

METHOD FOR JOINT ESTIMATION OF TARGET LOCALIZATION AND ENVIRONMENTAL SPREAD PARAMETER OF UNDERWATER WIRELESS SENSOR NETWORK

TECHNICAL FIELD

The present disclosure relates to the technical field of underwater wireless sensor network node localization, and in particular, to a method for jointing target localization and environmental spread parameter estimation.

BACKGROUND

As an important part of a marine three-dimensional monitoring system, an Underwater Wireless Sensor Network (UWSN) is an important foundation and support thereof, which can provide good technical means and information platform for marine security, marine environmental protection, safe navigation of intelligent ships and other activities. In the UWSN, if the sensed data does not carry location information, the data will become meaningless. Therefore, how to obtain more accurate position is not only a requirement for marine monitoring applications, but also a basis for studying other theoretical issues of UWSNs.

However, it is still a problem to be solved urgently about how to obtain more accurate position information in a complex and changeable underwater environment. On the one hand, a layered effect of a signal spread underwater makes the ranging and localization accuracy low; on the other hand, model parameters of the signal spread underwater, especially environmental spread factors, often change with changes in underwater temperature, humidity, and salinity, further increasing the localization error. The existing localization technology does not have a good solution to the above problems and cannot obtain high localization accuracy in an underwater environment where signals have layered effects and in the case where the environmental spread parameters are unknown.

In Chapter 5 of "Research on Key Technology for Target Node Localization in Marine Sensor Network" (Xiaojun Mei, Shanghai Maritime University), it is proposed to consider the spread of radio signals on the water surface in a 2D scenario. However, this paper does not take into account

the simultaneous existence of layered effects and absorption effects during underwater acoustic signal spread to build a ranging model, moreover, the underwater acoustic signal spread scenario is 3D, and the technical complexity and accuracy requirements for the underwater scenario also vary greatly. Therefore, this paper does not solve the problem of inaccurate localization of an underwater wireless sensor in the case where environmental spread parameters are unknown.

SUMMARY

An object of the present disclosure is to provide a method for joint estimation of target localization and environmental spread parameter of underwater wireless sensor network (TLESE) to solve the problem of reduced localization accuracy caused by signal layered effects and unknown environmental parameters in a highly dynamic marine environment. Through the implementation of the present disclosure, high localization accuracy can be obtained under the above harsh conditions.

In order to achieve the above object, the present disclosure is achieved through the following technical scheme:

A method for joint estimation of target localization and environmental spread parameter of underwater wireless sensor network, including following steps:

S1, according to a layered spread effect of an acoustic signal underwater, constructing a ranging model of received signal strength based on the Snell's law and ray tracing theorem; and combining the Snell's law is with the ray tracing theorem to acquire a distance d_i^t from the i^{th} anchor node to a target node:

$$d_i^t = -\left(ax_j^t + b\right) \frac{\theta^x - \theta^i}{a \cos \theta^x} \quad (1),$$

where a represents a gradient parameter, b represents a spread speed of an acoustic wave on the water surface, θ^i and θ^x represent received signal angles of the i^{th} anchor node and the target node respectively, and the value range thereof is $[-\pi/2, \pi/2]$; and constructing a ranging model of received signal strength according to the distance d_i^t to the target node;

S2, constructing an objective function using an environmental spread parameter and a target position as variables through multiple first-order Taylor series expansions;

S3, performing coarse-grained estimation on the variables using a dichotomy method; and

S4, performing linear expansion according to a value of the coarse-grained estimation and re-optimizing iteratively to further improve the accuracy of a solution, obtaining an estimated value of the target position:

$$\mathbf{x}_{\kappa+1}^t = \frac{1}{N} \sum_{i=1}^N \left(\mathbf{a}_i^t + d_i^t \frac{(\mathbf{x}_{\kappa}^t - \mathbf{a}_i^t)^T}{\|\mathbf{x}_{\kappa}^t - \mathbf{a}_i^t\|} \right) \quad (2),$$

where $\mathbf{x}_{\kappa}^t$ is the κ^{th} estimated value of the target position, $\mathbf{a}_i^t$ is the position of the i^{th} anchor node at time t , d_i^t is the distance between the i^{th} anchor node and the target node, and N is the number of buoy sensor nodes containing position information deployed underwater.

The according to a layered spread effect of an acoustic signal underwater, constructing a ranging model of received signal strength based on the Snell's law in step S1 specifically includes:

S11, if N buoy sensor nodes containing position information are deployed underwater, that is, anchor nodes and a target node, assuming that the position of the i^{th} anchor node at time t is $\mathbf{a}_i^t = [a_{i1}^t, a_{i2}^t, a_{i3}^t]^T$, and the position of the target node at time t is $\mathbf{x}^t = [x_1^t, x_2^t, x_3^t]^T$; assuming that all nodes are equipped with pressure sensors which are capable of accurately knowing their own depth information; according to a spread speed model of the acoustic signal underwater, obtaining:

$$C(z) = az + b \quad (3),$$

where a represents a gradient parameter; z represents a water depth; b represents a spread speed of an acoustic wave on the water surface; and $C(z)$ represents a solution function for the spread speed of the acoustic signal when the water depth is z ;

S12, according to the Snell's law, obtaining:

$$\frac{\cos \theta}{C(z)} = \frac{\cos \theta^i}{C(a_{i3}^t)} = \frac{\cos \theta^x}{C(x_3^t)} = k \quad (4),$$

where θ^i and θ^x represent received signal angles of the i^{th} anchor node and target node respectively, and the value range thereof is $[-\pi/2, \pi/2]$; and k represents a constant; and

S13, after acquiring the distance (i.e. the formula (1)) between the anchor node and the target node, obtaining a ranging model based on signal strength in an underwater environment through a

path loss model, which is:

$$P_{ri}^t = P_s^t - PL(d_0) - 10\alpha^t \log_{10} \frac{d_i^t}{d_0} - \alpha^f d_i^t + \gamma_i^t, \quad (5),$$

where P_{ri}^t represents a power of the target node received by the i^{th} anchor node at time t ; P_s^t represents a transmit power of the target node at time t ; $PL(d_0)$ represents a loss value when a reference distance is d_0 , d_0 is usually 1m; α^t represents an environmental spread parameter of a signal; d_i^t represents a distance between the i^{th} anchor node and the target node; γ_i^t represents a signal attenuation noise for the i^{th} anchor node at time t , assuming that the noise variance at each time is equal, which, if obeying the Gaussian distribution with zero mean and σ_i^2 variance, is expressed as $\gamma_i^t \sim G(0, \sigma_i^2)$; α^f represents an absorption factor of the signal, which is acquired according to a transmit frequency according to Thorpe's theorem, that is:

$$\alpha^f = 0.11 \frac{f^2}{1+f^2} + 44 \frac{f^2}{4100+f^2} + 2.75 \times 10^{-4} f^2 + 0.003 \quad (6).$$

The constructing an objective function using an environmental spread parameter and a target position as variables through multiple first-order Taylor series expansions in step S2 specifically includes:

S21, assuming $c_{\max} = \alpha^f d_{i_{\max}}^t$, where $d_{i_{\max}}^t$ is the maximum distance between the anchor node and the target node, according to the Euler's geometric theorem and signal processing principle, when the area of an underwater monitoring region is determined, determining the maximum error $c_{\max}$ caused by an absorption effect of the signal;

S22, performing term-shifting transformation according to the expression (4) obtained in S14, and squaring the distance between each anchor node and the target node at time t , obtaining:

$$(d_i^t)^2 = d_0^2 10^{\frac{\hat{P}_0^t - P_{ri}^t - c_{\max} + \gamma_i^t}{5\alpha^t}} \quad (7),$$

wherein d_i^t represents a distance between the i^{th} anchor node and the target node, γ_i^t represents a signal attenuation noise for the i^{th} anchor node at time t , d_0 represents a reference

distance, usually 1m, P_{ri}^t represents a power of the target node received by the i^{th} anchor node at time t , and α^t represents an environmental spread parameter of the signal;

S23, performing Taylor first-order expansion on $10^{\frac{\gamma_i^t}{5\alpha^t}}$ using an exponential function Taylor first-order expansion $a^x = 1 + x \ln a$, when the reference distance $d_0 = 1$ m, expressing formula (6) in S22 as:

$$(d_i^t)^2 \approx 10^{\frac{\hat{P}_0^t - P_{ri}^t - c_{\max}}{5\alpha^t}} \cdot \left(1 + \frac{\ln 10}{5\alpha^t} \gamma_i^t\right) \quad (8),$$

wherein d_i^t represents a distance between the i^{th} anchor node and the target node, γ_i^t represents a signal attenuation noise for the i^{th} anchor node at time t , P_{ri}^t represents a power of the target node received by the i^{th} anchor node at time t , and α^t represents an environmental spread parameter of the signal;

S24, introducing a parameter α_0^t , such that $\alpha^t = \alpha_0^t (1 + (\alpha^t - \alpha_0^t) / \alpha_0^t)$ and $\tilde{\delta} \triangleq (\alpha^t - \alpha_0^t) / \alpha_0^t$, where $\alpha_0^t \neq 0$; when $\tilde{\delta}$ is small, α_0^t approaches α^t , so formula (7) in S23 is transformed into:

$$(d_i^t)^2 \approx 10^{\frac{\hat{P}_0^t - P_{ri}^t - c_{\max}}{5\alpha_0^t(1+\tilde{\delta})}} \cdot \left(1 + \frac{\ln 10}{5\alpha^t} \gamma_i^t\right) \quad (9),$$

where d_i^t is represents a distance between the i^{th} anchor node and the target node, γ_i^t represents a signal attenuation noise for the i^{th} anchor node at time t , P_{ri}^t represents a power of the target node received by the i^{th} anchor node at time t , and α^t represents an environmental spread parameter of the signal;

S25, assuming that $|\tilde{\delta}|$ is small, that is, $1/(1+\tilde{\delta}) \approx 1 - \tilde{\delta}$, performing Taylor first-order expansion on $1/(1+\tilde{\delta})$, obtaining:

$$(d_i^t)^2 \approx \rho_i^{(1-\tilde{\delta})} \cdot \left(1 + \frac{\ln 10}{5\alpha^t} \gamma_i^t\right) \quad (10),$$

where $\rho_i = 10^{\frac{\hat{P}_0^t - P_{\text{th}}^t - c_{\text{max}}}{5\alpha_0^t}}$, d_i^t represents a distance between the i^{th} anchor node and the target node, γ_i^t represents a signal attenuation noise for the i^{th} anchor node at time t , and α^t represents an environmental spread parameter of the signal;

S26, however, since formula (9) in S25 is still nonlinear and difficult to solve, assuming that $|\tilde{\delta} \ln \rho_i|$ is small, performing Taylor first-order expansion on $\rho_i^{-\tilde{\delta}}$ using the exponential function Taylor first-order expansion, so formula (9) is further transformed into:

$$(d_i^t)^2 \approx \rho_i (1 - \tilde{\delta} \ln \rho_i) \cdot \left(1 + \frac{\ln 10}{5\alpha^t} \gamma_i^t\right) \quad (11),$$

where d_i^t represents a distance between the i^{th} anchor node and the target node, γ_i^t represents a signal attenuation noise for the i^{th} anchor node at time t , and α^t represents an environmental spread parameter of the signal, $\tilde{\delta}$ is as shown in S24;

S27, after obtaining the square of the distance, constructing a weight-based least squares framework:

$$\arg \min_{\mathbf{x}^t} \sum_{i=1}^N \omega_i \left[\|\mathbf{x}^t - \mathbf{a}_i^t\|^2 - (d_i^t)^2 \right]^2 \quad (12),$$

where $\omega_i = 1 - d_i^t / \sum_{i=1}^N d_i^t$, $\|\cdot\|$ represents a second-order norm, d_i^t represents a distance between the i^{th} anchor node and the target node, and $\mathbf{a}_i^t$ represents a position of the i^{th} anchor node at time t ; and

S28, making $\boldsymbol{\theta}^t = [\chi^t, x_1^t, x_2^t, x_3^t, \tilde{\delta}^t]^T$, expanding formula (11) in S27, obtaining:

$$\begin{aligned} J(\boldsymbol{\theta}^t) &= \min_{\boldsymbol{\theta}^t} \left\| \mathbf{w} (\mathbf{A} \boldsymbol{\theta}^t - \mathbf{B}) \right\|^2 \\ s.t. & (\boldsymbol{\theta}^t)^T \mathbf{D} \boldsymbol{\theta}^t + 2 \mathbf{f}^T \boldsymbol{\theta}^t = 0 \end{aligned} \quad (13),$$

where $\mathbf{w} = \text{diag}([\omega_1^2, \dots, \omega_N^2])$,

$$A = \begin{bmatrix} 1 & -2(\mathbf{a}_1^t)^T & \rho_1 \ln \rho_1 \\ \vdots & \vdots & \vdots \\ 1 & -2(\mathbf{a}_N^t)^T & \rho_N \ln \rho_N \end{bmatrix}, B = \begin{bmatrix} \rho_1 - \|\mathbf{a}_1^t\|^2 \\ \vdots \\ \rho_N - \|\mathbf{a}_N^t\|^2 \end{bmatrix}, D = \begin{bmatrix} 0 & \mathbf{0}_{1 \times k} & 0 \\ 0 & \mathbf{I}_k & 0 \\ 0 & \mathbf{0}_{1 \times k} & 0 \end{bmatrix}, \mathbf{f} = \begin{bmatrix} -\frac{1}{2} \\ \mathbf{0}_{(k+1) \times 1} \end{bmatrix} \quad (14),$$

$\mathbf{I}$ and $\mathbf{0}$ represent an identity matrix and a zero matrix respectively.

The performing coarse-grained estimation on the variables using a dichotomy method in step S3 specifically includes:

S31, introducing a multiplier λ and solving a value thereof at each time according to formula (14), that is:

$$\lambda = \left((A^T \omega A + \lambda D)^{-1} (A^T \omega B - \lambda \mathbf{f}) \right)^T D \left((A^T \omega A + \lambda D)^{-1} (A^T \omega B - \lambda \mathbf{f}) \right) \quad (15);$$

S32, solving an optimal multiplier λ^* at each time according to formula (15), that is:

$$\lambda^* = \max \left[-\text{diag} \left(A^T \omega A \right), \lambda \right] \quad (16);$$

S33, according to the obtained optimal multiplier λ^* , calculating a variable of the coarse-grained estimation, that is:

$$\theta^t = (A^T \omega A + \lambda^* D)^{-1} (A^T \omega B - \lambda^* \mathbf{f}) \quad (17); \text{ and}$$

S34, after calculating the variable θ^t , the estimated value of the environmental spread parameter is $\hat{\alpha}' = \alpha_0^t (1 + \theta^t|_{5,1})$, where $\theta^t|_{5,1}$ is the 5th row and 1st column of the variable to be calculated; in order to further optimize an estimated value of the path loss factor, calculating a mean of the path loss factor using the obtained position information, i.e. $\theta^t|_{2:4,1}$, that is:

$$\bar{\alpha}^t = \frac{\sum_{i=1}^N \hat{P}_0^t - P_{ri}^t - c_{\max}}{10 \log_{10} \prod_{i=1}^N \hat{d}_i^t} \quad (18),$$

where $\hat{d}_i^t$ represents estimated position information of an unknown node; $\theta^t|_{2:4,1}$ represents an estimated distance to the anchor node.

The performing linear expansion according to a value of the coarse-grained estimation and iteratively re-optimizing to further improve the accuracy of a solution, obtaining an estimated value of a target position in step S4 specifically includes:

S41, taking a coarse-grained target estimated value $\hat{\mathbf{x}}^t = \boldsymbol{\theta}^t|_{2,4,1}$ obtained according to S34 as an initial value, that is, $\hat{\mathbf{x}}_0^t$; according to the first-order Taylor series expansion, performing linear processing on the estimated value at the κ^{th} time, obtaining:

$$S(\mathbf{x}^t | \mathbf{x}_\kappa^t) = \sum_{i=1}^N \left(\|\mathbf{x}^t - \mathbf{a}_i^t\|^2 - 2d_i^t \|\mathbf{x}_\kappa^t - \mathbf{a}_i^t\| \right) - 2 \sum_{i=1}^N d_i^t \frac{(\mathbf{x}_\kappa^t - \mathbf{a}_i^t)^T}{\|\mathbf{x}_\kappa^t - \mathbf{a}_i^t\|} (\mathbf{x}^t - \mathbf{x}_\kappa^t) \quad (19); \text{ and}$$

S42, deriving and simplifying $\mathbf{x}^t$ according to formula (18) in S41, obtaining that the $(\kappa + 1)^{\text{th}}$ estimated value of the target position estimate is the formula (2).

Compared with the prior art, the present disclosure has the following advantages: a UWSN ranging model with layered effects and unknown environmental spread parameters is built, and the environmental spread parameters and the target position can be jointly estimated at each time, improving the localization accuracy. Compared with the paper "Research on Key Technology for Target Node Localization in Marine Sensor Network" in the prior art, the present disclosure takes into account the spread of underwater acoustic signals in a 3D scenario, and also takes into account the layered effects and absorption effects of signals spread underwater to construct a ranging model. In addition, during the solving process of the algorithm, the solving method proposed by the present disclosure fuses the dichotomy method and the linear re-optimization method based on Taylor series expansion, which is obviously different from the related methods in Chapter 5 of the paper. More importantly, the method proposed by the present disclosure is targeted at underwater scenario, and has significant improvements in computational complexity and accuracy. Therefore, the present disclosure is essentially different from Chapter 5 of the paper in localization scenario, model construction and algorithm solving process, and has better localization performance better than that of the method proposed in Chapter 5 of the paper.

BRIEF DESCRIPTION OF THE DRAWINGS

Figure 1 is a flow chart of a method for joint estimation of target localization and environmental spread parameter of underwater wireless sensor network of the present disclosure.

Figure 2(a) and Figure 2(b) show estimation errors corresponding to different numbers of anchor nodes in the present disclosure.

Figure 3(a) and Figure 3(b) show estimation errors of different anchor nodes in the present disclosure on a target position and an environmental spread parameter (path loss factor).

Figure 4(a) and Figure 4(b) show corresponding estimation errors of different absorption factors in the present disclosure on a target position and an environmental spread parameter (path loss factor).

DETAILED DESCRIPTION

The present disclosure will be further elaborated below by describing in detail a preferred specific implementation case in conjunction with the accompanying drawings.

Figure 1 shows a flow chart of a method for joint estimation of target localization and environmental spread parameter of underwater wireless sensor network, which specifically includes:

S1, according to a layered spread effect of an acoustic signal underwater, constructing a ranging model of received signal strength based on the Snell's law and ray tracing theorem; and combining the Snell's law with the ray tracing theorem to acquire a distance d_i^t from the i^{th} anchor node to a target node:

$$d_i^t = -\left(ax_3^t + b\right) \frac{\theta^x - \theta^i}{a \cos \theta^x} \quad (1),$$

where a represents a gradient parameter, b represents a spread speed of an acoustic wave on the water surface, θ^i and θ^x represent received signal angles of the i^{th} anchor node and the target node respectively, and the value range thereof is $[-\pi/2, \pi/2]$; and constructing a ranging model of received signal strength according to the distance d_i^t to the target node;

S2, constructing an objective function using an environmental spread parameter and a target position as variables through multiple first-order Taylor series expansions;

S3, performing coarse-grained estimation on the variables using a dichotomy method; and

S4, performing linear expansion according to a value of the coarse-grained estimation and re-optimizing iteratively to further improve the accuracy of a solution, obtaining an estimated value of a target position:

$$\mathbf{x}_{\kappa+1}^t = \frac{1}{N} \sum_{i=1}^N \left(\mathbf{a}_i^t + d_i^t \frac{(\mathbf{x}_{\kappa}^t - \mathbf{a}_i^t)^T}{\|\mathbf{x}_{\kappa}^t - \mathbf{a}_i^t\|} \right) \quad (2),$$

where $\mathbf{x}_{\kappa}^t$ is the κ^{th} estimated value of the target position, $\mathbf{a}_i^t$ is the position of the i^{th} anchor node at time t , d_i^t is a distance between the i^{th} anchor node and the target node, and N is the number of buoy sensor nodes containing position information deployed underwater.

In this implementation case, the step S1 specifically includes:

S11, if N buoy sensor nodes containing position information are deployed underwater, that is, anchor nodes and a target node, assuming that the position of the i^{th} anchor node at time t is $\mathbf{a}_i^t = [a_{i1}^t, a_{i2}^t, a_{i3}^t]^T$, and the position of the target node at time t is $\mathbf{x}^t = [x_1^t, x_2^t, x_3^t]^T$; assuming that all nodes are equipped with pressure sensors which are capable of accurately knowing their own depth information; according to a spread speed model of the acoustic signal underwater, obtaining:

$$C(z) = az + b \quad (3),$$

where a represents a gradient parameter; z represents a water depth; b represents a spread speed of an acoustic wave on the water surface; and $C(z)$ represents a solution function for the spread speed of the acoustic signal when the water depth is z ;

S12, according to the Snell's law, obtaining:

$$\frac{\cos \theta}{C(z)} = \frac{\cos \theta^i}{C(a_{i3}^t)} = \frac{\cos \theta^x}{C(x_3^t)} = k \quad (4),$$

where θ^i and θ^x represent received signal angles of the i^{th} anchor node and the target node respectively, and the value range thereof is $[-\pi/2, \pi/2]$; and k represents a constant; and

S13, after acquiring the distance (i.e. the formula (1)) between the anchor node and the target node, obtaining a ranging model based on signal strength in an underwater environment through a path loss model, which is:

$$P_{ri}^t = P_s^t - PL(d_0) - 10\alpha^t \log_{10} \frac{d_i^t}{d_0} - \alpha^f d_i^t + \gamma_i^t \quad (5),$$

where P_{ri}^t represents a power of the target node received by the i^{th} anchor node at time t ; P_s^t represents a transmit power of the target node at time t ; $PL(d_0)$ represents a loss value when a reference distance is d_0 , d_0 is usually 1m; α^f represents an environmental spread parameter of a signal; d_i^t represents a distance between the i^{th} anchor node and the target node; γ_i^t represents a signal attenuation noise for the i^{th} anchor node at time t , assuming that the noise variance at each time is equal, which, if obeying the Gaussian distribution with zero mean and σ_i^2 variance, is expressed as $\gamma_i^t \sim G(0, \sigma_i^2)$; α^f represents an absorption factor of the signal, which is acquired according to a transmit frequency according to Thorpe's theorem, that is:

$$\alpha^f = 0.11 \frac{f^2}{1+f^2} + 44 \frac{f^2}{4100+f^2} + 2.75 \times 10^{-4} f^2 + 0.003 \quad (6).$$

The step S2 specifically includes:

S21, assuming $c_{\max} = \alpha^f d_{i_{\max}}^t$, where $d_{i_{\max}}^t$ is the maximum distance between the anchor node and the target node, according to the Euler's geometric theorem and signal processing principle, when the area of an underwater monitoring region is determined, determining the maximum error $c_{\max}$ caused by an absorption effect of the signal;

S22, performing term-shifting transformation according to the expression (4) obtained in S14, and squaring the distance between each anchor node and the target node at time t , obtaining:

$$(d_i^t)^2 = d_0^2 10^{\frac{\hat{P}_0^t - P_{ri}^t - c_{\max} + \gamma_i^t}{5\alpha^t}} \quad (7),$$

where d_i^t represents a distance between the i^{th} anchor node and the target node, γ_i^t represents a signal attenuation noise for the i^{th} anchor node at time t , d_0 represents a reference distance, usually 1m, P_{ri}^t represents a power of the target node received by the i^{th} anchor node at time t , and

α represents an environmental spread parameter of the signal;

S23, performing Taylor first-order expansion on $10^{\frac{\gamma_i^t}{5\alpha^t}}$ using an exponential function Taylor first-order expansion $a^x = 1 + x \ln a$, when the reference distance $d_0 = 1$ m, expressing formula (6) in S22 as:

$$(d_i^t)^2 \approx 10^{\frac{\hat{P}_0^t - P_{ii}^t - c_{\max}}{5\alpha^t}} \cdot \left(1 + \frac{\ln 10}{5\alpha^t} \gamma_i^t\right) \quad (8),$$

where d_i^t represents a distance between the i^{th} anchor node and the target node, γ_i^t represents a signal attenuation noise for the i^{th} anchor node at time t , P_{ii}^t represents a power of the target node received by the i^{th} anchor node at time t , and α represents an environmental spread parameter of the signal;

S24, introducing a parameter α_0^t , such that $\alpha^t = \alpha_0^t (1 + (\alpha^t - \alpha_0^t) / \alpha_0^t)$ and $\tilde{\delta} \triangleq (\alpha^t - \alpha_0^t) / \alpha_0^t$, where $\alpha_0^t \neq 0$; when $\tilde{\delta}$ is small, α_0^t approaches α , so formula (7) in S23 is transformed into:

$$(d_i^t)^2 \approx 10^{\frac{\hat{P}_0^t - P_{ii}^t - c_{\max}}{5\alpha_0^t(1+\tilde{\delta})}} \cdot \left(1 + \frac{\ln 10}{5\alpha^t} \gamma_i^t\right) \quad (9),$$

where d_i^t represents a distance between the i^{th} anchor node and the target node, γ_i^t represents a signal attenuation noise for the i^{th} anchor node at time t , P_{ii}^t represents a power of the target node received by the i^{th} anchor node at time t , and α represents an environmental spread parameter of the signal;

S25, assuming that $|\tilde{\delta}|$ is small, that is, $1/(1+\tilde{\delta}) \approx 1-\tilde{\delta}$, performing Taylor first-order expansion on $1/(1+\tilde{\delta})$, obtaining:

$$(d_i^t)^2 \approx \rho_i^{(1-\tilde{\delta})} \cdot \left(1 + \frac{\ln 10}{5\alpha^t} \gamma_i^t\right) \quad (10),$$

where $\rho_i = 10^{\frac{\hat{P}_0 - P_{ii} - c_{\max}}{5\alpha_0^t}}$, d_i^t represents a distance between the i^{th} anchor node and the target node, γ_i^t represents a signal attenuation noise for the i^{th} anchor node at time t , and α^t represents an environmental spread parameter of the signal;

S26, however, since formula (9) in S25 is still nonlinear and difficult to solve, assuming that $|\tilde{\delta} \ln \rho_i|$ is small, performing Taylor first-order expansion on $\rho_i^{-\tilde{\delta}}$ using the exponential function Taylor first-order expansion, so formula (9) is further transformed into:

$$(d_i^t)^2 \approx \rho_i (1 - \tilde{\delta} \ln \rho_i) \cdot \left(1 + \frac{\ln 10}{5\alpha^t} \gamma_i^t\right) \quad (11),$$

where d_i^t represents a distance between the i^{th} anchor node and the target node, γ_i^t represents a signal attenuation noise for the i^{th} anchor node at time t , and α^t represents an environmental spread parameter of the signal, $\tilde{\delta}$ is as shown in S24;

S27, after obtaining the square of the distance, constructing a weight-based least squares framework:

$$\arg \min_{\mathbf{x}^t} \sum_{i=1}^N \omega_i \left[\|\mathbf{x}^t - \mathbf{a}_i^t\|^2 - (d_i^t)^2 \right]^2 \quad (12),$$

where $\omega_i = 1 - d_i^t / \sum_{i=1}^N d_i^t$, $\|\cdot\|$ represents a second-order norm, d_i^t represents a distance between the i^{th} anchor node and the target node, and $\mathbf{a}_i^t$ represents a position of the i^{th} anchor node at time t ;

S28, making $\boldsymbol{\theta}^t = [\chi^t, x_1^t, x_2^t, x_3^t, \tilde{\delta}]^T$, expanding formula (11) in S27, obtaining:

$$\begin{aligned} J(\boldsymbol{\theta}^t) &= \min_{\boldsymbol{\theta}^t} \left\| \mathbf{w}(\mathbf{A} \boldsymbol{\theta}^t - \mathbf{B}) \right\|^2 \\ s.t. & (\boldsymbol{\theta}^t)^T \mathbf{D} \boldsymbol{\theta}^t + 2 \mathbf{f}^T \boldsymbol{\theta}^t = 0 \end{aligned} \quad (13),$$

where $\boldsymbol{\omega} = \text{diag}([\omega_1^2, \dots, \omega_N^2])$,

$$\mathbf{A} = \begin{bmatrix} 1 & -2(\mathbf{a}_1^t)^T & \rho_1 \ln \rho_1 \\ \vdots & \vdots & \vdots \\ 1 & -2(\mathbf{a}_N^t)^T & \rho_N \ln \rho_N \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \rho_1 - \|\mathbf{a}_1^t\|^2 \\ \vdots \\ \rho_N - \|\mathbf{a}_N^t\|^2 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0 & \mathbf{0}_{1 \times k} & 0 \\ 0 & \mathbf{I}_k & 0 \\ 0 & \mathbf{0}_{1 \times k} & 0 \end{bmatrix}, \mathbf{f} = \begin{bmatrix} -\frac{1}{2} \\ \mathbf{0}_{(k+1) \times 1} \end{bmatrix} \quad (14),$$

$\mathbf{I}$ and $\mathbf{0}$ represent an identity matrix and a zero matrix respectively.

The step S3 specifically includes:

S31, introducing a multiplier λ and solving a value thereof at each time according to formula (14), that is:

$$\lambda = \left((A^T \omega A + \lambda D)^{-1} (A^T \omega B - \lambda f) \right)^T D \left((A^T \omega A + \lambda D)^{-1} (A^T \omega B - \lambda f) \right) \quad (15),$$

S32, solving an optimal multiplier λ^* at each time according to formula (15), that is:

$$\lambda^* = \max \left[-\text{diag} \left(A^T \omega A \right), \lambda \right] \quad (16),$$

S33, according to the obtained optimal multiplier λ^* , calculating a variable of the coarse-grained estimation, that is:

$$\theta^t = (A^T \omega A + \lambda^* D)^{-1} (A^T \omega B - \lambda^* f) \quad (17); \text{ and}$$

S34, after calculating the variable θ^t the estimated value of the environmental spread parameter is $\hat{\alpha}^t = \alpha_0^t (1 + \theta^t|_{5,1})$, where $\theta^t|_{5,1}$ is the 5th row and 1st column of the variable θ^t to be calculated; in order to further optimize an estimated value of the path loss factor, calculating a mean of the path loss factor using the obtained position information, i.e. $\theta^t|_{2:4,1}$, that is:

$$\bar{\alpha}^t = \frac{\sum_{i=1}^N \hat{P}_0^t - P_{ri}^t - c_{\max}}{10 \log_{10} \prod_{i=1}^N \hat{d}_i^t} \quad (18),$$

where $\hat{d}_i^t$ represents estimated position information of an unknown node; $\theta^t|_{2:4,1}$ represents an estimated distance to the anchor node.

The step S4 specifically includes:

S41, taking a coarse-grained target estimated value $\hat{\mathbf{x}}^t = \theta^t|_{2:4,1}$ obtained according to S33 as an initial value, that is $\hat{\mathbf{x}}_0^t$; according to the first-order Taylor series expansion, performing linear processing on the estimated value at the κ^{th} time, obtaining:

$$S(\mathbf{x}^t | \mathbf{x}_\kappa^t) = \sum_{i=1}^N \left(\|\mathbf{x}^t - \mathbf{a}_i^t\|^2 - 2d_i^t \|\mathbf{x}_\kappa^t - \mathbf{a}_i^t\| \right) - 2 \sum_{i=1}^N d_i^t \frac{(\mathbf{x}_\kappa^t - \mathbf{a}_i^t)^T}{\|\mathbf{x}_\kappa^t - \mathbf{a}_i^t\|} (\mathbf{x}^t - \mathbf{x}_\kappa^t) \quad (19); \text{ and}$$

S42, deriving and simplifying $\mathbf{x}^t$ according to formula (18) in S41, obtaining that the $(\kappa+1)^{\text{th}}$ estimated value of the target position is the formula (2).

In order to verify the effectiveness of the method of the present disclosure (referred to as TLESE algorithm), a simulation experiment is conducted in Matlab R2021b. For different scenarios, compared with the existing algorithms including USR, PLSE, and TSE, the minimum root mean square error (RMSE) is used as an evaluation index to evaluate the performance of the algorithm, that is:

$$RMSE_{\mathbf{x}^t} = \sqrt{\frac{1}{MC} \sum_{nu=1}^{MC} (\hat{\mathbf{x}}^t - \mathbf{x}^t)^2}, \quad RMSE_{\alpha^t} = \sqrt{\frac{1}{MC} \sum_{nu=1}^{MC} (\hat{\alpha}^t - \alpha^t)^2} \quad (20),$$

where MC represents the total number of Monte Carlo simulations; and nu represents the number of Monte Carlo simulations.

In addition, in order to simulate a dynamic scenario in a marine environment, the anchor node and target node positions of each Monte Carlo simulation are set to change in the simulation. Other fixed parameters are set as follows: $a = 0.1$, $b = 1473$ m/s, $P_0 = -55$ dBm, $MC = 1000$, $\kappa = 1000$, and the side length of the simulation region is 50 m. In addition, since the empirical value α^t is usually 2 to 6, if the value range of α_0^t is too small or too large, δ is also too small or too large accordingly, the formula (9) expanded by the first-order Taylor series in step S25 will be invalid, so the median of 4.75 is selected as the value of α_0^t , that is, $\alpha_0^t = 4.75$.

Figure 2(a) and Figure 2(b) show estimation errors of different noise on a target position and an environmental spread parameter in the case where $N = 8, \alpha^f = 0.06$. It can be seen from the figure that as the noise increases, a corresponding localization error also increases. In the estimation for the target position, the TLESE method proposed by the present disclosure has significantly better localization accuracy than other three methods because of fusing the linear iterative re-optimization method. The same result can be obtained from the environmental parameter estimation error in Figure 2(b). Although the environmental parameter estimation error in the

method proposed by the present disclosure increases as the noise increases, the overall estimation performance of the method is better than the other three methods.

Figure 3(a) and Figure 3(b) show estimation errors of different anchor nodes on a target position and an environmental spread parameter in the case where $\alpha^f = 0.06$, $\sigma_i^2 = 3$ dB. As the number of anchor nodes increases, the received signal strength (RSS) information that can be used to estimate underwater acoustic signal spread also increases, so the estimation error of each algorithm decreases as the number of anchor nodes increases. As can be seen from Figure 3(a), the localization accuracy of each algorithm is relatively close when $N = 6$, especially PLSE and TSE. As the number of anchor nodes continues to increase, the superiority of localization performance of the TLESE method gradually becomes apparent, and the position estimation accuracy of the TLESE method is better than the other three methods. This superiority is further illustrated in the environmental spread parameter estimation error in Figure 3(b).

Figure 4(a) and Figure 4(b) show estimation errors of different absorption factors on a target position and an environmental spread parameter in the case where $N = 8$, $\sigma_i^2 = 3$ dB. It can be seen from Figure 4(a) that the estimation performance of USR, PLSE and TLESE has good robustness to changes in the absorption factor, while the position estimation error of the TSE increases with the increase of the absorption factor. Among the four methods, the TLESE method proposed by the present disclosure has better position estimation performance. As can be seen in Figure 4(b), the estimation errors of USR, PLSE and TSE on environmental parameters increase as the absorption factor increases. On the other hand, for TLESE, the estimation error of the environmental spread parameter thereof decreases with the increase of absorption factors. Therefore, the TLESE method proposed by the present disclosure has better estimation performance for environmental parameters under different absorption factors.

Although the content of the present disclosure has been described in detail through the above preferred embodiments, it should be understood that the above description should not be considered as limiting the present disclosure. Various modifications and substitutions to the present disclosure will be apparent to those skilled in the art after reading the above content. Therefore, the protection scope of the present disclosure should be defined by the appended claims.

Claims

1. A method for jointing target localization and environmental spread parameter estimation of underwater wireless sensor network, comprising following steps:

S1, according to a layered spread effect of an acoustic signal underwater, constructing a ranging model of received signal strength based on the Snell's law and ray tracing theorem; and combining the Snell's law with the ray tracing theorem to acquire a distance d_i^t from the i^{th} anchor node to a target node:

$$d_i^t = -\left(ax_3^t + b\right) \frac{\theta^x - \theta^i}{a \cos \theta^x} \quad (1),$$

wherein a represents a gradient parameter, b represents a spread speed of an acoustic wave on the water surface, θ^i and θ^x represent received signal angles of the i^{th} anchor node and the target node respectively, and the value range thereof is $[-\pi/2, \pi/2]$; and constructing a ranging model of received signal strength according to the distance d_i^t to the target node;

S2, constructing an objective function using an environmental spread parameter and a target position as variables through multiple first-order Taylor series expansions;

S3, performing coarse-grained estimation on the variables using a dichotomy method; and

S4, performing linear expansion according to a value of the coarse-grained estimation and re-optimizing iteratively to further improve the accuracy of a solution, obtaining an estimated value of the target position:

$$\mathbf{x}_{\kappa+1}^t = \frac{1}{N} \sum_{i=1}^N \left(\mathbf{a}_i^t + d_i^t \frac{(\mathbf{x}_{\kappa}^t - \mathbf{a}_i^t)^T}{\|\mathbf{x}_{\kappa}^t - \mathbf{a}_i^t\|} \right) \quad (2),$$

wherein $\mathbf{x}_{\kappa}^t$ is the κ^{th} estimated value of the target position, $\mathbf{a}_i^t$ is the position of the i^{th} anchor node at time t , d_i^t is the distance between the i^{th} anchor node and the target node, and N is the number of buoy sensor nodes containing position information deployed underwater.

2. The method for jointing target localization and environmental spread parameter estimation TLESE of underwater wireless sensor network according to claim 1, wherein the according to a

layered spread effect of an acoustic signal underwater, constructing a ranging model of received signal strength based on the Snell's law in step S1 specifically comprises:

S11, if N buoy sensor nodes containing position information are deployed underwater, that is, anchor nodes and a target node, assuming that the position of the i^{th} anchor node at time t is $\mathbf{a}_i^t = [a_{i1}^t, a_{i2}^t, a_{i3}^t]^T$, and the position of the target node at time t is $\mathbf{x}^t = [x_1^t, x_2^t, x_3^t]^T$; assuming that all nodes are equipped with pressure sensors which are capable of accurately knowing their own depth information; according to a spread speed model of the acoustic signal underwater, obtaining:

$$C(z) = az + b \quad (3),$$

wherein a represents a gradient parameter; z represents a water depth; b represents a spread speed of an acoustic wave on the water surface; and $C(z)$ represents a solution function for the spread speed of the acoustic signal when the water depth is z ;

S12, according to the Snell's law, obtaining:

$$\frac{\cos \theta}{C(z)} = \frac{\cos \theta^i}{C(a_{i3}^t)} = \frac{\cos \theta^x}{C(x_3^t)} = k \quad (4),$$

wherein θ^i and θ^x represent received signal angles of the i^{th} anchor node and target node respectively, and the value range thereof is $[-\pi/2, \pi/2]$; and k represents a constant;

S13, after acquiring the distance (i.e. the formula (1)) between the anchor node and the target node, obtaining a ranging model based on signal strength in an underwater environment through a path loss model, which is:

$$P_{ri}^t = P_s^t - PL(d_0) - 10\alpha^t \log_{10} \frac{d_i^t}{d_0} - \alpha^f d_i^t + \gamma_i^t \quad (5),$$

wherein P_{ri}^t represents a power of the target node received by the i^{th} anchor node at time t ; P_s^t represents a transmit power of the target node at time t ; $PL(d_0)$ represents a loss value when a reference distance is d_0 , d_0 is usually 1m; α^t an environmental spread parameter of a signal; d_i^t represents a distance between the i^{th} anchor node and the target node; γ_i^t represents a signal attenuation noise for the i^{th} anchor node at time t , assuming that the noise variance at each time is

equal, which, if obeying the Gaussian distribution with zero mean and σ_i^2 variance, is expressed as $\gamma_i^t \sim G(0, \sigma_i^2)$; α^f represents an absorption factor of the signal, which is acquired according to a transmit frequency f according to Thorpe's theorem, that is:

$$\alpha^f = 0.11 \frac{f^2}{1+f^2} + 44 \frac{f^2}{4100+f^2} + 2.75 \times 10^{-4} f^2 + 0.003 \quad (6).$$

3. The method for joint estimation of target localization and environmental spread parameter of underwater wireless sensor network according to claim 2, wherein the constructing an objective function using an environmental spread parameter and a target position as variables through multiple first-order Taylor series expansions in step S2 specifically comprises:

S21, assuming $c_{\max} = \alpha^f d_{i_{\max}}^t$, where $d_{i_{\max}}^t$ is the maximum distance between the anchor node and the target node, according to the Euler's geometric theorem and signal processing principle, when the area of an underwater monitoring region is determined, determining the maximum error $c_{\max}$ caused by an absorption effect of the signal;

S22, performing term-shifting transformation according to the expression (4) obtained in S14, and squaring the distance between each anchor node and the target node at time t , obtaining:

$$\left(d_i^t\right)^2 = d_0^2 10^{\frac{\hat{P}_0^t - P_{ri}^t - c_{\max} + \gamma_i^t}{5\alpha^t}} \quad (7),$$

wherein d_i^t represents a distance between the i^{th} anchor node and the target node, γ_i^t represents a signal attenuation noise for the i^{th} anchor node at time t , d_0 represents a reference distance, usually 1m, P_{ri}^t represents a power of the target node received by the i^{th} anchor node at time t , and α^f represents an environmental spread parameter of the signal;

S23, performing Taylor first-order expansion on $10^{\frac{\gamma_i^t}{5\alpha^t}}$ using an exponential function Taylor first-order expansion $a^x = 1 + x \ln a$, when the reference distance $d_0 = 1$ m, expressing formula (6) in S22 as:

$$\left(d_i^t\right)^2 \approx 10^{\frac{\hat{P}_0^t - P_{ri}^t - c_{\max}}{5\alpha^t}} \cdot \left(1 + \frac{\ln 10}{5\alpha^t} \gamma_i^t\right) \quad (8),$$

wherein d_i^t represents a distance between the i^{th} anchor node and the target node, γ_i^t represents a signal attenuation noise for the i^{th} anchor node at time t , P_{ri}^t represents a power of the target node received by the i^{th} anchor node at time t , and α^t represents an environmental spread parameter of the signal;

S24, introducing a parameter α_0^t , such that $\alpha^t = \alpha_0^t \left(1 + (\alpha^t - \alpha_0^t) / \alpha_0^t\right)$ and $\tilde{\delta} \triangleq (\alpha^t - \alpha_0^t) / \alpha_0^t$, where $\alpha_0^t \neq 0$; when $\tilde{\delta}$ is small, α_0^t approaches α^t , so formula (7) in S23 is transformed into:

$$\left(d_i^t\right)^2 \approx 10^{\frac{\hat{P}_0^t - P_{ri}^t - c_{\max}}{5\alpha_0^t(1+\tilde{\delta})}} \cdot \left(1 + \frac{\ln 10}{5\alpha^t} \gamma_i^t\right) \quad (9),$$

wherein d_i^t represents a distance between the i^{th} anchor node and the target node, γ_i^t represents a signal attenuation noise for the i^{th} anchor node at time t , P_{ri}^t represents a power of the target node received by the i^{th} anchor node at time t , and α^t represents an environmental spread parameter of the signal;

S25, assuming that $|\tilde{\delta}|$ is small, that is, $1/(1+\tilde{\delta}) \approx 1-\tilde{\delta}$, performing Taylor first-order expansion on $1/(1+\tilde{\delta})$, obtaining:

$$\left(d_i^t\right)^2 \approx \rho_i^{(1-\tilde{\delta})} \cdot \left(1 + \frac{\ln 10}{5\alpha^t} \gamma_i^t\right) \quad (10),$$

wherein $\rho_i = 10^{\frac{\hat{P}_0^t - P_{ri}^t - c_{\max}}{5\alpha_0^t}}$, d_i^t represents a distance between the i^{th} anchor node and the target node, γ_i^t represents a signal attenuation noise for the i^{th} anchor node at time t , and α^t represents an environmental spread parameter of the signal;

S26, however, since formula (9) in S25 is still nonlinear and difficult to solve, assuming that

$|\tilde{\delta} \ln \rho_i|$ is small, performing Taylor first-order expansion on $\rho_i^{-\tilde{\delta}}$ using the exponential function Taylor first-order expansion, so formula (9) is further transformed into:

$$(d_i^t)^2 \approx \rho_i \left(1 - \tilde{\delta} \ln \rho_i\right) \cdot \left(1 + \frac{\ln 10}{5\alpha^t} \gamma_i^t\right) \quad (11),$$

wherein d_i^t represents a distance between the i^{th} anchor node and the target node, γ_i^t represents a signal attenuation noise for the i^{th} anchor node at the time t , and α^t represents an environmental spread parameter of the signal, $\tilde{\delta}$ is as shown in S24;

S27, after obtaining the square of the distance, constructing a weight-based least squares framework:

$$\arg \min_{\mathbf{x}^t} \sum_{i=1}^N \omega_i \left[\|\mathbf{x}^t - \mathbf{a}_i^t\|^2 - (d_i^t)^2 \right]^2 \quad (12),$$

wherein $\omega_i = 1 - d_i^t / \sum_{i=1}^N d_i^t$, $\|\cdot\|$ represents a second-order norm, d_i^t represents a distance between the i^{th} anchor node and the target node, and $\mathbf{a}_i^t$ represents a position of the i^{th} anchor node at time t ; and

S28, making $\boldsymbol{\theta}^t = [\chi^t, x_1^t, x_2^t, x_3^t, \tilde{\delta}^t]^T$, expanding formula (11) in S27, obtaining:

$$\begin{aligned} J(\boldsymbol{\theta}^t) &= \min_{\boldsymbol{\theta}^t} \left\| \mathbf{w}(\mathbf{A} \boldsymbol{\theta}^t - \mathbf{B}) \right\|^2 \\ s.t. (\boldsymbol{\theta}^t)^T \mathbf{D} \boldsymbol{\theta}^t + 2 \mathbf{f}^T \boldsymbol{\theta}^t &= 0 \end{aligned} \quad (13),$$

wherein $\boldsymbol{\omega} = \text{diag}([\omega_1^2, \dots, \omega_N^2])$,

$$\mathbf{A} = \begin{bmatrix} 1 & -2(\mathbf{a}_1^t)^T & \rho_1 \ln \rho_1 \\ \vdots & \vdots & \vdots \\ 1 & -2(\mathbf{a}_N^t)^T & \rho_N \ln \rho_N \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \rho_1 - \|\mathbf{a}_1^t\|^2 \\ \vdots \\ \rho_N - \|\mathbf{a}_N^t\|^2 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0 & \mathbf{0}_{1 \times k} & 0 \\ 0 & \mathbf{I}_k & 0 \\ 0 & \mathbf{0}_{1 \times k} & 0 \end{bmatrix}, \mathbf{f} = \begin{bmatrix} -\frac{1}{2} \\ \mathbf{0}_{(k+1) \times 1} \end{bmatrix} \quad (14),$$

$\mathbf{I}$ and $\mathbf{0}$ represent an identity matrix and a zero matrix respectively.

4. The method for joint estimation of target localization and environmental spread parameter of underwater wireless sensor network according to claim 3, wherein the performing coarse-grained estimation on the variables using a dichotomy method in step S3 specifically comprises:

S31, introducing a multiplier λ and solving a value thereof at each time according to formula (14), that is:

$$\lambda = \left((A^T \omega A + \lambda D)^{-1} (A^T \omega B - \lambda f) \right)^T D \left((A^T \omega A + \lambda D)^{-1} (A^T \omega B - \lambda f) \right) \quad (15);$$

S32, solving an optimal multiplier λ^* at each time according to formula (15), that is:

$$\lambda^* = \max \left[-\text{diag} (A^T \omega A), \lambda \right] \quad (16);$$

S33, according to the obtained optimal multiplier λ^* , calculating a variable of the coarse-grained estimation, that is:

$$\theta^t = (A^T \omega A + \lambda^* D)^{-1} (A^T \omega B - \lambda^* f) \quad (17); \text{ and}$$

S34, after calculating the variable θ^t , the estimated value of the environmental spread parameter is $\hat{\alpha}^t = \alpha_0^t (1 + \theta^t|_{5,1})$, wherein $\theta^t|_{5,1}$ is the 5th row and 1st column of the variable θ^t to be calculated; in order to further optimize an estimated value of the path loss factor, calculating a mean of the path loss factor using the obtained position information, i.e. $\theta^t|_{2:4,1}$, that is:

$$\bar{\alpha}^t = \frac{\sum_{i=1}^N \hat{P}_0^t - P_{ri}^t - c_{\max}}{10 \log_{10} \prod_{i=1}^N \hat{d}_i^t} \quad (18),$$

wherein $\hat{d}_i^t$ represents estimated position information of an unknown node; $\theta^t|_{2:4,1}$ represents an estimated distance to the anchor node.

5. The method for joint estimation of target localization and environmental spread parameter of underwater wireless sensor network according to claim 4, wherein the performing linear expansion according to a value of the coarse-grained estimation and re-optimizing iteratively to further improve the accuracy of a solution in step S4 specifically comprises:

S41, taking a coarse-grained target estimated value $\hat{\mathbf{x}}^t = \theta^t|_{2:4,1}$ obtained according to S33 as an initial value, that is, $\hat{\mathbf{x}}_0^t$; according to the first-order Taylor series expansion, performing linear processing on the estimated value at the κ^{th} time, obtaining:

$$S(\mathbf{x}^t | \mathbf{x}_\kappa^t) = \sum_{i=1}^N \left(\|\mathbf{x}^t - \mathbf{a}_i^t\|^2 - 2d_i^t \|\mathbf{x}_\kappa^t - \mathbf{a}_i^t\| \right) - 2 \sum_{i=1}^N d_i^t \frac{(\mathbf{x}_\kappa^t - \mathbf{a}_i^t)^T}{\|\mathbf{x}_\kappa^t - \mathbf{a}_i^t\|} (\mathbf{x}^t - \mathbf{x}_\kappa^t) \quad (19); \text{ and}$$

S42, deriving and simplifying $\mathbf{x}^t$ according to formula (18) in S41, obtaining that the $(\kappa+1)^{\text{th}}$ estimated value of the target position is the formula (2).