

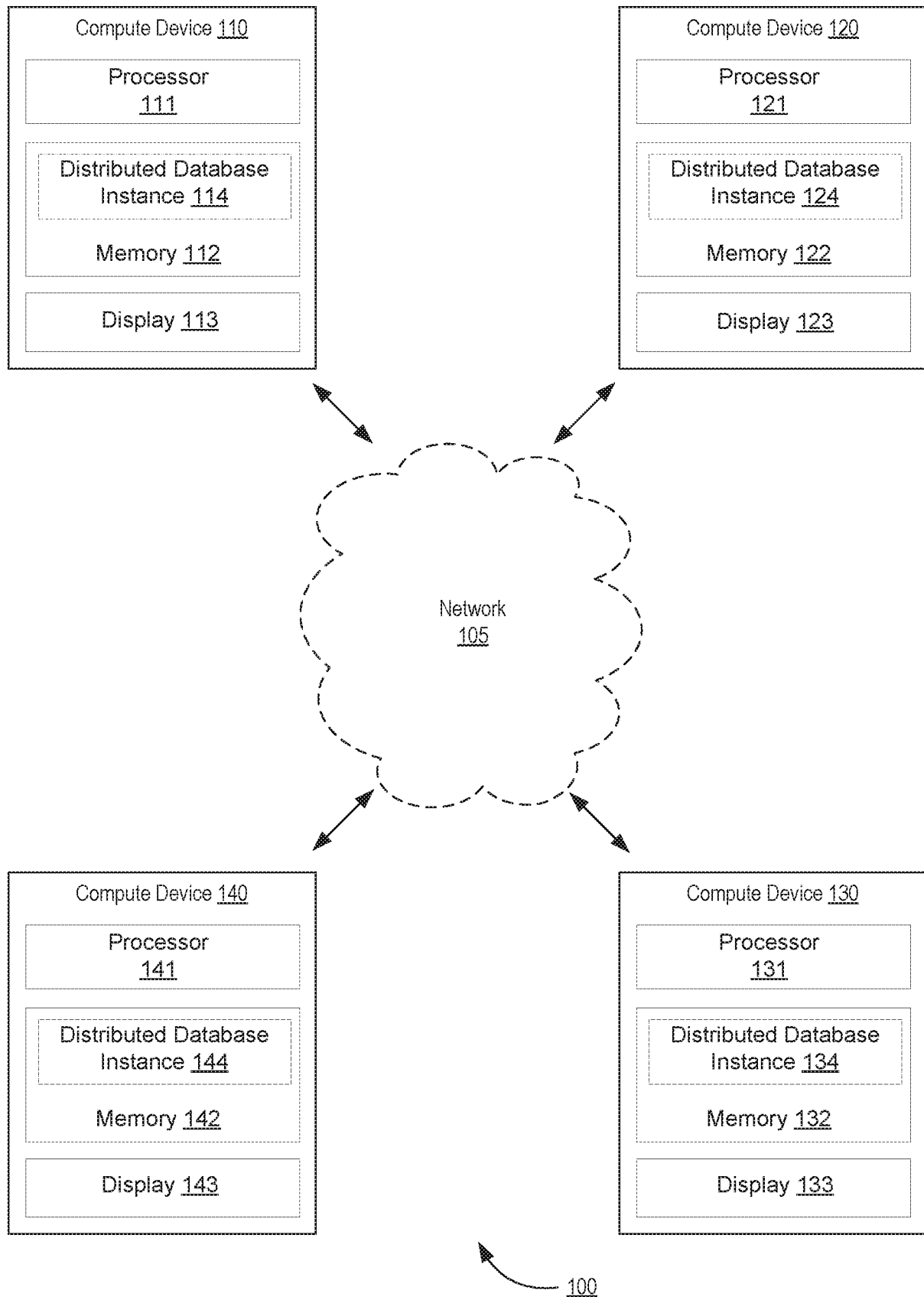


FIG. 1

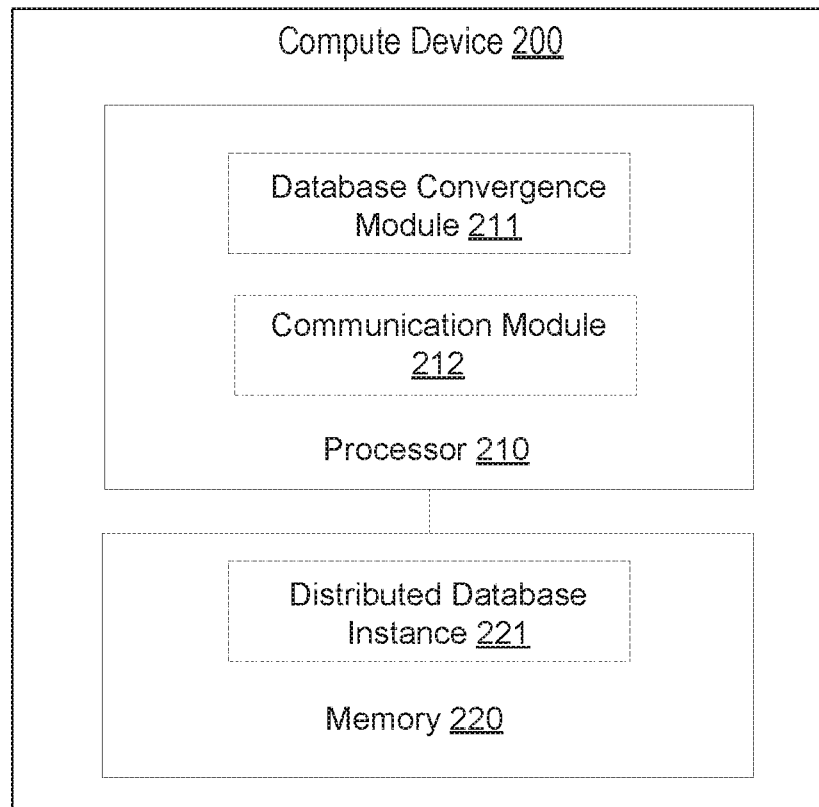


FIG. 2

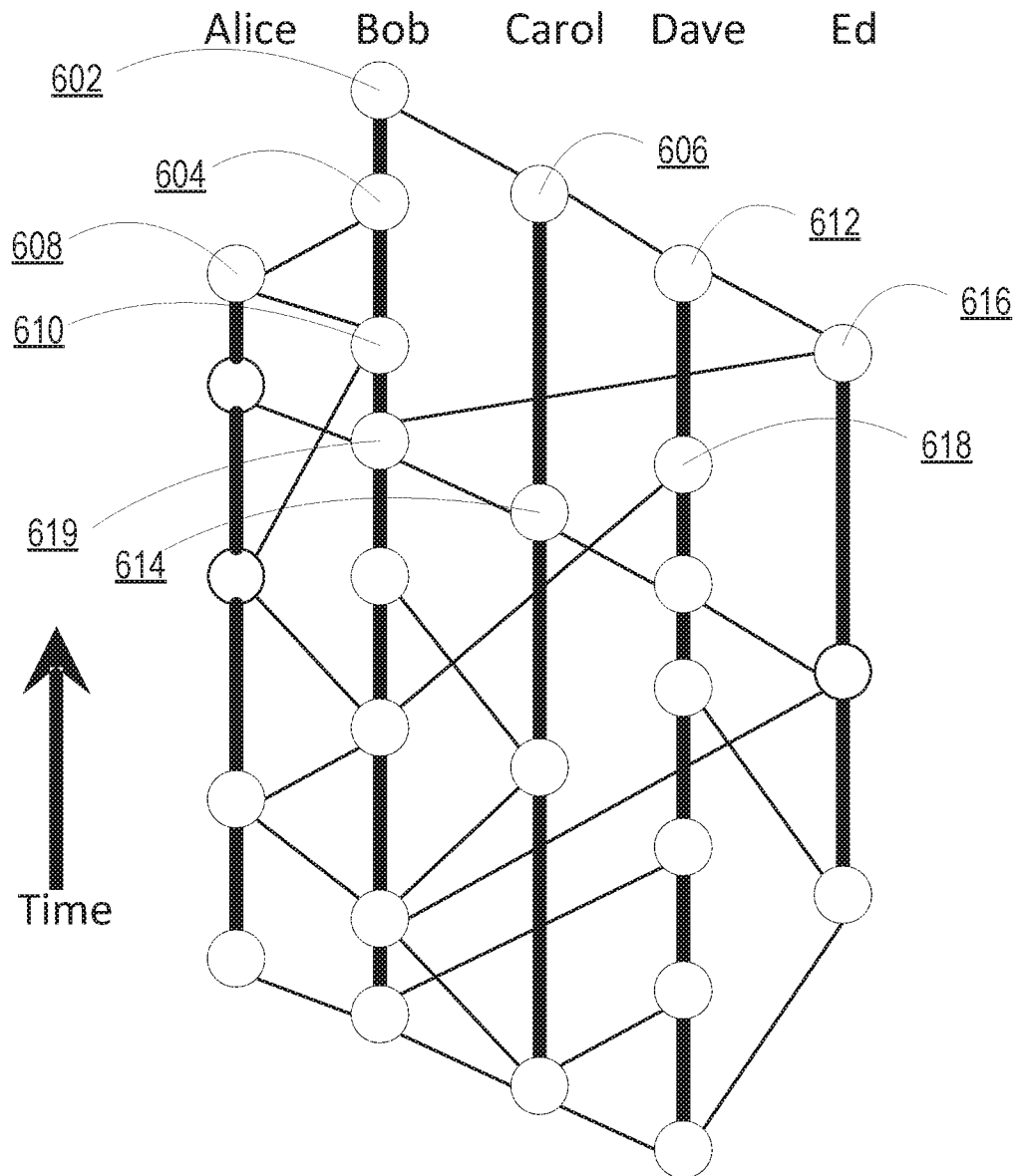
600

FIG. 3

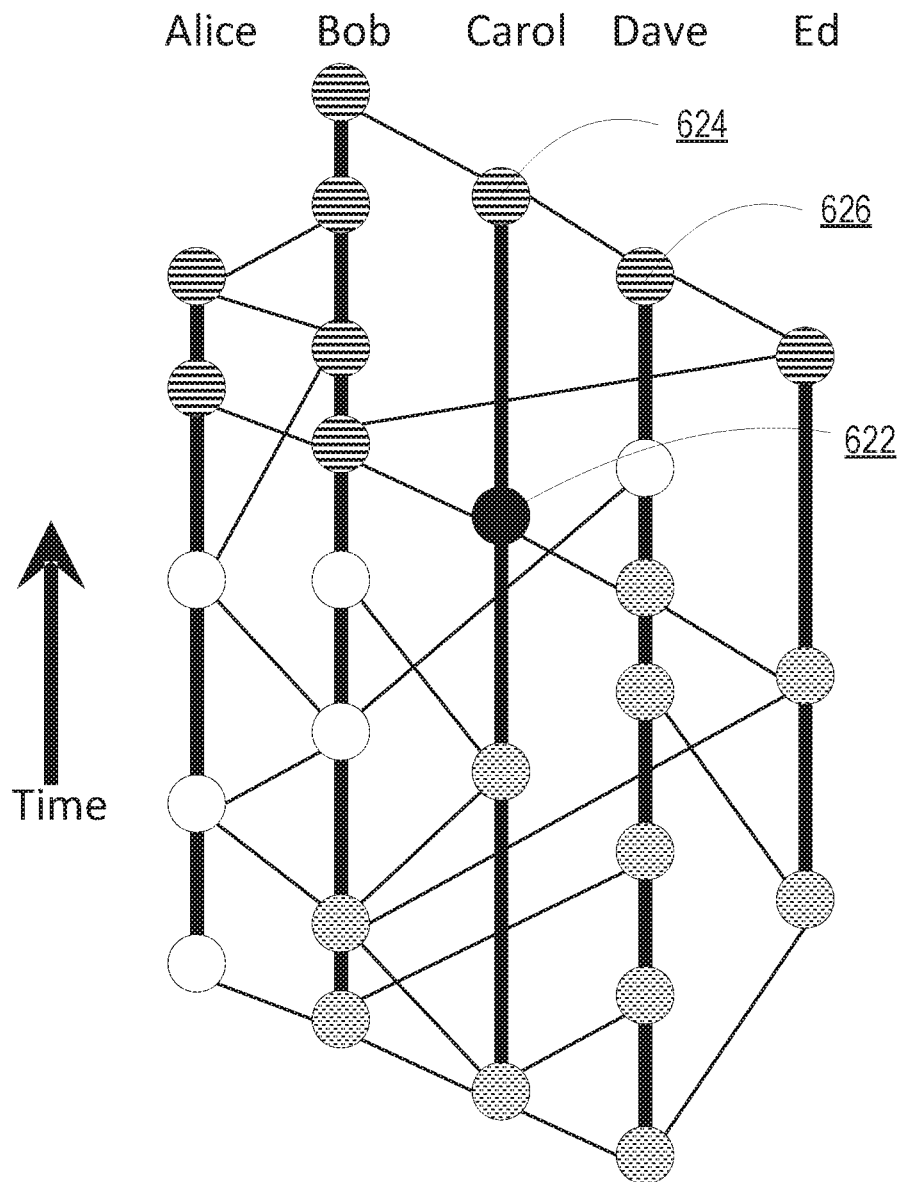
620

FIG. 4

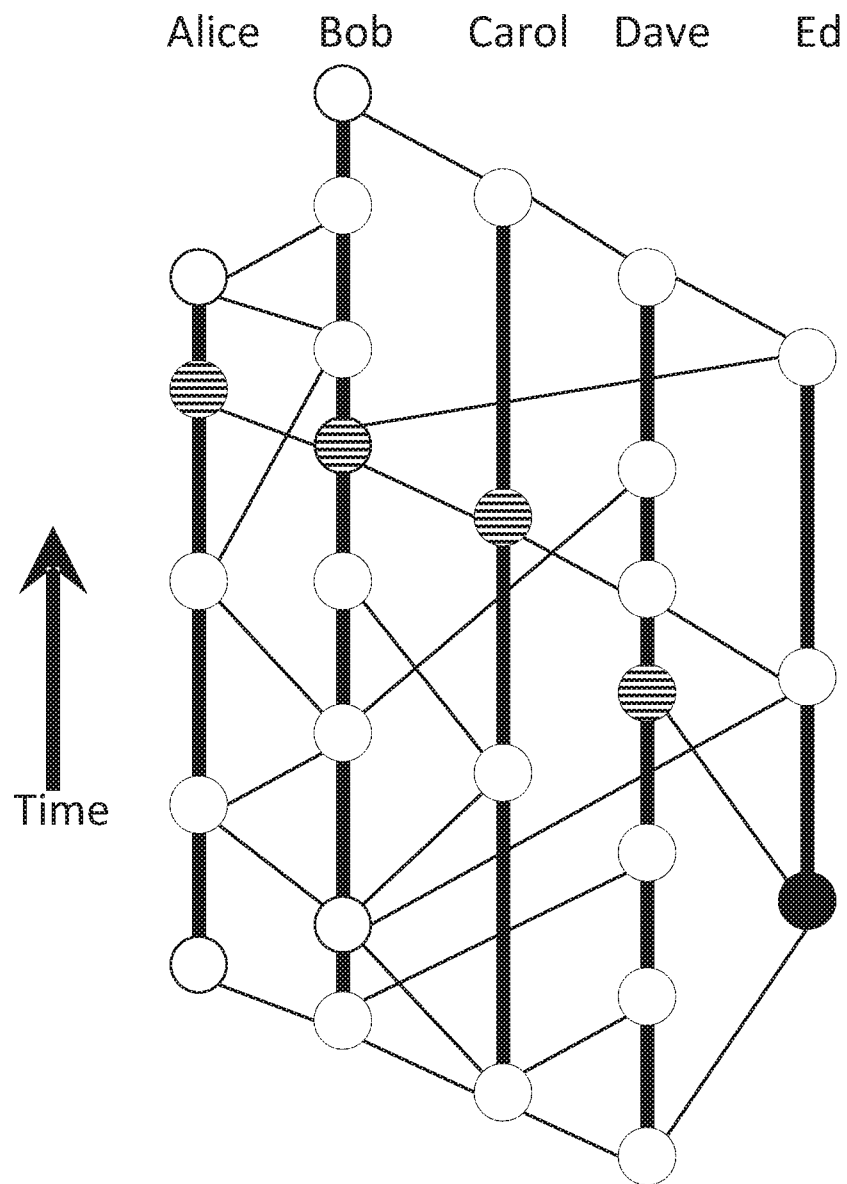


FIG. 5

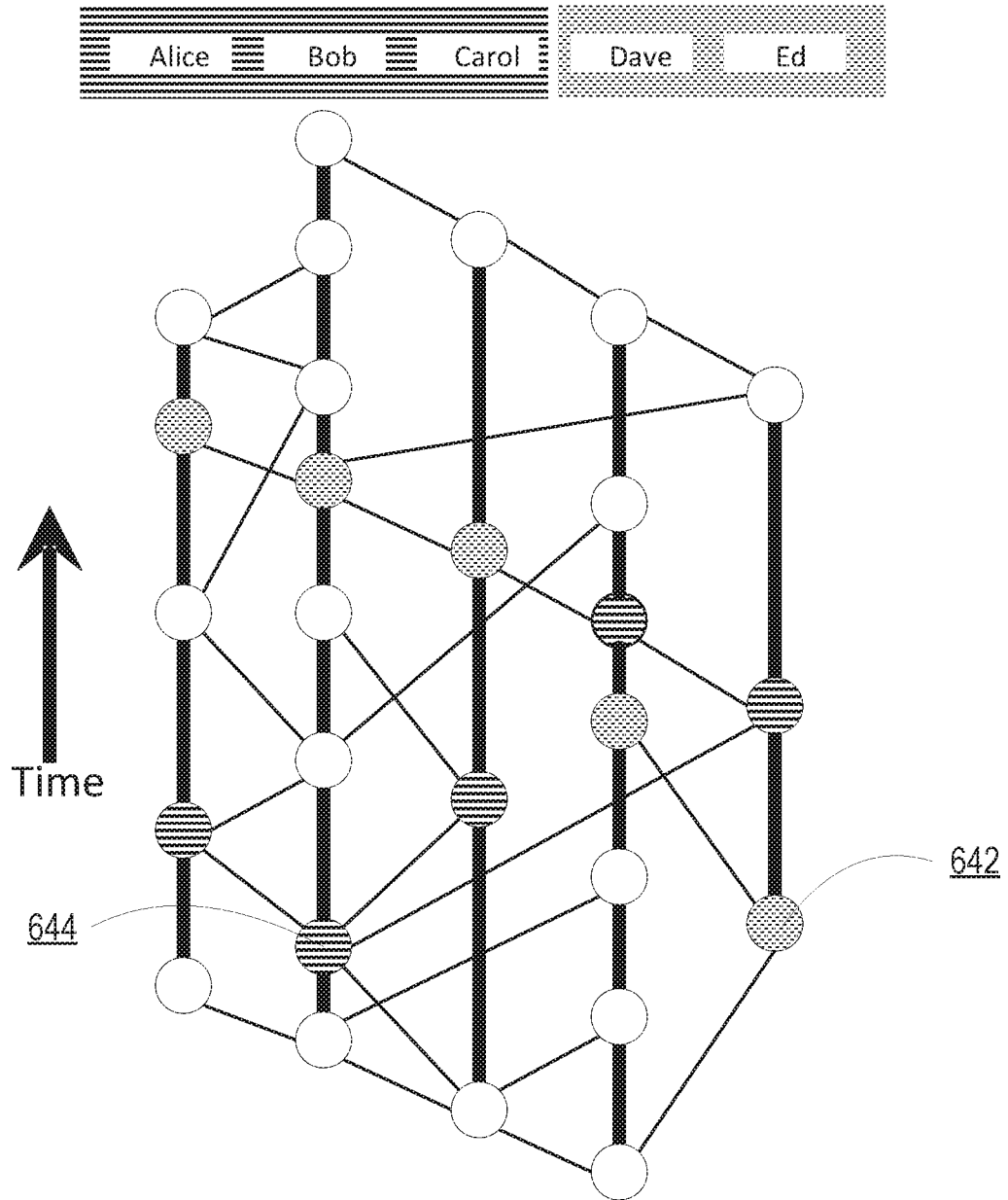
640

FIG. 6

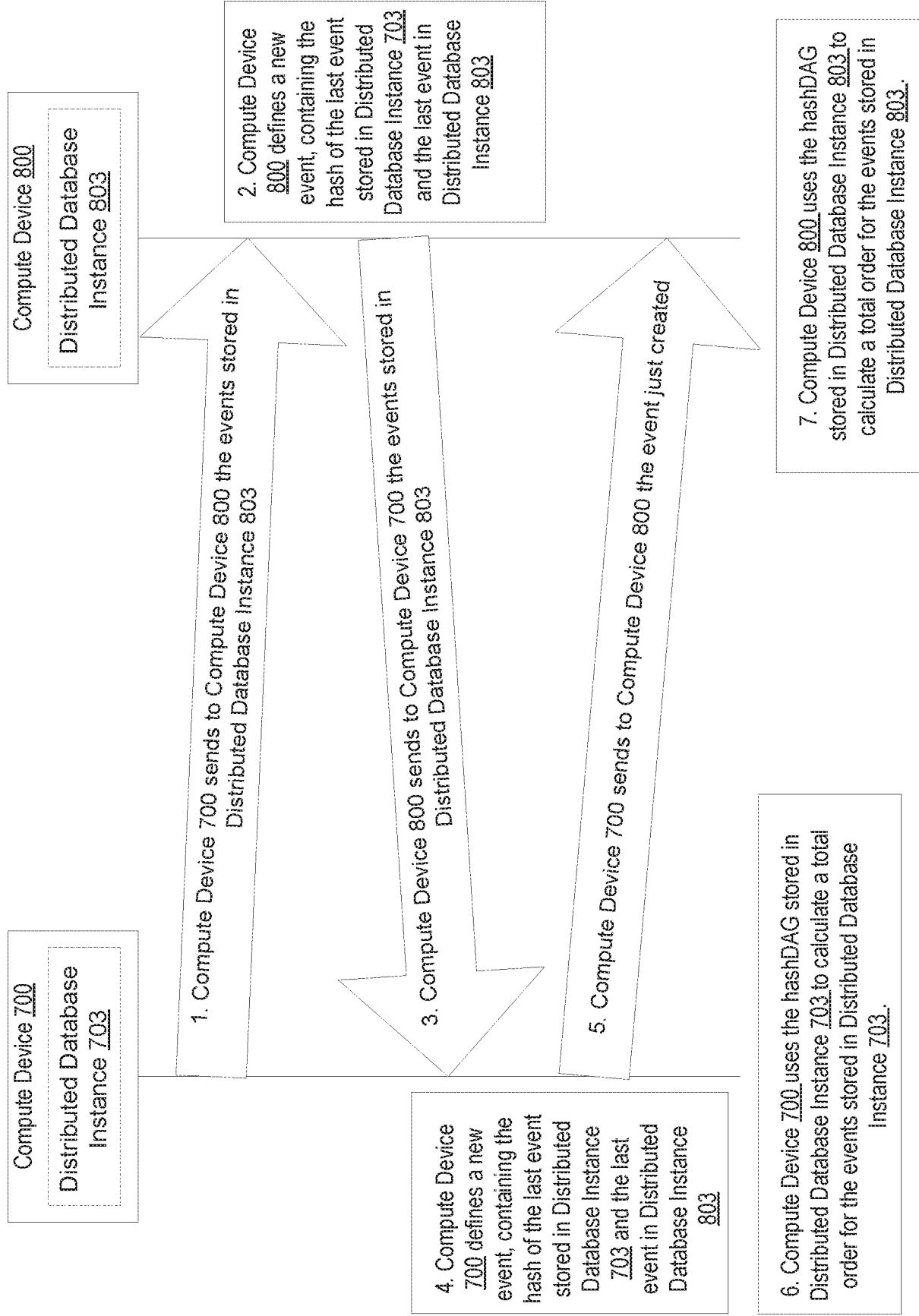


FIG. 7

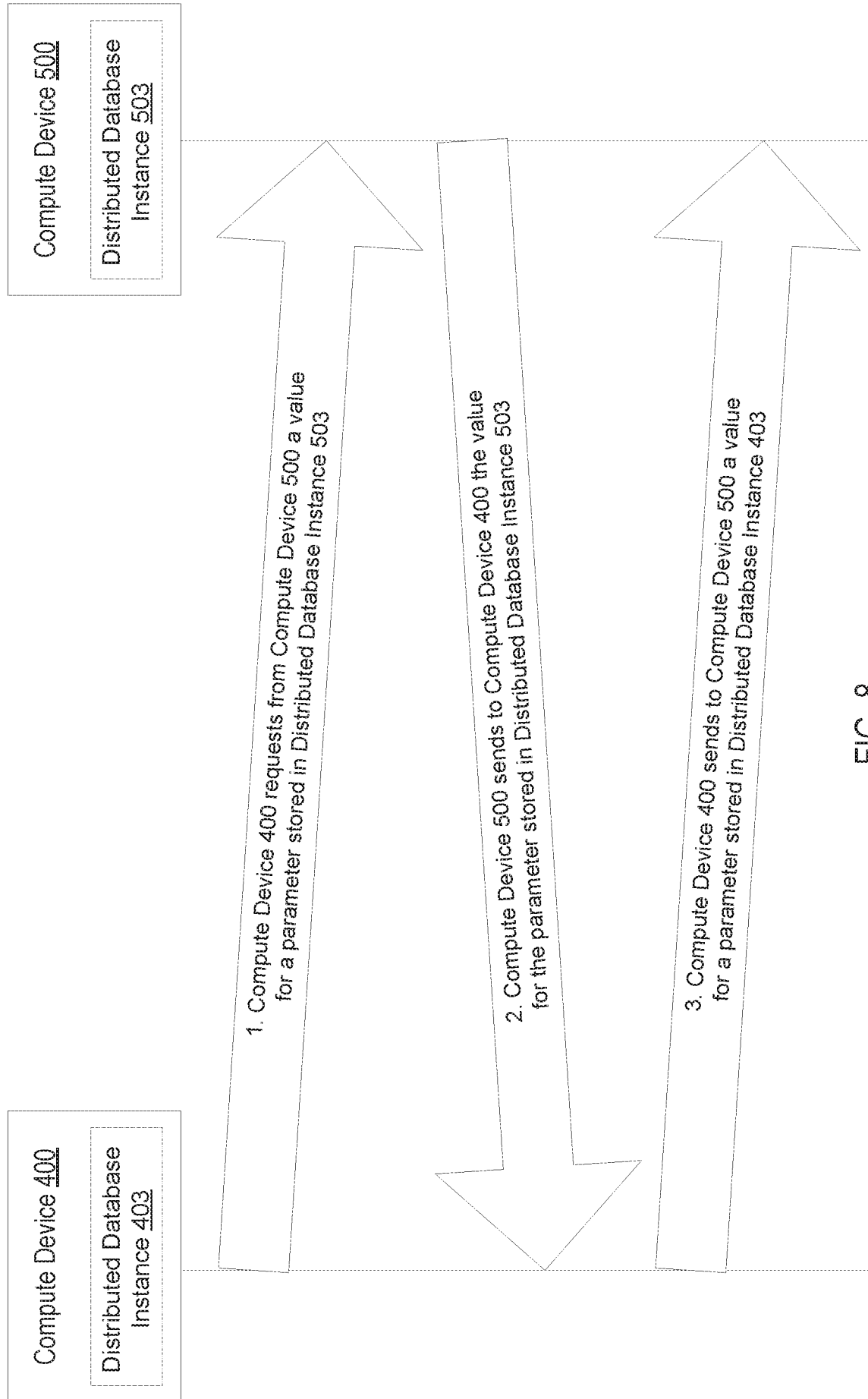


FIG. 8

YES	NO	NO	YES	YES
Value <u>411</u>	Value <u>412</u>	Value <u>413</u>	Value <u>414</u>	Value <u>415</u>

Vector of Values 410



FIG. 9a

{A, B, C, D}	{A, B, C, E}	{A, B, C, F}	{A, B, F, G}	{A, B, G, H}
Value <u>421</u>	Value <u>422</u>	Value <u>423</u>	Value <u>424</u>	Value <u>425</u>

Vector of Values 420



FIG. 9b

1. Alice 2. Bob 3. Chuck 4. David 5. Ed 6. Frank	1. Bob 2. Alice 3. Chuck 4. David 5. Ed 6. Frank	1. Bob 2. Alice 3. Chuck 4. David 5. Frank 6. Ed	1. Alice 2. Bob 3. Chuck 4. Ed 5. David 6. Frank	1. Alice 2. Bob 3. Ed 4. Chuck 5. David 6. Frank
Value <u>431</u>	Value <u>432</u>	Value <u>433</u>	Value <u>434</u>	Value <u>435</u>

Vector of Values 430



FIG. 9c

YES	YES	YES	YES	YES
Value <u>515</u>	Value <u>514</u>	Value <u>513</u>	Value <u>512</u>	Value <u>511</u>

Vector of Values 510

FIG. 10a

NO	YES	YES	YES	YES
Value <u>516</u>	Value <u>515</u>	Value <u>514</u>	Value <u>513</u>	Value <u>512</u>

Vector of Values 520

FIG. 10b

NO	NO	YES	YES	YES
Value <u>517</u>	Value <u>516</u>	Value <u>515</u>	Value <u>514</u>	Value <u>513</u>

Vector of Values 530

FIG. 10c

YES	NO	NO	YES	YES
Value <u>518</u>	Value <u>517</u>	Value <u>516</u>	Value <u>515</u>	Value <u>514</u>

Vector of Values 540

FIG. 10d

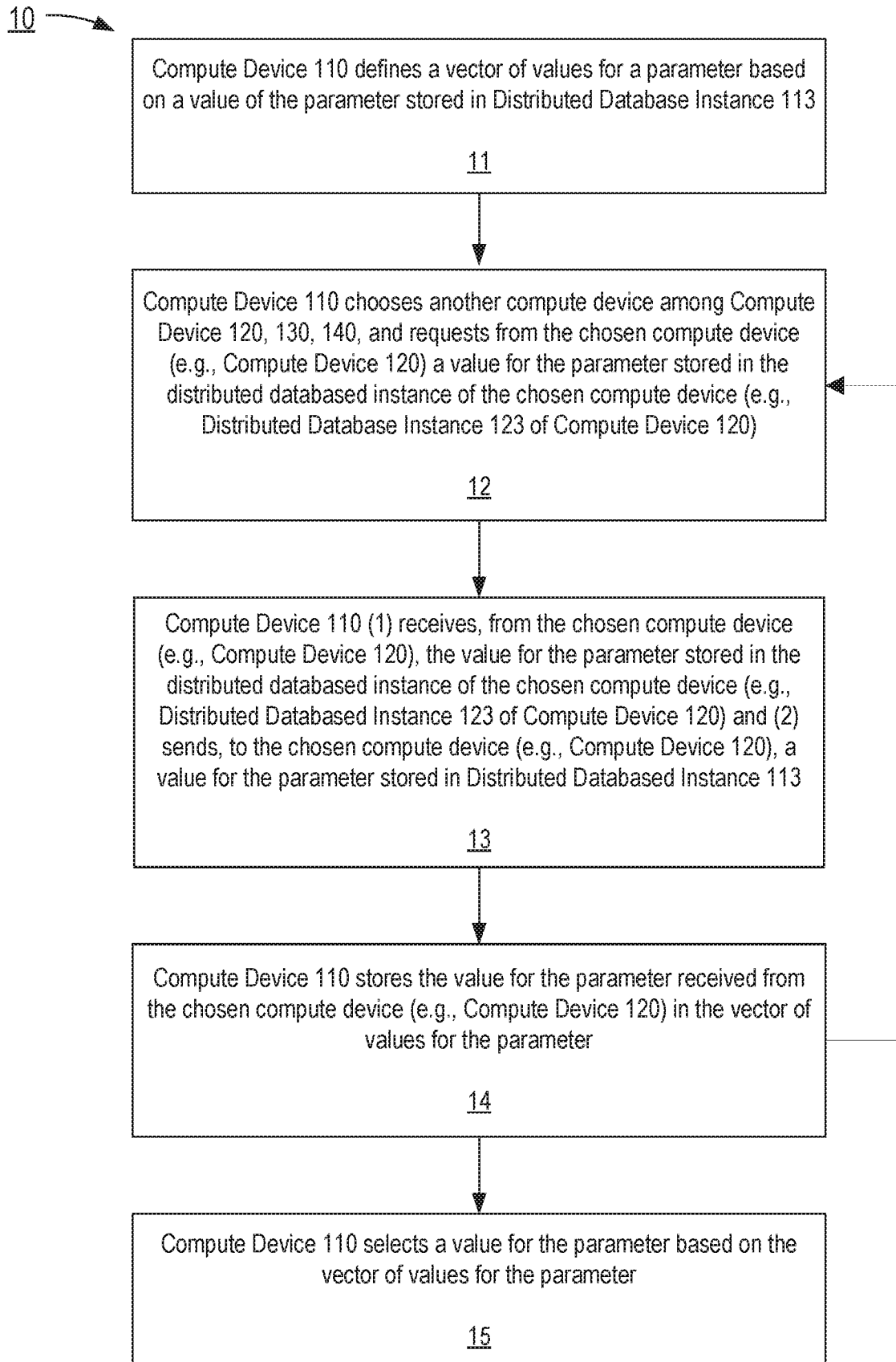


FIG. 11

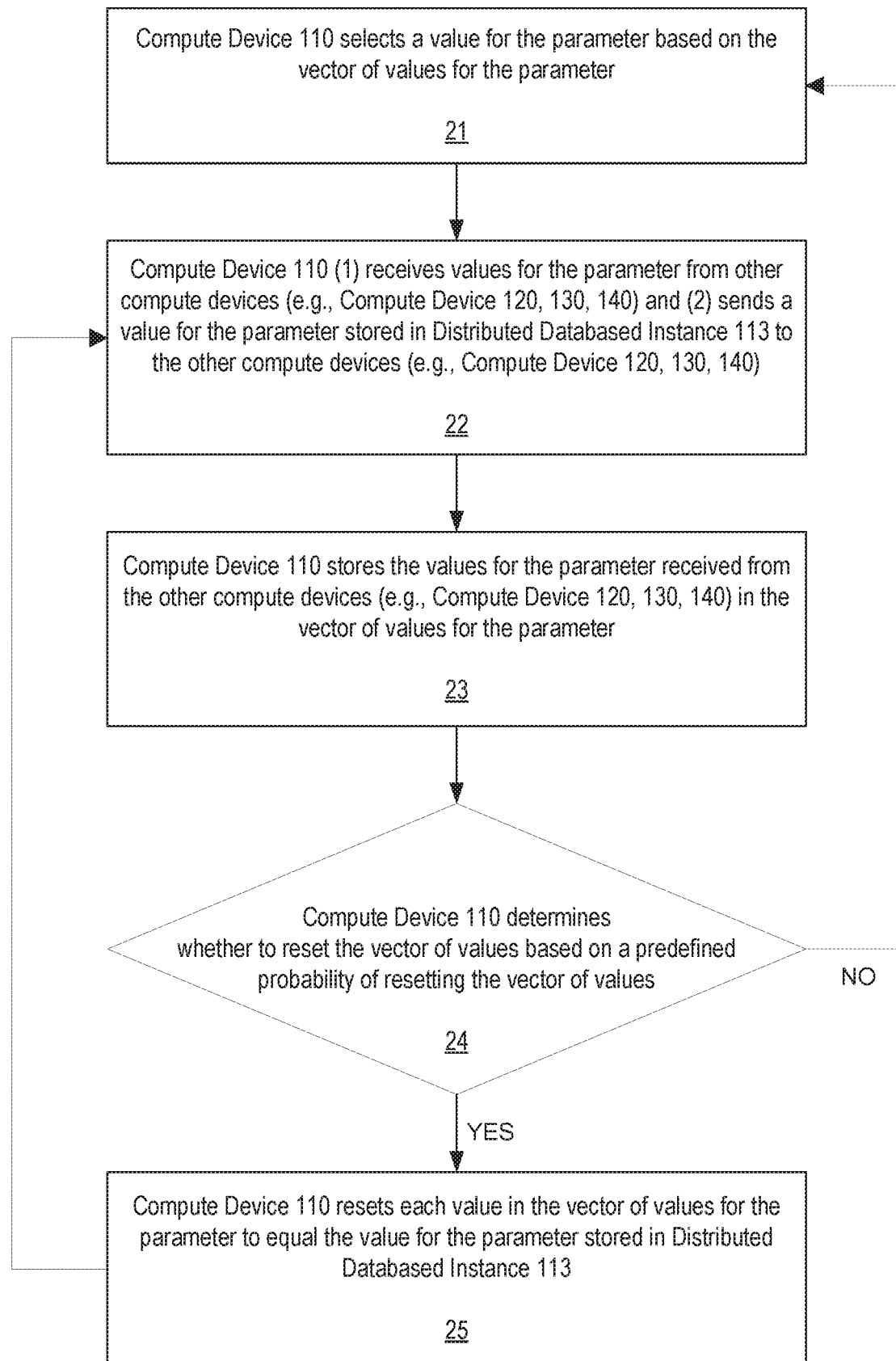
20

FIG. 12

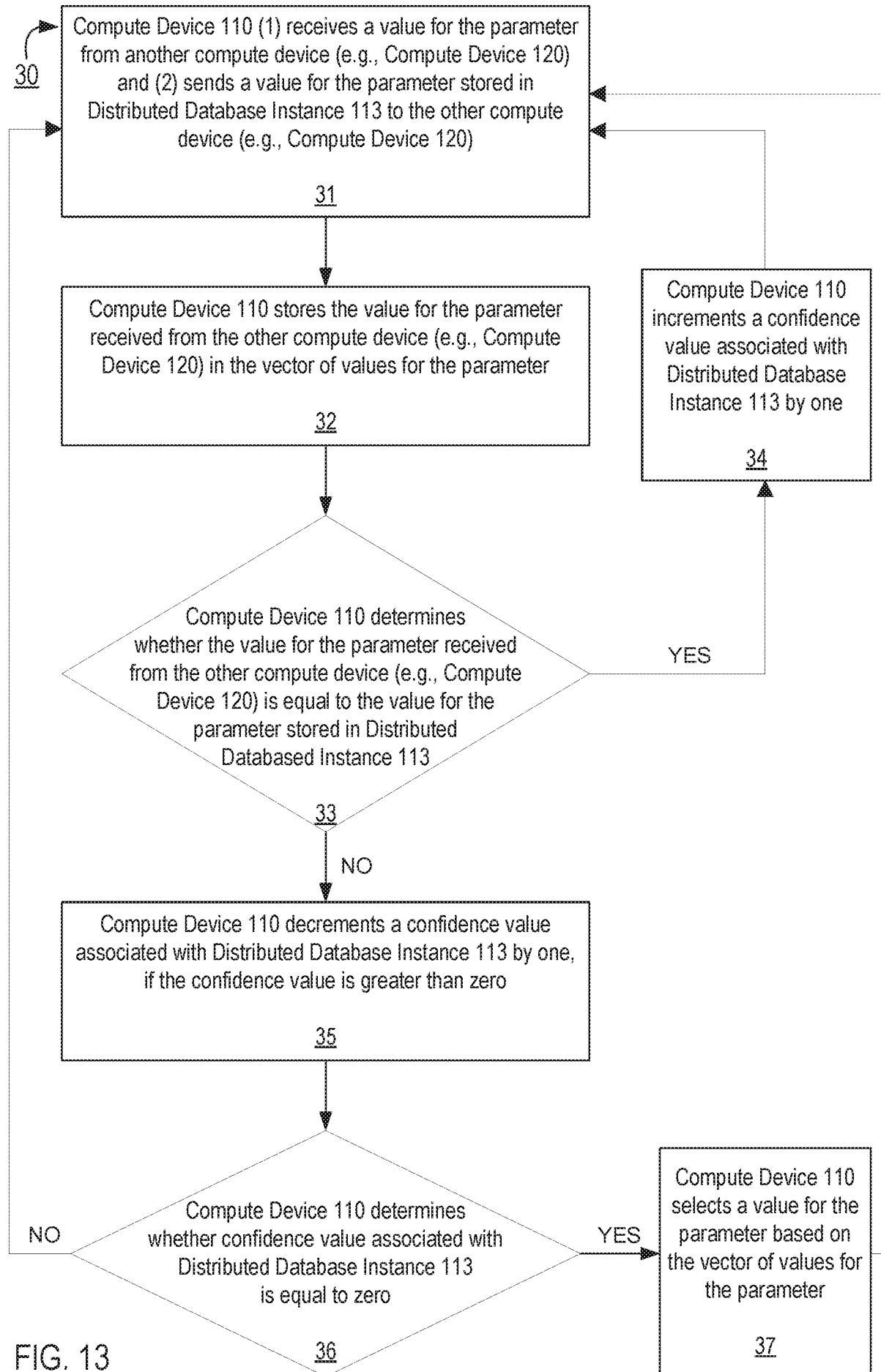


FIG. 13

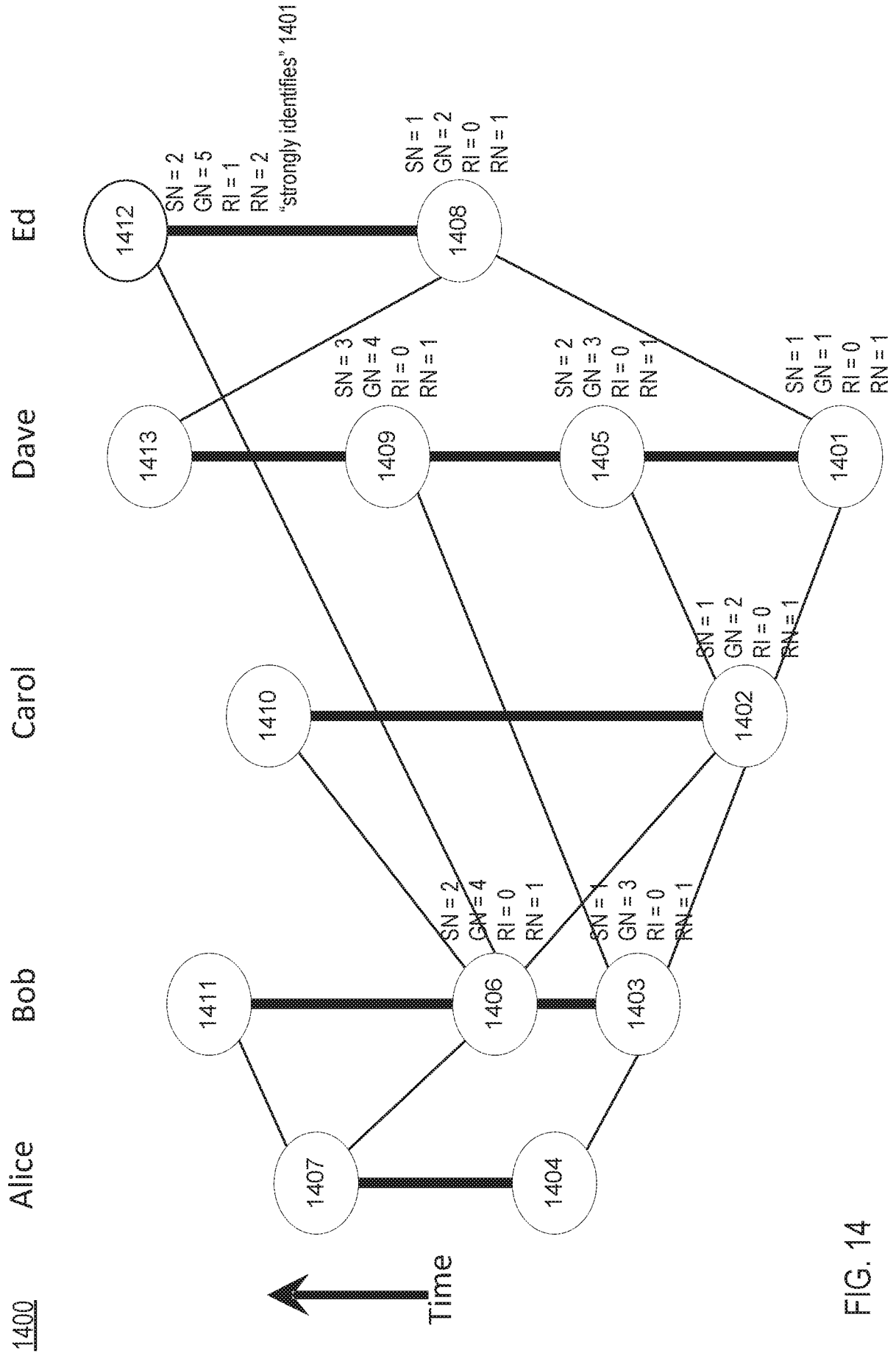
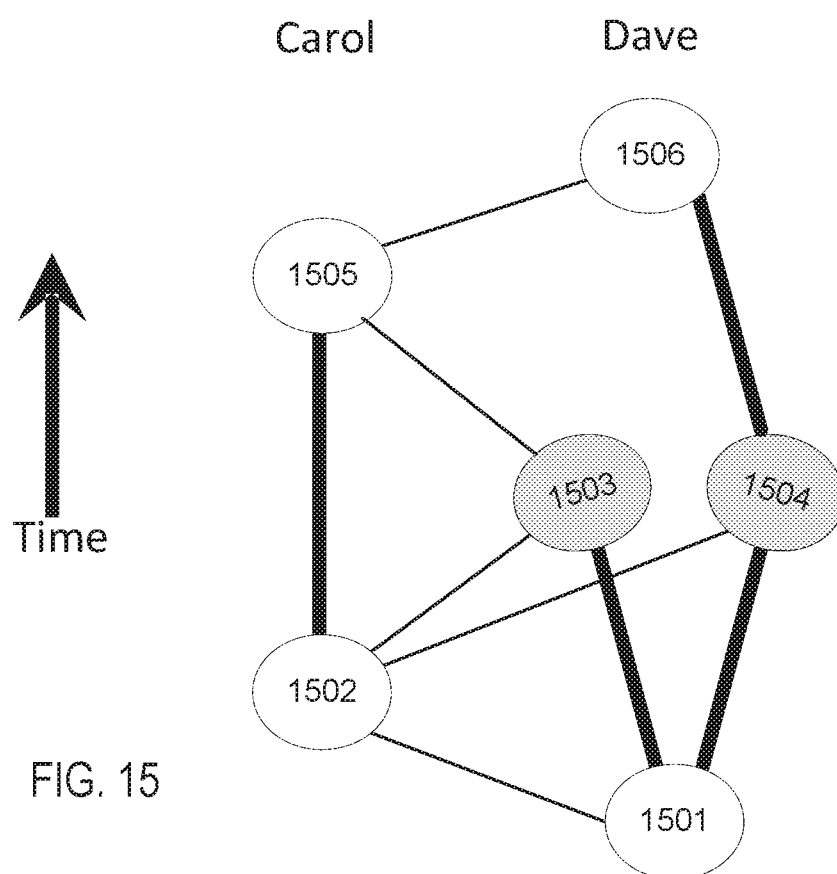


FIG. 14



An event is a tuple $e = \{d, h, t, c, s\}$ where:

d	$= data(e)$	$=$ the “payload” data, which may include transactions.
h	$= hashes(e)$	$=$ a list of hashes of the event’s parents, self-parent first.
t	$= time(e)$	$=$ creator’s claimed date and time of the event’s creation.
c	$= creator(e)$	$=$ creator’s ID number.
s	$= sig(e)$	$=$ creator’s digital signature of $\{d, h, t, c\}$.
n	$=$	the number of members in the population
m	$= 1 + \lfloor 2n/3 \rfloor$	
$first$	$=$	the unique event that has no parents
E	$=$	the set of all events
T	$=$	set of all possible $(time, date)$ pairs
$\mathbb{B}$	$=$	$\{true, false\}$
$\mathbb{N}$	$=$	$\{0, 1, 2, \dots\}$
ancestor	$:$	$E \times E \rightarrow B$
selfAncestor	$:$	$E \times E \rightarrow B$
see	$:$	$E \times E \rightarrow B$
stronglySee	$:$	$E \times E \rightarrow B$
parentRound	$:$	$E \rightarrow \mathbb{N}$
witness	$:$	$E \rightarrow \mathbb{B}$
round	$:$	$E \rightarrow \mathbb{N}$
roundDiff	$:$	$E \times E \rightarrow \mathbb{I}$
votes	$:$	$E \times E \times \mathbb{B} \rightarrow \mathbb{N}$
voteFraction	$:$	$E \times E \rightarrow \mathbb{R}$
vote	$:$	$E \times E \rightarrow \mathbb{B}$
decide	$:$	$E \times E \rightarrow \mathbb{B}$
allFamous	$:$	$\mathbb{I} \rightarrow 2^E$
famous	$:$	$E \rightarrow \mathbb{B}$
roundReceived	$:$	$E \rightarrow \mathbb{N}$
timeReceived	$:$	$E \rightarrow T$

FIG. 16a

$$\begin{aligned}
\text{ancestor}(x, y) &= (x = y) \vee (\exists z \in \text{parents}(x) : \text{ancestor}(z, y)) \\
\text{selfAncestor}(x, y) &= \text{ancestor}(x, y) \wedge ((\text{selfParent}(x) = y) \vee \text{selfAncestor}(\text{selfParent}(x), y)) \\
\text{see}(x, y) &= \text{ancestor}(x, y) \wedge \neg(\exists a, b, c \in E : \\
&\quad (\text{ancestor}(y, a) \wedge \text{ancestor}(y, b) \wedge c \in \text{parents}(x) \wedge c \in \text{parents}(b)) \wedge \\
&\quad \text{creator}(a) = \text{creator}(b) = \text{creator}(c)) \\
\text{stronglySee}(x, y) &= \text{see}(x, y) \wedge (\exists S \in 2^E : (|S| = m) \wedge (z \in S \iff (\text{see}(x, z) \wedge \text{see}(z, y)))) \\
\text{parentRound}(x) &= \begin{cases} 0 & \text{if } x = \text{first} \\ \max_{y \in \text{parents}(x)} \text{round}(y) & \text{otherwise} \end{cases} \\
\text{witness}(x) &= \exists S \in 2^E : (|S| = m \wedge \\
&\quad (\forall y \in S : (\text{round}(y) = \text{parentRound}(x) \wedge \text{stronglySee}(x, y)))) \\
\text{round}(x) &= \begin{cases} 1 + \text{parentRound}(x) & \text{if } \text{witness}(x) \\ \text{parentRound}(x) & \text{otherwise} \end{cases} \\
\text{roundDiff}(x, y) &= \text{round}(x) - \text{round}(y) \\
\text{votes}(x, y, v) &= |\{z \in E \mid \text{see}(x, y) \wedge \text{roundDiff}(x, z) = 1 \wedge \\
&\quad \text{stronglySee}(x, z) \wedge \text{vote}(z, y) = v\}| \\
\text{voteFraction}(x, y) &= \text{votes}(x, \text{true}) / (\text{votes}(x, \text{true}) + \text{votes}(x, \text{false})) \\
\text{vote}(x, y) &= \begin{cases} \text{see}(x, y) & \text{if } \text{roundDiff}(x, y) = 1 \\ (\text{voteFraction}(x, y) \geq 1/2) & \text{if } (\text{roundDiff}(x, y) \bmod 5 \neq 1) \vee \\ & |\text{voteFraction}(x, y) - 1/2| > 1/6 \\ (1 = \text{LSB}(\text{signature}(x))) & \text{otherwise} \end{cases} \\
\text{decide}(x, y) &= \text{vote}(x, y) \wedge (\text{roundDiff}(x, y) \bmod 5 \neq 1) \wedge (\text{voteFraction}(x, y) > 2/3) \\
\text{allFamous}(r) &= \{x \in E \mid \text{famous}(x) \wedge \text{round}(x) = r\} \\
\text{famous}(x) &= \text{witness}(x) \wedge \exists y \in E : \text{decide}(y, x) \\
\text{roundReceived}(x) &= \min_{r \in \mathbb{N}} (|\{y \in E \mid \text{round}(y) = r \wedge \text{famous}(y) \wedge \text{see}(y, x)\}| / \\
&\quad |\{y \in E \mid \text{round}(y) = r \wedge \text{famous}(y)\}| \geq 1/2) \\
\text{timeReceived}(x) &= \text{median}(\{\text{time}(y) \mid y \in E \wedge \text{see}(y, x) \wedge \\
&\quad (\exists z \in E : \text{round}(z) = \text{roundReceived}(x) \wedge \text{selfAncestor}(z, y)) \wedge \\
&\quad \neg(\exists w \in E : \text{selfAncestor}(y, w) \wedge \text{see}(w, x))\})
\end{aligned}$$

FIG. 16b

An event is a tuple $e = \{d, h, t, c, s\}$ where:

d	$= data(e)$	$=$ the “payload” data, which may include transactions.
h	$= hashes(e)$	$=$ a list of hashes of the event’s parents, self-parent first.
t	$= time(e)$	$=$ creator’s claimed date and time of the event’s creation.
i	$= creator(e)$	$=$ creator’s ID number.
s	$= sig(e)$	$=$ creator’s digital signature of $\{d, h, t, c\}$.
n	$=$	the number of members in the population
c	$=$	frequency of coin rounds (e.g., $c = 6$)
E	$=$	(the set of all events) $\cup \{\emptyset\}$
$\mathbb{T}$	$=$	set of all possible $(time, date)$ pairs
$\mathbb{B}$	$=$	$\{true, false\}$
$\mathbb{N}$	$=$	$\{0, 1, 2, \dots\}$
parents	:	$E \rightarrow 2^E$
selfParent	:	$E \rightarrow E$
ancestor	:	$E \times E \rightarrow \mathbb{B}$
selfAncestor	:	$E \times E \rightarrow \mathbb{B}$
see	:	$E \times E \rightarrow \mathbb{B}$
stronglySee	:	$E \times E \rightarrow \mathbb{B}$
parentRound	:	$E \rightarrow \mathbb{N}$
roundInc	:	$E \rightarrow \mathbb{B}$
round	:	$E \rightarrow \mathbb{N}$
witness	:	$E \rightarrow \mathbb{B}$
roundDiff	:	$E \times E \rightarrow \mathbb{I}$
votes	:	$E \times E \times \mathbb{B} \rightarrow \mathbb{N}$
fractTrue	:	$E \times E \rightarrow \mathbb{R}$
decide	:	$E \times E \rightarrow \mathbb{B}$
vote	:	$E \times E \rightarrow \mathbb{B}$
famous	:	$E \rightarrow \mathbb{B}$
roundReceived	:	$E \rightarrow \mathbb{N}$
timeReceived	:	$E \rightarrow \mathbb{T}$

FIG. 17a

$\text{parents}(x)$	$=$	set of parents of event x .
$\text{selfParent}(x)$	$=$	the self-parent of event x , or $\emptyset$ if none
$\text{ancestor}(x, y)$	$=$	$(x \neq \emptyset) \wedge ((x = y) \vee (\exists z \in \text{parents}(x) : \text{ancestor}(z, y)))$
$\text{selfAncestor}(x, y)$	$=$	$(x \neq \emptyset) \wedge ((x = y) \vee \text{selfAncestor}(\text{selfParent}(x), y))$
$\text{see}(x, y)$	$=$	$\text{ancestor}(x, y) \wedge \neg(\exists a, b \in E : \text{creator}(y) = \text{creator}(a) = \text{creator}(b) \wedge$ $\text{ancestor}(x, a) \wedge \text{ancestor}(x, b) \wedge \neg \text{selfAncestor}(a, b) \wedge \neg \text{selfAncestor}(b, a))$
$\text{stronglySee}(x, y)$	$=$	$\text{see}(x, y) \wedge (\exists S \in 2^E : (S > 2n/3) \wedge (z \in S \iff (\text{see}(x, z) \wedge \text{see}(z, y))))$
$\text{parentRound}(x)$	$=$	$\max(\{0\} \cup \{\text{round}(y) \mid y \in \text{parents}(x)\})$
$\text{roundInc}(x)$	$=$	$\exists S \in 2^E : (S > 2n/3 \wedge$ $(\forall y \in S : (\text{round}(y) = \text{parentRound}(x) \wedge \text{stronglySee}(x, y))))$
$\text{round}(x)$	$=$	$\text{parentRound}(x) + \begin{cases} 1 & \text{if } \text{roundInc}(x) \\ 0 & \text{otherwise} \end{cases}$
$\text{witness}(x)$	$=$	$(\text{selfParent}(x) = \emptyset) \vee (\text{round}(x) > \text{round}(\text{selfParent}(x)))$
$\text{roundDiff}(x, y)$	$=$	$\text{round}(x) - \text{round}(y)$
$\text{votes}(x, y, v)$	$=$	$ \{z \in E \mid \text{roundDiff}(x, z) = 1 \wedge \text{stronglySee}(x, z) \wedge \text{vote}(z, y) = v\} $
$\text{fractTrue}(x, y)$	$=$	$\frac{\text{votes}(x, y, \text{true})}{(\text{votes}(x, y, \text{true}) + \text{votes}(x, y, \text{false}))}$
$\text{decide}(x, y)$	$=$	$(x \neq \emptyset) \wedge (\text{roundDiff}(x, y) > 1) \wedge (\text{decide}(\text{selfParent}(x), y) \vee$ $(\text{witness}(x) \wedge (\text{roundDiff}(x, y) \bmod c \neq 1) \wedge \neg(\frac{1}{3} \leq \text{fractTrue}(x, y) \leq \frac{2n}{3})))$
$\text{vote}(x, y)$	$=$	$\begin{cases} \text{vote}(\text{selfParent}(x), y) & \text{if } (\neg \text{witness}(x)) \vee \text{decide}(\text{selfParent}(x), y) \\ 1 = \text{middleBit}(\text{signature}(x)) & \text{if } \text{witness}(x) \\ & \wedge \neg \text{decide}(\text{selfParent}(x), y) \\ & \wedge (\text{roundDiff}(x, y) \neq 1) \\ & \wedge (\text{roundDiff}(x, y) \bmod c = 1) \\ \text{fractTrue}(x, y) \geq \frac{1}{2} & \text{otherwise} \end{cases}$
$\text{famous}(x)$	$=$	$\text{witness}(x) \wedge \exists y \in E : \text{decide}(y, x) \wedge \text{vote}(y, x)$
$\text{roundReceived}(x)$	$=$	$\min_{r \in \mathbb{N}} \frac{ \{y \in E \mid (\text{round}(y) = r) \wedge \text{famous}(y) \wedge \text{see}(y, x)\} }{ \{y \in E \mid (\text{round}(y) = r) \wedge \text{famous}(y)\} } \geq 1/2$
$\text{timeReceived}(x)$	$=$	$\text{median}(\{\text{time}(y) \mid y \in E \wedge \text{see}(y, x) \wedge$ $(\exists z \in E : \text{round}(z) = \text{roundReceived}(x) \wedge \text{selfAncestor}(z, y)) \wedge$ $\neg(\exists w \in E : \text{selfAncestor}(y, w) \wedge \text{see}(w, x))\})$

FIG. 17b