

METHOD FOR PLANNING OPTIMAL PATH HAVING MINIMUM TARGET POSITIONING ERROR OF WATER SURFACE AUTONOMOUS VEHICLE

TECHNICAL FIELD

The present invention relates to the field of path planning for water surface autonomous vehicles, specifically to an optimal path planning method for water surface autonomous vehicles with minimal target positioning error.

BACKGROUND

As one of the important tools for exploring the ocean, autonomous underwater vehicles (AUVs) are widely used in scientific research and commercial tasks, such as ocean resource tracking, seabed mapping, ocean monitoring, etc. In these tasks, if the position coordinates of the data cannot be obtained, the data is meaningless. However, the underwater environment cannot receive GPS signals, and how to better locate AUVs (targets) through water surface autonomous vehicles (ASVs) on the water surface is an urgent problem to be solved.

The existing technology has a significant drawback, which is that it only considers the situation where noise in the signal propagation process follows a Gaussian distribution. However, the marine environment is complex and ever-changing, and during signal propagation, especially in underwater environments, it is highly susceptible to the influence of factors such as plumes and eddies, resulting in nonlinear non Gaussian noise. In this way, the existing technology will not be able to locate the target well, which may lead to inaccurate positioning and other situations. In addition, in the scenario where ASVs are used to locate targets, the initial position of the target is often known, which is difficult to achieve in highly dynamic marine environments, and the initial position of the target is often inaccurate. How to better utilize ASVs to locate targets in nonlinear non Gaussian noise and uncertain initial target positions, plan their optimal paths, and minimize positioning errors is a challenge.

SUMMARY

The purpose of the present invention is to provide an optimal path planning method for water surface autonomous vehicles with the minimum target positioning error, which solve the problem of optimal path planning for surface vessels when the prior information of the target position is unknown and there are nonlinear non-Gaussian noises present, making it impossible to obtain a closed-form expression for the objective function, thus resulting in the inability to solve for the minimum localization error.

In order to achieve the above objectives, the present invention is implemented through the following technical solutions:

A method for optimal path planning of water surface autonomous vehicles with minimum target positioning error, characterized by the following steps:

S1, Constructing a system model (motion model and observation model) for water surface autonomous vehicles;

S2, Derive the optimal geometric relationship between different numbers of the water surface autonomous vehicle and the target;

S3, Acquire prior position information of the target by a gridded area gridded area method (for one or two water surface autonomous vehicles) or by a positioning method (for three water surface autonomous vehicles);

S4, using a Monte Carlo strategy to particleize nonlinear non-Gaussian ranging noise and construct an objective function closed-loop expression;

S5, solving the objective function by combining a nonlinear dynamic programming algorithm to obtain an optimal path.

Step S1 involves constructing a system model (motion model and observation model) for a water surface autonomous vehicle, which specifically includes:

S11, considering the known situation of the target water depth, mapping the 3D scene to the 2D solution, and assuming that the coordinate reference frame is an east-north-down geodetic coordinate reference frame, the target's position at time t is

$\mathbf{B}^t = [b_1^t, b_2^t]^T$, the position of the i-th water surface autonomous vehicle (ASVs)

at time t is $\mathbf{A}_i^t = [\mathbf{a}_{i1}^t, \mathbf{a}_{i2}^t]^T, i = 1, \dots, N$, where T represents transposition; If ASVs can perceive their azimuth angle $\phi(t)$, taking into account the input speed $v(t)$ and the angular velocity $r(t)$, the state corresponding to the ASVS can be expressed as:

$$\dot{\mathbf{A}}_i^t = \begin{bmatrix} v(t) \cos(\phi_i(t)) \\ v(t) \sin(\phi_i(t)) \\ r_i(t) \end{bmatrix}. \quad (1)$$

defining two orthogonal unit vectors, i.e.

$$[g(\theta), O(\theta)] = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \quad (2)$$

The motion model of ASVs without considering flow velocity is:

$$\mathbf{A}_i^t = \begin{cases} \mathbf{A}_i^{t-1} + \frac{v(t)}{r_i(t)} (O(\phi_i(t-1)) - O(\phi_i(t))), & \text{if } r_i(t) \neq 0 \\ \mathbf{A}_i^{t-1} + v(t) (g(\phi_i(t)) - g(\phi_i(t-1))), & \text{Others} \end{cases}, \quad (3)$$

Among them, $\mathbf{A}_i^{t-1}$ and $\mathbf{A}_i^t$ respectively represent the position of the i -th ASVs at the previous moment and the current moment; the $\phi_i(t-1)$ and $\phi_i(t)$ respectively represent the azimuth information of the i -th ASVs at the previous and current moments; when the angular velocity is not 0, it indicates that ASVs are performing curved motion during the corresponding time period, conversely, when the angular velocity is 0, it means that ASVs are performing approximately linear motion;

S12, the ASVS is provided with a sensor capable of receiving signal strength, the position of the sensor and the target is calculated by obtaining the signal intensity value of the sensor, and the observation model of the system is further constructed through the path loss model, that is,

$$z_i^t = P_0 - 10\mathcal{G} \log_{10} \left(\frac{\|\mathbf{B}^t - \mathbf{A}_i^t\|}{d_0} \right) + \eta_i^t \quad (4)$$

Among them, the z_i^t represents the signal power value of the i -th ASVs receiving the target at time t ; the P_0 represents the power value of the target transmission (considering that the transmission power value is equal at every moment); the $\mathcal{G}$ represents the path loss factor; $\|\mathbf{B}^t - \mathbf{A}_i^t\|$ represents the distance between the i -th ASVs and the target at time t ; the d_0 represents the reference distance, usually one meter; the η_i^t represents

the corresponding Gaussian noise (considering that the noise is equal at every moment, i.e. $\eta'_i = \eta$), its mean is zero and its variance is σ .

Due to the complexity and variability of the marine environment, its noise is not only Gaussian noise, but may also contain outliers, resulting in non Gaussian noise. Introducing the noise pollution factor β , whose value range is $[0,1]$, the probability distribution $p(\eta)$ corresponding to non-Gaussian noise can be expressed as:

$$p(\eta) = (1 - \beta)\Omega(\eta; 0, \sigma^2) + \beta\Phi(\eta). \quad (5)$$

Among them, the $\Omega(\eta; 0, \sigma^2)$ represents Gaussian probability distribution; the $\Phi(\eta)$ represents the probability distribution of non-Gaussian anomalous noise.

The step S2 derives the optimal geometric relationship between the water surface autonomous vehicle and its target, which specifically includes:

S21, The optimal geometric relationship exists only when the number of ASVs exceeds 2;

S22, It is derived that the optimal geometric relationship for two ASVs scenarios is a relative target angle of $\pi/2$;

Inference 1: Let α_{12} be the angle between ASVs 1 and 2, and $\alpha_{12} = \alpha_{21}$, for any fixed $\|\mathbf{B}' - \mathbf{A}'_i\|$, let the angle at which the determinant of the Fisher information matrix is maximized be $\pi/2$, i.e. $\alpha_{12} = \alpha_{21} = \pi/2$;

Proof 1: Let $\gamma_i = \tau / \|\mathbf{B}' - \mathbf{A}'_i\|^2$, where $\tau = (\mathbf{I}_\eta)^t \cdot \mathcal{G}^2$, and there exists a constant $\gamma > 0$ such that $\gamma_2 = \zeta\gamma_1$, at each moment, its corresponding function can be expressed as:

$$J^t = J_1^t + J_2^t = \frac{\varepsilon}{4} \left[\left(\sum_{i=1}^2 \frac{1}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 - \left(\sum_{i=1}^2 \frac{\cos(2\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 - \left(\sum_{i=1}^2 \frac{\sin(2\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 \right] \quad (6)$$

To minimize the target positioning error, it is necessary to maximize the determinant of equation (6), which is equivalent to minimizing equation (7), that is:

$$\left(\sum_{i=1}^2 \frac{\cos(2\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 + \left(\sum_{i=1}^2 \frac{\sin(2\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2. \quad (7)$$

By transformation, we can further obtain:

$$\begin{aligned} & \gamma_1^2 \left[\sin(2\phi_1(t)) + \zeta \sin(2\phi_2(t))^2 + (\cos(2\phi_1(t)) + \zeta \cos(2\phi_2(t)))^2 \right] \\ &= \gamma_1^2 [1 + \zeta + \zeta \cos(2\phi_1(t) - 2\phi_2(t))] \end{aligned} \quad (8)$$

It is obvious that when $\cos(2\phi_1(t) - 2\phi_2(t)) = -1$, the equation (18) can take the minimum value. Therefore, for $\lambda \in \square$, $\phi_2(t) - \phi_1(t) = \lambda\pi - \frac{\pi}{2}$; for two ASVs, when their angle is $\pi/2$, the optimal geometric relationship can be obtained, that is, $\alpha_{12} = \alpha_{21} = \phi_2(t) - \phi_1(t) \bmod(\pi) = \frac{\pi}{2}$. Certificate completed.

S23, For three ASVs scenarios, the optimal geometric relationship angles relative to the target are derived, including: 1) when two ASVs have a relative target angle of 0 or π , and another ASVs has an angle of $\pi/2$ relative to these two ASVs; 2) Three ASVs have a relative target angle of $2\pi/3$; 3) Two ASVs have a relative target angle of $\pi/3$, and another ASVs has a relative target angle of $2\pi/3$. The derivation process is as follows:

Assuming α_{12} , α_{13} , and α_{23} are the angular relationships between the i -th ASVs and the l -th ASVs, where $i, l \in \{1, 2, 3\}$, $\gamma_i = \tau / \|\mathbf{B}^t - \mathbf{A}_i^t\|^2$, $\tau = (\mathbf{I}_\eta)^t \cdot \mathcal{G}^2$, $\gamma_{il} = \gamma_i \cdot \gamma_l$ and $\alpha_{12} = \alpha_{21}$, $\alpha_{13} = \alpha_{31}$, $\alpha_{23} = \alpha_{32}$, the angular relationship between the i -th ASVs and the l -th ASVs can be expressed as:

$$\begin{aligned}\alpha_{12} &= \frac{1}{2} \arccos \left(\frac{\gamma_{13}^2 \gamma_{23}^2 - \gamma_{12}^2 \gamma_{23}^2 - \gamma_{12}^2 \gamma_{13}^2}{2\gamma_{12}^2 \gamma_{13} \gamma_{23}} \right) \\ \alpha_{13} &= \frac{1}{2} \arccos \left(\frac{\gamma_{12}^2 \gamma_{23}^2 - \gamma_{12}^2 \gamma_{13}^2 - \gamma_{13}^2 \gamma_{23}^2}{2\gamma_{12} \gamma_{13}^2 \gamma_{23}} \right) \\ \alpha_{23} &= \pi - \alpha_{12} - \alpha_{13}.\end{aligned}\tag{9}$$

If $\gamma_l > \sum_{1 \leq i \leq 3, i \neq l} \gamma_i$, means that one ASVs approaches the target, then:

$$\phi_r = \pm \frac{\pi}{2} + \phi_i, r \in \{1, 2, 3\} \setminus i.\tag{10}$$

Equation (10) can be further expressed as:

$$\begin{cases} \alpha_{il} = \frac{\pi}{2}, & \text{if } \gamma_{il} > \gamma_{rl}, \forall r l \in \{12, 13, 23\} \setminus \{il\} \\ \alpha_{il} = 0 \text{ or } \pi, & \text{others} \end{cases},\tag{11}$$

Among them, the $\setminus$ represents the representation of reduced sets;

When $\gamma_1 = \gamma_2 = \gamma_3$, there exists a special optimal geometric relationship, namely

$$\alpha_{12} = \alpha_{13} = \alpha_{23} = 2\pi/3 \text{ or } \alpha_{12} = \alpha_{23} = \pi/3, \alpha_{13} = 2\pi/3.$$

Step S3 obtains prior position information of the target by a gridded area method (for

one or two water surface autonomous vehicles) or by a positioning method (for three water surface autonomous vehicles), which specifically includes:

S31, When there is only one or two ASVs, quantify the uncertainty of the prior position of the target, grid the relevant areas, and use the intersection points in the grid as the possible positions of the target, as shown in Figure 2.

S32, When there are 3 ASVs and the prior position of the target is obtained through positioning technology, if $d_0 = 1$ m is obtained, equation (4) of S12 can be transformed to obtain:

$$\left\| \mathbf{B}^t - \mathbf{A}_i^t \right\| = 10^{\frac{P_0 - z_i^t}{10g}} \cdot 10^{\frac{\eta_i^t}{10g}} \quad (12)$$

Subsequently, by expanding the first-order Taylor series, we obtain:

$$\left\| \mathbf{B}^t - \mathbf{A}_i^t \right\| \approx 10^{\frac{P_0 - z_i^t}{10g}} \cdot \left(1 + \frac{\ln 10}{10g} \eta \right) \quad (13)$$

Construct a nonlinear least squares expression by square distance operation and combining weights, that is:

$$\arg \min_{\mathbf{B}^t} \sum_{i=1}^N \omega_i \left(\left\| \mathbf{B}^t - \mathbf{A}_i^t \right\|^2 - \left(\hat{d}_i^t \right)^2 \right)^2 \quad (14)$$

Where the $\hat{d}_i^t$ represents the corresponding estimated distance; $\omega_i = 1 - \hat{d}_i^t / \sum_{i=1}^N \hat{d}_i^t$

After expanding equation (14), it can be further transformed into a generalized trust region subproblem, namely:

$$J(\mathbf{y}) = \minimize \left\| \omega(\wp \mathbf{y} - \kappa) \right\|^2, \text{s.t. } \mathbf{y}^T \mathbf{D} \mathbf{y} + 2\mathbf{f}^T \mathbf{y} = 0, \quad (15)$$

Among them, $\mathbf{y} = \left[\left(\mathbf{B}^t \right)^T, \left\| \mathbf{B}^t \right\|^2 \right]^T$; $\boldsymbol{\omega} = \text{diag} \left(\left[\omega_1^2, \dots, \omega_N^2 \right] \right)$;

$$\wp = \begin{bmatrix} \left(\mathbf{A}_1^t \right)^T & -1 \\ \vdots & \vdots \\ \left(\mathbf{A}_N^t \right)^T & -1 \end{bmatrix}; \kappa = \begin{bmatrix} \left(\left\| \mathbf{A}_1^t \right\|^2 - \left(\hat{d}_1^t \right)^2 \right) \\ \vdots \\ \left(\left\| \mathbf{A}_N^t \right\|^2 - \left(\hat{d}_N^t \right)^2 \right) \end{bmatrix}; \mathbf{D} = \begin{bmatrix} \mathbf{I}_2 & \mathbf{0}_{2 \times 1} \\ \mathbf{0}_{1 \times 2} & 0 \end{bmatrix}; \mathbf{f} = \begin{bmatrix} \mathbf{0}_{2 \times 1} \\ -\frac{1}{2} \end{bmatrix} \quad (16)$$

$\mathbf{I}$ and $\mathbf{0}$ represent the identity matrix and zero matrix, respectively.

After obtaining the expression (15) of the generalized trust region subproblem, the

binary method is used to solve and obtain the prior position of the target.

The step S4 utilizes Monte Carlo strategy to particle the nonlinear non Gaussian ranging noise and construct a closed-loop expression of the objective function, which specifically includes:

S41, In order to minimize the positioning estimation error, the Claudius lower bound constructor in parameter estimation theory is introduced. The corresponding Fisher information matrix at time t is:

$$FIM^t = \left[\left(\frac{\partial p(\mathbf{B}^t | z_i^t)}{\partial \mathbf{B}^t} \right)^T \sigma^{-1} \left(\frac{\partial p(\mathbf{B}^t | z_i^t)}{\partial \mathbf{B}^t} \right) \right] \\ = \xi \cdot \frac{\mathcal{G}^2}{\sigma^2} \begin{bmatrix} \sum_{i=1}^N \frac{\cos^2(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} & \sum_{i=1}^N \frac{\cos(\phi_i(t)) \cdot \sin(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \\ \sum_{i=1}^N \frac{\cos(\phi_i(t)) \cdot \sin(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} & \sum_{i=1}^N \frac{\sin^2(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \end{bmatrix} \quad (17)$$

Due to the presence of outliers, equation (17) often cannot obtain its closed-loop expression; Therefore, the present invention utilizes Monte Carlo strategy to particle the nonlinear non Gaussian noise, and equation (17) can be expressed as:

$$FIM^t = \xi \cdot \mathcal{G}^2 \cdot (I_\eta)^t \cdot \begin{bmatrix} FIM_1^1 & FIM_1^2 \\ FIM_2^1 & FIM_2^2 \end{bmatrix} \quad (18)$$

Among them,

$$FIM_1^1 = \sum_{i=1}^N \frac{\cos^2(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2}, FIM_1^2 = \sum_{i=1}^N \frac{\cos(\phi_i(t)) \cdot \sin(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \\ FIM_2^2 = \sum_{i=1}^N \frac{\sin^2(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2}, FIM_2^1 = \sum_{i=1}^N \frac{\cos(\phi_i(t)) \cdot \sin(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \quad (19)$$

$$(I_\eta)^t = E_{p(\eta)} \left(\frac{\left\{ \nabla_\eta [p(\eta)]^t \right\}^2}{[p^2(\eta)]^t} \right) = \int \frac{\left\{ \nabla_\eta [p(\eta)]^t \right\}^2}{[p^2(\eta)]^t} [p(\eta)]^t d\eta \quad (20)$$

The $\nabla_\eta [p(\eta)]^t$ represents the corresponding first-order degree; Equation (20) represents the use of Monte Carlo strategy to particle the nonlinear non-Gaussian noise operation.

S42, For ASVs, in order to minimize the target localization error, it is necessary to maximize the determinant of the Fisher information matrix, that is:

$$\max \left\{ \det(FIM^t) \right\} = \max \left\{ \frac{\Xi}{4} \left[\left(\sum_{i=1}^N \frac{1}{\|B^t - A_i^t\|^2} \right)^2 - \left(\sum_{i=1}^N \frac{\cos(2\phi_i(t))}{\|B^t - A_i^t\|^2} \right)^2 - \left(\sum_{i=1}^N \frac{\sin(2\phi_i(t))}{\|B^t - A_i^t\|^2} \right)^2 \right] \right\} \quad (21)$$

Among them, $\Xi = \xi^2 \left[(I_\eta)^t \right]^2 \mathcal{G}^4$;

By performing logarithmic operations on equation (21), the objective function can be expressed as:

$$\arg \max_{|r| \leq c} \sum_{t=1}^M J^t = \arg \max_{|r| \leq c} \sum_{t=1}^M \ln \det(FIM^t) \Big|_{B^t, A_i^t, \phi_i(t)} \quad (22)$$

From equation (22), it can be seen that if angular velocity is used as the variable, for the optimal path of ASVs, the corresponding parameter that needs to be solved at each moment is the corresponding angular velocity, that is:

$$\begin{bmatrix} r_1^*(1) & \cdots & r_N^*(1) \\ \vdots & \ddots & \vdots \\ r_1^*(M) & \cdots & r_N^*(M) \end{bmatrix} = \arg \max_{|r| \leq c} \sum_{t=1}^M J^t \quad (23)$$

Among them, the $r_i^*(t=1:M)$ represents the optimal angular velocity of the i-th ASVs corresponding to the time period from 1 to M; C represents the corresponding constraint.

S43, When there is only one ASVs, considering that all intersection points of the grid in the uncertain region are k, it is necessary to maximize the worst-case scenario in the uncertain region. Combined with the set of all intersection points in the grid, according to the optimization equation of equation (23), the objective function of having only one ASVs can be expressed as:

$$\begin{bmatrix} r_1^*(1) \\ \vdots \\ r_1^*(M) \end{bmatrix} = \arg \max_{|r| \leq c, i=1} \sum_{t=1}^M \left[\arg \min_{j=1, \dots, k} (J_j^t) \right] \quad (24)$$

S44, When there are 2 ASVs, similarly considering that all intersection points of the grid in the uncertain region are k, it is necessary to maximize the worst-case scenario in the uncertain region. Combined with the set of all intersection points in the grid,

according to the optimization equation in equation (23), the objective functions of the 2 ASVs can be expressed as:

$$\begin{bmatrix} r_1^*(1) & r_2^*(1) \\ \vdots & \vdots \\ r_1^*(M) & r_2^*(M) \end{bmatrix} = \arg \max_{|r| \leq c} \sum_{i=1}^2 \sum_{t=1}^M \left[\arg \min_{j=1, \dots, k} (J_j^t) \right] \quad (25)$$

S45, When there are 3 ASVs, the corresponding objective function is,

$$\begin{bmatrix} r_1^*(1) & r_2^*(1) & r_3^*(1) \\ \vdots & \vdots & \vdots \\ r_1^*(M) & r_2^*(M) & r_3^*(M) \end{bmatrix} = \arg \min_{|r| \leq c} \sum_{i=1}^3 \sum_{t=1}^M J_i^t \quad (26)$$

The step S5 combines a nonlinear dynamic programming algorithm to solve the objective function and obtain the optimal path, which specifically includes:

S51, Based on the objective functions obtained from S43, S44, and S45, the Matlab nonlinear dynamic programming optimization toolbox is used to solve for the angular velocity at t moment. Then, the ASVs are made to move to the next turning point according to this angular velocity, and this process is repeated to obtain the optimal path planning scheme for the ASVs within the specified time.

This invention has the following advantages over existing technologies: The method is applicable to solve the problem of optimal path planning for surface vessels when the prior information of the target position is unknown and there are nonlinear non-Gaussian noises present, making it impossible to obtain a closed-form expression for the objective function, thus resulting in the inability to solve for the minimum localization error.

BRIEF DESCRIPTION OF THE DRAWINGS

Figure 1 is a flowchart of the optimal path planning method for a water surface autonomous vehicle with the minimum target positioning error.

Figure 2 is a schematic diagram of the impact of outliers of different pollution factors under uniform distribution.

Figure 3 is a schematic diagram of the rasterized uncertain area of the present invention.

Figure 4 is a schematic diagram of the geometric relationship of ASVs relative to the target.

Figure 5 is the pseudocode for the optimal path acquisition algorithm for target localization of 1 ASVs in the present invention.

Figure 6 is the pseudocode for the optimal path acquisition algorithm for target localization of 2 ASVs in the present invention.

Figure 7 is the pseudocode for the positioning algorithm of the present invention.

Figure 8 is a flowchart for obtaining the optimal path for locating three ASVs targets in the present invention

Figure 9 shows the experimental parameters of the present invention for one ASVs scenario

Figure 10 shows the experimental parameters of the present invention for two ASVs scenario

Figure 11 shows the experimental parameters of the present invention for three ASVs scenario

Figure 12 shows the optimal path diagram of an ASV under different pollution factors in the present invention.

Figure 13 is the optimal path diagram for 2 ASVs when the pollution factor of the present invention is 0.4.

Figure 14 shows the optimal paths of 3 ASVs when the pollution factor is 0.4 for the present invention (dynamic target).

Figure 15 shows three ordinary S-shaped paths (dynamic targets) of ASVs with a pollution factor of 0.4 according to the present invention.

Figure 16 is a comparison chart of the positioning effect between the optimal path of the present invention and the normal S-shaped path.

DETAILED DESCRIPTION

The following, combined with the accompanying drawings, further elaborates on the present invention by detailing a preferred specific implementation case.

Figure 1 is a flowchart of the optimal path planning method for a water surface autonomous vehicle with the minimum target positioning error proposed by the present invention, which specifically includes:

S1, Constructing a system model (motion model and observation model) for water surface autonomous vehicles, the system model includes motion model and observation

model;

S2, deriving the optimal geometric relationship between water surface autonomous vehicle and its relative targets;

S3, Acquire prior position information of the target by a gridded area method (for one or two water surface autonomous vehicles) or by a positioning method (for three water surface autonomous vehicles);

S4, using a Monte Carlo strategy to particleize nonlinear non-Gaussian ranging noise and construct an objective function closed-loop expression;

S5, combining the nonlinear dynamic programming algorithm to solve the objective function to obtain the optimal path.

In this implementation case, step S1 specifically includes:

S11, considering the known situation of the target water depth, mapping the 3D scene to the 2D solution, and assuming that the coordinate reference frame is an east-north-down geodetic coordinate reference frame, the target's position at time t is $\mathbf{B}^t = [b_1^t, b_2^t]^T$, the position of the i -th water surface autonomous vehicle (ASVs) at time t is $\mathbf{A}_i^t = [a_{i1}^t, a_{i2}^t]^T$, $i = 1, \dots, N$, where the T represents transposition; If ASVs can perceive their azimuth angle $\phi(t)$, taking into account their input speed $v(t)$ and the angular velocity $r(t)$, the state corresponding to the ASVS can be expressed as:

$$\dot{\mathbf{A}}_i^t = \begin{bmatrix} v(t) \cos(\phi_i(t)) \\ v(t) \sin(\phi_i(t)) \\ r_i(t) \end{bmatrix} \quad (1)$$

defining two orthogonal unit vectors, i.e.

$$[g(\theta), O(\theta)] = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \quad (2)$$

The motion model of ASVs without considering flow velocity is:

$$\mathbf{A}_i^t = \begin{cases} \mathbf{A}_i^{t-1} + \frac{v(t)}{r_i(t)} (O(\phi_i(t-1)) - O(\phi_i(t))), & \text{if } r_i(t) \neq 0 \\ \mathbf{A}_i^{t-1} + v(t) (g(\phi_i(t)) - g(\phi_i(t-1))), & \text{Others} \end{cases} \quad (3)$$

Among them, $\mathbf{A}_i^{t-1}$ and $\mathbf{A}_i^t$ respectively represent the position of the i -th ASVs at the previous moment and the current moment; the $\phi_i(t-1)$ and $\phi_i(t)$ respectively represent

the azimuth information of the i -th ASVs at the previous and current moments; when the angular velocity is not 0, it indicates that ASVs are performing curved motion during the corresponding time period, conversely, when the angular velocity is 0, it means that ASVs are performing approximately linear motion.

S12, the ASVs is provided with a sensor capable of receiving the signal strength, the position of the sensor and the target is calculated by obtaining the signal intensity value of the sensor, and the observation model of the system is further constructed through the path loss model, that is,

$$z_i^t = P_0 - 10\mathcal{G} \log_{10} \left(\frac{\|\mathbf{B}^t - \mathbf{A}_i^t\|}{d_0} \right) + \eta_i^t \quad (4)$$

Among them, the z_i^t represents the signal power value of the i -th ASVs receiving the target at time t ; the P_0 represents the power value of the target transmission (considering that the transmission power value is equal at every moment); the $\mathcal{G}$ represents the path loss factor; the $\|\mathbf{B}^t - \mathbf{A}_i^t\|$ represents the distance between the i -th ASVs and the target at time t ; the d_0 represents the reference distance, usually one meter; the η_i^t represents the corresponding Gaussian noise (considering that the noise is equal at every moment, i.e. $\eta_i^t = \eta$), its mean is zero and its variance is σ .

Due to the complexity and variability of the marine environment, its noise is not only Gaussian noise, but may also contain outliers, resulting in non Gaussian noise. Introducing the noise pollution factor β , whose value range is $[0,1)$, the probability distribution $p(\eta)$ corresponding to non-Gaussian noise can be expressed as:

$$p(\eta) = (1 - \beta)\Omega(\eta; 0, \sigma^2) + \beta\Phi(\eta). \quad (5)$$

Among them, the $\Omega(\eta; 0, \sigma^2)$ represents Gaussian probability distribution; the $\Phi(\eta)$ represents the probability distribution of non Gaussian anomalous noise; Figure 2 shows the influence of different pollution factors β on the observed signal intensity under uniform distribution.

The step S2 specifically includes:

The optimal geometric relationship exists only when the number of ASVs exceeds 2, as shown in Figure 4.

S22, It is derived that the optimal geometric relationship for two ASVs scenarios is a relative target angle of $\pi/2$;

Inference 1: Let α_{12} be the angle between ASVs 1 and 2, and $\alpha_{12} = \alpha_{21}$, for any fixed $\|\mathbf{B}^t - \mathbf{A}_i^t\|$, let the angle at which the determinant of the Fisher information matrix is maximized be $\pi/2$, i.e. $\alpha_{12} = \alpha_{21} = \pi/2$.

Proof 1: Let $\gamma_i = \tau / \|\mathbf{B}^t - \mathbf{A}_i^t\|^2$, where $\tau = (\mathbf{I}_\eta)^t \cdot \mathcal{G}^2$, and there exists a constant $\gamma > 0$ such that $\gamma_2 = \zeta \gamma_1$, at each moment, its corresponding function can be expressed as:

$$J^t = J_1^t + J_2^t = \frac{\varepsilon}{4} \left[\left(\sum_{i=1}^2 \frac{1}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 - \left(\sum_{i=1}^2 \frac{\cos(2\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 - \left(\sum_{i=1}^2 \frac{\sin(2\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 \right] \quad (6)$$

To minimize the target positioning error, it is necessary to maximize the determinant of equation (6), which is equivalent to minimizing equation (7), that is:

$$\left(\sum_{i=1}^2 \frac{\cos(2\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 + \left(\sum_{i=1}^2 \frac{\sin(2\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 \quad (7)$$

By transformation, we can further obtain:

$$\begin{aligned} & \gamma_1^2 \left[\sin(2\phi_1(t)) + \zeta \sin(2\phi_2(t)) \right]^2 + \left[\cos(2\phi_1(t)) + \zeta \cos(2\phi_2(t)) \right]^2 \\ &= \gamma_1^2 [1 + \zeta + \zeta \cos(2\phi_1(t) - 2\phi_2(t))] \end{aligned} \quad (8)$$

It is obvious that when the $\cos(2\phi_1(t) - 2\phi_2(t)) = -1$, equation (18) can take the minimum value. Therefore, for $\lambda \in \square$, $\phi_2(t) - \phi_1(t) = \lambda\pi - \frac{\pi}{2}$; for two ASVs, when their angle is $\pi/2$, the optimal geometric relationship can be obtained, that is, $\alpha_{12} = \alpha_{21} = \phi_2(t) - \phi_1(t) \bmod(\pi) = \frac{\pi}{2}$. Certificate completed.

S23, For three ASVs scenarios, the optimal geometric relationship angles relative to the target are derived, including: 1) when two ASVs have a relative target angle of 0 or π , and another ASVs has an angle of $\pi/2$ relative to these two ASVs; 2) three ASVs have a relative target angle of $2\pi/3$; 3) two ASVs have a relative target angle of $\pi/3$, and the other ASVs has a relative target angle of $2\pi/3$. The derivation process is as follows:

Assuming α_{12} , α_{13} , and α_{23} are the angular relationships between the i -th ASVs and the l -th ASVs, where $i, l \in \{1, 2, 3\}$, $\gamma_i = \tau / \|\mathbf{B}^t - \mathbf{A}_i^t\|^2$, $\tau = (\mathbf{I}_\eta)^t \cdot \mathcal{G}^2$,

$\gamma_{il} = \gamma_i \cdot \gamma_l$ and $\alpha_{12} = \alpha_{21}$, $\alpha_{13} = \alpha_{31}$, $\alpha_{23} = \alpha_{32}$, the angular relationship between the i -th ASVs and the l -th ASVs can be expressed as:

$$\begin{aligned}\alpha_{12} &= \frac{1}{2} \arccos \left(\frac{\gamma_{13}^2 \gamma_{23}^2 - \gamma_{12}^2 \gamma_{23}^2 - \gamma_{12}^2 \gamma_{13}^2}{2 \gamma_{12}^2 \gamma_{13} \gamma_{23}} \right) \\ \alpha_{13} &= \frac{1}{2} \arccos \left(\frac{\gamma_{12}^2 \gamma_{23}^2 - \gamma_{12}^2 \gamma_{13}^2 - \gamma_{13}^2 \gamma_{23}^2}{2 \gamma_{12} \gamma_{13}^2 \gamma_{23}} \right) \\ \alpha_{23} &= \pi - \alpha_{12} - \alpha_{13}.\end{aligned}\tag{9}$$

If $\gamma_l > \sum_{1 \leq i \leq 3, i \neq l} \gamma_i$, means that a certain ASVs is close to the target, then:

$$\phi_r = \pm \frac{\pi}{2} + \phi_i, r \in \{1, 2, 3\} \setminus i.\tag{10}$$

Equation (10) can be further expressed as:

$$\begin{cases} \alpha_{il} = \frac{\pi}{2}, & \text{if } \gamma_{il} > \gamma_{rl}, \forall r l \in \{12, 13, 23\} \setminus \{il\} \\ \alpha_{il} = 0 \text{ or } \pi, & \text{others} \end{cases},\tag{11}$$

Among them, $\setminus$ represents the representation of reduced sets.

When $\gamma_1 = \gamma_2 = \gamma_3$, there exists a special optimal geometric relationship, namely

$$\alpha_{12} = \alpha_{13} = \alpha_{23} = 2\pi/3 \text{ or } \alpha_{12} = \alpha_{23} = \pi/3, \alpha_{13} = 2\pi/3.$$

Step S3 specifically includes:

S31, When there is only one ASVs or two ASVs, quantify the uncertainty of the unknown prior of the target, as shown in Figure 3, by rasterizing the relevant areas and taking the intersection points in the grid as the possible positions of the target.

S32, When there are 3 ASVs and the prior position of the target is obtained through positioning technology, if $d_0 = 1 \text{ m}$ is obtained, equation (4) of S12 can be transformed to obtain:

$$\|\mathbf{B}^t - \mathbf{A}_i^t\| = 10^{\frac{P_0 - z_i^t}{10\theta}} \cdot 10^{\frac{\eta_i^t}{10\theta}}.\tag{12}$$

Subsequently, by expanding the first-order Taylor series, we obtain:

$$\|\mathbf{B}^t - \mathbf{A}_i^t\| \approx 10^{\frac{P_0 - z_i^t}{10\theta}} \cdot \left(1 + \frac{\ln 10}{10\theta} \eta \right)\tag{13}$$

Construct a nonlinear least squares expression by square distance operation and combining weights, that is:

$$\arg \min_{\mathbf{B}^t} \sum_{i=1}^N \omega_i \left(\left\| \mathbf{B}^t - \mathbf{A}_i^t \right\|^2 - \left(\hat{d}_i^t \right)^2 \right)^2 \quad (14)$$

Where $\hat{d}_i^t$ represents the corresponding estimated distance; $\omega_i = 1 - \hat{d}_i^t / \sum_{i=1}^N \hat{d}_i^t$

After expanding equation (14), it can be further transformed into a generalized trust region subproblem, namely:

$$\begin{aligned} J(y) &= \text{minimize } \|\omega(\wp y - \kappa)\|^2, \\ \text{s.t. } y^T D y + 2f^T y &= 0, \end{aligned} \quad (15)$$

Among them, $y = \left[\left(\mathbf{B}^t \right)^T, \left\| \mathbf{B}^t \right\|^2 \right]^T$; $\omega = \text{diag} \left(\left[\omega_1^2, \dots, \omega_N^2 \right] \right)$;

$$\wp = \begin{bmatrix} \left(\mathbf{A}_1^t \right)^T & -1 \\ \vdots & \vdots \\ \left(\mathbf{A}_N^t \right)^T & -1 \end{bmatrix}; \kappa = \begin{bmatrix} \left(\left\| \mathbf{A}_1^t \right\|^2 - \left(\hat{d}_1^t \right)^2 \right) \\ \vdots \\ \left(\left\| \mathbf{A}_N^t \right\|^2 - \left(\hat{d}_N^t \right)^2 \right) \end{bmatrix}; \mathbf{D} = \begin{bmatrix} \mathbf{I}_2 & \mathbf{0}_{2 \times 1} \\ \mathbf{0}_{1 \times 2} & 0 \end{bmatrix}; \mathbf{f} = \begin{bmatrix} \mathbf{0}_{2 \times 1} \\ -\frac{1}{2} \end{bmatrix} \quad (16)$$

$\mathbf{I}$ and $\mathbf{0}$ represent the identity matrix and zero matrix, respectively.

After obtaining the expression (15) of the generalized trust region subproblem, the binary method is used to solve and obtain the prior position of the target. The corresponding pseudocode for the positioning technology is shown in Figure 7.

The step S4 utilizes Monte Carlo strategy to particle the nonlinear non Gaussian ranging noise and construct a closed-loop expression of the objective function, which specifically includes:

S41, In order to minimize the positioning estimation error, the Claudius lower bound constructor in parameter estimation theory is introduced. The corresponding Fisher information matrix at time t is:

$$\begin{aligned} FIM^t &= \left[\left(\frac{\partial p(\mathbf{B}^t | z_i^t)}{\partial \mathbf{B}^t} \right)^T \sigma^{-1} \left(\frac{\partial p(\mathbf{B}^t | z_i^t)}{\partial \mathbf{B}^t} \right) \right] \\ &= \xi \cdot \frac{\sigma^2}{\sigma^2} \begin{bmatrix} \sum_{i=1}^N \frac{\cos^2(\phi_i(t))}{\left\| \mathbf{B}^t - \mathbf{A}_i^t \right\|^2} & \sum_{i=1}^N \frac{\cos(\phi_i(t)) \cdot \sin(\phi_i(t))}{\left\| \mathbf{B}^t - \mathbf{A}_i^t \right\|^2} \\ \sum_{i=1}^N \frac{\cos(\phi_i(t)) \cdot \sin(\phi_i(t))}{\left\| \mathbf{B}^t - \mathbf{A}_i^t \right\|^2} & \sum_{i=1}^N \frac{\sin^2(\phi_i(t))}{\left\| \mathbf{B}^t - \mathbf{A}_i^t \right\|^2} \end{bmatrix} \quad (17) \end{aligned}$$

Due to the presence of outliers, equation (17) often cannot obtain its closed-loop expression; Therefore, the present invention utilizes Monte Carlo strategy to particle the nonlinear non Gaussian noise, and equation (17) can be expressed as:

$$FIM^t = \xi \cdot \mathcal{G}^2 \cdot (I_\eta)^t \cdot \begin{bmatrix} FIM_1^1 & FIM_1^2 \\ FIM_2^1 & FIM_2^2 \end{bmatrix}, \quad (18)$$

Among them,

$$\begin{aligned} FIM_1^1 &= \sum_{i=1}^N \frac{\cos^2(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2}, FIM_1^2 = \sum_{i=1}^N \frac{\cos(\phi_i(t)) \cdot \sin(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \\ FIM_2^2 &= \sum_{i=1}^N \frac{\sin^2(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2}, FIM_2^1 = \sum_{i=1}^N \frac{\cos(\phi_i(t)) \cdot \sin(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \end{aligned} \quad (19)$$

$$(I_\eta)^t = E_{p(\eta)} \left(\frac{\left\{ \nabla_\eta [p(\eta)]^t \right\}^2}{[p^2(\eta)]^t} \right) = \int \frac{\left\{ \nabla_\eta [p(\eta)]^t \right\}^2}{[p^2(\eta)]^t} [p(\eta)]^t d\eta \quad (20)$$

The $\nabla_\eta [p(\eta)]^t$ represents the corresponding first-order degree; Equation (20) represents the use of Monte Carlo strategy to particle the nonlinear non Gaussian noise operation.

S42, For ASVs, in order to minimize the target localization error, it is necessary to maximize the determinant of the Fisher information matrix, that is:

$$\max \{ \det(FIM^t) \} = \max \left\{ \frac{\Xi}{4} \left[\left(\sum_{i=1}^N \frac{1}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 - \left(\sum_{i=1}^N \frac{\cos(2\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 - \left(\sum_{i=1}^N \frac{\sin(2\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 \right] \right\} \quad (21)$$

Among them, $\Xi = \xi^2 \left[(I_\eta)^t \right]^2 \mathcal{G}^4$;

By performing logarithmic operations on equation (21), the objective function can be expressed as:

$$\arg \max_{|r| \leq c} \sum_{t=1}^M J^t = \arg \max_{|r| \leq c} \sum_{t=1}^M \ln \det(FIM^t) \Big|_{\mathbf{B}^t, \mathbf{A}_i^t, \phi_i(t)} \quad (22)$$

From equation (22), it can be seen that if angular velocity is used as the variable, for the optimal path of ASVs, the corresponding parameter that needs to be solved at each moment is the corresponding angular velocity, that is:

$$\begin{bmatrix} r_1^*(1) & \cdots & r_N^*(1) \\ \vdots & \ddots & \vdots \\ r_1^*(M) & \cdots & r_N^*(M) \end{bmatrix} = \arg \max_{|r| \leq c} \sum_{t=1}^M J^t \quad (23)$$

Among them, $r_i^*(t=1:M)$ represents the optimal angular velocity of the i -th ASVs corresponding to the time period from 1 to M ; C represents the corresponding constraint.

S43, When there is only one ASVs, considering that all intersection points of the grid in the uncertain region are k , it is necessary to maximize the worst-case scenario in the uncertain region. Combined with the set of all intersection points in the grid, according to the optimization equation of equation (23), the objective function of having only one ASVs can be expressed as:

$$\begin{bmatrix} r_1^*(1) \\ \vdots \\ r_1^*(M) \end{bmatrix} = \arg \max_{|r| \leq c, i=1} \sum_{t=1}^M \left[\arg \min_{j=1, \dots, k} (J_j^t) \right] \quad (24)$$

S44, When there are 2 ASVs, considering that all intersection points of the grid in the uncertain region are k , it is necessary to maximize the worst-case scenario in the uncertain region. Combined with the set of all intersection points in the grid, according to the optimization equation in equation (23), the objective functions of the 2 ASVs can be expressed as:

$$\begin{bmatrix} r_1^*(1) & r_2^*(1) \\ \vdots & \vdots \\ r_1^*(M) & r_2^*(M) \end{bmatrix} = \arg \max_{|r| \leq c} \sum_{i=1}^2 \sum_{t=1}^M \left[\arg \min_{j=1, \dots, k} (J_j^t) \right] \quad (25)$$

S45, When there are 3 ASVs, the corresponding objective function is:

$$\begin{bmatrix} r_1^*(1) & r_2^*(1) & r_3^*(1) \\ \vdots & \vdots & \vdots \\ r_1^*(M) & r_2^*(M) & r_3^*(M) \end{bmatrix} = \arg \min_{|r| \leq c} \sum_{i=1}^3 \sum_{t=1}^M J_i^t \quad (26)$$

The step S5 combines a nonlinear dynamic programming algorithm to solve the objective function and obtain the optimal path, which specifically includes:

S51, Based on the objective functions obtained from S43, S44, and S45, the Matlab nonlinear dynamic programming optimization toolbox is used to solve for the angular velocity at each moment; then, the ASVs are made to move to the next turning point according to this angular velocity, and this process is repeated to obtain the optimal path planning scheme for the ASVs within the specified time; The pseudocode for optimal

path planning with the minimum positioning error for one ASVs target is shown in Figure 5; The pseudocode for optimal path planning with the minimum positioning error for two ASVs targets is shown in Figure 6; The optimal path planning process with the minimum positioning error for three ASVs targets is shown in Figure 8. To visualize the path planning method provided by the present invention, simulation was conducted in Matlab R2021b with the following parameter settings: $P_0 = -55 \text{ dBm}$, $g = 3$, $\sigma = 2 \text{ dB}$, $I = 1000$, $r \in [-20^\circ, 20^\circ] \text{ rad/s}$. In addition, the Monte Carlo strategy for granulating nonlinear non Gaussian noise was executed 100 times with a particle count of $N_c = 1000$.

When there is only one ASVs scenario, relevant simulations are conducted, and the parameter settings are shown in Figure 9. Under this parameter, the optimal path for target localization corresponding to different pollution factors is shown in Figure 12. From Figure 12, it can be seen that as the pollution factor increases, the trajectory radius of the optimal path also increases. The corresponding path will gradually approach the target, because the observation model calculates the distance between the target and ASVs based on signal strength. The closer to the target, the stronger the signal strength and the smaller the signal loss, which is beneficial for minimizing positioning errors.

Simulate two ASVs scenarios, with relevant parameter settings shown in Figure 10. According to experiments with only one ASVs scenario, the larger the pollution factor, the larger the trajectory radius. To this end, the median was taken in two ASVs scenarios, that is, when the pollution factor was set to 0.4, the trajectories of the optimal paths of the two ASVs were visualized, as shown in Figure 13. From Figure 13, it can be seen that the optimal paths of the two ASVs gradually approach the uncertain region of the target over time.

To explore the optimal path in three ASVs scenarios, simulations were conducted with the relevant parameter settings shown in Figure 11. Unlike the previous two scenarios, in order to better illustrate the effectiveness of the present invention in providing the optimal path, the target position presents dynamic changes. The optimal path diagram is shown in Figure 14, which presents a curved motion trend around the target. To better compare the optimal positioning accuracy under the optimal path, the scenario of the commonly used S-shaped ordinary path is compared (as shown in Figure 15). Using positioning technology to calculate the error at each moment, as shown in

Figure 16, it can be seen from Figure 16 that under the optimal path proposed by the present invention, the positioning error is significantly better than the commonly used S-shaped path strategy at present.

Although the content of the present invention has been described in detail through the preferred embodiments described above, it should be recognized that the above description should not be construed as limiting the present invention. After reading the above content, various modifications and alternatives to the present invention will be apparent to those skilled in the art. Therefore, the scope of protection of the present invention should be limited by the appended claims.

What is claimed is:

1. An optimal path planning method for water surface autonomous vehicle with minimum target positioning error, comprising the following steps:

S1, constructing a system model of water surface autonomous vehicle including motion model and observation model; establishing said motion model by the position of the surface autonomous vehicle at the last time, the azimuth at the current time, velocity and angular velocity; constructing a path loss model as an observation model by using the signal power value of the target received by the water surface autonomous vehicle at the current moment;

S2, deriving the optimal geometric relationship between different numbers of the water surface autonomous vehicle and its relative targets;

S3, obtains prior position information of target for one or two water surface autonomous vehicles through gridded area method, or for three water surface autonomous vehicle through positioning technologies;

S4, using Monte Carlo strategy to particleize nonlinear non-Gaussian ranging noise and construct closed-loop expression of objective function;

S5, combining the nonlinear dynamic programming algorithm to solve the objective function to obtain the optimal path.

2. The optimal path planning method for water surface autonomous vehicle with minimum target positioning error according to claim 1, wherein the constructing a system model of water surface autonomous vehicle including motion model and observation model in step S1 specifically comprises:

S11, considering the known situation of the target water depth, mapping the 3D scene to the 2D solution, and assuming that the coordinate reference frame is an east-north-down geodetic coordinate reference frame, the target's position at time t is $\mathbf{B}^t = [b_1^t, b_2^t]^T$, the position of the i -th water surface autonomous vehicle(ASVs) at time t is $\mathbf{A}_i^t = [a_{i1}^t, a_{i2}^t]^T, i=1, \dots, N$, where the T represents transposition; if ASVs can perceive azimuth angle $\phi(t)$, taking into account their input speed $v(t)$ and the angular velocity $r(t)$, the state corresponding to the ASVS can be expressed as:

$$\dot{A}_i^t = \begin{bmatrix} v(t) \cos(\phi_i(t)) \\ v(t) \sin(\phi_i(t)) \\ r_i(t) \end{bmatrix} \quad (1)$$

defining two orthogonal unit vectors, i.e.

$$[g(\theta), O(\theta)] = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \quad (2)$$

the motion model of ASVs without considering flow velocity is:

$$A_i^t = \begin{cases} A_i^{t-1} + \frac{v(t)}{r_i(t)} (O(\phi_i(t-1)) - O(\phi_i(t))), & \text{if } r_i(t) \neq 0 \\ A_i^{t-1} + v(t) (g(\phi_i(t)) - g(\phi_i(t-1))), & \text{others} \end{cases}, \quad (3)$$

among them, the A_i^{t-1} and A_i^t respectively represent the position of the i-th ASVs at the previous moment and the current moment; the $\phi_i(t-1)$ and $\phi_i(t)$ respectively represent the azimuth information of the i-th ASVs at the previous and current moments; when the angular velocity is not 0, it indicates that ASVs are performing curved motion during the corresponding time period, conversely, when the angular velocity is 0, it means that ASVs are performing approximately linear motion;

S12, the ASVs is provided with a sensor capable of receiving signal strength, the position of the sensor and the target is calculated by obtaining the signal intensity value of the sensor, and the observation model of the system is further constructed through the path loss model, that is,

$$z_i^t = P_0 - 10\mathcal{G} \log_{10} \left(\frac{\|B^t - A_i^t\|}{d_0} \right) + \eta_i^t \quad (4)$$

among them, the z_i^t represents the signal power value of the i-th ASVs receiving the target at time t; the P_0 represents the power value of the target transmission (considering that the transmission power value is equal at every moment); the $\mathcal{G}$ represents the path loss factor; the $\|B^t - A_i^t\|$ represents the distance between the i-th ASVs and the target at time t; the d_0 represents the reference distance, usually one meter; the η_i^t represents the corresponding Gaussian noise (considering that the noise is equal at every moment, i.e. $\eta_i^t = \eta$), its mean is zero and its variance is σ ;

introducing the noise pollution factor β , whose value range is $[0,1)$, the probability distribution $p(\eta)$ corresponding to non-Gaussian noise can be expressed as:

$$p(\eta) = (1 - \beta)\Omega(\eta; 0, \sigma^2) + \beta\Phi(\eta) \quad (5)$$

among them, the $\Omega(\eta; 0, \sigma^2)$ represents Gaussian probability distribution; the $\Phi(\eta)$ represents the probability distribution of non-Gaussian anomalous noise.

3. The optimal path planning method for water surface autonomous vehicle with minimum target positioning error according to claim 2, wherein the step S2 derives the optimal geometric relationship between the water surface autonomous vehicle and its target, specifically comprises:

S21, the optimal geometric relationship exists only when the number of ASVs exceeds 2;

S22, For two ASVs scenarios, it is derived that the relative target angle $\pi/2$ is the optimal geometric relationship;

inference 1: let α_{12} be the angle between ASVs 1 and 2, and $\alpha_{12} = \alpha_{21}$, for any fixed $\|\mathbf{B}' - \mathbf{A}_i'\|$, let the angle at which the determinant of the Fisher information matrix is maximized be $\pi/2$, i.e. $\alpha_{12} = \alpha_{21} = \pi/2$;

proof 1: let $\gamma_i = \tau / \|\mathbf{B}' - \mathbf{A}_i'\|^2$, where $\tau = (\mathbf{I}_\eta)^t \cdot \mathcal{G}^2$, and there exists a constant $\gamma > 0$ such that $\gamma_2 = \zeta \gamma_1$, at each moment, its corresponding function can be expressed as:

$$J^t = J_1^t + J_2^t = \frac{\varepsilon}{4} \left[\left(\sum_{i=1}^2 \frac{1}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 - \left(\sum_{i=1}^2 \frac{\cos(2\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 - \left(\sum_{i=1}^2 \frac{\sin(2\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 \right] \quad (6)$$

to minimize the target positioning error, it is necessary to maximize the determinant of equation (6), which is equivalent to minimizing equation (7), that is:

$$\left(\sum_{i=1}^2 \frac{\cos(2\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 + \left(\sum_{i=1}^2 \frac{\sin(2\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 \quad (7)$$

by transformation, we can further obtain:

$$\gamma_1^2 \left[\sin(2\phi_1(t)) + \zeta \sin(2\phi_2(t))^2 + (\cos(2\phi_1(t)) + \zeta \cos(2\phi_2(t)))^2 \right]$$

$$= \gamma_1^2 [1 + \zeta + \zeta \cos(2\phi_1(t) - 2\phi_2(t))] \quad (8)$$

it is obvious that when $\cos(2\phi_1(t) - 2\phi_2(t)) = -1$, the equation (18) can take the minimum value; therefore, for $\lambda \in \mathbb{N}$, $\phi_2(t) - \phi_1(t) = \lambda\pi - \frac{\pi}{2}$; for two ASVs, when their angle is $\pi/2$, the optimal geometric relationship can be obtained, that is, $\alpha_{12} = \alpha_{21} = \phi_2(t) - \phi_1(t) \bmod(\pi) = \frac{\pi}{2}$;

S23, for three ASVs scenarios, the optimal geometric relationship angles relative to the target are derived, including: 1) when two ASVs have a relative target angle of 0 or π , and another ASVs has an angle of $\pi/2$ relative to these two ASVs; 2) three ASVs have a relative target angle of $2\pi/3$; 3) two ASVs have a relative target angle of $\pi/3$, and another ASVs has a relative target angle of $2\pi/3$, the derivation process is as follows:

assuming α_{12} , α_{13} , and α_{23} are the angular relationships between the i -th ASVs and the l -th ASVs, where $i, l \in \{1, 2, 3\}$, $\gamma_i = \tau / \|\mathbf{B}^t - \mathbf{A}_i^t\|^2$, $\tau = (I_\eta)^t \cdot \mathcal{G}^2$, $\gamma_{il} = \gamma_i \cdot \gamma_l$ and $\alpha_{12} = \alpha_{21}$, $\alpha_{13} = \alpha_{31}$, $\alpha_{23} = \alpha_{32}$, the angular relationship between the i -th ASVs and the l -th ASVs can be expressed as:

$$\begin{aligned} \alpha_{12} &= \frac{1}{2} \arccos \left(\frac{\gamma_{13}^2 \gamma_{23}^2 - \gamma_{12}^2 \gamma_{23}^2 - \gamma_{12}^2 \gamma_{13}^2}{2\gamma_{12}^2 \gamma_{13} \gamma_{23}} \right) \\ \alpha_{13} &= \frac{1}{2} \arccos \left(\frac{\gamma_{12}^2 \gamma_{23}^2 - \gamma_{12}^2 \gamma_{13}^2 - \gamma_{13}^2 \gamma_{23}^2}{2\gamma_{12} \gamma_{13}^2 \gamma_{23}} \right) \\ \alpha_{23} &= \pi - \alpha_{12} - \alpha_{13}. \end{aligned} \quad (9)$$

if $\gamma_l > \sum_{1 \leq i \leq 3, i \neq l} \gamma_i$, means that one ASVs approaches the target, then:

$$\phi_r = \pm \frac{\pi}{2} + \phi_i, r \in \{1, 2, 3\} \setminus i \quad (10)$$

equation (10) can be further expressed as:

$$\begin{cases} \alpha_{il} = \frac{\pi}{2}, & \text{if } \gamma_{il} > \gamma_{rl}, \forall r l \in \{12, 13, 23\} \setminus \{il\} \\ \alpha_{il} = 0 \text{ or } \pi, & \text{others} \end{cases} \quad (11)$$

among them, the $\setminus$ represents the representation of reduced sets;

when $\gamma_1 = \gamma_2 = \gamma_3$, there exists a special optimal geometric relationship, namely

$$\alpha_{12} = \alpha_{13} = \alpha_{23} = 2\pi/3 \quad \text{or} \quad \alpha_{12} = \alpha_{23} = \pi/3, \quad \alpha_{13} = 2\pi/3.$$

4. The optimal path planning method for water surface autonomous vehicle with minimum target positioning error in claim 3, characterized in that step S3 obtains prior position information of target for one or two water surface autonomous vehicles through gridded area method, or for three water surface autonomous vehicle through positioning technologies, specifically comprising:

S31, when there is only one or two ASVs, quantify the uncertainty of the prior position of the target, grid the relevant areas, and use the intersection points in the grid as the possible positions of the target;

S32, when there are 3 ASVs and the prior position of the target is obtained through positioning technology, if $d_0 = 1$ m is obtained, equation (4) of S12 can be transformed to obtain:

$$\| \mathbf{B}^t - \mathbf{A}_i^t \| = 10^{\frac{P_0 - z_i^t}{10g}} \cdot 10^{\frac{\eta_i^t}{10g}}, \quad (12)$$

subsequently, by expanding the first-order Taylor series, we obtain:

$$\| \mathbf{B}^t - \mathbf{A}_i^t \| \approx 10^{\frac{P_0 - z_i^t}{10g}} \cdot \left(1 + \frac{\ln 10}{10g} \eta \right). \quad (13)$$

construct a nonlinear least squares expression by square distance operation and combining weights, that is:

$$\arg \min_{\mathbf{B}^t} \sum_{i=1}^N \omega_i \left(\| \mathbf{B}^t - \mathbf{A}_i^t \|^2 - (\hat{d}_i^t)^2 \right)^2 \quad (14)$$

where the $\hat{d}_i^t$ represents the corresponding estimated distance;

$$\omega_i = 1 - \hat{d}_i^t / \sum_{i=1}^N \hat{d}_i^t$$

after expanding equation (14), it can be further transformed into a generalized trust region subproblem, namely:

$$\begin{aligned} J(\mathbf{y}) &= \minimize \|\omega(\varphi \mathbf{y} - \kappa)\|^2, \\ \text{s.t. } &\mathbf{y}^T D \mathbf{y} + 2\mathbf{f}^T \mathbf{y} = 0, \end{aligned} \quad (15)$$

among them, $\mathbf{y} = \left[(\mathbf{B}^t)^T, \|\mathbf{B}^t\|^2 \right]^T$; $\boldsymbol{\omega} = \text{diag} \left([\omega_1^2, \dots, \omega_N^2] \right)$;

$$\boldsymbol{\varphi} = \begin{bmatrix} (\mathbf{A}'_1)^T & -1 \\ \vdots & \vdots \\ (\mathbf{A}'_N)^T & -1 \end{bmatrix}; \boldsymbol{\kappa} = \begin{bmatrix} \left(\|\mathbf{A}'_1\|^2 - (\hat{d}'_1)^2 \right) \\ \vdots \\ \left(\|\mathbf{A}'_N\|^2 - (\hat{d}'_N)^2 \right) \end{bmatrix}; \mathbf{D} = \begin{bmatrix} \mathbf{I}_2 & \mathbf{0}_{2 \times 1} \\ \mathbf{0}_{1 \times 2} & 0 \end{bmatrix}; \mathbf{f} = \begin{bmatrix} \mathbf{0}_{2 \times 1} \\ -\frac{1}{2} \end{bmatrix} \quad (16)$$

$\mathbf{I}$ and $\mathbf{0}$ represent the identity matrix and zero matrix, respectively,

after obtaining the expression (15) of the generalized trust region subproblem, the binary method is used to solve and obtain the prior position of the target.

5. The optimal path planning method for water surface autonomous vehicle with minimum target positioning error in claim 4, characterized in that step S4 utilizes Monte Carlo strategy to particle the nonlinear non Gaussian ranging noise and construct a closed-loop expression of the objective function, which specifically includes:

S41, in order to minimize the positioning estimation error, the Claudius lower bound constructor in parameter estimation theory is introduced; the corresponding Fisher information matrix at time t is:

$$\begin{aligned} FIM^t &= \left[\left(\frac{\partial p(\mathbf{B}^t | \mathbf{z}_i^t)}{\partial \mathbf{B}^t} \right)^T \sigma^{-1} \left(\frac{\partial p(\mathbf{B}^t | \mathbf{z}_i^t)}{\partial \mathbf{B}^t} \right) \right] \\ &= \xi \cdot \frac{\mathcal{G}^2}{\sigma^2} \begin{bmatrix} \sum_{i=1}^N \frac{\cos^2(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}'_i\|^2} & \sum_{i=1}^N \frac{\cos(\phi_i(t)) \cdot \sin(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}'_i\|^2} \\ \sum_{i=1}^N \frac{\cos(\phi_i(t)) \cdot \sin(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}'_i\|^2} & \sum_{i=1}^N \frac{\sin^2(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}'_i\|^2} \end{bmatrix} \quad (17) \end{aligned}$$

due to the presence of outliers, equation (17) often cannot obtain its closed-loop expression; therefore, the present invention utilizes Monte Carlo strategy to particle the nonlinear non Gaussian noise, and equation (17) can be expressed as:

$$FIM^t = \xi \cdot \mathcal{G}^2 \cdot (\mathbf{I}_\eta)^t \cdot \begin{bmatrix} FIM_1^1 & FIM_1^2 \\ FIM_2^1 & FIM_2^2 \end{bmatrix} \quad (18)$$

Among them,

$$\begin{aligned} FIM_1^1 &= \sum_{i=1}^N \frac{\cos^2(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}'_i\|^2}, FIM_1^2 = \sum_{i=1}^N \frac{\cos(\phi_i(t)) \cdot \sin(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}'_i\|^2} \\ FIM_2^1 &= \sum_{i=1}^N \frac{\sin^2(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}'_i\|^2}, FIM_2^2 = \sum_{i=1}^N \frac{\cos(\phi_i(t)) \cdot \sin(\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}'_i\|^2} \end{aligned} \quad (19)$$

$$(I_\eta)^t = E_{p(\eta)} \left(\frac{\left\{ \nabla_\eta [p(\eta)]^t \right\}^2}{[p^2(\eta)]^t} \right) = \int \frac{\left\{ \nabla_\eta [p(\eta)]^t \right\}^2}{[p^2(\eta)]^t} [p(\eta)]^t d\eta \quad (20)$$

the $\nabla_\eta [p(\eta)]^t$ represents the corresponding first-order degree; equation (20) represents the use of Monte Carlo strategy to particle the nonlinear non-Gaussian noise operation;

S42, for ASVs, in order to minimize the target localization error, it is necessary to maximize the determinant of the Fisher information matrix, that is:

$$\max \{ \det(FIM^t) \} = \max \left\{ \frac{\Xi}{4} \left[\left(\sum_{i=1}^N \frac{1}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 - \left(\sum_{i=1}^N \frac{\cos(2\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 - \left(\sum_{i=1}^N \frac{\sin(2\phi_i(t))}{\|\mathbf{B}^t - \mathbf{A}_i^t\|^2} \right)^2 \right] \right\} \quad (21)$$

among them, $\Xi = \xi^2 \left[(I_\eta)^t \right]^2 \mathcal{G}^4$;

by performing logarithmic operations on equation (21), the objective function can be expressed as:

$$\arg \max_{|r| \leq c} \sum_{t=1}^M J^t = \arg \max_{|r| \leq c} \sum_{t=1}^M \ln \det(FIM^t) \Big|_{\mathbf{B}^t, \mathbf{A}_i^t, \phi_i(t)} \quad (22)$$

from equation (22), it can be seen that if angular velocity is used as the variable, for the optimal path of ASVs, the corresponding parameter that needs to be solved at each moment is the corresponding angular velocity, that is:

$$\begin{bmatrix} r_1^*(1) & \cdots & r_N^*(1) \\ \vdots & \ddots & \vdots \\ r_1^*(M) & \cdots & r_N^*(M) \end{bmatrix} = \arg \max_{|r| \leq c} \sum_{t=1}^M J^t \quad (23)$$

among them, the $r_i^*(t=1:M)$ represents the optimal angular velocity of the i-th ASVs corresponding to the time period from 1 to M; C represents the corresponding constraint;

S43, when there is only one ASVs, considering that all intersection points of the grid in the uncertain region are k, it is necessary to maximize the worst-case scenario in the uncertain region; combined with the set of all intersection points in the grid, according to the optimization equation of equation (23), the objective function of having only one ASVs can be expressed as:

$$\begin{bmatrix} r_1^*(1) \\ \vdots \\ r_1^*(M) \end{bmatrix} = \arg \max_{|r| \leq c, i=1} \sum_{t=1}^M \left[\arg \min_{j=1, \dots, k} (J_j^t) \right] \quad (24)$$

S44, when there are 2 ASVs, similarly considering that all intersection points of the grid in the uncertain region are k , it is necessary to maximize the worst-case scenario in the uncertain region; combined with the set of all intersection points in the grid, according to the optimization equation in equation (23), the objective functions of the 2 ASVs can be expressed as:

$$\begin{bmatrix} r_1^*(1) & r_2^*(1) \\ \vdots & \vdots \\ r_1^*(M) & r_2^*(M) \end{bmatrix} = \arg \max_{|r| \leq c} \sum_{i=1}^2 \sum_{t=1}^M \left[\arg \min_{j=1, \dots, k} (J_j^t) \right] \quad (25)$$

S45, when there are 3 ASVs, the corresponding objective function is,

$$\begin{bmatrix} r_1^*(1) & r_2^*(1) & r_3^*(1) \\ \vdots & \vdots & \vdots \\ r_1^*(M) & r_2^*(M) & r_3^*(M) \end{bmatrix} = \arg \min_{|r| \leq c} \sum_{i=1}^3 \sum_{t=1}^M J_i^t \quad (26) \text{。}$$

6. The optimal path planning method for water surface autonomous vehicle with minimum target positioning error in claim 5, characterized in that step S5 combines a nonlinear dynamic programming algorithm to solve the objective function and obtain the optimal path, which specifically includes:

S51, based on the objective functions obtained from S43, S44, and S45, the Matlab nonlinear dynamic programming optimization toolbox is used to solve for the angular velocity at t moment; then, the ASVs are made to move to the next turning point according to this angular velocity, and this process is repeated to obtain the optimal path planning scheme for the ASVs within the specified time.