Fig. 1

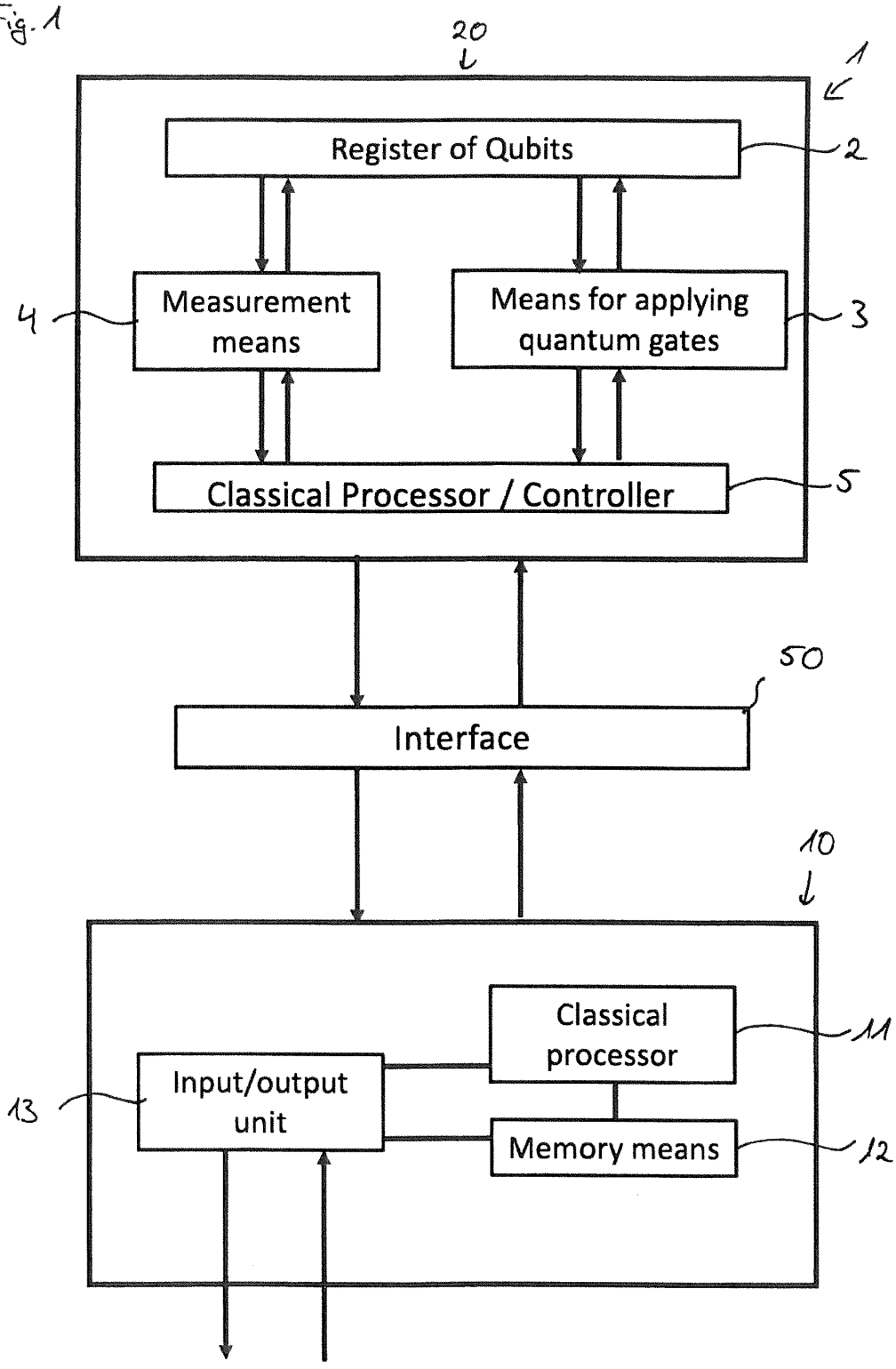

20
1
Register of Qubits
2
Measurement means
4
Means for applying quantum gates
3
Classical Processor / Controller
5
50
Interface
10
Classical processor
11
Input/output unit
13
Memory means
12

Fig. 2 – Part 1

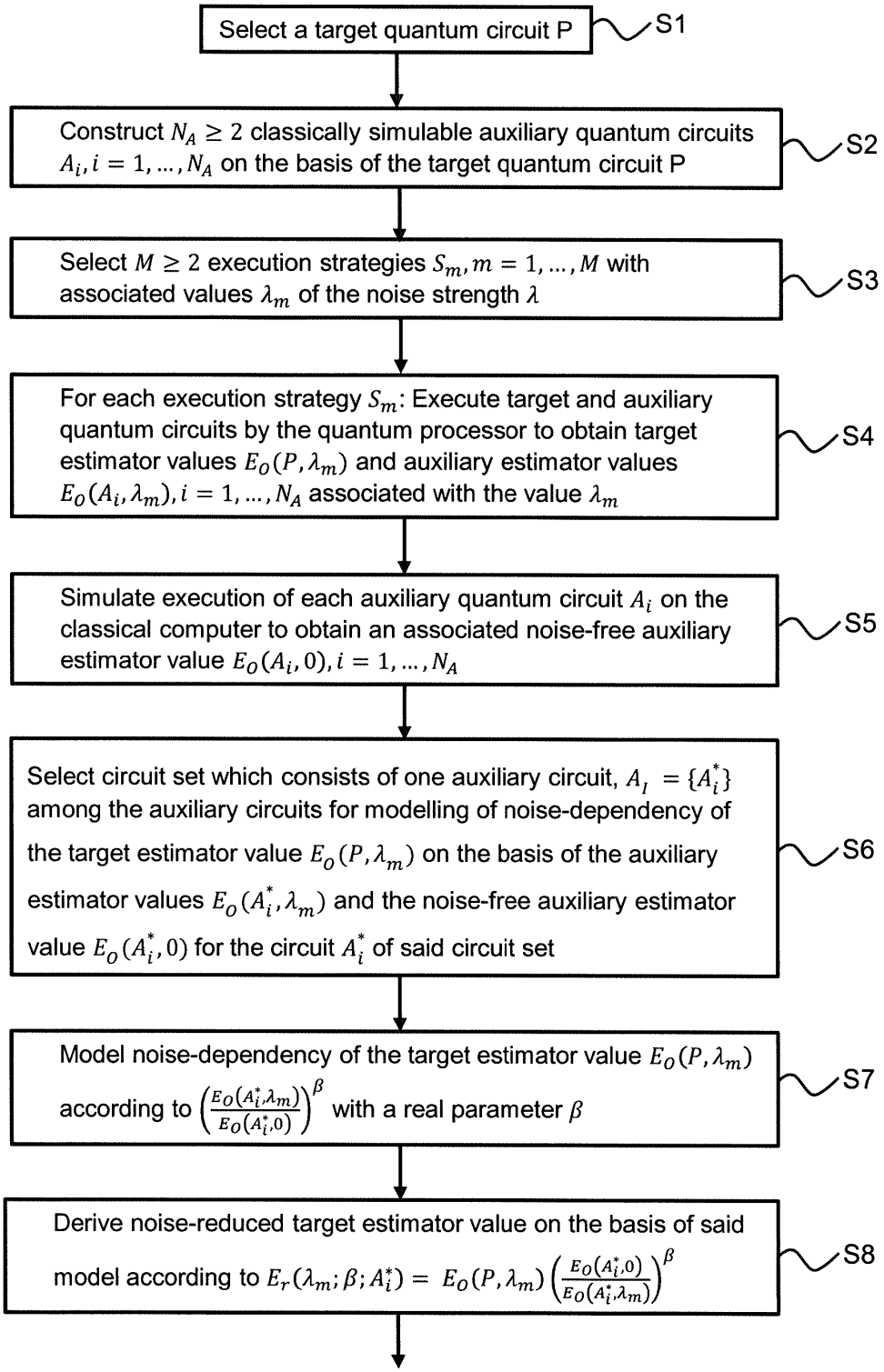

Select a target quantum circuit P
S1
Construct $N_A \geq 2$ classically simulable auxiliary quantum circuits $A_i, i = 1, \ldots, N_A$ on the basis of the target quantum circuit P
S2
Select $M \geq 2$ execution strategies $S_m, m = 1, \ldots, M$ with associated values λ_m of the noise strength λ
S3
For each execution strategy S_m: Execute target and auxiliary quantum circuits by the quantum processor to obtain target estimator values $E_O(P, \lambda_m)$ and auxiliary estimator values $E_O(A_i, \lambda_m), i = 1, \ldots, N_A$ associated with the value λ_m
S4
Simulate execution of each auxiliary quantum circuit A_i on the classical computer to obtain an associated noise-free auxiliary estimator value $E_O(A_i, 0), i = 1, \ldots, N_A$
S5
Select circuit set which consists of one auxiliary circuit, $A_l = \{A_i^*\}$ among the auxiliary circuits for modelling of noise-dependency of the target estimator value $E_O(P, \lambda_m)$ on the basis of the auxiliary estimator values $E_O(A_i^*, \lambda_m)$ and the noise-free auxiliary estimator value $E_O(A_i^*, 0)$ for the circuit A_i^* of said circuit set
S6
Model noise-dependency of the target estimator value $E_O(P, \lambda_m)$ according to $\left(\frac{E_O(A_i^*, \lambda_m)}{E_O(A_i^*, 0)}\right)^\beta$ with a real parameter β
S7
Derive noise-reduced target estimator value on the basis of said model according to $E_r(\lambda_m; \beta; A_i^*) = E_O(P, \lambda_m)\left(\frac{E_O(A_i^*, 0)}{E_O(A_i^*, \lambda_m)}\right)^\beta$
S8

Fig. 2 – Part 2

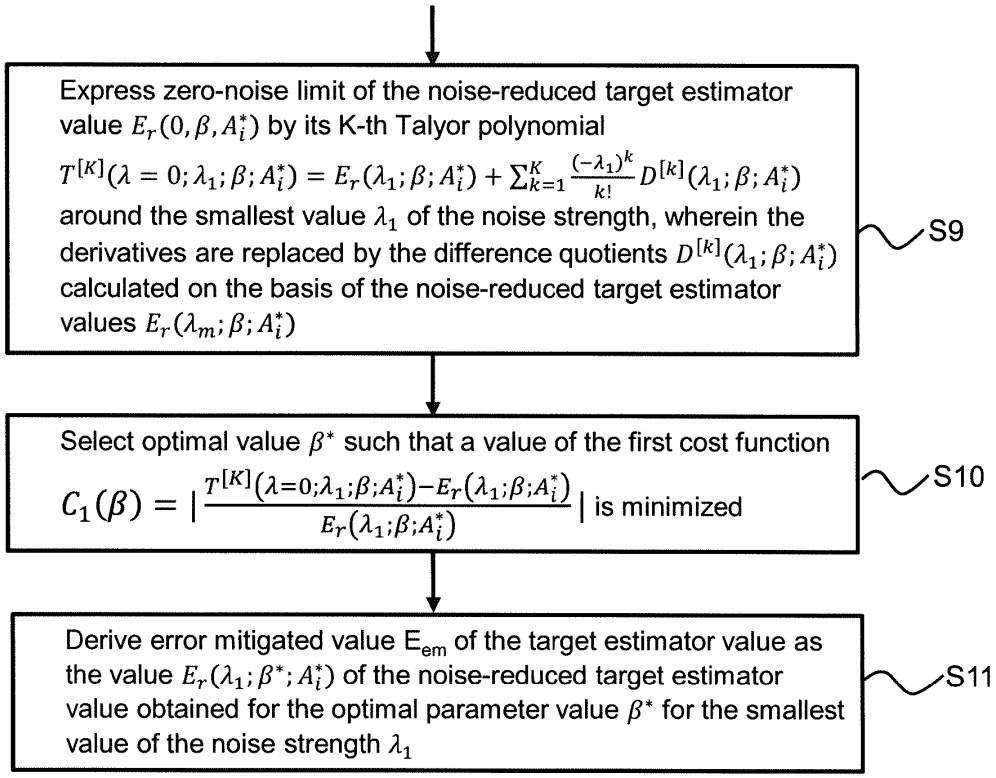

Express zero-noise limit of the noise-reduced target estimator value $E_r(0,\beta,A_i^*)$ by its K-th Talyor polynomial $T^{[K]}(\lambda=0;\lambda_1;\beta;A_i^*) = E_r(\lambda_1;\beta;A_i^*) + \sum_{k=1}^{K} \frac{(-\lambda_1)^k}{k!} D^{[k]}(\lambda_1;\beta;A_i^*)$ around the smallest value λ_1 of the noise strength, wherein the derivatives are replaced by the difference quotients $D^{[k]}(\lambda_1;\beta;A_i^*)$ calculated on the basis of the noise-reduced target estimator values $E_r(\lambda_m;\beta;A_i^*)$
S9
Select optimal value β^* such that a value of the first cost function $C_1(\beta) = \left|\frac{T^{[K]}(\lambda=0;\lambda_1;\beta;A_i^*) - E_r(\lambda_1;\beta;A_i^*)}{E_r(\lambda_1;\beta;A_i^*)}\right|$ is minimized
S10
Derive error mitigated value E_{em} of the target estimator value as the value $E_r(\lambda_1;\beta^*;A_i^*)$ of the noise-reduced target estimator value obtained for the optimal parameter value β^* for the smallest value of the noise strength λ_1
S11

Fig. 3 – Part 1

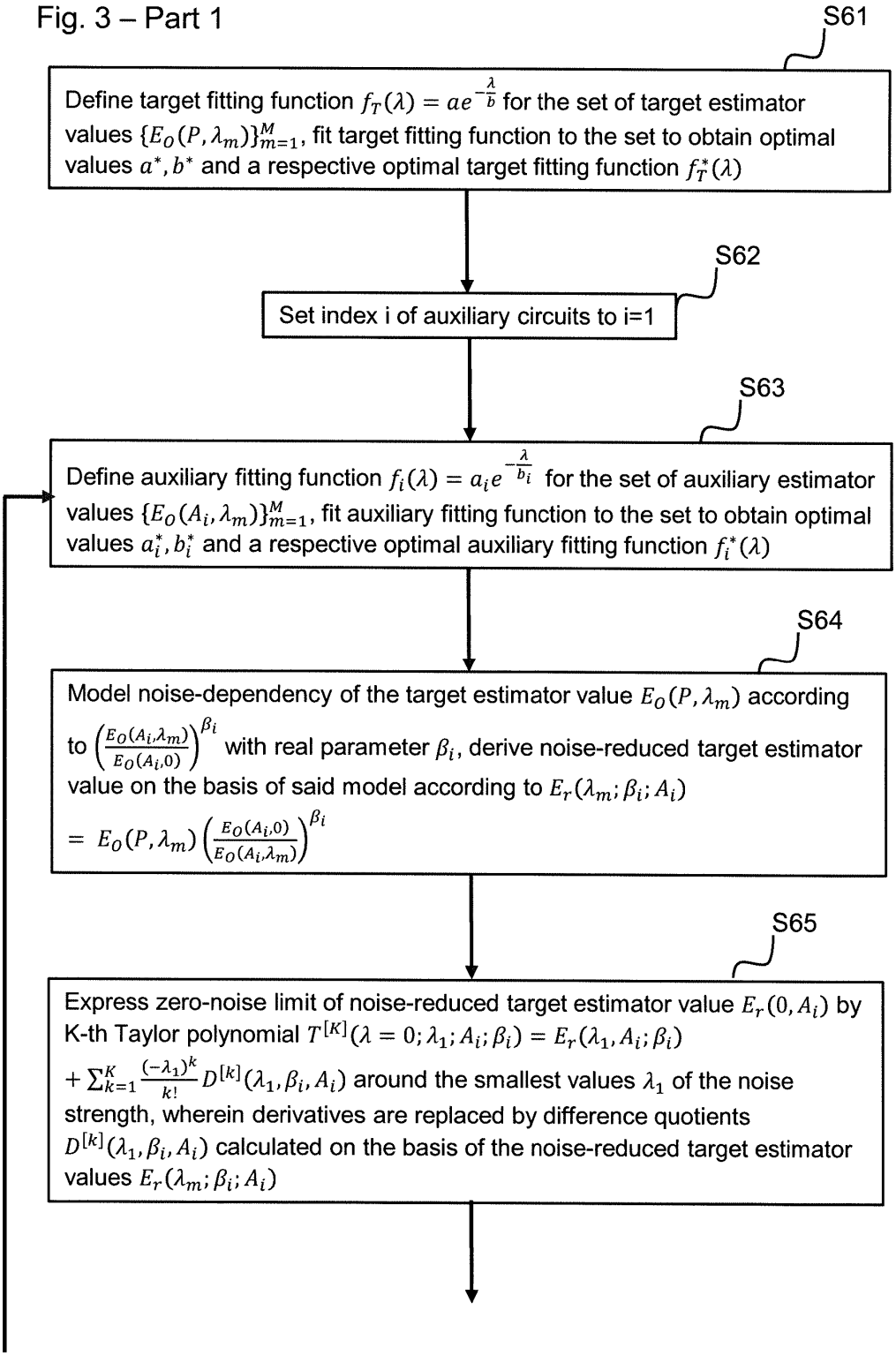

S61
Define target fitting function $f_T(\lambda) = ae^{-\frac{\lambda}{b}}$ for the set of target estimator values $\{E_O(P,\lambda_m)\}_{m=1}^{M}$, fit target fitting function to the set to obtain optimal values a^*, b^* and a respective optimal target fitting function $f_T^*(\lambda)$
S62
Set index i of auxiliary circuits to i=1
S63
Define auxiliary fitting function $f_i(\lambda) = a_i e^{-\frac{\lambda}{b_i}}$ for the set of auxiliary estimator values $\{E_O(A_i,\lambda_m)\}_{m=1}^{M}$, fit auxiliary fitting function to the set to obtain optimal values a_i^*, b_i^* and a respective optimal auxiliary fitting function $f_i^*(\lambda)$
S64
Model noise-dependency of the target estimator value $E_O(P,\lambda_m)$ according to $\left(\frac{E_O(A_i,\lambda_m)}{E_O(A_i,0)}\right)^{\beta_i}$ with real parameter β_i, derive noise-reduced target estimator value on the basis of said model according to $E_r(\lambda_m;\beta_i;A_i) = E_O(P,\lambda_m)\left(\frac{E_O(A_i,0)}{E_O(A_i,\lambda_m)}\right)^{\beta_i}$
S65
Express zero-noise limit of noise-reduced target estimator value $E_r(0,A_i)$ by K-th Taylor polynomial $T^{[K]}(\lambda=0;\lambda_1;A_i;\beta_i) = E_r(\lambda_1,A_i;\beta_i) + \sum_{k=1}^{K}\frac{(-\lambda_1)^k}{k!}D^{[k]}(\lambda_1,\beta_i,A_i)$ around the smallest values λ_1 of the noise strength, wherein derivatives are replaced by difference quotients $D^{[k]}(\lambda_1,\beta_i,A_i)$ calculated on the basis of the noise-reduced target estimator values $E_r(\lambda_m;\beta_i;A_i)$

Fig. 3 – Part 2

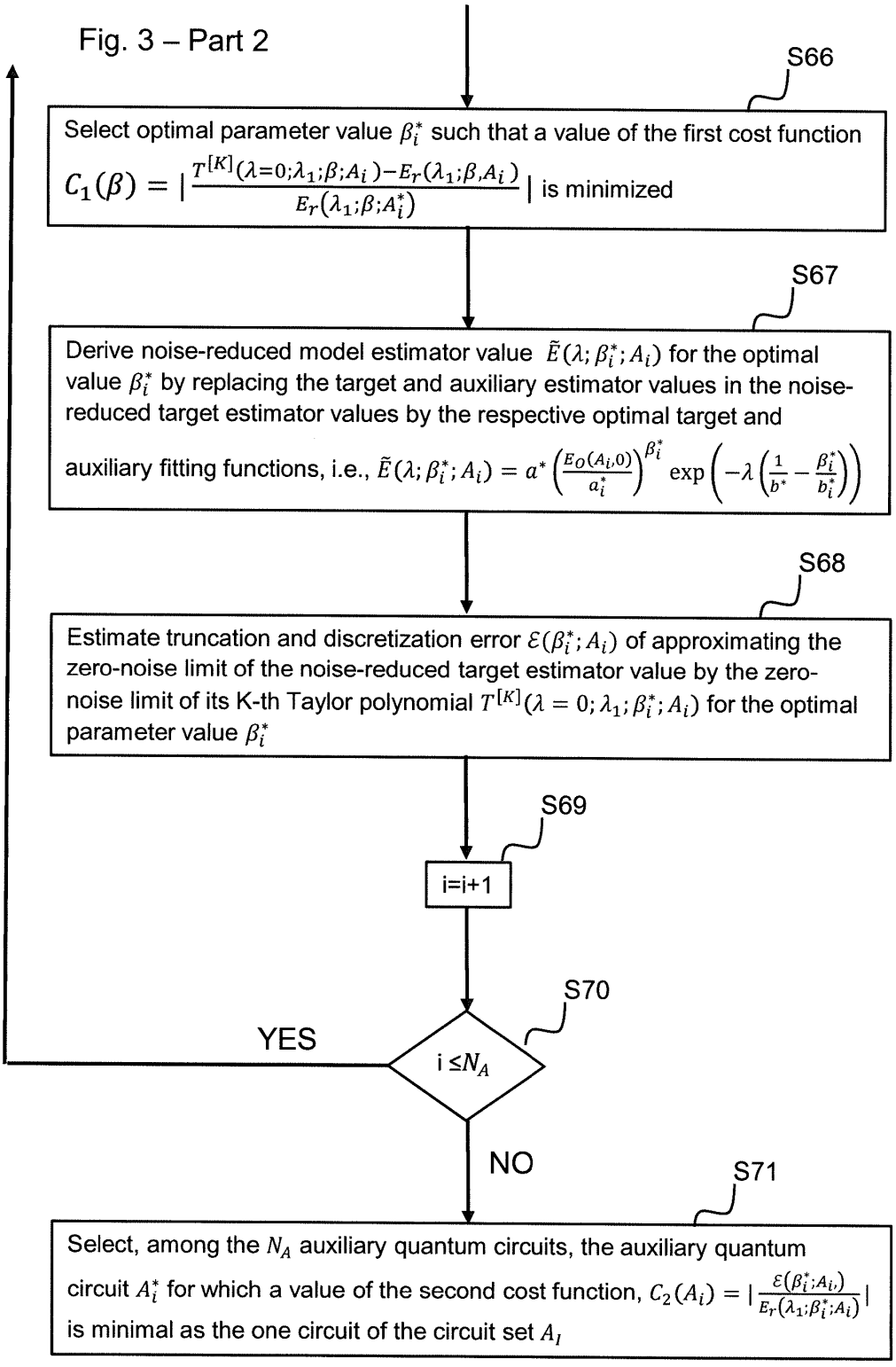

S66
Select optimal parameter value β_i^* such that a value of the first cost function $C_1(\beta) = \left| \frac{T^{[K]}(\lambda=0;\lambda_1;\beta;A_i) - E_r(\lambda_1;\beta,A_i)}{E_r(\lambda_1;\beta;A_i^*)} \right|$ is minimized
S67
Derive noise-reduced model estimator value $\tilde{E}(\lambda;\beta_i^*;A_i)$ for the optimal value β_i^* by replacing the target and auxiliary estimator values in the noise-reduced target estimator values by the respective optimal target and auxiliary fitting functions, i.e., $\tilde{E}(\lambda;\beta_i^*;A_i) = a^* \left(\frac{E_O(A_i,0)}{a_i^*}\right)^{\beta_i^*} \exp\left(-\lambda\left(\frac{1}{b^*} - \frac{\beta_i^*}{b_i^*}\right)\right)$
S68
Estimate truncation and discretization error $\mathcal{E}(\beta_i^*;A_i)$ of approximating the zero-noise limit of the noise-reduced target estimator value by the zero-noise limit of its K-th Taylor polynomial $T^{[K]}(\lambda=0;\lambda_1;\beta_i^*;A_i)$ for the optimal parameter value β_i^*
S69
i=i+1
S70
YES
i ≤N_A
NO
S71
Select, among the N_A auxiliary quantum circuits, the auxiliary quantum circuit A_i^* for which a value of the second cost function, $C_2(A_i) = \left| \frac{\mathcal{E}(\beta_i^*;A_i)}{E_r(\lambda_1;\beta_i^*;A_i)} \right|$ is minimal as the one circuit of the circuit set A_I

Fig. 4a

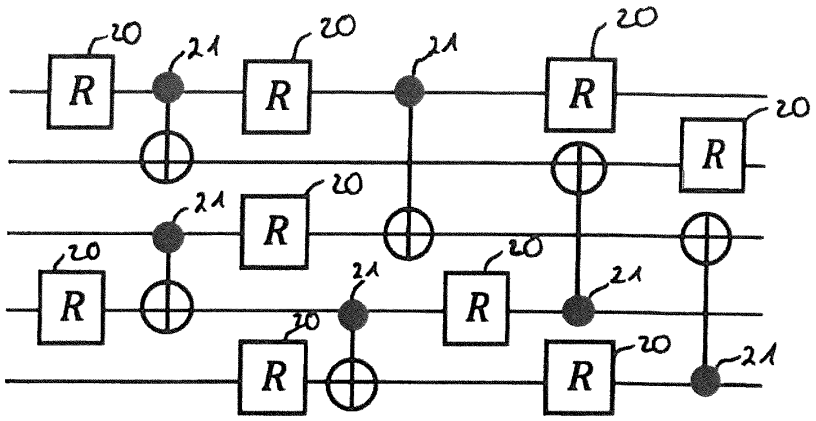


Fig. 4b

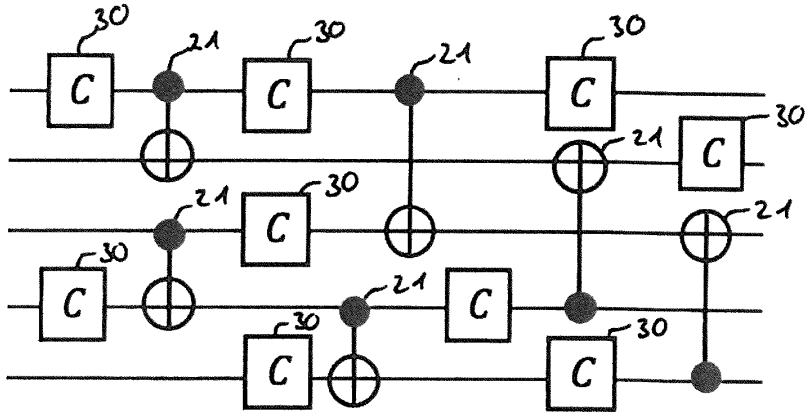


Fig. 4c

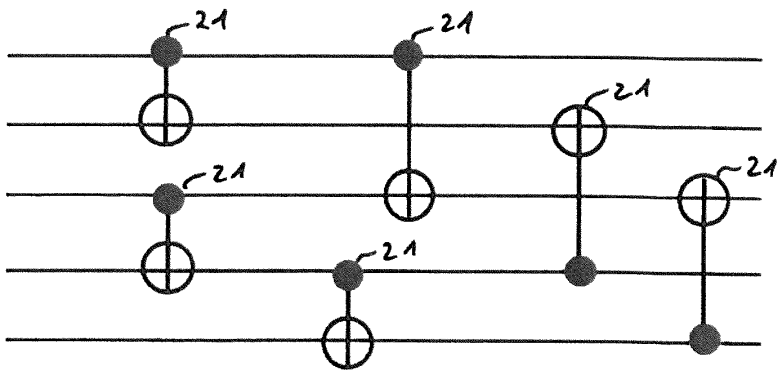


Fig. 5a

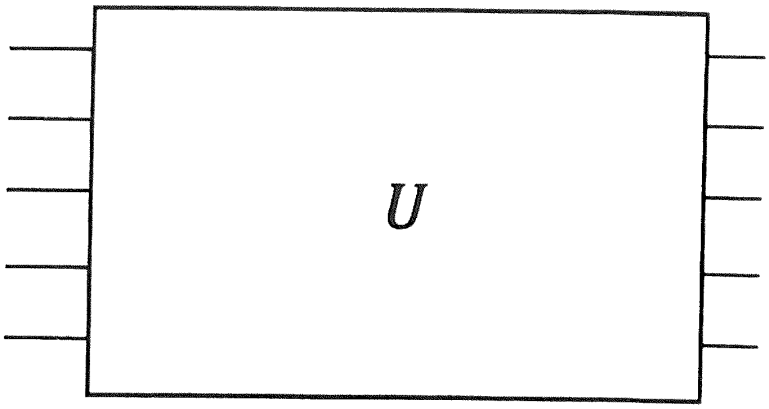


Fig. 5b

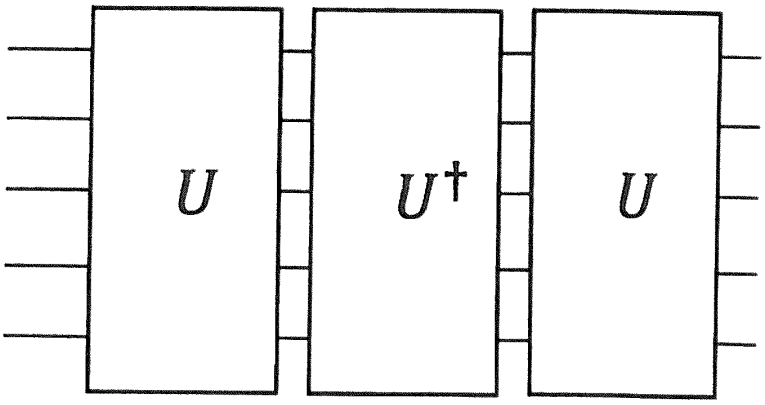


Fig. 5c

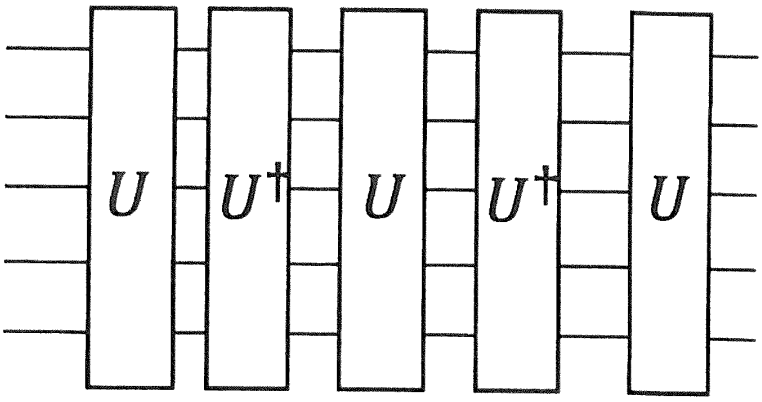


Fig. 6a

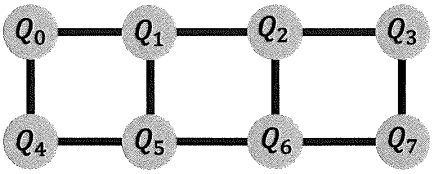


Fig. 6b

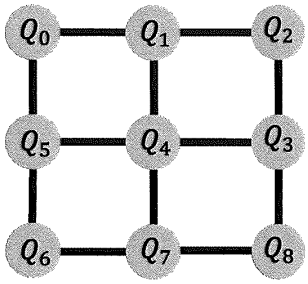


Fig. 6c

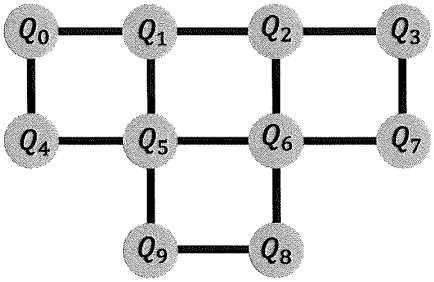


Fig. 6d

Noise profile	value
T_1	$[15-39]\ \mu s$
T_2	$[9\ -38]\ \mu s$
Excited state population	$[2-5]\times 10^{-2}$
Single qubit gate time	40 ns
Two qubit gate time	100 ns
Single qubit gate depolarizing rate	$[1-11]\times 10^{-4}$
Two qubit gate depolarizing rate	2×10^{-2}

Fig. 7a

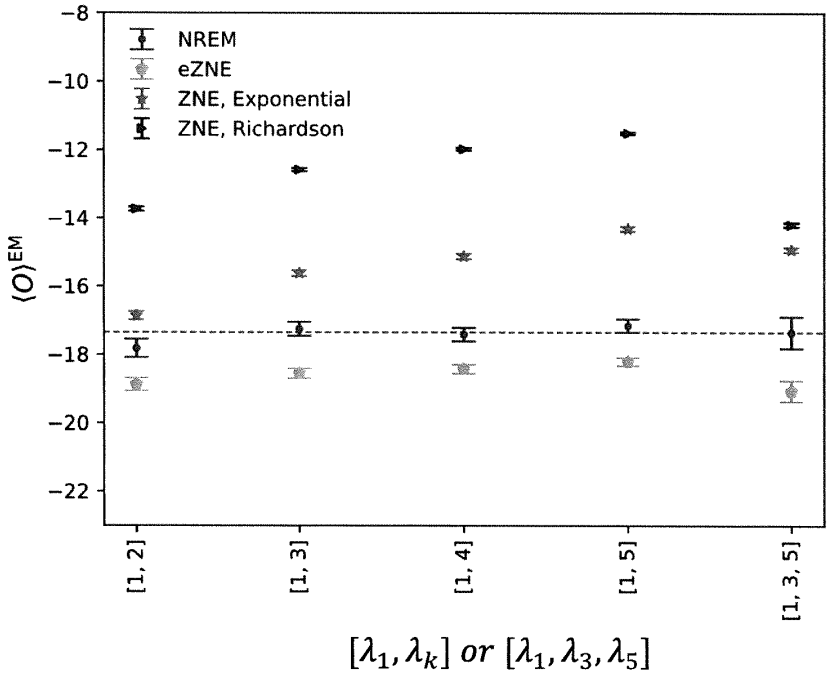

NREM
eZNE
ZNE, Exponential
ZNE, Richardson
⟨O⟩EM
−8
−10
−12
−14
−16
−18
−20
−22
[1, 2]
[1, 3]
[1, 4]
[1, 5]
[1, 3, 5]
$[\lambda_1, \lambda_k]$ or $[\lambda_1, \lambda_3, \lambda_5]$

Fig. 7b

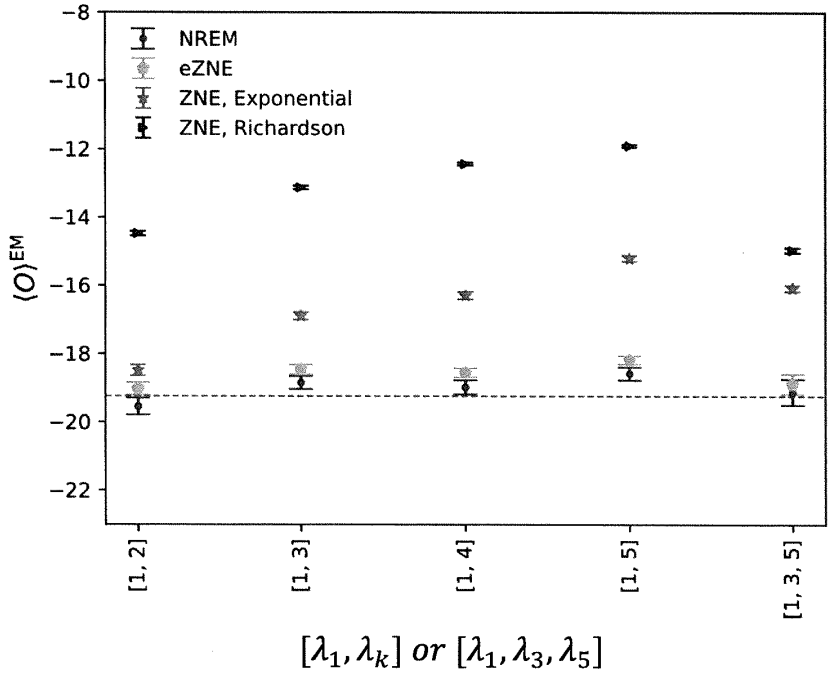
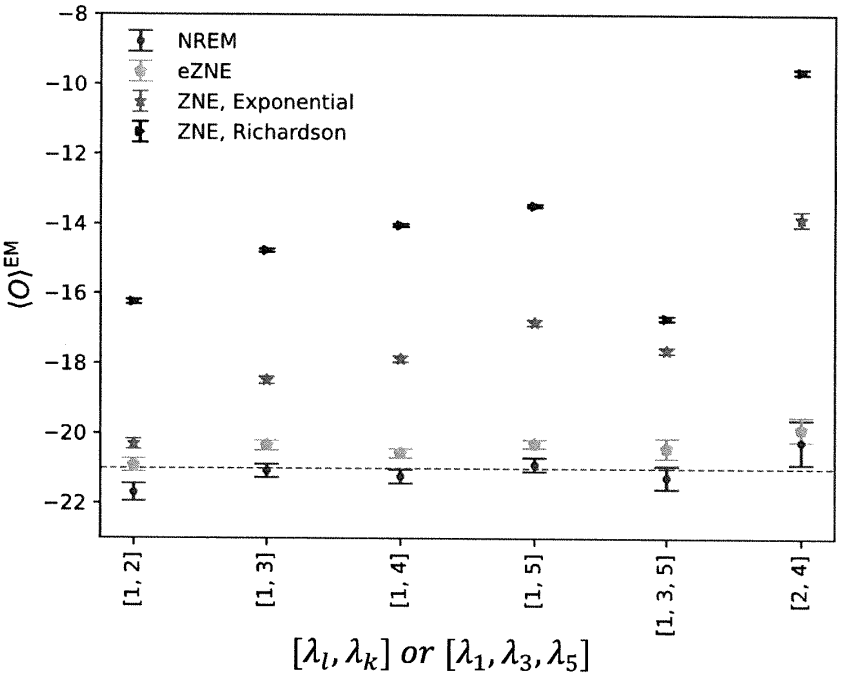

NREM
eZNE
ZNE, Exponential
ZNE, Richardson
⟨O⟩EM
−8
−10
−12
−14
−16
−18
−20
−22
[1, 2]
[1, 3]
[1, 4]
[1, 5]
[1, 3, 5]
$[\lambda_1, \lambda_k]$ or $[\lambda_1, \lambda_3, \lambda_5]$

Fig. 7c


NREM
eZNE
ZNE, Exponential
ZNE, Richardson
−8
−10
−12
−14
−16
−18
−20
−22
$\langle O \rangle^{EM}$
[1, 2]
[1, 3]
[1, 4]
[1, 5]
[1, 3, 5]
[2, 4]
$[\lambda_l, \lambda_k]$ or $[\lambda_1, \lambda_3, \lambda_5]$