CLAIMS

1. A zirconocene-titanocene catalyst system comprising a zirconocene catalyst and a titanocene catalyst, wherein the zirconocene catalyst comprises a product of an activation reaction of $((R^1)_x$-cyclopentadienyl)$((R^2)_y$-cyclopentadienyl)zirconium dichloride/dibromide/dialkyl and an alkylaluminoxane, wherein subscript x is 1 or 2; subscript y is 0, 1 or 2; and each R^1 and R^2 independently is methyl, ethyl, a normal-$(C_3$-$C_{10})$alkyl (linear), or an iso-$(C_3$-$C_{10})$alkyl; and wherein the titanocene catalyst comprises a product of an activation reaction of bis(cyclopentadienyl)titanium dichloride with a trialkylaluminum; wherein the zirconocene-titanocene catalyst system is characterized by a weight/weight ratio of trialkylaluminum to the zirconocene catalyst from 0.005 to 0.25; and wherein the zirconocene-titanocene catalyst system is characterized by a weight/weight ratio of bis(cyclopentadienyl)titanium dichloride to the zirconocene catalyst from 0.001 to 0.05.

2. The zirconocene-titanocene catalyst system of claim 1 further comprising a metal carboxylate salt, wherein the metal carboxylate salt is represented by the formula: $MQ_m(O_2CR)_n$, wherein M is a metal atom of Group 2 or Group 13 of the Periodic table of Elements; Q is a halogen, hydroxy, alkyl, alkoxy, aryloxy, siloxy, silyl, or sulfonate group; R is a $(C_5$-$C_{30})$hydrocarbyl; subscript m is an integer from 0 to 3; subscript n is an integer from 1 to 3; and the sum of subscripts m and n is equal to the valence of M.

3. The zirconocene-titanocene catalyst system of claim 1 or 2 characterized by any one of limitations (i) to (vi): (i) subscript x is 1 and subscript y is 0, (ii) subscripts x and y are each 1, (iii) subscript x is 1 and subscript y is 2, (iv) subscript x is 2 and subscript y is 0, (v) subscript x is 2 and subscript y is 1, (vi) subscript x is 2 and subscript y is 2.

4. The zirconocene-titanocene catalyst system of any one of claims 1 to 3 characterized by any one of limitations (i) to (viii): (i) the $((R^1)_x$-cyclopentadienyl)$((R^2)_y$-cyclopentadienyl)zirconium dichloride/dibromide/dialkyl is selected from bis(butylcyclopentadienyl)zirconium dichloride, bis(butylcyclopentadienyl)zirconium dibromide, bis(butylcyclopentadienyl)zirconium dimethyl, and bis(butylcyclopentadienyl)zirconium diethyl; (ii) the $((R^1)_x$-cyclopentadienyl)$((R^2)_y$-cyclopentadienyl)zirconium dichloride/dibromide/dialkyl is bis(butylcyclopentadienyl)zirconium dichloride; (iii) the $((R^1)_x$-cyclopentadienyl)$((R^2)_y$-cyclopentadienyl)zirconium dichloride/dibromide/dialkyl is bis(butylcyclopentadienyl)zirconium dimethyl; (iv) the $((R^1)_x$-cyclopentadienyl)$((R^2)_y$-

cyclopentadienyl)zirconium dichloride/dibromide/dialkyl is bis(butylcyclopentadienyl)zirconium diethyl; (v) the $((R^1)_x$-cyclopentadienyl)$((R^2)_y$-cyclopentadienyl)zirconium dichloride/dibromide/dialkyl is selected from bis(1-methyl-3-butylcyclopentadienyl)zirconium dichloride, bis(1-methyl-3-butylcyclopentadienyl)zirconium dibromide, bis(1-methyl-3-butylcyclopentadienyl)zirconium dimethyl, and bis(1-methyl-3-butylcyclopentadienyl)zirconium diethyl; (vi) the $((R^1)_x$-cyclopentadienyl)$((R^2)_y$-cyclopentadienyl)zirconium dichloride/dibromide/dialkyl is bis(1-methyl-3-butylcyclopentadienyl)zirconium dichloride; (vii) the $((R^1)_x$-cyclopentadienyl)$((R^2)_y$-cyclopentadienyl)zirconium dichloride/dibromide/dialkyl is bis(1-methyl-3-butylcyclopentadienyl)zirconium dimethyl; and (viii) the $((R^1)_x$-cyclopentadienyl)$((R^2)_y$-cyclopentadienyl)zirconium dichloride/dibromide/dialkyl is bis(1-methyl-3-butylcyclopentadienyl)zirconium diethyl.

5. The zirconocene-titanocene catalyst system of any one of claims 1 to 4 wherein the trialkylaluminum is selected from any one of limitations (i) to (vii): (i) tri($(C_1$-$C_8)$alkyl)aluminum, (ii) tri($(C_3$-$C_7)$alkyl)aluminum, (iii) tri($(C_4$-$C_6)$alkyl)aluminum, (iv) tri((C_4)alkyl)aluminum, (v) tri((C_6)alkyl)aluminum, (vi) tri(2-methylpropyl)aluminum, and (vii) tri(hexyl)aluminum.

6. The zirconocene-titanocene catalyst system of any one of claims 1 to 5 wherein the zirconocene catalyst is supported on a carrier material.

7. The zirconocene-titanocene catalyst system of any one of claims 1 to 5 wherein the zirconocene catalyst and, optionally, the titanocene catalyst, is spray-dried on a carrier material.

8. A method of making a zirconocene-titanocene catalyst system, the method comprising contacting the $((R^1)_x$-cyclopentadienyl)$((R^2)_y$-cyclopentadienyl)zirconium dichloride/dibromide/dialkyl with an alkylaluminoxane and, optionally, a carrier material and, optionally, a metal carboxylate salt of the formula: $MQ_m(O_2CR)_n$, wherein M, Q, R, m and n are as defined above, to give zirconocene catalyst, and then contacting the zirconocene catalyst with a titanocene catalyst made by an activation reaction of bis(cyclopentadienyl)titanium dichloride with a trialkylaluminum; thereby giving the titanium catalyst and the zirconocene-titanocene catalyst system.

9. A method of making a polyethylene composition, the method comprising contacting ethylene and optionally zero, one, or more $(C_3$-$C_{20})$alpha-olefin with the zirconocene-titanocene catalyst system of any one of claims 1 to 8 in a polymerization reactor to

generate a polymerization reaction giving a polyethylene composition comprising a polyethylene homopolymer or ethylene/(C_3-C_{20})alpha-olefin copolymer, respectively, and the zirconocene-titanocene catalyst system, or a by-product thereof.

10. The method of claim 9 characterized by any one of limitations (i) to (iv): (i) externally-sourced molecular hydrogen gas (H_2) is not added into the polymerization reactor and is not present during the contacting step of the method; (ii) the method further comprises adding externally-sourced H_2 gas into the polymerization reactor during the contacting step of the method; (iii) the method is free of (C_3-C_{20})alpha-olefin and makes the polyethylene homopolymer, which contains constituent units that are derived from ethylene only; (iv) the method further comprises one or more (C_3-C_{20})alpha-olefin and makes the ethylene/(C_3-C_{20})alpha-olefin copolymer, which contains monomeric constituent units that are derived from ethylene and comonomeric constituent units that are derived from one or more (C_3-C_{20})alpha-olefin comonomer(s), respectively.

11. The method of claim 9 or 10 comprising a gas phase polymerization optionally in the presence of added external molecular hydrogen gas (H_2), optionally in the presence of an induced condensing agent (ICA); and in one, two or more gas phase polymerization reactors under (co)polymerizing conditions, thereby making the polyethylene composition; wherein the (co)polymerizing conditions comprise a reaction temperature from 80 degrees (°) to 110° Celsius (C.); a molar ratio of the molecular hydrogen gas to the ethylene from 0.00001 to 0.25; and a molar ratio of the comonomer to the ethylene from 0.001 to 0.20.

12. The method of any one of claims 9 to 11 wherein prior to the contacting step the method further comprises any one of steps (i) to (iii): (i) premixing the zirconocene catalyst and the titanocene catalyst in a separate mixing vessel to make a premixture thereof, aging the premixture for from 2 hours to 7 days to make an aged premixture, and then feeding the aged premixture into the polymerization reactor; (ii) premixing the zirconocene catalyst and the titanocene catalyst with each other in a mixer to make an unaged premixture thereof, and within 120 minutes of the premixing, feeding the unaged premixture into the polymerization reactor; and (iii) feeding the zirconocene catalyst and the titanocene catalyst separately via separate reactor inlets into the polymerization reactor, thereby making the zirconocene-titanocene catalyst system *in situ* in the polymerization reactor.