METHOD AND DATA SYSTEM FOR DETERMINING FINANCIAL INSTRUMENTS FOR USE IN FUNDING OF A LOAN.

INTRODUCTION

This invention relates to a method and a data processing system for calculating the type, the number, and the volume of financial instruments for funding a loan with equivalent proceeds to a debtor, the loan being designed to be at least partially refinanced during the remaining term to maturity of the loan. By the method according to the invention, the remaining term to maturity of the loan is also determined at the beginning of each period such that the debtor's payments on the loan during the entire term to maturity of the loan are within a band defined by a set of maximum and minimum limits which may be determined for each period, and such that the remaining term of the loan is within a band defined by a maximum limit and a minimum limit. Furthermore, a special financial instrument is determined which is designed to ensure that given maximum limits for payments on the loan and term to maturity are observed. The results of the method according to the invention may be used by a lender, e.g. a financing institution such as a mortgage credit institution, to ensure that such a loan is funded such that both interest rate risk and imbalances in the payment flows are prevented or minimized. By applying the results of the method according to the invention, the lender may thus create a hedge between the lending and the funding.

When refinancing a loan, other financial instruments than the instruments forming the basis of the volume of the original loan may be used, for which reason, in connection with the refinancing, an adjustment of the interest rate on the loan

may be necessary in relation to the interest level applicable at the time of the refinancing. Loans which are fully or partially refinanced during the term to maturity of the loan are thus termed Loan with Adjustable Interest Rates (LAIR). One example of the financial instruments is non-callable bullet bonds. In the following, the financial instruments are also called funding instruments, just as funding volume is also used as a term for the financial instruments constituting the volume.

BACKGROUND OF THE INVENTION AND INTRODUCTION TO THE INVENTION.

In the Danish mortgage credit market callable loans have historically been far the predominant type of loans and, therefore, callable bonds in a pure "pass through" have been equally predominant on the bond side. For a number of years, up to the withdrawal in 1985 by the Danish Ministry of Housing of the permission to grant cash loans, mortgage credit institutions also offered the so-called loans with adjustable interest rates. The previous loans with adjustable interest rates were characterized by:

1) a long-term credit commitment.
2) funding every fifth year by the issue of bonds with a term to maturity of 1 to 5 years.
3) the interest rate being fixed in successive periods of 5 years.
4) the underlying bonds with a term to maturity of 1 to 5 years being non-callable. The debtor is in a position to terminate the loan at par prior to the next interest rate adjustment.

The Danish loans with adjustable interest rates did not prove very successful in that only per milles of the total lending by mortgage credit institutions was granted as loans with adjustable interest rates. The reasons were, presumably, that the call premium was insignificant in those years due to a

very large difference between the market rate and the coupon rate and in addition, the investors were not as aware of the problem as they are today. Therefore, the difference in interest rates between callable and non-callable bonds was not sufficiently large in itself to make loans with adjustable interest rates attractive. Furthermore, the product was not transparent seen from the point of view of the borrower. An aspect which might have had some influence at times was that a continued rise in the Danish interest level was expected so that the borrowers did not expect a loan with adjustable interest rates to be advantageous in the long run. Finally, the previous structure of loans with adjustable interest rates involved an arbitrary and unpredictable interest rate risk every fifth year. Most likely, these conditions explain the poor supply in those years.

In June 1993 certain Danish tax laws were changed such that the mortgage credit institutions were, in actual fact, once again given the opportunity of offering loans with adjustable interest rates.

This offers the prospect of changing the long-term mortgage market such that in future the funding products will also be attractive to foreign investors. One prerequisite is, in all probability, that in future bonds are offered in conformity with international practice, e.g. as non-callable bullet bonds. It has therefore been of interest to examine whether variants of loans with adjustable interest rates can be made attractive to the borrowers.

The traditional loans with adjustable interest rates are connected with a risk of, in principle, unlimited, intermittent jumps in the interest rate on the loan. To many borrowers, especially in the segment of private customers, this risk must be assumed to be unacceptable, particularly in view of the consequences as to the borrower's liquidity of a rise in the interest rate on the loan to a very high level. It is therefore of interest to consider whether the structure of

loans with adjustable interest rates can be combined with an adjustable term to maturity in which the interest rate on the loan rising or falling, respectively, affects the debtor payments only to a predetermined extent defined by a set of maximum and minimum limits, whereas the remaining term to maturity of the loan is varied in accordance with the interest rate on the loan.

Typically, there is, however, both a maximum limit and a minimum limit for the range of the remaining term to maturity, which may be determined by both the borrower, the lender, and by public authorities or legislation.

A characteristic feature of traditional loans with adjustable interest rates was a match between the term to maturity of the last maturing funding instrument and the period of time between interest rate adjustments, viz. 5 years. If this precondition is abolished, the way is paved for an, in principle, far wider range of opportunities as to funding and interest rate adjustment.

Thus, the opportunity arises of securing a gradual adjustment of the borrowing costs to the market rate with an adjustment time depending on the maximum term to maturity of the underlying interest rate adjustment bond and on the weight with which the individual interest rate adjustment bond is included. This principle will, just as the above-mentioned possibility of an adjustable term to maturity, reduce the risk of large intermittent changes in the interest rate on the loan characterizing the traditional loans with adjustable interest rates.

If the short-term interest rate is systematically minimum than the long-term interest rate, it will be possible to reduce the long-term borrowing costs for the borrowers. Furthermore, the borrowing costs may, as mentioned above, be reduced relative to callable bonds due to the absence of a

call right and via increased liquidity and internationalization of sales.

Whether it is possible to counter an interest rate adjustment by adjusting the remaining term to maturity of the loan depends on the determined maximum and minimum limits for payments on the loan and term to maturity, as well as on the extent to which the remaining debt of the loan is adjusted to the market rate at the time of the adjustment of the interest rate. The traditional loans with adjustable interest rates were characterized by the remaining debt of the loan being 100 per cent adjusted to the market rate every fifth year. Partly by allowing other frequencies with which the interest rate adjustment is performed, and partly by allowing only a partial adjustment of the interest rate of the remaining debt of the loan, larger changes in the interest rate on the loan than in the original structure will be compatible with the maximum and minimum limits for the payments on the loan. Therefore, it should be possible to combine a partial adjustment of the interest rate of the remaining debt of the loan, as well as other interest rate adjustment frequencies, with an adjustable term to maturity.

In connection with loans with adjustable interest rates, relations to the balance principle must be mentioned. It is a leading principle in the legal regulation of the activities of Danish mortgage credit institutions that the institutions must not expose themselves to interest rate and funding risks. On the face of it, the structure of loans with adjustable interest rates is contrary to these basic principles, the funding side having a substantially shorter term to maturity than has the lending side. Traditional loans with adjustable interest rates are nevertheless regarded as lying within the balance principle seen in the perspective that borrowers accept to pay any interest rate that may occur in connection with a future refinancing. In principle, therefore, this is a "pass through" which does not inflict any risk upon the lender.

In the funding of a new type of loans with adjustable interest rates by a range of e.g. non-callable bullet bonds, four conditions must be fulfilled according to present Danish practice and legislation:

1. The volume of the individual volumes of each of the financial instruments on the creditor side of the loan must be determined such that the market price of the financial instruments equals the volume of the loan on the debtor side.

2. The debtor's interest rate on the loan must be determined such that the interest rate on the loan is based on the yield to maturity of the funding portfolio, said yield to maturity being given by the interest rate at which the present value of a future payment flow for funding instruments equals the remaining debt on the debtor's loan.

3. The requirement with respect to a balance between all payments on the debtor side and payments on the creditor side must be fulfilled.

4. Furthermore, the statutory requirement with respect to terms to maturity and repayment profile must be fulfilled, also for loans with adjustable interest rates with adjustable terms to maturity.

In former calculations of volumes for traditional loans with adjustable interest rates, no allowance was made for the requirement as to the interest rate on the loan mentioned herein.

In the funding of traditional loans with adjustable interest rates, there was an unambiguous connection between the maximum terms to maturity of the funding instruments and the interest rate adjustment period. This structure may briefly be explained as follows: The funding principle was based on

the assumption that there was a 5 year period in which the debtor's interest rate was fixed. The traditional loans with adjustable interest rates were funded by the debtor by the issue of interest rate adjustment bonds with terms to maturity to maturity of 1 to 5 years.

This funding principle is, however, not compatible with the desire for issuing a range of e.g. 10 non-callable bullet bonds with terms to maturity of 1 to 10 years, and at the same time keeping the duration of the interest rate adjustment period at e.g. 1-2 years.

Thus, in Denmark there is an interest in a general funding principle comprising funding by the above range of non-callable bonds or other financial instruments suitable for the purpose. Presently, in international financial markets there is no tradition of a close connection between the lending and the funding of loans. Nevertheless, the broad applicability of a principle linking the loan with a range of financial instruments must be presumed to give rise also to an international interest in a general funding principle of the type described herein.

Thus, the funding principle may e.g. be used in mark-to market pricing of loans and claims otherwise not traded. By applying the principle, it will be possible to determine a portfolio of traded financial instruments with an equivalent payment flow on the basis of which the loan or the claim may be priced in accordance with observed market prices.

Similarly, the funding principle may be applied to risk management of loans and claims, the principle being applicable to the determination of a hedge consisting of a portfolio of financial instruments, as well as to the pricing of such hedge. In recent years, the trend has been towards a higher degree of attention being paid to financial risks, including the possibility of hedging these risks, so it is

within this area in particular that the international interest in the funding principle is expected.

However, one technical problem in connection with such a general funding principle has been that there was no knowledge of an effective general calculation method for computerized calculation of the volume of financial instruments or funding volumes for funding a loan, the loan being at least partially refinanced during the remaining term to maturity of the loan, in which the calculation result must fulfil both the requirement that lending institutions must not expose themselves to interest rate and funding risks, or at least they must not or will not expose themselves to such risks above certain maximum limits, and be able to contribute to minimizing the costs for the borrower such that the loan with adjustable interest rates is as inexpensive as possible within the given preconditions.

In Danish patent application no. 0165/96 and international patent application no. PCT/DK97/00044, such convenient computerized method for calculating the volume of financial instruments or funding volumes for funding a loan of the type described herein. In the following, this type of loan is referred to as "Loans with Adjustable Interest Rates" (LAIR I).

A further development of the invention described in the above patent applications has the following background:

In connection with Loans with Adjustable Interest Rates, it is sometimes considered a problem that there is a risk of the interest rate rising so much and for so long that it works through to the borrower who may experience an increase in his payments on the loan, at least during part of the term to maturity of the loan, above the level that he can or wishes to pay. It would be desirable to provide a possibility of calculating the loans in such a way that instead of an increase in payments on the loan, or in combination with a

minor increase in payments on the loan, the term to maturity was prolonged so that the borrower would be able to pay the payments on the loan in view of his current financial position.

Danish patent applications nos. 233/97, 308/97 and 770/97 concern this further development and relate to a method by which not only the above parameters may be determined, but by which requirements may also be laid down with respect to maximum (or minimum) payments on the loan for the debtor in one or more periods during the term to maturity of the loan, the term to maturity of the loan optionally being calculated as adjusted to these requirements. Conversely, it will be possible by the method according to these patent applications to lay down requirements with respect to the maximum (or minimum) term to maturity of the loan, and then calculate an adjusted payment on the loan. By the method according to the mentioned patent applications, calculation results of a high value may be achieved, which means, inter alia, that a high degree of stability in the volumes of the calculated payments on the loan is achieved despite relatively large interest rate fluctuations in the different funding periods being input. In the following, the type of loan calculated by means of a method as described in Danish patent applications nos. 233/97, 308/97 and 770/97 is referred to as "Loans with Adjustable Interest Rates II" (LAIR II).

In connection with Loans with Adjustable Interest Rates (LAIR), it is sometimes considered a problem that the payments on the loan may rise to a level above that which the debtor can pay. There are alternative types of loans which have some characteristics in common with a LAIR and which, as a facility, contain a maximum limit on the payments on the loan.

Turning towards the American bond market which is traditionally seen as the most developed bond market in the world, there are types of loans in which a short-term interest rate

is combined with a ceiling on the possible interest rate to a varying degree.

A first example is "step up" bonds. "Step up" bonds are long-term bonds for which the coupon rate changes periodically according to a predetermined pattern. Typically, the pattern is based on the structure of the forward rates. If the forward rate structure is rising, the coupon rate will typically also rise over time.

The adjustment to the structure of the forward rate means that, in principle, "step up" bonds will carry the short-term interest rate initially. In periods with a rising yield curve, the debtor may thus gain an interest rate advantage comparable to the interest rate advantage of a LAIR.

In certain cases, the changes in the coupon rate are combined with a call right. The debtor thus gets the possibility of prepaying the remaining debt at par in connection with the change in the coupon rate. The loan is thereby in the nature of a loan with a short-term interest rate combined with an option on the future interest rate, and the comparison to a LAIR combined with an option on the interest rate adjustments springs to mind.

The use of "step up" bonds for funding mortgage loans is, as far as it is known, limited. "Step up"-bonds have mainly been used in the high-risk bond market, where the lower coupon is initially to secure the debtor's financial survival in the short run.

Another example of a similar type of loan is "adjustable rate mortgages" which are found in the American mortgage market. "Adjustable rate mortgages" are loans in which the interest rate is pegged to an interest rate index optionally added an interest differential as a reflection of a credit risk or the like.

The interest rate index may be e.g. a "treasury"-based index with a term to maturity of 1/2 year, 1 year, or 5 years. The interest rate on the loan is adjusted at fixed intervals typically of the same length as the interest rate index. Thus, the loan has characteristics in common with a LAIR I.

A variant of the "adjustable rate mortgages" has as a facility a band in the interest rate index. The interest rate is bound upwards by a "cap", whereas a "floor" sets a minimum limit for the interest rate. The interest rate on the loan will thus float within a band during the entire term to maturity of the loan.

Both "step up" bonds and "adjustable rate mortgages" are characterized by the hedging of the debtor's risk of rising payments on the loan being built into the underlying bonds.

This structure has certain advantages. Once the "step up" bonds or the "adjustable rate mortgages" are accepted by the investors, the entire funding side of the loan is in conformity with the market.

On the other hand, the structure is considered to have several disadvantages.

Firstly, the tight linking of the debtor and bond sides inhibits the flexibility on the debtor side. For each loan with individual characteristics on the debtor side, a bond with individual characteristics must exist on the funding side. An adjustment of the debtor side to individual debtor preferences will thus quickly lead to a situation in which a wealth of different "step up" bonds or "adjustable rate mortgages" must be opened.

Secondly, it is subject to some uncertainty whether the "step up" bonds or the "adjustable rate mortgages" can attain the necessary market conformity within a foreseeable number of years.

In connection with the above loans with adjustable interest rates with adjustable terms to maturity, situations may occur in which the interest rate has increased such that the payments on the loan cannot be kept within an established maximum limit even if the term to maturity is prolonged, e.g. because of statutory limits for the duration of the term to maturity. Against this background, it may be considered to supplement the applied financial instruments with a special financial instrument, hereinafter termed a "payment guarantee instrument", from which payments are made in situations in which the maximum limits for payments on the loan and term to maturity would otherwise have been exceeded. It may be convenient also to consider making payments to the instrument in situations in which payments on the loan and term to maturity would otherwise have fallen below their minimum limits.

The present invention permits an appropriate and realistically practicable computerized calculation of the above parameters which are calculated in accordance with the above patent applications, as well as further calculation of payments from (or to) a "payment guarantee instrument" of the above type. As will appear from the following, it is possible by use of available calculation methods to determine such a price for such instrument that a professional market might buy the instrument or a volume of instruments corresponding to a volume of loans.

The payment guarantee instrument is considered particularly convenient when apart from granting payments to the debtor in situations in which agreed maximum limits for payments on the loan and term to maturity are exceeded, it also receives payments from the debtor in situations in which payments on the loan and term to maturity would otherwise have fallen below their minimum limits. Therefore, this type of payment guarantee instrument is in particular the basis of the following explanation of the method according to the invention, even if it is understood that a payment guarantee instrument not designed to receive payments from the debtor could also

be included and treated in the same way by the method according to the invention.

Thus, the invention relates to a method for determining, by means of a first computer system, the type, the number, and the volume of financial instruments for funding a loan, determining the term to maturity and payment profile of the loan, and further determining the payments on a payment guarantee instrument designed to ensure that the payments on the loan and the term to maturity of the loan do not exceed predetermined limits, and from which instrument payments are made to the debtor in situations in which the maximum limits for payments on the loan and term to maturity would otherwise have been exceeded, the loan being designed to be at least partially refinanced during the remaining term to maturity of the loan,

- requirements having been laid down stipulating that
  - the term to maturity of the loan is not longer than a predetermined maximum limit nor less than a predetermined minimum limit,
  - debtor's payments on the loan are within predetermined limits,

- requirements having been laid down stipulating a maximum permissible difference in balance between, on the one hand, payments on the loan and refinancing amounts and, on the other hand, net payments to the owner of the financial instruments applied for the funding, and payments to and from the payment guarantee instrument,
- requirements having been laid down stipulating a maximum permissible difference in proceeds between, on the one hand, the sum of the market price of the volume of the financial instruments applied for the funding of the loan, and payments to and from the payment guarantee instrument and, on the other hand, the volume of the loan,

- and requirements optionally having been laid down stipulating a maximum permissible difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding,

said method comprising

(a) inputting and storing, in a memory or a storage medium of the computer system, a first set of data specifying the parameters: the volume and the repayment profile of the loan,

(b) inputting and storing, in a memory or a storage medium of the computer system, a second set of data specifying

(i) a maximum and a minimum limit for the debtor's payments on the loan in each of a number of periods collectively covering the term to maturity of the loan,

(ii) a maximum and a minimum limit for the term to maturity of the loan, and

(iii) optionally, a desired/intended payment on the loan or a desired/intended term to maturity when the maximum and the minimum limits for the payment in the first period are not equivalent (i), or when the maximum and the minimum limits for the term to maturity are not equivalent (ii),

(c) inputting and storing, in a memory or a storage medium of the computer system, a third set of data specifying a desired/intended refinancing profile, such as one or more point(s) in time at which refinancing is to take place, and the amount of the remaining debt to be refinanced at said point(s) in time,

and/or said third set of data specifying a desired/intended funding profile, such as a desired/intended

number of financial instruments applied for the funding together with their type and volumes,

(d) inputting and storing, in a memory or a storage medium of the computer system, a fourth set of data comprising a maximum permissible difference in balance within a predetermined period, a maximum permissible difference in proceeds and, optionally, a maximum permissible difference in interest rates equivalent to the difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding and, optionally, the payment guarantee instrument,

(e) determining and storing, in a memory or a storage medium of the computer system, a fifth set of data specifying a selected number of financial instruments with inherent characteristics such as the type, the price/market price, and the date of the price/market price,

(f) determining and storing, in a memory or a storage medium of the computer system, a sixth set of data representing a first profile of the interest rate on the loan and either a first term to maturity profile or a first payment profile of the loan,

(g) calculating and storing, in a memory or a storage medium of the computer system, a seventh set of data representing

- a first term to maturity profile or a first payment profile (depending on what was determined under (f)) corresponding to interest and repayments on the part of the debtor,
- and a first remaining debt profile,

said term to maturity profile or payment profile, as well as the remaining debt profile, being calculated on the basis of

- the volume and repayment profile of the loan as input under (a),
- the set of data input under (b),

- the refinancing profile and/or the funding profile input under (c)
- and the profile of the interest rate on the loan and either the payment profile or the term to maturity profile determined under (f),

(g1) if necessary/if desired, calculating and storing, in a memory or a storage medium of the computer system, an eighth set of data representing payments (positive, zero or negative) on the payment guarantee instrument, the requirements with respect to a maximum permissible difference in balance and a maximum permissible difference in proceeds, as well as the limits for payments on the loan and term to maturity, always being fulfilled.

(h) selecting a number of financial instruments among the financial instruments stored under (e), and calculating and storing a ninth set of data specifying these selected financial instruments with their volumes, for use in the funding of the loan, said ninth set of data being calculated on the basis of

- the payment profile determined under (f) or calculated under (g) and
- the remaining debt profile calculated under (g),
- the payments on the payment guarantee instrument optionally calculated under (g1),
- the refinancing profile input under (c) and/or the funding profile input under (c),
- the set of data input under (b),
- the requirements input under (d), and
- in the case of a refinancing where financial instruments from a previous funding have not yet matured, the type, the number, and the volume of these instruments,

one or more recalculations being made if necessary, including if necessary, selection of a new number of the financial instruments stored under (e),

storing, in a memory or a storage medium of the computer system, after each recalculation

- the recalculated profile of the interest rate on the loan,
- the recalculated term to maturity profile,
- the recalculated payment profile,
- the recalculated remaining debt profile, and
- the selected financial instruments with their calculated volumes,

until all the conditions stated under (b) and (d) have been fulfilled,
and the payments on the payment guarantee instrument optionally being calculated in accordance with (g1), and the recalculated payments being stored in a memory or a storage medium of the computer system after each recalculation,

after which, if desired, the thus determined combination of the type, the number and the volume of the financial instruments for funding the loan,

- together with the calculated term to maturity,
- together with the calculated payment profile,
- optionally, together with the payments on the payment guarantee instrument,
- preferably, together with the calculated interest rate on the loan, and
- preferably, together with the calculated remaining debt profile,

is output, transferred to a storage medium or sent to another computer system.

In the following, the type of loan calculated by means of a method according to the invention is referred to as "Loans with Adjustable Interest Rates III" (LAIR III).

Apart from the input, determined and/or calculated data being stored in a memory or on a storage medium, the data may be output to a display or a printer. The memories applied may be

e.g. electronic memories such as ROM, PROM, EEPROM or RAM, and the storage media may be e.g. tapes, discs or CD-ROM.

It will also be possible to input data for use in or resulting from the data processing according to the invention in one set of memories or storage media which may form part of the first or a second computer system, and to transfer these data to a second set of memories or storage media which may form part of the second or the first computer system, these data being optionally transferred e.g. via a data transmission line or a net combining the first and the second computer systems, or in a wireless manner, e.g. electromagnetically or optically.

The method according to the invention calculates, inter alia, the volume of the individual financial instruments to be sold to fund the loan. These volumes are normally not whole or round volumes, and in certain cases they may be fairly small. Usually, the institution issuing the loan solves the divisibility problem by pooling many small loans when financial instruments are sold in the market. The lending institution makes an exact registration of the individual financial instruments applied for the funding of each individual loan with their volumes.

When in the present description and claims it is stated that by the method according to the invention, the volume, the type and the number of financial instruments for funding a loan, the term to maturity and payment profile of the loan and optionally the payments on the payment guarantee instrument are determined, this indicates that the information resulting from the method according to the invention may be used e.g. as the basis of the actual physical action of the lender (e.g. a mortgage credit institution) issuing/selling the said financial instruments. The information resulting from the method according to the invention may of course also be applied for pricing a loan in connection with a loan offer, and/or for calculating the lender's risk profile, e.g.

with a view to countering the risk by means of a hedge, or for other analysis purposes etc., without the said financial instruments actually being issued.

It will be understood that the sequence of the inputs/determinations/storage operations (a)-(e) stated above is arbitrary and, therefore, the sequence of letters do not specify an equivalent compulsory sequence of the steps. Step (f) may also be carried out at an arbitrary stage in the sequence unless it is chosen, as is often preferred, to have the computer calculate a first guess at a profile of the interest rate on the loan, and either a first term to maturity profile or a first payment profile, in which case step (f) will definitely follow step (e). Instead of expressing that data are input/determined and stored in the individual steps, it may simply be expressed (and should be considered equivalent to the first expression form) that by the method according to the invention, calculations are made by means of the computer system on the basis of stored inputs of sets of data (a)-(f). It will also be understood that these and other inputs to be used in order to start the individual calculations, e.g. the first profile of the interest rate on the loan, and either a term to maturity profile or a payment profile (f), may be, as mentioned above, a guess or an initial value which is also performed/determined by means of the computer system according the predetermined rules, and stored/applied as an initial value. Another example of data being either input or guessed/calculated is the desired/intended payment or the desired/intended term to maturity under (b) (iv); if no initial value thereof has been input/stored, the computer system is conveniently designed to "guess" or calculate a value according to an established rule, e.g. as an average of the values stored under (b) (i) and (b) (ii).

A number of the inputs mentioned above are inputs applying to a corresponding period. This is the case e.g. for the maximum permissible difference in balance and the payment limits. In

these cases, either the corresponding period is input, said input applying, or the period has already been generally input to the computer system. For annuity loans, the period mentioned under (b)(i) is preferably a refinancing period, which will therefore normally be a default in the computer system but, in principle, this period may be any period desired by the debtor, said period normally being input together with the mentioned limits.

The requirement with respect to the maximum permissible difference in balance is linked to a period which, depending on the legislation or the practice which is to form the basis in connection with the calculations, may be a calendar year, a year not following the calendar year but comprising the time of a payment to the creditor, or another period either comprising or not comprising the time of a payment to the creditor. In Denmark a strict balance requirement must be fulfilled per calendar year.

In the calculation of data corresponding to the volume of the financial instruments applied for the funding, the requirement with respect to maximum permissible difference in balance is, according to the current Danish rules of mortgage loans, given by a strict balance, i.e. no appreciable difference in balance occurs or, to put it differently, the difference is practically zero. However, the method according to the invention may also be used where a certain difference in balance is tolerated or perhaps even desired, this tolerance or this positive difference in balance then being stored as part of the data set in (d).

In the calculation according to the invention, both the requirement with respect to the difference in proceeds, the requirement with respect to the difference in interest rates as well as the requirement with respect to the difference in balance may be specified in different ways. Data may e.g. be input, specifying a direct maximum permissible difference in balance between, on the one hand, the sum of the market price

of the volume of the financial instruments applied for the funding of the loan and payments to and from the payment guarantee instrument and, on the other hand, the volume of the loan, as well as data may be input, specifying a direct maximum permissible difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding, and data directly specifying a maximum permissible difference in balance. The requirement with respect to maximum permissible difference in proceeds may also be input as data specifying a convergence condition for the difference in proceeds, and/or the requirement with respect to maximum permissible difference between the interest rate on the loan and the yield to maturity may be given by input data specifying a convergence condition for the difference in interest rates, and/or the requirement with respect to maximum permissible difference in balance may be given by a convergence condition for the difference in balance.

Usually, when obtaining the loan there is no match between, on the one hand, the disbursement date of the loan and/or the prepayment date of the loan and, on the other hand, the settlement date of the financial instruments, for which reason correction is preferably performed in the calculation by the method according to the invention of a possible difference between, on the one hand, the disbursement date of the loan and/or the prepayment date of the loan and, on the other hand, the settlement date of the financial instruments by proportionally correcting for the already elapsed part or the remaining part of the disbursement period and the prepayment period, respectively. Here, data specifying a correction factor for use in the calculation may e.g. be input or calculated.

In practice, one may select to have the maturity of the loan coincide with a creditor payment date, which would normally require that the last element of the term to maturity profile is calculated as adjusted to attain this coincidence. Here,

it is most appropriate that the term to maturity is calculated prolonged, which in turn implies that the payment on the loan falls, for which reason the minimum limit for the payments on the loan on the loan in the last funding period is suspended in the calculation.

The calculation method according to the present invention is also applicable in situations in which the input data specifies that more than one debtor payment on the loan will be made within one creditor payment period.

Under (c) information is to be input or be available, concerning the point or points in time at which refinancing is desired to take place, and concerning the amount to be refinanced at said points in time. In one instance, which is important in practice, the input data specify that full refinancing of the remaining debt is performed at the end of a predetermined period which is shorter than the term to maturity of the loan, and in a second important instance, the input data specify that refinancing of the remaining debt is performed with a fixed annual fraction.

The method according to the invention may be applied for determining the number and the volume of the financial instruments, the term to maturity and the payment profile in the situation in which the loan is to be calculated for the first time, i.e. in the first funding situation, as well as in the situation in which a refinancing is to be calculated. The expression funding thus covers both "new funding" and "refinancing". Apart from the parameters mentioned under (a)-(f), information concerning the type, the number, and the volume of the financial instruments which have not yet matured at the time of refinancing is included in the calculations in the refinancing situation. This information is often stored in the computer system from the previous calculation, but inputting this information is evidently within the scope of the invention. It will be understood that the parameters under (a)-(f) are parameters which are related to

the funding situation in question, so that for the case in which a refinancing is calculated, they are naturally related to the remaining debt of the loan as the volume of the loan and to the remaining term to maturity of the loan as the term to maturity of the loan.

Reference in the claims to the "remaining term to maturity" or "term to maturity", means - depending on the context - the remaining term to maturity or the term to maturity which is the basis of the first calculation in the funding period for which the calculation is performed.

The result of the method according to the invention as defined above is usually at least one set of data which may be applied in the next funding situation, whether this situation is the first funding period of the loan, or a later refinancing situation.

The expression "term to maturity profile" is related to a term to maturity being calculated by the method according to the invention, as mentioned above, usually for each funding or refinancing period. Hence, the expression term to maturity profile refers to the series of terms to maturity which being assigned to the refinancing period at each calculation in connection with a refinancing.

The expression "profile of the interest rate on the loan" is similarly related to a calculation of the interest rate on the loan being performed by the method according to the invention, usually for each funding or refinancing period. Hence, the expression profile of the interest rate on the loan refers to the series of interest rates on the loan being assigned to funding periods at each calculation in connection with the refinancing.

In the present description and claims, the term "financial instruments" has the meaning normally used and covers thus

e.g. all types of interest rate related claims, i.e. all types of bonds, including zero-coupon bonds.

When by means of the method according to the invention, the calculations are performed with financial instruments which are not directly interest-bearing, first, a calculation is conveniently performed of the expected payment flows such that a calculation of an internal interest rate may be performed, causing the payment flow or flows or the likely payment flow or flows to be expressed in parameters corresponding to the above-mentioned parameters for interest-bearing claims, primarily a yield to maturity. Thus, e.g. for an option which has a price of DKK 100 and which has a probability of 50 per cent of resulting in proceeds amounting to DKK 210 at the end of a term to maturity of one year and a probability of 50 per cent of resulting in proceeds amounting to DKK 0, this can be done by a purely statistical calculation of the average proceeds of DKK 105 and by formulation of the relevant parameters as a price of DKK 100, a quotation of 100 and an interest rate of 5 per cent per annum which - together with the interest rate on the other financial instruments applied - is to constitute the basis on which it is checked whether the requirement with respect to maximum permissible interest difference has been fulfilled. These parameters may then be input to the computer system. Alternatively, and often preferably, the data being stored as characteristics of the instruments in section (a) above may be data directly defining the financial instruments in question, and the computer system may be adapted to perform a conversion into parameters characterizing an interest-bearing claim according to predetermined principles. In the case of CAPS or FLOORS, the procedure is similar as the same payment flows may be expressed by corresponding interest-bearing instruments, the characteristics of which may then be stored as stated in section (e), or the computer system may preferably be adapted to perform a conversion into parameters characterizing an interest-bearing claim according to predetermined principles. It will be understood that in each individual

case, the calculation may also be performed on the basis of the prerequisite that in the individual funding or refinancing situation, a combination of different types of financial instruments is used, the characteristics to be applied to the calculation then being specified for each type of instrument. In this situation, the fulfilment of the requirement with respect to the maximum permissible difference in interest rates is preferably checked on the basis of an overall calculation based on the total payment flows from all the financial instruments applied. Alternatively, a weighted average of the interest rates of the individual instruments may be applied.

By the method according to the invention, calculations may thus be performed on the basis of different types of financial instruments or funding volumes, but in one instance, which is important in practice, the calculation is performed on the basis of bonds with a maximum term to maturity corresponding to the refinancing period. The bonds are usually non-callable bullet bonds, also including zero-coupon bonds. The method according to the invention may, as explained above, also be used advantageously in the calculation in connection with other types of financial instruments, such as e.g. serial bonds or annuity bonds.

The type of a financial instrument refers conventionally to the combination of all the basic information or basic data collectively and unambiguously defining the said financial instrument, for bullet bonds thus the nominal volume, the nominal interest rate, the date of maturity, all interest rate payment dates, and the ex-coupon date, i.e. the deadline determining who will receive the next interest rate payment on the bond, as well as optionally the day count convention, i.e. the formula applied to the conversion of the payment flows of the bond into an annual yield to maturity. The number of financial instruments specify how many different financial instruments to be stated as applied. The volume specifies how many entities of the individual financial

instruments, or the volume of nominal amount of the individual financial instrument to be indicated as applied.

In accordance with common practice, the expressions "repayment profile", "remaining debt profile" and "payment profile" specify the development over time in repayments, remaining debt and payments on the loan, respectively.

The repayment profile may follow the annuity loan principle as well as the serial loan principle. In addition, any arbitrary placing in time of the repayments is naturally possible. For types of loans with an repayment profile depending on the interest rate on the loan, the repayment profile may be determined either on the basis of the interest rate on the loan applying at the time in question, or on the basis of the original interest rate on the loan, or on the basis of an arbitrarily determined interest rate.

The expressions "financing profile" and "funding profile" respectively specify the type, the number, and the volume of the financial instruments applied for the funding. In the present description and claims, the expression may be used about the desired or intended funding profile which is input and stored under (c), and which might not be fulfilled, as well as about the accurate funding profile which is the result of the calculations following application of the method.

Here, the expression "refinancing profile" specifies at which points in time and with which amounts the loan is to be refinanced.

It should be noted that in some cases, the desired/intended refinancing profile stored as a second set of data under (c) above may be rewritten as a funding profile, viz. as a number of financial instruments with their type and volumes. An indication of a desired annual interest rate adjustment percentage of 100 may e.g. be rewritten into the loan being

desired to be funded exclusively through sales of bullet bonds with a term to maturity of 1 year. It will be understood that the invention also extends to the situation in which such a rewriting has taken place in the data stored under (c).

The method according to the invention may be implemented in different ways, as simultaneous calculation of all parameters to be calculated may be performed, except that payments to and from the payment guarantee instrument are always to be calculated following the calculation of the payment on the loan (and normally the term), or a division of the calculations according to different criteria may be performed. In a presently preferred embodiment which is described in detail in the example section below, the method comprises obligatorily an "inner loop" determining volumes of the underlying financial instruments and the interest rate on the loan in consideration of the balance condition and the proceeds condition, and an "outer loop" determining the term to maturity in consideration of the maximum and minimum limits for the payment on the loan. If payments on the loan and term to maturity are within the established limits, the calculation may be seen to be concluded, but if the limits for payments on the loan and term to maturity are not observed, particularly if the term to maturity, in consideration of the limits for the payments on the loan, is not within the accepted interval, a model is activated which calculates payments to or from the payment guarantee instrument such that the conditions with respect to payments on the loan and term to maturity are fulfilled.

It will be understood that the invention also extends to an embodiment in which the term to maturity is calculated at the same time as the other parameters in the "inner loop". This may e.g. be carried out by increasing the dimension of the iteration procedure by one. (In a preferred embodiment, a Gauss-Newton iteration algorithm is applied).

In connection with the maturity of a loan, an immediate result of the calculations may indicate that the date of maturity does not coincide with the date of maturity of the last maturing financial instrument considered. It is naturally possible to apply such a result but in a preferred embodiment, the date of maturity of the loan is corrected such that it corresponds to the date of maturity of the last maturing financial instrument. The correction comprises determining whether the term to maturity is to be round up to a creditor payment date (a date of maturity of a financial instrument) or be round down to the preceding creditor payment date (a date of maturity of a financial instrument one period earlier). In this case, the adjustment of the date of maturity may preferably be performed as follows:

When the set of data under (c) specifies that calculations are to be performed for the case in which full refinancing of the remaining debt is to be performed periodically with a predetermined period which is shorter than the term to maturity of the loan, and the remaining term to maturity of the loan is shorter than the period which according to (c) elapses between two successive interest rate adjustments, and the remaining term to maturity does not correspond to the maturity of the last maturing financial instrument selected under (h), but it is desired that the loan matures at the same time as the maturity of the last maturing financial instrument selected under (h), the term to maturity may conveniently be determined by the method according to the invention as

(i) the term to maturity prolonged as little as possible to a date of maturity of one or more of the selected financial instruments, provided the payment profile does not thereby exceed the minimum limit for the payments on the loans as specified under (b) (i), or

(ii) the term to maturity shortened as little as possible to a date of maturity of one or more of

the selected financial instruments, provided the payment profile does not thereby exceed the maximum limit for the payments on the loans as specified under (b) (i), and provided the condition under (i) is not fulfilled, or

(iii) the term to maturity prolonged as little as possible to a date of maturity of one or more of the selected financial instruments, if none of the conditions specified under (i) and (ii) is fulfilled.

When on the other hand the set of data (c) specifies that calculations are to be performed for the case in which partial refinancing of the remaining debt is to be performed periodically with a predetermined period which is shorter than the term to maturity of the loan, e.g. in such a way that the refinancing equals a fixed fraction of the remaining debt of the loan, and the remaining term to maturity of the loan is less than or equal to a set value and it is desired that the loan matures no later than the date of maturity stated under (e) of one or more of the funding instruments applied for the refinancing of the loan, the term to maturity is conveniently determined by the method according to the invention as the term to maturity prolonged as little as possible to a date of maturity of one or more funding instruments.

As indicated above, it is preferred at present to implement the method according to the invention by use of an "outer loop" and an "inner loop", the specified recalculations being performed in the "outer loop" on the basis of a first term to maturity profile, changing the term to maturity at substantially each recalculation until the payment on the loan in each funding period is within the limits established in (b) (i),

the determination of the type, the number and the volume of the financial instruments for funding the loan being calculated and recalculated at each iteration over the

term to maturity until the relevant variables with respect to the type, the number and the volume of the financial instruments are established in observance of the other requirements/conditions/desires,

after which, if the term to maturity for which the payment profile is within the limits established therefor is not within the limits specified in (b) (ii) for the term to maturity, the payments on the payment guarantee instrument are calculated such that the limit for the term to maturity as well as the limits for the payments on the loan are observed.

The calculation of the payments on the payment guarantee instrument is conveniently performed on the basis of an interest rate on the loan which is recalculated such that the limits for payments on the loan as well as term to maturity are observed, and wherein either resulting differences in the payments on the debtor side and the payments on the financial instruments or resulting differences in the market price of sold financial instruments and the funding demands correspond to the payments on the payment guarantee instrument.

The funding demand is defined at the disbursement of the loan as the volume of the loan and at the adjustment of the interest rate of the loan as the amount at which the requirement with respect to maximum permissible difference in balance is fulfilled in the year immediately preceding.

Conveniently, the payments on the payment guarantee instrument correspond to the differences in the market price of sold financial instruments and the funding demand resulting from the recalculation, the volume of the financial instruments being determined such that the requirement with respect to maximum permissible difference in balance is fulfilled.

It will be understood that this embodiment of the method extends to a series of recalculations in the outer loop, each of these recalculations normally occasioning a series of

recalculations in the inner loop. The recalculations in the inner loop are performed each time until the conditions of the inner loop are fulfilled.

A distinguishing feature of the inner loop in the embodiments preferred at present is that its calculations are based on an interest rate on the loan. However, it will be understood that any mathematical expression which represents the interest rate on the loan may be applied, e.g. the remaining debt profile, the payment profile or the repayment profile of an annuity loan, or the payment profile of a bullet loan or a serial loan, the other calculation parameters being adjusted thereto in accordance with common and obvious mathematical principles. It will also be understood that, in principle, a first calculation of the number, the type and the volume of the financial instruments may be performed prior to the determination of the first value of the interest rate on the loan, but even if this is the case, calculations and optionally recalculations are subsequently to be performed, e.g. of the interest rate on the loan according to the principles stated above. Thus, it would be possible under (f) to replace the profile of the interest rate on the loan by the volume of the financial instruments and under (g) either to calculate the profile of the interest rate on the loan or recalculate the volume of the financial instruments.

A possible equivalent way of expressing steps (f) and (g) could thus be to replace, in those two steps, the profile of the interest rate on the loan by the volume of the financial instruments, in such a way, however, that the volume of the financial instruments fulfils the proceeds criterion, and then calculate the interest rate on the loan in step (h).

If no instrument is selected in (e) with payment in the period up to the next point in time at which a refinancing is to be performed according to the refinancing profile input under (c), the calculations in the inner model in the preferred embodiment relate to a situation in which the result

for at least one of the financial instruments applied for the funding will be negative, i.e. corresponding to the borrower having to buy one or more financial instruments in the next period in order to fulfil the balance requirement. As will appear from the following, it is preferred at present that steps are taken to change the calculations, so that they do not result in negative volumes of the financial instruments.

In the cases in which it is stated in the input refinancing profile that full refinancing is to be performed, the financial instruments applied for the refinancing may e.g. be calculated in the inner model in the same way as the financial instruments applied for the initial funding, in other words, it would be possible to perform a new calculation according to the method of the volume of financial instruments for funding a new loan, the volume of the new loan corresponding to the amount to be refinanced.

In another embodiment of the inner loop, it may be specified in the input data corresponding to the refinancing profile that a partial refinancing of the remaining debt is to be performed. In the inner model a solution may be found to the volume of the financial instruments constituting the volume, if it has been input e.g. that refinancing is to be performed periodically with a predetermined period which is shorter than the term to maturity of the loan. A solution may also be calculated if it is specified that periodic refinancing of a fraction of the remaining debt of the loan is to be performed, the denominator of the fraction corresponding to the whole number of years of the financial instrument having the longest term to maturity when the loan was obtained. Here, the selected period may be e.g. 1 year, but other periods such as 2, 4, 5, 6 or 10 years may be selected. Furthermore, periods corresponding to a whole number of months, e.g. 2, 3, 4 and 6 months may be selected.

In connection with a full or a partial refinancing, it is normally necessary in the inner loop to calculate with one or

more new refinancing instruments not included in the range of initial financial instruments constituting the range of funding volumes which were applied according to the given data when the loan was obtained or in a previous refinancing of the loan. Normally, these new refinancing instruments have such a term to maturity that they mature at a later point in time than the points in time at which the initial financial instruments mature.

In the partial refinancing, the refinancing in the inner loop may further comprise a funding by use of additional funding for the financial instruments or funding volumes remaining at the time of the refinancing. In the following, the volume of such additional funding and new refinancing instruments are also designated as the addition to the volume of the financial instruments.

The calculation method according to the present invention will also provide a solution to the volumes of the additions to the financial instruments applied for the refinancing. When calculating the volume of these additions, data comprising possible new refinancing instruments within the range of selected financial instruments must be input. In the calculation in the case of a refinancing, the proceeds criterion may e.g. be given as a requirement with respect to the difference between, on the one hand, a funding demand given by the balance requirement and, on the other hand, the sum of the market price of the addition to the financial instruments.

As mentioned above, the issue of new financial instruments, as well as additional issue of financial instruments already applied may be made in connection with a refinancing. However, in theory, it will also be possible to repurchase the financial instruments already applied, but this involves a number of inconveniences, inter alia, an additional depreciation risk on the part of the borrower and problems pertaining to the mortgages, for which reason repurchase is not effected in practice.

Therefore, according to a preferred embodiment of the method, the volume of the additions to the financial instruments will be calculated in consideration of the volumes of the previously applied financial instruments remaining at the time of refinancing.

When in the present description and claims reference is made to payments from the payment guarantee instrument and payments to the payment guarantee instrument, this is not necessarily to be taken to represent a direct "communication" between the debtor and the instrument. In most cases, it is convenient that the payments from the payment guarantee instrument is experienced by the debtor as a reduction in the interest rate payable, possibly via an increase in the market price of the financial instruments to be applied in the refinancing, and that the payments to the payment guarantee instrument is experienced by the debtor as an increase in the interest rate.

For a number of reasons further discussed in the example section, it is convenient that the payment guarantee instrument has a price or a value of zero. This may be achieved by the desired/intended term to maturity of the loan being input under (b) (iii) and/or the limits for the payments on the loan and/or the limits for the term to maturity are established such that the present value of the payments on the payment guarantee instrument is zero.

The calculation of the present value of the payments on the payment guarantee instrument may conveniently be performed by use of a stochastic yield curve model. The stochastic yield curve model is preferably calibrated to a yield curve which is determined at the time of calculation.
The stochastic yield curve model is conveniently formulated in discrete time and implemented in a yield curve lattice, appropriately in e.g. a trinomial lattice according to Hull & White (references to Hull & White in the present text comprise: "On derivatives. A compilation of articles by John

Hull and Alan White", Risk Publications 1996) in which the yield curve model in a preferred embodiment is the extended Vasicek model. A more detailed discussion of these aspects are found in the example section.

In an embodiment of the method according to the invention, also called Type F in the subsequent detailed description, it is specified by the set of data under (c) that calculations are to be performed for the case in which full refinancing of the remaining debt is to be performed periodically with a predetermined period which is shorter than the term to maturity of the loan. If the limits for payments on the loan as well as term to maturity are fulfilled, and the payment on the payment guarantee instrument is at the same time zero, the method for determining the volumes of financial instruments specified in step (h) comprises calculating the difference in proceeds of the calculated volumes of the financial instruments applied for the funding, and/or calculating an interest rate adjustment of the interest rate on the loan, said interest rate adjustment preferably being calculated in consideration of the calculated difference in proceeds, calculations being performed as to whether the interest rate adjustment is so small that the requirement with respect to maximum permissible difference in proceeds or a convergence condition for the difference in proceeds is fulfilled. This situation which will manifest itself in practice in many, and perhaps most, of the calculations for which the method according to the invention will be applied, is described in more detail in the following.

It should be noted that when a requirement with respect to a difference in proceeds of 0 or very close to 0 is fulfilled, the difference in interest rates is automatically 0 or very close to zero, meaning that the requirement with respect to difference in interest rates may conveniently be omitted from the input starting conditions in this situation.

This is the reason why the input concerning maximum permissible difference in interest rates is stated as being optional, whereas the input concerning maximum permissible difference in proceeds is stated as being compulsory in all situations. It should be noted, however, that a compulsory input concerning maximum permissible difference in proceeds may be fulfilled by inputting information which is completely equivalent thereto, e.g. an interest rate input, and that the present invention naturally extends to such substitutions.

If the requirements or conditions laid down with respect to the difference in proceeds or the difference in interest rates are not fulfilled, the recalculations in this part of Type F comprise one or more interest rate iterations, each interest rate iteration comprising

calculating and storing, in a memory or a storage medium of the computer, data specifying a new interest rate on the loan which is preferably based on the previous interest rate on the loan and the calculated interest rate adjustment,

calculating and storing, in a memory or a storage medium of the computer, data specifying a new payment profile and remaining debt profile for the debtor, said payment profile and remaining debt profile being calculated in consideration of the new interest rate on the loan, the volume, the term to maturity, and the repayment profile of the loan as input under (a), and the refinancing profile and/or the funding profile input under (b), and

calculating and storing, in a memory or a storage medium of the computer system, data specifying a new set of volumes of the financial instruments applied for the funding.

Here, the interest rate iteration is preferably performed applying a numerical optimization algorithm or by "grid search".

Examples of numerical optimization algorithms are a Gauss-Newton algorithm, a Gauss algorithm, a Newton-Ramphson algorithm, a quadratic hill climbing algorithm, a quasi-

Newton algorithm, a maximum likelihood algorithm, a method of scoring algorithm and a BHHH algorithm. A Gauss-Newton algorithm has proved very appropriate.

When the relevant requirement or requirements with respect to maximum permissible difference in proceeds and/or maximum permissible difference in interest rates is/are fulfilled in the Type F embodiment, it is convenient to determine whether all the calculated volumes of financial instruments are positive. If the calculated set of volumes comprises at least one negative volume, the result as such may optionally be applied, implying that the calculation specifies that the borrower is to buy one or more financial instruments in order for the balance requirement to be fulfilled. As mentioned above, this is normally not preferred, for which reason in this case the method according to the invention normally further comprise either

i) selecting a new number of financial instruments among the financial instruments stored under (e), one or more of the instruments in the new number of instruments being determined such that the payments on this/these instrument(s) are due relatively later compared to the original number of financial instruments, after which a recalculation is performed as stated in connection with the description herein and in the following of the Type F embodiment, or

ii) the negative volume or volumes is/are assigned the value zero after which a recalculation is performed as stated in connection with the description herein and in the following of the Type F embodiment.

Where the input data specify that partial refinancing is to be performed, the volume of the financial instruments applied for the funding or the refinancing is calculated in a preferred embodiment of the inner loop such that they follow the development in the remaining debt given by the remaining debt profile. This calculation may comprise the use of a first function which is adjusted to the remaining debt profile as explained in the following. If e.g. the input data indicate a

difference between, on the one hand, the disbursement date and/or prepayment date of the loan and, on the other hand, the settlement date of the financial instruments, the volume(s) of one or more financial instruments may be determined in the calculation according to a preferred embodiment of the invention such that this instrument or these instruments does/do not follow the polynomial function, but contributes/contribute to a solution to the above marginal conditions.

In a preferred embodiment of the invention, it is determined in the inner loop whether the thus calculated volume of financial instruments fulfils one or more predetermined convergence conditions. If such condition or conditions are not fulfilled, one or more iterations may be calculated until the new set of financial instruments fulfils one or more of the convergence conditions.

In a preferred embodiment of the inner loop, the function coefficients are calculated on the basis of a calculated difference in proceeds, and/or a calculated refinancing difference preferably corresponding to the difference between, on the one hand, a funding demand given by the balance requirement and, on the other hand, a desired refinancing. The function coefficients may be found either analytically or by iteration.

If it is determined in the calculation in this embodiment of the inner loop that the volume of financial instruments calculated in connection with the funding or the refinancing do not fulfil the requirements laid down by the input data with respect to the difference in interest rates, one or more recalculations in the form of interest rate iterations will be performed in a preferred embodiment, a new interest rate on the loan being determined or calculated, after which a new set of financial instruments is calculated. Interest rate iteration is performed until the requirements with respect to difference in interest rates have been fulfilled. In the

description below, examples are given with a detailed explanation of this embodiment. The situation in which the function coefficients to the function expression, which is adjusted to the development in the remaining debt, are found by iteration, as well as the situation in which the function coefficients are found analytically are described.

A more detailed description follows of the situation in which the set of data (c) specifies that calculations are to be performed for the case in which partial refinancing of the remaining debt is to be performed periodically with a predetermined period which is shorter than the term to maturity of the loan, e.g. in such a way that the refinancing corresponds to fixed fraction of the remaining debt of the loan. In this embodiment generally termed Type P in the following detailed description, some or all of the financial instruments applied for the funding in the first calculation in step (g) is/are calculated in the inner loop in the case in which the limits for payments on the loan as well as term to maturity are fulfilled, and the payment on the payment guarantee instrument is at the same time zero such that they substantially follow a shifted level remaining debt profile, after which recalculations are made, if necessary, until all the condition mentioned under (d) have been fulfilled. This situation which will often apply to one or more periods when applying the method according to the invention is explained in more detail in the following.

The adjustment to a shifted level remaining debt profile are conveniently performed by the volume of some or all the financial instruments being calculated in the calculation in step (h) and optionally in one or more recalculations in step (h) by use of a function which is adjusted to a shifted level remaining debt profile. This function is conveniently a polynomial function with a maximum degree which is one (1) less than the number of financial instruments applied.

The polynomial function is conveniently calculated by use of a statistical curve-fitting method. The least squares' method has been found to be an appropriate statistical curve-fitting method, but other statistical curve-fitting methods, such as other maximum likelihood methods or the cubic splines method would also be useful for this application.

In a preferred embodiment of the invention, a calculation (as opposed to an iteration) is performed of one or more coefficients in the polynomial function. The calculation is performed such that the subsequent determination of the adjustments to the volumes of the financial instruments fulfils the requirements with respect to maximum permissible difference in proceeds, said adjustments corresponding to the difference between the polynomial function and the volumes of the financial instruments already issued and, if possible, the requirement with respect to maximum permissible difference in balance, the actual interest rate adjustment fraction at the same time corresponding to the desired interest rate adjustment fraction.

If coefficients are calculated in the polynomial function such that one or more of the resulting adjustments to the volumes of the financial instruments is/are negative, said adjustments will not be used, which is indicated by an adjustment of an indicator function. In this embodiment, the indicator function is an M-dimensional vector in which the elements have either the value one or the value zero, and the value zero indicates that the said financial instrument is not applied for the funding. On the basis of the adjusted indicator function, a new calculation of one or more coefficients is performed in the polynomial function, the resulting adjustments to the volumes of the financial instruments are checked and, if necessary, the indicator function is readjusted.

The calculation of coefficients in the polynomial function and the adjustment of the indicator function continue until either

all adjustments to the financial instruments are non-negative (viz. positive or zero)

or

either the first element of the indicator function has the value zero, or the sum of the elements in the indicator function is strictly less than 2, in each of which cases only one coefficient is calculated in the polynomial function such that the resulting range of adjustments to the volumes of the financial instruments fulfil the requirement with respect to maximum difference in proceeds; the resulting adjustment of the interest rate will be determined by a residual calculation in accordance with the requirement with respect to maximum permissible difference in balance.

It is also possible to adjust only one element at a time in the indicator function.

The above-mentioned analytical method for determining the function coefficients in the polynomial function is a method which is easy to calculate and hence time-saving.

On the other hand, the function coefficients may also be calculated by iteration as discussed in the now immediately subsequent sections.

In the embodiment termed Type P, the recalculations of all or some of the data mentioned in (g) and (h), and/or one or more function coefficients for the function representing the shifted level remaining debt profile, and/or the interest rate on the loan in the inner loop are performed by iteration carried out by applying numerical optimization algorithms or by grid search. One of the optimization algorithms mentioned

above in connection with Type F may be used as optimization algorithm here as well, and here the optimization algorithm is also conveniently a Gauss-Newton algorithm.

If the requirements laid down with respect to the difference in proceeds and/or the difference in interest rates and/or the difference in balance calculated in consideration of the refinancing profile input under (c) are not fulfilled, the recalculations in the inner loop may comprise one or more iterations in connection with the Type P embodiment, each iteration comprising

calculating and storing data specifying a new interest rate on the loan, and/or

calculating and storing data specifying a new payment profile and a new remaining debt profile for the debtor, said payment profile and remaining debt profile being calculated in consideration of the new interest rate on the loan, the volume, the term to maturity and the repayment profile of the loan as input under (a), and the refinancing profile and/or the funding profile input under (c), and/or

calculating and storing data specifying a new set of coefficients for the function which is adjusted to the shifted level remaining debt profile, and/or

calculating and storing data specifying a new set of volumes of the financial instruments applied for the funding, said new set of volumes being calculated on the basis of the financial instruments already issued for the funding and the new payment profile and remaining debt profile, as well as the requirement with respect to maximum permissible difference in balance.

One may choose to iterate over the proceeds requirement and the difference in balance widened in consideration of the refinancing profile input under (c), and iteration over the interest rate on the loan is not performed until the two requirements have been fulfilled. It will be understood that the iteration may be performed in an arbitrary order, and

that the iteration of the function applied, the so-called trend function, may also be included herein.

In this case, the method in step (h) in the inner loop may comprise determining whether the calculated volumes of financial instruments fulfil at least two of two or more predetermined convergence conditions which are preferably calculated in consideration of a calculated difference in proceeds and a difference in balance calculated in consideration of the refinancing profile input under (c), and in cases in which the calculated volumes of financial instruments do not fulfil these conditions, the recalculations comprise one or more iterations of the coefficients for the function which is adjusted to a shifted level remaining debt profile, each iteration comprising

calculating and storing data specifying two or more new function coefficients for the function representing the shifted level remaining debt profile,

calculating and storing data specifying a new set of volumes of the financial instruments applied for the funding, said new set of volumes being calculated in consideration of the new function representing the shifted level remaining debt profile.

determining whether the new set of calculated volumes of financial instruments fulfils the at least two or more predetermined convergence conditions until the new set of calculated volumes of financial instruments fulfils these conditions. The new function coefficient or function coefficients may conveniently be calculated in consideration of the calculated difference in proceeds and a difference in balance calculated in consideration of the refinancing profile input under (c).

A calculation of the difference between the interest rate on the loan and the yield to maturity of the calculated volumes of the financial instruments may be performed in the inner loop, calculations being performed as to whether the difference in interest rates is so small that that is fulfils the

requirement with respect to maximum permissible difference in interest rates or a convergence condition for the difference in interest rates, and in cases in which the requirements laid down are not fulfilled, the recalculations may comprise one or more interest rate iterations, each interest rate iteration comprising

calculating and storing an interest rate adjustment, said interest rate adjustment preferably being calculated in consideration of the difference between the interest rate on the loan and the yield to maturity of the calculated volumes of the financial instruments, e.g. by use of a Gauss-Newton algorithm,

calculating and storing data specifying a new interest rate on the loan which is preferably based on the previous interest rate on the loan and the calculated adjustment of the interest rate on the loan,

calculating and storing data specifying a new payment profile and a new remaining debt profile for the debtor, said payment profile and remaining debt profile being calculated in consideration of the new interest rate on the loan, the volume, the term to maturity and the repayment profile of the loan as input under (a), and the refinancing profile and/or the funding profile input under (c),

calculating and storing data specifying a new set of coefficients for the function which is adjusted to the shifted level remaining debt profile, and

calculating and storing data specifying a new set of volumes of the financial instruments applied for the funding.

In connection with the calculations in the inner loop of Type P, it is also within the ambit of the invention at a time to determine whether the calculated volumes of financial instruments fulfil at least three of three or more predetermined convergence conditions which are preferably calculated in consideration of a calculated difference in proceeds, a difference in balance calculated in consideration of the refinancing profile input under (c), and a maximum permissible difference in interest rates, and, in cases in which

the calculated volumes of financial instruments do not fulfil these conditions, have said recalculations comprise one or more iterations, each iteration comprising

calculating and storing an interest rate adjustment, said interest rate adjustment preferably being calculated in consideration of the difference between the interest rate on the loan and the yield to maturity of the calculated volumes of the financial instruments,

calculating and storing data specifying a new interest rate on the loan which is preferably based on the previous interest rate on the loan and the calculated adjustment of the interest rate on the loan,

calculating and storing data specifying a new payment profile and a new remaining debt profile for the debtor, said payment profile and remaining debt profile being calculated in consideration of the new interest rate on the loan, the volume, the term to maturity and the repayment profile of the loan as input under (a) and the refinancing profile and/or the funding profile input under (c),

calculating and storing data specifying a new set of coefficients for the function which is adjusted to the shifted level remaining debt profile, and

calculating and storing data specifying a new set of volumes of the financial instruments applied for the funding, said new set of volumes being calculated in consideration of the new function representing the shifted level remaining debt profile.

determining whether the new set of calculated volumes of financial instruments fulfils said at least three or more predetermined convergence conditions,

In this instance, the iterations may also be performed applying a numerical optimization algorithm, preferably e.g. a three-dimensional Gauss-Newton algorithm.

In cases in which the calculated set of volumes in the calculations in the inner model of Type P comprises at least one negative volume, the negative volume or volumes may conveni-

ently be assigned the value 0 so as to avoid negative volumes in the result, cf. above remarks on the normally undesired situation in which it is stated that the borrower is to buy financial instruments, after which the calculations are continued on the basis of the thus determined volumes of the financial instruments.

According to one embodiment of the invention, it will also be possible to determine the volume of the financial instruments applied for the loan in the cases in which data corresponding to a funding profile or a financing profile desired by the debtor have been input, said funding or financing profile comprising the desired financial instruments. In this case the calculations may also comprise calculation as to whether the volume of the financial instruments in the specified funding profile fulfils the requirement with respect to maximum permissible difference in proceeds, and in the case in which the specified volumes do not fulfil this requirement, one or more adjustments to the previously specified volumes are made according to a preferred embodiment of the method, adjustments being made until the new set of financial instruments fulfils the requirement with respect to maximum permissible difference in proceeds.

Apart from the calculation of the proceeds requirement, here it is also preferred that a calculation is performed as to whether the requirement with respect to maximum permissible difference in balance is fulfilled, and in the case in which the found volumes do not fulfil this requirement, one or more calculations of new financial instruments for at least one of the financial instruments not fulfilling the requirement with respect to maximum permissible difference in balance.

Here, the calculation of new financial instruments is preferably performed for one or more financial instruments to which repayments are payable in a period in which the requirement with respect to maximum permissible difference in balance is not fulfilled. In a preferred embodiment, the calculation

will be performed for one or more financial instruments to which repayments are payable in the last period in which the requirement with respect to maximum permissible difference in balance is not fulfilled. Here, the calculation of the new financial instruments may preferably be based on the difference in balance for the periods in which the corresponding, previously found financial instruments do not fulfil the requirement with respect to maximum permissible difference in balance.

Generally, it applies to the method according to the invention that in many situations it will be possible, after a result has been obtained, to perform a new calculation on the basis of other instruments to establish whether a less expensive loan may thereby be achieved.

The range of financial instruments determined under (e) is selected among a number of available financial instruments. It will be understood that this number of instruments may, if desired, be input to a data base in the computer system or be available via a network, and that the determination may, if desired, be performed automatically or semi-automatically by means of the computer system according to predetermined criteria or functions.

The invention also relates to a data processing system, such as a computer system for determining the type, the number and the volume of financial instruments for funding a loan, determining the term to maturity and payment profile of the loan, and further determining the payments on a payment guarantee instrument designed to ensure that the payments on the loan and the term to maturity of the loan do not exceed predetermined limits, and from which instrument payments are made to the debtor in situations in which the maximum limits for payments on the loan and term to maturity would otherwise have been exceeded, the loan being designed to be at least partially refinanced during the remaining term to maturity of the loan,

- requirements having been laid down stipulating that
  - the term to maturity of the loan is not longer than a predetermined maximum limit nor less than a predetermined minimum limit,
  - debtor's payments on the loan are within predetermined limits,

- requirements having been laid down stipulating a maximum permissible difference in balance between, on the one hand, payments on the loan and refinancing amounts and, on the other hand, net payments to the owner of the financial instruments applied for the funding, and payments to and from the payment guarantee instrument,
- requirements having been laid down stipulating a maximum permissible difference in proceeds between, on the one hand, the sum of the market price of the volumes of the financial instruments applied for the funding of the loan, and payments to and from the payment guarantee instrument, and, on the other hand, the volume of the loan,
- and requirements optionally having been laid down stipulating a maximum permissible difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding,

said data processing system comprising

(a) means, typically input means and a memory or a storage medium, for inputting and storing a first set of data specifying the parameters: the volume and the repayment profile of the loan,

(b) means, typically input means and a memory or a storage medium, for inputting and storing a second set of data specifying

(i) a maximum and a minimum limit for the debtor's payments on the loan in each of a number of periods

collectively covering the term to maturity of the loan,

(ii) a maximum and a minimum limit for the term to maturity of the loan, and

(iii) optionally, a desired/intended payment on the loan or a desired/intended term to maturity when the maximum and the minimum limits for the payments in the first period are not equivalent (i) or when the maximum and the minimum limits for the term to maturity are not equivalent (ii),

(c) means, typically input means and a memory or a storage medium, for inputting and storing a third set of data specifying a desired/intended refinancing profile, such as one or more point(s) in time at which refinancing is to take place, and specifying the amount of the remaining debt to be refinanced at said point(s) in time,

and/or said third set of data specifying a desired/intended funding profile, such as a desired/intended number of financial instruments applied for the funding together with their type and volumes,

(d) means, typically input means and a memory or a storage medium, for inputting and storing a fourth set of data comprising a maximum permissible difference in balance within a predetermined period, a maximum permissible difference in proceeds and, optionally, a maximum permissible difference in interest rates equivalent to the difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding and, optionally, the payment guarantee instrument,

(e) means, typically input means and a memory or a storage medium, for determining and storing a fifth set of data specifying a selected number of financial instruments with inherent characteristics such as the type, the price/market price, and the date of the price/market price,

(f) means, typically input means and a memory or a storage medium, for determining and storing a sixth set of data representing a first profile of the interest rate on the loan and either a first term to maturity profile or a first payment profile of the loan,

(g) means, typically calculating means and a memory or a storage medium, for calculating and storing a seventh set of data representing

- a first term to maturity profile or a first payment profile (depending on what was determined under (f)) corresponding to interest and repayments for the debtor
- and a first remaining debt profile,

said term to maturity profile or payment profile, as well as the remaining debt profile, being calculated on the basis of

- the volume and repayment profile of the loan as input under (a),
- the set of data input under (b),
- the refinancing profile and/or the funding profile input under (c)
- and the profile of the interest rate on the loan and either the payment profile or the term to maturity profile established under (f),

(g1) means, typically calculating means and a memory or a storage medium for, if necessary/desired, calculating and storing an eighth set of data representing payments (positive, zero or negative) on the payment guarantee instrument, the requirements with respect to maximum permissible difference in balance and maximum permissible difference in proceeds, as well as the limits for payments on the loan and term to maturity, always being fulfilled,

(h) means, typically calculating means and a memory or a storage medium, for selecting a number of financial instruments among the financial instruments stored under (e), and calculating and storing a ninth set of data specifying these selected financial instruments with their volumes for use in

the funding of the loan, said ninth set of data being calculated on the basis of

- the payment profile established under (f) or calculated under (g) and
- the remaining debt profile calculated under (g),
- the payments on the payment guarantee instrument optionally calculated under (g1),
- the refinancing profile input under (c) and/or the funding profile input under (c),
- the set of data input under (b),
- the requirements input under (d), and
- in the case of a refinancing where financial instruments from a previous funding have not yet matured, the type, the number and the volume of these instruments,

said means being adapted to perform, if necessary, one or more recalculations, including, if necessary, selecting a new number of the financial instruments stored under (e), said means further being adapted to store, after each recalculation,

- the recalculated profile of the interest rate on the loan,
- the recalculated term to maturity profile,
- the recalculated payment profile,
- the recalculated remaining debt profile, and
- the selected financial instruments with their calculated volumes,

until all the conditions stated under (b) and (d) have been fulfilled,

and the means further being adapted to optionally recalculate the payments on the payment guarantee instrument in accordance with (g1), and store, after each recalculation, the recalculated payments in the memory or the storage medium,

means for outputting the hereby determined combination of the type, the number, and the volume of financial instruments for funding the loan,

- together with the calculated term to maturity,

- together with the calculated payment profile,
- optionally, together with the payments on the payment guarantee instrument,
- preferably, together with the calculated interest rate on the loan, and
- preferably, together with the calculated remaining debt profile,

or means for transferring the combination, if desired, to a storage medium or sending it to another computer system.

A computer system which may be applied for the method according to the present invention may comprise means for inputting and storing the data necessary for the calculations. The input means may comprise a keyboard or a mouse, a scanner, a microphone or the like but may also comprise means for carrying out electronic inputting via a storage medium or via a network. As mentioned above, the storage media may be electronic memories such as ROM, PROM, EEPROM or RAM, or storage media such as tapes, discs or CD-ROM.

Furthermore, the system comprises calculating means adapted to perform the calculations necessary for the implementation of the method. Here, the calculating means may typically comprise one or more microprocessors.

The system may be a computer system programmed such that the system is capable of performing the calculations necessary for the implementation of the method according to the invention. In this connection it will be understood that there may be different embodiments of the system meaning that these different embodiments are adapted to perform the calculations specified in the different embodiments of the method according to the invention mentioned above and in the claims.

Embodiments and details of the method and the system according to the present invention further appear from the claims and the detailed description in connection with the drawing

and the example section. The example section contains - apart from preferred examples of the method according to the invention - a description of a number of preconditions for the invention, and of a number of preferred applications of the method according to the invention, and of the results obtained by the method.

In the example section, the expression "bonds" is used about a financial instrument in the ordinary meaning of the word. Thus, the expression covers all types of interest-bearing and non-interest-bearing claims, including financial instruments and bonds.

In the example section, the expression "financial instrument" is used about a payments guarantee instrument as previously defined.

Brief description of the drawings

Figure 1 shows an example of a lattice structure and tree structure of a binomial model. A tighter structure is achieved in the trinomial models.

Figure 2 shows the connection between the continuous structure and the lattice structure. The average value for the interest rate is determined by the initial yield curve to which the model is calibrated. This appears from the figure by the graph (1). In a deterministic model, pricing is performed solely on the basis of this graph. The connection is illustrated in principle. No calculations form the basis of the figure.

Figure 3 shows an example of the dynamic adjustment of the lattice structure.

Figure 4 shows an example of the yield curve for r* prior to the calibration to the initial, observed yield curve.

Figure 5 shows the possible impact of the recalculation of the interest rate on the loan on the payment profile and remaining debt profile of the loan.

Figure 6 shows the calculation of probabilities in the lattice by means of Bayes' rule. In the figure the shown lattice is calibrated to a flat yield curve.

Figure 7 shows the flow diagram of the model for a LAIR III type F.

Figure 8 shows the determination of the next value of the interest rate on the loan in the iteration according to the Gauss-Newton algorithm.

Figure 9 shows the flow diagram of the model for a LAIR III type P.

Figure 10 shows an example of the adjustment of the trend function. The volumes shown are not calculated. The loan could be a LAIR type P20,0 in which extreme yield curve has resulted in the volume of the bond with a term to maturity of four years being disproportionate, for which reason the trend function "breaks".

Figure 11 shows the possible pattern of the payments on the financial instrument. In the upper part of the lattice, the payments are positive due to the high interest rate. In the lower part of the lattice, the low interest rate implies negative payments on the instrument.

Figure 12 shows the pricing of the financial instrument in each node according to the backward induction principle.

Figure 13 shows the flow diagram of the model for quoting the limits for payments on the loan and term to maturity.

Figure 14 shows the flow diagram of the model for Type F' in the case in which the limits for payments on the loan and term to maturity are compatible, payments on the financial instrument thus not being necessary.

Figure 15 shows the flow diagram of the model for Type F' in the case in which the limits for payments on the loan and term to maturity are incompatible, payments on the financial instrument thus being necessary.

Figure 16 shows the alternative modelling of a LAIR III type P.

Figure 17 shows an example of the initial adjustment of the trend function. The trend function is parallel-shifted upwards when determining G_1 as G_1=1.25. In this way the model obtains better information on the marginal issue in the individual years.

1. Modelling the yield curve

1.0 Introduction

The underlying yield curve is important for the pricing of financial claims (claims are to be taken in a broad sense as securities, debt and financial instruments). The yield curve is an expression of the interest rate of different claims as a function of a selected characteristic feature. Usually, the selected characteristic feature is the remaining term to maturity or duration of the claim, and thus it is the horizontal yield curve. By contrast, the vertical yield curve is an expression of the interest rate of claims with identical terms to maturity but with different credit risks, liquidity, or the like.

In order for claims with different cash flows to be priced, the horizontal yield curve is most often formulated as a zero-coupon yield curve. Zero-coupon rates express the interest rate of a claim with only one payment in the entire term to maturity of the claim. Claims with different cash flows may thus be seen as different portfolios of zero-coupons, and once the zero-coupon yield curve has been determined, a pricing of any known cash flow is possible.

The pricing of the bonds and the financial instrument underlying the LAIR III is based on the horizontal zero-coupon yield curve. Therefore, a model thereof will be set up in the following.

The finance theory has many different suggestions as to the modelling of the yield curve. The different suggestions deviate by including, in different ways, factors, such as volatility, observed market prices etc. At the same time, the models are widely different in their degree of operationability, which should be considered fairly important for this purpose. Prior to the presentation and description of

the selected model, aspects of the selection of yield curve model are explained without this developing into a review of recent yield curve theory, however.

1.1 Aspects of the selection of yield curve

The selection of yield curve rests on a number of criteria, which are reviewed in the following.

- Firstly, the question of the modelling of the stochastics in the model is to be considered.

The possibilities in the finance theory range from completely disregarding the stochastics in the deterministic models to a modelling of several stochastic variables in the so-called multiple factor models.

- Secondly, it has to be clarified whether the modelling is to be based on an equilibrium model or a no arbitrage model.

- Thirdly, the question of the handling of the volatility in the interest rates is to be clarified.

- Fourthly, the pricing of the specific financial instrument lays down a number of requirements as to the modelling of the yield curve.

1.1.1 Stochastic modelling of the yield curve

The use of stochastic processes in the economic theory is generally justifiable in that no reliable models with perfect predictability may be set up.

It is a prerequisite for perfect predictability, firstly, that all economic connections are known and, secondly, that new information does not become available to the agents of the

economy on a current basis. For obvious reasons, perfect predictability is therefore considered unattainable.

In the financial markets in particular, there is a long way from perfect predictability to the facts of the world. The financial markets react currently to a very large quantity of information, and the reaction patterns change on a current basis. Thus, a deterministic modelling of the future development in the interest rate will be very insufficient indeed, and will not provide the basis of a reliable pricing.

By introducing stochastics in the description of the future interest rate, a random principle is actually left to rule. Therefore, it is far from immaterial how the stochastics are introduced.

In the main part of the finance theory, the stochastics are introduced in such a way that the interest rate will approach an (equilibrium) level in the long term, whereas the short-term movement may fluctuate quite significantly around the long-term trend. This modelling of the stochastics seems plausible assessed on the basis of economic principles. In the short term, the disclosure of more or less irrelevant information may influence the formation of interest rates solely because economic agents predict the reaction of other agents etc., whereas in the long term, the interest rate will converge towards an equilibrium level which is not affected by the irrelevant information. Realization of the equilibrium level requires, however, that no new information is revealed for a period of time, and will thus not necessarily occur.

One category of stochastic processes fulfilling the above-mentioned properties is the so-called Ito processes. In its general form, the Ito process is formulated by (1.1).

(1.1) $$dx = \mu(t,x)dt + \sigma(t,x)dZ(t)$$

where x is the state variable and t is time. In the Ito process the drift is given by $\mu(t,x)dt$, whereas the diffusion of the process - i.e. the fluctuations around the long-term trend - is given by $\sigma(t,x)dZ(t)$.

In (1.1) dZ-(t) is a so-called Wiener process. (The process is also termed a generalized Brownian motion). The Wiener process is to be seen as the counterpart to a random walk in continuous time, and is thus a random walk in continuous time. In differential form, the Wiener process complies with the following equation.

(1.2) $$dZ(t) = \lim_{\Delta t \to 0} \varepsilon \sqrt{\Delta t} \equiv \varepsilon \sqrt{dt}$$

This connection is interesting in relation to the value of the state variable of the Ito process at a future point in time t. An expression thereof is obtained by writing (1.1) in integral form.

(1.3) $$x_t = x_0 + \int_0^t \mu(\tau, x)d\tau + \int_0^t \sigma(\tau, x)dZ(\tau)$$

It appears from (1.2) and (1.3) that the drift term will dominate the diffusion in the long term as a result of the higher order of dt in the drift term (dt) as compared with the diffusion ($\sqrt{dt}$).

The Wiener process, and thus also the Ito process, are also characterized by being Markovian. A stochastic process X_1 is said to be Markovian if

(1.4) $$P(x_{t_n} | x_{t_1}, \ldots, x_{t_{n-1}}) = P(x_{t_n} | x_{t_{n-1}})$$

A Markovian process has no memory. Only the immediately preceding value x_{t-1} is crucial for the value of the process in

the current period x_t, whereas all other preceding values of the process are immaterial.

When setting up a stochastic yield curve model, it is exceptionally important that the process is Markovian. This may be interpreted as path independence. For Markovian processes, it is sufficient at any future point in time to know only the current interest rate to determine the further developments. The development in the movement up to the current period is thus immaterial. If a discrete modelling method is followed, it is thus possible to set up a lattice instead of a tree causing the number of nodes to be reduced drastically, even though the lattice spans a corresponding interval of possible interest rates.

Examples of the lattice and tree structures appear from figure 1.

The Ito process given by (1.1) defines a one-factor model, the only variable in the process being x. In the light of the interaction of the interest rate with a wide range of other economic variables, it would be desirable, from a theoretical point of view, to have other factors influence the process for the interest rate.

The finance theory has accepted this challenge in the so-called multiple factor models which include factors such as inflation, the interest rate in other countries, or similar factors in the stochastic process for the interest rate. In the nature of the case, also the included other factors are described by a stochastic process. A prominent example is Heath, Jarrow and Morton (1991) "Bond Pricing and the structure of interest rates: A new methodology for contingent claims valuation" Working paper Cornell University.

The multiple factor models have obvious theoretical advantages, but suffer from the weakness of not being Markovian. This means that in practice, the models may be operationalized only to a small extent, which is a central property for this purpose. Therefore, the modelling is limited to a one-factor model in the following.

1.1.2 Equilibrium models versus no arbitrage models

Stochastic yield curve models are generally divided into two categories.

The first category consists of equilibrium models. The basis of these models is of a microeconomic nature. The yield curve is determined in accordance with the preferences of the agents so as to provide a balance in the capital markets.

However, it may be difficult to determine the preference structure for the agents of the economy. First, the preference structure should reflect the degree of risk aversion of the agents, said risk aversion traditionally causing opinions to differ.

The advantage of the equilibrium models is that as soon as the preference structure is described, all claims may be priced. The use of parameters in the models is thus limited. An example of the equilibrium model is the CIR model (Cox, Ingersoll and Ross 1985) "A theory of the term structure of interest rates", Econometrica 53) and the Vasicek model (Vasicek (1977) "An equilibrium characterization of the term structure", Journal of Financial Economics 5).

The problems related to modelling the preference structure of the agents have led to the development of a new category of models, the so-called no arbitrage models.

The no arbitrage models are characterized in that the modelling of the future interest rate is calibrated to an observed initial yield curve and optionally to a volatility structure. The modelling of the future yield curve is thus no arbitrage, as no possibilities of arbitrage occur between the observed prices and the claim prices fixed in the model.

However, the theoretical basis of the no arbitrage models is not significantly different from the balance models, as no arbitrage must be considered a prerequisite for an economic equilibrium.

With the no arbitrage models, the modelling of the yield curve is subject to a more narrow interpretation. The future yield curves, which may be calculated in the model, are not to be interpreted as an actual prediction of the future development in interest rates, but as a calculation of what the observed prices reveal about the expectations of the financial markets for one to agree with or not. With this interpretation it becomes legitimate not to include macroeconomic conditions in the modelling of the yield curve.

The no arbitrage models have gained ground to an increasing extent in recent years, especially in practical applications. It may be an obvious advantage that the yield curve model is calibrated to observed yield curves, causing an actual estimate of the preference structure to be rendered superfluous. At the same time, the risk of obtaining a pricing which is distorted in relation to the market is reduced.

Thus, in the following, the focus is solely on no arbitrage modellings.

1.1.3 Volatility

In the Ito process, the volatility in the future interest rates is given by the diffusion term.

$$\sigma(t,x)dZ(t)$$

where the diffusion coefficient σ may depend on t and x. Consequently, both the yield curve and the volatility structure change over time according to the Ito process.

In the no arbitrage models, the formulation of the diffusion coefficient as time-dependent opens up the prospect of the modelling of the yield curve being calibrated to the initial yield curve as well as the volatility structure. At first glance, this also seems like a natural element in the setup of a reliable yield curve model.

However, Hull and White (1996)/(1994a) (Hull and White (1996) is a collection of previously published articles. (1996)/(1994a) refers to Hull and White's article from 1994 that is included in the collection of articles. This reference is used henceforth) adduce an argument against the diffusion coefficient being time-dependent.

In simulations of the extended Vasicek model, which will be dealt with later on, the volatility structure proves to develop very differently from the traditional perception of the volatility. The future volatility structure is particularly sensitive to the initial estimate of the volatility of claims with a long term to maturity.

Hull & White compare the time-dependent diffusion coefficient with an excessive parameterization of the model and conclude, on this basis, that the most reliable results are obtained with a value of σ(t,x) which is not time-dependent. It is preferred that the following recommendation is followed in a method according to the invention.

1.1.4. Requirements with respect to the modelling of the yield curve derived from the financial instrument

The pricing of the financial instrument lays down requirements with respect to the modelling of the yield curve.

The most important requirement is that the modelling is to be performed in discrete time. The financial instrument is characterized in that the payments on the instrument are dependent on the other variables on the debtor and funding sides of the loan, said variables being dependent of the yield curve. This implies that the payments are determined at each adjustment of the interest rate for the period up to the next adjustment of the interest rate on the basis of the yield curve. This pattern cannot immediately be described in a model in continuous time.

For the modelling of the stochastics, the transition to discrete time means that the continuous process for the interest rate must be approximated by the discrete expression

$$\Delta x = \mu(t,x)\Delta t + \sigma(t,x)\Delta Z(t) \qquad (1.5)$$

Even though the majority of the stochastic yield curve models are based on the Ito process, far from all stochastic interest rate models may be adjusted to discrete time.

In a series of modellings, Hull and White have developed a general frame in which different, originally continuous yield curve models may be made discrete and be implemented in a trinomial lattice. Further, the model frame distinguishes itself by being more operational than other discrete yield curve models. Thus, there are good arguments in favour of following Hull and White's (1996) approach to a stochastic modelling of the yield curve in discrete time

1.2 The currently preferred model - Hull & White's extended Vasicek model

In the following the selected stochastic modelling of the yield curve is presented.

First, the general Ito process is specified, and the issue of negative interest rates is discussed. Then the structure of the lattice in the trinomial model is explained, and the adjustment of the process to the lattice is deduced. Eventually, the analytical results of the model are deduced, the most important result being the deduction of the yield curve in each individual point in the lattice.

1.2.1 The stochastic process for the interest rate

In order to be able to claim a stochastic process for the interest rate, the Ito process must be concretized, the term and diffusion coefficients having to be specified.

Hull and White's very general model frame permits an implementation of a number of yield curve models. In its general form the process for the interest rate may be set up as in (1.6)

(1.6) $$dr = a(b-r)dt + \sigma r^{\beta} dZ(t)$$

The formulation of the term coefficient in (1.6) corresponds to a so-called *Ornstein Uhlenbeck* process characterized by being mean reverting. It follows from the formulation that the interest rate will be drawn towards an equilibrium level expressed by the parameter b at a velocity a.

Further, it follows from the specification that a must be less than 1 in order for the interest rate to converge. The value of a, together with the diffusion term, will determine the volatility of the interest rate. A high value of a (close to 1) implies that the interest rate quickly returns to the

equilibrium level. Thus, the volatility of the interest rate is limited.

The value of β is crucial to the properties of the model.

If $\beta=0$, Vasicek's (1977) model appears. This model is characterized in that the diffusion is not dependent in the level of the interest rate. The relatively simple model has a number of favourable analytical properties (see e.g. Hull and White (1996)/(1990)), primarily in relation to the pricing of European options.

One disadvantage of Vasicek's model is that the negative interest rates are not excluded, but will occur in the model with a positive predictability. The occurrence of negative interest rates constitutes a technical problem, as an argumentation based on arbitrage arguments can hardly be extended to a situation with negative interest rates.

In practice, the problem is manageable. Firstly, the likelihood of negative interest rates will be limited by a realistic determination of the parameters of the model. Secondly, a situation has arisen in practice, in which the interest rates have been 0 (zero) or negative as a consequence of imperfections of the market. At the same time, the model - like most other models - paves the way for very high interest rates. The focus on the possibility of negative interest rates with merely a minor probability may therefore seem exaggerated.

It appears from the following that Hull and White's method for implementing the model contains a facility which reduces the possibility of negative interest rates.

The probability of negative interest rates may be reduced to 0 (zero) at a second specification of β. If a value within the interval $]0.1[$ is assigned to β, the effect of the diffusion of the interest rate process will grow drastically if the interest rate approaches 0 (zero). With the probability of 1, the interest rate will thus be increased by the diffusion before it assumes a negative value. $r=0$ will thus for $\beta \in]0.1[$ constitute a reflective barrier to the interest rate. Instead of a reflective barrier, an absorbent barrier could be modelled, in which the interest rate remains 0 (zero) following the observation of $r=0$.

In the CIR model, the value ½ is assigned to β, and thus the CIR model do not allow negative interest rates with a positive probability. Thus, in comparison with the Vasicek model, the CIR model has an obvious theoretical advantage. In practice, the formulation of the diffusion results in the model being difficult to implement. To Hull and White this is a crucial argument in favour of applying Vasicek's model as a basis, said argument being the one currently preferred to follow in a method according to the invention.

As formulated in (1.6) the model cannot immediately be calibrated to an initial yield curve. Hull and White (1996)/(1994a) makes the model no arbitrage by introducing a further time-dependent parameter in the drift of the model. Thus, the model appears as follows:

(1.7) $$dr=(\theta(t)-ar)dt+\sigma dZ(t)$$

In this formulation the equilibrium level is thus seen to be decided by $\theta(t)$ which is determined on the basis of the initial yield curve. (1.6) is termed the extended Vasicek model, and in the following, this model will be implemented in

a discrete trinomial model. That is to say that (1.7) in Hull and White is approximated by

(1.8) $$\Delta r=(\theta(t)-ar)\Delta t+\sigma\Delta Z(t)$$

1.2.2 The lattice structure in the trinomial model

Hull and White (1996)/(1994a) implement the extended Vasicek model in a trinomial lattice.

The idea underlying the implementation of the model in a trinomial lattice is that the lattice is to reflect the development in the underlying continuous interest rate process. In the underlying continuous process for the interest rate, a continuous distribution of the adjustment to the interest rate will exist for every t, the distribution, the average, and the variance being determined by the continuous process for the interest rate.

The continuous distribution is approximated in the lattice by a discrete distribution consisting of an increasing number of nodes. From each node there are three branching possibilities: up, middle, and down. The probabilities of each of these results are determined in each node such that the process in the discrete lattice develops (approximately) in the same way as the underlying continuous process. However, a difference will always occur as a consequence of the transition from a continuous to a discrete distribution.

The difference becomes evident in the Vasicek model in which the adjustment to the interest rate at any point in time will follow a continuous normal distribution. This follows from the specification of the drift and diffusion coefficients. Thus, in the continuous distribution of the interest rate, no maximum and minimum limits for the adjustments will exist, cf.

the discussion of the positive probability of negative interest rates in section 1.2.1.

In the lattice there will be a maximum and a minimum limit for the adjustment to the interest rate for a given value of t. This follows from the distribution being discrete. Hence, the lattice will not span an interval of adjustments as wide as will the continuous distribution.

The connection between the underlying, continuous process and the lattice structure is illustrated in figure 2.

Hull and White do not only adjust the probabilities to the drift and volatility of the process, but also to the branching structure. In the model the adjustment of the branching structure to the drift of the process is introduced by the parameter

$$k=\{-1,0,1\}$$

which expresses whether the branching is to be pushed up or down. The adjustment of the branching in the lattice is performed for extreme values of the interest rate. For a very high interest rate in the lattice, the drift downwards of the process - which is mean reverting cf. section 1.2.1 - may be so strong that the discrete distribution will be able to match only the underlying continuous distribution by one or two of the probabilities being negative in the said node. Thus, the probabilities will not be probabilities in a theoretical sense. This problem is solved by the branching being pushed downwards. Similarly, for extremely low interest rates, a situation might occur in which the branching is to be pushed upwards.

Depending on the parameters of the model and the yield curve to which the model is calibrated, the minimum limit will

fluctuate. It is not a foregone conclusion that the minimum limit is determined at a positive interest rate (≥0). Thus, the branching procedure does not preclude for certain the occurrence of negative interest rates in the lattice.

In figure 2 it is presumed that k assumes the value 0 in all nodes, so that the branching structure is fixed in the entire lattice. As an example of the adjustment of the lattice structure, it is presumed in figure 3 that k has the value -1 in the upper node (2) and 1 in the lower node (3) for t=2.

At the same time, the adjustment of the lattice structure has the favourable feature that the range of the lattice is limited. The model thus obtains a higher degree of operationability, as it is not necessary to operate with extreme interest rates, which is also uninteresting in practice.

By contrast, the adjustment constitutes a (minor) theoretical problem for the pricing. The adjustments of the branching structure and the probabilities are performed such that the interest rate process still corresponds to the underlying continuous distribution. In so far as the payments on the financial instruments do not follow the same distribution, i.e. they are not linear in the interest rates, the adjustment will result in an imbalance in the determination of the payments. Whether this imbalance affects the pricing is difficult to assess. Hull and White's model frame is applied in many situations, and e.g. in the pricing of options that must be considered very sensitive, indeed, to these imbalances. Thus, the consensus is that these imbalances may be ignored.

1.2.3 Deduction of the probabilities and the branching structure

In Hull and White (1996)/(1994a) and (1996)/(1996), the deduction of the probabilities and the branching structure is performed in two stages. First, the lattice structure of the model that is not calibrated to the initial yield curve is determined. In this model, the probabilities of the different results and the branching are determined. Then the calibration is introduced as a displacement of the lattice.

Hull and White begin by considering the continuous interest rate process for dr*, which appears by setting $\theta(t)$ and the initial value of r to 0 (zero) in (1.7).

(1.9) $$dr^* = -ar^*dt + \sigma dZ(t)$$

The adjustment to the interest rate during a time interval of Δt is distributed in a normal manner. The following assumptions are made about the distribution of the discrete adjustments

(1.10) $$\Delta r^* = r^*(t+\Delta t) - r^*(t) \sim N(r^*(t)E, V)$$ where

(1.11) $$r^*(t)E \equiv (e^{-a\Delta t} - 1)r^*(t) \cong -ar^*(t)\Delta t$$ and

$$V \equiv \sigma^2 \frac{1 - e^{2a\Delta t}}{2a} \cong \sigma^2 \Delta t$$

are the average and the variance, respectively, in the distribution of the adjustments.

$\Delta(t)$ is the **step size** in the lattice. The step size may be determined arbitrarily in consideration of the claim that is being priced. The determination of the step size is discussed in more detail under the pricing of the financial instrument in section 3.

The adjustments of the process to the interest rate are kept fixed in the entire lattice. Since the lattice is based on an

initial value of 0 (zero) for the interest rate, the lattice is symmetrical at $r^*(t)=0$.

A value for Δr^* is to be determined. On the basis of numerical analysis, Hull and White recommend to set

$$\Delta r^* = \sqrt{3V}$$

which is here followed without further argumentation.

In the lattice each node may be described at the point in time and at the interest rate, i.e. (t,r). The fixed values for both Δt and Δr^* open the prospect of a more appropriate notation based on the adjustments. Each node may thus be described by (g,h), where g denotes the number of periods elapsed and h denotes the number of up results. This gives the following relations

(1.12) $$t = g\Delta t \Rightarrow g = \frac{t}{\Delta t} \quad \text{and} \quad r^* = h\Delta r^* \Rightarrow h = \frac{r^*}{\Delta r^*}$$

for g=0,1,2,... and h=0,±1±2... The probabilities of up, middle, or down branching are denoted P_o, P_m and P_n. Three requirements may be laid down with respect to the probabilities. Formally, it is the fulfilment of these three requirements which makes P_o, P_m and P_n eligible for being perceived as probabilities.

Firstly, the average of the adjustment in each node in the discrete lattice is to correspond to the average of the underlying continuous process $r^*(t)E$. This may be formalized to

(1.13) $$P_o(k+1)\Delta r^* + P_m k\Delta r^* + P_n(k-1)\Delta r^* = h\Delta r^* E$$

It should be noted that the expression applies to the average of the adjustment to the interest rate and not of the interest

rate as in e.g. Hull and White (1996)/(1993). Thus, h is not included on the left-hand side of the relation. h appearing on the left-hand side of the relation is due to the average of the adjustment being level-dependent.

Secondly, the expressions of the variance must be in accordance.

$$P_o(k+1)^2\Delta r^{*2} + P_m k^2 \Delta r^{*2} + P_n(k-1)^2\Delta r^{*2} = V + (h\Delta r^* E)^2 \quad (1.14)$$

The expression follows from the relation $E[X^2]-E[X]^2=VAR[X]$. Finally and thirdly, the probabilities are to sum to one.

$$P_o + P_m + P_n = 1 \quad (1.15)$$

Thus, the probabilities may be found as the solution to three equations with three unknown quantities. The probabilities may thus be found as the solution to the matrix equation

$$\mathbf{AB=C} \quad \text{where} \quad (1.16)$$

$$\mathbf{A} = \begin{vmatrix} k+1 & k & k-1 \\ (k+1)^2\Delta r^{*2} & k^2\Delta r^{*2} & (k-1)^2\Delta r^{*2} \\ 1 & 1 & 1 \end{vmatrix}$$

$$\mathbf{B} = \begin{vmatrix} P_o \\ P_m \\ P_n \end{vmatrix} \quad \text{and} \quad (1.17)$$

$$\mathbf{C} = \begin{vmatrix} hE \\ V+(h\Delta r^* E)^2 \\ 1 \end{vmatrix}$$

In (1.13) it applies that Δr^* may be reduced out in (1.13). The probabilities are found by inverting the A matrix.

(1.18) $$\mathbf{A}^{-1}=\begin{bmatrix} \frac{2k-1}{2} & \frac{1}{2\Delta r^{*2}} & \frac{k(k-1)}{2} \\ 2k & -\frac{1}{\Delta r^{*2}} & (1-k)(1+k) \\ \frac{2k+1}{2} & \frac{1}{2\Delta r^{*2}} & \frac{k(k+1)}{2} \end{bmatrix}$$

The probabilities may then be found by $\mathbf{A}^{-1}\mathbf{C}$.

(1.19) $$P_o=\frac{1-2k}{2}hE+\frac{1}{2\Delta r^{*2}}\left(V+(h\Delta r^{*}E)^2\right)+\frac{k(k-1}{2}$$

(1.20) $$P_m=2khE-\frac{1}{\Delta r^{*2}}\left(V+(h\Delta r^{*}E)^2\right)+(1-k)(1+k$$

(1.21) $$P_n=\frac{-1-2k}{2}hE+\frac{1}{2\Delta r^{*2}}\left(V+(h\Delta r^{*}E)^2\right)+\frac{k(k+1}{2}$$

As mentioned previously, k may assume the values {-1,0,1} in the lattice. If the possible values for k are inserted in (1.19) to (1.21), the actual probabilities for the branching are deduced, at the same time applying that

$$V=\tfrac{1}{3}\Delta r^{*2}$$

In nodes in which the branching is normal, i.e. k=0, the following probabilities apply

(1.22) $$P_o=\tfrac{1}{6}+\tfrac{1}{2}(h^2E^2+hE)$$

(1.23) $$P_m=\tfrac{2}{3}-h^2E^2$$

(1.24) $$P_n=\tfrac{1}{6}+\tfrac{1}{2}(h^2E^2-hE)$$

It appears from (1.22) and (1.24) that the probabilities of an up and a down branching are symmetrical functions of E, as could be expected when the branching is normal.

In nodes in which the branching is increasing, i.e. for k=1, the following probabilities may be determined.

(1.25) $$P_o=\tfrac{1}{6}+\tfrac{1}{2}(h^2E^2-hE)$$

(1.26) $$P_m=-\tfrac{1}{3}-h^2E^2+2hE$$

(1.27) $P_n = \frac{7}{6} + \frac{1}{2}(h^2E^2 - 3hE)$

Finally, the probabilities at a decreasing branching for k=-1 are given by

(1.28) $P_o = \frac{7}{6} + \frac{1}{2}(h^2E^2 + 3hE)$

(1.29) $P_m = -\frac{1}{3} - h^2E^2 - 2hE$

(1.30) $P_n = \frac{1}{6} + \frac{1}{2}(h^2E^2 + hE)$

The only factor lacking in the determination of the lattice structure is the determination of a value for k. For a sufficiently large value of h - and thus for a high interest rate - the drift downwards of the process is so strong that one or more of the probabilities is/are immediately to be assigned a negative value in order to fulfil the requirement given in (1.13). For this value of h, k=-1.

Similarly, a low value of h forms a floor below the interest rates in the lattice. Thus, the lattice structure will appear as shown in figure 4.

For which values of h the branching structure is to be changed is determined by the probabilities on the basis of a requirement that these must not be negative. The probabilities for all values of k are to be tested, not only a maximum value of h having to be determined prior to the branching being pushed downwards, but also a lower value of h, and vice versa

Notwithstanding the value of h, the probabilities for both P_o and P_n will be positive. Therefore, the focus is now on P_m. It has proved convenient to define the variable $x = \frac{h}{E}$. For k=0 we have

(1.31) $-x^2 + \frac{2}{3} \geq 0 \Rightarrow -0,8165 \leq x \leq 0,8165$

and for k=1

(1.32) $$-x^2-\frac{3}{2}x+\frac{7}{6}\geq 0 \Rightarrow 0,1835\leq x\leq 1,8165$$

and for k=-1

(1.33) $$-x^2+2x-\frac{1}{3}\geq 0 \Rightarrow -0,1835\leq x\leq -1,8165$$

The definition of E means that it will always assume negative values for a>0. This means that the maximum limit for h (h_{max}) is to be found among the negative values of x. The maximum limit for h is given by a integer value fulfilling

(1.34) $$\frac{-0,1835}{E}\leq h_{max}\leq\frac{-0,8165}{E}$$

Similarly, the minimum limit for h is given by

(1.35) $$\frac{0,8165}{E}\leq h_{min}\leq\frac{0,1835}{E}$$

Once again the limits are symmetrical, as expected.

1.2.4 The adjustment to the initial yield curve

The calibration to an initial yield curve is performed by a new lattice being formed as a displacement in the vertical plane of the old lattice. The displacement is determined such that the lattice prices zero-coupon bonds in accordance with the observed zero-coupon rates constituting the initial zero-coupon yield curve. Thus, the underlying process for the interest rate is once again given by

(1.8) $$dr=(\theta(t)-ar)dt+\sigma dZ(t)$$

The displacement of the lattice is introduced by the parameter α^*_g which is time-dependent but not level-dependent. For a given g, the displacement of all nodes will thus be identical, making it possible to determine the interest rate in each node

as the interest rate in the previous lattice plus α_g. The lattice thus maintains its symmetrical structure at h=0.

The determination of α_g is carried out on the basis of $\hat{\theta}g(\Delta t)$ $\theta(g\Delta t)$ which is an estimate for $\theta(g\Delta t)$. In the previous lattice, t=0 is in the vertical centre of the lattice. Further, r*=0 for t=0. This means that the drift (for h=0) in the new lattice is given by

$$\hat{\theta}(g\Delta t) - a\alpha_g .$$

From time $(g-1)\Delta t$ to $g\Delta t$, an adjustment of the interest rate in the new lattice is given by

$$(r^*(t)+\alpha_g)-(r^*(t)+\alpha_{g-1})=\alpha_g-\alpha_{g-1}$$

The adjustment of the interest rate is to correspond to the drift of the process. Hence, the following relation between $\hat{\theta}g(\Delta t)$ and α_g.

(1.36) $$\left(\hat{\theta}(g\Delta t)-a\alpha_g\right)\Delta t=\alpha_g-\alpha_{g-1} \Leftrightarrow \hat{\theta}(g\Delta t)=\frac{\alpha_g-\alpha_{g-1}}{\Delta t}+a\alpha_g$$

where

(1.37) $$\lim_{\Delta t\to 0}\hat{\theta}(g\Delta t)=\theta(t)$$

It follows from (1.36) and (1.37) that the calibration of the interest rate process is in place when an expression for α_g has been found. This expression is found by following a forward induction method.

First, a new set of probabilities is to be introduced in the lattice. Let $q(h^*,h^{**})$ denote the probability of a movement from the node (g^*,h^*) to (g^*+1,h^{**}). The q-probabilities are merely rewritings of P_o, P_m, and P_n which serve to facilitate the notation, the branching being built into q(.).

An *Arrow Debreu* asset with the price (Q(g,h) is then to be introduced. This asset is characterized by having solely the payment 1 in the node (g,h), whereas no payments from the asset are found in any other node. Thus, the price of the asset will reflect the discounting up to $g\Delta t$ as well as the probability with which (g,h) is expected to occur. It follows from the definition of the *Arrow Debreu* asset that Q(0,0)=1 as the payments of the asset are secure

The advantage of applying *Arrow Debreu* assets may be illustrated by an arbitrage argument showing the determination of $Q(g^*+1,h^*)$ given $Q(g^*,h^*)$for all h.

The state (g^*+1,h^*) is obtainable in three ways from the nodes (g^*,h^*-1), (g^*,h^*) and (g^*,h^*+1). The argument is based on the normal branching but may be immediately extended to situations in which the node is obtainable in less or more ways. In the node (g^*,h^*-1), the probability of (g^*+1,h^*) being realized is given by $q(h^*-1,h^*)$. In the node the interest rate is given by $r(g^*,h^*-1)$. The expected value of the payment 1 in (g^*+1,h^*) is thus (g^*,h^*-1) in the node.

(1.38) $$q(h^*-1,h^*)e^{-r(g^*,h^*-1)\Delta t}$$

and in the node (0,0)

(1.39) $$Q(g^*,h^*-1)q(h^*-1,h^*)e^{-r(g^*,h^*-1)\Delta t}$$

Similarly, the values in the other nodes are discounted to (0,0) given by

(1.40) $$Q(g^*,h^*)q(h^*,h^*)e^{-r(g^*,h^*)\Delta t}$$ i (g^*,h^*) and

(1.41) $$Q(g^*,h^*+1)q(h^*+1,h^*)e^{-r(g^*,h^*+1)\Delta t}$$ i (g^*,h^*+1)

In a state of arbitrage equilibrium, the sum of these values is to correspond exactly to the *Arrow Debreu* asset with the payment 1 in (g^*+1,h^*), as the expected payment is the same. Consequently, in a state of no arbitrage, it must apply that the sum of (1.39) to (1.41) equals $Q(g^*+1,h^*)$. This may be generalized to

$$Q(g^*+1,h^*)=\sum_{h=-1}^{1} Q(g^*,h)q(h,h^*)e^{-r(g^*,h)\Delta t} \tag{1.42}$$

If the notation is adjusted to the specific problem and to all possible branching structures, (1.42) appears as follows

$$Q(g^*+1,h^*)=\sum_{h} Q(g^*,h)q(h,h^*)e^{-(\alpha_{g^*}+h\Delta r)\Delta t} \tag{1.43}$$

The deduction of (1.43) follows a forward induction argument, as it is possible on the basis of Q(0,0) to determine Q(1,-1), Q(1,0) and Q(1,1), etc. A corresponding forward induction argument is applied in the determination of α_g such that the α's are determined successively for g=0,1,2,...

First, α_0 is to be determined on the basis of the zero-coupon bond with a maturity of $1\Delta t$. The zero-coupon bond is assumed to have the observed price

$$P(0,1\Delta t)=e^{1R(0,\Delta t)\Delta t} \tag{1.44}$$

where R(.) is the observed zero-coupon rate. In the interest rate lattice, the bond is to have the price

$$Q(0,0)e^{-\alpha_0\Delta t}=e^{-\alpha_0\Delta t} \tag{1.45}$$

as Q(0,0)=1. The q probabilities are not included in the expression. The zero-coupon has payments in all the nodes at time $1\Delta t$. Therefore, it is of secondary importance for the

payment which node is realized after (0,0). It follows from (1.45) that

(1.46) $$e^{-R(0,\Delta t)\Delta t} = e^{-\alpha_0 \Delta t} \Leftrightarrow \alpha_0 = R(0,\Delta t)$$

This was immediately apparent since the old lattice is constructed in such a way that r=0 in (0,0). The same method is applied to the determination of α_1

(1.47) $$P(0,2\Delta t) = Q(1,-1)e^{-(\alpha_1-\Delta r^*)\Delta t} + Q(1,0)e^{-\alpha_1 \Delta t} + Q(1,1)e^{-(\alpha_1+\Delta r^*)\Delta t}$$

which, reduced for α_1, provides the expression

(1.48) $$\alpha_1 = \frac{1}{\Delta t}\left(\log\left(\sum_{h=-1}^{1} Q(1,h)e^{-h\Delta r^* \Delta t}\right) - \log(P(0,2\Delta t))\right)$$

The expressions in (1.47) and (1.48) may be generalized into an expression for α_{g^*}. On the assumption that Q(g,h) is determined for all g $g \leq g^*$, α_{g^*} is to fulfil

(1.49) $$P(0,g^*\Delta t) = \sum_h Q(g^*,h)e^{-\left(\alpha_{g^*}+h\Delta r^*\right)\Delta t}$$

which solved for α_{g^*} produces the expression

(1.50) $$\alpha_{g^*} = \frac{1}{\Delta t}\left(\log\sum_h Q(g^*,h)e^{-(h\Delta r^*)\Delta t} - \log P(0,g^*\Delta t)\right)$$

where Q(.) is given by (1.43) and P(.) is the observed price of the zero-coupon bond.

The lattice has thus been calibrated to the initial yield curve. One example of the possible appearance of the lattice is shown in figure 3.

1.2.5 The results of the model

The interest rate modelled above is defined over a period of Δt. It is thus a short-term interest rate, depending on the exogenous selection of the step size in the lattice, but not as short as in continuous models in which the instant interest rate is modelled.

The modelled interest rate may be sufficient for the pricing of claims in which the payments are independent of the prevailing yield curve. In such instances, the modelled interest rate is applied to the discounting of the payments via the lattice, causing the present value, and hence the price, to be determined.

However, it may apply to the specific problem that the payments may be dependent on the yield curve. The payments are determined as the loss or gain in proceeds in connection with the bond funding of the loan, provided the payments on the loan and the term to maturity are within the allowed limits, cf. section 2. Since the loan may be funded by bonds with a maximum term to maturity of 11 years, the payments on the instrument will consequently to a considerable extent depend on the interest rate curve, and not just on the Δt interest rate. In each node in the lattice, therefore, a yield curve, and not just an interest rate, is to be determined.

Hull and White (1996)/(1996) deduce an expression for the yield curve, which may be calculated in each individual node. The deduction goes via Ito's lemma which is too comprehensive to be reviewed herein. Therefore, the focus is solely on Hull and White's deduction of the expression.

For this purpose, the price of a zero-coupon bond at time t with maturity at time $T \geq t$ is defined as P(t,T). P(t,T) will then fulfil

(1.51) $$P(t,T) = A(t,T)e^{-B(t,T)\rho}$$ where

(1.52) $$\log A(t, T) = \log \frac{P(0, T)}{P(0, t)} + B(t, T)F(0, t) - \frac{\sigma^2}{4a}(1 - e^{-2at})B(t, T)^2$$

(1.53) $$B(t, T) = \frac{1}{a}(1 - e^{a(T-t)})$$

It follows from the expressions that P(T,T)=1, as B(T,T)=0 and A(T,T)=1. This is an obvious result, the price of the secure payment of 1 being 1 in a state of no arbitrage. P(T,T)=1 is also the marginal condition for the solution to the partial differential equation under Ito's lemma, and is therefore fulfilled by definition.

In the expression of A(t,T) F(0,t) is included, which is the instant forward rate at time t seen from time 0. F(0,t) is given by

(1.54) $$F(0, t) = -\frac{\partial \log P(0, t)}{\partial t}$$

Thus, the deduction of the future yield curve presupposes that the estimated initial yield curve (P(0,t)) is given by an expression which may be differentiated.

Further, the future zero-coupon rates are seen to depend on ρ which is the instant interest rate, whereas r in the lattice is a Δt period interest rate. However, a conversion from r to ρ is possible by means of (1.51). (1.51) provides the possibility of calculating a longer interest rate on the basis of a short-term interest rate. Let (t,T) be given by (t,t+Δt).

(1.55) $$P(t, t+\Delta t) = A(t, t+\Delta t)e^{-B(t, t+\Delta t)\rho}$$

As $P(t,t+\Delta t) = e^{-r\Delta t}$ it follows from (1.55) that

(1.56) $$\rho = \frac{r\Delta t + A(t, t+\Delta t)}{B(t, t+\Delta t)}$$

Subsequently, this expression may be inserted in (1.51)

(1.57) $$P(t,T)=A(t,T)e^{-B(t,T)\frac{r\Delta t+\lambda(t,\,t+\Delta t)}{\delta(t,\,t+\Delta t)}}$$

According to (1.51), the zero-coupon rate from time t to T may be calculated on the basis of the relationship between the interest rate at time 0 of zero-coupon bonds with maturity at time t and T, respectively. This implies that it is impossible to calculate zero-coupon rates stretching further forward than the initial yield curve which typically corresponds to the length of the lattice.

1.2.6 Estimation of the initial yield curve and the parameters of the model.

A few comments are to be made concerning the estimation of the initial yield curve.

Firstly, it has to be clarified what bonds are applied for the estimation. Government bonds are often applied, optionally together with secured money market investments with a very low risk The resulting yield curve is hence essentially risk-free. At the same time, liquidity premiums do usually not form part of the estimated yield curve, as the government bonds applied are typically liquid securities.

It should be expected that it applies to the specific problem that the financial instruments have a credit premium as well as a liquidity premium, as the instruments are most often not in conformity with the market. The yield curve is also used in the calculation of future prices of the underlying bonds which are also priced with an interest differential in relation to similar government bonds.

This is not specifically allowed for in a preferred modelling of the yield curve. However, credit risk and liquidity premiums may be introduced in the pricing by the initial yield curve being estimated on the basis of similar securities.

However, it would be of great importance that correction for the term to maturity effect is performed, so that a credit premium, which was high at the time of the estimation, of a bond with a long term to maturity, does not make itself felt in the future short-term interest rates. Consequently, the built-it credit and liquidity premiums are to be fairly constant for the different terms to maturity.

In the determination of the parameters of the model, Hull and White (1996)/(1994a) recommend a very general, but not very operational principle. According to Hull and White, the parameters of the model are to be determined such that the volume of

$$\sum_i (P_i - V_i)^2$$

is minimized, where P_i is the market price of the ith financial claim, and V_i is the pricing in the model of the said asset.

On an operational level, the parameterization of the model is a question of determining values for a and σ which are both crucial for the volatility of the interest rates.

σ determines the volatility of the short interest rate. In the long term, the drift will, as already mentioned, dominate the diffusion and hence the part of the volatility originating from σ. σ should therefore be estimated on the basis of the observation of the volatility in the short end of the interest rate spectrum.

a determines how fast the interest rate approaches the equilibrium level which is determined in the model by the initial yield curve. Thus, the relationship between the volatility of the short interest rate and the volatility of the long interest rate is decisive for the determination of a.

Therefore, a should be determined on the basis of observations of this relationship.

1.3 Summary

The basis of the pricing of the financial instrument in a LAIR III is a modelling of the future interest rate.

Firstly, the future interest rate will be decisive for the volume of the payments on the financial instrument. Secondly, the future interest rate will be decisive for the present value of the future payments, and hence for the price of the instrument. The modelling of the future interest rate is thus an important part of the method according to the invention.

The applied model is Hull and White's extended Vasicek model which has a number of theoretical as well as practical advantages.

Firstly, the interest rate in the model is considered stochastic. Perfect predictability should be precluded for obvious reasons. Therefore, a reliable modelling of the interest rate must involve stochastics. The stochastics is introduced in the model via the general Ito process which has a number of favourable characteristics. It follows from the Ito process that the interest rate i the short term will fluctuate about an equilibrium level which is approached by the interest rate in the long term. And the Ito process is Markovian, which permits the implementation of the model in a discrete interest rate lattice.

Secondly, Hull and White's extended Vasicek model belongs to the category of no arbitrage models. This means that the model may be calibrated to an initial, observed yield curve. Thus, provision is made for the theoretical prices of the model being in accordance with the observed market prices. The

calibration also implies that an estimation of the risk aversion of the agent is unnecessary.

Thirdly, Hull and White's model frame is characterized by a high degree of operationability. Firstly, the model frame permits the setting up of continuous stochastic processes for the interest rate in discrete time. Secondly, the model may be implemented in a discrete trinomial lattice. The very possibility of implementation in a discrete lattice is essential in the determination of the payments on the financial instrument.

The construction of the interest rate lattice constitutes the most important part of the model. The lattice structure is adjusted to the drift in the stochastic process on a current basis such that the lattice structure reflects the initial yield curve. The lattice structure may also be made dynamic so that a modelling of extremely low or extremely high interest rates is avoided.

An essential result of the model is the deduction of the yield curve in each node in the lattice. The payments on the financial instrument depend not only on the short-term interest rate but on the entire yield curve. Therefore, the result is a prerequisite for the determination of the payments of the instrument.

2. Modelling the debtor and funding sides of a LAIR III

2.0 Introduction

In this section, all variables on the debtor side as well as the funding side of the loan are modelled.

A LAIR III comprises a financial instrument combined with a LAIR II or a LAIR I. A LAIR I may be perceived as a special case of a LAIR II. In the situation in which identical maximum and minimum limits are determined for the term to maturity, these limits will be fixed during the entire term to maturity of the loan. Thus, said LAIR II degenerates to a LAIR I.

When setting up a model, this implies that it is sufficient to model the debtor and funding sides of a LAIR II. Via the determination of input to the model, a LAIR I emerges by itself. If it is concluded, however, that a LAIR III is to function solely as an extension of LAIR I, it is possible to reduce the scope of the model.

The model for a LAIR III consists of a model for a LAIR II, as well as an extension managing the financial instrument. Prior to the review of the model, the section contains a more verbal review of central aspects of the model. In section 2.1 aspects regarding an adjustable term to maturity, including the determination of the term to maturity, the term to maturity concept, etc. are explained. In addition, it is discussed in the section how the limits for the term to maturity and the payments on the loan are determined in consideration of the debtor's costs pertaining to the financial instrument.

Section 2.2 contains a general description of how to hedge the limits of the loan in combination with a LAIR II. This is may

be done in different ways, each of which is discussed before the established method is described.

In section 2.3 the implementation of the model in the lattice is explained. The model set up calculates the debtor and the funding sides of one interest rate adjustment period at a time. Therefore, the calculations in the model are to be performed in each node which coincides with an interest rate adjustment, which requires e.g. a determination of the input to the model in each node. A method for determining input is thus deduced in the section.

Section 2.4 contains a review of the adjustment of the interest rate on a LAIR. The adjustment of the interest rate on a LAIR (for both LAIR I, II and III) divides the product into two types of loans, LAIR type f and type p, respectively, with very different characteristics with regard to the determination of the bond volumes, etc. Consequently, a distinction must be made in the modelling between these two types of loans.

Subsequently, the models are set up for type F and then type P. Furthermore, it should be noted that appendix A contains the modelling of a variant of type f which occurs with certain structures of input. Moreover, appendix B contains an alternative method for modelling type P.

2.1 Adjustable term to maturity - the general problem

If, for one moment, the payments on the financial instrument are not taken into account, the interest rate on the loan will rise and fall in line with the interest rate level at the time of the interest rate adjustment. A falling interest rate is not a problem for the debtor (the interest rate risk of the remaining debt of the loan not being taken into account), and, therefore, a falling interest rate does not give rise to considerations regarding the product. However, the adjustment to a higher interest rate level constitutes a potential

problem for the debtor. A rising interest rate may influence the loan in two ways.

- Firstly, the payments on the loan may increase.

- Secondly, the term to maturity of the loan may increase.

The optimum method seen from debtor's point of view cannot be generally established but depends on the individual, and possibly level-dependent, preferences of the debtor. Thus, it is imaginable that the debtor prefers that minor increases in the interest rate affect the payments on the loan, the adjustment to large increases in the interest rate being performed via the term to maturity of the loan.

A LAIR with an adjustable term to maturity is characterized in that the payments on the loan float within a band defined by a set of maximum and minimum limits for the payments on the loan. The payments fluctuate as the loan is interest rate adjusted to the prevailing market rate. The limits are denoted

$$YD_J^{max} \text{ og } YD_J^{min}$$

respectively, YD generally denoting the debtor's payments on the loan, and J indexing the interest rate adjustment periods. J=0 specifies the disbursement date of the loan, and J=M specifies the most recent interest rate adjustment such that

$$0 \leq J \leq M$$

The fluctuation within the band is ensured by a correction of the term to maturity of the loan when the payments on the loan would otherwise have fluctuated outside the band. The possible corrections are defined on an interval defined by a maximum and a minimum limit for the term to maturity. Similarly, the limits for the term to maturity are denoted L^{max} and L^{min}, L generally denoting the term to maturity.

No requirements in the model stipulate that the term to maturity is in integer years or payment periods. The possible corrections of the term to maturity are defined on a continuous interval limited by L^{max} and L^{min}. This is necessary if it is to be possible at each adjustment of the interest rate on the loan to calculate a payment which is within a relatively narrow band. At the maturity of the loan, however, the term to maturity is corrected such that the loan matures on 1 January at the same time as the underlying bonds.

The fact that the possible corrections of the term to maturity are defined on a continuous interval also opens up the possibility of offering the borrower fixed payments on the loan. In the model fixed payments on the loan correspond to the maximum and minimum limits for the payments on the loan being equivalent such that

$$YD_j^{max}=YD_j^{min}$$

In the model, there is no general need of distinguishing between fixed payments on the loan and payments on the loan within a maximum and a minimum limit. For a LAIR III identical determination of the maximum and the minimum limits is conditional on the initial term to maturity being very far from the maximum limit if the financial instrument is not to be too cost-intensive.

2.1.1 The limits for payments on the loan and term to maturity

The limits for payments on the loan and term to maturity are, basically, selected by the debtor, and are thus considered exogenous in the model. However, in practice, a need will arise of the limits being determined with a view to the resulting pricing of the financial instrument, which will be elaborated on in section 2.1.2.

The notation permits the definition of different limits for each interest rate adjustment period during the term to maturity of the loan. In the case of annuity loans, it would be sensible to have a fixed maximum limit and a fixed minimum limit corresponding to

(2.1) $$YD_0^{max} = YD_1^{max} =, , , = YD_M^{max} \text{ and } YD_0^{min} = YD_1^{min} =, , , = YD_M^{min}$$

The subsigns are maintained in the notation in order not to lose generality. In principle, the notation also opens up the prospect of having the serial loans being covered by LAIR III. The payment profile of a serial loan is decreasing, for which reason fixed limits for the payments on the loan would produce inconvenient results. By defining a decreasing band over time such that

$$YD_0^{max} \geq YD_1^{max} \geq YD_2^{max} \geq \ldots \geq YD_M^{max} \quad \text{and}$$

$$YD_0^{min} \geq YD_1^{min} \geq YD_2^{min} \geq \ldots \geq YD_M^{min}$$

sensible results may be obtained.

The limits for the term to maturity are typically determined by legislative or credit policy considerations, and are thus given exogenously. However, the possibility for the debtor of defining more narrow limits than stipulated by the exogenous conditions is kept open.

But, the term to maturity concept of a LAIR with an adjustable term to maturity paves the way for several possible interpretations of limits for the term to maturity. The adjustable term to maturity means that for a loan there will be a sequence of terms to maturity given by

(2.2) $$L_0 \neq L_1 \neq \ldots \neq L_J \neq \ldots \neq L_M$$ where

L_J specifies the term to maturity at the Jth adjustment of the interest rate

On the face of it, it would be most obvious to demand that the term to maturity at each interest rate adjustment fulfils

(2.3) $$\forall J : L^{min} \leq L_J \leq L^{max}$$

The actual term to maturity of the loan defined as the period from the disbursement to the maturity is, however, given by L_M. Hence, another possible interpretation of the limits for the term to maturity is that only L_M is to be imposed on these limits, whereas it is allowed that either $L_J > L^{max}$ or $L_J < L^{min}$ for J<M. The possibility of adjusting the term to maturity is hereby enhanced.

If this interpretation is followed, a method must be established for determining L_M for each J<M. If L_M is calculated on the assumption that the interest rate is falling, this would pave the way for values of L_J for J<M which is high above L^{max}, the assumed fall in interest rates subsequently causing the term to maturity to be at the level allowed. This would clearly be contrary to the intentions of the provisions of the Danish Mortgage Credit Act.

A more reasonable assumption in the calculation of L_M would be a future unchanged yield curve. The fact that a future unchanged yield curve opens up the prospect of $L_J > L^{max}$ for J<M, while $L_M \leq L^{max}$ is due to a shift in the composition of the bond portfolio underlying the loan. The funding of a LAIR in several bonds means that the future interest rate on the loan will change not only as a result of the adjustments of the yield to maturity of the bonds, but also as a result of the current changes in the distribution of the bond volumes provided the yield curve is not horizontal. If the yield curve is rising, the effect thereof is a falling interest rate on the loan at the end of the term to maturity of the loan, when

the funding is effected in shorter and shorter bonds. Thus, the implicit fall in the interest rate opens up the prospect that $L_J > L^{max}$ for J<M at the same time as $L_M \leq L^{max}$.

However, a calculation of L_M on the assumption that the yield curve is unchanged will complicate the implementation of the model in the trinomial lattice. Thus, in each node not only calculations for the forthcoming adjustment of the interest rate on the loan are to be performed, but for the entire remaining term to maturity of the loan, if solely the limits for the payments on the loan are applied to L_M. At the same time, numerical tests show that the variation between L_J and L_M is limited. The possibility of applying only the limits for the term to maturity to L_M is therefore not a part of the currently preferred embodiment according to the invention.

If the limits for the term to maturity are determined identically, the possibility of adjusting the term to maturity would be exhausted in advance. The term to maturity is thus fixed, and said LAIR II will degenerate to a LAIR I. The same result could be obtained by determining the limits for the payments on the loan at ∞ (infinite) and 0 (zero) respectively. However, the loan is thereby unable to function in combination with the financial instrument, as there are no limits to be hedged.

2.1.2 The determination of the limits for payments on the loan and term to maturity

The very purpose of the introduction of the financial instrument is to ensure that the debtor's payments on the loan does not exceed the band. Payments outside the band are so to speak covered by the instrument. This implies that payments from the instrument are conditional on

- firstly, the interest rate rising so that YD_J^{max} becomes binding and the term to maturity is prolonged, and

- secondly, the term to maturity having reached the maximum limit, excluding the possibility of further prolonging said term to maturity

whereas payments to the financial instrument, by contrast, are conditional on

- firstly, the interest rate falling so that YD_J^{min} becomes binding and the term to maturity is shortened, and

- secondly, the term to maturity being shortened to the minimum limit, meaning that said term to maturity cannot be further shortened

Apart from exogenous conditions such as the prevailing yield curve, the pricing of the financial instrument will depend on the initial term to maturity of the loan, the maximum and minimum limits for the term to maturity, and the maximum and minimum limits for the payments on the loan.

The model must be so flexible that an arbitrary determination of all variables within the legislative and credit policy framework is possible. A favourable solution would be that the financial instrument has the price 0 (zero) at the disbursement of the loan, causing the variables of the loan to be interdependent.

The initial term to maturity of the loan may be applied for the determination of a level for the payment initially. In the case of a rising yield curve, the positive price of hedging the maximum limit for the payments on the loan will be significantly higher (numerically) than the negative price of hedging the minimum limit. In order to achieve a negative price of the hedging of the minimum limit which is as high as possible, this price may be conveniently fixed as the initial

payment on the loan. At the same time, the minimum limit for the term to maturity is fixed conveniently as the initial term to maturity.

Even a marginal fall in interest rates will thereby trigger payments to the financial instrument. Thus, the minimum limits provide, to the greatest extent possible, the possibility of determining a maximum limit for the payments on the loan at a relatively low level. This possibility is supported by a determination of the maximum limit for the term to maturity at a level as high as possible within the legislative and credit policy framework.

If the variables of the loan are determined according to the mentioned directions, the maximum limit for the payments on the loan is determined unambiguously on the secondary condition that the price of the instrument is 0 (zero). The directions imply that the maximum limit will be determined at the lowest possible level.

However, the model is to allow that the limits are determined according to other directions determined by the debtor. Therefore, the limits will be considered exogenous. When pricing the financial instrument in section 6, the above method for determining the limits is modelled.

2.2 The financial instrument

As described above, the financial instrument is to prevent the payments on the loan from fluctuating outside the band defined by the maximum and minimum limits.

There are, however, several methods which may be applied for the hedging of the maximum and minimum limits. Depending on the method selected, the instrument will be more or less in conformity with the market, which must be considered very

important for the possibilities of the product. By contrast, the debtor side of the loan must not suffer from the hedging of the limits for the payments on the loan being designed with a view to market conformity.

In the following, two methods are identified which may be followed in the introduction of the financial instrument for hedging the limits. The emphasis is on advantages and disadvantages of the method.

2.2.1 A cap/floor approach

Firstly, the hedging of the limits for the payments on the loan may be performed by a cap/floor approach.

According to this approach, the financial instrument is defined directly by the payments which is outside the band in the model for an adjustable term to maturity. Thus, this would be a complete hedging of the risk of fluctuations outside the band.

The model may be used with adjustable as well as fixed terms to maturity - i.e. as a further development of both a LAIR I and a LAIR II. In principle, the model will be a pure further development of either an underlying LAIR I or LAIR II, which renders implementation of the model relatively easy, as the underlying LAIR I or LAIR II may, in principle, be calculated in an existing model.

The legislative and fiscal conditions mean that most conveniently, the financial instrument exists only on the debtor side of the loan. On the basis of the payments from the underlying LAIR corrected by the payments on the financial instrument, a new debtor side is thus to be calculated.

In the situation in which the payments from the financial instrument are positive (the maximum limit for the term to

maturity is thus binding), the recalculated debtor payment is to be smaller than on the underlying loan. This is achieved by reducing the interest rate. The new interest rate on the loan may be determined in a relatively simple way as the interest rate which in the expression for the calculation of an annuity produces a payment corresponding to the maximum limit given the term to maturity.

However, the recalculation of the interest rate on the loan do not only affect the volume of the payment but also the distribution thereof on interest and repayments. It follows from the expression for the calculation of an annuity that a lowering of the interest rate increases the volume of the repayment, both relatively and absolutely. Consequently, the remaining debt of the loan at the end of the interest rate adjustment period will be smaller on the recalculated debtor side than on the underlying loan. The consequences of the recalculation for the payments and term to maturity profiles in principle are illustrated in figure 5.

The reduced remaining debt at the end of the interest rate adjustment period expressed by the difference between (4) and (5) in the figure gives rise to imbalances. Bonds corresponding to the remaining debt of the underlying loan mature on the bond side, whereas the interest rate adjustment amount on the debtor side is calculated on the basis of the reduced remaining debt.

The imbalance may be equalized by the reduction in the remaining debt also being covered by the payments of the financial instrument. However, this produces a very distorted development in the payments, the fixed payments for each payment date in the interest rate adjustment period thus being supplemented with a large payment at the end of the period.

Alternatively, the imbalance may be equalized by the repayment profile being fixed. The fixing may be performed at the disbursement of the loan, which is a model that has previously been applied for floating rate mortgage credit loans. However, the model does not allow an adjustable term to maturity and must, on this basis, be precluded. Secondly, the fixing may be performed at the beginning of each interest rate adjustment period so that e.g. the repayment profile of the underlying loan is maintained. This model is difficult to comprehend for the debtor and will result in either the payments or the interest on the loan not being constant during the interest rate adjustment period.

Apart from the consequences outlined, the approach implies that the funding side of the loan is made artificially large in certain cases. The approach implies that bonds are issued, the payments of which are covered by the financial instrument. This does not represent a problem but must be considered a less than optimum solution from a theoretical point of view.

2.2.2 An approach based on put or call options

The other method for hedging the limits for the payments on the loan follows an approach comparable to a collar, i.e. a combination of a long position in put options for hedging the maximum limit and a short position in call options for hedging the minimum limit.

The approach implies that the payments on the financial instrument are not defined directly by he payment profile of the underlying loan, but instead as the necessary reduction of the volume of underlying bonds ensuring that the payments on the loan are within the band defined by the maximum and minimum limits.

The approach is based on the development of the adjustment of the interest rate on a LAIR.

At the beginning of each interest rate adjustment period, a funding demand arises as a result of the adjustment of the interest rate on the loan at the end of the preceding interest rate adjustment period. At the interest rate adjustment, the underlying volume of bonds matures fully or partially (depending on the interest rate adjustment fraction) on the bond side. On the debtor side, these payments correspond to a new volume of underlying bonds being issued. The market price thereof equals the nominal value of the bonds which have only just matured corrected for bond repayments on account, so that a balance between the payments is achieved.

The market price of the sold bonds influences the interest rate on the loan in the interest rate adjustment period. At a low market price, the extent of the necessary sale of bonds increases. The larger volume of bonds causes larger payments on the bond side and thereby also on the debtor side, the balance principle having to be respected. The interest rate on the loan is thus increased.

The mechanics of the adjustment of the interest rate on a LAIR means that a reduction in the volume of the payments, so that a maximum limit for the payments on the loan is observed, is obtainable by means of a smaller sale of bonds at the beginning of the interest rate adjustment period. A loss in proceeds will thereby occur at the beginning of the period in the relationship between, on the one hand, the nominal value of the mature bonds and, on the other hand, the market price of the sold bonds. According to the approach, this loss in proceeds defines the payments from the financial instrument.

Correspondingly, a gain in proceeds may occur if an increased volume of bonds is sold to enhance the volume of the payments such that a minimum limit for the payments on the loan is

observed. The gain in proceeds constitutes the payments to the financial instrument.

Firstly, the approach implies that payments on the financial instrument will be concentrated at the beginning of the interest rate adjustment periods. Consequently, there will be no current payments on the instrument, which will facilitate the pricing.

Secondly, it follows from the approach that the bond side and the debtor side of the loan will be in agreement on a current basis. The hedging of the limits is not in the nature of a further development equalizing differences between the bond side and the debtor side of the loan, but is, however, an integrated part of the funding side. This also speaks in favour of the approach.

The financial instrument formed by losses and gains in proceeds is comparable to a collar.

The hedging of the maximum limit by means of covering the loss in proceeds corresponds to the debtor having a long position in put options with an exercise price corresponding to the interest rate on the loan when the maximum limit has been observed. Similarly, the hedging of the minimum limit corresponds to a short position in a call option with an exercise price corresponding to the interest rate on the loan when the minimum limit is binding.

In order for the financial instrument to be formed as a portfolio of put options in conformity with the market, a number of preconditions must be met.

Firstly, the composition of the bond portfolio underlying the loan must be known at all future interest rate adjustments. If the distribution of bonds in the individual years is known, it

naturally follows that it will be impossible to take positions in options on the individual bonds.

However, if the distribution of bonds on the individual years is to be determined *ex ante*, this requires that the amortization to maturity is known *ex ante*. This precludes adjustable terms to maturity as well as annuities calculated with a floating interest rate. Consequently, conformity with the market presupposes that the loan degenerates from a LAIR II to a LAIR I with a fixed repayment profile.

Secondly, characteristics of the underlying bonds up to the maturity must be known. Among these, the coupon rates of the bonds must be known, which constitutes a problem in practice as a floating minimum interest rate over time may result in floating coupon rates over time.

In comparison with the financial instrument defined directly by the payments outside the band, it must be assessed that the financial instrument described above should obtain a far higher degree of conformity with the market, primarily as a consequence of the concentration of the payments on the instrument at one point in time in each interest rate adjustment period.

On the basis of the obvious advantages, the focus will, in the following, be on an implementation of the last-mentioned approach for hedging the limits for the payments of the loan.

2.3 Remaining debt, term to maturity and volume lattice

The model for calculating the debtor and funding sides of the loan must be implemented in the trinomial lattice set up in section 1.

At each interest rate adjustment, the payments of the debtor and bond sides are calculated in each node on the basis of the

yield curve in the node in question. However, the debtor side as well as the bond side depend not only on the yield curve. A number of variables determined at the preceding interest rate adjustment will also affect the current interest rate adjustment.

Firstly, the volume of the remaining debt and the interest rate adjustment amount at the end of the preceding interest rate adjustment will affect the current interest rate adjustment. Subsequently, the term to maturity in the preceding interest rate adjustment is also to be used in the calculation of the current interest rate adjustment. Basically, the term to maturity is not changed provided the maximum and minimum limits for the payment are not exceeded. Further, for type P the bond volumes from the preceding interest rate adjustment and the associated coupon rates are to be input to the model because of the rolling movement in the bond funding, cf. section 2.4.

In the lattice, it is usually impossible to identify unambiguously the node for the preceding interest rate adjustment. Typically, a node may be reached in several ways for which reason it is not unambiguous in which node - and thus under which yield curve - the preceding interest rate adjustment was calculated. Only in the nodes in the lattice which may be reached by an unambiguous path, will the input to the calculation of the current interest rate adjustment be unambiguously given.

In the other nodes, the said inputs must be determined as an expected value of the nodes from which the current node may be reached with a positive probability. In other words, input are determined by a projection through the lattice in which the probabilities of the different branching structures are weighted.

The projection is complicated by the fact that there is not necessarily a one-to-one correlation between the step size in the lattice (Δt) and the length of the interest rate adjustment periods. Typically, there are four annual payment dates on the debtor side for which reason a step size greater than ¼ may produce inaccurate results. However, the interest rate adjustment periods may last up to 10 years for which reason there may be many nodes between each interest rate adjustment.

In order not to complicate the projection unduly, it is preferable to project inputs in each node in the lattice, though an adjustment of the interest rate on the loan is not performed in the node in question. The projection may thus follow a forward induction method comparable to that applied in section 1.

First, the vector x(g,h) is defined. The elements in the vector are constituted by the above input variables, i.e.

x={remaining debt and interest rate adjustment amount at end of period; term to maturity of the loan; bond volumes at end of period, coupon rates associated to the volumes}

For type F the bond volumes at end of period will all assume the value 0 (zero), and there will be a match between the remaining debt and the interest rate adjustment amounts.

In the projection, Bayes' rule is applied for the determination of the probabilities. Bayes' rule expresses the probability of an event having occurred by a specific path as probabilities of said path seen from time 0 (zero) divided by the cumulative probability of the event, also seen from time 0 (zero).

The cumulative probabilities $\bar{q}(g,h)$ are determined according to the principle illustrated in figure 6.

The method in the figure leads to the following general expression

(2.4) $$\bar{q}(g,h)=\sum_k \bar{q}(g-1,k)q(k,h)$$

It follows from (2.4) that the cumulative probabilities are to be found successively for g=0,1,2... corresponding to the forward induction method. With the definition of the cumulative probabilities, Bayes' rule may be formulated. Let

(2.5) $$P^B((g-1,k)|(g,h))$$

denote the probability of the immediately preceding node being (g -1,k) provided one is in (g,h). Then it follows from Bayes' rule that

(2.6) $$P^B((g-1,k)|(g,h))=\frac{\bar{q}(g-1,k)q(k,h)}{\bar{q}(g,h)}$$

Consequently, the expected value of x in each node is found by the following expression

(2.7) $$x(g,h)=\sum_k P^B((g-1,k)|(g,h))x(g-1,k)$$

This expression is also based on forward induction. x (0,0) will be exogenously determined. On the basis of x (0,0) it will be possible to determine successively the value of x for g=1,2,3... until the projection has been performed through the entire lattice.

Up to the first interest rate adjustment, x (g,h) will have the same value in all nodes, as only the debtor and bond sides of the loan have been calculated, viz. in (0,0). In connection with the first interest rate adjustment, new values are

assigned to x (g,h) on the basis of the interest rate adjustment period, etc. Apart from the changes resulting from the weighting over the different probabilities, x will thus change values for the values of g which coincide with an interest rate adjustment.

2.4 The adjustment of the interest rate on a LAIR

The adjustment of the interest rate on a LAIR may follow two patterns, for which reason it is necessary to distinguish between two types of LAIR.

- Firstly, the interest rate adjustment may take place at a fixed frequency as 100 per cent of the remaining debt of the loan at the interest rate adjustment. This type of loan is termed LAIR type F
- Secondly, the interest rate adjustment may take place annually as a fixed fraction of the remaining debt at beginning of period in the year in question. This type of loan is termed LAIR type P.

It is preferred to perform the interest rate adjustment in connection with a payment date on the bond side of the loan regardless of the fraction and frequency of the interest rate adjustment.

Especially the bond side is different for the two types of loans. For LAIR type F, the full adjustment of the interest rate on the loan means that all underlying bonds mature at the interest rate adjustment. Thus, bonds with a term to maturity longer than the period up to the next interest rate adjustment period will not be issued.

For a LAIR type P, the partial adjustment of the interest rate on the loan means that the bond issue follows a rolling movement. The majority of the bonds are issued with a term to maturity longer than the period up to the next interest rate

adjustment. The rolling movement complicates the calculation of the bond side, as e.g. bonds already issued are to be taken into account currently at each interest rate adjustment.

The differences in the organization of the bond side are so extensive that it is necessary to distinguish between the two types of loans in the modelling a LAIR III. In the following, the models for type F and then type P are set up.

The necessity of operating with both a LAIR type F and type P stems from the preferences of the debtors. Where a debtor with a LAIR type f is exposed to a relatively large risk at few points in time, a debtor with a LAIR type P is exposed to risk more often, said risk being limited in scope, however. With the introduction of a maximum limit for the payments on the loan, one may argue for a limitation of the number of types of loans to either type F or type P. Which type of loan is most advantageous in combination with a hedging of the risk of a fluctuating payment on the loan depends, however, on several factors, e.g. the yield curve. A limitation of the types of loans may thus have inconvenient consequences.

2.5 The model for a LAIR III type F

A preferred model for a LAIR III type F is built on the conditions laid down for a solution to the model. Thus, the description of the model begins by a formulation of these conditions. Then the problem is formulated, which problem is solved by the model, and the general model structure is described in section 2.5.2. The specific model is set up in section 2.5.3. A variant of the LAIR type F - type F* - is described in the appendix.

2.5.1 The conditions in the model

In each interest rate adjustment period, the debtor and funding sides are subject to four conditions.

- Firstly, the term to maturity of the loan must be within the interval from L^{min} to L^{max}
- Secondly, the debtor's payments on the loan must be within the band defined by the maximum and minimum limits.

These conditions are discussed above. However, the conditions need to be concretized and formalized.

(2.3) $$\forall J : L^{min} \leq L_J \leq L^{max}$$

(2.8) $$\forall J : YD_J^{min} \leq YD_J(.) \leq YD_J^{max}$$

Neither (2.3) nor (2.8) are operational and these conditions will be insufficient to determine the term to maturity of the loan generally. The conditions are rendered operational by also requiring that corrections of the term to maturity are minimized in scope. If the payments on the loan are obviously larger than the maximum limit, the term to maturity is corrected such that the payments on the loan correspond exactly to the maximum limit, and vice versa. L_J must thus further fulfil the condition

(2.9) $$L_J \in \operatorname{argmin}(|L_J - L_{J-1}| \,|\, YD_J^{min} \leq YD_J(.) \leq YD_J^{max}, L^{min} \leq L_J \leq L^{max})$$

for $1 \leq J \leq M$

- Thirdly, the payments on the funding side of the loan are to balance the total payments on the debtor side.

On the funding side of the loan, the current payments comprise coupon rates and mature bonds. In order to fulfil the requirement, bonds in all years with maturity before the next interest rate adjustment are sold (to begin with, the type F* variant of type F is ignored). Consequently, bonds will mature, not just in connection with the adjustments of the interest rate on the loan.

On the debtor side of the loan, the payments are constituted by the debtor's payments of interest and repayment. In addition, the remaining debt on the debtor side is included in the payments at the end of the interest rate adjustment period. The remaining debt is interest rate adjusted by bonds being sold once again at the beginning of the next interest rate adjustment period. In the current interest rate adjustment period, the interest rate adjustment amount balances the volume of mature bonds.

The balance condition may be formalized. Thus, it applies for every year (j') until the interest rate adjustment that

(2.10) $$YD_J(j') = H_J(j') + \sum_{j=j'}^{m} R_J^N(j) H_J(j), \quad j' < m \text{ where}$$

j	(not to be confused with J) denotes the year within the interest rate adjustment period and numbers the funding volumes. Therefore, j is assigned the value 0 following each adjustment of the interest rate on the loan. In the notation, there is thus a direct connection between the year and the bond maturing in this year.
$YD_J(j)$	is the debtor's payments on the loan in year j following the Jth adjustment of the interest rate on the loan.
$H_J(j)$	is the jth bond volume at a given point in time.
$R_J^N(j)$	is the nominal interest rate on the jth bond volume.
m	(not to be confused with M) is the number of bonds at the beginning of the interest rate adjustment period and at the same time in the next interest rate adjustment period.

That j may denote year as well as funding volumes is due solely to the fact that the bonds have only one annual creditor payment date on 1 January. If the number of annual creditor payment dates is adjusted, the notation is to be changed.

On the debtor side it is possible, however, to have both 1 and 4 annual payment dates. Consequently, j cannot denote the debtor payment dates as well. The notation is facilitated by the debtor's payments on the loan being specified by $YD_J(j)$ where a summing over the payment dates within the year are performed. Let n denote the number of debtor payment dates per year

(2.11) $$YD_J(j) = \sum_{i=1}^{n} AFD_J(i) + \sum_{i=1}^{n} \frac{R_J^K}{n} RG_J(i-1)$$ where

$R_J^K(j)$	is the interest rate on the loan
$RG_J(j)$	is the remaining debt of the loan at time j
$AFD_J(i)$	is the debtor's repayment in payment period i
i	denotes the payment dates within the year, i.e. i=1,2,...,n

(2.10) does not include the year of the disbursement of the loan in which the payments on the underlying bonds will be reduced by the accrued interest. Therefore, a regulation factor is introduced in (2.10). At the disbursement of the loan, (2.10) is formulated as

(2.12) $$YD_J(1) = H_J(1) + REG_J \sum_{j=1}^{m} R_J^N(j) H_J(j)$$ where

REG_J	is a regulation factor determining the amount of the interest payment the creditor is to receive from the debtor on the next creditor payment date. REG_J is determined as that part of the year the loan has existed measured from 30 November when the bonds turn ex-coupon. Therefore, REG_J may assume values between 1/12 (if the loan is obtained on 30 November) and 13/12 (if the loan is obtained on 1 December, so that year 1 lasts 13 months). For J>0 REG_J=1, the adjustment of the interest rate on the loan being performed on the creditor payment date.

In addition, (2.10) is to be modified for the years in which the loan is interest rate adjusted, cf. j'<m, the interest rate adjustment amount being included in the payments on the debtor side of the loan.

(2.13) $$YD_J(m) + RG_J(m) = H_J(m) + R_J^N(m)H_J(m)$$

by means of which the total balance conditions may be formulated

(2.12) Year 1 $$YD_J(1) = H_J(1) + REG_J \sum_{j=1}^{m} R_J^N(j)H_J(j)$$

(2.10) Year 2 $$YD_J(2) = H_J(2) + \sum_{j=2}^{m} R_J^N(j)H_J(j)$$

$$\vdots$$

(2.13) Year m $$YD_J(m) + RG_J(m) = H_J(m) + R_J^N(m)H_J(m)$$

- Forthly, the market price of the sold bonds and the payments from the financial instrument must cover the funding demand of the debtor in connection with the adjustment of the interest rate on the loan at the end of the preceding interest rate adjustment period.

The requirement is termed the proceeds condition. Payments on the financial instrument are conditional on either the maximum limit for payments on the loan and term to maturity being binding, or the corresponding minimum limits being binding. In these situations, the payments on the financial instrument are calculated as the residue between the funding demand and the market price of the sold bonds. The proceeds condition may then be formulated as

(2.14) $$F_J = RG_J(0) - \sum_{j=1}^{m} K_J(j)H_J(j)$$ where

F_J is the payments on the financial instrument

$K_J(j)$ is the market price of the jth funding instrument

Thus, the proceeds condition will be fulfilled by definition, corresponding to the criterion being suspended.

The market prices of the underlying bonds are observable at the disbursement of the loan, but at all other points in time the calculation is performed on the basis of the modelled yield curve. Consequently, the model is dependent on the interest rate via the market prices.

In all other situations, there will not be payments on the financial instrument, making it possible to formulate the proceeds condition as a balance between the market price of sold bonds and the funding demand. This may be formulated as

$$RG_J(0) = \sum_{j=1}^{m} K_J(j) H_J(j), \qquad (2.15)$$

$RG_J(0)$ is the remaining debt at beginning of period, corresponding, at the disbursement of the loan, to the volume of the loan, and at the adjustments of the interest rate on the loan to the refinancing amount, the entire remaining debt being interest rate adjusted.

The coupon rates of the underlying bonds are included in the balance conditions as well as in the proceeds condition. At the disbursement of the loan, the coupon rates of each bond are known. At the future adjustments of the interest rate, however, the coupon rate will depend on the policy of the lending institute with regard to the opening of bonds and, equally important, on the development in the minimum interest rate, fiscal factors in practice precluding coupon rates under the minimum interest rate.

The fixing of a minimum interest rate is governed by subsection 3 of section 7 of the Danish Gains on Securities and Foreign Currency Act. The minimum interest rate is fixed every six months on the basis of a 20 trading days' average of

the bond yield of all fixed-interest bonds denominated in kroner listed on the Copenhagen Stock Exchange. The bond volume is weighted in the calculation of the average. Callable bonds above price 100 are not included in the calculation of the minimum interest rate. The calculated average bond yield is regulated by the factor $\frac{7}{8}$ and is rounded down to the next integer percentage. The rounded down interest rate constitutes the minimum interest rate

Consequently, the minimum interest rate will not only vary with the development in the interest rates but also with the composition of the open bonds. One example thereof could be observed when in October 1996 the minimum interest rate was reduced extraordinarily to 4 per cent following the closing in August 1996 of the very large mortgage series with maturity in 2026.

An accurate prediction of the minimum interest rate on the basis of the yield curve is thus difficult. Going further back, a reasonable approximation of the minimum interest rate seems to be the 10-year interest rate less 1 percentage point and round down to the next integer percentage. This approximation will not be very accurate in the current situation which must be considered extraordinary due to the very steep yield curve.

The approximation of the minimum interest rate constitutes a floor below the future coupon rates. However, it is not given in advance that the coupon rate is determined such that it corresponds to this floor. One relevant consideration in the policy for opening new bonds could be that the bonds must be applicable for as long as possible, the volume and liquidity of the bonds being maximized. This consideration speaks in favour of the bonds being opened with a coupon above the minimum interest rate so that an increase in the minimum interest rate does not close the bonds. Product considerations

also speak in favour of the bond being opened with a price close to 100.

In the model the future coupon rates are determined such that they fulfil the requirement with respect to a minimum interest rate and such that the price is as close to 100 as possible.

2.5.2 The general structure of the model

Collectively, the conditions define the problem that the model solves. The problem may be formulated as follows:

The problem for a LAIR III type F

Determine the term to maturity of the loan, the interest rate on the loan, the volumes of the underlying bonds, and the volume of the payments on the financial instrument such that

1. the term to maturity is within the maximum and the minimum limits, c.f. (2.3)
2. the payments on the loan are within the maximum and the minimum limits, cf. (2.8)
3. the balance conditions are fulfilled, cf. (2.10), (2,12) and (2.13)
4. the proceeds condition is fulfilled, cf. (2.15) and
5. the payments on the financial instrument fulfil the condition given by (2.14)

The number of variables in the model may be immediately determined at m+3, m volumes, an interest rate on the loan, a term to maturity, and the volume of the payments on the financial instrument having to be determined. However, payments from the financial instrument are conditional on the term to maturity having reached its maximum or minimum allowed value, thus no longer being adjustable. Actually, the number of variables thus amounts to m+2.

The balance condition must be fulfilled every year in the interest rate adjustment period and thereby defines m equations. The proceeds condition defines another equation. If there are no payments from the financial instrument, (2.15) will determine the market price of the sold bonds. However, if there are payments from the financial instrument, (2.14) which was deduced on the basis of the proceeds condition will determine the volume of the payments. Finally, (2.9) will define a last equation determining the term to maturity such that (2.3) and (2.8) are fulfilled. Thus, the model solves m+2 equations with m+2 variables.

In the degenerated model in which the term to maturity is kept fixed, the number of variables and the number of equations are reduced by 1, the term to maturity not constituting a variable and (2.9) losing its significance. However, basically, the problem will not be modified.

The problem has a simultaneous as well as a recursive structure. The simultaneous structure appears when there are no payments on the financial instrument.

For a given term to maturity, the determination of the bond volumes will be determined by the proceeds condition as well as by the payments on the loan via the balance conditions. If one takes as a starting point any interest rate on the loan, the payments may be calculated. With reference to the balance conditions, the payments determine the volumes of the underlying bonds. The proceeds of the sale of the bonds may then be compared to the funding demand. If there is a deficit, more bonds must be sold and the interest rate on the loan must be increased, so that the large payments on the bond side are covered by the payments of the debtor side. If, on the contrary, there is a profit from the bond issue, the interest on the loan may be lowered. It will always be possible to find an unambiguous and positive interest rate on the loan, which

solves the problem for the given term to maturity. This is due to the proceeds of the sale of bonds being a strictly rising function of the interest on the loan, assuming that the prices of the underlying bonds are positive.

When a solution has been found that complies with both the balance conditions and the proceeds condition, the calculated debtor payments on the loan may be compared to the limits for payments on the loan, and a correction of the term to maturity is performed. The determination of the interest rate on the loan and the volumes of the underlying bonds is then to be repeated. Similarly, it is always possible to find a term to maturity that solves the problem, the payments on the loan being a strictly declining function of the term to maturity.

In so far as the necessary correction of the term to maturity causes the term to maturity to exceed the allowed interval, the financial instrument is activated. Thereby the model changes to a recursive structure. First, the interest on the loan is determined by the requirement as to the payments on the loan, one of the limits being binding. In the next step, the payments on the loan determine an unambiguous value for each volume. The proceeds of the sold bonds may then be calculated, causing the payments on the financial instrument to be determined at the same time.

The flow chart of the model appears from figure 7. As indicated above, and as appears from the figure, the model is divided into an inner and an outer loop. The inner loop consists of steps 2 to 11. In these steps volumes of the underlying bonds and the interest rate on the loan are determined with reference to the balance conditions and the proceeds criterion. The outer loop is constituted by step 1 and steps 12 to 16. The loop determines the term to maturity with reference to the band for the payments on the loan. Finally, steps 17 to 22 constitute the recursive model

determining the volume of the financial instrument. The individual steps are described in the following.

2.5.3 The steps of the model

Step 1 - Determine the term to maturity within the interval from L^{min} to L^{max}

In step 1 the outer loop of the model begins. In step 1 the model determines an initial value for the term to maturity of the loan.

At the disbursement of the loan the term to maturity is determined on the basis of the input from the debtor. It will be natural to require that the debtor specifies an intended term to maturity (the term to maturity is intended, a realization of the term to maturity presupposing that future rises and falls in interest rates cancel out) which may be applied in the model immediately.

However, it may be argued that it is convenient that instead the debtor has the possibility of determining an intended payment on the loan, which is easier to relate to the limits for the payments on the loan. In this situation, the initial term to maturity is determined as

$$L_0 = -\ln\left(\frac{1-\frac{RG_0(0)R_0^K}{YD_0(1)}}{1+R_0^K}\right) \qquad (2.16)$$

where $YD_0(1)$ is the payment on the loan during the first integer year of the loan. The interest on the loan is approximated initially by the yield to maturity of the last maturing bond underlying the loan.

At adjustments of the interest rate on the loan the term to maturity is basically maintained in relation to the term to maturity at the preceding adjustment of the interest rate.

Therefore, the initial value for L_J is determined by the input vector which is projected through the lattice, cf. section 2.3.

At the last adjustment of the interest rate on the loan (J=M), the term to maturity should be corrected such that the loan matures at the same time as the underlying bonds. In so far as it is possible, the correction is performed such that the limits for the payments on the loan are observed. However, one cannot preclude the possibility of the maximum limit being exceeded by shortening the term to maturity to the next creditor payment date, whereas the minimum limit is exceeded by prolonging the term to maturity. In this situation, the term to maturity is prolonged, as a payment on the loan which is too low must be considered a minor problem for the debtor than a payment on the loan which is too high. If the payment falls below the minimum limit, payments on the financial instrument are not triggered in this instance. Payments from the financial instrument presuppose that at the same time the term to maturity conflicts with the minimum limit. With the definition of the minimum limit (cf. subsequently in the description of step 1), a shortening of the term to maturity within the year will not solve such a conflict.

L_M is determined according to the following directions.

1. Prolong L_M if

(2.17) $$YD_M(j)\big|_{\bar{L}_M} > YD_M^{\min}$$

where $\bar{L}_M$ denotes the term to maturity prolonged to the immediately subsequent creditor payment date. Otherwise move on to 2.

2. Shorten L_M to the immediately preceding creditor payment date, if

(2.18) $$YD_M(j)\big|_{L_M} < YD_M^{max}$$

where L_M denotes the shortened term to maturity. Otherwise move on to 3.

3. Prolong the term to maturity to the next creditor payment date.

Step 3 in the procedure ensures that a solution is always found, so that the calculations do not continue infinitely. The steps imply that it is necessary to operate with two terms to maturity in the model, $\bar{L}_M$ and L_M, respectively.

If the loan is a LAIR I for which the term to maturity is fixed given by $L^{max}=L_j=L^{min}$, the term to maturity is naturally determined in the model as the fixed term to maturity.

Step 1 being the first step in the model, all inputs are further to be input to the model. These are constituted by

the funding side
the number of bonds (and thus also the interest rate adjustment frequency), the nominal yield of each instrument, the number of annual creditor payment periods and creditor payment dates, and the market price of each instrument.

the debtor side
the disbursement date, the volume or the remaining debt, the number of annual debtor payment periods and debtor payment dates, the maximum and minimum limits for the payments on the loan and, optionally, an intended payment on the loan or term to maturity, cf. above.

Furthermore, values for inc. are input. inc are used in the Gauss-Newton algorithm applied at the iterations in the model. Finally, values are to be assigned to ε. ε is a set of accuracy parameters specifying a minimum permissible deviation from the various requirements.

When inputting data, it is checked that

$$\forall J : YD_J^{max} \geq YD_J^{min} \geq 0 \quad \text{and} \quad L^{max} \geq L^{min} \geq 0$$

Since the loan is required to mature at a creditor payment date, it is required that the input values for L^{max} and L^{min} comply with a creditor payment date. If the loan is disbursed e.g. on 30 June, the model requires that L^{max} and L^{min} are specified by an integer number of years +½.

Step 2 - Determine the initial interest rate on the loan

The inner loop of the model is initiated by an initial determination of the interest rate on the loan. The yield to maturity of the last maturing bond is applied as an approximation for the interest rate on the loan, the main part of the issue being effected in this bond. The last maturing bond has a term to maturity of m years, m being determined as

(2.19) m=max [the number of bonds initially; the remaining term to maturity rounded up]

such that bonds with maturity later than the maturity of the loan are not issued.

Step 3 - Calculate debtor payments on the loan

Once an interest rate on the loan has been determined, the debtor payments on the loan until the next adjustment of the interest rate may be calculated both for

$$R_J^K \text{ and } R_J^K + inc$$

The repayment profile and the remaining debt profile until the next adjustment of the interest rate are thereby also determined.

Step 4 - Calculate the volumes

The calculated payment profile and remaining debt at end of period permit the determination, via the balance conditions, of the volumes of the underlying bonds. In a matrix form, the balance conditions may be formulated

(2.20) $\mathbf{YD_J+RG_J=AH_J}$ where

$\mathbf{YD_J} = (YD_J(1), YD_J(2), \ldots, YD_J(m))$,
$\mathbf{RG_J} = (0, 0, \ldots 0, RG_J(m))$ and
$\mathbf{H_J} = (H_J(1), H_J(2) \ldots, H_J(m))$ are m×1 vectors,

and where **A** is defined as an m×m upper triangular matrix given by

$$\mathbf{A}=\begin{vmatrix} 1+REG_JR_J^N(1) & REG_JR_J^N(2) & REG_JR_J^N(3) & . & . & REG_JR_J^N(m) \\ 0 & 1+R_J^N(2) & R_J^N(3) & R_J^N(4) & . & R_J^N(m) \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 1+R_J^N(m) \end{vmatrix}$$

The m bond volumes are found by

(2.21) $$\mathbf{H_J=[A^TA]^{-1}A^T[YD_J+RG_J]}$$

In principle, $\mathbf{[A^TA]^{-1}A^T}$ may be replaced by $\mathbf{A^{-1}}$ in (2.21) as long as **A** is quadratic. The rewriting $\mathbf{[A^TA]^{-1}A^T}$ is necessary only if more bonds are introduced. Thus, the rewriting is only a method by which non-quadratic matrices may be inverted approximatively.

Step 5 - Determine the proceeds function

The difference between the funding demand and the market price of the bond volumes defines a proceeds function given by

(2.22) $$f(R_J^K)=RG_J(0)-\sum_{j=1}^{m}K_J(j)H_J(j)$$

The value of the proceeds function equals the payments on the financial instrument, cf. (2.14). F_J and $f(R_J^K)$ should however be kept apart. The proceeds condition is fulfilled for $f(R^K) = 0$.

Step 6 - Calculate adjustment to the interest rate on the loan

On the basis of the value of the proceeds function, an adjustment to the interest rate on the loan is calculated. The calculation of the adjustment is performed prior to the test of the proceeds condition. The adjustment constitutes a better measure as to whether the iterative procedure is to continue, or whether the interest rate on the loan has converged. Thus, a situation may arise in which the proceeds condition deviates marginally between the funding demand and the market price of the sold bonds, meaning that the model is immediately to continue, but a zero adjustment to the interest rate is calculated, said interest rate having converged. The calculation of the adjustment to the interest rate on the loan follows the Gauss-Newton algorithm.

(2.23) $\Delta R_J^K = \left[[D^T D]^{-1} D^T g\right] \circ j_v^{\circ -}$ for

(2.24) $j_v = [\mathrm{Diag} J_a^T J_a]^h$

(2.25) $D = [J_a^T J_a] \circ [j_v^T j_v]^{\circ -1}$

(2.26) $g = [J_a^T F(L_J)] \circ j_v^{\circ -1}$

(2.27) $J_a = [F(L_J) - F(L_J + inc)] \circ inc^{\circ -1}$ where

D^TD	is the Hesse matrix
g	is the gradient
J_a	is the Jacoby matrix
j_v	is the diagonal elements

$A \circ B^{\circ -1}$ is the Schur product of two matrices meaning that the elements of the matrix are divided one by one.

(2.23) to (2.27) define the multi-dimensional Gauss-Newton algorithm. In the specific example, the problem is one-dimensional meaning that the expressions may be reduced. D, J_a, and j_v all have the dimension 11. Thus, it applies that

(2.28) $j_v = J_a$

as $j_v = [\mathrm{Diag}J_a^T J_a]^{1/2} = [J_a^2]^{1/2} = J_a$. If $j_v = J_a$ is inserted in the expression for D, the result is

(2.29) $$D = \frac{J_a^T J_a}{j_v j_v^T} = \frac{J_a}{j_v^2} = 1$$

The expression for g may also be reduced to

(2.30) $$g = \frac{J_a^T F(R_J^K)}{j_v} = f(R_J^K)$$

If the reduced expressions are inserted in (2.23), the result is as follows

(2.31) $$\Delta R_J^K = \frac{[D^T D]^{-1} D^T g}{j_v} \Rightarrow \Delta R_J^K = f(R_J^K) \frac{inc}{f(R_J^K) - f(R_J^K + inc)}$$

The calculation of the adjustment to the interest rate on the loan by means of the reduced Gauss-Newton given by (2.31) is illustrated in figure 8.

The basic idea of the algorithm is to apply the secant through two points on the graph of f(.) for $\left(R_J^K, f(R_J^K)\right)$ and $\left(R_J^K + inc, f(R_J^K + inc)\right)$ respectively. In the figure, these points are given by (6) and (7). When the secant has been determined, the intersection of the secant with the x axis (8) is determined. The intersection is the next guess as to the interest rate on the loan in the iterative process. If f(.) is strictly declining, the algorithm will always reach a solution. This will be the case as a rising interest rate increases the proceeds of the loan, and vice versa. The figure shows that inc is not to be seen as an accuracy parameter. On the contrary, inc determines the step size in the iteration procedure.

Step 7 - Is the proceeds condition fulfilled?

The convergence on the interest rate is tested on the basis of the adjustment. One of two conditions is to be fulfilled in order for the interest rate to have converged.

(2.32) $$\frac{|\Delta R_J^K|}{\varepsilon|R_J^K|} < \varepsilon$$

(2.33) $$\left|\frac{[f(R_J^K)^2 - f(R_J^K + \Delta R_J^K)^2]}{f(R_J^K)^2}\right| < \varepsilon$$

If one of the conditions is fulfilled, the interest rate is accepted and the model moves on to step 8. Otherwise the interest rate is corrected in step 9.

Step 8 - Correct the interest rate on the loan

The interest rate on the loan is corrected by the adjustment, and steps 3 to 7 are repeated for $R_J^K + \Delta R_J^K$.

Step 9 - Are all volumes positive?

One cannot preclude the possibility that one or more of the volumes are negative. The immediate interpretation thereof is that the debtor is to buy bonds underlying the loan, which is theoretically possible, but in practice this is not a viable solution. The problem is solved by the loan changing characteristics to a type F* loan in step 11.

However, if all the bond volumes are positive, the model moves on to step 12.

Step 10 - Shift to F*

This step sends the model on to the F* variant which is described in the appendix. Information concerning the loan is input to the F* model.

The negative bond volumes are produced as a result of the interest rate on the loan being significantly lower than the coupon rate of the underlying bonds. Another possibility is, therefore, that new coupon rates are determined for the bonds. However, it is usually preferable, in practice, that the loan is calculated as a type F*.

Step 11 - Calculate adjustment to the term to maturity

On the basis of the relationship between the calculated debtor payments on the loan and the limits for the payments on the loan, the model calculates an adjustment to the term to maturity in step 12. As in steps 6 and 7, the convergence of the iteration is determined on the basis of the adjustment, for which reason the adjustment is calculated before the calculated debtor payments on the loan are compared to the limits for the payments on the loan.

The calculation of the adjustment follows a procedure corresponding to that applied in step 6. First, a function f(.) is defined, which measures the distance between the calculated debtor payments on the loan and the band.

However, it is necessary to distinguish between the maximum limit and the minimum limit for the payments on the loan when f(.) is defined. If e.g. the debtor payments on the loan are above the maximum limit, it follows from (2.9) that a correction of the term to maturity is to be performed such that the payments on the loan correspond exactly to the maximum limit. If the payments on the loan become strictly lower than the maximum limit, the correction is thus too large. f(.) is to be defined such that both positive and negative distances are measured at each limit. f(.) is given by

$$f(L_J) = \begin{cases} YD_J(j) - YD_J^{max} & : YD_J(j) > YD_J^{max} \\ 0 & : YD_J^{min} \le YD_J(j) \le YD_J^{max} \\ YD_J(j) - YD_J^{min} & : YD_J(j) < YD_J^{min} \end{cases} \qquad (2.34)$$

For J=0 the payments on the loan are regulated in (2.34) such that the term to maturity is evaluated on the basis of payments on the loan during an entire year. The adjustment to the term to maturity ΔL_J is calculated by means of the Gauss-Newton algorithm meaning that

(2.35) $$\Delta L_J = \left[[D^T D]^{-1} D^T g \right] \circ j_V$$

where the notation is taken over from step 6. The problem is one-dimensional and (2.32) is thus reduced to

(2.36) $$\Delta L_J = f(L_J) \frac{inc}{f(L_J) - f(L_J + inc)}$$

In the calculation of the adjustment, account should be taken of the special procedure for the determination of the term to maturity at the maturity of the loan, i.e. for J=M. The procedure determines that the term to maturity is prolonged to $\bar{L}_M$ if the payment is not thereby lower than allowed. However, the possibility that the term to maturity is to be further prolonged in order to observe the maximum limit cannot be precluded. f(.) is given by

(2.37) $$f(\bar{L}_M) = YD_M(j)\big|_{\bar{L}_M} - YD_M^{\max} \quad \text{for} \quad YD_M(j)\big|_{\bar{L}_M} > YD_M^{\max}$$

and otherwise $f(\bar{L}_M) = 0$. Similarly, a shortening of the term to maturity to $\bar{L}_M$ may be insufficient to ensure payments on the loan above the minimum limit. In this situation, f(.) is given by

(2.38) $$f(\underline{L}_M) = YD_M(j)\big|_{\underline{L}_M} - YD_M^{\min} \quad \text{for} \quad YD_M(j)\big|_{\underline{L}_M} < YD_M^{\min}$$

and otherwise $f(\underline{L}_M) = 0$. The last possibility is that

(2.39) $$YD_M(j)\big|_{\bar{L}_M} < YD_M^{\min} \quad \text{and} \quad YD_M(j)\big|_{\underline{L}_M} > YD_M^{\max}$$

In this situation, the term to maturity is prolonged to $\bar{L}_M$ and it is accepted that the payments on the loan are lower than

the minimum limit, causing payments to the financial instrument to occur. Consequently, no adjustment to L_M are to be calculated, and $f(L_M)=0$ will apply.

The calculation of the adjustments at the maturity of the loan ensures that subsequently it is not necessary to correct the term to maturity plus the adjustment such that the loan matures in a creditor payment period. This correction is built into the adjustment.

It should be noted that in the calculation of the adjustment to the term to maturity no account is normally taken of the limits for the term to maturity. If an adjustment is calculated which causes the term to maturity to exceed the allowed limits, payments on the financial instrument are triggered. Therefore, the range of the adjustments must not be limited.

Even though $L^{min}=L^{max}$ such that the loan has a fixed term to maturity, an adjustment is calculated. In this situation, any $\Delta L_J \neq 0$ implies that the fixed term to maturity is incompatible with the limits for payments on the loan, for which reason payments on the financial instrument are required. If ΔL_J is set equal to 0 without regard to the payments on the loan, the model loses information as to when payments on the instrument are necessary.

Step 12 - Are the payments on the loan within the interval from YD_J^{min} to YD_J^{max}?

Step 12 determines whether the model is to continue the iteration over terms to maturity in the outer loop, or whether a solution has been found, which fulfils the requirements imposed on the repayments on the loan.

The term to maturity has converged if $f(L_J)$ fulfils one of the convergence conditions

(2.40) $$\frac{|\Delta L_J|}{\varepsilon|L_J|} < \varepsilon$$

(2.41) $$\left|\frac{\left[f(L_J)^2 - f(L_J + \Delta L_J)^2\right]}{f(L_J)^2}\right| < 1$$

If one of the above convergence conditions is fulfilled, the term to maturity of the loan has converged and the calculations of the debtor and bond sides of the loan are completed. The model is finalized in step 13. If neither of the conditions are fulfilled, however, the model moves on to step 14.

Step 13 - The loan has been calculated!

The model reaches step 13 only if the payments on the loan and the term to maturity are within the established limits. Thus, in this case there are no payments from the financial instrument, and the debtor and bond sides of the loan will have immediate characteristics in common with a LAIR II with respect to the volumes of the payments.

Step 14 - Is the corrected term to maturity within the interval from L^{min} to L^{max}?

If the term to maturity of the loan has not converged and the debtor payments on the loan are thus outside the band, it is assumed that the term to maturity is to be corrected.

However, a correction causing the term to maturity to exceed the maximum limit will not bring the model closer to a solution. In this case, the limits for payments on the loan and term to maturity are mutually incompatible and the financial instrument must be activated.

If the calculated correction causes the term to maturity to exceed the maximum limit, the term to maturity is set to the maximum limit, and the model moves on to step 16. In the

reverse situation in which the correction causes the term to maturity to exceed the minimum limit, the term to maturity is set equal to the minimum limit and, similarly, the model moves on to step 16.

If the corrected term to maturity does not exceed neither the maximum limit nor the minimum limit, the model corrects the term to maturity by the adjustment in step 15.

Step 15 - Correct the term to maturity by the adjustment
The term to maturity is corrected by the calculated adjustment, after which steps 2 to 12 of the model are repeated.

Step 16 - Determine the interest rate on the loan such that the payments are within the interval
The model reaches step 16 only if the yield curve causes the limits for payments on the loan and term to maturity to be in mutual conflict. In the following steps, the payment on the financial instrument is to be calculated. Thus, step 16 is the first step in the recursive model structure.

In this part of the model, the proceeds condition no longer constitutes a binding requirement in the determination of the bond volumes. On the contrary, one of the limits for the payments on the loan enters the model as a binding condition, cf. the description of the conditions in the model.

In step 16 an interest rate on the loan is to be determined such that the payments on the loan are exactly on the binding limit. If the term to maturity has reached its maximum at the same time as the maximum limit for the payments on the loan being binding, an interest rate on the loan is to be determined which produces the maximum permissible payment on the loan. The interest rate must fulfil

(2.42) $$\frac{R_J^K RG_J(0)}{1-(1+R_J^K)^{L^{max}-\varphi}} = YD_J^{max} \quad \text{for} \quad YD_J(j)|_{L^{max}} > YD_J^{max}$$

where

φ is the elapsed part of the term to maturity. $L_J-\varphi$ is thus the remaining term to maturity of the loan

R_J^K may not be isolated in (2.42), but may simply be found by a numerical method.

In the reverse situation in which a falling interest rate has meant that the minimum limit for the payments on the loan cannot be observed simultaneous with the minimum limit for the term to maturity being observed, the interest rate is to fulfil

(2.43) $$\frac{R_J^K RG_J(0)}{1-(1+R_J^K)^{L^{min}-\varphi}} = YD_J^{min} \quad \text{for} \quad YD_J(j)|_{L^{min}} < YD_J^{min}$$

The annuity payment is strictly rising in the interest rate. This means that a solution will always be found to (2.42) and (2.43).

Step 17 - Calculate the volumes

The calculation of the interest rate on the loan in step 16 permits the determination of the payment profile and the remaining debt profile of the loan. As in step 4, the balance conditions may subsequently be applied for the calculation of the volumes in each bond. They are found as a solution to the matrix equation

(2.21) $$\mathbf{H}_J = [\mathbf{A}^T\mathbf{A}]^{-1}\mathbf{A}^T[\mathbf{YD}_J + \mathbf{RG}_J]$$

where $\mathbf{H}_J$ is a vector of volumes, $\mathbf{A}$ is the payment matrix of the bonds, $\mathbf{YD}_J$ is a vector constituted by the annual debtor payments on the loan, and $\mathbf{RG}_J$ is a vector in which the last element is the remaining debt at the end of the interest rate

adjustment period and the other elements are 0 (zero), cf. step 4.

Step 18 - Are all volumes positive?

Once again it must be ascertained that no volumes are negative, which will have unfortunate consequences for the debtor. Negative bond volumes are treated as in steps 9 and 10 in the model for F^*.

However, if all volumes are positive, the model moves on to the step 20.

Step 19 - Shift to F^*

If the presence of negative volumes is established in step 18, move on to the model for F^*. However, there will be a difference between whether the model for F^* is called from step 10 at which the financial instrument has not yet been activated, or from step 19. The information as to the step from which the model is coming is therefore input together with the upper data in the F^* model.

Step 20 - Calculate the proceeds

Step 17 determined the volumes of the underlying bonds. The proceeds of the bond sale may thus be calculated as

(2.44) $$\sum_{j=1}^{m} K_J(j)H_J(j)$$

The proceeds of the bond sale must be compared to the funding demand with a view to determining the payments on the financial instrument. The payments are given by

(2.14) $$F_J = RG_J(0) - \sum_{j=1}^{m} K_J(j)H_J(j)$$

If the maximum limits for the payments on the loan and term to maturity, respectively, are mutually incompatible F_J will

assume a positive value, the bond volumes in step 17 having been reduced, and vice versa.

Step 21 - The loan and the instrument have been calculated!
With the determination of the payments on the financial instrument, all variables on both the debtor and the funding sides of the loan have been calculated for the interest rate adjustment period in question.

The model has thereby found a solution which fulfils all the formulated requirements.

The division of the model into a simultaneous and a recursive structure, respectively, is necessary because of the design of the financial instrument. Pointing to possible improvements of the model, an integration of the inner and the outer loop in the simultaneous structure of the model is possible. Thus, the model will have to iterate simultaneously over the interest rate on the loan and the term to maturity in a two-dimensional Gauss-Newton algorithm.

An integration of the inner and outer loops in the model will obstruct the interaction with the recursive part of the model. The model reaches the recursive part when the interest rate on the loan has converged, but a convergent solution for the term to maturity is not found, and in a two-dimensional iteration it cannot be taken for granted that the interest rate converges prior to the term to maturity.

2.6 The model for a LAIR III type P
For a LAIR type P, a partial adjustment of the interest rate of the remaining debt of the loan is performed each year. It is intended that the partial adjustment of the interest rate constitutes a fixed fraction selected by the debtor at the disbursement of the loan.

On the funding side, the intended interest rate adjustment fraction is decisive for the range of the underlying bond portfolio. If it is intended to have an adjustment of the interest rate on the loan of 10 per cent annually, it is convenient to sell bonds with terms to maturity of up to 10 years. After 10 years, the loan will be fully interest rate adjusted and at the same time the last of the bonds originally issued will mature. If the number of bonds are called m_0, the interest rate adjustment fraction may be expressed as $\frac{1}{m_0}$. The period until the loan has been fully interest rate adjusted is termed the funding period.

Unlike a LAIR type F, in the case of a type P, the interaction is close between the funding side of the loan from interest rate adjustment to interest rate adjustment. The partial adjustment of the interest rate implies that some of the underlying bonds do not yet mature. In the determination of the bond volumes, account should be taken of the bonds previously issued as well as of the future issue. This consideration complicates the model.

The interaction between the individual adjustments of the interest rate on the loan means that the conditions of the model have an intertemporal aspect. The conditions for a solution of the model are described in section 2.6.1. In 2.6.2. the problem is formulated and the general structure is described before the actual model is set up in section 2.6.3. There are several methods for solving the problem for type P. An alternative model is described in appendix B.

2.6.1 Conditions in the model

As in the model for type F, a solution of the P model is subject to a number of requirements.

- Firstly, the term to maturity of the loan must observe the maximum and minimum limits for the term to maturity.

- Secondly, the debtor's payments on the loan must observe the maximum and minimum limits for the payments on the loan.

The conditions may be formulated as

(2.3) $$\forall J : L^{min} \leq L_J \leq L^{max}$$

(2.8) $$\forall J : YD_J^{min} \leq YD_J(.) \leq YD_J^{max}$$

whereas corrections of the term to maturity as in the model for type F must fulfil

(2.9) $$L_J \in \operatorname{argmin}(|L_J - L_{J-1}| \,|\, YD_J^{min} \leq YD_J(.) \leq YD_J^{max}, L^{min} \leq L_J \leq L^{max})$$

making (2.3) and (2.8) operational. However, the definition of J in the conditions should be noticed. The annual adjustment of the interest rate means that the requirements are to be evaluated solely for the current interest rate adjustment period, whereas the conditions for J+1, J+2,..., J+m_0 are not evaluated.

If e.g. an upwards correction of the term to maturity is to be calculated following a rise in the interest rate, it is important that this correction is calculated solely on the basis of the payments on the loan in year J (the annual adjustments of the interest rate mean that there is equivalence between year and J, which will be applied in the notation). If the years J+1, J+2,..., J+m_0 are also included in the calculation of the correction, the latter will immediately be larger, the rise in the interest rate gradually making itself felt and increasing the payments on the loan. But the larger correction implies that in year J, the payments on the loan are lower than the maximum limit contrary to (2.9). Therefore, it is necessary that the evaluation of the above conditions is limited to the current interest rate adjustment period, even though further corrections of the term to

maturity etc. must be anticipated at the future adjustments of the interest rate.

- Thirdly, the payments on the funding side of the loan are to balance the payments on the debtor side of the loan.

On the debtor side, the payments are constituted by the payments on the loan and refinancing amounts, and on the funding side the payments are constituted by coupon rates and mature bonds.

The annual adjustment of the interest rate means that the balance condition is given another role in the model. At the end of each year, the total payments on the funding side of the loan are known. By determining the refinancing amount residually, the balance condition is fulfilled by definition, the extent of the bond sale being determinable on the basis of a balance requirement.

The fact that the refinancing amounts are determined residually means, however, that the actual adjustment of the interest rate may deviate from what was intended. The deviations from the intended adjustment of the interest rate should naturally be limited, as the choice of the debtor should be respected in so far as possible. The balance condition may therefore be interpreted as a condition of accordance between the actual and the intended adjustment of the interest rate. Henceforth, this condition is termed the interest rate adjustment condition.

A deviation between the actual and the intended adjustment of the interest rate will result in one or more of the calculated volumes being negative, which, as in the model for type F, is not accepted. In this situation, said volumes are assigned the value 0 (zero), causing the interest rate adjustment fraction to increase in relation to the intended level.

The payments in the interest rate adjustment condition are known for certain only for one year at a time. Already at the next interest rate adjustment, the payments on the loan must be expected to change in line with the development in interest rates. Consequently, the interest rate adjustment condition will also change. The interest rate adjustment condition may therefore be formulated as

(2.45) Year 1 $$YD_J(1)+REG_J^D\frac{RG_J(0)}{m_0}=H_J(1)+REG_J\sum_{j=1}^{m}R_J^N(j)H_J(j)$$

Both m_0 and m denote the number of bonds in the underlying portfolio. In general, m_0 and m will be in agreement. At the maturity of the loan, m is gradually scaled down, whereas m_0 is constant, for which reason it is necessary to distinguish between the two parameters.

The issue of bonds must also be arranged with regard to the remaining years in the funding period, so that the possibility of the intended adjustment of the interest rate being respected at the future adjustments of the interest rate is not reduced. The expected future conditions may be formulated as

(2.46) Year 2 $$YD_J(2)+REG_J^D\frac{RG_J(0)}{m_0}=H_J(2)+REG_J\sum_{j=2}^{m+1}R_J^N(j)H_J(j)$$

⋮

(2.47) Year m $$YD_J(m)+REG_J^D\frac{RG_J(0)}{m_0}=H_J(m)+REG_J\sum_{j=m}^{2m-1}R_J^N(j)H_J(m)$$

where

REG_J^D is a regulation factor for the interest rate adjustment percentage in the first year following the disbursement of the loan. In the disbursement year, the interest rate adjustment percentage is written down depending on the quarter after which the loan is disbursed. Therefore, the factor may assume the values {¼,½,¾,1}. For J>0 the factor will have the value 1.

The payments in (2.46) and (2.47) are not known, and (2.46) and (2.47) may thus not be applied directly as conditions in the model.

$H_J(j)$ expresses the total bond volume in the jth bond. Thus, a summing is performed over the issue in the bond in the current and the preceding interest rate adjustment periods. One cannot preclude that the issue in the jth bond will have a different coupon rate depending on the date of the issue, for which reason coupon rates are indexed by J. A summing is performed over all issued bonds in the calculation of the total coupon payments.

If (2.45) is considered by itself, the problem is manageable. m_0 volumes are to be determined under one condition, assuming for a moment that the proceeds condition is fulfilled. This results in an infinite number of possible solutions.

If year 2 is considered, an arbitrary distribution of the bond issue in the individual years will however cause problems. If a large part of the bonds were issued as 2-year bonds, the right-hand side of (2.46) will be large. A reduction of (2.46) requires a negative sale of the now 1-year bond year, which is not accepted. On the contrary, the left-hand side of (2.46) must adjust to the right-hand side. The only possibility is that the interest rate adjustment fraction is increased causing the intended adjustment of the interest rate not to be respected. If, by contrast, a limited volume of bonds in the originally 2-year bond was sold, the problem would simply arise at a later stage.

In order to be able to respect the intended adjustment of the interest rate also at the future adjustments of the interest rate, the distribution of the bond issue in the individual bonds must follow a dynamic strategy involving a long-term perspective.

The regard to the future interest rate adjustment fraction stipulates that a decreasing fraction of the expected payments on the debtor side is funded by bond issues in the current period. Thus, the volume of the payments may be adjusted on a current basis in line with the development in interest rates, and the possibility of respecting the intended interest rate adjustment is optimized.

In the model, the falling profile in the bond issue is ensured by the marginal issue in each bond being determined by a trend function which is estimated on the basis of the profile in the intended adjustment of the interest rate and the bonds already issued, so that negative volumes are avoided.

The trend function is adjusted to the interest rate adjustment condition and, if possible, to the proceeds condition such that the marginal issue fulfils these conditions. An adjustment to the proceeds condition presupposes that the proceeds condition is not already applied for determining the payments on the financial instrument, cf. below. In such cases, the trend function is adjusted solely to the interest rate adjustment condition.

- Fourthly, the proceeds of the sale of bonds and payments on the financial instrument must balance the funding demand.

If the financial instrument is active, the proceeds condition defines the payments on the instrument as in the model for type F. The payments on the financial instrument are thus given by

(2.48) $$F_J = Fin_J(0) - \sum_{j=1}^{m} K_J(j) M_J(j) \quad \text{where}$$

$Fin_J(0)$ denotes the funding demand at the beginning of the interest rate adjustment period

$M_J(j)$ (not to be confused with M) denotes the marginal issue of the period in the jth bond.

The notation has been changed as compared with (2.14). As in the F model, the funding demand is constituted by the disbursement of the volume of the loan. However, at the interest rate adjustments, the funding demand is defined by the balance condition of the preceding year. Therefore, the funding demand may be formulated as

(2.49)

$$Fin_J(0) = \begin{cases} volume & disbursement \\ \sum_{j=1}^{m} H_{J-1}(j) R^N_{J-1}(j) + H_{J-1}(1) - YD_{J-1}(1) & adjustment\ of\ the\ interest\ rate \end{cases}$$

If the actual adjustment of the interest rate corresponds to what was intended, it applies in (2.49) that

$$(2.50) \qquad \sum_{j=1}^{m} H_{J-1}(j) R^N_{J-1}(j) + H_{J-1}(1) - YD_{J-1}(1) = \frac{RG_{J-1}(1)}{m_0}$$

Further, the notation has been changed, a distinction being made between the marginal issue and bonds already issued. This distinction is necessary, as the proceeds naturally corresponds only to the market price of the marginal issue. The marginal issue may be defined as

$$(2.51) \qquad M_J(j) = H_J(j) - H_{J-1}(j+1)$$

i.e. as the adjustment to the jth bond.

If the financial instrument is inactive, corresponding to the limits for payments on the loan and term to maturity being compatible, the proceeds condition is given by

$$(2.52) \qquad Fin_J(0) = \sum_{j=1}^{m} K_J(j) M_J(j)$$

- Fifth, the interest rate on the loan must correspond to the yield to maturity of the underlying bond portfolio.

A LAIR type P is characterized in that there is not a one-to-one connection between the debtor and the funding sides. Consequently, it is not without ambiguity when the payments of the debtor side in year j is funded by the issue of bonds. In principle, this paves the way for an imbalance which will make itself felt in the interest rate on the loan. Therefore, a condition with respect to a connection between the interest rate on the loan and the yield to maturity of the underlying bonds must be imposed on the solution to the model. Full accordance is not required. Differences in the number of payment dates on the debtor side and the funding side mean that a certain difference between the interest rate on the loan and the yield to maturity of the bonds will occur, said difference not being due to imbalances.

In situations in which the financial instrument is active, the interest rate on the loan is determined with regard to the limits for the payments on the loan. It is not thereby possible to maintain the condition with respect to a connection between the interest rates on the debtor and funding sides, for which reason the requirement is suspended.

2.6.2 The general structure of the model

The model for type P solves the problem

The problem for type P

Determine the term to maturity of the loan, the interest rate on the loan, the volumes of the underlying bonds, and the volume of the payments on the financial instrument such that

1. *the term to maturity is within the maximum limit and minimum limits, cf. (2.3)*
2. *the payments on the loan are within the maximum and minimum limits, cf. (2.8)*

3. *the interest rate adjustment condition is fulfilled, cf. (2.45)*
4. *the proceeds condition is fulfilled, cf. (2.52)*
5. *the payments on the financial instrument fulfil the condition given by (2.48), and*
6. *the interest rate on the loan fulfils the condition with respect to a connection with the yield to maturity of the bond portfolio.*

The number of variables in the model constitutes m+2, m volumes, an interest rate on the loan and a term to maturity or a payment on the financial instrument having to be determined.

On the basis of the two first conditions in the problem formulation, (2.9) defines an equation to be fulfilled by the solution with respect to the determination of the term to maturity.

If, at first, a situation is considered in which the financial instrument is inactive, the interest rate adjustment condition and the proceeds condition each define an equation. However, via the trend function which is adjusted to the two conditions, m equations are formed which determines the volumes of the underlying bonds. Finally, in this situation the interest rate condition will constitute yet another equation causing the conditions to lead to a total m+2 equations, including (2.9), as required.

In the situation in which the financial instrument is active, the proceeds condition determines the payment on the financial instrument, and, therefore, the condition is not included in the adjustment of the trend function. The trend function is adjusted solely to the interest rate adjustment condition which, in this situation, leads to m equations. The interest

condition is discontinued, causing the conditions to lead to m+2 equations in this situation as well.

It appears from the above that the construction based on the trend function is necessary in order to achieve equality between the number of variables and equations in the model. Without the trend function, the equation system would be sub-determined with an infinite number of possible solutions.

The model for type P has the same simultaneous and recursive structure as has the model for type F. If the financial instrument is inactive, the model is simultaneous. In an outer loop, iteration over the term to maturity is performed, whereas an inner loop iterates over the interest rate on the loan until the interest rate adjustment condition and the proceeds condition have been fulfilled. The iteration in the inner loop follows a two-step procedure. First, the bond volumes are calculated via the trend function given the interest rate, and then the convergence of the interest rate is tested against the yield to maturity of the bond portfolio. The inner loop is thus more extensive than in the model for type F.

The recursive structure occurs if the limits for payments on the loan and term to maturity collide under the given yield curve and there are thus payments from the financial instrument.

In the first step in the recursive structure, the interest rate on the loan is determined as a function of the established payments on the loan. Then a new trend function is defined which estimates the remaining debt profile of the loan under the recalculated interest rate. The trend function is adjusted to the interest rate adjustment condition which is thus also respected in a solution of the model with an active financial instrument.

Finally, the payments on the financial instrument are determined on the basis of the proceeds condition. The flow chart of the model is shown in figure 9.

2.6.3 The steps of the model

Step 1 - Determine the term to maturity within the interval from L^{min} to L^{max}

Step 1 determines an initial value of the iteration over the term to maturity in the outer loop.

At the disbursement of the loan, the debtor's input determines an initial value for the term to maturity. As in the model for type F, the possibility of the debtor selecting an intended payment on the loan must be kept open, in which case the term to maturity is given by

$$L_0 = -\ln\left(\frac{1-\frac{RG_0(0)}{YD_0(1)}}{1+R_0^K}\right) \qquad (2.16)$$

The interest rate on the loan is unknown and is approximated by the initial interest rate on the loan determined in step 2.

At adjustments of the interest rate on the loan, the term to maturity is taken over from the preceding adjustment of the interest rate. Therefore, the initial term to maturity may be read in the input vector, cf. section 2.3.

At the maturity of the loan, the term to maturity is to be determined according to special directions. The current issue of bonds with maturity after the next adjustment of the interest rate means that the term to maturity must not be shortened uncritically when the loan is approaching the year of maturity. A risk would thereby arise that the loan matures before a volume of the underlying bonds, causing imbalances in

the payments to be inevitable. Thus, when the model is entering the last funding period of the loan for $J \geq M - m_o$, the term to maturity must, in general, not be shortened.

Consequently, it is impossible in the case of a fall in interest rates to maintain the debtor's payments on the loan at the minimum limit by shortening the term to maturity. A payment on the loan below the minimum limit will not necessarily trigger payments to the financial instrument, as this also requires that the term to maturity has reached the minimum limit.

The adjustment of the term to maturity of the loan such that the maturity coincides with a payment date on the funding side may thus be effected solely as a prolongation of the term to maturity of the loan to the next payment date of the underlying funding.

However, it is inadequate first to adjust the term to maturity for J=M. As soon as M is within the funding period, the model funds payments in the last year. Therefore, it is necessary at an earlier stage to correct the term to maturity. Consequently, a parameter *closing time* is introduced, which specifies when the model is to be adjusted to the time of maturity of the loan for the first time, i.e.

(2.53) $$\forall J \geq M - m_0 : L_J = \overline{L}_J$$

Closing time must be determined on the basis of the type of loan. Therefore, it will not make any sense to set closing time > m_0. Closing time = m_0 will provide the most convenient maturity, but implies, however, that the payment on the loan may exceed the minimum limit for a long period.

If the loan has a fixed term to maturity, said fixed term to maturity is immediately input.

The total input to the model is input in step 1

the funding side
the number of bonds (and thus also the interest rate adjustment percentage), the nominal interest rate of each bond, the number of annual creditor payment periods and creditor payment dates, and the market price of each bond, volumes of bonds already issued.

the debtor side
the disbursement date, the volume or the remaining debt, the funding demand, the number of annual debtor payment periods and debtor payment dates, the maximum and minimum limits for the payments on the loan, the maximum and minimum limits for the term to maturity and, optionally, an intended payment on the loan or term to maturity.

In addition, values for inc and closing time are input. Finally, the validity of the limits is checked as in the model for type F.

Step 2 - Determine the initial interest rate on the loan. Determine m

As a start value for the iteration in the inner loop, the following approximation is applied

(2.54) $$R_0^K = \frac{\sum_{t=1}^{m} \frac{1}{m} \cdot t \cdot r(0,t)}{\sum_{t=1}^{m} \frac{1}{m} \cdot t}$$ where

t	denotes the term to maturity of the underlying bonds
r(0,t)	is the yield to maturity at time 0 of the bond with maturity at time t. At time 0, the yields to maturity may be observed. In other situations, the yields to maturity are calculated on the basis of the zero-coupon rate structure estimated by the model.

(2.54) approximates the interest rate on the loan by a term-weighted average of the yields to maturity of the underlying bonds. (2.54) presupposes that a value for m is determined. Typically the value is set to $m=m_0$. In two situations, however, another determination of m is necessary.

Firstly, the interest rate adjustment fraction is written down in the year of disbursement via the REG_J^D-faktoren factor in relation to the quarter in which the loan was disbursed. Consequently, another year will elapse before the loan is fully interest rate adjusted, and the funding period is thus to be prolonged by one year. Therefore, a variable TILT is introduced where

(2.55)

$$TILT=\left\{\begin{array}{ll} 1 & \textit{if the loan is disbursed in the period from January to November} \\ 0 & \textit{if the loan is disbursed in December} \end{array}\right\}$$

such that

(2.56) $$m=m_0+TILT$$

TILT also indicates that a special procedure is to be applied in the determination of the volumes.

Secondly, m is to be adjusted such that bonds with maturity later than the maturity of the loan are not issued. Collectively, m is determined as

(2.57) $$m=\min[L_J-\varphi; m_0+TILT]$$

where $L_J-\varphi$ is the remaining term to maturity of the loan.

Step 3 - Calculate the debtor payments on the loan

The model calculates a development in the payments on the loan and the remaining debt on the basis of the interest rate on the loan.

Step 4 - Calculate the loan for m=1

If the number of bonds is limited to 1 - corresponding to the loan maturing in one year - the problem is considerably simplified. The funding demand is to be funded by sale of only the 1-year bond. The marginal issue is given by

(2.58) $$M_J(1)=\frac{Fin_J(0)}{K_J(1)} \qquad \text{for J=M}$$

As no further adjustments of the interest rate on the loan are to be performed, the nature of the interest rate adjustment condition changes. That the loan is not interest rate adjusted at end of period may be interpreted as the interest rate adjustment amount at end of period having to be 0 (zero) in the condition. Thus, the condition has the same contents as the balance condition, i.e. the payments on the loan are to be determined as

(2.59) $$YD_J(1)=(1+R_J^N(1))H_J(1) \qquad \text{for J=M}$$

which determines an unambiguous interest rate on the loan. The convergence of the interest rate is tested in step 12.

Step 5 - Define a trend function

For m>1 the bond volumes are determined by the trend function to be defined in step 5. The trend function estimates the future intended interest rate adjustment amounts corrected for the funding already issued. Therefore, the trend function must have a functional form which is appropriate for estimating the development in the remaining debt of the loan to be followed by the interest rate adjustment amounts. A satisfactory estimation is achieved by applying a polynomial of (q-1)th degree, causing the trend function to have the form

(2.60) $$a_0+a_1(j-1)+a_2(j-1)^2+\ldots+a_{q-1}(j-1)^{q-1}$$

for j=1,2,..., m. The trend function depends on (j-1) and not on j. This is due to the fact that the intended adjustment of the interest rate is given as a fraction of the remaining debt at beginning of period. Therefore, the jth volume is decided on the basis of the remaining debt at time j-1.

The degree of the polynomial must not exceed the number of bond volumes minus 1, whereby the polynomial would have too

many degrees of freedom in the determination of the coefficients. Therefore, the degree is limited by m. In the special TILT procedure in which the number of volumes is increased, it is not necessary also to increase the degree of the polynomial, for which reason

$$q \leq m - TILT \tag{2.61}$$

Generally, the degree of the polynomial is maximized such that q=m-TILT, causing a perfect estimation of the future intended interest rate adjustment amounts to be achieved.

Step 6 - Determine the coefficients in the trend function

The actual estimation of the future intended adjustment of the interest rate is performed in step 6 in which the coefficients of the trend function are estimated. For each j=1, 2..., m the value of the trend function is to balance the intended adjustment of the interest rate less the bond volume already issued such that negative marginal issue does not occur. The coefficients in the trend function must fulfil

$$a_0 + a_1(j-1) + a_2(j-1)^2 + \ldots + a_{q-1}(j-1)^{q-1} = \max\left[REG_J^D \frac{RG_J(j-1)}{m_0}, H_J(0,j)\right] \tag{2.62}$$

for j=1,...,m. (2.62) is solved by the matrix equation

$$(a_0, a_1, \ldots, a_{q-1}) = [\mathbf{B}^T\mathbf{B}]^{-1}\mathbf{B}^T \max\left[REG_J^D \frac{RG_J(j-1)}{m_0}, H_J(0,j)\right] \tag{2.63}$$

where $(a_0, a_1, \ldots, a_{q-1})$ is a qx1 vector, max(.) forms an mx1 vector, and **B** is an mxq vector given by

$$\mathbf{B} = \begin{vmatrix} 1 & j_1-1 & (j_1-1)^2 & . & (j_1-1)^{q-1} \\ 1 & j_2-1 & (j_2-1)^2 & . & (j_2-1)^{q-1} \\ . & . & . & . & . \\ 1 & j_m-1 & (j_m-1)^2 & . & (j_m-1)^{q-1} \end{vmatrix} \tag{2.64}$$

$$= \begin{vmatrix} 1 & 0 & 0 & . & 0 \\ 1 & 1 & 1 & . & 1 \\ 1 & 2 & 4 & . & 2^{q-1} \\ . & . & . & . & . \\ 1 & m-1 & (m-1)^2 & . & (m-1)^{q-1} \end{vmatrix}$$

as $(j_0, j_1, \ldots, j_m) = (1, \ldots, m)$

Step 7 - Determine G_0 and G_1

In step 7, the trend function is to be adjusted so that the resulting bond volumes fulfil the interest rate adjustment condition as well as the proceeds condition. The adjustment is performed by two factors G_0 and G_1 being added to the trend function, said factors being determined on the basis of the two conditions. The trend function appears as follows

(2.65) $$G_0 a_0 + G_1 a_1 (j-1) + a_2 (j-1)^2 + \ldots + a_{q-1} (j-1)^{q-1}$$

G_0 shifts the trend function in the vertical plane, whereas G_1 influences the slope of the function. Generally, it will thus be possible to find a solution which fulfils both conditions. The adjustment of the trend function is illustrated in figure 10.

The bond issue is to be arranged according to a long-term strategy, cf. section 2.6.1. On the operational plane, the long-term strategy may be interpreted as a declining part of the expected future payments having to be funded by means of bond issue here and now. The total volume outstanding which determines the volume of the adjustment of the interest rate, can thereby be adjusted such that the intended level is realized. The long-term strategy is introduced in the model via G_0 and G_1. The primary task for G_0 is to adjust the issue of the bond with the shortest term to maturity such that the interest rate adjustment condition is fulfilled.

By adjusting the value of G_1, the declining funding fraction of the future payments is secured. G_1 is determined primarily with

a view to a fulfilment of the proceeds condition. Typically, the slope of the curve is to be increased in order for the proceeds not to exceed the funding demand. Thereby a declining funding profile is also secured.

The trend function added G_0 and G_1 may be divided into a (G_0,G_1) variant part and a (G_0,G_1) invariant part. The variant part is denoted $X_J(j)$, whereas the invariant part is denoted $Y_J(j)$ given by

$$X_J(j) = G_0 a_0 + G_1 a_1 (j-1) \quad \text{and}$$

(2.66)

$$Y_J(j) = a_2(j-1)^2 + a_2(j-1)^3 + \ldots + a_{q-1}(j-1)^{q-1}$$

The marginal issue is determined as the value of the trend function of the j in question corrected for bonds already issued in the bond in question. However, M(j) is to be positive. For j=1, 2,..., m is $M_J(j)$ given by

(2.67)

$$M_J(j) = \max(0; G_0 a_0 + G_1 a_1 (j-1) + a_2(j-1)^2 + \ldots + a_{q-1}(j-1)^{q-1} - H_J(0, j)$$

By means of (2.66) the expression may be formulated as

(2.68) $$M_J(j) = \max[0; X_J(j) + Y_J(j) - H_J(0, j)]$$

G_0 and G_1 cannot be isolated in (2.68) as long as the expression comprises a max-function. Therefore, an indicator function $I_J(j)$ j=1, 2,..., m is implemented with the value 0 (zero) if the jth bond volume is to be assigned the value 0 (zero), and otherwise with the value 1. Thus, the indicator function forms an m-dimensional vector.

The point of departure is that all bonds are to be applied, for which reason $I_J(j)$ is assigned the value

(2.69) $$I_J(j) = (1,1,\ldots,1)$$

If negative volumes are produced, a new value is assigned to $I_J(j)$ in step 9. With the introduction of the indicator function, (2.68) may be formulated as

(2.70)
$$M_J(j) = I_J(j)[X_J(j) + Y_J(j) - H_J(0,j)] =$$
$$I_J(j)X_J(j) + I_J(j)(Y_J(j) - H_J(0,j))$$

If it is to be possible to fulfil both the interest rate adjustment condition and the proceeds condition, at least two bonds must be available. That is to say that at least two bonds have not previously been assigned the value 0 (zero). At the same time, one of the two bonds must be that with the shortest term to maturity, as it is otherwise impossible to fulfil the interest rate adjustment condition. This may be formulated as

(2.71)
$$\sum_{j=1}^{m} I_J(j) \geq 2 \quad \text{and} \quad I_J(1) > 0$$

If both conditions in (2.71) are fulfilled, G_0 and G_1 are to be found as the solution to both the interest rate adjustment condition and the proceeds condition given by

(2.45)
$$YD_J(1) + REG_J^D \frac{RG(0)}{m_0} = H_J(1) + REG_J \sum_{j=1}^{m} R_J^N(j) H_J(j)$$

where $H_J(1) = H_J(0,1) + M_J(1)$ and $H_J(j) = M_J(j) + H_J(0,j)$ and

(2.52)
$$Fin_J(0) = \sum_{j=1}^{m} K_J(j) M_J(j)$$

(2.70) may be inserted in (2.45) for $M_J(1)$ and $M_J(j)$.

(2.72)
$$YD_J(1) + REG_J^D \frac{RG_J(0)}{m_0} =$$
$$H_J(0,1) + I_J(1)X_J(1) + I_J(1)(Y_J(1) - H_J(0,1)) +$$

$$REG_J \sum_{j=1}^{m} R_J^N(j)[I_J(j)X_J(j) + I_J(j)(Y_J(j) - H_J(0,j)) + H_J(0,j)]$$

The expression is solved for X (j)

$$I_J(1)X_J(1) + REG_J \sum_{j=1}^{m} R_J^N(j) I_J(j) X_J(j) =$$

(2.73)
$$YD_J(1) + REG_J^D \frac{RG_J(0)}{m_0} - H_J(0,1) - I_J(1)(Y_J(1) - H_J(0,1)) -$$

$$REG_J \sum_{j=1}^{m} R_J^N(j)[I_J(j)(Y_J(j) - H_J(0,j)) + H_J(0,j)]$$

The right-hand side of (2.73) defines the variable Z_1. Correspondingly, (2.70) may be inserted in the proceeds criterion. This leads to the expression

(2.74)
$$Fin_J(0) = \sum_{j=1}^{m} K_J(j)[I_J(j)X_J(j) + I_J(j)(Y_J(j) - H_J(0,j))]$$

(2.74) is solved for $X_J(j)$

(2.75)
$$\sum_{j=1}^{m} K_J(j) I_J(j) X_J(j) = Fin_J(0) - \sum_{j=1}^{m} K_J(j) I_J(j)(Y_J(j) - H_J(0,j))$$

The right-hand side of this expression is denoted Z_2. (2.73) and (2.75) define two equations with two unknown quantities G_0 and G_1. The solution for the equation system depends on TILT. If the interest rate adjustment fraction has been written down, TILT has the value 1. In this situation the first volume must be calculated explicitly. This is due to the fact that the volume of the first adjustment of the interest rate may be as small as ¼ of the other adjustments of the interest rate, which may hardly be comprised by the trend function and the adjustment thereof.

First, G_0 and G_1 are found in the general situation in which TILT has the value 0 (zero).

$X_J(j)$ is defined in (2.66) as that part of the trend function which depends on G_0 and G_1. In matrix form XJ (j) may be written as

(2.76) **X=aG**

where **X**=($X_J(1)$, $X_J(2)$,..., $X_J(m)$) is a 1xm vector, G = (G_0, G_1) is a 2x1 vector and **a** is an mx2 matrix given by

$$\mathbf{a} = \begin{vmatrix} a_0 & 0 \\ a_0 & a_1 \\ a_0 & 2a_1 \\ \vdots & \vdots \\ a_0 & (m-1)a_1 \end{vmatrix}$$

Then a matrix **K** is defined such that **K**x**X**, and thereby **K**x**a**x**G** constitutes the left-hand sides in (2.73) and (2.75). **K** is given by

$$(2.77)\quad \mathbf{K} = \begin{bmatrix} I_J(1)(1+REG_JR_J^N(1)) & I_J(2)REG_JR_J^N(2) & \cdots & I_J(m)REG_JR_J^N(m) \\ I_J(1)K_J(1) & I_J(2)K_J(2) & \cdots & I_J(m)K_J(m) \end{bmatrix}$$

With the definition of **K**, the interest rate adjustment condition and the proceeds condition given by (2.73) and (2.75) may be written in a matrix form, in which it applies that the right-hand sides of (2.73) and (2.75) are given by Z_1 and Z_2.

(2.78) **Z=KaG**

where **Z** = (Z_1, Z_2) is a 2x1 vector. **Ka** forms an mxm matrix, which is invertible. Therefore, G_0 and G_1 may be found as

(2.79) $\mathbf{G} = [\mathbf{Ka}]^{-1}\mathbf{Z}$

which determines the trend function. However, if TILT has the value 1, $M_J(1)$ must be determined explicitly as

$$M_J(j)=\max(0;(W_J(1),G_0a_0+a_1(j-1)+a_2(j-1)^2+ \quad (2.80)$$
$$\ldots+a_{q-1}(j-1)^{q-1})-H_J(0,j))$$

Again the trend function may be divided into a variant and an invariant part given by

$$X_J(j)=\begin{cases} G_0a_0 & \forall j>1 \\ W_J(1) & j=1 \end{cases} \quad \text{and} \quad (2.81)$$
$$Y_J(j)=a_1(j-1)+a_2(j-1)^2+\ldots+a_{q-1}(j-1)^{q-1}$$

$W_J(1)$ and G_0 must be determined such that the resulting volumes comply with the interest rate adjustment condition and the proceeds condition. It is not necessary to suspend the conditions during the TILT procedure. The proceeds condition is observed by definition, and the very objective of the TILT procedure is to secure the interest rate adjustment condition. The nature of the conditions does not change as a result of TILT, and the conditions are thus, unchanged, given by (2.73) and (2.75), which are not repeated here. The only change in the conditions is the definition of $X_J(j)$ and $Y_J(j)$, which is here given by (2.81). The right-hand sides of (2.73) and (2.75) are denoted Z_1^* and Z_2^*, respectively. $X_J(j)$ may be expressed in the matrix form

(2.82) $\mathbf{X}=\mathbf{a}^*\mathbf{G}^*$ where

(2.83) $\mathbf{G}^* = (G_0, W_J(1))$ and $\mathbf{a}^*=\begin{vmatrix} 0 & 1 \\ a_0 & 0 \\ \vdots & \vdots \\ a_0 & 0 \end{vmatrix}$

Analogously to the above, (2.73) and (2.75) lead to the expression

(2.84) $\mathbf{Z}^*=\mathbf{K}\mathbf{a}^*\mathbf{G}^*$

where $\mathbf{Z}^*=(Z_1^*,Z_2^*)$ and $\mathbf{K}$ are, unchanged, given by (2.75). As before, the matrix equation is solved by

(2.85) $$G^* = [Ka^*]^{-1}Z^*$$

causing the trend function to be determined. However, if either

(2.86) $$\sum_{j=1}^{m} I(j) < 2 \quad \text{or} \quad I(1) = 0$$

only the proceeds condition can be fulfilled, whereas the balance condition is suspended. It follows from (2.86) that either the model has only one bond at its disposal or the first bond is not available. In the first case, it is possible to fulfil only one condition in which case priority is given to the proceeds condition. In the second case, observance of the interest rate adjustment condition requires that the first volume is negative, which is not accepted. Thus, a suspension of the interest rate adjustment condition is necessary.

(2.86) having been fulfilled, G_0 and G_1 must, in principle, be determined on the basis of only one condition resulting in an infinite number of solutions. Therefore, a fixed value is to be assigned to either G_0 or G_1. It is immediately preferable that G_0 is maintained as the free variable. The adjustment to the proceeds condition via G_1 could produce inappropriate solutions in which e.g. the bond issue follows a rising profile. A fixed value is thus assigned to G_1, said fixed value being determined by numerical analysis.

The fixing of G_1 implies that now G_1 belongs to the invariant part of the trend function. The definition of $X_J(j)$ and $Y_J(j)$ is thus changed to

(2.87) $$X_J(j) = G_0 a_0 \quad \text{and}$$

$$Y_J(j) = G_1 a_1 (j-1) + a_2 (j-1)^2 + \ldots + a_{q-1} (j-1)^{q-1}$$

G_0 is determined in accordance with the proceeds condition which, solved for $X_J(j)$, is given

$$\sum_{j=1}^{m} K_J(j) I_J(j) X_J(j) = Fin_J(0) - \sum_{j=1}^{m} K_J(j) I_J(j) (Y_J(j) - H_J(0, j)) \tag{2.75}$$

$X_J(j)$ is one-dimensional which paves the way for a relatively simple solution of (2.75). In the proceeds condition, the expression for $X_J(j)$ is inserted given by (2.87)

$$G_0 a_0 \sum_{j=1}^{m} K_J(j) I_J(j) = Fin_J(0) - \sum_{j=1}^{m} K_J(j) I_J(j) (Y_J(j) - H_J(0, j)) \tag{2.88}$$

G_0 may then be isolated

$$G_0 = \frac{Fin_J(0) - \sum_{j=1}^{m} K_J(j) I_J(j) (Y_J(j) - H_J(0, j))}{a_0 \sum_{j=1}^{m} K_J(j) I_J(j)} \tag{2.89}$$

and thereby determined. By insertion of G_0 in the balance requirement, the interest rate adjustment is calculated residually in the first year. It is not necessary to determine G_0 for TILT=1. TILT=1 will occur only at the disbursement of the loan. At this point in time, the volumes of the already issued bonds are all 0 (zero), causing the marginal issue not to be negative. By definition, (2.86) will thereby not be fulfilled.

Step 8 - Determine the volumes

The marginal issue is determined by

$$M_J(j) = G_0 a_0 + G_1 a_1 (j-1) + a_2 (j-1)^2 + \ldots + a_{q-1} (j-1)^{q-1} - H_J(0, j) \tag{2.90}$$

for TILT=0 and as

$$M_J(j) = (W_J(1), G_0 a_0 + a_1 (j-1) + a_2 (j-1)^2 + \ldots +$$

(2.91)

$$a_{q-1}(j-1)^{q-1}-H_J(0,j))$$

for TILT=1. Both expressions are defined for j=1, 2,..., m. A max-condition is not included in the expressions for $M_0(j)$. Indeed, negative volumes are to be observable, so that adjustments of the indicator function and G_0 and G_1 are possible.

Step 9 - Are all volumes positive?

Step 9 determines whether the calculated volumes are applicable, or whether an adjustment of the indicator function and subsequent recalculations of the volumes are necessary. If the marginal issue is negative for just one j, i.e.

(2.92) $$\exists j \in \{1,2,\ldots,m\} : M_J(j) < 0$$

the indicator function is adjusted in step 10. Otherwise, the model moves on to step 11.

Step 10 - Adjust the indicator function

For the bonds in which a negative marginal issue was established in step 9, the indicator function is adjusted such that

(2.93) $$I_J(j)=\begin{cases} 0 & \forall j : M_J(j) < 0 \\ 1 & \forall j : M_J(j) \geq 0 \end{cases}$$

Normally, nothing is gained by setting the volumes to 0 (zero) one by one. When a volume is set to zero, fewer bonds in the remaining years must be sold in order to fulfil the proceeds condition. Thus, the volumes of all the remaining bonds will typically decrease. Once a volume has turned negative, it is likely to become even more negative for each operation of assigning 0 value(s) via the indicator function.

Step 11 - Calculate adjustment to the interest rate on the loan

According to the interest rate condition, there must be a certain degree of accordance between the interest rate on the loan and the yield to maturity of the underlying bond portfolio. Therefore, an adjustment to the interest rate on the loan is calculated in step 11 on the basis of the deviation in relation to the yield to maturity of the bonds.

The condition does not require full accordance, differences in the number of payment periods on the debtor and bond sides having to be corrected for. Therefore, the interest rate on the loan must be corrected for a "prepayment effect". At the same time, there is a tradition that the interest rate on the loan is calculated according to the money market convention. The adjustment is calculated on the basis of the function

(2.94) $$f(R_J^K) = \sum_{i=1}^{n} \frac{R_N^K RG_J(i-1)}{n RG_J(0)} - REG_J r^p$$ where

r^p	is the yield to maturity of the bond portfolio
n	is the number of payment periods per year
i	indexes the payment periods within the year

If (2.94) is inserted in the reduced Gauss-Newton algorithm, the adjustment may be calculated as

(2.95) $$\Delta R_J^K = \frac{inc}{f(R_J^K) - f(R_J^K + inc)}$$

Step 12 - Has the interest rate on the loan converged?

The mathematical convergence of the interest rate on the loan is tested by

(2.32) $$\frac{|\Delta R_J^K|}{\varepsilon |R_J^K|} < \varepsilon$$

$$\left|\frac{f(R_J^K)^2 - f(R_J^K + \Delta R_J^K)^2}{f(R_J^K)^2}\right| < \varepsilon \qquad (2.33)$$

where merely one of the conditions must be fulfilled in order that the interest rate has converged and the model can proceed to step 14. Otherwise, the model proceeds to step 13.

Step 13 - Adjust the interest rate on the loan

The interest rate on the loan is adjusted by the calculated adjustment. Then the model repeats step 3.

Step 14 - Calculate adjustment to the term to maturity

The adjustment to the term to maturity is calculated as a function $F(L_J)$ of the deviation of the payments on the loan from the established limits.

If the payments on the loan are within the band defined by the maximum and the minimum limits, no adjustment to the term to maturity is calculated, corresponding to $\Delta L_J=0$. If the payments on the loan exceed their maximum permissible value, a positive adjustment to the term to maturity is calculated on the basis of the relationship between the calculated payments on the loan and the maximum limit. However, if the payments on the loan are too low, a negative adjustment is calculated as a function of the relationship between the payments on the loan and the minimum limit. f (L_J) is given by

$$f(L_J) = \begin{cases} YD_J(j) - YD_J^{max} & : YD_J(j) > YD_J^{max} \\ 0 & : YD_J^{min} \le YD_J(j) \le YD_J^{max} \\ YD_J(j) - YD_J^{min} & : YD_J(j) < YD_J^{min} \end{cases} \qquad (2.34)$$

for $J<M-m_0$ where all payments on the loan are for integer years. The adjustment is calculated by use of the Gauss-Newton algorithm which may be written in its reduced form as

$$\Delta L_J = f(L_J)\frac{inc}{F(L_J) - f(L_J + inc)} \qquad (2.36)$$

The term to maturity must not be shortened for $J \geq M - m_0$. The definition of $f(L_J)$ is thus changed to

$$f(L_J) = \begin{cases} YD_J(j) - YD_J^{max} & : YD_J(j) > YD_J^{max} \\ 0 & : YD_J(j) \leq YD_J^{max} \end{cases} \quad (2.96)$$

whereas the adjustment is calculated, unchanged, by (2.36). If the term to maturity of the loan is fixed, an adjustment to the term to maturity is calculated after all according to the above directions. This is the only way of determining whether the payments on the loan are compatible with the fixed term to maturity.

Step 15 - Are the payments on the loan within the interval from YD_J^{min} to YD_J^{max}?

The calculated debtor payments on the loan are to be compared with the limits for the payments on the loan. This is done by testing the mathematical convergence of the adjustment to the term to maturity given by

$$\frac{|\Delta L_J|}{\varepsilon |L_J|} < \varepsilon \quad (2.40)$$

$$\left| \frac{f(L_J)^2 - f(L_J + \Delta L_J)^2}{f(L_J)^2} \right| < \varepsilon \quad (2.41)$$

where ΔL_J must fulfil merely one of the conditions for the term to maturity having converged.

Step 16 - The loan has been calculated!

If the term to maturity has converged in step 15, all variables on the debtor and funding sides of the loan have been calculated. Step 16 presupposes that a solution has been found which fulfils the limits for payments on the loan as well as term to maturity. Thus, the payments on the financial instrument are 0 (zero).

Step 17 - Is the corrected term to maturity within the interval from L^{min} to L^{max}?

Step 17 determines whether payments on the financial instrument are necessary in order that the limits for payments on the loan as well as term to maturity can be observed.

In step 15 it was concluded that the term to maturity has not yet converged. If the term to maturity corrected by the adjustment calculated in step 14 is within the allowed limits, the adjustment is applied, and the debtor and funding sides of the loan are recalculated in the simultaneous model in which the financial instrument has the payment 0 (zero) by definition.

If a correction of the term to maturity by the calculated adjustment will cause the term to maturity to exceed the allowed value, the model continues in the recursive structure instead.

Step 18 - Correct the term to maturity by an adjustment

The calculated adjustment may be applied, and steps 2 to 17 of the model are repeated with the corrected term to maturity.

Step 19 - Determine the interest rate on the loan such that the payments on the loan are within the interval

Step 19 initiates the recursive part of the model in which the financial instrument is active. The interest rate on the loan must be determined such that the payments on the loan correspond to the relevant limit. That is to say that the interest rate on the loan is determined by a numerical method, so that the relevant condition of the following two conditions is fulfilled.

$$\frac{R_J^K RG_J(0)}{1-(1+R_J^K)^{L^{max}-\varphi}} = YD_J^{max} \quad \text{for} \quad YD_J(j)|_{L^{max}} > YD_J^{max} \tag{2.42}$$

(2.43) $$\frac{R_J^K RG_J(0)}{1-(1+R_J^K)^{L^{min}-\varphi}} = YD_J^{min} \quad \text{for} \quad YD_J(j)\big|_{L^{min}} < YD_J^{min}$$

It should be noted that throughout the recursive model, TILT has the value 0 (zero), as the loan is presumed to be within the limits for payments on the loan and term to maturity at the disbursement. Similarly, REG_J and REG_J^D may also be omitted.

Step 20 - Define a trend function

As in the simultaneous model, the basis of the determination of the bond volumes is a trend function estimating the intended interest rate adjustment profile. First, the trend function is defined, unchanged, as

(2.60) $$a_0 + a_1(j-1)^2 + \ldots + a_{q-1}(j-1)^{q-1}$$

for j=1,2,...,m and $q \leq m$

Step 21 - Determine the coefficients in the trend function

$(a_0, a_1, \ldots, a_{q-1})$ must be estimated on the basis of the intended interest rate adjustment profile which may be calculated on the basis of the recalculated interest rate on the loan. The coefficients are found by the matrix equation

(2.97) $$(a_0, a_1, \ldots, a_{q-1}) = [\mathbf{B}^T\mathbf{B}]^{-1}\mathbf{B}^T \max\left[\frac{RG_J(j-1)}{m_0}, H_J(0,j)\right]$$

(2.97) has been changed as compared to (2.63), as REG^D may be omitted since $REG_J^D = 1$ by definition. **B** is, unchanged, given by (2.64).

Step 22 - Determine G_0 such that the interest rate adjustment condition is fulfilled

In the following step in the model, the proceeds condition is to be applied to determine the payments on the financial instrument. Therefore, the trend function is only adjusted to

the interest rate adjustment condition, for which reason only one factor G_0 may be determined.

Alternatively, G_0 could be maintained, whereas G_1 is used as an adjustment variable. G_1 determines the slope of the trend function, which has a greater impact on the long-term bonds in the portfolio than on the very short-term bond which is decisive for the interest rate adjustment condition. Thus, by adjusting G_1 to the condition, there is a risk of large imbalances occurring in the issue of long-term bonds.

The trend function is divided into a G_0 variant part and a G_0 invariant part given by

$$X_J(j) = G_0 a_0 \text{ and}$$

$$Y_J(j) = G_1 a_1 (j-1) + a_2 (j-1)^2 + \ldots + a_{q-1} (j-1)^{q-1} \tag{2.98}$$

for j=1,2,...,m. G_1 is maintained in the invariant part. By numerical analysis, a fixed value for G_1 may be determined providing the trend function with an appropriate slope. It is not given in advance that G_1 is to be assigned the same value as in step 7. Firstly, the trend function is to be adjusted to another payment profile and secondly, there will be a difference in the value of G_0, depending on whether said value is determined on the basis of the interest rate adjustment condition as is the case here, or the proceeds condition as in step 7.

The marginal issue is given by

$$M_J(j) = \max(0; G_0 a_0 + G_1 a_1 (j-1) + a_2 (j-1)^2 + \ldots + a_{q-1} (j-1)^{q-1} - H_J(0, j)) \tag{2.99}$$

which by use of (2.98) may be formulated as

$$M_J(j) = I_J(j) X_J(j) + I_J(j)[Y_J(j) - H_J(0, j)] \tag{2.100}$$

where $I_J(j)$ is the indicator function as defined in step 7. The interest rate adjustment condition is given by

$$YD_J(1)+\frac{RG_J(0)}{m_0}=H_J(1)+\sum_{j=1}^{m}R_J^N(j)H_J(j) \qquad (2.102)$$

The interest rate adjustment condition is inserted in (2.100), which produces the expression

$$\begin{aligned} &I_J(1)X_J(1)+\sum_{j=1}^{m}R_J^N(j)I_J(j)X(j)=YD_J(1)+\frac{RG_J(0)}{m_0}-H_J(0,1) \\ &-I_J(1)[Y_J(1)-H_J(0,1)]-\sum_{j=1}^{m}R_J^N(j)[I_J(j)(Y_J(j)-H_J(0,j))+H_J(0,j)] \end{aligned} \qquad (2.103)$$

$X_J(j)=G_0a_0$ is inserted in the expression

$$I_J(1)G_0a_0+\sum_{j=1}^{m}R_J^N(j)I_J(j)G_0a_0=Z^{**} \qquad (2.104)$$

where Z^{**} is the right-hand side of (2.103). Solved for G_0, (2.104) appears as follows

$$G_0=\frac{Z^{**}}{I_J(1)a_0+\sum_{j=1}^{m}R_J^N(j)I_J(j)a_0} \qquad (2.105)$$

The determination of G_0 permits the calculation of the distribution on the individual bonds in step 23.

Step 23 - Determine the volumes

The marginal issue is given by

$$\begin{aligned} M_J(j)&=G_0a_0+G_1a_1(j-1)+a_2(j-1)^2+\ldots \\ &+a_{q-1}(j-1)^{q-1}-H_J(0,j) \end{aligned} \qquad (2.90)$$

Step 24 - Are all volumes positive

(2.90) paves the way for negative marginal volumes for one or more bonds. If

$$\exists j \in \{1, 2, \ldots, m\} : M_J(j) < 0 \qquad (2.92)$$

the trend function is adjusted in step 25. Otherwise, the model proceeds to step 26.

Step 25 - Adjust the indicator function

The indicator function is adjusted according to the following directions

$$I_J(j) = \begin{cases} 0 & \forall j : M_J(j) < 0 \\ 1 & \forall j : M_J(j) \geq 0 \end{cases} \qquad (2.93)$$

Step 26 - Calculate the proceeds condition

The proceeds of the determined marginal issue of bonds may be computed to

$$\sum_{j=1}^{m} K_J(j) M_J(j) \qquad (2.106)$$

However, in relation to the funding demand, one may record either a loss in proceeds if the maximum limits for payments on the loan and term to maturity were incompatible, or a gain in proceeds in the opposite situation in which the minimum limits are incompatible.

The loss or gain in proceeds defines the payments on the financial instrument which is thus given by

$$F_J = Fin_J(0) - \sum_{j=1}^{m} K_J(j) M_J(j) \qquad (2.48)$$

$Fin_J(0)$ is given by (2.49).

Step 27 - The loan and the instrument have been calculated!

All variables on the debtor side as well as the funding side of the loan have been determined for the interest rate adjustment period in question.

A variant of the model steps 5 to 10 is described in appendix B. In terms of formulae, the variant is easier, but it is more difficult to calculate. This is because in the model, G_0 and G_1 are determined by an iterative procedure which, on the one hand, stipulates smaller requirements as to formulae but which, on the other hand, requires more calculation time.

2.7 Summary

A LAIR III introduces a band for the debtor's payments on the loan. The band is defined by a maximum limit and a minimum limit which are determined according to the choice of the debtor.

To begin with, the limits for the payments on the loan are defended by the adjustable term to maturity. If the payments on the loan are floating outside the band, the term to maturity is corrected. The corrections of the term to maturity are determined such that the payments on the loan only just moves within the band . The term to maturity is thus adjusted as little as possible.

If the term to maturity reaches either its maximum or minimum value, the possibilities of adjusting the term to maturity are exhausted. If the interest rate further rises or falls, respectively, the limits for the payments on the loan are secured via the financial instrument.

The payments from the financial instrument are thereby conditional on the maximum limits for payments on the loan and term to maturity having both been reached. Similarly, payments to the instrument will occur only if the minimum limits for payments on the loan and term to maturity are incompatible.

Thus, the adjustable term to maturity functions as a buffer, which reduces the price of the financial instrument.

The method described implies that the payments on the financial instrument are determined either as the gain or loss in proceeds at the beginning of every interest rate adjustment period. The loss or gain in proceeds is produced by the model funding, via a bond issue, a payment on the loan within the band. If the limits are incompatible, the proceeds of the bond issue does not correspond to the funding demand. The residue defines the payments on the financial instrument.

The residual calculation of the payments on the financial instrument implies that the model will fulfil by definition the balance principle of the Danish Mortgage Credit Act without imbalances. The volume of the product is not thereby limited by the provisions in the law.

In the section a model is set up for calculating the debtor side as well as the funding side of the loan. If the limits are compatible, the model has a simultaneous structure, all variables being mutually dependent in the model. However, if the limits are incompatible, the model changes to a recursive structure, cf. the residual calculation of the payments on the financial instrument.

A special situation arises if the model calculates negative bond volumes. Negative bond volumes are not accepted, as the debtor is to buy bonds in this situation. The situation is dealt with in different ways, depending on the interest rate adjustment pattern of the loan. For type P negative bond volumes may imply that the intended interest rate adjustment is not respected.

3. The pricing of the financial instrument

3.0 Introduction

In this section the financial instrument is to be priced. The price may be both positive, 0 (zero) and negative. Thus, the economic interpretation of prices is applied. In the preceding sections it was described how the yield curve is modelled, then how the payments on the financial instrument are found as a function of the interest rate. With these conditions in order, the final pricing of the instrument is possible.

The price of the financial instrument is calculated as the present value in the lattice of the payments on the instrument. By pricing the instrument as the present value of the payments, two significant assumptions are implicitly made.

- Firstly, it is implicitly assumed that the market for the financial instrument is characterized by no arbitrage.

- Secondly, it is assumed that the market discounts the payments on the instrument in the same way as does the vendor of the instrument.

If the above conditions are not fulfilled, the market will either calculate another present value or price the instrument with a deviation in relation to the present value. In both cases the price of the market will deviate from the price predicted by the model set up.

Therefore, the price that may be calculated in the model is to be considered primarily as a theoretical price. In practice, especially factors such as credit risk and liquidity premiums are to influence the price mechanism of the instrument, for which reason the theoretical price must be corrected before it is applied for practical purposes.

The correction is difficult to model as it rests on factors that are not dependent on the interest rate. To a great extent the correction should also be based on observations of credit and liquidity premiums in the market. In the following, the attention is thus focused on the determination of the theoretical price.

Section 3.1 describes a method for calculating the theoretical price of the instrument in the lattice. The method applies a backward induction principle which is immediately easier to handle than is the forward induction principle previously described.

There are obvious advantages connected to the instrument having the price 0 (zero) at the disbursement, said advantages being described in greater detail in section 3.2. This means that the limits are to be quoted as it is known e.g. from swap interest rates, etc. In section 3.3 a model for quoting the limits is set up.

The price of the instrument is not only relevant at the disbursement of the loan. In the case of a prepayment of the loan, a price of the instrument is also to be computed in the calculation of the prepayment amount. The computation of the prepayment amount is discussed in section 3.4.

3.1 Method for pricing the financial instrument

The method for pricing the financial instrument is that all payments on the instrument in the interest rate lattice are discounted to time 0 (zero), cf. the introduction.

In each node the payment on the financial instrument is determined by the model which was set up in section 2. If the node coincides with an adjustment of the interest rate on the loan, the payments on the instrument depend on the mutual

compatibility of the limits under the given yield curve in the node.

If the maximum limits for payments on the loan and term to maturity are mutually incompatible, the payments on the instrument are positive seen from the point of view of the debtor. If both the maximum and minimum limits are fulfilled, there are no payments on the instrument, whereas minimum limits in mutual conflict trigger negative payments on the instrument. The possible pattern in the signs of the payments is shown in figure 11. The figure is simplified, the underlying yield curve being assumed to be flat, causing the lattice to be symmetrical around the initial interest rate. It is further assumed that the limits are symmetrical. Finally, the step size in the lattice corresponds to an interest rate adjustment period.

At all other points in time at which the nodes do not coincide with an adjustment of the interest rate, the payments on the instrument is by definition 0 (zero).

The adjustable term to maturity means that, in principle, the size of the lattice is not known at the disbursement of the loan. In the lower section of the lattice, the term to maturity of the loan - and thus the term to maturity of the financial instrument - will be relatively close to L^{min}, whereas the term to maturity in the upper section of the lattice will be relatively close to L^{max}. That the instrument has matured means, in practice, that there are no payments on the instrument. It is therefore not necessary to allow for the date of maturity in the pricing, the payments of 0 (zero) not influencing the pricing on the assumption of no arbitrage. Thus, the lattice must be dimensioned such that all h contain the maximum term to maturity of the loan.

In the construction of the interest rate lattice, a forward induction principle is applied. This is due to the interest rate being forward path dependent, corresponding to the interest rate tomorrow depending on the interest rate today. Similarly, the projection of input to the model in section 2 is performed by forward induction, the input being known at time 0 (zero) and being calculable successively for g=1, g=2, etc.

This method cannot be applied here. Since it is the price at time 0 (zero) which is to be determined, a method based thereon is inapplicable.

By contrast, a method will be used in the following which is based on a backward induction principle. By following a backward induction principle, the method is based on the end nodes of the lattice, i.e. on the nodes in which g assumes its maximum value. There are no future payments on the financial instrument in the end nodes. Under no arbitrage, the theoretical price of the financial instrument must correspond to the payments in the node, as there are no future payments on the instrument.

In the immediately preceding nodes, the value may then be found as the discounted expected value in the next period plus payments in the node. The expected value is calculated by applying the probabilities of the different branching structures, and the interest rate in the node is used in the discounting. Thus, the pricing will be in accordance with the assumption of no arbitrage. An expression of the price may then be deduced successively for the preceding period, etc. until the price in (0,0) has been determined.

The backward induction principle may thus be interpreted as the value of the instrument today depending on the value tomorrow.

The pricing by use of the backward induction principle may be formalized. First, the payments on the financial instrument are to be defined. In the model in section 2, said payments on the financial instrument were determined as

(2.48) $$F_J = Fin_J(0) - \sum_{j=1}^{m} K_J(j) M_J(j)$$

In the following, the notation is changed, the payments having to be related to a node in the lattice. Furthermore, the payments on the financial instrument may at an advantage be seen as a function of the initial term to maturity of the loan, the limits for payments on the loan and term to maturity. Thus, the notation is changed to

(3.1) $$F_{(g,h)}(L_0, YD_J^{min}, YD_J^{max}, L^{min}, L^{max}) \equiv F_J$$

The basis of the pricing is the end nodes for $g=g^{max}$. If $P_{(g,h)}$ denotes the theoretical price of the instrument in the node (g,h), it applies in the end nodes that

(3.2) $$P_{(g^{max},h)} = F_{(g^{max},h)}(L_0, YD_J^{min}, YD_J^{max}, L^{min}, L^{max})$$

For $g=g^{max}-1$, i.e. in the immediately preceding nodes, the expected discounted value thereof is given by

(3.3) $$(P_o P_{(g+1,h+1+k)} + P_m P_{(g+1,h+k)} + P_n P_{(g+1,h-1+k)}) e^{-r\Delta t}$$

r is the Δt period interest rate, causing $e^{-r\Delta t}$ to form the discounting factor from time $t+\Delta t$ to time t. P_o, P_m and P_n denote the probability of an upwards, middle, or downwards branching. The parameter k is seen to be included in the expression. From section 4 it is remembered that k shifts the branching upwards or downwards. The parameter thus determines the nodes over which the expected price in the next period is to be calculated.

The pricing according to the backward induction principle is illustrated in figure 12.

The backward induction implies that in each node only one expected value of three nodes is to be calculated in the next period. In the trinomial lattice, there are always three possible branching structures from each node, whereas one node may be reached from a varying number of nodes. This further implies that in the backward induction argument it is adequate to operate with the simple P probabilities rather than the more complex q probabilities which were deduced in section 2.3.

The price of the instrument in (g,h) further depends on the payment in nodes and is thus given by

$$P_{(g,h)} = F_{(g,h)}(L_0, YD_J^{min}, YD_J^{max}, L^{min}, L^{max}) + (P_o P_{(g+1,h+1+k)} + P_m P_{(g+1,h+k)} + P_n P_{(g+1,h-1+k)})e^{-r\Delta t} \quad (3.4)$$

The expression (3.3) applies in general. Provided that $P_{(g+1,h)}$ has been determined for all h, $P_{(g,h)}$ may be found for all h by means of (3.3). Thus, (3.3) is to be applied successively for

$$g = g^{max}-1,\ g^{max}-2, \ldots, 0$$

until a theoretical price of the financial instrument has been calculated in all nodes.

(3.4) is seen not to depend on the step size in the lattice Δt which is thus only bound by Δt not being allowed to exceed the length of the interest rate adjustment period of the loan.

A low value of Δt will increase the accuracy of the pricing. By assigning a low value to Δt, the value of Δt falls

simultaneously, cf. section 4.2.3. (Δt, Δr) will thereby span a tighter lattice in which the distance between each node is very small. The discrete distribution of the interest rates for each t will thus approach the continuous distribution.

On the other hand, a low value of Δt reduces the calculation rate of the model. Δt does not directly affect the number of adjustments of the interest rate to be calculated by the model, but via Δr a shorter step size will indirectly increase the number of calculations significantly. The determination of Δt is thereby a trade off between, on the one hand, the accuracy in the pricing and, on the other hand, the calculation rate of the model.

3.2 Quoting the limits in the model

In principle, the limits are to be selected by the debtor and are thus seen as exogenous in the model. Depending on the selection, a price of the financial instrument may be fixed by the method described above. The debtor must accept this price as a result of the selected limits.

However, there are several good arguments in favour of applying a combination of maximum and minimum limits ensuring a price of 0 (zero) on the financial instrument at the disbursement of the loan.

Firstly, one cannot preclude the possibility that a funding of an optionally positive price could not be performed by the issue of mortgage credit bonds. The rules of determining the volume of loans and the underlying bond issue are presently being revised, and it cannot be precluded that the result will be a close linking between the lending limits and the bond issue. In that case, it will not be possible to equalize a positive price of the instrument by selling a large volume of

bonds. Thus, the debtor will not be able to have the same proceeds as in another type of mortgage loans.

Secondly, there will be a risk that the debtor fixes the limits at unrealistic levels and does not, therefore, benefit fully from the product unless the limits are quoted.

Thirdly, an initial price of 0 (zero) binds the future prepayment costs to a certain extent. This argument is elaborated on in section 3.4.

A possible procedure for determining the limits was outlined in section 2.1.2. The procedure will here briefly be repeated.

- The initial term to maturity is determined by the debtor.
- The maximum limit for the term to maturity is determined as the legislative or credit policy maximum.
- The minimum limit for the term to maturity is determined as the initial term to maturity of the loan.
- The maximum limit for the term to maturity is determined such that the price of the instrument is 0 (zero).

The procedure means that the maximum limit is determined at the lowest level possible given the initial term to maturity of the loan. By following the procedure, the maximum limit is a function of the initial term to maturity. Thus, there is a trade off between, on the one hand, the debtor's payment here and now and, on the other hand, the risk of future increases in the payments on the loan.

3.3 Model for quoting the maximum limit for the payments on the loan

The quoting of the maximum limit for the payments on the loan is impeded by the complex connection between the limits for the payments on the loan and the payments on the financial instrument.

Thus, the path from the limits to the payments on the financial instrument goes via the modelling of the debtor and funding sides in section 2 and the pricing as described above. Hence, a simple functional connection between the limits and the payments on the instrument for use in the quoting cannot be deduced.

The complex connection means that an iterative procedure is to be included in the quoting. The iteration may be outlined as follows.

For given values of the other outputs, a maximum limit for the payments on the loan can be guessed. On the basis of the guess, a price of the instrument is calculated. Then an adjustment to the maximum limit is determined on the basis of the deviation of the price from 0 (zero). If the adjustment has converged, the iteration may be ended - otherwise the iteration continues with a corrected maximum limit for the payments on the loan. The structure of the iteration may be described by the flow chart in figure 13.

The steps of the model are described in the following.

Step 1 - Determine the initial maximum limit for the payments on the loan

The iteration over the maximum limit is initiated by an initial value for YD_J^{max}. Each iteration involves a pricing of the financial instrument. Thus, great importance is to be attached to the initial value being determined such that the number of iterations are limited, as the pricing is difficult to calculate, cf. the preceding section.

The quoting of the maximum limit for the payments on the loan depends on the initial term to maturity and the maximum limit for the term to maturity, if the other limits are determined in accordance with the directions in section 3.2. Further, the yield curve will also affect the quoting.

The plurality of factors combined with the complex connections in the model makes it difficult to deduce a simple approximative connection to be applied in all situations. In substitution a connection may be estimated by an empirical method on the basis of pricing operations.

Step 2 - Calculate the theoretical price

The theoretical price is calculated for the determined value of YD_J^{max} In each node in the lattice, the payments on the instrument are calculated by the model in section 2. The payments may be inserted in (3.4), and the price of the instrument may be calculated.

Step 3 - Calculate adjustment to the maximum limit for the payments on the loan

In step 3 an adjustment to is calculated such that the price is approached to 0 (zero). The calculation follows the Gauss-Newton algorithm. The first step in the calculation of an adjustment is to define the function

$$f(YD_J^{max}) = P_{(0,0)} - 0 \qquad (3.5)$$

measuring the deviation of the price from 0 (zero). ΔYD_J^{max} is found by the reduced Gauss-Newton expression

$$\Delta YD_J^{max} = f(YD_J^{max}) \frac{inc}{f(YD_J^{max}) - f(YD_J^{max} + inc)} \qquad (3.6)$$

inc denotes the step size in the iteration. Thus, if the iteration is to converge by a reasonable velocity, inc must not be valued very low in relation to YD_J^{max}. Conversely, a

value of inc which is too large could mean that the iteration becomes divergent. In the model inc is determined as

(3.7) $inc=0,0001RG_0(0)$

i.e. as 0,1 per milles of the volume of the loan.

Step 4 – Has the maximum limit for the payments on the loan converged

Step 4 determines whether the iteration has converged and the price is sufficiently close to 0 (zero), or whether the iterative process is to be continued. The question is determined by the mathematical convergence of the adjustment. One of the following two conditions must be fulfilled in order for the iteration to have converged.

(3.9) $$\frac{\Delta YD_J^{max}}{\varepsilon|YD_J^{max}|}<\varepsilon$$

(3.10) $$\left|\frac{f(YD_J^{max})^2-f(YD_J^{max}+\Delta YD_J^{max})^2}{f(YD_J^{max})^2}\right|<\varepsilon$$

Step 5 – Correct the maximum limit by an adjustment

If the adjustment to YD_J^{max} has not converged, the adjustment is applied. In step 3, a new price of the financial instrument is calculated for the correcting maximum limit

$$YD_J^{max}+\Delta YD_J^{max}$$

Step 6 – The maximum limit has been quoted

If the adjustment to the maximum limit has converged and the price is thus sufficiently close to 0 (zero), the calculations of the model have been completed. The maximum limit has thus been quoted.

A maximum limit for the payments on the loan is quoted in the model, whereas the notation invites the limit to vary from interest rate adjustment to interest rate adjustment via the

indexation J. Thereby, the model aims primarily at the situation in which

$$YD_0^{max} = YD_1^{max} = \ldots = YD_M^{max}$$

which must also be considered the most interesting situation. It will not be possible to quote the limits independently of each other. Thus, if the limits are determined independently, there will be an infinite number of solutions to the quoting problem. The number of solutions may be reduced to a singleton by determining the limits independently of each other such that if YD_0^{max} is well defined, $\forall J > 0 : YD_J^{max}$ is also well defined. The quoting problem is thereby determined, and it will be possible to quote the limits by the method described above.

It must be expected that the model will always be able to find a maximum limit for the payments on the loan, which ensures a theoretical price of 0 (zero) of the financial instrument.

Firstly, the bond issue is a strictly rising function of the maximum limit for the payments on the loan provided that the limits for payments on the loan and term to maturity are incompatible. This connection follows from the balance condition.

Secondly, the payments on the financial instrument is a strictly declining function of the bond issue and thus of the maximum limit for the payments on the loan. This follows from the proceeds condition.

Finally and thirdly, it follows from (3.3) that the price of the financial instrument is a strictly rising function of the payments on the financial instrument. The theoretical price is thus a strictly declining function of the maximum limit for the payments on the loan.

3.4 The debtor's prepayment costs

Normally, the bonds underlying a LAIR are non-callable. Therefore, the debtor cannot prepay his LAIR at par. However, a LAIR may be prepaid by handing in the underlying volume of bonds, or in cash, the lending institution calculating the prepayment costs as the market price of the underlying bonds. However, it is possible to prepay interest rate adjustment amounts at par in cash.

That the bonds are non-callable means that the market price of the underlying bonds is not bound by a maximum limit. However, the short term to maturity and resulting short duration of the bonds reduce the market price sensitivity of the remaining debt, for which reason the prepayment amount typically fluctuates less than in the case of traditional types of loans. Hence, the debtor's interest rate risk is limited.

When prepaying a LAIR III, the underlying bonds as well as the financial instrument must be prepaid if imbalances in the payments are to be avoided. Thus, the price of the financial instrument at the date of prepayment will be included in the computation of the debtor's prepayment costs together with the current market price of the underlying bonds. Normally, this precludes other forms of prepayment than prepayment in cash.

If the interest rate has risen to a level in the upper section of the lattice, the payments on the instrument will be positive seen from the point of view of the debtor. The debtor's position in put options is thereby in the money, and the debtor will thus be able to obtain a positive price of the instrument, the opposite party avoiding future payments.

If, by contrast, the interest rate is in the lower section of the lattice, prepayment of the financial instrument will be connected with costs on the part of the debtor. In this situation, the debtor's short position in call options will

thereby be in the money and the payments on the instrument will be negative, the opposite party selling future payments.

Thus, the price of the financial instrument will follow the same pattern as the market price of the underlying bonds. Thereby, the fluctuations in the debtor's prepayment costs increase with the introduction of the financial instrument.

Ensuring that the debtor's payments on the loan are within a band may be interpreted as a fixing of the interest rate during the term to maturity of the loan, which increases the duration of the loan. Thus, the fluctuations in the prepayment amount will also increase.

The quoting of the limits at the disbursement of the loan such that the initial price of the financial instrument is 0 (zero) will, to a certain extent, limit the fluctuations. Thus, the interest rate is to move away from the initial level in order for the instrument to have a positive or a negative price.

An obvious possibility is that a maximum limit for the price of the instrument in connection with a prepayment is built into the instrument. The maximum limit could optionally be combined with a minimum limit in order to reduce the costs related to the maximum limit.

However, it will be difficult to price this facility. In this situation, the price should reflect the conversion behaviour of the debtor which must be modelled, if need be, on the basis of the observed prepayment rates under different yield curves. However, the experience in this area is very limited, LAIR having existed as a product only since October 1996. At the same time, the yield curve in this period has been very stable.

A pricing of a financial instrument with a maximum limit for the prepayment costs as a facility could thus be arbitrary.

3.5 Summary

The theoretical price of the financial instrument is determined as the discounted value of the future payments at time 0 (zero). The discounting is performed in the interest rate lattice, causing the price to be in accordance with the observed yield curve. The theoretical price is thereby no arbitrage.

The theoretical price may be seen as a function of the mutual relationship between the limits of the loan, the term to maturity of the loan at the disbursement, and the yield curve. Therefore, it is possible to influence the pricing via the determination of the limits.

There are good arguments in favour of the instrument obtaining a price 0 (zero) at the disbursement via the determination of the limits. Firstly, the debtor is thereby secured the full proceeds of the loan and secondly, the limits will be determined at a sensible level. Hence, the limits are quoted as it is known from swap interest rates, etc.

The price of 0 (zero) at the disbursement does not imply that the financial instrument will have the price 0 (zero) at all future points in time. In the upper section of the lattice, the financial instrument will have a positive price seen from the point of view of the debtor, whereas the price in the lower section of the lattice will be negative. If the debtor wishes to prepay his LAIR, the prepayment amount will further fall if the interest rate has risen, and vice versa. This is a consequence of the fact that securing the debtor's future payments on the loan increases the duration of the loan.

Appendix A Type F^+

A.0 Introduction

It will not be always possible to calculate a number of positive bond volumes fulfilling the proceeds and interest rate adjustment conditions of a LAIR type F.

If the last maturing bond underlying a LAIR type f has a price over 100, the nominal issue is less than the volume of the loan on the debtor side. Thus, the debtor obtains a capital gain which affects the balance between the payments in the last year in the interest rate adjustment period when the remaining debt of the loan is to be interest rate adjusted simultaneous with the bonds maturing at price 100.

If the difference in the volume of the nominal issue and the volume of the loan is larger than the debtor's total repayments up to the adjustment of the interest rate, the balance condition cannot immediately be fulfilled, the interest rate on the loan being unchanged. In this situation, the payments of the bond side are larger than the payments of the debtor side.

One possibility is that the interest rate on the loan is determined in accordance with the balance condition in the last year of the interest rate adjustment period, but the interest rate on the loan will then have to float within each interest rate adjustment period, making the loan an interest rate shift loan. Pursuant to the present provisions of the Danish Gains on Securities and Foreign Currency Act, interest rate shift loans are treated asymmetrically for fiscal purposes. This road has thus been barred.

Another possibility is that the loan is funded solely in the bond with maturity in year m simultaneous with the introduction of a so-called minimum refinancing.

The idea of the minimum refinancing is to transfer payments from the previous years to the last year of the interest rate adjustment period in which a surplus in the payments on the debtor side was immediately established. This is carried out by a deficit in the debtor payments on the loan being funded by a new issue in the bond with maturity in year m such that the surplus is gradually reduced. If the bond is closed, e.g. as a result of an adjusted minimum interest rate, the issue is carried out in a new bond with maturity in year m.

A special problem is connected to the determination of the interest rate on the loan. In connection with the minimum interest rate adjustment, the interest rate on the loan is recalculated year by year, i.e. also within each interest rate adjustment period.

The minimum refinancing is calculated each year as the residue between the payments of the debtor and funding sides. Therefore, the interest rate on the loan is only bound in the last year of the interest rate adjustment period. In the remaining years, an arbitrary determination of the interest rate will merely change the minimum refinancing. However, the consequence of an unfortunate determination of the interest rate is a drastic adjustment of the interest rate in the last year in order to fulfil the balance condition. One objective is, therefore, to select a method for determining the interest rate on the loan, which results in a stable development. The specified method is thus one appropriate method among others. The interest rate could e.g. be determined as the bond yield, etc.

The recalculation opens up the possibility of the payment breaking during the interest rate adjustment period, which does not trigger payments from the financial instrument, however, unless the term to maturity of the loan is incompatible with the limits for the payments on the loan. However, the fluctuations in the interest rate are typically limited, as the minimum refinancing is indeed minimal.

It is necessary to distinguish between two situations in the model for type F^*.

If the model is called from step 10 of the F model, the financial instrument is inactive. In that situation, the proceeds condition may be applied, in a relatively simple manner, to the determination of the volume of this one bond in which the loan is funded. Thus, the model has a recursive structure based on the proceeds condition.

If, by contrast, the model is called from step 19 in the model for type F, the proceeds condition is to be applied to the determination of the payments on the financial instrument which is now active. Therefore, the structure of the F^* model must be altered such that the balance condition is applied in the determination of the bond volume. Again, the structure of the model will, however, be recursive.

In the following description, a distinction is made between two situations.

A.1 The type F^* model for compatible limits

First, a model is set up for calculating the debtor and funding sides of the loan in a situation in which the limits of the loan are compatible, and the financial instrument thus inactive. Hence, the starting point is step 10 of the F model.

The flow chart of the model is shown in figure 14.

It appears that the F* model returns to step 11 of the F model when the bond volume, the interest rate and payments on the loan have been calculated. Thus, the F* model is covered by the iteration over the term to maturity in the outer loop in the F model. Therefore, the term to maturity may be seen as given in the F* model.

The steps of the model must be applied both at the ordinary adjustment of the interest rate and at the minimum refinancing. A distinction between the two situations is made in the individual steps.

Step 10 of the F model
All information concerning the loan, including the term to maturity, is input from step 10 of the F model.

Step 1 - Calculate the volume by means of the proceeds condition
The volume of the mth bond is determined at the date of disbursement of the loan, or immediately following an ordinary adjustment of the interest rate as

(A.1) $$H_J(m) = \frac{RG_J(0)}{K_J(m)} \quad \text{and} \quad H_J(j) = 0 \text{ for } j < m$$

The volume of m is taken over from the F model. This does not affect m, as the issue is made in only one bond. In the remaining years until the ordinary adjustment of the interest rate, the minimum refinancing is given by

(A.2) $$M_J^j(m) = \frac{Fin_J(j)}{K_J^j(m)} \quad \text{for } j=1,2,\ldots,m-1$$

such that the proceeds condition is fulfilled. The top sign j indexes the minimum refinancing. j runs until m-1, a minimum adjustment of the interest rate not being performed in

connection with the ordinary adjustment of the interest rate at the end of the period.

It should be noted that j gets a slightly different meaning, j being set to zero only in connection with the ordinary adjustments of the interest rate. Thus, bonds are issued at time j=0,1,2,.., m-1 in connection with the minimum refinancing, and not just at time j=0 at the ordinary adjustments of the interest rates.

The funding demand in (A.2) is given by

(A.3) $$Fin_J(j) = R_J^N(m)H_J(j,m) - YD_J(j) \text{ for } 1 \le j \le m-1$$

where $H_J(j,m)$ is the bond volume already issued at time j.

Step 2 - Calculate the interest rate on the loan by means of the balance condition

The interest rate on the loan must be determined on the basis of a condition for a global balance between the total payments of the debtor side over all m years and the payments of the funding side in the same period. Hence, the interest rate on the loan must fulfil

(A.4) $$\sum_{i=j+1}^{m} YD_J(i) + RG_J(m) = (m-j)R_J^N(m)\left(M_J^j(m) + H_J(j,m)\right)$$

$M_J^j(m)+H_J(j,m)$ constitutes the total bond volume issued in the mth year following the minimum adjustment of the interest rate. The interest rate on the loan is included in the payments on the loan as well as in the remaining debt and cannot immediately be isolated. Therefore, the interest rate is found by a numerical method. It should be noted that (A.4) does not allow for the future minimum refinancing. It is the very idea that the interest rate is determined at a level which is too low such that a deficit occurs, and hence a minimum refinancing.

Step 3 - Calculate the debtor payments on the loan
The determined interest rate determines an unambiguous profile of the debtor's payments on the loan and remaining debt which are calculated in this step.

Step 4 - Shift to step 11 of the F model
All variables on the debtor and funding sides are determined, for which reason the model may return to step 11 of the F model in which an adjustment to the term to maturity of the loan is calculated.

A.2 The type F* model for incompatible limits
If the limits for payments on the loan and term to maturity, respectively, are incompatible, the financial instrument is activated.

The minimum adjustment of the interest rate means that several bonds are issued during the interest rate adjustment period, causing the payments on the loan to increase until the last year of the interest rate adjustment period in which the capital gain is realized and the payments on the loan decrease.

The rising payments on the loan constitute a problem in the situation in which the maximum limit for the payments on the loan is binding. Thus, it will not be possible to determine a payment on the loan which constantly corresponds to the maximum limit. Exceeding the limits cannot be built into the model at an advantage either. Indeed, the basic idea of the minimum refinancing is to transfer payments to the interest rate adjustment year. At the same time, an expansion of the financial instrument to include these payments as well makes the characteristics of the instrument difficult to comprehend. The frequency of payments on the instrument will thus vary - not just in line with whether the limits may be fulfilled -

but also in line with the model shifting between F and F*. Therefore, exceeding the limits during the interest rate adjustment period must be accepted. At the same time, the extent to which the limits are exceeded will be limited.

In connection with the minimum refinancing, payments on the financial instrument are thus not to be calculated, causing the structure for j>0 to be the same as with compatible limits. The model is shown in figure16.

Step 1 - Calculate the interest rate on the loan such that the payments on the loan are within the interval

The interest rate on the loan is to be determined such that the payments on the loan up to the first minimum adjustment of the interest rate correspond to the binding limit. The interest rate on the loan must fulfil the relevant one of the conditions

(A.5) $$\frac{R_J^K RG_J(0)}{1-(1+R_J^K)^{L^{max}-\varphi}} = YD_J^{max} \quad \text{for} \quad YD_J(0)\big|_{L^{max}} > YD_J^{max}$$

(A.6) $$\frac{R_J^K RG_J(0)}{1-(1+R_J^K)^{L^{min}-\varphi}} = YD_J^{min} \quad \text{for} \quad YD_J(0)\big|_{L^{min}} > YD_J^{min}$$

Step 2 - Calculate the volume by means of the balance condition

The volume of the mth bond is determined on the basis of a requirement with respect to balance in the payments of the debtor and funding sides, the payment profile and the remaining debt profile on the debtor side being calculated with an interest rate determined in step 1. The volume must fulfil

(A.7) $$\sum_{j=1}^{m} YD_J(j) + RG_J(m) = mR_J^N(m)H_J^0(m)$$

Step 3 - Calculate the proceeds

On the basis of the volume, the proceeds may be computed, which also defines the payment on the financial instrument. The payment on the financial instrument is given by

(A.8) $$F_J = Fin_J(0) - K_J^0(m)H_J^0(m)$$

The variables of the loan have thereby been calculated for j=0. In step 4, the model continues the calculation of the minimum refinancing for j=1, 2,..., m-1

Step 4 - Calculate the marginal issue by means of a proceeds condition

As there are no payments on the financial instrument in connection with the minimum refinancing, the proceeds condition may be applied to the determination of the marginal issue.. $M_J^j(m)$ must fulfil

(A.2) $$M_J^j(m) = \frac{Fin_J(j)}{K_J^j(m)} \quad \text{for } j=1,2,\ldots,m-1$$

where $Fin_J(j)$ is given by (A.3)

Step 5 - Calculate the interest on the loan by means of the balance condition

The interest rate on the loan is calculated by the expression

(A.4) $$\sum_{i=j+1}^{m} YD_J(i) + RG_J(m) = (m-j)R_J^N(m)\left(M_J^j(m) + H_J(j,m)\right)$$

for j=1,2,...,m-1 which is solved by a numerical method.

Step 6 - Calculate the debtor payments on the loan

Eventually, the payments on the loan on the debtor side may be calculated.

Step 7 - The loan and the financial instrument have been calculated!

When steps 1 to 3 have been applied for j=0 and steps 4 to 6 have been applied for j=1, 2,..., m-1, all variables have been calculated. In this situation, it is not necessary to return to the model for type F.

The implementation of the model in the trinomial lattice with a view to the pricing of the financial instrument is not entirely without problems.

In the lattice, the debtor and funding sides of the loan are calculated in the nodes which coincide with an ordinary adjustment of the interest rate on the loan. The calculation is performed partly to determine payments on the financial instrument and partly to determine the development in the remaining debt and term to maturity of the loan with a view to the calculation of the loan at the next adjustment of the interest rate. This information is projected through the lattice.
If e.g. a LAIR III type F5 is considered, the loan is to be calculated in the lattice at intervals of 5 years. This means that the projection of the relevant information is performed regularly over five years.

However, if the loan changes its characteristics to a LAIR III type F5* because of the yield curve, the debtor and funding sides of the loan must be calculated each year as a result of the minimum refinancing. The minimum refinancing is dependent on the interest rate and should thus be performed in the lattice in accordance with the modelled development in interest rates.

However, a calculation at intervals of 1 year means that in the lattice, projection is performed over periods of both 5 years and 1 year. As far as we know, no methods exist for dealing with this situation.

The problem is most obviously solved by a calculation of the debtor and funding sides being performed at each ordinary adjustment of the interest rate for the whole period until the next ordinary adjustment of the interest rate, as is the case for type F. This requires an assumption concerning the development in the interest rate of the m'th bond.

A yield curve is determined in each node, which yield curve also defines an implicit forward rate. As an approximation of the development in the lattice, it is convenient to apply the implicit forward rate in the determination of $K_J^j(m)$. The approximation should not affect the pricing of the financial instrument, as the minimum refinancing is indeed minimal.

Appendix B The variant of the model for type P

B.0 Introduction

Appendix B contains a description of an alternative modelling of the debtor and funding sides of a LAIR III type P.

The alternative modelling may replace steps 2 to 13 in the model for type P. Thus, it is only the inner loop in the simultaneous model structure (the part of the model in which there are no payments on the financial instrument) which is different in the two variants of the model. The link to other steps in the model is thus the same.

The alternative modelling has several characteristics in common with the model in section 2.

Firstly, the interest rate on the loan is applied as an iteration variable. For a given interest rate on the loan, the debtor payments on the loan and volumes are calculated such that the proceeds condition and the balance condition are fulfilled. Then the convergence of the interest rate determines whether the debtor payments on the loan and volumes are to be recalculated in a new iteration.

Secondly, the volumes are determined by use of a trend function. However, the adjustment of the trend function to the proceeds and balance conditions is performed in an iterative procedure in which the factors G_0 and G_1 are iteration variables.

The iterative procedure for determining G_0 and G_1 facilitates the alternative modelling in terms of formulae. Conversely, the iterative procedure reduces the calculation rate of the model.

B.1 The general structure of the model

The determination of G_0 and G_1 by iteration means that another loop is added to the model.

In the outer loop, iteration is performed over the term to maturity. In the central loop, the model iterates over the interest rate on the loan, and in an inner loop, the model iterates over G_0 and G_1 until the proceeds and interest rate adjustment conditions are observed.

The structure of the model appears from the flow chart in figure 16. The flow chart shows steps only for the part of the model in which iterations over the interest rate on the loan and over G_0 and G_1 are performed, the other steps being, as already mentioned, identical to the model for type P which was described in section 2. (Furthermore, certain of the steps in figure 16 are identical to the steps in the model in section 2. However, these steps are included, as the description would otherwise be difficult to comprehend).

The handling of negative bond volumes is different in the two variants of the model. In this variant of the model, the iteration continues in the inner loop until both the proceeds condition and the balance condition are fulfilled for positive volumes. If a solution is not found, the model will end the iteration after 30 attempts, after which a correction of the volumes is performed such that the proceeds condition is fulfilled. This appears from the flow chart in that the model may leave step k and move directly to step m without checking the conditions in step 1 after 30 iterations.

B.2 The steps of the model

Step a - Determine the initial interest rate on the loan. Determine m

On the basis of the term to maturity which has been determined in the outer loop, the model determines a value for m as

(B.1) $$m=\min[L_J-\varphi; m_0+TILT]$$

L_0 - φ denoting the remaining term to maturity of the loan. TILT specifies that a further bond is to be applied at the disbursement of the loan due to the reduced interest rate adjustment percentage at the first adjustment of the interest rate. At the disbursement of the loan, TILT has the value

(B.2)

$$TILT=\left\{\begin{array}{ll} 1 & \textit{if the loan is disbursed in the period from January to November} \\ 0 & \textit{if the loan is disbursed in December} \end{array}\right\}$$

for J=0. For J>0 TILT=0.

The initial value of the interest rate on the loan is determined as a weighted average of the yield to maturity of the underlying bond portfolio.

(B.3) $$R_J^K=\frac{\sum_{t=1}^{m}\frac{1}{m}\cdot t\cdot r(0,t)}{\sum_{t=1}^{m}\frac{1}{m}\cdot t}$$

Step b -Calculate the debtor payments on the loan

The payment profile and the remaining debt profile on the debtor side of the loan may be calculated as a function of the term to maturity and the interest rate.

Step c - Is m=1, m=2, or m>2?

The model may continue in three different ways.

If m=1, the volume of the 1-year bond and an interest rate on the loan are calculated in a relatively simple manner in step d.

If m=2, the model moves on to step e in which bond volumes and an interest rate on the loan are calculated without the use of

iteration. A LAIR Type P50 will be calculated in this step until the maturity of the loan is only 1 year away. Other types of loans are calculated in step e when the remaining term to maturity is 2 years.

If m>2, the model moves on to step f in which a trend function is defined.

Step e - Determine the volume of m=1

If m=1, the funding may be determined on the basis of the proceeds condition as

(B.3) $$M_J(1) = \frac{Fin_J(0)}{K_J(1)}$$

$Fin_J(0)$ denotes the current funding demand at the time of calculation, which funding demand is defined by the payments in the preceding interest rate adjustment period such that the balance condition is fulfilled.

(B.4)

$$Fin_J(0) = \begin{cases} Volume & disbursement \\ \sum_{j=1}^{m} H_{J-1}(j)R^N_{J-1}(j) + H_{J-1}(1) - YD_{J-1}(1) & adjustment\ on\ the\ interest\ rate \end{cases}$$

Step e presupposes that J=M, for which reason the first part of the definition is irrelevant here but is nevertheless included for later use.

As the loan is not to be interest rate adjusted at a later stage, it follows from the balance condition that

(B.5) $$YD_M(1) = [1 + R^N_J(1)]H_M(1)$$

which determines an unambiguous interest rate on the loan.

Step f - Determine the volumes of m=2

If m=2, the interest rate adjustment condition may be formulated as

(B.6) $$YD_J(1)+REG_J^D\frac{RG_J(0)}{m_0}=H_J(1)+REG_JR_J^N(1)H_J(1)+REG_JR_J^N(2)H_J(2)$$

and the proceeds condition as

(B.7) $$M_J(1)K_J(1)+M_J(2)K_J(2)=Fin_J(0)$$

which collectively define two equations with two unknown quantities. (B.6) and (B.7) may be described by the matrix equation

(B.8) $$\mathbf{C_JM_J}=\mathbf{D_J}$$

where $\mathbf{M_J}=(M_J(1),\ M_J(2))$ is a 2x1 vector, and

(B.9) $$\mathbf{D_J}=\left[YD_J(1)+REG_J^D\frac{RG_J(0)}{m_0}-(1+REG_JR_J^N(1))H_J(0,1)-REG_JR_J^N(2)H_J(0,1),\ Fin_J(0)\right]$$

is also a 2x1 vector. In the setup of (B.9), the relation is applied.

$$H_J(j)=M_J(j)+H_J(0,j)$$

$\mathbf{C_J}$ is a 2x2 matrix given by

(B.10) $$\mathbf{C_J}=\begin{bmatrix}1+REG_JR_J^N(1) & REG_JR_J^N(2)\\ K_J(1) & K_J(2)\end{bmatrix}$$

C is quadratic and may thus be inverted, which produces the solution

(B.11) $$\mathbf{M_J}=\max[0;\mathbf{C_J^{-1}D_J}]$$

strictly negative funding volumes (<0) not being accepted. The max-condition may mean that the volumes do not fulfil both conditions. If a negative volume is assigned the value 0 (zero) in the max-condition, the model will overfund the funding demand, causing the proceeds condition not to be fulfilled. If it is the volume of the 1-year bond which is

assigned the value 0 (zero), nor the interest rate adjustment condition will be fulfilled. Therefore, the model moves on to step m in which the proceeds condition is checked.

Step f - Define a trend function

The volumes are determined by means of a trend function which is defined as

$$a_0+a_1(j-1)+a_2(j-1)^2+\ldots+a_{q-1}(j-1)^{q-1} \tag{B.12}$$

for j=1,2,...,m, where $q \leq m-TILT$ as in the model in section 2.

Step g - Determine coefficients in the trend function

The coefficients of the trend function are estimated such that the value of the trend function corresponds to the volumes given the intended adjustment of the interest rate and the bond volume already issued in each bond.

$$a_0+a_1(j-1)+a_2(j-1)^2+\ldots+a_{q-1}(j-1)^{q-1} = \max\left[REG_J^D\frac{RG_J(j-1)}{m_0}, H_J(0,j)\right] \tag{B.13}$$

A max-condition is included in the expression, negative volumes not being accepted. The coefficients are determined by the matrix equation

$$(a_0, a_1, \ldots, a_{q-1}) = [\mathbf{B}^T\mathbf{B}]^{-1}\mathbf{B}^T\max\left[REG_J^D\frac{RG_J(j-1)}{m_0}, H_J(0,j)\right] \tag{B.14}$$

The matrix B is given by (2.64)

Step h - Guess at an increment to the coefficients

In this step, two factors are added to the trend function. The factors function as an increment to the trend function, so that the trend function is adjusted to the interest rate adjustment condition and the proceeds condition.

G_0 and G_1 are added to the trend function as factors to a_0 and a_1.

(B.15) $$G_0 a_0 + G_1 a_1 (j-1) + a_2 (j-1)^2 + \ldots + a_{q-1} (j-1)^{q-1}$$

G_0 effects a vertical parallel shift of the trend function in the $(H_J(j), j)$ plane, whereas G_1 influences the slope of the trend function.

As a start value for the iteration,

(B.16) $G_0=1,25$ and $G_1=1$

are set.

The trend function is thereby shifted upwards. This supplies the model with information concerning the relationship between the marginal issue in the individual bonds. If the marginal volume is assigned the value 0 (zero) in a bond, the model does not immediately obtain information as to the extent to which the trend function is to be changed for the volume to be positive once again. By shifting the trend function initially in the iteration, this information is provided. The principle is shown in figure B.2

$H_J(0,4)$ - in the figure (9) - is disproportionately large, so according to the trend function, $M_J(4)$ will be 0 (zero). If the marginal issue in all bonds is increased (10), the model determines the relationship between $M_J(4)$ and the other marginal volumes.

<u>If TILT=1</u>, and the model thus operates with an extra bond, only one G variable is introduced in step h. Due to the special interest rate adjustment profile, the first funding volume is estimated explicitly by a variable Z which is described in more detail in the next step.

Step i - The volumes are determined

In this step, the inner loop of the model is initiated, in which inner loop the final values of G_0 and G_1 are determined by iteration such that the trend function is adjusted to the interest rate adjustment condition and the proceeds condition. The fact that step i initiates the inner loop implies that a_0, $a_1, \ldots, a_{q-1}$ are only estimated once for each iteration over the interest rate in the central loop. Similarly, $(G_0, G_1)=(1{,}25, 1)$ is merely an initial guess.

On the basis of the trend function, the marginal issue is determined by

$$M_J(j) = \max[0;[G_0a_0 + G_1a_1(j-1) + a_2(j-1)^2 + \ldots + a_{q-1}(j-1)^{q-1}] - H_J(0,j)] \quad \text{(B.17)}$$

for j=1, 2,..., m. At the disbursement of the loan, it is once again applied that $H_0(j)=M_0(j)+H_0(0,j)$ and $H_0(0,j)=0$.

For <u>TILT=1</u>, the first volume $M_0(1)$ is determined as

$$M_0(1)=Z-H_0(0,j) \quad \text{(B.18)}$$

The rest of the marginal issue is determined by the trend function in which only G_0 is included. The total marginal issue is thus given by

$$M_0(j) = \max[0;[Z,G_0a_0 + a_1(j-1) + a_2(j-1)^2 + \ldots + a_{q-1}(j-1)^{q-1}] - H_0(0,j)] \quad \text{(B.19)}$$

for j=1, 2,..., m. In parallel thereto, the volumes are determined for

(B.20) $((G_0+inc, G_1), (G_0, G_1+inc)$.

for use in the Gauss-Newton algorithm.

Step j - Calculate proceeds and balance conditions

On the basis of the calculated volumes, the interest rate adjustment condition and the proceeds condition are evaluated, so that a possible adjustment to G_0 and G_1 may be calculated in the next step. The interest rate adjustment condition and the proceeds condition are given by

(B.21) $$YD_J(1)+REG_J^D\frac{RG_J(0)}{m_0}=H_J(1)+REG_J\sum_{j=1}^{m}R_J^N(j)H_J(j) \text{ and}$$

(B.22) $$Fin_J(0)=\sum_{j=1}^{m}K_J(j)M_J(j)$$

respectively.

Step k - Calculate adjustment of increment

In step k an adjustment of the increment variables (G_0, G_1) is calculated on the basis of the interest rate adjustment condition and the proceeds condition which were calculated in step j.

A function $f(G_0, G_1)$ is defined, which measures the deviation from the conditions given (G_0, G_1).

(B.23)
$$f(G_0, G_1)=\left[Fin_J(0)-\sum_{j=1}^{m}K_J(j)M_J(j),\right.$$
$$\left.H_J(1)+REG_J\sum_{j=1}^{m}R_J^N(j)H_J(j)-YD_J(1)-REG_J^D\frac{RG_J(0)}{m_0}\right]$$

The adjustment of (G_0, G_1) is given by

(B.24) $$\Delta(G_0, G_1)=\frac{[D^TD]^{-1}D^Tg}{j_v}$$

where D, g, j_v fulfil (2.24) to (2.27), and the Jacoby matrix is given by

(B.25)
$$J_a = \frac{1}{inc}\begin{bmatrix} f_1(G_0, G_1) - f_1(G_0 + inc, G_1) & f_1(G_0, G_1) - f_1(G_0, G_1 + inc) \\ f_2(G_0, G_1) - f_2(G_0 + inc, G_1) & f_2(G_0, G_1) - f_2(G_0, G_1 + inc) \end{bmatrix}$$

where the subsign on f(.) specifies the part of the argument in (B.23) which is evaluated. That is to say that f_i (G_0, G_1) determines the value of the proceeds condition for (G_0,G_1).

For TILT=1, an adjustment to (Z,G_0) is determined instead.

Step 1 - Are the conditions fulfilled?

Step 1 determines whether (G_0, G_1) has converged such that the interest rate adjustment condition and the proceeds condition are fulfilled, or whether the iteration in the inner loop is to continue.

The mathematical convergence for (G_0, G_1) is determined by

(B.26)
$$\frac{|\Delta(G_0, G_1)|}{\varepsilon|G_0, G_1|} < \varepsilon$$

(B.27)
$$\left|\frac{f(G_0, G_1)^2 - f((G_0, G_1) + \Delta(G_0, G_1))^2}{f(G_0, G_1)^2}\right| < \varepsilon$$

where merely one of the conditions must be fulfilled in order that (G_0, G_1) has converged and the volumes may be applied.

Step m - Is the proceeds condition fulfilled?

Step m reaches the model in three ways.

Firstly, the model may reach step 1 when a convergent solution has been found. In this situation, the solution fulfils the proceeds condition, and the model moves on to step o.

Secondly, the model may reach step m after 30 iterations in the inner loop without convergence. In this situation, the model will not fulfil both conditions, but it cannot be precluded that the proceeds condition is fulfilled. This is clarified before the model moves on to either step o or step n.

Finally and thirdly, the model may reach step m from step e for m=2.

The precondition for moving on to step o is given by

(B.28) $$\left| Fin_J(0) - \sum_{j=1}^{m} K_J(j) M_J(j) \right| < \varepsilon$$

If the condition is not fulfilled, the volumes are corrected in step n.

Step n - Adjust the volumes

Step n adjusts the volumes such that at the proceeds condition is definitely fulfilled. In principle, three situations may occur

1. The model overfunds, i.e. the volumes must be reduced

$$\mathbf{M}_J^* = \left(\frac{Fin_J(0)}{\sum_{j=1}^{m} K_J(j) M_J(j)} \right) \mathbf{M}_J$$

for

(B.29)

$$\mathbf{M}_J = (M_J(1), M_J(2), \ldots, M_J(m))$$
$$\mathbf{M}_J^* = (M_J^*(1), M_J^*(2), \ldots, M_J^*(m))$$

where M* marks the adjusted funding volumes.

2. The model underfunds, so that the volumes must be increased. The first volume is maintained such that the interest rate adjustment percentage is not increased more than absolutely necessary. The other funding volumes are adjusted by

$$\mathbf{M}_J^* = \left(1, \frac{Fin_J(1) - K_J(1)M_J(1)}{\sum_{j=2}^{m} K_J(j)M_J(j)}\right)\mathbf{M}_J \tag{B.30}$$

where $\mathbf{M}$ and $\mathbf{M}^*$ are defined as above.

3. Finally, the volumes may sum up to 0 (zero). Then the interest rate adjustment is funded solely in the bond with the shortest term to maturity, i.e.

$$M_J^*(j) = \begin{cases} \dfrac{Fin_J(0)}{K_J(1)} & : j = 1 \\ 0 & : j = 2, 3, \ldots, m \end{cases} \tag{B.31}$$

Step o - Calculate adjustment to the interest rate on the loan
An adjustment to the interest rate on the loan is calculated according to the same directions as in the other variant of the model.

The adjustment is calculated in relation to the yield to maturity of the portfolio of underlying bonds corrected for the effect of the typically different number of payment periods on the debtor and creditor sides. First, the function f(.) is defined as

$$f(R_J^K) = \sum_{i=1}^{n} \frac{R_J^K}{n} \frac{RG_J(i-1)}{RG_J(0)} - REG_J r^p \tag{B.32}$$

The adjustment is calculated by the reduced Gauss-Newton which was deduced in section 2.

$$\Delta R_J^K = \frac{inc}{f(R_J^K) - f(R_J^K + inc)} \tag{B.33}$$

Step p - Has the interest rate on the loan converged ?

It is to be determined in step p whether the interest rate on the loan has also converged, or whether the iteration in the central loop is to continue.

The mathematical convergence of the interest rate is examined by

$$\frac{|\Delta R_J^K|}{\varepsilon|R_J^K|} < \varepsilon \qquad \text{(B.34)}$$

$$\left|\frac{f(R_J^K)^2 - f(R_J^K + \Delta R_J^K)^2}{f(R_J^K)^2}\right| < \varepsilon \qquad \text{(B.35)}$$

where merely one of the conditions need to be fulfilled in order for the interest rate to have converged. In that case, the model continues in step r.

Step q - Correct the interest rate on the loan

If neither (B.34) nor (B.35) were fulfilled in step p, the interest rate on the loan is corrected by the calculated adjustment.

Step r - The loan has been calculated

If the interest rate has converged, all variables have been calculated for the given term to maturity. Having been implemented in the model for a LAIR III type P, the model moves on to 14 from here.

Claims

1. A method for determining, by means of a first computer system, the type, the number and the volume of financial instruments for funding a loan, determining the term to maturity and the payment profile of the loan, and further determining the payments on a payment guarantee instrument designed to ensure that the payments on the loan and the term to maturity of the loan do not exceed predetermined limits, and from which instrument payments are made to the debtor in situations in which the maximum limits for payments on the loan and term to maturity would otherwise have been exceeded, the loan being designed to be at least partially refinanced during the remaining term to maturity of the loan,

- requirements having been laid down stipulating that
  - the term to maturity of the loan is not longer than a predetermined maximum limit nor less than a predetermined minimum limit,
  - debtor's payments on the loan are within predetermined limits,

- requirements having been laid down stipulating a maximum permissible difference in balance between, on the one hand, payments on the loan and refinancing amounts and, on the other hand, net payments to the owner of the financial instruments applied for the funding, and payments to and from the payment guarantee instrument,
- requirements having been laid down stipulating a maximum permissible difference in proceeds between, on the one hand, the sum of the market price of the volume of the financial instruments applied for the funding of the loan and payments to and from the payment guarantee instrument and, on the other hand, the volume of the loan,
- and requirements optionally having been laid down stipulating a maximum permissible difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding,



said method comprising

(a) inputting and storing, in a memory or a storage medium of the computer system, a first set of data specifying the parameters: the volume and the repayment profile of the loan,

(b) inputting and storing, in a memory or a storage medium of the computer system, a second set of data specifying

(i) a maximum and a minimum limit for the debtor's payments on the loan in each of a number of periods collectively covering the term to maturity of the loan,

(ii) a maximum and a minimum limit for the term to maturity of the loan, and

(iii) optionally, a desired/intended payment on the loan or a desired/intended term to maturity when the maximum and the minimum limits for the payments in the first period are not equivalent (i) or when the maximum and the minimum limits for the term to maturity are not equivalent (ii),

(c) inputting and storing, in a memory or a storage medium of the computer system, a third set of data specifying a desired/intended refinancing profile, such as one or more point(s) in time at which refinancing is to take place, and specifying the amount of the remaining debt to be refinanced at said point(s) in time,

and/or said third set of data specifying a desired/intended funding profile, such as a desired/intended number of financial instruments applied for the funding together with their type and volumes,

(d) inputting and storing, in a memory or a storage medium of the computer system, a fourth set of data comprising a maximum permissible difference in balance within a

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predetermined period, a maximum permissible difference in proceeds and, optionally, a maximum permissible difference in interest rates equivalent to the difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding and, optionally, the payment guarantee instrument,

(e) determining and storing, in a memory or a storage medium of the computer system, a fifth set of data specifying a selected number of financial instruments with inherent characteristics such as the type, the price/market price, and the date of the price/market price,

(f) determining and storing, in a memory or a storage medium of the computer system, a sixth set of data representing a first profile of the interest rate on the loan and either a first term to maturity profile or a first payment profile of the loan,

(g) calculating and storing, in a memory or a storage medium of the computer system, a seventh set of data representing

- a first term to maturity profile or a first payment profile (depending on what was determined under (f)) corresponding to interest and repayments for the debtor
- and a first remaining debt profile,

said term to maturity profile or payment profile, as well as the remaining debt profile, being calculated on the basis of

- the volume and repayment profile of the loan as input under (a),
- the set of data input under (b),
- the refinancing profile and/or the funding profile input under (c)
- and the profile of the interest rate on the loan and either the payment profile or the term to maturity profile determined under (f),

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(g1) if necessary/desired, calculating and storing, in a memory or a storage medium of the computer system, an eighth set of data representing payments (positive, zero or negative) on the payment guarantee instrument, the requirements with respect to a maximum permissible difference in balance and a maximum permissible difference in proceeds, as well as the limits for payments on the loan and term to maturity, always being fulfilled.

(h) selecting a number of financial instruments among the financial instruments stored under (e), and calculating and storing a ninth set of data specifying these selected financial instruments with their volumes, for appliance in the funding of the loan, said ninth set of data being calculated on the basis of

- the payment profile determined under (f) or calculated under (g) and
- the remaining debt profile calculated under (g),
- the payments on the payment guarantee instrument optionally calculated under (g1),
- the refinancing profile input under (c) and/or the funding profile input under (c),
- the set of data input under (b),
- the requirements input under (d), and
- in the case of a refinancing where financial instruments from a previous funding have not yet matured, the type, the number and the volume of these instruments,

one or more recalculations being made if necessary, including if necessary selection of a new number of the financial instruments stored under (e),
storing, in a memory or a storage medium of the computer system, after each recalculation

- the recalculated profile of the interest rate on the loan,
- the recalculated term to maturity profile,
- the recalculated payment profile,
- the recalculated remaining debt profile, and

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- the selected financial instruments with their calculated volumes,

until all the conditions stated under (b) or (d) have been fulfilled,

and the payments on the payment guarantee instrument optionally being recalculated in accordance with (g1), and the recalculated payments being stored in a memory or a storage medium of the computer system after each recalculation,

after which, if desired, the thus determined combination of the type, the number and the volume of the financial instruments for funding the loan,

- together with the calculated term to maturity,
- together with the calculated payment profile,
- optionally, together with the payments on the payment guarantee instrument,
- preferably, together with the calculated interest rate on the loan, and
- preferably, together with the calculated remaining debt profile,

is output, transferred to a storage medium or sent to another computer system.

2. A method according to claim 1, wherein calculations are performed for all future funding periods op to maturity of the loan.

3. A method according to claim 2, wherein the result or results of the calculations for one or more later funding periods is/are used in the calculations for one or more previous funding periods.

4. A method according to claim 1, 2 or 3, wherein said recalculations are performed on the basis of a first term to maturity profile, changing the term to maturity at substantially each recalculation until the payment on the loan for

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each funding period is within the limits specified in (b) (i),

the determination of the type, the number and the volume of the financial instruments for funding the loan being calculated and recalculated at each iteration over the term to maturity until the relevant variables with respect to the type, the number and the volume of the financial instruments are established in observance of the other requirements/conditions/desires,

after which, if the term to maturity for which the payment profile is within the limits established therefor is not within the limits specified in (b) (ii) for the term to maturity, the payments on the payment guarantee instrument are calculated such that the limit for the term to maturity as well as the limits for the payments are observed.

5. A method according to claim 4, wherein the calculation of the payments on the payment guarantee instrument is performed on the basis of an interest rate on the loan which is recalculated so that the limits for the payments as well as the term to maturity are observed, and wherein either resulting differences in the payments on the debtor side and the payments on the financial instruments or resulting differences in the market price of sold financial instruments and the funding demand correspond to the payments on the payment guarantee instrument.

6. A method according to claim 5, wherein the payments on the payment guarantee instrument correspond to the differences in the market price of sold financial instruments and the funding demand resulting from the recalculation, the volume of the financial instruments being determined such that the requirement with respect to maximum permissible difference in balance is fulfilled.

7. A method according to claim 5 or claim 6, wherein the desired/intended term to maturity of the loan input under (b) (iii) and/or the limits for the payments and/or the limits

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for the term to maturity is/are determined such that the present value of the payments on the payment guarantee instrument is zero.

8. A method according to claim 5, 6 or 7, wherein the calculation of the present value of the payments on the payment guarantee instrument is performed by use of a stochastic yield curve model.

9. A method according to claim 8, wherein the stochastic yield curve model is calibrated to a yield curve which is determined at the time of calculation.

10. A method according to claim 8, wherein the stochastic yield curve model is formulated in discrete time and implemented in an interest rate lattice.

11. A method according to claim 10, wherein the implementation of the yield curve model formulated in discrete time is effected in a trinomial lattice according to Hull & White.

12. A method according to claim 11, wherein the yield curve model is the extended Vasicek model.

13. A method according to any one of claims 1-12, wherein the financial instruments in (e) are determined such that at least one financial instrument is one on which payment falls due within the first period for which a maximum permissible difference in balance applies.

14. A method according to any one of claims 1-13, wherein the requirement with respect to maximum permissible difference in proceeds is given by a convergence condition for the difference in proceeds, and/or the requirement with respect to maximum permissible difference in interest rates is given by a convergence condition for the difference in interest rates, and/or the requirement with respect to maximum permissible

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difference in balance is given by a convergence condition for the difference in balance.

15. A method according to any one of claims 1-14, wherein correction is additionally made, in the calculation, for any difference between, on the one hand, the disbursement date of the loan and/or the prepayment date of the loan and, on the other hand, the settlement date of the financial instruments by proportionally correcting for the already elapsed part or the remaining part of the disbursement period and the prepayment period, respectively.

16. A method according to any one of claims 1-15, wherein the term to maturity,

when the set of data under (c) specifies that a calculation is to be performed for the case in which full refinancing of the remaining debt is to be performed periodically with a predetermined period which is shorter than the term to maturity of the loan, and the remaining term to maturity of the loan is shorter than the period which according to (c) elapses between two successive interest rate adjustments, and the remaining term to maturity does not correspond to the maturity of the last maturing financial instrument selected under (h), but it is desired that the loan matures at the same time as the last maturing financial instrument selected under (h),

is determined as

(i) the term to maturity prolonged as little as possible to a date of maturity of one or more of the selected financial instruments, provided the payment profile does not thereby exceed the minimum limit for the payments on the loans as specified under (b) (i), or

(ii) the term to maturity shortened as little as possible to a date of maturity of one or more of the selected financial instruments, provided the payment profile

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does not thereby exceed the maximum limit for the payments on the loans as specified under (b) (i); and provided the condition under (i) is not fulfilled, or

(iii) the term to maturity prolonged as little as possible to a date of maturity of one or more of the selected financial instruments, if none of the conditions specified under (i) and (ii) is fulfilled.

17. A method according to any one of claims 1-16, wherein the set of data under (c) specifies that a calculation is to be performed for the case in which full refinancing of the remaining debt is to be performed periodically with a predetermined period which is shorter than the term to maturity of the loan, said method comprising, in the determination of the volumes of financial instruments specified in step (h) in the case in which the limits for the payment as well as the term to maturity are observed and the payment on the payment guarantee instrument is at the same time zero, calculating the difference in proceeds for the calculated volumes of the financial instruments applied for the funding and/or calculating an adjustment of the interest rate on the loan, said interest rate adjustment preferably being calculated in consideration of the calculated difference in proceeds, calculations being performed as to whether the interest rate adjustment is so small that the interest rate on the loan fulfils the requirement with respect to maximum permissible difference in interest rates or a convergence condition for the difference in interest rates, or as to whether the interest rate adjustment is so small that the requirement with respect to maximum permissible difference in proceeds or a convergence condition for the difference in proceeds is fulfilled.

18. A method according to claim 17, wherein, in cases in which the requirements or conditions laid down with respect to the difference in proceeds or the difference in interest rates are not fulfilled, the recalculations comprise one or

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more interest rate iterations, each interest rate iteration comprising

calculating and storing, in a memory or a storage medium of the computer, data specifying a new interest rate on the loan which is preferably based on the previous interest rate on the loan and the calculated interest rate adjustment,

calculating and storing, in a memory or a storage medium of the computer, data specifying a new payment profile and a new remaining debt profile for the debtor, said payment profile and remaining debt profile being calculated in consideration of the new interest rate on the loan, the volume and payment profile of the loan as input under (a), and the refinancing profile and/or the funding profile as input under (c), as well as the term to maturity, and

calculating and storing, in a memory or a storage medium of the computer system, data specifying a new set of volumes of the financial instruments applied for the funding.

19. A method according to claim 17 or 18 , wherein the interest rate iteration is performed applying a numerical optimization algorithm or by "grid search".

20. A method according to claim 19, wherein the optimization algorithm is a Gauss-Newton algorithm.

21. A method according to any one of claims 17-20, which method, when the relevant requirement(s) with respect to maximum permissible difference in proceeds and/or the requirement with respect to maximum permissible difference in interest rates are fulfilled, further comprises

determining whether all the calculated volumes of financial instruments are positive, and

in cases in which the calculated set of volumes comprises at least one negative volume, further comprising either

i) selecting a new number of financial instruments among the financial instruments stored under (e), one or more of the instruments in the new number of instruments being determined such that the payments on this/these instrument(s) are

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due relatively later compared to the original number of financial instruments, after which a recalculation is performed in accordance with any one of claims 17-20, or

ii) setting the negative volume or volumes equal to 0, after which a recalculation is performed in accordance with any one of claims 17-20.

22. A method according to any one of claims 1-14, wherein the term to maturity,

when the set of data under (c) specifies that a calculation is to be performed for the case in which partial refinancing of the remaining debt is to be performed periodically with a predetermined period which is shorter than the term to maturity of the loan, e.g. in such a way that the refinancing corresponds to a fixed fraction of the remaining debt of the loan, and the remaining term to maturity of the loan is less than or equal to a set value and it is desired that the loan matures no later than the date of maturity specified under (e) for one or more of the financial instruments applied for the refinancing of the loan,

is determined as the term to maturity prolonged as little as possible to a date of maturity of one or more financial instruments.

23. A method according to any one of claims 1-14 and 22, wherein the set of data (c) specifies that a calculation is to be performed for the case in which partial refinancing of the remaining debt is to be performed periodically with a predetermined period which is shorter than the term to maturity of the loan, e.g. in such a way that the refinancing corresponds to a fixed fraction of the remaining debt of the loan, in which method the volume of some or all of the financial instruments applied for the funding in the first calculation i step (h) in the case in which the limits for payments as well as term to maturity are observed and the pay-

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ment on the payment guarantee instrument is at the same time zero, is calculated such that they substantially follow a shifted level remaining debt profile, after which recalculations are performed, if necessary, until all the conditions specified under (d) are fulfilled.

24. A method according to claim 23, wherein the volume of some or all the financial instruments are calculated in the calculation in step (h) by use of a function which is adjusted to a shifted level remaining debt profile.

25. A method according to claim 24, wherein the volume of some or all of the financial instruments in one or more of the recalculations optionally made in step (h) are calculated by applying a function which is adjusted to a shifted level remaining debt profile.

26. A method according to claim 24 or 25, wherein the function is a polynomial function with a maximum degree which is one (1) less than the number of financial instruments applied.

27. A method according to claim 26, wherein the polynomial function is calculated applying a statistical curve-fitting method.

28. A method according to claim 27, wherein the statistical curve-fitting method is the least squares' method.

29. A method according to any one of claims 23-28. wherein recalculation of all or some of the data mentioned in (g) and (h) and/or one or more function coefficients for the function representing the shifted level remaining debt profile and/or the interest rate on the loan is performed by use of iteration carried out applying numerical optimization algorithms or by grid search.

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30. A method according to claim 29, wherein the optimization algorithm is a Gauss-Newton algorithm.

31. A method according to any one of claims 23-30, wherein, in cases in which the requirements or conditions laid down with respect to the difference in proceeds and/or the difference in interest rates and/or the difference in balance calculated in consideration of the refinancing profile input under (c) are not fulfilled, the recalculations comprise one or more iterations, each iteration comprising

calculating and storing data specifying a new interest rate on the loan and/or

calculating and storing data specifying a new payment profile and a new remaining debt profile for the debtor, said payment profile and remaining debt profile being calculated in consideration of the new interest rate on the loan, the volume and repayment profile of the loan as input under (a) and the refinancing profile and/or the funding profile input under (c), together with the term to maturity, and/or

calculating and storing data specifying a new set of coefficients for the function which is adjusted to the shifted level remaining debt profile, and/or

calculating and storing data specifying a new set of volumes of the financial instruments applied for the funding, said new set of volumes being calculated on the basis of the financial instruments already issued for the funding, the new payment profile and the new remaining debt profile.

32. A method according to any one of claims 23-31, comprising, in step (h), determining whether the calculated volumes of financial instruments fulfil at least two of two or more predetermined convergence conditions which preferably are calculated in consideration of a calculated difference in proceeds and a difference in balance calculated in consideration of the refinancing profile input under (c), and in cases in which the calculated volumes of financial instruments do not fulfil these conditions, the recalculations comprise one or more iterations of the coefficients for the

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function which is adjusted to a shifted level remaining debt profile, each iteration comprising

calculating and storing data specifying two or more new function coefficients for the function representing the shifted level remaining debt profile,

calculating and storing data specifying a new set of volumes of the financial instruments applied for the funding, said new set of volumes being calculated in consideration of the new function representing the shifted level remaining debt profile,

determining whether the new set of calculated volumes of financial instruments fulfils said at least two or more predetermined convergence conditions,

until the new set of calculated volumes of financial instruments fulfils these conditions.

33. A method according to claim 32, wherein the new function coefficient or coefficients is/are calculated in consideration of the calculated difference in proceeds and a difference in balance calculated in consideration of the refinancing profile input under (c).

34. A method according to claim 32 or 33, comprising calculating the difference between the interest rate on the loan and the yield to maturity of the calculated volumes of the financial instruments, calculations being performed as to whether the difference in interest rates is so small that it fulfils the requirement with respect to maximum permissible difference in interest rates or a convergence condition for the difference in interest rates.

35. A method according to claim 34, wherein, in cases in which the requirements or conditions laid down with respect to the difference in interest rates are not fulfilled, the recalculations comprise one or more interest rate iterations, each interest rate iteration comprising

calculating and storing an interest rate adjustment, said interest rate adjustment preferably being calculated in

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consideration of the difference between the interest rate on the loan and the yield to maturity of the calculated volumes of the financial instruments, e.g. by use of a Gauss-Newton algorithm,

calculating and storing data specifying a new interest rate on the loan which is preferably based on the previous interest rate on the loan and the calculated adjustment of the interest rate on the loan.

calculating and storing data specifying a new payment profile and a new remaining debt profile for the debtor, said payment profile and remaining debt profile being calculated in consideration of the new interest rate on the loan, the volume and repayment profile of the loan as input under (a), the refinancing profile and/or the funding profile input under (c), and the term to maturity,

calculating and storing data specifying a new set of coefficients for the function which is adjusted to the shifted level remaining debt profile, and

calculating and storing data specifying a new set of volumes of the financial instruments applied for the funding.

36. A method according to any one of claims 23-31, which method comprises determining whether the calculated volumes of financial instruments fulfil at least three of three or more predetermined convergence conditions which are preferably calculated in consideration of a calculated difference in proceeds, a difference in balance calculated in consideration of the refinancing profile input under (c) and a maximum permissible difference in interest rates, and, in cases in which the calculated volumes of financial instruments, the calculated payment profile and the calculated term to maturity do not fulfil these conditions, said recalculations comprising one or more iterations, each iteration comprising

calculating and storing an interest rate adjustment, said interest rate adjustment preferably being calculated in consideration of the difference between the interest rate on the loan and the yield to maturity of the calculated volumes of the financial instruments,

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calculating and storing data specifying a new interest rate on the loan which is preferably based on the previous interest rate on the loan and the calculated adjustment of the interest rate on the loan,

calculating and storing data specifying a new payment profile and a new remaining debt profile for the debtor, said payment profile and remaining debt profile being calculated in consideration of the new interest rate on the loan, the volume and repayment profile of the loan as input under (a), the refinancing profile and/or the funding profile input under (c), and the term to maturity,

calculating and storing data specifying a new set of coefficients for the function which is adjusted to the shifted level remaining debt profile, and

calculating and storing data specifying a new set of volumes of the financial instruments applied for the funding, said new set of volumes being calculated in consideration of the new function representing the shifted level remaining debt profile.

determining whether the new set of calculated volumes of financial instruments fulfils said at least three or more predetermined convergence conditions.

37. Method according to any one of claims 23-36, wherein, in cases in which the calculated set of volumes comprises at least one negative volume,
the negative volume or volumes is/are assigned the value 0, after which the calculations are continued on the basis of the thus determined volumes of the financial instruments.

38. A method according to claim 37, which method comprises calculating whether the volumes of the financial instruments fulfil the requirement with respect to maximum permissible difference in proceeds, and, in cases in which the calculated volumes do not fulfil this requirement, one or more adjustments to the previously calculated positive volumes of financial instruments being made, adjustments being made until the new set of volumes of financial instruments fulfils the

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requirement with respect to maximum permissible difference in proceeds and the requirement with respect to maximum permissible difference in balance.

39. A method according to any one of claims 23-28, wherein the determination of one or more function coefficients for the function representing the shifted level remaining debt profile is performed analytically.

40. A method according to claim 39, wherein recalculation of all or some of the data mentioned under (g) and (h) and/or recalculation of the interest rate on the loan is performed by iteration carried out by applying numerical optimization algorithms or by grid search.

41. A method according to claim 40, wherein the optimization algorithm is a Gauss-Newton algorithm.

42. A method according to any one of claims 39-41, wherein, in cases in which the requirements or conditions laid down with respect to the difference in interest rates are not fulfilled, said recalculations comprising one or more interest rate iterations, each iteration comprising

calculating and storing data specifying a new interest rate on the loan, and/or

calculating and storing data specifying a new payment profile and a new remaining debt profile for the debtor, said payment profile and remaining debt profile being calculated in consideration of the new interest rate on the loan, the volume and repayment profile of the loan as input under (a), the refinancing profile and/or the funding profile input under (c), and the term to maturity, and/or

calculating and storing data specifying a new set of coefficients for the function which is adjusted to the shifted level remaining debt profile, and/or

calculating and storing data specifying a new set of volumes of the financial instruments applied for the funding, said new set of volumes being calculated on the basis of the

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financial instruments already issued for the funding, and the new payment profile and remaining debt profile.

43. A method according to any one of claims 39-42, comprising, in step (h), determining the volumes of the financial instruments by analytical calculation of one or more of the coefficients for the function which is adjusted to a shifted level remaining debt profile, so that the conditions under (d) with respect to maximum permissible difference in proceeds and maximum permissible difference in balance in consideration of the refinancing profile input under (c) are fulfilled.

44. A method according to any one of claims 39-43, wherein, in cases in which the calculated volumes of the financial instruments applied comprise at least one negative volume, the negative volume or volumes is/are assigned the value 0, collectively or one at a time, the calculations being continued after such a or each operation of assigning 0 value(s) on the basis of the thus determined volumes of the financial instruments applied.

45. A method according to claim 44, wherein the calculated coefficients for the function adjusted to the shifted level remaining debt profile are regulated by an indicator function specifying either that a financial instrument is to be applied or that an instrument is not to be applied, said indicator function being adjusted at each operation of assigning 0 value(s), and wherein the function coefficients are calculated such that both the proceeds criterion and the balance criterion calculated in consideration of the refinancing profile mentioned under (c) are fulfilled in the case in which the indicator function specifies that two or more financial instruments are to be applied, at least one of which matures within the next predetermined period in which it is specified that partial refinancing of the remaining debt is to take place, both the proceeds criterion and the balance criterion being fulfilled in all other cases.

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46. A method according to any one of claims 23-45, wherein the volume/volumes of one or more financial instruments, particularly the instrument with the shortest term to maturity and/or the instrument with the longest term to maturity, is/are not calculated by use of the function representing the remaining debt profile, but is/are determined separately to ensure that the actual refinancing profile corresponds to that input under (c).

47. A data processing system, such as a computer system, for determining the type, the number and the volumes of financial instruments for funding a loan, determining the term to maturity and payment profile of the loan, and further determining the payments on a payment guarantee instrument designed to ensure that the payments on the loan and the term to maturity of the loan do not exceed predetermined limits, and from which instrument payments are made to the debtor in situations in which the maximum limits for payments on the loan and term to maturity would otherwise have been exceeded, the loan being designed to be at least partially refinanced during the remaining term to maturity of the loan,

- requirements having been laid down stipulating that
  - the term to maturity of the loan is not longer than a predetermined maximum limit nor less than a predetermined minimum limit,
  - debtor's payments on the loan are within predetermined limits,

- requirements having been laid down stipulating a maximum permissible difference in balance between, on the one hand, payments on the loan and refinancing amounts and, on the other hand, net payments to the owner of the financial instruments applied for the funding, and payments to and from the payment guarantee instrument,
- requirements having been laid down stipulating a maximum permissible difference in proceeds between, on the one hand, the sum of the market price of the volumes of the

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financial instruments applied for the funding of the loan, and payments to and from the payment guarantee instrument, and, on the other hand, the volume of the loan,

- and requirements optionally having been laid down stipulating a maximum permissible difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding,

said data processing system comprising

(a) means, typically input means and a memory or a storage medium, for inputting and storing a first set of data specifying the parameters: the volume and the repayment profile of the loan,

(b) means, typically input means and a memory or a storage medium, for inputting and storing a second set of data specifying

(i) a maximum and a minimum limit for the debtor's payments on the loan for each of a number of periods collectively covering the term to maturity of the loan,

(ii) a maximum and a minimum limit for the term to maturity of the loan, and

(iii) optionally, a desired/intended payment on the loan or a desired/intended term to maturity when the maximum and the minimum limits for the payments in the first period are not equivalent (i) or when the maximum and the minimum limits for the term to maturity are not equivalent (ii),

(c) means, typically input means and a memory or a storage medium, for inputting and storing a third set of data specifying a desired/intended refinancing profile, such as one or more point(s) in time at which refinancing is to take

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place, and specifying the amount of the remaining debt to be refinanced at said point(s) in time,

and/or said third set of data specifying a desired/intended funding profile, such as a desired/intended number of financial instruments applied for the funding together with their type and volumes,

(d) means, typically input means and a memory or a storage medium, for inputting and storing a fourth set of data comprising a maximum permissible difference in balance within a predetermined period, a maximum permissible difference in proceeds and, optionally, a maximum permissible difference in interest rates equivalent to the difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding and, optionally, the payment guarantee instrument,

(e) means, typically input means and a memory or a storage medium, for determining and storing a fifth set of data specifying a selected number of financial instruments with inherent characteristics such as the type, the price/market price, and the date of the price/market price,

(f) means, typically input means and a memory or a storage medium, for determining and storing a sixth set of data representing a first profile of the interest rate on the loan and either a first term to maturity profile or a first payment profile of the loan,

(g) means, typically calculating means and a memory or a storage medium, for calculating and storing a seventh set of data representing

- a first term to maturity profile or a first payment profile (depending on what was determined under (f)) corresponding to interest and repayments for the debtor
- and a first remaining debt profile,

said term to maturity profile or payment profile, as well as the remaining debt profile, being calculated on the basis of

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- the volume and repayment profile of the loan as input under (a),
- the set of data input under (b),
- the refinancing profile and/or the funding profile input under (c)
- and the profile of the interest rate on the loan and either the payment profile or the term to maturity profile established under (f),

(g1) means, typically calculating means and a memory or a storage medium for, if necessary/desired, calculating and storing an eighth set of data representing payments (positive, zero or negative) on the payment guarantee instrument, the requirements with respect to maximum permissible difference in balance and maximum permissible difference in proceeds, as well as the limits for payments on the loan and term to maturity, always being fulfilled,

(h) means, typically calculating means and a memory or a storage medium, for selecting a number of financial instruments among the financial instruments stored under (e), and calculating and storing a ninth set of data specifying these selected financial instruments with their volumes for appliance in the funding of the loan, said ninth set of data being calculated on the basis of

- the payment profile established under (f) or calculated under (g) and
- the remaining debt profile calculated under (g),
- the payments on the payment guarantee instrument optionally calculated under (g1),
- the refinancing profile input under (c) and/or the funding profile input under (c),
- the set of data input under (b),
- the requirements input under (d), and
- in the case of a refinancing where financial instruments from a previous funding have not yet matured, the type, the number and the volume of these instruments,

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said means being adapted to perform, if necessary, one or more recalculations, including, if necessary, selecting a new number of the financial instruments stored under (e), said means further being adapted to store, after each recalculation,

- the recalculated profile of the interest rate on the loan,
- the recalculated term to maturity profile,
- the recalculated payment profile,
- the recalculated remaining debt profile, and
- the selected financial instruments with their calculated volumes,

until all the conditions specified under (b) and (d) have been fulfilled,

and said means further being adapted to optionally recalculate the payments on the payment guarantee instrument in accordance with (g1) and to store, after each recalculation, the recalculated payments in the memory or the storage medium,

means for outputting the hereby determined combination of the type, the number, and the volume of the financial instruments for funding the loan,

- together with the calculated term to maturity,
- together with the calculated payment profile,
- optionally, together with the payments on the payment guarantee instrument,
- preferably, together with the calculated interest rate on the loan, and
- preferably, together with the calculated remaining debt profile,

or means for transferring the combination, if desired, to a storage medium or sending it to another computer system.

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