(k) = \frac{\partial}{\partial v} f(v) \quad \text{where } v = \frac{u(k)}{y(k)}$$

Therefore, when equations (A1-10) are solved for l, p , we obtain control (A1-9) that satisfies equation (A1-8). Since equation (A1-8) satisfies Hamilton-Jacobi-Bellman equation (A1-7), it is concluded that (A1-9) is indeed quadratic regulator optimal controller $u^*(\cdot)$.

Take note of the following points. Firstly, there is no prerequisite for the form of function $f(\cdot)$ in (A1-1) or (A1-2). Unlike the conventional QR optimal control schemes, where (A1-1) or (A1-2) is usually required to be linear so that explicit solution of optimal controller can be solved, this invention allows $f(\cdot)$ to be of any mathematical function.

Secondly, Hamilton-Jacob-Bellman equation can be solved and explicit optimal control solution can be obtained as shown in (A1-9). According to existing control schemes, most nonlinear process Hamilton-Jacob-Bellman equations are unsolvable that optimal controllers can not be obtained explicitly. If the optimal controller cannot be solved explicitly, it is difficult to apply it in industrial processes. Even if it can be solved by approximate methods, these approximate optimal controllers are often found to be effective only in special cases.

Lastly, finding QRH optimal controller is not a complex task: solving equation (A1-10) to find l, p and the explicit optimal controller is obtained. (A1-10) is a nonlinear equation, there are many numerical methods, for example Matlab, that are available to solve these equations accurately. Furthermore, only once it is required to solve nonlinear solutions (A1-10) to find the unknown parameters l, p . Any future adjustments of manipulated variable $u(\cdot)$ can be calculated by simply multiplying constant value l with controlled variables $y(\cdot)$. Such simple scheme is most appropriate to be applied in industrial process control.

APPENDIX 2. QRH MAPPING EQUATION

It is intended in this invention to apply HOPOC to other nonlinear processes control, and many nonlinear processes are non-QRH. In this appendix, a mapping equation is derived from Euler's Homogeneous Theorem, which is a partial differential equation. By solving the mapping equation we obtain the desired mapping functions that project non-QRH process into QRH, thus resolving the difficulty of applying HOPOC in nonlinear processes.

QRH Mapping Equation: For any nonlinear process (A2-1)

$$y(k+1) = f(y(k), u(k)) \quad (A2-1)$$

Suppose there exist a pair of mapping functions $\Phi(\cdot), \Gamma(\cdot)$

$$z = \Phi(y) \quad w = \Gamma(y, u)$$

Such that the pair of mapped variables $z(\cdot), w(\cdot)$ are projected to form a QRH process, then mapping functions $\Phi(\cdot), \Gamma(\cdot)$ must and shall satisfy the following partial differential equation (A2-2)

$$[\Phi \Phi_f f_y - \Phi_y \Phi^f] \Gamma_u - \Phi \Phi_f f_u \Gamma_y + \Phi_f \Phi_y f_u \Gamma = 0 \quad (A2-2)$$

Where:

$$\Phi^f = \Phi(f); \Phi_y = \frac{\partial \Phi(y)}{\partial y}; \Phi_f = \frac{\partial \Phi(f)}{\partial f}$$

$$\Gamma_u = \frac{\partial \Gamma(y, u)}{\partial u}; \Gamma_y = \frac{\partial \Gamma(y, u)}{\partial y}$$

$$f_y = \frac{\partial f(y, u)}{\partial y}; f_u = \frac{\partial f(y, u)}{\partial u}$$

Note that equation (A2-2) is necessary and sufficient for the mapping functions. That is to say, when mapping functions satisfy equation (A2-2), they can project non-QRH process into QRH; whereas any mapping function which is able to project non-QRH process into QRH, must satisfy mapping equation (A2-2).

Firstly , let's derive the necessary condition: if Φ, Γ project non-QRH process into QRH, they must satisfy equation (A2-2).

For nonlinear process (A2-1), under the mapping functions Φ, Γ , we obtain mapping functions as below

$$z(k+1) = \Phi(y(k+1)) = \Phi(f(y(k), u(k))) = z(k) \frac{\Phi(f(y(k), u(k)))}{z(k)} \quad (\text{A2-3})$$

HOPOC requires $\frac{\Phi(f(y(k), u(k)))}{z(k)}$ be QRH to $\Phi(y)/\Gamma(y, u)$ or z/w , hence from Euler's Homogeneous Theorem it is found:

$$z(k) \frac{\partial}{\partial z} \left(\frac{\Phi(f(y(k), u(k)))}{z(k)} \right) + w(k) \frac{\partial}{\partial w} \left(\frac{\Phi(f(y(k), u(k)))}{z(k)} \right) = 0 \quad (\text{A2-4})$$

Simplifying derivation with following notations:

$$\Phi = \Phi(y) = z(k) \quad \Gamma = \Gamma(y, u) = w(k) \quad \Phi^f = \Phi(f)$$

And

$$\Phi^f = \Phi(f); \Phi_y = \frac{\partial \Phi(y)}{\partial y}; \Phi_f = \frac{\partial \Phi(f)}{\partial f}$$

$$\Gamma_u = \frac{\partial \Gamma(y, u)}{\partial u}; \Gamma_y = \frac{\partial \Gamma(y, u)}{\partial y}$$

$$f_y = \frac{\partial f(y, u)}{\partial y}; f_u = \frac{\partial f(y, u)}{\partial u}$$

We have:

$$\frac{\partial}{\partial z} \left(\frac{\Phi(f(y(k), u(k)))}{z(k)} \right) = \frac{\Phi \Phi_f f_y \Gamma_u - \Phi \Phi_f f_u \Gamma_y - \Phi_y \Phi^f \Gamma_u}{\Phi^2 \Phi_y \Gamma_u} \quad (\text{A2-5})$$

$$\frac{\partial}{\partial w} \left(\frac{\Phi(f(y(k), u(k)))}{z(k)} \right) = \frac{f_u \Phi_f}{\Phi \Gamma_u} \quad (\text{A2-6})$$

Substitute (A2-5) and (A2-6) into (A2-4) we have the mapping equation (A2-2),

$$[\Phi \Phi_f f_y - \Phi_y \Phi^f] \Gamma_u - \Phi \Phi_f f_u \Gamma_y + \Phi_f \Phi_y f_u \Gamma = 0$$

Which implies that proper mapping functions must satisfy equation (A2-2)

Secondly, let's consider the sufficient condition: if Φ, Γ satisfy equation (A2-2), they shall project non-QRH process into QRH.

From equation (A2-2), (A2-5) and (A2-6), we have the equation (A2-4) rewritten as below:

$$z(k) \frac{\partial}{\partial z} \left(\frac{\Phi(f(y(k), u(k)))}{z(k)} \right) + w(k) \frac{\partial}{\partial w} \left(\frac{\Phi(f(y(k), u(k)))}{z(k)} \right) = 0$$

Suppose that:

$$H(z, w) = \frac{\Phi(f(y(k), u(k)))}{z(k)}$$

Equation (A2-4) can be written as:

$$z \frac{\partial H(z, w)}{\partial z} + w \frac{\partial H(z, w)}{\partial w} = 0$$

Solving partial differential equation of $H(z, w)$ to z, w , we have the solution:

$$H(z, w) = F\left(\frac{w}{z}\right)$$

Where $F(\cdot)$ is arbitrary function. This indicates that mapping functions satisfy equation (A2-2) project non-QRH process into QRH indeed.

Non-QRH processes being able to be projected into QRH is an important feature, as it enables applying of HOPOC to find the explicit optimal controller for the projected QRH process. In Appendix 3, solution procedure is developed for solving the mapping functions from QRH mapping equation (A2-2).

APPENDIX 3. QRH MAPPING FUNCTIONS

This invention suggests one solution procedure to solve the equation (A2-2), as developed in the following.

QRH Mapping Functions: Suppose there exist mapping functions Φ, Γ satisfying equation (A2-2), which is rewritten as (A3-1)

$$[\Phi \Phi_f f_y - \Phi_y \Phi^f] \Gamma_u - \Phi \Phi_f f_u \Gamma_y + \Phi_f \Phi_y f_u \Gamma = 0 \quad (A3-1)$$

The solution can be found as (A3-1)

$$\Gamma = \xi\left(\frac{\Phi^f}{\Phi}\right) \exp\left(\int \frac{\Phi_y \Phi^f - \Phi \Phi_f f_y}{\Phi_f \Phi_y f_u} dp\right) \quad (A3-2)$$

Where $\Phi(\cdot)$ and $\xi(\cdot)$ are arbitrary functions and:

$$\begin{cases} p = \frac{\Phi^f}{\Phi} \\ q = u \end{cases}$$

Let's fix the function $\Phi(\cdot)$ and regard (A3-1) as equation of $\Gamma(\cdot)$, this equation becomes first order partial differential equation of $\Gamma(\cdot)$ to variables of y, u . To solve this first order partial differential equation, introduce the following pair of variables p, q

$$\begin{cases} p = \frac{\Phi^f}{\Phi} \\ q = u \end{cases}$$

And let:

$$\Gamma_p = \frac{\partial \Gamma(p, q)}{\partial p}$$

By some calculations we have the following simplified equation (A3-3)

$$[\Phi \Phi_f f_y - \Phi_y \Phi^f] \Gamma_p + \Phi_f \Phi_y f_u \Gamma = 0 \quad (A3-3)$$

Solving equation (A3-3), we have the following solution

$$\Gamma = \xi\left(\frac{\Phi^f}{\Phi}\right) \exp\left(\int \frac{\Phi_y \Phi^f - \Phi \Phi_f f_y}{\Phi_f \Phi_y f_u} dp\right) \quad (A3-4)$$

It is found in above solution procedure, that there are infinite $\Phi(\cdot)$ and $\xi(\cdot)$ functions, which allows the liberty of selecting suitable functions for the mappings. This feature is useful. Not all mapping functions are acceptable if we seek to design a robust controller. It is definite that the projected QRH process is robust under the quadratic regulator optimal control scheme. But such optimal controller need be reversed by mapping functions, as demonstrated in (9) of the “Description of the preferred embodiment”. It is not necessary that such reversed nonlinear controller is still robust. Only suitable mapping functions ensure the stability and robustness, as revealed in Appendix 4.

APPENDIX 4. HOPOC SCHEME

With above mathematical preparations, it is adequate hereby to illustrate the detailed solution procedure of the invented HOPOC scheme, which is developed in the procedure of the following sequential steps.

HOPOC Scheme: The invented HOPOC scheme may be applied for designing robust controller for nonlinear processes in the following steps:

Step-1: Use Euler's Homogeneous Theorem to verify if the given process is QRH

Step-2: When the process is QRH, apply the method of Appendix 1 to design robust QR optimal controller

Step-3: When the process is non-QRH, apply the method of Appendix 2 and Appendix 3 to find mapping functions to project the process into QRH

Step-4: The mapping function must be selected such that $(z, w) = 0$ must imply $(y, u) = 0$

Step-5: For the projected QRH process, apply the method of Appendix 1 to obtain quadratic regulator optimal controller

Step-6: Use mapping functions to reverse quadratic regulator optimal controller, that a robust controller for the nonlinear process is obtained

Step-7: Apply the controller of Step-2 or Step-6 to the process for industrial process control

All steps are apparent by now, except Step-4 which may not be straightforward. We will have more discussions on this.

It is known that the projected QRH process is stable under quadratic regulator optimal control. This means projected controlled variable z and manipulated variable w will eventually converge to zero. With the feature of Step-4, the convergence of (z, w) implies the convergence of (y, u) . Since (y, u) eventually converges to zero, by the definition of control system stability, we conclude that the original process of (y, u) is indeed stable.

It is also known that the projected QRH process is robust under quadratic regulator optimal control. This indicates that there exist areas surrounding the original model parameters such that, under model mismatch, the projected QRH process is still stable. Step-4 indicates that mapping function shall project this area of coordinates system (z, w) to the corresponding new area of coordinates (y, u) surrounding the projected areas, that when model mismatch happens in new areas, the system is still stable, or in another word, the original system under this control is robust.

From above descriptions and derivations, it can be seen that a novel method of process control has been provided, whereby an industrial process is identified to be QRH or non-QRH, that a non-QRH process can be projected into QRH by mapping functions, and quadratic regulator optimal controller can explicitly be solved for the projected QRH process which can in turn be reversed to yield a suitable controller for the original process, thus achieving robust control for the industrial process.

It will be understood that various modifications of this control technique may occur to those skilled in the art and it is intended that this invention be limited only by the scope of the claims.

I claim:

1. A method of providing robust controllers to processes having plurality of controlled variables and manipulated variables, said method comprising the steps of:
 - a) Obtaining models of said processes which are quadratic regulator homogeneous;
 - b) Applying optimization methods to said models to calculate values of said manipulated variables;

- c) Adjusting said manipulated variables by adjustments of said calculated values for moving said controlled variables to predetermined values;
- 2. Method of claim 1, wherein step of obtaining models comprising at least one of the followings:
 - a) Applying theories and laws of nature to said processes having plurality of said controlled variables and said manipulated variables, to obtain first principle continuous or discrete models;
 - b) Measuring variations in said manipulated variables and effects of said variations on said controlled variables, applying statistics methods on said measurements to obtain empirical models at discrete interval of time;
- 3. Method of claim 2 further comprising verifying said first principle models or said empirical models are said quadratic regulator homogeneous.
- 4. Method of claim 2 further comprising verifying said first principle models or said empirical models are non quadratic regulator homogeneous, for deriving mapping functions to convert said control variables and said manipulated variables into projected controlled variables and projected manipulated variables, said models consisting of said projected controlled variables and said projected manipulated variables are said quadratic regulator homogeneous.