Non-Patent Document 6: G. Glentis, K. Berberidis, and S. Theodoridis, Efficient least squares adaptive algorithms for FIR transversal filtering, IEEE Signal Processing Magazine, 16, 4, pp. 13-41, 1999

SUMMARY

Provided is a system identification method for performing fast real-time identification for a system from input/output data, the method comprising estimating a state estimation value $x^{\wedge}_{k|k}$ of a state x_k of the system from a received signal u_k of an interface in an acoustic system or a communication system and from an observation signal y_k of an echo or a feedback by using a filter robust to disturbance in such a manner that a maximum energy gain from the disturbance to a filter error is, as an evaluation criterion, set smaller than a given upper limit γ_f^2, and canceling out the echo or the feedback using a result of the state estimation value $x^{\wedge}_{k|k}$, wherein the filter is, by Expressions (49) to (62) below, provided to a state space model represented by Expressions (11) to (13) below to estimate the state estimation value $x^{\wedge}_{k|k}$ of the state x_k of the system:

CLAIMS

1. A system identification method for performing fast real-time identification for a system from input/output data, the method comprising:

estimating a state estimation value $x^{\wedge}_{k|k}$ of a state x_k of the system from a received signal u_k of an interface in an acoustic system or a communication system and from an observation signal y_k of an echo or a feedback by using a filter robust to disturbance in such a manner that a maximum energy gain from the disturbance to a filter error is, as an evaluation criterion, set smaller than a given upper limit γ_f^2; and

canceling out the echo or the feedback using a result of the state estimation value $x^{\wedge}_{k|k}$, wherein

the filter is, by Expressions (49) to (62) below, provided to a state space model represented by Expressions (11) to (13) below to estimate the state estimation value $x^{\wedge}_{k|k}$ of the state x_k of the system:

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \boldsymbol{G}_k \boldsymbol{w}_k, \qquad \boldsymbol{w}_k . \boldsymbol{x}_k \in \mathcal{R}^N \tag{11}$$

$$y_k = \boldsymbol{H}_k \boldsymbol{x}_k + v_k, \qquad y_k, v_k \in \mathcal{R} \tag{12}$$

$$z_k = \boldsymbol{H}_k \boldsymbol{x}_k, \qquad z_k \in \mathcal{R},\ \boldsymbol{H}_k \in \mathcal{R}^{1\times N} \tag{13}$$

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k-1|k-1} + \boldsymbol{K}_{s,k}(y_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k-1|k-1}) \tag{49}$$

$$\boldsymbol{K}_{s,k} = \frac{\tilde{\boldsymbol{K}}_k(:,1)}{\boldsymbol{R}_{e,k}(1,1)} \tag{50}$$

$$\boldsymbol{b}_k = \boldsymbol{b}_{k-1} - \rho^{-1}\tilde{\boldsymbol{K}}_k \boldsymbol{W} \underline{\boldsymbol{e}}_k \tag{51}$$

$$\underline{S}_k = \rho \underline{S}_{k-1} + \underline{\boldsymbol{e}}_k^T \boldsymbol{W} \underline{\tilde{\boldsymbol{e}}}_k \tag{52}$$

$$\underline{\boldsymbol{e}}_k = \boldsymbol{\chi}_k^{-1} \underline{\tilde{\boldsymbol{e}}}_k \tag{53}$$

$$\boldsymbol{\chi}_k = \rho^{-1} \boldsymbol{R}_{e,k} \boldsymbol{W} \tag{54}$$

$$\boldsymbol{R}_{e,k} = \boldsymbol{R}_{e,k-1} + \frac{\tilde{\boldsymbol{e}}_k \tilde{\boldsymbol{e}}_k^T}{S_{k-1}} - \underline{\tilde{\boldsymbol{e}}}_k \tilde{\boldsymbol{\mu}}_k \tag{55}$$

$$\underline{\tilde{\boldsymbol{e}}}_k = \underline{S}_{k-1} \tilde{\boldsymbol{\mu}}_k^T \tag{56}$$

$$\tilde{\boldsymbol{K}}_k = \tilde{\boldsymbol{m}}_k - \boldsymbol{b}_{k-1} \tilde{\boldsymbol{\mu}}_k \tag{57}$$

$$\begin{bmatrix} \tilde{\boldsymbol{m}}_k \\ \tilde{\boldsymbol{\mu}}_k \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \tilde{\boldsymbol{K}}_{k-1} \end{bmatrix} + \begin{bmatrix} 1 \\ \boldsymbol{a}_{k-1} \end{bmatrix} \frac{\tilde{\boldsymbol{e}}_k^T}{S_{k-1}} \tag{58}$$

$$\boldsymbol{a}_k = \boldsymbol{a}_{k-1} - \rho^{-1} \tilde{\boldsymbol{K}}_{k-1} \boldsymbol{W} \boldsymbol{e}_k \tag{59}$$

$$S_k = \rho S_{k-1} + \boldsymbol{e}_k^T \boldsymbol{W} \tilde{\boldsymbol{e}}_k \tag{60}$$

$$\boldsymbol{e}_k = \boldsymbol{\chi}_{k-1}^{-1} \tilde{\boldsymbol{e}}_k \tag{61}$$

$$\tilde{\boldsymbol{e}}_k = \boldsymbol{c}_k + \boldsymbol{C}_{k-1} \boldsymbol{a}_{k-1} \tag{62}$$

where C_k and W are defined by the following expressions:

$$\boldsymbol{C}_k = \begin{bmatrix} \boldsymbol{H}_k \\ \boldsymbol{H}_k \end{bmatrix}, \quad \boldsymbol{W} = \begin{bmatrix} 1 & 0 \\ 0 & -\gamma_f^{-2} \end{bmatrix},$$

$\boldsymbol{c}_k \in \mathcal{R}^{2\times 1}$ is a first column vector of $\boldsymbol{C}_k = [\boldsymbol{c}_k, \ldots, \boldsymbol{c}_{k-N+1}]$,

$$\breve{\boldsymbol{C}}_k = \begin{bmatrix} u_k & \cdots & u_{k-N} \\ & & \\ u_k & \cdots & u_{k-N} \end{bmatrix} \tag{63}$$

$$\breve{\boldsymbol{C}}_k(:,1) = \boldsymbol{c}_k = [u_k \; u_k]^T, \quad \breve{\boldsymbol{C}}_k(:,i) = \breve{\boldsymbol{C}}_{k-1}(:,i-1), \quad i = 2, \cdots, N+1 \tag{64}$$

$H_k = [u_k, \ldots, u_{k-N+1}]^T$,

x_k: a state vector or merely a state; unknown, and this is a target for estimation,

x_0: an initial state; unknown,

w_k: system noise; unknown,

v_k: observation noise; unknown,

y_k: an observation signal; taken as filter input, and known,

z_k: an output signal; unknown,

G_k: a drive matrix; known upon execution,

H_k: an observation matrix, known,

$x^{\wedge}_{k|k}$: an estimation value, using observation signals y_0 to y_k, of the state x_k (an impulse response of an unknown system) at time k; provided by a filter equation,

$x^{\wedge}_{k+1|k}$: a single-step prediction value, using the observation signals y_0 to y_k, of a state x_{k+1} at the time k; provided by the filter equation,

$x^{\wedge}_{-1|-1}$: an initial estimation value of the state; unknown in a normal situation, but zero is used for convenience,

$K_{s,k}$: a filter gain; obtained from a matrix $\Sigma^{\wedge}_{k|k-1}$,

$R_{e,k}$: an auxiliary variable; a 2×2 matrix,

χ_k: a conversion factor; a 2×2 matrix,

N: a dimension (a tap number) of the state vector; provided in advance,

$\tilde{\mu}_k$: an N+1-th 1×2 row vector of K^U_k; obtained by K^U_k,

m_k: an N×2 matrix including first to N-th rows of K^U_k; obtained by K^U_k,

W: a weighting matrix; determined by γ_f,

a_k: a forward linear prediction factor; an N-dimensional column vector,

b_k: a backward linear prediction factor; an N-dimensional column vector,

e_k: an ex-post forward prediction error; a two-dimensional column vector,

$\underline{e}_k$: an ex-post backward prediction error; a two-dimensional column vector, and

γ_f: an attenuation level; provided upon designing.

2. A system identification method for performing fast real-time identification for a system from input/output data, the method comprising:

estimating a state estimation value $x^{\wedge}_{k|k}$ of a state x_k of the system from a received signal u_k of an interface in an acoustic system or a communication system and from an observation signal y_k of an echo or a feedback by using a filter robust to disturbance in such a manner that a maximum energy gain from the disturbance to a filter error is, as an evaluation criterion, set smaller than a given upper limit γ_f^2; and

canceling out the echo or the feedback using a result of the state estimation value $x^{\wedge}_{k|k}$, wherein

the filter is, by Expressions (49), (50), (55), (58), (60) to (62) below, provided to a state space model represented by Expressions (11) to (13) below to estimate the state estimation value $x^{\wedge}_{k|k}$ of the state x_k of the system:

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \boldsymbol{G}_k \boldsymbol{w}_k, \qquad \boldsymbol{w}_k . \boldsymbol{x}_k \in \mathcal{R}^N \tag{11}$$

$$y_k = \boldsymbol{H}_k \boldsymbol{x}_k + v_k, \qquad y_k, v_k \in \mathcal{R} \tag{12}$$

$$z_k = \boldsymbol{H}_k \boldsymbol{x}_k, \qquad z_k \in \mathcal{R},\ \boldsymbol{H}_k \in \mathcal{R}^{1\times N} \tag{13}$$

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k-1|k-1} + \mu \frac{\tilde{\boldsymbol{K}}_k(:,1)}{\boldsymbol{R}_{e,k}(1,1)} (y_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k-1|k-1}) \tag{49), (50}$$

$$\boldsymbol{R}_{e,k} = \boldsymbol{R}_{e.k-1} + \frac{\boldsymbol{c}_k \boldsymbol{c}_k^T}{S_{k-1}} - \boldsymbol{c}_{k-N} \tilde{\boldsymbol{\mu}}_k \tag{55}$$

$$\begin{bmatrix} \tilde{\boldsymbol{K}}_k \\ \tilde{\boldsymbol{\mu}}_k \end{bmatrix} = \begin{bmatrix} \boldsymbol{c}_k / S_{k-1} \\ \tilde{\boldsymbol{K}}_{k-1} \end{bmatrix} \tag{58}$$

$$S_k = \rho S_{k-1} + \boldsymbol{c}_k^T \boldsymbol{W} \tilde{\boldsymbol{c}}_k = \rho S_{k-1} + (1 - \gamma_f^{-2}) u_k^2 \tag{60}$$

$$\tilde{\boldsymbol{e}}_k = \boldsymbol{c}_k, \quad \boldsymbol{e}_k = \boldsymbol{c}_k, \quad \boldsymbol{c}_k = [u_k\ u_k]^T \tag{61), (62}$$

where C_k and W are defined by the following expressions:

$$\boldsymbol{C}_k = \begin{bmatrix} \boldsymbol{H}_k \\ \boldsymbol{H}_k \end{bmatrix}, \quad \boldsymbol{W} = \begin{bmatrix} 1 & 0 \\ 0 & -\gamma_f^{-2} \end{bmatrix},$$

$\boldsymbol{c}_k \in \mathcal{R}^{2\times 1}$ is a first column vector of $\boldsymbol{C}_k = [\boldsymbol{c}_k, \ldots, \boldsymbol{c}_{k-N+1}]$,

$$\check{\boldsymbol{C}}_k = \begin{bmatrix} u_k & \cdots & u_{k-N} \\ \\ u_k & \cdots & u_{k-N} \end{bmatrix} \tag{63}$$

$$\check{\boldsymbol{C}}_k(:,1) = \boldsymbol{c}_k = [u_k \; u_k]^T, \quad \check{\boldsymbol{C}}_k(:,i) = \check{\boldsymbol{C}}_{k-1}(:,i-1), \quad i = 2, \cdots, N+1 \tag{64}$$

$H_k = [u_k, \ldots, u_{k-N+1}]^T$,

x_k: a state vector or merely a state; unknown, and this is a target for estimation,

x_0: an initial state; unknown,

w_k: system noise; unknown,

v_k: observation noise; unknown,

y_k: an observation signal; taken as filter input, and known,

z_k: an output signal; unknown,

G_k: a drive matrix; known upon execution,

H_k: an observation matrix, known,

$x^\wedge_{k|k}$: an estimation value, using observation signals y_0 to y_k, of the state x_k (an impulse response of an unknown system) at time k; provided by a filter equation,

$x^\wedge_{k+1|k}$: a single-step prediction value, using the observation signals y_0 to y_k, of a state x_{k+1} at the time k; provided by the filter equation,

$x^\wedge_{-1|-1}$: an initial estimation value of the state; unknown in a normal situation, but zero is used for convenience,

$K_{s,k}$: a filter gain; obtained from a matrix $\Sigma^\wedge_{k|k-1}$,

$R_{e,k}$: an auxiliary variable; a 2×2 matrix,

N: a dimension (a tap number) of the state vector; provided in advance,

$\tilde{\mu}_k$: an N+1-th 1×2 row vector of K^U_k; obtained by K^U_k,

W: a weighting matrix; determined by γ_f,

e_k: an ex-post forward prediction error; a two-dimensional column vector,

$\underline{e}_k$: an ex-post backward prediction error; a two-dimensional column vector,

γ_f: an attenuation level; provided upon designing, and

μ: a step size (a relaxation factor); a real number of $\mu > 0$, and provided in advance.

3. A system identification method for performing fast real-time identification for a system from input/output data, the method comprising:

estimating a state estimation value $x^{\wedge}_{k|k}$ of a state x_k of the system from a received signal u_k of an interface in an acoustic system or a communication system and from an observation signal y_k of an echo or a feedback by using a filter robust to disturbance in such a manner that a maximum energy gain from the disturbance to a filter error is, as an evaluation criterion, set smaller than a given upper limit γ_f^2; and

canceling out the echo or the feedback using a result of the state estimation value $x^{\wedge}_{k|k}$, wherein

the filter is, by Expressions (49), (50), (55), (58) below, provided to a state space model represented by Expressions (11) to (13) below to estimate the state estimation value $x^{\wedge}_{k|k}$ of the state x_k of the system:

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{x}_k + \boldsymbol{G}_k \boldsymbol{w}_k, \qquad \boldsymbol{w}_k, \boldsymbol{x}_k \in \mathcal{R}^N && (11) \\ y_k &= \boldsymbol{H}_k \boldsymbol{x}_k + v_k, \qquad y_k, v_k \in \mathcal{R} && (12) \\ z_k &= \boldsymbol{H}_k \boldsymbol{x}_k, \qquad z_k \in \mathcal{R},\ \boldsymbol{H}_k \in \mathcal{R}^{1\times N} && (13) \end{aligned}$$

$$\begin{aligned} \hat{\boldsymbol{x}}_{k|k} &= \hat{\boldsymbol{x}}_{k-1|k-1} + \mu \frac{\tilde{\boldsymbol{K}}_k(:,1)}{\boldsymbol{R}_{e,k}(1,1)} (y_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k-1|k-1}) && (49).(50) \\ \boldsymbol{R}_{e,k} &= \boldsymbol{R}_{e,k-1} + \epsilon_0 \boldsymbol{c}_k \boldsymbol{c}_k^T - \boldsymbol{c}_{k-N} \tilde{\boldsymbol{\mu}}_k && (55) \\ \begin{bmatrix} \tilde{\boldsymbol{K}}_k \\ \tilde{\boldsymbol{\mu}}_k \end{bmatrix} &= \begin{bmatrix} \epsilon_0 \boldsymbol{c}_k \\ \tilde{\boldsymbol{K}}_{k-1} \end{bmatrix} && (58) \end{aligned}$$

where C_k and W are defined by the following expressions:

$$\boldsymbol{C}_k = \begin{bmatrix} \boldsymbol{H}_k \\ \boldsymbol{H}_k \end{bmatrix}, \quad \boldsymbol{W} = \begin{bmatrix} 1 & 0 \\ 0 & -\gamma_f^{-2} \end{bmatrix},$$

$\boldsymbol{c}_k \in \mathcal{R}^{2\times 1}$ is a first column vector of $\boldsymbol{C}_k = [\boldsymbol{c}_k, \ldots, \boldsymbol{c}_{k-N+1}]$,

$$\breve{\boldsymbol{C}}_k = \begin{bmatrix} u_k & \cdots & u_{k-N} \\ u_k & \cdots & u_{k-N} \end{bmatrix} \qquad (63)$$

$$\breve{\boldsymbol{C}}_k(:,1) = \boldsymbol{c}_k = [u_k\ u_k]^T, \quad \breve{\boldsymbol{C}}_k(:,i) = \breve{\boldsymbol{C}}_{k-1}(:,i-1), \quad i = 2, \cdots, N+1 \qquad (64)$$

$H_k = [u_k, \ldots, u_{k-N+1}]^T$,

x_k: a state vector or merely a state; unknown, and this is a target for estimation,

x_0: an initial state; unknown,

w_k: system noise; unknown,

v_k: observation noise; unknown,

y_k: an observation signal; taken as filter input, and known,

z_k: an output signal; unknown,

G_k: a drive matrix; known upon execution,

H_k: an observation matrix, known,

$x^{\wedge}_{k|k}$: an estimation value, using observation signals y_0 to y_k, of the state x_k (an impulse response of an unknown system) at time k; provided by a filter equation,

$x^{\wedge}_{k+1|k}$: a single-step prediction value, using the observation signals y_0 to y_k, of a state x_{k+1} at the time k; provided by the filter equation,

$x^{\wedge}_{-1|-1}$: an initial estimation value of the state; unknown in a normal situation, but zero is used for convenience,

$K_{s,k}$: a filter gain; obtained from a matrix $\Sigma^{\wedge}_{k|k-1}$,

$R_{e,k}$: an auxiliary variable; a 2×2 matrix,

N: a dimension (a tap number) of the state vector; provided in advance,

$\tilde{\mu}_k$: an N+1-th 1×2 row vector of K^U_k; obtained by K^U_k,

W: a weighting matrix; determined by γ_f,

e_k: an ex-post forward prediction error; a two-dimensional column vector,

$\underline{e}_k$: an ex-post backward prediction error; a two-dimensional column vector,

ε_0: an optional positive constant; $\varepsilon_0 > 0$, and provided in advance,

γ_f: an attenuation level; provided upon designing, and

μ: a step size (a relaxation factor); a real number of $\mu > 0$, and provided in advance.

4. A system identification device for performing the system identification method according to claim 1, comprising:

a filter robust to disturbance, by setting a maximum energy gain from the disturbance to a filter error , as an evaluation criterion, smaller than a given upper limit γ_f^2,

wherein the filter is, by Expressions (49) to (62) below, provided to a state space model represented by Expressions (11) to (13) below to estimate a state estimation value $x^{\wedge}_{k|k}$ of a state x_k of the system:

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \boldsymbol{G}_k \boldsymbol{w}_k, \qquad \boldsymbol{w}_k . \boldsymbol{x}_k \in \mathcal{R}^N \tag{11}$$

$$y_k = \boldsymbol{H}_k \boldsymbol{x}_k + v_k, \qquad y_k, v_k \in \mathcal{R} \tag{12}$$

$$z_k = \boldsymbol{H}_k \boldsymbol{x}_k, \qquad z_k \in \mathcal{R},\ \boldsymbol{H}_k \in \mathcal{R}^{1\times N} \tag{13}$$

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k-1|k-1} + \boldsymbol{K}_{s,k}(y_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k-1|k-1}) \tag{49}$$

$$\boldsymbol{K}_{s,k} = \frac{\tilde{\boldsymbol{K}}_k(:,1)}{\boldsymbol{R}_{e,k}(1,1)} \tag{50}$$

$$\boldsymbol{b}_k = \boldsymbol{b}_{k-1} - \rho^{-1}\tilde{\boldsymbol{K}}_k \boldsymbol{W} \underline{\boldsymbol{e}}_k \tag{51}$$

$$\underline{S}_k = \rho \underline{S}_{k-1} + \underline{\boldsymbol{e}}_k^T \boldsymbol{W} \underline{\tilde{\boldsymbol{e}}}_k \tag{52}$$

$$\underline{\boldsymbol{e}}_k = \boldsymbol{\chi}_k^{-1} \underline{\tilde{\boldsymbol{e}}}_k \tag{53}$$

$$\boldsymbol{\chi}_k = \rho^{-1} \boldsymbol{R}_{e,k} \boldsymbol{W} \tag{54}$$

$$\boldsymbol{R}_{e,k} = \boldsymbol{R}_{e,k-1} + \frac{\tilde{\boldsymbol{e}}_k \tilde{\boldsymbol{e}}_k^T}{S_{k-1}} - \underline{\tilde{\boldsymbol{e}}}_k \tilde{\boldsymbol{\mu}}_k \tag{55}$$

$$\underline{\tilde{\boldsymbol{e}}}_k = \underline{S}_{k-1} \tilde{\boldsymbol{\mu}}_k^T \tag{56}$$

$$\tilde{\boldsymbol{K}}_k = \tilde{\boldsymbol{m}}_k - \boldsymbol{b}_{k-1} \tilde{\boldsymbol{\mu}}_k \tag{57}$$

$$\begin{bmatrix} \tilde{\boldsymbol{m}}_k \\ \tilde{\boldsymbol{\mu}}_k \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \tilde{\boldsymbol{K}}_{k-1} \end{bmatrix} + \begin{bmatrix} 1 \\ \boldsymbol{a}_{k-1} \end{bmatrix} \frac{\tilde{\boldsymbol{e}}_k^T}{S_{k-1}} \tag{58}$$

$$\boldsymbol{a}_k = \boldsymbol{a}_{k-1} - \rho^{-1} \tilde{\boldsymbol{K}}_{k-1} \boldsymbol{W} \boldsymbol{e}_k \tag{59}$$

$$S_k = \rho S_{k-1} + \boldsymbol{e}_k^T \boldsymbol{W} \tilde{\boldsymbol{e}}_k \tag{60}$$

$$\boldsymbol{e}_k = \boldsymbol{\chi}_{k-1}^{-1} \tilde{\boldsymbol{e}}_k \tag{61}$$

$$\tilde{\boldsymbol{e}}_k = \boldsymbol{c}_k + \boldsymbol{C}_{k-1} \boldsymbol{a}_{k-1} \tag{62}$$

where C_k and W are defined by the following expressions:

$$\boldsymbol{C}_k = \begin{bmatrix} \boldsymbol{H}_k \\ \boldsymbol{H}_k \end{bmatrix}, \quad \boldsymbol{W} = \begin{bmatrix} 1 & 0 \\ 0 & -\gamma_f^{-2} \end{bmatrix},$$

$\boldsymbol{c}_k \in \mathcal{R}^{2\times 1}$ is a first column vector of $\boldsymbol{C}_k = [\boldsymbol{c}_k, \ldots, \boldsymbol{c}_{k-N+1}]$,

$$\breve{\boldsymbol{C}}_k = \begin{bmatrix} u_k & \cdots & u_{k-N} \\ \\ u_k & \cdots & u_{k-N} \end{bmatrix} \tag{63}$$

$$\breve{\boldsymbol{C}}_k(:,1) = \boldsymbol{c}_k = [u_k\ u_k]^T, \quad \breve{\boldsymbol{C}}_k(:,i) = \breve{\boldsymbol{C}}_{k-1}(:,i-1), \quad i = 2, \cdots, N+1 \tag{64}$$

$H_k = [u_k, \ldots, u_{k-N+1}]^T$,

x_k: a state vector or merely a state; unknown, and this is a target for estimation,

x_0: an initial state; unknown,

w_k: system noise; unknown,

v_k: observation noise; unknown,

y_k: an observation signal; taken as filter input, and known,

z_k: an output signal; unknown,

G_k: a drive matrix; known upon execution,

H_k: an observation matrix, known,

$x^{\wedge}_{k|k}$: an estimation value, using observation signals y_0 to y_k, of the state x_k (an impulse response of an unknown system) at time k; provided by a filter equation,

$x^{\wedge}_{k+1|k}$: a single-step prediction value, using the observation signals y_0 to y_k, of a state x_{k+1} at the time k; provided by the filter equation,

$x^{\wedge}_{-1|-1}$: an initial estimation value of the state; unknown in a normal situation, but zero is used for convenience,

$K_{s,k}$: a filter gain; obtained from a matrix $\Sigma^{\wedge}_{k|k-1}$,

$R_{e,k}$: an auxiliary variable; a 2×2 matrix,

χ_k: a conversion factor; a 2×2 matrix,

N: a dimension (a tap number) of the state vector; provided in advance,

$\tilde{\mu}_k$: an N+1-th 1×2 row vector of K^U_k; obtained by K^U_k,

m_k: an N×2 matrix including first to N-th rows of K^U_k; obtained by K^U_k,

W: a weighting matrix; determined by γ_f,

a_k: a forward linear prediction factor; an N-dimensional column vector,

b_k: a backward linear prediction factor; an N-dimensional column vector,

e_k: an ex-post forward prediction error; a two-dimensional column vector,

$\underline{e}_k$: an ex-post backward prediction error; a two-dimensional column vector, and

γ_f: an attenuation level; provided upon designing.

5. A system identification device for performing the system identification method according to claim 2, comprising:

a filter robust to disturbance, by setting a maximum energy gain from the disturbance to a filter error, as an evaluation criterion, smaller than a given upper limit γ_f^2,

wherein the filter is, by Expressions (49), (50), (55), (58), (60) to (62) below, provided to a state space model represented by Expressions (11) to (13) below to estimate a state estimation value $x^{\wedge}_{k|k}$ of a state x_k of the system:

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \boldsymbol{G}_k \boldsymbol{w}_k, \qquad \boldsymbol{w}_k . \boldsymbol{x}_k \in \mathcal{R}^N \tag{11}$$

$$y_k = \boldsymbol{H}_k \boldsymbol{x}_k + v_k, \qquad y_k, v_k \in \mathcal{R} \tag{12}$$

$$z_k = \boldsymbol{H}_k \boldsymbol{x}_k, \qquad z_k \in \mathcal{R},\ \boldsymbol{H}_k \in \mathcal{R}^{1 \times N} \tag{13}$$

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k-1|k-1} + \mu \frac{\tilde{\boldsymbol{K}}_k(:,1)}{\boldsymbol{R}_{e,k}(1,1)} (y_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k-1|k-1}) \tag{49), (50}$$

$$\boldsymbol{R}_{e,k} = \boldsymbol{R}_{e.k-1} + \frac{\boldsymbol{c}_k \boldsymbol{c}_k^T}{S_{k-1}} - \boldsymbol{c}_{k-N} \tilde{\boldsymbol{\mu}}_k \tag{55}$$

$$\begin{bmatrix} \tilde{\boldsymbol{K}}_k \\ \tilde{\boldsymbol{\mu}}_k \end{bmatrix} = \begin{bmatrix} \boldsymbol{c}_k / S_{k-1} \\ \tilde{\boldsymbol{K}}_{k-1} \end{bmatrix} \tag{58}$$

$$S_k = \rho S_{k-1} + \boldsymbol{c}_k^T \boldsymbol{W} \tilde{\boldsymbol{c}}_k = \rho S_{k-1} + (1 - \gamma_f^{-2}) u_k^2 \tag{60}$$

$$\tilde{\boldsymbol{e}}_k = \boldsymbol{c}_k, \quad \boldsymbol{e}_k = \boldsymbol{c}_k, \quad \boldsymbol{c}_k = [u_k\ u_k]^T \tag{61), (62}$$

where C_k and W are defined by the following expressions:

$$\boldsymbol{C}_k = \begin{bmatrix} \boldsymbol{H}_k \\ \boldsymbol{H}_k \end{bmatrix}, \quad \boldsymbol{W} = \begin{bmatrix} 1 & 0 \\ 0 & -\gamma_f^{-2} \end{bmatrix},$$

$\boldsymbol{c}_k \in \mathcal{R}^{2 \times 1}$ is a first column vector of $\boldsymbol{C}_k = [\boldsymbol{c}_k, \ldots, \boldsymbol{c}_{k-N+1}]$,

$$\breve{\boldsymbol{C}}_k = \begin{bmatrix} u_k & \cdots & u_{k-N} \\ & & \\ u_k & \cdots & u_{k-N} \end{bmatrix} \tag{63}$$

$$\breve{\boldsymbol{C}}_k(:,1) = \boldsymbol{c}_k = [u_k\ u_k]^T, \quad \breve{\boldsymbol{C}}_k(:,i) = \breve{\boldsymbol{C}}_{k-1}(:,i-1), \quad i = 2, \cdots, N+1 \tag{64}$$

$H_k = [u_k, \ldots, u_{k-N+1}]^T$,

x_k: a state vector or merely a state; unknown, and this is a target for estimation,

x_0: an initial state; unknown,

w_k: system noise; unknown,

v_k: observation noise; unknown,

y_k: an observation signal; taken as filter input, and known,

z_k: an output signal; unknown,

G_k: a drive matrix; known upon execution,

H_k: an observation matrix, known,

$x^\wedge_{k|k}$: an estimation value, using observation signals y_0 to y_k, of the state x_k (an impulse response of an unknown system) at time k; provided by a filter equation,

$x^{\wedge}_{k+1|k}$: a single-step prediction value, using the observation signals y_0 to y_k, of a state x_{k+1} at the time k; provided by the filter equation,

$x^{\wedge}_{-1|-1}$: an initial estimation value of the state; unknown in a normal situation, but zero is used for convenience,

$K_{s,k}$: a filter gain; obtained from a matrix $\Sigma^{\wedge}_{k|k-1}$,

$R_{e,k}$: an auxiliary variable; a 2×2 matrix,

N: a dimension (a tap number) of the state vector; provided in advance,

$\tilde{\mu}_k$: an N+1-th 1×2 row vector of K^U_k; obtained by K^U_k,

W: a weighting matrix; determined by γ_f,

e_k: an ex-post forward prediction error; a two-dimensional column vector,

$\underline{e}_k$: an ex-post backward prediction error; a two-dimensional column vector,

γ_f: an attenuation level; provided upon designing, and

μ: a step size (a relaxation factor); a real number of $\mu > 0$, and provided in advance.

6. The system identification device according to claim 5, wherein

for Expressions (58), (55), (49), (50), the following expressions modified from Expressions (58), (55), (49), (50) are used according to Expression (60):

$$S_k = \rho S_{k-1} + (1 - \gamma_f^{-2}) u_k^2$$

$$\tilde{\boldsymbol{K}}_k(:,1) = \begin{bmatrix} u_k / S_{k-1} \\ \tilde{\boldsymbol{K}}_{k-1}(1:N-1,1) \end{bmatrix}$$

$$R_{e,k}(1,1) = R_{e,k-1}(1,1) + \frac{u_k^2}{S_{k-1}} - u_{k-N} \tilde{\boldsymbol{K}}_{k-1}(N,1)$$

$$\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_{k-1} + \mu \tilde{\boldsymbol{K}}_k(:,1) \frac{y_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k-1}}{R_{e,k}(1,1)}$$

where μ is the step size and a real number of $\mu > 0$.

7. The system identification device according to claim 5, wherein

the filter sets $\gamma_f = 1.0$, $\rho = 1.0$, $S_k = S_{k-1} = S_{-1} = 1/\varepsilon_0$ to use degenerated expressions including Expressions (49), (50), (55), (58) below for the state space model represented by Expressions (11) to (13) below:

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \boldsymbol{G}_k \boldsymbol{w}_k, \qquad \boldsymbol{w}_k, \boldsymbol{x}_k \in \mathcal{R}^N \tag{11}$$

$$y_k = \boldsymbol{H}_k \boldsymbol{x}_k + v_k, \qquad y_k, v_k \in \mathcal{R} \tag{12}$$

$$z_k = \boldsymbol{H}_k \boldsymbol{x}_k, \qquad z_k \in \mathcal{R},\ \boldsymbol{H}_k \in \mathcal{R}^{1\times N} \tag{13}$$

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k-1|k-1} + \mu \frac{\tilde{\boldsymbol{K}}_k(:,1)}{\boldsymbol{R}_{e,k}(1,1)} (y_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k-1|k-1}) \tag{49). (50}$$

$$\boldsymbol{R}_{e,k} = \boldsymbol{R}_{e,k-1} + \epsilon_0 \boldsymbol{c}_k \boldsymbol{c}_k^T - \boldsymbol{c}_{k-N} \tilde{\boldsymbol{\mu}}_k \tag{55}$$

$$\begin{bmatrix} \tilde{\boldsymbol{K}}_k \\ \tilde{\boldsymbol{\mu}}_k \end{bmatrix} = \begin{bmatrix} \epsilon_0 \boldsymbol{c}_k \\ \tilde{\boldsymbol{K}}_{k-1} \end{bmatrix} \tag{58}$$

where C_k and W are defined by the following expressions:

$$\boldsymbol{C}_k = \begin{bmatrix} \boldsymbol{H}_k \\ \boldsymbol{H}_k \end{bmatrix}, \quad \boldsymbol{W} = \begin{bmatrix} 1 & 0 \\ 0 & -\breve{\gamma}_f^{-2} \end{bmatrix},$$

$\boldsymbol{c}_k \in \mathcal{R}^{2\times 1}$ is the first column vector of $\boldsymbol{C}_k = [\boldsymbol{c}_k, \ldots, \boldsymbol{c}_{k-N+1}]$,

$$\breve{\boldsymbol{C}}_k = \begin{bmatrix} u_k & \cdots & u_{k-N} \\ u_k & \cdots & u_{k-N} \end{bmatrix} \tag{63}$$

$$\breve{\boldsymbol{C}}_k(:,1) = \boldsymbol{c}_k = [u_k\ u_k]^T, \quad \breve{\boldsymbol{C}}_k(:,i) = \breve{\boldsymbol{C}}_{k-1}(:,i-1), \quad i = 2, \cdots, N+1 \tag{64}$$

$H_k = [u_k, \ldots, u_{k-N+1}]^T$,

ε_0: an optional positive constant; $\varepsilon_0 > 0$, and provided in advance, and

μ: the step size (the relaxation factor); a real number of $\mu > 0$, and provided in advance.

8. A system identification device for performing the system identification method according to claim 3, comprising:

a filter robust to disturbance, by setting a maximum energy gain from the disturbance to a filter error, as an evaluation criterion, smaller than a given upper limit γ_f^2,

wherein the filter is, by Expressions (49), (50), (55), (58) below, provided to a state space model represented by Expressions (11) to (13) below to estimate a state estimation value $x^{\wedge}_{k|k}$ of a state x_k of the system:

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \boldsymbol{G}_k \boldsymbol{w}_k, \qquad \boldsymbol{w}_k . \boldsymbol{x}_k \in \mathcal{R}^N \tag{11}$$

$$y_k = \boldsymbol{H}_k \boldsymbol{x}_k + v_k, \qquad y_k, v_k \in \mathcal{R} \tag{12}$$

$$z_k = \boldsymbol{H}_k \boldsymbol{x}_k, \qquad z_k \in \mathcal{R},\ \boldsymbol{H}_k \in \mathcal{R}^{1 \times N} \tag{13}$$

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k-1|k-1} + \mu \frac{\tilde{\boldsymbol{K}}_k(:,1)}{\boldsymbol{R}_{e,k}(1,1)} (y_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k-1|k-1}) \tag{49), (50}$$

$$\boldsymbol{R}_{e,k} = \boldsymbol{R}_{e,k-1} + \epsilon_0 \boldsymbol{c}_k \boldsymbol{c}_k^T - \boldsymbol{c}_{k-N} \tilde{\boldsymbol{\mu}}_k \tag{55}$$

$$\begin{bmatrix} \tilde{\boldsymbol{K}}_k \\ \tilde{\boldsymbol{\mu}}_k \end{bmatrix} = \begin{bmatrix} \epsilon_0 \boldsymbol{c}_k \\ \tilde{\boldsymbol{K}}_{k-1} \end{bmatrix} \tag{58}$$

where C_k and W are defined by the following expressions:

$$\boldsymbol{C}_k = \begin{bmatrix} \boldsymbol{H}_k \\ \boldsymbol{H}_k \end{bmatrix}, \quad \boldsymbol{W} = \begin{bmatrix} 1 & 0 \\ 0 & -\breve{\gamma}_f^{-2} \end{bmatrix},$$

$\boldsymbol{c}_k \in \mathcal{R}^{2\times 1}$ is a first column vector of $\boldsymbol{C}_k = [\boldsymbol{c}_k, \ldots, \boldsymbol{c}_{k-N+1}]$,

$$\breve{\boldsymbol{C}}_k = \begin{bmatrix} u_k & \cdots & u_{k-N} \\ u_k & \cdots & u_{k-N} \end{bmatrix} \tag{63}$$

$$\breve{\boldsymbol{C}}_k(:,1) = \boldsymbol{c}_k = [u_k\ u_k]^T, \quad \breve{\boldsymbol{C}}_k(:,i) = \breve{\boldsymbol{C}}_{k-1}(:,i-1), \quad i = 2, \cdots, N+1 \tag{64}$$

$H_k = [u_k, \ldots, u_{k-N+1}]^T$,

x_k: a state vector or merely a state; unknown, and this is a target for estimation,

x_0: an initial state; unknown,

w_k: system noise; unknown,

v_k: observation noise; unknown,

y_k: an observation signal; taken as filter input, and known,

z_k: an output signal; unknown,

G_k: a drive matrix; known upon execution,

H_k: an observation matrix, known,

$x^{\wedge}_{k|k}$: an estimation value, using observation signals y_0 to y_k, of the state x_k (an impulse response of an unknown system) at time k; provided by a filter equation,

$x^{\wedge}_{k+1|k}$: a single-step prediction value, using the observation signals y_0 to y_k, of a state x_{k+1} at the time k; provided by the filter equation,

$x^{\wedge}_{-1|-1}$: an initial estimation value of the state; unknown in a normal situation, but zero is used for convenience,

$K_{s,k}$: a filter gain; obtained from a matrix $\Sigma^{\wedge}_{k|k-1}$,

$R_{e,k}$: an auxiliary variable; a 2×2 matrix,

N: a dimension (a tap number) of the state vector; provided in advance,

$\tilde{\mu}_k$: an N+1-th 1×2 row vector of K^U_k; obtained by K^U_k,

W: a weighting matrix; determined by γ_f,

e_k: an ex-post forward prediction error; a two-dimensional column vector,

$\underline{e}_k$: an ex-post backward prediction error; a two-dimensional column vector,

ε_0: an optional positive constant; $\varepsilon_0 > 0$, and provided in advance,

γ_f: an attenuation level; provided upon designing, and

μ: a step size (a relaxation factor); a real number of $\mu > 0$, and provided in advance.

9. The system identification device according to claim 8, wherein

for Expressions (58), (55), (49), (50), the following expressions modified from Expressions (58), (55), (49), (50) are used:

$$\tilde{\boldsymbol{K}}_k(:,1) = \begin{bmatrix} \epsilon_0 u_k \\ \tilde{\boldsymbol{K}}_{k-1}(1:N-1,1) \end{bmatrix}$$

$$R_{e,k}(1,1) = R_{e,k-1}(1,1) + \epsilon_0 u_k^2 - u_{k-N}\tilde{\boldsymbol{K}}_{k-1}(N,1)$$

$$\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_{k-1} + \mu\tilde{\boldsymbol{K}}_k(:,1)\frac{y_k - \boldsymbol{H}_k\hat{\boldsymbol{x}}_{k-1}}{R_{e,k}(1,1)}$$

where μ is the step size and a real number of $\mu > 0$.

10. The system identification device according to claim 4, further comprising:

a processing section configured to execute the filter,

wherein the processing section performs

a first step of reading an initial state of a recursive expression from a storage section or receiving the initial state from an input section or an interface section, thereby initializing a recursive variable of the filter,

a second step of determining whether or not the time k exceeds a preset data length L, thereby initializing and restarting the filter gain or terminating processing when the time k exceeds the data length L,

a third step of receiving, when the time k is equal to or less than the data length L, an input signal u_k from the input section or the interface section or reading the input signal u_k from the storage section, thereby updating an augmented observation matrix C^U_k by means of the input signal u_k and a shift characteristic $C^U_k = [c_k, C_{k-1}] = [C_k, c_{k-N}]$,

a fourth step of executing a filter algorithm per unit time, the filter algorithm including processing of calculating the expressions for providing the filter with a second term on a right-hand side of Expression (49) and a relaxation factor μ (μ is a real number of $\mu > 0$) in descending order from Expression (62) to Expression (49),

a fifth step of updating a filter equation including Expression (49) and storing the state estimation value $x^{\wedge}_{k|k}$ in the storage section and/or outputting the state estimation value $x^{\wedge}_{k|k}$ from an output section or the interface section, and

a sixth step of advancing the time k and repeating the processing of the first to fourth steps.

11. The system identification device according to claim 7, further comprising:

a processing section configured to execute the filter,

wherein the processing section performs

a first step of reading an initial state of a recursive expression from a storage section or receiving the initial state from an input section or an interface section, thereby initializing a recursive variable of the filter,

a second step of determining whether or not the time k exceeds a preset data length L, thereby initializing and restarting the filter gain or terminating processing when the time k exceeds the data length L,

a third step of receiving, when the time k is equal to or less than the data length L, an input signal u_k from the input section or the interface section or reading the input signal u_k from the storage section, thereby updating an augmented observation matrix C^U_k by means of the input signal u_k and a shift characteristic $C^U_k = [c_k, C_{k-1}] = [C_k, c_{k-N}]$,

a fourth step of executing a filter algorithm per unit step size, the filter algorithm including processing of calculating the expressions for providing the filter with Expression (49) and

the step size μ (μ is a real number of μ > 0) in descending order from Expression (58) to Expression (49),

a fifth step of updating a filter equation including Expression (49) and storing the state estimation value $x^{\wedge}_{k|k}$ in the storage section and/or outputting the state estimation value $x^{\wedge}_{k|k}$ from an output section or the interface section, and

a sixth step of advancing the time k and repeating the processing of the first to fourth steps.

12. The system identification device according to claim 4, wherein

the evaluation criterion satisfies Expression (14) below for the state space model:

$$\sup_{\boldsymbol{x}_0,\{\boldsymbol{w}_i\},\{v_i\}} \frac{\sum_{i=0}^{k} \|e_{f,i}\|^2/\rho}{\|\boldsymbol{x}_0 - \check{\boldsymbol{x}}_0\|^2_{\boldsymbol{\Sigma}_0^{-1}} + \sum_{i=0}^{k} \|\boldsymbol{w}_i\|^2 + \sum_{i=0}^{k} \|v_i\|^2/\rho} < \gamma_f^2 \tag{14}$$

where $e_{f,i}$: a filter error; $e_{f,i} = z^{v}_{i|i} - H_i x_i$, $z^{v}_{i|i}$ is an output estimation value $H_i x^{\wedge}_{i|i}$,

Σ_0^{-1}: an inverse matrix of a weighting matrix representing uncertainty of the state; Σ_0 is known,

ρ: a forgetting factor; when γ_f is determined, determined by $\rho = 1 - \chi(\gamma_f)$, and can be also determined independently of γ_f, and

γ_f: the attenuation level; provided upon designing.

13. The system identification device according to claim 10, wherein

the processing section of the filter executes, at the third step, a step of determining a scalar existence condition of an expression below before or after execution of the processing, and when Expression (30) is not satisfied, terminates or restarts the processing:

$$-\varrho\hat{\Xi}_i + \rho\gamma_f^2 > 0, \quad i = 0, \ldots, k \tag{30}$$

where $\varrho, \hat{\Xi}_i$ are defined by expressions below:

$$\varrho = 1 - \gamma_f^2, \quad \hat{\Xi}_i = \frac{\rho \boldsymbol{H}_i \boldsymbol{K}_{s,i}}{1 - \boldsymbol{H}_i \boldsymbol{K}_{s,i}} = \frac{\rho \boldsymbol{H}_i \boldsymbol{K}_i(:,1)}{1 - (1-\gamma_f{}^2)\boldsymbol{H}_i \boldsymbol{K}_i(:,1)} \tag{31}$$

14. The system identification device according to claim 10, wherein

the processing section of the filter further calculates, at the fifth step, an output estimation value $z^{v}_{k|k}$,$z^{v}_{k|k-1}$ according to Expression (48) below, thereby storing the output estimation value in a storage section and/or outputting the output estimation value from the output section or an interface section:

$$z^{\wedge}_{k|k} = H_k x^{\wedge}_{k|k} \quad (48)$$

where $z^{\wedge}_{k|k}$: an estimation value $H_k x^{\wedge}_{k|k}$, using the observation signals y_0 to y_k, of output z_k at the time k; provided by the filter equation, and

$z^{\wedge}_{k|k-1}$: a single-step prediction value $H_k x^{\wedge}_{k-1|k-1}$, using the observation signals y_0 to y_{k-1}, of the output z_k at the time k; provided by the filter equation.

15. The system identification device according to claim 4, wherein

the filter is applied to obtain the state estimation value $x^{\wedge}_{k|k}$, and

pseudo echo $d^{\wedge}_k$ is estimated as in Expression (66) below, and is subtracted from echo to cancel the echo:

$$\hat{d}_k = \sum_{i=0}^{N-1} \hat{h}_i[k] u_{k-i}, \quad k = 0, 1, 2, \cdots \qquad (66)$$

where $[h^{\wedge}_0[k], \ldots, h^{\wedge}_{N-1}[k]]^T$ is $x^{\wedge}_{k-1|k-1}$ to $x^{\wedge}_{k|k}$, u_k is a reception signal at time t_k (= kT;T is a sampling cycle), and N is the tap number (known).

16. A non-transitory computer-readable recording medium recording a system identification program for causing a computer to execute the system identification method according to claim 1, wherein

the system identification program causes the computer to execute

a step of providing, by a processing section, a filter to a state space model represented by Expressions (11) to (13) below according to Expressions (49) to (62) below,

a step of reading, by the processing section, an initial state of a recursive expression from a storage section or receiving the initial state from an input section or an interface section, thereby initializing a recursive variable of the filter,

a step of setting, by the processing section, the filter such that a maximum energy gain from disturbance to a filter error is, as an evaluation criterion, set smaller than a given upper limit γ_f^2 to provide a filter robust to the disturbance and calculating the expressions for providing the filter in descending order to estimate a state estimate value $x^{\wedge}_{k|k}$ of a state x_k of the system, and

a step of storing, by the processing section, the state estimate value $x^{\wedge}_{k|k}$ in the storage section and/or outputting the state estimate value $x^{\wedge}_{k|k}$ from an output section or the interface section:

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{x}_k + \boldsymbol{G}_k \boldsymbol{w}_k, \qquad \boldsymbol{w}_k . \boldsymbol{x}_k \in \mathcal{R}^N && (11) \\ y_k &= \boldsymbol{H}_k \boldsymbol{x}_k + v_k, \qquad y_k, v_k \in \mathcal{R} && (12) \\ z_k &= \boldsymbol{H}_k \boldsymbol{x}_k, \qquad z_k \in \mathcal{R},\ \boldsymbol{H}_k \in \mathcal{R}^{1\times N} && (13) \end{aligned}$$

$$\begin{aligned} \hat{\boldsymbol{x}}_{k|k} &= \hat{\boldsymbol{x}}_{k-1|k-1} + \boldsymbol{K}_{s,k}(y_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k-1|k-1}) && (49) \\ \boldsymbol{K}_{s,k} &= \frac{\tilde{\boldsymbol{K}}_k(:,1)}{\boldsymbol{R}_{e,k}(1,1)} && (50) \\ \boldsymbol{b}_k &= \boldsymbol{b}_{k-1} - \rho^{-1}\tilde{\boldsymbol{K}}_k \boldsymbol{W} \underline{\boldsymbol{e}}_k && (51) \\ \underline{S}_k &= \rho \underline{S}_{k-1} + \underline{\boldsymbol{e}}_k^T \boldsymbol{W} \underline{\tilde{\boldsymbol{e}}}_k && (52) \\ \underline{\boldsymbol{e}}_k &= \boldsymbol{\chi}_k^{-1} \underline{\tilde{\boldsymbol{e}}}_k && (53) \\ \boldsymbol{\chi}_k &= \rho^{-1} \boldsymbol{R}_{e,k} \boldsymbol{W} && (54) \\ \boldsymbol{R}_{e,k} &= \boldsymbol{R}_{e,k-1} + \frac{\tilde{\boldsymbol{e}}_k \tilde{\boldsymbol{e}}_k^T}{S_{k-1}} - \underline{\tilde{\boldsymbol{e}}}_k \tilde{\boldsymbol{\mu}}_k && (55) \\ \underline{\tilde{\boldsymbol{e}}}_k &= \underline{S}_{k-1} \tilde{\boldsymbol{\mu}}_k^T && (56) \\ \tilde{\boldsymbol{K}}_k &= \tilde{\boldsymbol{m}}_k - \boldsymbol{b}_{k-1} \tilde{\boldsymbol{\mu}}_k && (57) \\ \begin{bmatrix} \tilde{\boldsymbol{m}}_k \\ \tilde{\boldsymbol{\mu}}_k \end{bmatrix} &= \begin{bmatrix} \boldsymbol{0} \\ \tilde{\boldsymbol{K}}_{k-1} \end{bmatrix} + \begin{bmatrix} 1 \\ \boldsymbol{a}_{k-1} \end{bmatrix} \frac{\tilde{\boldsymbol{e}}_k^T}{S_{k-1}} && (58) \\ \boldsymbol{a}_k &= \boldsymbol{a}_{k-1} - \rho^{-1} \tilde{\boldsymbol{K}}_{k-1} \boldsymbol{W} \boldsymbol{e}_k && (59) \\ S_k &= \rho S_{k-1} + \boldsymbol{e}_k^T \boldsymbol{W} \tilde{\boldsymbol{e}}_k && (60) \\ \boldsymbol{e}_k &= \boldsymbol{\chi}_{k-1}^{-1} \tilde{\boldsymbol{e}}_k && (61) \\ \tilde{\boldsymbol{e}}_k &= \boldsymbol{c}_k + \boldsymbol{C}_{k-1} \boldsymbol{a}_{k-1} && (62) \end{aligned}$$

where C_k and W are defined by the following expressions:

$$\boldsymbol{C}_k = \begin{bmatrix} \boldsymbol{H}_k \\ \boldsymbol{H}_k \end{bmatrix}, \quad \boldsymbol{W} = \begin{bmatrix} 1 & 0 \\ 0 & -\gamma_f^{-2} \end{bmatrix},$$

$\boldsymbol{c}_k \in \mathcal{R}^{2\times 1}$ is a first column vector of $\boldsymbol{C}_k = [\boldsymbol{c}_k, \ldots, \boldsymbol{c}_{k-N+1}]$,

$$\breve{\boldsymbol{C}}_k = \begin{bmatrix} u_k & \cdots & u_{k-N} \\ \\ u_k & \cdots & u_{k-N} \end{bmatrix} \tag{63}$$

$$\breve{\boldsymbol{C}}_k(:,1) = \boldsymbol{c}_k = [u_k\ u_k]^T, \quad \breve{\boldsymbol{C}}_k(:,i) = \breve{\boldsymbol{C}}_{k-1}(:,i-1), \quad i = 2, \cdots, N+1 \tag{64}$$

$H_k = [u_k, \ldots, u_{k-N+1}]^T$,

x_k: a state vector or merely a state; unknown, and this is a target for estimation,

x_0: an initial state; unknown,

w_k: system noise; unknown,

v_k: observation noise; unknown,

y_k: an observation signal; taken as filter input, and known,

z_k: an output signal; unknown,

G_k: a drive matrix; known upon execution,

H_k: an observation matrix, known,

$x^{\wedge}_{k|k}$: an estimation value, using observation signals y_0 to y_k, of the state x_k (an impulse response of an unknown system) at time k; provided by a filter equation,

$x^{\wedge}_{k+1|k}$: a single-step prediction value, using the observation signals y_0 to y_k, of a state x_{k+1} at the time k; provided by the filter equation,

$x^{\wedge}_{-1|-1}$: an initial estimation value of the state; unknown in a normal situation, but zero is used for convenience,

$K_{s,k}$: a filter gain; obtained from a matrix $\Sigma^{\wedge}_{k|k-1}$,

$R_{e,k}$: an auxiliary variable; a 2×2 matrix,

χ_k: a conversion factor; a 2×2 matrix,

N: a dimension (a tap number) of the state vector; provided in advance,

$\tilde{\mu}_k$: an N+1-th 1×2 row vector of K^U_k; obtained by K^U_k,

m_k: an N×2 matrix including first to N-th rows of K^U_k; obtained by K^U_k,

W: a weighting matrix; determined by γ_f,

a_k: a forward linear prediction factor; an N-dimensional column vector,

b_k: a backward linear prediction factor; an N-dimensional column vector,

e_k: an ex-post forward prediction error; a two-dimensional column vector,

$\underline{e}_k$: an ex-post backward prediction error; a two-dimensional column vector, and

γ_f: an attenuation level; provided upon designing.

17. A non-transitory computer-readable recording medium recording a system identification program for causing a computer to execute the system identification method according to claim 2, wherein

the system identification program causes the computer to execute

a step of providing, by a processing section, a filter to a state space model represented by Expressions (11) to (13) below according to Expressions (49), (50), (55), (58), (60) to (62) below,

a step of reading, by the processing section, an initial state of a recursive expression from a storage section or receiving the initial state from an input section or an interface section, thereby initializing a recursive variable of the filter,

a step of setting, by the processing section, the filter such that a maximum energy gain from disturbance to a filter error is, as an evaluation criterion, set smaller than a given upper limit γ_f^2 to provide a filter robust to the disturbance and calculating the expressions for providing the filter in descending order to estimate a state estimate value $x^{\wedge}_{k|k}$ of a state x_k of the system, and

a step of storing, by the processing section, the state estimate value $x^{\wedge}_{k|k}$ in the storage section and/or outputting the state estimate value $x^{\wedge}_{k|k}$ from an output section or the interface section:

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \boldsymbol{G}_k \boldsymbol{w}_k, \qquad \boldsymbol{w}_k . \boldsymbol{x}_k \in \mathcal{R}^N \tag{11}$$

$$y_k = \boldsymbol{H}_k \boldsymbol{x}_k + v_k, \qquad y_k, v_k \in \mathcal{R} \tag{12}$$

$$z_k = \boldsymbol{H}_k \boldsymbol{x}_k, \qquad z_k \in \mathcal{R},\ \boldsymbol{H}_k \in \mathcal{R}^{1\times N} \tag{13}$$

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k-1|k-1} + \mu \frac{\tilde{\boldsymbol{K}}_k(:,1)}{\boldsymbol{R}_{e,k}(1,1)} (y_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k-1|k-1}) \tag{49), (50}$$

$$\boldsymbol{R}_{e,k} = \boldsymbol{R}_{e.k-1} + \frac{\boldsymbol{c}_k \boldsymbol{c}_k^T}{S_{k-1}} - \boldsymbol{c}_{k-N} \tilde{\boldsymbol{\mu}}_k \tag{55}$$

$$\begin{bmatrix} \tilde{\boldsymbol{K}}_k \\ \tilde{\boldsymbol{\mu}}_k \end{bmatrix} = \begin{bmatrix} \boldsymbol{c}_k / S_{k\ 1} \\ \tilde{\boldsymbol{K}}_{k-1} \end{bmatrix} \tag{58}$$

$$S_k = \rho S_{k-1} + \boldsymbol{c}_k^T \boldsymbol{W} \tilde{\boldsymbol{c}}_k = \rho S_{k-1} + (1 - \gamma_f^{-2}) u_k^2 \tag{60}$$

$$\tilde{\boldsymbol{e}}_k = \boldsymbol{c}_k, \quad \boldsymbol{e}_k = \boldsymbol{c}_k, \quad \boldsymbol{c}_k = [u_k\ u_k]^T \tag{61), (62}$$

where C_k and W are defined by the following expressions:

$$\boldsymbol{C}_k = \begin{bmatrix} \boldsymbol{H}_k \\ \boldsymbol{H}_k \end{bmatrix}, \quad \boldsymbol{W} = \begin{bmatrix} 1 & 0 \\ 0 & -\gamma_f^{-2} \end{bmatrix},$$

$\boldsymbol{c}_k \in \mathcal{R}^{2\times 1}$ is a first column vector of $\boldsymbol{C}_k = [\boldsymbol{c}_k, \ldots, \boldsymbol{c}_{k-N+1}]$,

$$\check{\boldsymbol{C}}_k = \begin{bmatrix} u_k & \cdots & u_{k-N} \\ \\ u_k & \cdots & u_{k-N} \end{bmatrix} \tag{63}$$

$$\check{\boldsymbol{C}}_k(:,1) \quad = \quad \boldsymbol{c}_k = [u_k \ u_k]^T, \quad \check{\boldsymbol{C}}_k(:,i) = \check{\boldsymbol{C}}_{k-1}(:,i-1), \quad i = 2, \cdots, N+1 \tag{64}$$

$H_k = [u_k, \ldots, u_{k-N+1}]^T$,

x_k: a state vector or merely a state; unknown, and this is a target for estimation,

x_0: an initial state; unknown,

w_k: system noise; unknown,

v_k: observation noise; unknown,

y_k: an observation signal; taken as filter input, and known,

z_k: an output signal; unknown,

G_k: a drive matrix; known upon execution,

H_k: an observation matrix, known,

$x^{\wedge}_{k|k}$: an estimation value, using observation signals y_0 to y_k, of the state x_k (an impulse response of an unknown system) at time k; provided by a filter equation,

$x^{\wedge}_{k+1|k}$: a single-step prediction value, using the observation signals y_0 to y_k, of a state x_{k+1} at the time k; provided by the filter equation,

$x^{\wedge}_{-1|-1}$: an initial estimation value of the state; unknown in a normal situation, but zero is used for convenience,

$K_{s,k}$: a filter gain; obtained from a matrix $\Sigma^{\wedge}_{k|k-1}$,

$R_{e,k}$: an auxiliary variable; a 2×2 matrix,

N: a dimension (a tap number) of the state vector; provided in advance,

$\tilde{\mu}_k$: an N+1-th 1×2 row vector of K^U_k; obtained by K^U_k,

W: a weighting matrix; determined by γ_f,

e_k: an ex-post forward prediction error; a two-dimensional column vector,

$\underline{e}_k$: an ex-post backward prediction error; a two-dimensional column vector,

γ_f: an attenuation level; provided upon designing, and

μ: a step size (a relaxation factor); a real number of $\mu > 0$, and provided in advance.

18. A non-transitory computer-readable recording medium recording a system identification program for causing a computer to execute the system identification method according to claim 3, wherein

the system identification program causes the computer to execute

a step of providing, by a processing section, a filter to a state space model represented by Expressions (11) to (13) below according to Expressions (49), (50), (55), (58) below,

a step of reading, by the processing section, an initial state of a recursive expression from a storage section or receiving the initial state from an input section or an interface section, thereby initializing a recursive variable of the filter,

a step of setting, by the processing section, the filter such that a maximum energy gain from disturbance to a filter error is, as an evaluation criterion, set smaller than a given upper limit γ_f^2 to provide a filter robust to the disturbance and calculating the expressions for providing the filter in descending order to estimate a state estimate value $x^{\wedge}_{k|k}$ of a state x_k of the system, and

a step of storing, by the processing section, the state estimate value $x^{\wedge}_{k|k}$ in the storage section and/or outputting the state estimate value $x^{\wedge}_{k|k}$ from an output section or the interface section:

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{x}_k + \boldsymbol{G}_k \boldsymbol{w}_k, \qquad \boldsymbol{w}_k, \boldsymbol{x}_k \in \mathcal{R}^N && (11) \\ y_k &= \boldsymbol{H}_k \boldsymbol{x}_k + v_k, \qquad y_k, v_k \in \mathcal{R} && (12) \\ z_k &= \boldsymbol{H}_k \boldsymbol{x}_k, \qquad z_k \in \mathcal{R},\ \boldsymbol{H}_k \in \mathcal{R}^{1\times N} && (13) \end{aligned}$$

$$\begin{aligned} \hat{\boldsymbol{x}}_{k|k} &= \hat{\boldsymbol{x}}_{k-1|k-1} + \mu \frac{\tilde{\boldsymbol{K}}_k(:,1)}{\boldsymbol{R}_{e,k}(1,1)} (y_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k-1|k-1}) && (49), (50) \\ \boldsymbol{R}_{e,k} &= \boldsymbol{R}_{e,k-1} + \epsilon_0 \boldsymbol{c}_k \boldsymbol{c}_k^T - \boldsymbol{c}_{k-N} \tilde{\boldsymbol{\mu}}_k && (55) \\ \begin{bmatrix} \tilde{\boldsymbol{K}}_k \\ \tilde{\boldsymbol{\mu}}_k \end{bmatrix} &= \begin{bmatrix} \epsilon_0 \boldsymbol{c}_k \\ \tilde{\boldsymbol{K}}_{k-1} \end{bmatrix} && (58) \end{aligned}$$

where C_k and W are defined by the following expressions:

$$\boldsymbol{C}_k = \begin{bmatrix} \boldsymbol{H}_k \\ \boldsymbol{H}_k \end{bmatrix}, \quad \boldsymbol{W} = \begin{bmatrix} 1 & 0 \\ 0 & -\gamma_f^{-2} \end{bmatrix},$$

$\boldsymbol{c}_k \in \mathcal{R}^{2\times 1}$ is a first column vector of $\boldsymbol{C}_k = [\boldsymbol{c}_k, \ldots, \boldsymbol{c}_{k-N+1}]$,

$$\breve{\boldsymbol{C}}_k = \begin{bmatrix} u_k & \cdots & u_{k-N} \\ & & \\ u_k & \cdots & u_{k-N} \end{bmatrix} \tag{63}$$

$$\breve{\boldsymbol{C}}_k(:,1) \quad = \quad \boldsymbol{c}_k = [u_k \; u_k]^T, \quad \breve{\boldsymbol{C}}_k(:,i) = \breve{\boldsymbol{C}}_{k-1}(:,i-1), \quad i = 2, \cdots, N+1 \tag{64}$$

$H_k = [u_k, \ldots, u_{k-N+1}]^T$,

x_k: a state vector or merely a state; unknown, and this is a target for estimation,

x_0: an initial state; unknown,

w_k: system noise; unknown,

v_k: observation noise; unknown,

y_k: an observation signal; taken as filter input, and known,

z_k: an output signal; unknown,

G_k: a drive matrix; known upon execution,

H_k: an observation matrix, known,

$x^{\wedge}_{k|k}$: an estimation value, using observation signals y_0 to y_k, of the state x_k (an impulse response of an unknown system) at time k; provided by a filter equation,

$x^{\wedge}_{k+1|k}$: a single-step prediction value, using the observation signals y_0 to y_k, of a state x_{k+1} at the time k; provided by the filter equation,

$x^{\wedge}_{-1|-1}$: an initial estimation value of the state; unknown in a normal situation, but zero is used for convenience,

$K_{s,k}$: a filter gain; obtained from a matrix $\Sigma^{\wedge}_{k|k-1}$,

$R_{e,k}$: an auxiliary variable; a 2×2 matrix,

N: a dimension (a tap number) of the state vector; provided in advance,

$\mu^{\sim}_k$: an N+1-th 1×2 row vector of K^U_k; obtained by K^U_k,

W: a weighting matrix; determined by γ_f,

e_k: an ex-post forward prediction error; a two-dimensional column vector,

$\underline{e}_k$: an ex-post backward prediction error; a two-dimensional column vector,

ε_0: an optional positive constant; $\varepsilon_0 > 0$, and provided in advance,

γ_f: an attenuation level; provided upon designing, and

μ: a step size (a relaxation factor); a real number of $\mu > 0$, and provided in advance.