WHAT IS CLAIMED IS:

1. A method comprising:

   contacting a feedstock comprising purified used oil with an extraction solvent to perform continuous liquid-liquid solvent extraction, to produce an extract stream comprising the extraction solvent and an extract dissolved in the extraction solvent, wherein the feedstock and the extraction solvent are agitated by a variable speed agitator during the solvent extraction at a selected agitation speed;

   separating the extract from the extraction solvent; and

   subjecting the extract to a continuous flow liquid phase hydrogenation treatment to produce an oil product having a viscosity index of at least 80.

2. The method of claim 1, wherein the liquid phase hydrogenation treatment comprises:

   adding a diluent to the extract to increase solubility of hydrogen in the extract, thus forming a liquid mixture comprising the diluent and the extract;

   adding hydrogen to the liquid mixture to dissolve the hydrogen in the liquid mixture; and

   heating the liquid mixture with dissolved hydrogen in the presence of a hydrogenation catalyst to saturate unsaturates in the liquid mixture, and removing sulfur and aromatics from the liquid mixture, thus forming the oil product.

3. The method of claim 2, wherein the extract comprises phosphorus, and silicon, and the continuous flow liquid phase hydrogenation treatment comprises removing phosphorus and silicon from the liquid mixture before exposing the liquid mixture to the hydrogenation catalyst.

4. The method of ~~any one of~~ claim~~s~~ 1 ~~to 3~~, wherein the extract comprises aromatics, and the continuous flow liquid phase hydrogenation treatment comprises removing

aromatics from the extract~~oil product~~.

5. The method of any one of claims 1 to 4, wherein the extraction solvent comprises n-methyl-2-pyrrolidone.

6. The method of any one of claims 1 to 5, wherein the oil product comprises at least 90 wt% saturates.

7. The method of claim 6, wherein the oil product comprises at least 95 wt% of saturates.

8. The method of any one of claims 1 to 7, wherein the oil product comprises less than 0.03 wt% of sulfur.

9. The method of any one of claims 1 to 8, wherein the oil product has a viscosity index of at least 120.

10. The method of any one of claims 1 to 9, wherein the purified used oil comprises purified used motor oil.

11. The method of any one of claims 1 to 10, wherein the purified used oil comprises purified used industrial oil.

12. The method of any one of claims 1 to 11, comprising purifying used oil to produce the feedstock.

13. The method of claim 12, wherein the purifying comprises subjecting the used oil to distillation to form the feedstock comprising a distillate from the distillation.

14. The method of any one of claims 1 to 13, comprising forming countercurrents of the feedstock and the extraction solvent in a solvent extraction column, wherein the agitation speed and the flow rates of the feedstock and extraction solvent into the solvent extraction column~~vessel~~ are adjusted independently based on a quality or property of the feedstock.

15. A system comprising:

    a purification unit configured to purify used oil, to form a feedstock comprising purified used oil;

    a continuous counter-current liquid-liquid extraction column for extracting an extract from the feedstock using an extraction agent, the extraction column comprising an agitator configured to agitate the feedstock and the extraction solvent flowing through the extraction column at a variable agitation speed; and

    a continuous flow liquid phase hydrogenation unit for hydroprocessing the extract extracted by the extraction column to produce an oil product.

16. The system of claim 15, wherein the continuous flow liquid phase hydrogenation unit comprises:

    a hydrogenation reactor comprising a hydrogenation catalyst;

    a transport line in fluid communication with the solvent extraction column and the hydrogenation reactor, for transporting the extract from the solvent extraction column to the hydrogenation reactor;

    a diluent inlet on the transport line, for introducing a diluent into the extract flowing through the transport line to form a liquid mixture comprising the extract and the diluent; and

    a hydrogen inlet on the transport line located downstream of the diluent inlet, for introducing hydrogen into the liquid mixture.

17. The system of claim 16, wherein the liquid phase hydrogenation unit further comprises a guard bed located on the transport line between the diluent inlet and the hydrogen inlet, the guard bed configured to remove at least phosphorus and silicon from the liquid mixture.

18. The system of any one of claims 15 to 17, wherein the hydrogenation catalyst comprises palladium, gold, or nickel.

19. The system of any one of claims 15 to 18, wherein the purification unit comprises one or more distillation columns.