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Machine, Method And Medium For Dynamic Optimization For Resource Allocation

Background Of The Invention

Field of the Invention

The present invention relates to a machine, method and medium for allocating resources over a plurality of time periods, taking into account a plurality of interrelated processes such as marketing, procurement and production.

Related Art

Today's businesses generally includes a procurement process, production process, and marketing process. The standard practice of these businesses is to optimize each of these processes separately with little or no regard to interdependencies among processes. Individual process decision is made in a disjointed manner with no consideration to the total business operation. Though manual handoffs exist, it is very poorly linked and contrary to best-in-class technology. This has created problems of inefficiency in most large and mid-size corporations. In addition to a lack of consideration of sufficient interdependencies, each process is optimized separately using static optimization techniques such that a system (comprising products or services made up of various resources and configured based upon customer demand) designed for a given time period t is not interdependent with systems designed for, e.g., times t+1 or t-1. In effect, the problem with the static optimization causes a business process to make reactive business decisions as opposed to proactive business decisions.

Thus, the problem existing with the majority of optimization tools existing today in industries is that they are based on static optimization algorithms that do not reflect true costs or provide any competitive advantage in the face of fierce competition in various markets. This is because the competition influences the elasticity of demand and price and makes the overall business environment very dynamic. For example, fluctuating demand in retail industries has a continuous impact on the shipping schedule among suppliers, warehouses and retail stores. In

the auto industry, changes in demand at the sales outlets have a bearing on the production floor schedules and procurement of spare parts. However, the frequency of change and variables of change differ from one industry to the other. In telecommunication services industry, changes in demand similarly have an impact on the existing capacity, procurement of additional capacity or production operation.

As competition rises in markets in the coming decades, companies will demand more real time information of resource costs and competitive pricing. Static optimization, however, only yields a cost-optimal system in a snap-shot of time, usually at present, without regard to future growth or decline in any service type, time value of money, introduction of new technology or change in customer demographics. The problem with the static optimization is that what is optimum today may not be optimum tomorrow. It causes a business to make reactive business decisions as opposed to proactive business decisions. It also relies heavily on human judgement and involvement and calls for creation of many unnecessary subprocesses within a business process or many unnecessary processes within a business. Business decisions that are made based on many of these business processes, are poor and sub-optimal with respect to a multi-year planning horizon. Specifically, when a static optimization technique is run at time period zero (i.e. now), it cannot solve resource rearrangement problems for future time periods.

It is true that current static optimization techniques can analyze a future time period as if it were the current time period (i.e. time period t=0). However, this still only deals with a single period, and does not take into account that effect that other time periods will have on it. Being in the current time period, this technique cannot produce optimal decisions that will occur at future time periods. Nonetheless, utilization of such static techniques have been the standard practice of businesses and industries. For example, retail, manufacturing, telecommunications and service industries rely heavily on static optimization techniques.

In the airline industry, an optimum fleet schedule is very important to the financial well-being of this industry. Efforts to enhance efficiency and charge "the right" price for seats have been the subject of reports such as "Yield Management at American Airlines" by Smith et al. However these reports (and the airline industry, generally), nonetheless applies mostly static optimization techniques. This leaves them vulnerable to the inclement business conditions for future time periods. Knowing about optimal business activities of the future time periods will put any business at a competitive edge. This is why a static optimization is not adequate in fiercely competitive marketplace.

Some industries referred to above that do not apply strictly "static" optimization instead utilize "pseudo-dynamic" optimization which is sometimes misconstrued as true dynamic optimization. Though the pseudo-dynamic optimization is also based on a multi-period planning horizon, many characteristics of static optimization heavily bias the results of this pseudo-dynamic optimization. For example, the output of static optimization is used as input to pseudo-dynamic optimization. Sometimes the results of static optimization are used to approximate the future activities at a very macro-level which can be significantly sub-optimal. This again calls for creation of additional business processes and more human intervention to respond to the business conditions in more reactive manner.

Thus, what is needed is a scheme for using dynamic optimization techniques such that systems contemplated for implementation at various times are interrelated, and that the procurement, production and marketing processes are interrelated as well.

Summary Of The Invention

The present invention overcomes the deficiencies of the above-noted schemes by providing a machine, method and medium for allocating resources over given time periods. Specifically, the present invention receives various requests (e.g., from customers) for products and/or services, and determines a preferred scheme for allocating resources, over a plurality of time periods, to provide the requested products and/or services. Marketing, procurement and production processes are taken into account in the analysis. The end result is a set of systems indicating how the resources should be deployed over the various time periods to satisfy customer demand. In doing this, the interrelationships between the time periods are taken into account, and a desired pricing scheme for the various products and services delivered (which may vary from time period to time period) is determined.

Brief Description Of The Drawings

Various objects, features, and attendant advantages of the present invention can be more fully appreciated as the same become better understood with reference to the following detailed description of the present invention when considered in connection with the accompanying drawings, in which:

FIGURE 1 depicts a specific telecommunications example implemented as a result of some embodiments of the present invention.

FIGURES 2-4 is a flowchart depicting methods of operation as contemplated by some embodiments of the present invention.

FIGURES 5-6 are charts representing a specific test implementation in the telecommunications area as implemented by some embodiments of the present invention.

FIGURE 7 is a block diagram of software components and interfaces as contemplated by some embodiments of the present invention.

FIGURE 8 depicts a moving horizon chart.

FIGURE 9 is a block diagram of a computer environment and components thereof as contemplated by some embodiments (and environments thereof) of the present invention.

Detailed Description

The present invention relates to a machine, method and medium for allocating resources over a plurality of time periods, taking into account a plurality of interrelated processes such as marketing, procurement and production.

As indicated in the Background section above, surviving in any competitive environment requires real-time integration of fundamental business processes such as procurement, engineering, production operation, marketing and sales. Such integration will allow flow of materials and information from process to process on a real-time basis. A multi-period dynamic optimization technique assures that flow of materials and information occurs in a cost-optimal way. In addition, "yield management" techniques can also be applied to stem fierce competition.

The utilization of capacity in a production-ready resource is driven by demand of different products or services. For example, customers sometimes do not like certain products or services and might return them sooner or after a period of using them if a return policy allows. The nature of returning products or services might vary from industry to industry. Certain products or services can never be returned. Customers sometimes leave or go to competitors for pricing reasons, technological reasons, political reasons, etc. Existing resource capacities carried from the previous time period also need to be considered for allocation. Decline in demand of some products or services might create idle resource capacities, some of which might come back to inventory for use.

In some embodiments contemplated by the present invention, a dynamic systems model is used to create systems containing the aforementioned products or services, over a plurality of time periods. Within each time period, the way in which

products or services flow from one, e.g., production plant or warehouse to another or from one process to the other are additional dimensions of the model. The products or services that are currently occupying a particular resource capacity or a series of resource capacities in a given time period might migrate to a different resource capacity or series of resource capacities in another time period.

The above concepts as applied to, for example, the telecommunications industry, will be primarily discussed below. It should be understood, however, that the present invention contemplates application in any number of other industries and technologies as well.

Thus, using a telecommunications industry example, the routing of a data circuit via certain network nodes and network facilities (each of which is an example of a resource) might be optimal in one time period to provide a certain service, but not optimal in another time period due to fluctuations of demand or other competing resources. The same data circuit might be rerouted through a different set of network nodes or facilities in another time period. Circuit demand also requires flow of optimal network facilities and network apparatus at different network nodes. Such a flow is taking place across procurement, engineering and production operation to support sales and marketing in a given time period.

As can be appreciated from the above, it is contemplated that the various requirements of customers over a plurality of time periods are taken into account, as is the "inventory" situation regarding availability of the resources needed to fulfill customer needs. Thus, marketing, procurement and production over the time periods all need to be taken into account. In that way, systems that need to be implemented over various time periods can be mapped out and planned, all at once, in advance. Naturally, there is some change to be expected as, for example, customers cancel orders, etc. Nonetheless, a relatively accurate picture of how systems will be implemented over a plurality of future time periods can be anticipated with significant accuracy and efficiency, so that a company can anticipate how it should prepare to allocate resources. The specific implementation of this concept will be discussed in greater detail with regard to various equations and flow charts described further below.

Figure 1 depicts an example of a specific multi-period dynamic systems model concerning telecommunications technology, illustrating the effects of the present invention. Referring to Figure 1, the four "bubbles" represent systems within four different discrete time periods (1, 2, 3 and T). Within each time period "bubble" are various resources used to configure products or services requested by customers.

Referring to time period 1 (102) as an example, a Point Of Presence (POP) 124 is shown. A POP can be defined as a major circuit office into which large numbers of circuits (e.g., telephone lines) can feed into, and from which smaller lines typically emanate. Three types of lines often used are DS0 lines (often referred to as "voice grade"), T1 lines (which have 24 times the bandwidth as a DS0 line) and DS3 lines (which have 28 times the bandwidth as T1 lines).

Theoretically, each customer could be directly attached to the POP 124, but that would create great inefficiencies. Consequently, local serving offices (LSOs) can be created, typically containing multiplexers of some type. These multiplexers allow conversion from a line of relatively large bandwidth to multiple lines of smaller bandwidth. Thus, for example, a DS3/DS1 multiplexer 110 is shown converting a DS3 to a T1 (and, although not shown, is capable of having another 27 T1 lines attached to it). Similarly, T1 multiplexer 108 is shown converting the T1 line to a DS0 line (and is capable of converting another 23).

The "end product" in this example are the products and services received by customer locations, such as those shown at 104 and 106. Here, the service that customer location 106 receives is an A1.5 circuit (which has the bandwidth of a T1 line), and customer 104 receives a DS0 circuit. As can be appreciated, depending upon demand, each DS3/DS1 multiplexer can be thought of as having a capacity of 28 T1 (or A1.5) "slots" (i.e., line inputs), and thus if 20 of them are being used, it has 8 to spare. Similarly, the T1 multiplexer has a capacity of 24 DS0 lines it can provide to customers.

At time period 2 (160), additional DS0 lines (in this case, representing services to customers) have been added to T1 multiplexer 120. There are not, however, enough DS0 lines to warrant putting in a T1 line directly between the POP 124 and LSO 128. In any event, some embodiments of the present invention keep track of, and account for, the used and spare "slot" in the various multiplexers, as well as the facilities (e.g., cables and multiplexers) themselves, when setting up products and services of time period 2 (160). Also, where necessary, in the event that the required resources do not exist at time period 2 (i.e., they were not available to be carried over from the previous time period, or there was no spare because all slots were used), then they must be produced. Thus, in time period 2, while there are enough "slots" within, say, T1 multiplexer 120 to service the additional customer locations with DS0 services, additional DS0 cables are needed. While there is no "carry-over" of this resource as there are for the slots in the T1 multiplexer 120, this one actually has to be procured from another source (e.g., obtained from a different

location, bought, manufactured, etc.).

At time period 3 (170), it can be seen that there was enough of a demand between POP 124 and LSO 128 (which included T1 multiplexer 108) to warrant the installation of a T1 line 174. Note that what was T1 multiplexer 120 during time period 2 is now a DS3/DS1 multiplexer 172. In addition, an additional T1 multiplexer 176 was warranted. Thus, that additional resource does not pass from the previous time period (i.e., it did not exist at that previous time period, as did multiplexer 108), and thus needs to be obtained, manufactured, etc. Though not explicitly shown, the additional T1 multiplexer 176 is a result of additional customer demand for more DS0 lines (i.e., more DS0 services).

Lastly, time period T (180) shows the result of even additional demand, where even more and higher-band width multiplexers are required within the system to supply customers with desired services.

The resources that can be used to implement various products and services as contemplated by some embodiments of the present invention can be classified in various orders of criticality. The ranking of a particular resource depends upon its necessity as a component of a given product or service, and is contemplated to be assigned a ranking based on some logical criteria. As an example, resources can be assigned as primary, secondary or tertiary. In the telecommunications example above for time period 1 (102), in order to provide the A1.5 service to customer location 106, it may be logical to designate the A1.5 standalone line as a primary resource, since it is absolutely needed. The DS3 multiplexer 110 can be thought of as secondary, since that particular component is not as important (e.g., the A1.5 line could, potentially, come directly from the POP 124). Then, the T1 multiplexer 108 could be thought of as tertiary, since it is potentially of some, but less importance. Of course, it should be understood that the present invention contemplates that resources can be ranked in various different ways and to various numbers of degrees.

With the above in mind, a generalized mathematical systems model follows:

A Generalized Mathematical Model

Let the primary resources be represented by a set of vectors as $(\alpha_d^c, \alpha_o^c, \alpha_p^c)$ where c signifies whether it is in the existing inventory or to be procured. $c = 0$ means the resource is in the existing inventory and $c = 1$ means the resource is to be either leased or purchased. The αs with subscripts, d, o and p, denote different types of primary resources. α_d^c represents standalone primary resources which are utilized for direct channel production operation and product or service distribution. These primary resources need not be assembled with any secondary or tertiary resource components to support the production of products or services. Each unit of these primary resources generally produces a unit of products or services and, therefore, is very expensive for mass production. α_o^c represents a type of primary resources which are assembled with tertiary resource components to support mass production of a specific type of products or services. Thus, unit cost of producing this type of products or services becomes cheaper. α_p^c represents primary resources which need to be assembled with both secondary resource components and tertiary resource components in a cascading manner to support mass production of multiple types of products or services, and, consequently, reduce the unit cost of production even further. In the following mathematical model, i is used as a subscript to indicate an element in the vector representing any of the above types of primary resources. The aforementioned secondary and tertiary resource components are denoted by two sets of vectors, β^c and γ^c, respectively. Two indices, h and l, are used to indicate secondary resource components and tertiary resource components respectively. The types of assemblies among primary, secondary and tertiary resources determine what kinds of products or services are produced. For example, assembly of α_o^c primary resources with γ^c tertiary resources produces bulk quantities of products or services which can be symbolized by ρ_g vector. These same products or services, ρ_g, can also be produced in bulk quantities if primary resources, α_p^c, are

assembled with secondary resources, β^c, which are then assembled with tertiary resources, γ^c. On the other hand, assembly of primary resources, α_p^c, with secondary resource components, β^c, produces bulk quantities of products or services, which can be symbolized by ρ_f vector. The m subscript is used to identify an element belonging to one of these product or service vectors i.e. $m \in (\rho_g, \rho_f)$. There are also different types of warehouses, production plants or distribution routes for which j is an index. In this model, j is of two types, standalone and bulk, denoted by v_d and v_b, respectively. Warehouses, production plants or distribution routes that are associated with standalone primary resources, α_d^c, are designated by vector, v_d. Warehouses, production plants or distribution routes that are associated with primary resources, α_o^c and α_p^c, are designated by vector, v_b.

An important aspect of a dynamic resource allocation problem is rearrangement activity, r. Products or services, m, which are assigned to primary resource i' of the j' –th warehouse, production plant or distribution route in one time period can migrate to primary resource i of warehouse, production plant or distribution route j in another time period to maintain cost-optimal production process, where $(j' = j) \cup (j' \neq j)$. i' can also be represented by $i(\delta)$ which implies that products or services produced by δ primary resources in production plant or distributionin route j' in one time period, can migrate to primary resource $i \in \alpha$ of production plant or distribution route j in another discrete time period when $i \neq i'$ and $(j' = j) \cup (j' \neq j)$.

t defines a time period and k is used to indicate customer demand locations for products or services, $\rho = (\rho_g, \rho_f)$. Let us assume that $t = 1, 2, .., T$; $j = 1, 2, ..., J$; $k = 1, 2, ..., K$; $m = 1, 2, ..., M$; and $i = 1, 2, ..., I$ where $i \in (\alpha_d^c, \alpha_o^c, \alpha_p^c)$.

Eight types of mathematical equations, which lay the foundation for solving a truly dynamic resource allocation problem, are presented below . These types of core equations offer the basic properties required to mimic the physical phenomena across business processes such as, marketing & sales, production and procurement. These

equations can be categorized as objective function, procurement or spare balance equation, usage balance equation, product or service inventory equation, product or service demand equation, rearrangement equation, decision equation to rearrange or not to rearrange and initial inventory equation. In addition to these equations, non-negativity and integer constraints are to be applied.

1 .The Linear Objective Function

The objective function is to be minimized. The cost coefficients have been converted to net present value over a planning horizon for a given interest rate.

$$\text{Minimize } Z = \sum_{t}\sum_{j\in v_b}\sum_{k}\sum_{m\in\rho_g}\sum_{l}\sum_{h}\sum_{i\in\alpha_p^c}(\varepsilon_{tjkmlhi}\,x_{tjkmlhi}+\varepsilon'_{tjkmlhi}\,\zeta_{tjkmlhi})+\sum_{t}\sum_{j\in v_b}\sum_{k}\sum_{m\in\rho_g}$$

$$\sum_{l}\sum_{i\in\alpha_o^c}(\varepsilon_{tjkmli}\,x_{tjkmli}+\varepsilon'_{tjkmli}\,\zeta_{tjkmli})+\sum_{t}\sum_{j\in v_b}\sum_{k}\sum_{m\in\rho_f}\sum_{h}\sum_{i\in\alpha_p^c}(\varepsilon_{tjkmhi}\,x_{tjkmhi}+\varepsilon'_{tjkmhi}\,\zeta_{tjkmhi})+$$

$$\sum_{t}\sum_{j\in v_d}\sum_{k}\sum_{m\in(\rho_g,\rho_f)}\sum_{i\in\alpha_d^c}(\varepsilon_{tjkmi}\,x_{tjkmi}+\varepsilon'_{tjkmi}\,\zeta_{tjkmi})+\sum_{t}\sum_{j\in v_b}\sum_{k}\sum_{m\in\rho_g}\sum_{l}\sum_{h}\sum_{i\in\alpha_p^c}$$

$$\sum_{i'\in i(\delta)_j'}\lambda_{tjkmlhi,i'j'}\,r_{tjkmlhi,i'j'}+\sum_{t}\sum_{j\in v_b}\sum_{k}\sum_{m\in\rho_g}\sum_{l}\sum_{i\in\alpha_o^c}\sum_{i'\in i(\delta)_j'}\lambda_{tjkmli,i'j'}\,r_{tjkmli,i'j'}$$

$$+\sum_{t}\sum_{j\in v_b}\sum_{k}\sum_{m\in\rho_f}\sum_{h}\sum_{i\in\alpha_p^c}\sum_{i'\in i(\delta)_j'}\lambda_{tjkmhi,i'j'}\,r_{tjkmhi,i'j'}+\sum_{i\in\alpha_p^c}\sum_{h}\sum_{l}\sum_{j\in v_b}\sum_{t}\mu_{ihljt}\cdot$$

$$R_{ihljt}+\sum_{i\in\alpha_o^c}\sum_{l}\sum_{j\in v_b}\sum_{t}\mu_{iljt}\,R_{iljt}+\sum_{i\in\alpha_p^c}\sum_{h}\sum_{j\in v_b}\sum_{t}\mu_{ihjt}\,R_{ihjt}+\sum_{i\in(\alpha_d^c,\,\alpha_o^c,\,\alpha_p^c)}$$

$$\sum_{j\in(v_b,v_d)}\sum_{t}\mu_{ijt}\,R_{ijt}+\sum_{i\in\alpha_p^c}\sum_{h}\sum_{l}\sum_{j\in v_b}\sum_{t}\xi_{ihljt}(S_{ihljt}+U_{ihljt})+\sum_{i\in\alpha_o^c}\sum_{l}\sum_{j\in v_b}\sum_{t}\xi_{iljt}(S_{iljt}+U_{iljt})$$

$$+\sum_{i\in\alpha_p^c}\sum_{h}\sum_{j\in v_b}\sum_{t}\xi_{ihjt}(S_{ihjt}+U_{ihjt})+\sum_{i\in(\alpha_d^c,\,\alpha_o^c,\,\alpha_p^c)}\sum_{j\in(v_b,\,v_d)}\sum_{t}\xi_{ijt}(S_{ijt}+U_{ijt}) \qquad (1)$$

where $\varepsilon_{tjkmlhi}$ =unit cost of producing a product or service, $m \in \rho_g$, in plant or warehouse j and supported by tertiary resource components, l, secondary resource components, h, and primary resource components, i

$\varepsilon'_{tjkmlhi}$ =unit cost of holding a product or service, $m \in \rho_g$ in plant or warehouse j and supported by tertiary resource components, l, secondary resource components, h, and primary resource components, i

ε_{tjkmli} =unit cost of producing a product or service, $m \in \rho_g$, in plant or warehouse j and supported by tertiary resource components, l, and primary resource components, i

ε'_{tjkmli} =unit cost of holding a product or service, $m \in \rho_g$ in plant or warehouse j and supported by tertiary resource components, l, and primary resource components, i

ε_{tjkmhi} =unit cost of producing a product or service, $m \in \rho_f$, in plant or warehouse j and supported by secondary resource components, h, and primary resource components, i

ε'_{tjkmhi} =unit cost of holding a product or service, $m \in \rho_f$, in plant or warehouse j and supported by secondary resource components, h, and primary resource components, i

ε_{tjkmi} =unit cost of producing a product or service, $m \in (\rho_g, \rho_f)$ in plant or warehouse, $j \in v_d$, and supported by primary resource components, $i \in \alpha_d^c$,

ε'_{tjkmi} =unit cost of holding a product or service, $m \in (\rho_g, \rho_f)$ in plant or warehouse, $j \in v_d$, and supported by primary resource components, $i \in \alpha_d^c$,

$\lambda_{tjkmlhi,j'j}$ =unit cost of migrating a product or service, $m \in \rho_g$, from $i' \in i(\delta)$ of plant j' to $i \in \alpha_p^c$ of plant j which requires both tertiary resource components l and secondary resource components h

$\lambda_{tjkmli,j'j}$ =unit cost of migrating a product or service, $m \in \rho_g$, from $i' \in i(\delta)$ of plant j' to $i \in \alpha_o^c$ of plant j which requires tertiary resource components l.

$\lambda_{ijkmhi'j'}$ =unit cost of migrating a product or service, $m \in \rho_f$, from $i' \in i(\delta)$ of plant j' to $i \in \alpha_p^c$ of plant j which requires secondary resource components h.

μ_{ihljt} =unit cost of tertiary resource components l which is assembled with secondary resource components h which is then assembled with primary resource components $i \in \alpha_p^c$

μ_{iljt} =unit cost of tertiary resource components l which is assembled with primary resource components $i \in \alpha_o^c$

μ_{ihjt} =unit cost of secondary resource components h which is assembled with primary resource components $i \in \alpha_p^c$

μ_{ijt} =unit cost of primary resource components $i \in (\alpha_d^c, \alpha_o^c, \alpha_p^c)$

ξ_{thljt} =unit cost of holding a slot, spare or used, in a tertiary resource component l which is assembled with secondary resource components h which is them assembled with primary resource components $i \in \alpha_p^c$

ξ_{iljt} =unit cost of holding a slot, spare or used, in a tertiary resource component l which is assembled with primary resource components $i \in \alpha_o^c$

ξ_{ihjt} =unit cost of holding a slot, spare or used, in a secondary resource component h which is assembled with primary resource components $i \in \alpha_p^c$

ξ_{ijt} =unit cost of holding a slot, spare or used, in primary resource components $i \in (\alpha_d^c, \alpha_o^c, \alpha_p^c)$

x = demand-driven activity quantities, ζ = product or service inventory quantities, r = rearrangement activity quantities, R = new resource quantities, S = spare resource quantities, U = used resource quantities, T = capacity of a unit of resource.

2. Procurement/Spare Balance Equations

These equations drive procurement decisions. It provides with optimal purchase or lease decisions of tertiary, secondary and primary resource components. If spares

exist in the existing inventory of these resources being carried from $(t-1)$ period, products or services utilize these spares in the existing production capacity before deciding to purchase or lease any additional resource components.

Tertiary Resource Components:

The equation (2) determines the number of tertiary resource components, R_{ihljt} required to support $m \in \rho_g$ products or services. Both demand variables x and rearrangement variables r are two main drivers to size the tertiary resource components, $l \in \gamma^c$. These tertiary components are assembled with secondary components, $h \in \beta^e$ which are also assembled with primary resource components $i \in \alpha_p^e$.

$$\sum_{k}\sum_{m\in\rho_g}\{x_{gkmlht}+\sum_{j'\in\delta j}\sum r_{gkmlhtj'j}\}+S_{ihljt}=T_l.R_{ihljt}+S_{ihlj(t-1)} \qquad (2)$$

when $l \in \gamma^0$, $T_l.R_{ihljt}=0$ and $l \in \gamma^1$, R_{ihljt} might have a positive value. j is associated with warehouses, plants or routes where concentration of producing various products or services m takes place, i.e. $j \in v_b.j'$ refers to all possible warehouses, plants or routes including j.

Equation (3) sizes the number of tertiary resource components, R_{iljt} which can be assembled with primary resource components $i \in \alpha_o^e$ to produce products or services $m \in \rho_g$. The equation also keeps track of spare slots, S_{iljt} at time period t. All spares are used first before new tertiary resource components can be purchased or leased.

If $l \in \gamma^0$, $T_l.R_{iljt}=0$ and if $l \in \gamma^1$, R_{iljt} might have a positive integer value.

$$\sum_{k}\sum_{m\in\rho_g}\{x_{gkmli}+\sum_{j'\in\delta j}\sum r_{gkmlij'j}\}+S_{iljt}=T_l.R_{iljt}+S_{ilj(t-1)} \text{ when } i \in \alpha_o^e \qquad (3)$$

where j is associated with warehouses, plants or routes producing a large concentration of various products or services m, i.e. $j \in v_b.j'$ refers to all possible warehouses, plants or routes including j .

Secondary Resource Components:

The number of tertiary resource components, R_{ihljt}, estimated from equation (2), demand-driven variables x and rearrangement variables r drive the number of secondary resource components R_{ihjt}. The tertiary resource components l cascade into secondary resource components h which can then cascade into primary resource components i to support products or services $m \in \rho_g$. Additionally, assembly of secondary resource components h with primary resource components i produces products or services $m \in \rho_f$. Equation (4) comprises of such variables as well as spare balances of production slots in secondary resource components.

$$R_{ihljt}+\sum_{k}\sum_{m\in\rho_f}\{x_{tjkmht}+\sum_{j'}\sum_{j}r_{tjkmht,j'j}\}+S_{ihjt}=T_h.R_{ihjt}+S_{ihj(t-1)} \qquad (4)$$

when $i \in \alpha_p^c$, $h \in \beta^c$ and $l \in \gamma^c$. j is associated with warehouses, plants or routes where a large concentration of various products or services m is produced, i.e. $j \in v_b$. j' refers to all possible warehouses, plants or routes including j.

If initial inventory of secondary resource components exists i.e. $h \in \beta^0$, $T_h.R_{ihjt} = 0$. Spares of secondary resources are used first before additional secondary resources are purchased or leased. If $h \in \beta^1$, R_{ihjt} might have a non-zero positive value.

Primary Resource Components:

The following three equations (5), (6), and (7) are used to determine the number of primary resources, R_{ijt}, to be purchased or leased. These three equations are based on three different categories of primary resources ($\alpha_d^c, \alpha_o^c, \alpha_p^c$) as stated earlier. In each case, $R_{ijt} = 0$ when $i \in (\alpha_d^0, \alpha_o^0, \alpha_p^0)$ i.e. when existing inventory exists in each of the three categories. Spares in the existing inventory are shown in the equations by $S_{ij(t-1)}$ which means that spares of primary resource i are carried from $(t-1)$ period in a warehouse, route or plant j.

Equation (5) determines if standalone primary resource components $i \in \alpha_d^c$ are to be procured and quantities of such components, R_{ijt} . The equation also updates the standalone primary resource spares, S_{ijt}. Here j represents warehouses, plants or routes where products or services m are produced and distributed on a standalone

basis, i.e. $j \in v_d$.

$$x_{ijkmi} + S_{ijt} = T_i.R_{ijt} + S_{ij(t-1)} \text{ when } i \in \alpha_d^c \text{ and } m \in (\rho_g, \rho_f) \quad (5)$$

Equation (6) makes procurement decisions of primary resource components $i \in \alpha_o^c$ which are assembled with tertiary resource components $l \in \gamma^c$ to produce products or services $m \in \rho_g$. R_{ijt} is the number of primary resource components i and S_{ijt} updates spare slots of the primary resource components i at time period t. Here j represents warehouses, plants or routes producing a large concentration of product m at any given time period t.

$$T_l.R_{ilit} + S_{ijt} = T_i.R_{ijt} + S_{ij(t-1)} \text{ when } i \in \alpha_o^c, m \in \rho_g \text{ and } j \in v_b \quad (6)$$

Equation (7) makes the procurement decisions of primary resource components $i \in \alpha_p^c$. The number of primary resource components, R_{ijt}, is optimally determined. Each unit of this primary resource components requires single or multiple secondary resource components and tertiary resource components to produce products or services $m \in (\rho_g, \rho_f)$. The equation also updates spares, S_{ijt}, from one time period to the next. Here j represents warehouses, plants or routes producing or distributing a large quantities of various products or services m.

$$T_h.R_{ihjt} + S_{ijt} = T_i.R_{ijt} + S_{ij(t-1)} \text{ when } i \in \alpha_p^c, m \in (\rho_g, \rho_f) \text{ and } j \in v_b \quad (7)$$

3. Usage Balance Equation

The equations in this section update the usage quantities of production slots in each type of resources. The left-hand side of the equations comprises of variables such as, demand-driven variables x, rearrangement variables r, usage variables U defining used slots being carried from $(t-1)$ period and relevant sub-components R. The right-hand side of these equations is the variable U symbolizing used production slots at time period t.

Tertiary Resource Components:

Equation (8) updates used production slots of tertiary resource components $l \in \gamma^c$ which are assembled with secondary resource components $h \in \beta^c$ and primary

resource components $i \in \alpha_p^c$ to produce products or services $m \in \rho_g$. The demand-driven products or services variables x, rearrangement variables r and used production slots carried from $(t-1)$ period, $U_{ihlj(t-1)}$ are added to derive the used production slots at time period t, U_{ihljt}. Here j is warehouses, plants or routes producing or distributing a large concentration of products or services m, i.e. $j \in v_b$. j' refers to all possible warehouses, plants or routes including j.

$$\sum_k \sum_{m \in \rho_g} \{x_{tjkmlhi} + \sum_{j' \in \delta j} \sum r_{tjkmlhi,j'j}\} + U_{ihlj(t-1)} \leq U_{ihljt} \qquad (8)$$

Equation (9) updates used production slots of tertiary resource components $l \in \gamma^e$ at each time period to support products or services $m \in \rho_g$. The demand-driven product or service assignment x, rearrangement variables r and used production slots carried from $(t-1)$ period, $U_{ilj(t-1)}$ are added to derive the used production slots at time period t, U_{iljt}. In this case, tertiary resource component l is assembled with primary resource component i, and j represents warehouses, plants or routes producing or distributing a large quantities of product or service m. j' is all possible warehouses, plants or routes including j.

$$\sum_k \sum_{m \in \rho_g} \{x_{tjkmli} + \sum_{j' \in \delta j} \sum r_{tjkmli,j'j}\} + U_{ilj(t-1)} \leq U_{iljt} \text{ where } i \in \alpha_o \text{ and } j \in v_b \qquad (9)$$

Secondary Resource Components:

Equation (10) updates used production slots of secondary resource components $h \in \beta^e$ at each time period to support products or services $m \in (\rho_g, \rho_f)$. The tertiary resource components R_{ihljt}, demand-driven service or product assignment x, rearrangement variables r and used production slots carried from $(t-1)$ period, $U_{ihj(t-1)}$ are added to derive the used production slots at time period t, U_{ihjt}. In this

$$R_{ihljt} + \sum_k \sum_{m \in \rho_f} \{x_{tjkmht} + \sum_{j'} \sum r_{tjkmhi,j'j}\} + U_{ihj(t-1)} \leq U_{ihjt} \quad \text{when } i \in \alpha_p^c \qquad (10)$$

equation, j represents warehouses, plants or routes producing or distributing a large quantities of products or services m, i.e. $j \in v_b$. j' represents all possible warehouses, plants or routes including j.

Primary Resource Components:

The following three equations [(11), (12), (13)] update the used production slots of three different types of primary resource components $\alpha_d^c, \alpha_o^c, \alpha_p^c$ corresponding to the procurement equations (5), (6) and (7) respectively. The first equation (11) updates usage of primary resource components α_d^c which are standalone primary resource components to produce standalone products or services $m \in (\rho_g, \rho_f)$

$$x_{ijkmt} + U_{ij(t-1)} \leq U_{ijt} \text{ when } i \in \alpha_d^c \quad \text{and } j\text{th warehouse,} \qquad (11)$$

plant or route is associated with standalone primary resources, i.e. $j \in \nu_d$.

Equation (12) updates usage of production slots in primary resource components α_o^c which produce products or services $m \in \rho_g$ jointly with tertiary resource components $l \in \gamma^c$. R_{iljt} =number of tertiary resource components l which can be assembled with primary resource components $i \in \alpha_o^c$. Here j represents warehouses, plants or routes producing or distributing a large quantities of product or service m.

$$T_l.R_{iljt} + U_{ij(t-1)} \leq U_{ijt} \quad \text{where } j \in \nu_b \qquad (12)$$

Equation (13) updates usage of production slots in primary resource components α_p^c which can be assembled with secondary resource components $h \in \beta^c$ to produce products or services $m \in (\rho_g, \rho_f)$. j represents warehouses, plants or routes from where a large concentration of production or distribution can occur for products or services m.

$$T_h.R_{ihjt} + U_{ij(t-1)} \leq U_{ijt} \quad \text{where } i \in \alpha_p^c \text{ and } j \in \nu_b \qquad (13)$$

4. Product or Service Inventory Equation

Equations (14), (15), (16) and (17) show how inventory of products or services are kept updated. This knowledge of carrying inventory from one time period to another, can be used to rearrange from one primary resource of one plant to another primary resource of another plant or the same plant.

Equation (14) updates inventory of products or services, $m \in \rho_g$, at time period t. These products or services are jointly produced by tertiary resource components, $l \in \gamma^e$, secondary resource components, $h \in \beta^e$, and primary resource components $i \in \alpha_p^e$. j signifies warehouse, plants or routes which can produce or distribute in large quantities of products or services as concentrated production or distribution points. j' represents all possible warehouses, plants or routes including j.

$$x_{tjlmlhi} + \sum_{i' \in \delta j} \sum_{j'} r_{tjlmlhi,i'j'} + \zeta_{(t-1)jlmlhi} \leq \zeta_{tjlmlhi} \text{ where } j \in v_b \qquad (14)$$

Equation (15) updates inventory of products or services, $m \in \rho_g$, at time period t. These products or services are jointly produced by tertiary resource components, $l \in \gamma^e$, and primary resource components, $i \in \alpha_o^e$. j represents warehouses, plants or routes which can produce or distribute large quantities of products or services at a given time period t as concentrated production or distribution points. j' represents all possible warehouses, plants or routes including j.

$$x_{tjlmli} + \sum_{i' \in \delta j} \sum_{j'} r_{tjlmli,i'j'} + \zeta_{(t-1)jlmli} \leq \zeta_{tjlmli} \text{ where } j \in v_b \qquad (15)$$

Equation (16) updates inventory of products or services, $m \in \rho_f$, at time period t. This inventory is jointly produced by secondary resource components, $h \in \beta^e$, and primary resource components, $i \in \alpha_p^e$. The equation is made of demand activities x, rearrangement activities r and inventory being carried from $(t-1)$ period i.e. $\zeta_{(t-1)jlmhi}$. In this equation, j represents warehouses, plants or routes producing large quantities of products or services at any given time period t as concentrated production or distribution centers, i.e. $j \in v_b$. j' represents all possible warehouese, plants or routes including j.

$$x_{tjlmhi} + \sum_{i' \in \delta j} \sum_{j'} r_{tjlmhi,i'j'} + \zeta_{(t-1)jlmhi} \leq \zeta_{tjlmhi} \text{ when } m \in \rho_f \text{ and } i \in \alpha_p^e \qquad (16)$$

Equation (17) updates inventory of products or services, $m \in (\rho_g, \rho_f)$, produced by standalone primary resource components, $i \in \alpha_d^c$. j, in this case, represents warehouses, plants or routes which are considered to be standalone production or distribution centers.

$$x_{tjkmi} + \zeta_{(t-1)jkmi} \leq \zeta_{tjkmi} \text{ where } m \in (\rho_g, \rho_f), i \in \alpha_d^c \text{ and } j \in v_d \qquad (17)$$

5. Product or Service Demand Equation

Equations (18) and (19) capture customer demand, a_{tkm}, in the variables x. The subscripts t, k and m refer to time period, customer demand locations and type of products or services respectively. Demand can be distributed across jth warehouse, production plant or route. $\sum_j$ in the left hand side, drives such a distribution mechanism. j, here, includes all possible production or distribution centers such as, both concentrated and standalone production or distribution centers i.e. $j \in (v_b, v_d)$. Equation (18) captures demand for products or services, $m \in \rho_g$, which can be produced by any of the three ways as follows: an assembly of tertiary resource components, $l \in \gamma^c$, secondary resource components, $h \in \beta^c$, and primary resource components, $i \in \alpha_p^c$, an assembly of tertiary resource components, $l \in \gamma^c$, and primary resource components, $i \in \alpha_o^c$, and standalone primary resource components, $i \in \alpha_d^c$. The first two ways are related to $j \in v_b$, concentrated production or distribution centers and the last one is related to $j \in v_d$, standalone production, distribution or routes.

$$\sum_{j \in v_b} \left(\sum_{i \in \alpha_p^c} x_{tjkmlhi} + \sum_{i \in \alpha_o^c} x_{tjkmli} \right) + \sum_{j \in v_d} \sum_{i \in \alpha_d^c} x_{tjkmi} = a_{tkm} \qquad (18)$$

Equation (19) is a set of demand equations related to products or services, $m \in \rho_f$, which are produced by any of the following two ways: an assembly of secondary resource components, $h \in \beta^c$, with primary resource components, $i \in \alpha_p^c$, and standalone primary resource components, $i \in \alpha_d^c$, for $m \in \rho_f$.

$$\sum_{j \in v_b} \sum_{i \in \alpha_p^c} x_{tjkmhi} + \sum_{j \in v_d} \sum_{i \in \alpha_d^c} x_{tjkmi} = a_{tkm} \quad \text{where } j \in v_b \text{ represents concentrated production} \qquad (19)$$

or distribution centers, or hubbing routes that are associated with primary resource components $i \in \alpha_p^c$ and $j \in v_d$ represents production or distribution centers, or standalone routes that are associated with standalone primary resource components

$i \in \alpha_d^c$.

6. Rearrangement Equation

Equations (20) to (25) set the rearrangement variables so that products or services can migrate from one primary resource of a production plant to another primary resource of another production plant or the same plant at a discrete time period t, if it makes economic sense.

Equation (20) defines the rearrangement equations for products or services $m \in \rho_g$ which require an assembly of tertiary resource components l, secondary resource components h and primary resource components i. Migration occurs from primary resource components i' of production plant j' to primary resource components i of production plant j where $j' = j$ or $j' \neq j$, and both j and j' belong to v_b i.e. concentrated production or distribution centers.

$$\sum_i \sum_j r_{tjkmlhi,j'j} \leq \zeta_{(t-1)j'kmlhi'} \text{ where } m \in \rho_g,\ l \in \gamma^c,\ h \in \beta^c,\ i \in \alpha_p^c \text{ and } i' \in \alpha_p^c(\delta) \quad (20)$$

Equation (21) defines the rearrangement equations for products or services $m \in \rho_g$ where migration occurs from primary resource components $i' \in \alpha_d^c$ of production plant j' to primary resource components $i \in \alpha_p^c$ of production plant j. The products or services carried by standalone primary resources i' of $j' \in v_d$ migrate to primary resource component i of $j \in v_b$ which requires an assembly of tertiary resource components l and secondary resource components h. $j \in v_b$ in the left-hand side of the equation and $j' \in v_d$ where $j = j'$ or $j \neq j'$.

$$\sum_i \sum_j r_{tjkmlhi,j'j} \leq \zeta_{(t-1)j'kml'} \text{ where } m \in \rho_g,\ l \in \gamma^c,\ h \in \beta^c,\ i \in \alpha_p^c \text{ and } i' \in \alpha_d^c \quad (21)$$

Equation (22) defines the rearrangement variables for products or services, $m \in \rho_g$, which require an assembly of tertiary resource components l and primary resource components i. Products or services, m, roll over from primary resource components i' of production plant j' to primary resource components i of production plant j.

$$\sum_i \sum_j r_{tjkmli,j'j} \leq \zeta_{(t-1)j'kmli'} \text{ where } m \in \rho_g,\ l \in \gamma^c,\ i \in \alpha_o^c,\ i' \in \alpha_o^c(\delta) \text{ and } j \in v_b \quad (22)$$

Equation (23) defines the rearrangement variables for products or services, $m \in \rho_g$, where migration occurs from primary resource components $i' \in \alpha_d^c$ of production plant $j' \in \nu_d$ to primary resource components $i \in \alpha_o^c$ of production plant j. The products or services carried by standalone primary resources i' of production plant $j' \in \nu_d$, migrate to primary resource component i of production plant $j \in \nu_b$, which requires an assembly with tertiary resource components $l.j \in \nu_b$ in the left-hand side of the equation and $j' \in \nu_d$ where $(j = j') \cup (j \neq j')$.

$$\sum_i \sum_j r_{tjkmli,i'j'} \leq \zeta_{(t-1)j'km i'} \text{ where } m \in \rho_g,\ l \in \gamma^e,\ i \in \alpha_o^c \text{ and } i' \in \alpha_d^c \qquad (23)$$

Equation (24) defines the rearrangement variables for products or services, $m \in \rho_f$, which require an assembly of secondary resource components, β^e, with primary resource components, $i \in \alpha_p^c$. Products or services, m, roll over from primary resource component i' of production plant j' to primary resource components i of production plant j where $(j = j') \cup (j \neq j')$.

$$\sum_i \sum_j r_{tjkmhi,i'j'} \leq \zeta_{(t-1)j'kmhi'} \text{ where } m \in \rho_f,\ h \in \beta^e,\ i \in \alpha_p^c,\ i' \in \alpha_p^c(\delta) \qquad (24)$$

Equation (25) defines the rearrangement variables for products or services, $m \in \rho_f$, where migration occurs from primary resource components $i' \in \alpha_d^c$ of production plant j' to primary resource components $i \in \alpha_p^c$ of production plant j. The products or services carried by standalone primary resources i' of production plant j', migrate to primary resource component i of production plant j, which requires an assembly with secondary resource components, $h \in \beta^e.j \in \nu_b$ in the left-hand side of the equation and $j' \in \nu_d$ where j can be same as j' or different from j' i.e., $(j = j') \cup (j \neq j')$.

$$\sum_i \sum_j r_{tjkmhi,i'j'} \leq \zeta_{(t-1)j'km i'} \text{ where } m \in \rho_f,\ h \in \beta^e,\ i \in \alpha_p^c \text{ and } i' \in \alpha_d^c \qquad (25)$$

7. Decision to rearrange or not to rearrange

Equations (26), (27), (28), (29),(30) and (31) are the main drivers of determining if products or services, m, being produced by primary resource components, i',

of production plant j' need to be rolled over to primary resource component i of production plant j at time period t, or continue to be produced by primary resource components, i'. at production plant j'.

Equation (26) signifies a decision to rearrange or not to rearrange products or services $m \in \rho_g$ produced by primary resource component, $i' \in \alpha_p^c$, at production plant j', together with tertiary resource component l and secondary resource component h. As a result of this rearrangement, products or services m migrate from primary resource component i' of production plant j' to primary resource component i of production plant j or stay with primary resource component i' at production plant j'. $j'(\phi)$ is the set of production plants from where migration takes place to production plant j'.

$$\zeta_{tj'lmlhi'} = x_{tj'lmlhi'} + \sum_{i'(\delta)} \sum_{j'(\phi)} r_{tj'lmlhi'i'(\delta)j'(\phi)} + \zeta_{(t-1)j'lmlhi'} - \sum_{i} \sum_{j} r_{tjlmlhi'j'} \qquad (26)$$

where $m \in \rho_g$, $l \in \gamma^e$, $h \in \beta^e$, $j \in v_b$ and $i' \in \alpha_p^c$, but $i' \notin i'(\delta)$ and $i \neq i'$.

Equation (27) signifies a decision to rearrange or not to rearrange products or services, $m \in \rho_g$, being produced by standalone primary resource components $i' \in \alpha_d^c$. These products or services, m, can migrate to non-standalone primary resource components $i \in \alpha_p^c$ which require to be assembled with tertiary resource components l and secondary resource components h.

$$\zeta_{tj'lmi'} = x_{tj'lmi'} + \zeta_{(t-1)j'lmi'} - \sum_{i} \sum_{j} r_{tjlmlhi'j'} \text{ where } j' \in v_d \qquad (27)$$

and $j \in v_b$.

Equation (28) signifies a decision to rearrange or not to rearrange products or services $m \in \rho_g$ produced by primary resource components, $i' \in \alpha_o^c$, together with tertiary resource components, $l \in \gamma^e$. As a result of this rearrangement, products or services m migrate from primary resource components i' of production plant j' to primary resource component i of production plant j or stay with primary resource component i' at production plant or warehouse j'.

$$\zeta_{tj'kmli'} = x_{tj'kmli'} + \sum_{i'(\delta)} \sum_{j'(\phi)} r_{tj'kmli'i'(\delta)j'(\phi)} + \zeta_{(t-1)j'kmli'} - \sum_{i} \sum_{j} r_{tjkmlij'i'} \qquad (28)$$

where $m \in \rho_g$, $l \in \gamma^c$, $j \in \nu_b$ and $i' \in \alpha_o^c$, but $i' \notin i'(\delta)$ and $i \neq i' j'(\phi)$ is a set of production plants from where products or services m migrate to production plant j'.

Equation (29) signifies a decision to rearrange or not to rearrange products or services, $m \in \rho_g$, being produced by standalone primary resource components $i' \in \alpha_d^c$. These products or services, m, can migrate to non-standalone primary resource components $i \in \alpha_o^c$ which require to be assembled with tertiary resource components l where $j' \in \nu_d$ and $j \in \nu_b$.

$$\zeta_{tj'kmi'} = x_{tj'kmi'} + \zeta_{(t-1)j'kmi'} - \sum_{l} \sum_{j} r_{tjkmlij'i'} \quad \text{where } i \neq i', i' \in \alpha_d^c \text{ and } m \in \rho_g \qquad (29)$$

Equation (30) determines whether or not to roll over products or services, $m \in \rho_f$, from primary resource components $i' \in \alpha_p^c$ of production plant j' to primary resource component $i \in \alpha_p^c$ of production plant j where $i \neq i'$ and $j = j'$ or $j \neq j'$. These primary resource components, i and i', need to be assembled with secondary resource component, $h \in \beta^c$.

$$\zeta_{tj'kmhi'} = x_{tj'kmhi'} + \sum_{i'(\delta)} \sum_{j'(\phi)} r_{tj'kmhi'i'(\delta)j'(\phi)} + \zeta_{(t-1)j'kmhi'} - \sum_{i} \sum_{j} r_{tjkmhij'i'} \qquad (30)$$

where $m \in \rho_f$, $j \in \nu_b$, $h \in \beta^c$ and $i' \in \alpha_p^c$, but $i' \notin i'(\delta)$ and $i \neq i' j'(\phi)$ is a set of warehouses, production plants or routes from where products or services migrate to warehouse, production plant or route j'.

Equation (31) determines whether or not to roll over products or services, $m \in \rho_f$ from standalone primary resource components, $i' \in \alpha_d^c$, of production plant j' to

primary resource component $i \in \alpha_p^c$, of production plant j, which need to be assembled with secondary resource component, $h \in \beta^c$ where $j' \in \nu_d$ and $j \in \nu_b$.

$$\zeta_{qj'kmi'} = x_{qj'kmi'} + \zeta_{(t-1)j'kmi'} - \sum_{i}\sum_{j} r_{tjkmhi,i'j'} \quad \text{when } i \neq i' \text{ and } m \in \rho_f \qquad (31)$$

8. Initial Inventory When t=0:

Tertiary Resource Components:

The equation (32) stores spare quantities, E_{thljt}, of tertiary resource units, l, that can be assembled with secondary resource components, h, and primary resource components, $i \in \alpha_p^c$. The variables under which spare quantities are stored, are S_{thljt}.

$$S_{thljt} = E_{thljt} \text{ where } t = 0 \text{ and } j \in \nu_b \qquad (32)$$

The equation (33) stores the used quantities, F_{thljt} of tertiary resource units, l, that can be assembled with secondary resource components, h, and primary resource components, $i \in \alpha_p^c$. The variables under which used quantities are stored, are U_{thljt}.

$$U_{thljt} = F_{thljt} \text{ where } t = 0 \text{ and } j \in \nu_b \qquad (33)$$

The equation (34) stores spare quantities, E_{iljt}, of tertiary resource units, l, that can be assembled with primary resource components, $i \in \alpha_o^c$. The variables under which spare quantities are stored, are S_{iljt}.

$$S_{iljt} = E_{iljt} \text{ where } t = 0 \text{ and } j \in \nu_b \qquad (34)$$

The equation (35) stores the used quantities, F_{iljt} of tertiary resource units, l, that can be assembled with primary resource components, $i \in \alpha_o^c$. The variables under which used quantities are stored, are U_{iljt}.

$$U_{iljt} = F_{iljt} \text{ where } t = 0 \text{ and } j \in \nu_b. \qquad (35)$$

Secondary Resource Components:

The equation (36) stores spare quantities, E_{ihjt}, of secondary resource units, h, that can be assembled with primary resource components, $i \in \alpha_p^c$. The variables under which spare quantities are stored, are S_{ihjt}

$$S_{ihjt} = E_{ihjt} \text{ where } t = 0 \text{ and } j \in v_b. \quad (36)$$

The equation (37) stores the used quantities, F_{ihjt} of secondary resource units, h, that can be assembled with primary resource components, $i \in \alpha_p^c$. The variables under which used quantities are stored, are U_{ihjt}.

$$U_{ihjt} = F_{ihjt} \text{ where } t = 0 \text{ and } j \in v_b. \quad (37)$$

Primary Resource Components:

The equation (38) stores spare quantities, E_{ijt} of primary resource units, $i \in (\alpha_d^c, \alpha_o^c, \alpha_p^c)$. The variables under which spare quantities are stored, are S_{ijt}.

$$S_{ijt} = E_{ijt} \text{ where } t = 0 \text{ and } j \in (v_b, v_d) \quad (38)$$

The equation (39) stores used quantities, F_{ijt} of primary resource units, $i \in (\alpha_d^c, \alpha_o^c, \alpha_p^c)$. The variables under which used quantities are stored, are U_{ijt}.

$$U_{ijt} = F_{ijt} \text{ where } t = 0 \text{ and } j \in (v_b, v_d) \quad (39)$$

9. Non-negativity and Integer Constraints:

The equation (40) defines that vectors, $\underline{x}$, $\underline{r}$, $\underline{R}$, $\underline{S}$, $\underline{U}$, and $\underline{\zeta}$ are non-negative, $\underline{x}$, $\underline{r}$ and $\underline{R}$ are integer variables and the remaining variables are positive and continuous.

$$\bar{x} \geq 0,\ \bar{r} \geq 0,\ \bar{R} \geq 0,\ \bar{S} \geq 0,\ \bar{U} \geq 0,\ \bar{\zeta} \geq 0, \text{ and } \bar{x},\ \bar{r} \text{ and } \bar{R} \text{ are integers.} \quad (40)$$

Methods contemplated by some embodiments of the present invention with relation to the above-noted equations are now described with regard to Figures 2-4. Referring first to Figure 2, pertinent resources and activities (for example, customer demand and available/requested equipment) are captured as a function of time across business processes (that is, they are captured across the plurality of time periods and across the marketing, procurement and production processes), as indicated by a block 204. Then, the procedure that follows below is implemented across the various time periods, as indicated by a block 206.

The next step is that various variables at T=0 are initialized (using equations 32-39) as indicated by a block 208. Also, at this point, it is assumed that a decision has been made concerning which resources are ranked as, for example, primary, secondary, and tertiary. Then, for all pertinent primary resources and activities (as indicated by a block 210) a determination is made as to whether a given primary resource (e.g., a DS3 line) requires a secondary resource (e.g., a DS3 multiplexer) to support a product or service, M (e.g., the A1.5 service), as indicated by a block 212. If the answer is "yes," a determination is then made as to whether the secondary resource requires tertiary resource l to support the product or service, as indicated by a block 214. If it does, then a determination is made as to whether a "spare" one of those tertiary resources l exist that can be carried over from the resources existing in time period t-1 to the current time period t at issue. This is determined using equations 2, 8, 14 and 20, as indicated by a block 216. If a "spare" does exist, then the pertinent activities are performed using that resource, as indicated by a block 220. If not, then it is performed using a new resource.

It should be realized that the definition of "spare" resource needs to be decided upon in some way (e.g., whether it means that the resource is available immediately, locally, from a nearby factory, etc). Thus, the line of demarcation between a spare and a new resource needs to be determined depending upon the current environment. For example, any resource currently in the possession of a business either due to lease or purchase can be defined as spare if the resource is idle. Any new order that is coming from suppliers (outside the business) can be defined as a new resource.

Whether or not the answer to block 214 is yes or no, reaching that block in the first place meant that a determination had already been made that the answer to block 212 was yes, and thus in either case a determination needs to be made as to whether a "spare" exists in secondary resource h that can be carried from the previous time period t-1 to the current one, using equations 4, 10, 16 and 24 as

indicated by a block 218. Again, if a spare exists, pertinent activities are performed with that secondary spare resource, as indicated by a block 224, and otherwise performed with a new resource, as indicated by a block 226.

Similar determinations occur with for the remaining possible combinations of dependencies, as indicated by blocks 228-322 shown on Figures 2 and 3. The next step after those blocks, in all cases, is to assign various resources in accordance with the demand, as a result of the analysis given in the previous boxes of the flowchart. This is done using equations 18 and 19, as indicated by a block 324 on Figure 3. Then, a determination is made as to whether it is more economical to keep the products or services assigned to the primary resources. This is determined using equations 26-31, as indicated by a block 326. If it is more economical, the products or services remain assigned as they were, as indicated by a block 328. Otherwise, a rearrangement of products or services is made, again using equations 26-31, as indicated by a block 330.

Referring now to Fig. 4, the next step is to determine whether all pertinent primary resources and activities have been considered. This is a kind of "sanity check" to make sure all was covered. This is indicated by a block 402.

Decision block 404 indicates that the above-noted analysis needs to be implemented for each time period t. Once that is done, the various systems implemented during the various time periods are further optimized, as indicated by a block 406. This could be done in any number of ways, for example, using an integer programming solver using CPLEX (made by CPLEX optimization, Inc. of Incline Village, NV) or using heuristic techniques.

The next step is that various pricing algorithms are used to determine "favorable" pricing of the various services over the time periods t, in view of such factors as availability. Algorithms exist for doing this, such as the virtual pricing techniques described in U.S. patent 5,270,921, which is incorporated by reference herein.

The next step is to determine whether there may be too much variation in demand between each time period t since the last analysis due to pricing effects on the market. If there is, then optimize the system again. Multiple runs of all the previous steps need to be performed starting from box 204 if there are multiple times of demand changes in a period at which the system is being optimized for future periods.

The next step is to determine whether a price-performance mark has been met, as indicated by blocks 412 and 414. Lastly, a determination is made as to whether the reference T = 0 has changed, in which case the analysis would have to be re-performed) as indicated by a block 416).

Of course, it should be understood that Figures 2-4 and the associated description above is merely by way of example, and that the present invention contemplates any number of different configurations, additional steps and/or sequence of steps as depicted.

As a specific example of usage of the present invention, the invention was applied to test the procurement process, production process and marketing process of AT&T's access network at a prototype level. The inventory of access network facilities and equipment changes over time due to rise of demand in one service and decline in another, introduction of new technology and changes in customer demographics. Three different services, ASDS (Accunet Spectrum Digital Services), Accunet T1.5 (supporting dedicated T1.5 service) and NODAL T1.5 (supporting business switched services such as, SDN, 800, etc.) were used to test this model. The other services such as, CCS (consumer communications services) can also be supported and new services can be added. At the present time, AT&T leases facilities and network equipment from regional bell operating companies (RBOCs) to provide customers access to its long distance network. It also returns facilities to bell companies when these facilities are not needed. In addition, migration of these services from standalone facilities to high cap or from one high cap to another high cap is needed to manage the network costs effectively, and thereby maximize the asset utilization and revenue. Flow of materials between AT&T and its vendors, and within AT&T's production and marketing processes can best be optimized by a dynamic optimization technique.

A time-variant dynamic systems model can best mimic this flow of materials across business processes as a function of time. The model makes optimal decisions of procurement process, production process and marketing process at discrete time periods. The technique was applied to one-level hubbing architecture in the AT&T access network. It is assumed that the intermediate hubbing architecture would be solved from the solution of the one-level hubbing problem.

In a one-level hubbing scenario, multiple LEC (local exchange carriers) offices are considered to be demand locations. It is assumed that demand of different telecommunication services originating at these locations is destined to multiple AT&T serving offices via a route j. In a one-level hubbing scenario, a route vector

consists of direct routes i.e., between k demand location and AT&T serving offices and via-routes (i.e., demand from k location might be routed to an AT&T serving office via another LEC office, a potential hub candidate). Rearrangement activities can take place to maintain the optimum state of the network. Three services, previously mentioned, are produced by leasing standalones, or high cap facilities (e.g. T1.5 carrying DS0, DS3 or DS3+s carrying both DS0 channels and DS1 channels). If the circuits are leased standalones over time and reach break-even with T1.5 or the lowest cost DS3 alternative, the standalones get rolled over to T1.5 or DS3 at that time. If the circuits are routed via a LEC hub-AT&T POP pair, j, in one time period, they might migrate to a higher level capacity in the same jth route, or another jth route, provided sufficient growth exists and cost-optimality criteria is satisfied.

The Pacific Bell territory has been chosen for this study. Figure 5 shows different access alternatives available in the Pacific Bell tariff. T1 multiplexers are considered to be tertiary resource components when they are cascaded into DS3 multiplexers to support the ASDS (DS0 type service). However, a T1 multiplexer is also used in conjunction with a T1 facility, designated as a "primary" resource component. DS3 multiplexers are secondary resource components which are cascaded into high cap DS3 facilities to support ACCUNET T1.5 and NODAL T1.5 services. High capacity facilities are of different types, and come in n different numbers of DS3 packs (n x DS3) or T1.5 facility. T1.5, 1 x DS3, 3 x DS3 and 12 x DS3 were used as primary resource components in this study in addition to standalone options. These types of resources might be available as initial inventory in the existing access network of AT&T, an important input to a multi-period dynamic systems model.

Figures 6 a-c account for possible migration activities of services among primary resources. The column i is the primary resource where services from primary resource i' would migrate to. In a growth situation, migration of asds, accunet and nodal can occur from standalones or lower capacity facility to higher capacity facility. The reverse is true with respect to declining demand of a service. However, this model currently addresses growth only and can be easily modified to include declining demand within the same mathematical principles.

Figure 7 displays some embodiments contemplated by the present invention of a high-level software architecture for deploying dynamic resource allocation techniques. In the example depicted by this figure, the marketing and sales force is required to gather customer-specific requirements and store them in a

customer requirements database 702. These requirements are time-dependent and so they should be time-tagged. Customer demand, product or service specific features and customer expected arrival time or departure time are some of these requirements.

The procurement process is required to gather vendor-specific resources or technology alternatives and their costs. The production process is required to gather production-related requirements and their costs. This information should be stored in a procurement/production database 704. In addition, the procurement process should identify possible warehouse or plant locations. For example, in telecommunication industry they are the local serving offices, network concentration locations or AT&T POPs. This information should also be time-tagged based on their availability.

A dynamic systems model 706 is contemplated to contain the facilities and various modules by which the above-noted equations can be implemented. Some embodiments of the present invention contemplate that this implementation can utilize the C programming language making use of CPLEX callable libraries. Of course, it should be understood that any number of programming languages and libraries can also be used.

The dynamic systems model 706 takes both the customer requirements from marketing and sales, production operation requirements from the production process and the procurement requirements from procurement as its input. The model then creates time-variant optimum results which are stored in a master database 708. The optimum procurement results are sent to the vendors for order processing. The yield management information is accessed by marketing and sales for product or service sales. The production process takes the optimum production schedule to assure that it produces finished products just-in time for delivery to customers. The input and output to and from the marketing, production, procurement (and inventory control) entities is done through user interface 710 and screens 712-718.

The whole cycle of optimization run is repeated based on a moving horizon policy to keep the inventory at optimum state. In a moving horizon policy, an optimization cycle gets repeated at the current time period (period 0) of the moved horizon as illustrated in Figure 8.

Example of hardware environments as contemplated by some embodiments of the present invention are now discussed with regard to Figure 9. Referring to Figure 9, a computer environment 900 shown can be a single or multiple-user environments, and can utilize one or more processors. In addition, computer environment 900 can be of any number of computer types, including

microcomputers, workstations, minicomputers, mainframes and massively parallel processing computers. Examples of microcomputers/workstations include the IBM PC manufactured from IBM Corporation of Armonk, NY, the Macintosh from Apple Computer and the SGI R4400 by Silicon Graphics of Mountain View, California; Examples of minicomputers include the VAX 750 from Digital Equipment Corporation of Maynard, Massachusetts and the AS/400 from IBM Corporation; an example of a mainframe is the System-390 series from IBM; and an example of a massively parallel processing computer is the Cray T3D from Cray Research of Eagan, Minnesota. Of course, any number of other types of computer systems are also contemplated for use with the present invention. Also, the present invention contemplates that two or more of such computer environments can be linked and used together.

In addition, any number of operating systems can be used with regard to the computer environment used with the present invention, including any of the various versions of Unix, VMS from Digital Equipment, OS/2 from IBM and/or DOS or Windows from Microsoft Corporation of Redmond, WA, or the Macintosh OS from Apple Computer.

Still referring to Figure 9, some embodiments of the present invention contemplate that some type of communications link such as a bus 902 allows various components of the computer environment 900 to communicate. As shown, one or more processor(s) 904 are in communication with bus 902. Some embodiments of the present invention contemplate that the processor(s) 904 can be those typically found in the computers mentioned above, or they can be any number of other types of processor(s). For example, with regard to microcomputers, any of the Intel 80X86 series, Pentium or any other CPU developed in Intel's future line of processors are contemplated.

Also in communication with bus 902 is a display device 905. This can be any type of display device capable of displaying video information, and can also act as an input device where a touch screen is used.

Memory/storage device(s) 906 is also shown as being in communication with bus 902. Some embodiments of the present invention contemplate that the memory/storage device(s) 906 could be any number of different types of transient, permanent or semi-permanent computer-readable mediums for storing information, including RAM, ROM, EPROM, magnetic, electronic, atomic or optical (including holographic) storage, some combination thereof, etc. Software component(s) 912 are shown as being associated with memory/storage 906. In addition, some

embodiments of the present invention also contemplate that the computer-readable medium could include any type of transmission scheme for communicating information.

The precise nature of software component(s) 912 are envisioned to vary depending upon the function and location with which the associated computer environment 900 is used. Some embodiments of the present invention contemplate that they would include the dynamic systems model 706 and databases 702, 704 and 708.

One or more input devices 908 are also envisioned to be associated with computer environment 900. Any number of different types of input devices such as a mouse, keyboard, etc., can be used.

In addition, a communications link 910 is also shown as being in communication with bus 902. This can be used, for example, to communicate with marketing and/or production and/or procurement sites, where any of them are at a remote location from the computer environment 900.

It should be understood that Figure 9 and the associated description above is merely an example, and that the present invention contemplates any number of different configurations and/or different components as well.

In general, it should be emphasized that the various components of embodiments of the present invention can be implemented in hardware, software or a combination thereof. In such embodiments, the various components and steps would be implemented in hardware and/or software to perform the functions of the present invention. Any presently available or future developed computer software language and/or hardware components can be employed in such embodiments of the present invention. For example, at least some of the functionality mentioned above could be implemented using the C or C++ programming languages.

It is also to be appreciated and understood that the specific embodiments of the invention described hereinbefore are merely illustrative of the general principles of the invention. Various modifications may be made by those skilled in the art consistent with the principles set forth hereinbefore.

Claims:

1. A computer-implemented method for allocating resources over a plurality of time periods t, comprising the steps of:

(1) receiving resource requirements to be implemented in each of the plurality of time periods;

(2) receiving resource availability information indicating anticipated resource availability during each of the plurality of time periods;

(3) configuring a system at each time period t based upon said steps (1) and (2), wherein the resources used at each time period t are interdependent with the other plurality of time periods;

(4) determining the pricing of a product or service, wherein said product or service is a subset of said system, based upon availability of the resources at each time period t.

2. The computer-implemented method of claim 1, wherein said step (3) comprises, at each time period t, the steps of:

(a) determining, for a specified product or service, which resources can be classified as primary and secondary;

(b) determining which of said secondary resources are required, in conjunction with said primary resources, to provide said specified product or service;

(c) for each said required secondary resource, determining whether said secondary resource is available or whether it requires procurement;

(d) for each primary resource, determining whether the primary resource is available or whether it requires procurement.

3. The computer-implemented method of claim 2, further comprising the steps of:

(e) determining which resources can be classified as tertiary;

(f) determining which of said tertiary resources are required, in conjunction with said primary resources or said secondary resources, to provide said specified product or service;

(g) for each said required tertiary resource, determining whether said tertiary resource is available or whether it requires procurement.

4. The computer-implemented method of claim 2, wherein said primary or secondary resources obtained from intercompany sources are designated as

* C L M S *

available resources.

5. The computer-implemented method of claim 1, further comprising the step, after said step (3), of optimizing each said system at each time period t by analyzing all resources over all of said plurality of time periods.

6. A computer-readable medium for allocating resources over a plurality of time periods t, comprising:

a first module, wherein said first module receives resource requirements to be implemented in each of the plurality of time periods;

a second module, wherein said second module receives resource availability information indicating anticipated resource availability during each of the plurality of time periods;

a third module, wherein said third module configures a system at each time period t based upon input from said first and second modules, wherein the resources used at each time period t are interdependent with the other plurality of time periods;

a fourth module, wherein said fourth module determines the pricing of a product or service, wherein said product or service is a subset of said system, based upon availability of the resources at each time period t.

7. The computer-readable medium of claim 6, wherein said third module comprises, at each time period t:

a fifth module, wherein said fifth module determines, for a specified product or service, which resources can be classified as primary and secondary;

a sixth module, wherein said sixth module determines which of said secondary resources are required, in conjunction with said primary resources, to provide said specified product or service;

a seventh module, wherein said seventh module determines, for each said required secondary resource, whether said secondary resource is available or whether it requires procurement;

an eighth module, wherein said eighth module determines, for each primary resource, whether the primary resource is available or whether it requires procurement.

8. The computer-readable medium of claim 7, further comprising:

* C L M S *

a ninth module, wherein said ninth module determines which resources can be classified as tertiary;

a tenth module, wherein said tenth module determines which of said tertiary resources are required, in conjunction with said primary resources or said secondary resources, to provide said specified product or service;

an eleventh module, wherein said eleventh module determines, for each said required tertiary resource, whether said tertiary resource is available or whether it requires procurement.

9. The computer-readable medium of claim 7, wherein said primary or secondary resources obtained from intercompany sources are designated as available resources.

10. The computer-readable medium of claim 6, further comprising an optimization module, wherein said optimization module optimizes each said system at each time period t by analyzing all resources over all of said plurality of time periods.

11. A machine for allocating resources over a plurality of time periods t, comprising:

first means for receiving resource requirements to be implemented in each of the plurality of time periods;

second means for receiving resource availability information indicating anticipated resource availability during each of the plurality of time periods;

third means for configuring a system at each time period t based upon said first and second means, wherein the resources used at each time period t are interdependent with the other plurality of time periods;

fourth means for determining the pricing of a product or service, wherein said product or service is a subset of said system, based upon availability of the resources at each time period t.

12. The machine of claim 11, wherein third means comprises, at each time period t:

fifth means for determining, for a specified product or service, which resources can be classified as primary and secondary;

sixth means for determining which of said secondary resources are required, in conjunction with said primary resources, to provide said specified product or service;

seventh means for determining, for each said required secondary resource, whether said secondary resource is available or whether it requires procurement;

eighth means for determining, for each primary resource, whether the primary resource is available or whether it requires procurement.

13. The machine of claim 12, further comprising:

ninth means for determining which resources can be classified as tertiary;

tenth means for determining which of said tertiary resources are required, in conjunction with said primary resources or said secondary resources, to provide said specified product or service;

eleventh means for each said required tertiary resource, determining whether said tertiary resource is available or whether it requires procurement.

14. The machine of claim 12, wherein said primary or secondary resources obtained from intercompany sources are designated as available resources.

15. The machine of claim 11, further comprising optimizing means for optimizing each said system at each time period t by analyzing all resources over all of said plurality of time periods.