

SYSTEM, DEVICE AND METHOD FOR IMPROVING A DEFINED PROPERTY OF TRANSFORM-DOMAIN SIGNALS

Related Application

5 This application is a continuation-in-part of US Application No. 09/058,671 (Attorney Docket No. CX098006), filed April 10, 1998, which is hereby incorporated by reference in its entirety.

Field Of Invention

10 This invention relates to a system, device and method for improving a defined property of transform-domain signals, and more particularly to a system and method for reducing the peak-to-average energy ratio (PAR) of time-domain signals.

Background Of Invention

15 The large time-domain peak-to-average energy ratio (PAR) of discrete multitone (DMT) signals is often cited as a major disadvantage of DMT systems. This problem exists in systems using other modulation schemes as well, such as in systems using orthogonal frequency division multiplexing (OFDM) and orthogonal quadrature amplitude modulation (OQAM).

20 A large PAR requires implementation of a high-precision digital-to-analog converter (DAC), or else requires the system to be tolerant of signal distortion (clipping) introduced when input signals exceed the DAC range. For a fixed DAC precision, scaling the input signal so that signal values are always within range may result in excessive quantization noise; on the other hand, insufficient signal
25 scaling may result in excessive clipping noise.

30 A number of approaches to reduce the time-domain peak amplitude of DMT and OFDM symbols have been proposed. These techniques can be divided into three classes. In the first class, multiple symbols are used to represent the same data and side information, transmitted on reserved tones, is used to tell the receiver which of the symbols was transmitted. For example in J.S. Chow, J.A.C. Bingham, and M.S. Flowers, "Mitigating clipping noise in multicarrier systems," *Proc. 1997 Int. Conf. Commun. (ICC'97)*, pp. 715-719, June 1997, if the peak of the DMT symbol is too high, the DMT symbol is scaled and reserved tones are used to relay the scaling factor to the receiver.
35 This technique reduces the signal-to-noise ratio (SNR) of the transmitted symbol and thus results in an increased bit error rate. In Djokovic, "PAR

reduction without noise enhancement", submission T1E1.4/97 270 to ADSL Standard Issue 2, Sept. 1997, the transmitter chooses between the original DMT symbol and its conjugate formed by scrambling the original symbol. In D.J. Mestdagh and P.M. Spruyt, "A method to reduce the probability of clipping in DMT-based transceivers," *IEEE Trans. On Commun.*, vol. 44. 1234-1238, Oct. 1996, a pseudo-random phase sequence is added to the original DMT symbol. The most significant drawback with this class of techniques is that the transmitter must relay side information about the transmitted symbol to the receiver. In addition to incurring a data rate loss or an increase in bandwidth, if the side information is corrupted, the entire DMT symbol will be destroyed.

The second class of PAR reduction techniques is based on determining sequences which have good PAR properties. See for example, S. Shepherd, J. Orriss, and S. Barton, "Asymptotic limits in peak envelope power reduction by redundant coding in orthogonal frequency-division multiplex modulation," *IEEE Trans. on Commun.*, vol. 46, pp. 5-10, Jan. 1998. These methods generally involve removing "bad" time-domain sequences from the set of possible transmitted symbols and thus result in a data rate loss. Furthermore, these methods require mapping the data to the "good" symbols. This map is generally accomplished via a lookup table. The size of the required lookup table becomes impractical in a DMT system with many tones and large constellation sizes.

In the third class of schemes, PAR reduction is achieved via a redundant signal representation, in which a given data block can be represented by any of a number of possible transmitted signals from some equivalence class, with the "most desirable" class representative—in this case, a representative with small time-domain

peak value—chosen for transmission. In such schemes, the receiver is designed to operate "modulo equivalence classes" producing the data block associated with an equivalence class whenever it detects an element of that class. In this way, the receiver requires no knowledge of the precise algorithm used to select a class representative at the transmitter. One way to operate "modulo

equivalence classes" in the DMT case is to have the receiver simply ignore the contents of various frequency bins. See A. Gatherer and M. Polley, "Controlling clipping probability in DMT transmission," *Conf. Record 31st Asilomar Conf. On Sign. Sys. And Comp.*, pp. xx-yy, Nov. 1997; A. Gatherer and M. Polley, "Proposed PAR Reduction Techniques for G.lite," Universal ADSL Technical

Group Contribution TG/98-025, Feb. 4, 1998; J. Tellado and J.M. Cioffi, "PAR reduction in multicarrier transmission systems," contribution 97-367 to T1E1.4 standards committee, Dec. 1997. For any given data block, the transmitter can place values in these unused bins to minimize (as far as possible) the peak value of the transmitted time-domain signal. These techniques incur a significant data rate loss since several bins are not used to transmit data.

Therefore, there exists a need for a PAR reduction technique which utilizes the data carrying or complex frequency bins in DMT modulation schemes to reduce PAR without affecting the data rate and which also utilizes the unused frequency bins to further reduce PAR. There is also a need for such a technique which can be applied generally to other modulation schemes and to improve other properties of the time-domain, or, in general, the transform-domain signal.

Brief Description of the Drawings

FIG. 1A is a schematic block diagram of a DMT transmitter configured according to this invention;

FIG. 1B is a schematic block diagram of an alternative DMT transmitter configuration according to this invention;

FIG. 2 is an expanded signal point constellation in accordance with this invention;

FIG. 3 is an alternative expanded signal point constellation in accordance with this invention;

FIG. 4 is a block diagram of the perturbation selector of FIG. 1A;

FIG. 5 is an illustration of the perturbation vectors of the modulo M_k perturbation device of FIG. 4;

FIG. 6 is a flow diagram illustrating a perturbation vector search used by the modulo M_k perturbation device of FIG. 4;

FIG. 7 is a flow diagram illustrating an alternative perturbation vector search used by the modulo M_k perturbation device of FIG. 4;

FIG. 8 depicts a set of perturbation vectors according to this invention;

FIG. 9 depicts a three dimensional table used in the perturbation vector search illustrated in the flow diagram of FIG. 10;

FIG. 10 is a flow diagram illustrating another alternative perturbation vector search used by the modulo M_k perturbation device of FIG. 4;

FIG. 11 is a schematic block diagram of the receiver depicted in FIGS. 1A and 1B;

FIG. 12 is a schematic block diagram of an alternative perturbation selector according to this invention;

5 FIG. 13 is a schematic block diagram of another alternative perturbation selector according to this invention;

FIG. 14 is a flow diagram of a perturbation vector search performed by the alternative perturbation selectors of FIGS. 15-18;

10 FIG. 15 is a flow diagram illustrating the operation of another alternative perturbation selector according to this invention;

FIG. 16 is a flow diagram illustrating the operation of yet another alternative perturbation selector according to this invention;

FIG. 17 is flow diagram illustrating the operation of another alternative perturbation selector according to this invention; and

15 FIG. 18 is flow diagram illustrating the operation of another alternative perturbation selector according to this invention.

Description of a Preferred Embodiment

20 The present invention is generally directed to a system and method for improving a defined property of a signal after block transformation, hereinafter referred to as a transform-domain signal. In order to provide a more readily understandable description of the invention we describe herein an actual application of the invention to reduce the peak-to-average energy (PAR) of the time-domain signal (more generally referred to herein as the transform-domain
25 signal) in discrete multitone modulation schemes. It will be apparent to those skilled in the art that the invention is generally applicable to other modulation schemes, such as orthogonal frequency division multiplexing (OFDM), orthogonal quadrature amplitude modulation (OQAM), and discrete wavelength multitone (DWMT). Moreover, it will be apparent to those skilled in the art that
30 the invention may be used to improve other defined properties in the transform-domain signal in addition to PAR.

There is shown in FIG. 1A a DMT transmitter 10 that includes a signal mapper 12 which receives input data and outputs a sequence of blocks of frequency-domain symbols X , (X_0 - X_{N-1}). Each symbol in a block corresponds to
35 a different frequency bin and for each frequency bin signal mapper 12 selects a symbol from a constellation of points. Signal mapper 12 chooses the constellation for each frequency bin based on the channel quality for the

frequency bin. The channel quality is typically determined by probing the channel during a training sequence. The size of the constellation, and hence the number of input data bits that can be represented by the symbol chosen from the constellation, is dependent upon the quality of the channel in the frequency range of the bin. A channel having better quality can use a denser constellation with more points and therefore more bits can be transmitted with each symbol. Thus, the number of input data symbols represented by a block of symbols is dependent upon the quality of the channel.

As an example, in a 32-point DMT system the output of the signal mapper 12 is X_0 - X_{31} . However, for asymmetrical digital subscriber line (ADSL) systems using DMT, there are no symbols transmitted in the zero (X_0) and Nyquist (X_{16}) frequency bins. There are symbols transmitted in the lower fifteen complex frequency bins (X_1 - X_{15}) and the upper fifteen complex frequency bins (X_{17} - X_{31}) are selected as the complex conjugate images of the lower fifteen bins so that the resulting frequency-domain signal possesses the Hermitian symmetry needed to make the time-domain signal real-valued.

Inverse Discrete Fourier Transform (IDFT) device 14 receives each block or vector of frequency-domain symbols X , X_0 - X_{N-1} , and transforms them into blocks or vectors of time-domain symbols x , x_0 - x_{N-1} , which are provided to perturbation selector 16. IDFT device 14 and perturbation selector 16 together form perturbation transform device 17. In other applications different types of invertible transform devices may be used and the outputs of the IDFT and the other invertible transform devices may generally be referred to as the transform-domain symbols.

Perturbation selector 16 perturbs the time-domain symbols, as described in detail below, in order to reduce the PAR (or, more generally, improve a defined property) of the transmitted symbols and outputs blocks or vectors of perturbed or modified time-domain symbols y , y_0 - y_{N-1} . Perturbed or modified time-domain symbols y are provided to parallel to serial converter 18 and these symbols are transmitted in serial form over the channel to receiver 19.

It should be noted that although perturbation selector 16 perturbs the time-domain symbols, or more generally the transform-domain symbols, this is not a necessary limitation of this invention. As shown in FIG. 1B, transmitter 10' has a perturbation selector 16' which perturbs the symbols prior to IDFT device 14, i.e. in this example, the frequency-domain symbols would be modified instead of the time-domain symbols. Perturbation selector 16' and IDFT 14 together form perturbation transform device 17'.

As described above, the prior art techniques for reducing PAR do so by either placing values in "unused" frequency bins and/or complex frequency bins which transmit data. In some cases the channel quality may be so poor in some frequency bins that several complex frequency bins in addition to the DC and Nyquist bins are not used to transmit data. However, in many practical cases the number of complex frequency bins that can not transmit data is small or zero. Therefore, these prior art schemes generally use complex frequency bins that could have transmitted data to reduce PAR. Thus, these schemes often exhibit a significant data rate loss.

Perturbation selector 16, according to this invention, uses the complex data carrying frequency bins to reduce PAR without affecting the data rate using a technique referred to herein as modulo M_k perturbation and additionally uses the unused frequency bins (DC, Nyquist and $N/4$) in novel ways to further reduce PAR. Of course, the PAR reduction techniques described in this specific DMT example are directly applicable to improve other defined properties in a transform-domain signal in DMT systems and systems using other modulation schemes, as will be apparent to those skilled in the art.

Modulo M_k Perturbation

With this approach, the signal constellations are expanded relative to the minimum necessary constellation size to support the given data rate in each complex frequency bin and the expanded constellation is partitioned into equivalence classes. The receiver 19, FIGS. 1A and 1B, is designed to detect equivalence classes (not individual constellation points or symbols). This provides the transmitter some flexibility in choosing the transmitted symbols. This extra degree of freedom can be used to optimize some objective function of the resulting signal—in particular, the time-domain peak amplitude. The objective function is optimized without a reduction in the transmitted bit rate, but the transmit power must be increased to accommodate the additional points in the expanded signal constellations. However, by limiting the size of the expanded constellations as described below, significant PAR reduction can be achieved with almost no increase in the average signal power, so the net performance penalty is negligible.

To illustrate the general idea of an expanded constellation with a specific example, consider a DMT system transmitting at a rate of two bits per frequency bin. Such a specific example, expanded 4-QAM (quadrature amplitude modulation) constellation 20, is depicted in FIG. 2. Expanded

constellation 20 includes a base constellation 22 (the constellation in the innermost dashed box) containing points A, B, C and D, from which the symbols are chosen by signal mapper 12, FIG.1. Expanded constellation 20 also includes expansion areas 24, 26, 28 and 30 each containing four points labeled A-D. All of the constellation points having the same label, i.e. A, are elements of the same equivalence class and a receiver receiving any one of the points in the equivalence class decodes the point into the same data. As described in detail below, by perturbing one or more of the frequency bins by selecting a point in the equivalence class outside of the base constellation, the time-domain peak value can be reduced.

The following describes how the base constellations may be expanded to form equivalence classes of constellation points. In the frequency-domain, every block of symbols X contains N symbols, (X_0-X_{N-1}) , each of which is modulated on a separate carrier frequency. The transmitted time-domain signal vector is given by the inverse Fourier Transform as follows:

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi nk/N}, n = 0, \dots, N-1 \quad (1)$$

i.e., $x=F^H X$, where F is the Fourier Transform matrix and H denotes Hermitian transpose. To ensure that x is real, X must be chosen to have complex conjugate symmetry, i.e., $X_k=X_{N-k}^*$, where $X_N=X_0$, implying that X_0 is real, as is $X_{N/2}$ when N is even.

First consider the complex symbols X_k , $k=1,2,\dots,(N/2)-1, (N/2)+1,\dots,N-1$. Generally these complex symbols X_k are chosen from an L_k^2 -QAM constellation, corresponding to transmitting $2\log_2 L_k$ bits on complex frequency k . Each L_k^2 -QAM constellation may alternatively be described as the Cartesian product of two independent L_k -PAM constellations, one for each real and imaginary component of symbol X_k . A value m_k is defined as the largest of the PAM signal magnitudes in the k th channel and d_k is the distance between the PAM symbols. A value M_k is equal to $2m_k+d_k$. The value M_k is used to define the equivalence class points in the expanded constellation.

In order to define the expanded constellation, the base constellation may be expanded to include for each base constellation point all points that are congruent to each of the base constellation points modulo- M_k in either of the two real and imaginary dimensions. All points congruent modulo- M_k are considered to be in the same equivalence class. See, for example, that one of the points "A" in expansion area 24 outside of the base constellation 22 is $+M_k$ from point "A" in base constellation 22. Also, another point "A" in expansion

area 26 is $+jM_k$ from point "A" in base constellation 22. The receiver implements a modulo- M_k operation in the real and imaginary components of the k th complex frequency so that any translation of the received symbol by a multiple of M_k in dimension k is transparent to the receiver. It should be noted that other expanded constellations are possible and will be apparent to those skilled in the art.

In theory, each equivalence class could contain an infinite number of points which are multiples of M_k away from the point in the base constellation; however, as described below, it is practical to choose the points that are close (e.g., $1M_k$) to the points in the base constellation to achieve PAR reduction while minimizing the transmitter power requirements.

Alternatively, the base constellation 22 of FIG. 2 may be expanded to constellation 40, FIG. 3. This expanded constellation includes base constellation 42 and expansion areas 44, 46, 48, 50, 52 and 54. Here, in the k th complex frequency bin, valid nonzero perturbations are drawn from the set $\{M, -M, M/2 + jM, M/2 - jM, -M/2 + jM, -M/2 - jM\}$, where $M = M_k$. Effectively this amounts to permitting shifts by elements of the two-dimensional lattice Λ generated by:

$$\frac{M}{2} \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \quad (2)$$

20

The receiver would then be required to operate modulo Λ . A wide variety of similar possible perturbation sets, involving other sublattices of the integer lattice in one, two, or more dimensions, are possible. For signal constellations based on other lattices, for example, the hexagonal lattice in two dimensions, still other perturbations sets are appropriate, as will be apparent to persons skilled in the art.

Referring now to FIG. 4, the perturbation selector 16 is shown in more detail to include modulo M_k perturbation device 60, which perturbs the block of time-domain symbols x to form a plurality of blocks of modified symbols z_i ($z_{i,0}$ - $z_{i,N-1}$). There is an unused frequency bin perturbation device 62, which further modifies the blocks of symbols z_i by adding energy to the unused (DC and Nyquist) frequency bins, forming blocks of symbols z_i' . And, there is a selection device 64, which selects the block of modified symbols z_i' that has the minimum peak, in the case of PAR reduction, or generally, selects the block of symbols which most improves the defined property.

Modulo M_k perturbation device 60 modifies the block of time-domain symbols x by adding to it all valid perturbation vectors v_i , $(v_{i,0}-v_{i,N-1})$, forming modified blocks or vectors, z_i , of time-domain symbols. As discussed below, the modified blocks or vectors of symbols, z_i , are further modified by unused
 5 frequency bin perturbation device to form vectors, z_i' , from which the vector with the minimum peak is chosen and its symbols are transmitted as output symbols y , y_0-y_{N-1} . Perturbation selector 16 operates in the time-domain to perturb the symbols, however, it can be readily modified to operate in the frequency-domain for use in transmitter 10' depicted in FIG. 1B.

10 The perturbation vectors are most easily described in the frequency-domain. Any perturbation vector $V=(V_0-V_{N-1})$, such that each V_k is an integer multiple of M_k in the k th dimension, may be added to the block of symbols X , (X_0-X_{N-1}) , as a valid perturbation vector as is done in perturbation selector 16', FIG. 1B. To preserve the complex conjugate symmetry of the transmitted
 15 signal vector, it is further required that $V_k=V_{N-k}^*$, for $k=0, 1, \dots, N-1$. As noted above, with the implementation of perturbation selector 16, FIG. 4, the perturbation vector is added in the time-domain; therefore, all valid perturbation vectors V_i must be transformed from the frequency to the time-domain and added to the blocks of time-domain symbols x .

20 Since the requirement for a valid perturbation vector is that each V_k be an integer multiple of M_k in the k th dimension, there are therefore an infinite number of valid perturbation vectors. By perturbing the frequency-domain symbols X , (X_0-X_{N-1}) , in this manner, the resulting perturbed or modified symbols are in the same equivalence class as the unmodified symbols X , (X_0-X_{N-1}) .
 25 When the perturbations are added in the time-domain, if each symbol in the block of perturbed or modified time-domain symbols is transformed to the frequency-domain, they will be in the same equivalence class as the frequency-domain symbols X , (X_0-X_{N-1}) , which produce the unmodified time-domain symbols x , (x_0-x_{N-1}) .

30 For many invertible transforms, and in particular for an inverse discrete Fourier transform with transform size greater than four (4), it is not feasible (or even impossible) to search over all valid vectors, but a search over a limited set of vectors (such as vectors which perturb only one real or imaginary component of one frequency bin by $\pm M_k$) can achieve a significant PAR reduction. This
 35 involves first perturbing only the real or imaginary component of a single frequency. Each complex frequency perturbation that is defined to be non-zero corresponds to adding a sine wave to the time-domain symbol. The peak of the

sine wave is determined by the magnitude of the perturbation which must be an integer multiple of M_k . Since we do not want the perturbation vector to induce additional peaks in the time-domain signal, it has been determined that small perturbations, i.e., $\pm M_k$, are useful.

5 Searching over a larger set of vectors (such as vectors which perturb two frequency components, either both in one bin or one in two bins) does improve performance but increases the search complexity. For ease of description, we will describe the case where valid perturbation vectors are vectors which perturb only one real or imaginary component of one frequency bin by $\pm M_k$.

10 The extension to larger sets of perturbation vectors will be readily apparent to those skilled in the art from the following description.

Signal mapper 12 maps input data to a block of (not necessarily optimum) N symbols or class representatives which are chosen from the base constellations, and then modulo M_k perturbation device 60 operates in the transform-domain by searching for the best valid perturbation, where a perturbation is valid if it transforms a given block of symbols to another block of symbols, which are symbol by symbol in the same equivalence class.

15 With N frequency bins, two of them being unused (zero and Nyquist), there are $N-2$ frequency bins which can be perturbed. Each frequency bin can be perturbed by one of four non-zero perturbations, namely, $+M_k$, $-M_k$, $+jM_k$ and $-jM_k$, where $j=\sqrt{-1}$. In the specific case of an inverse DFT, the time-domain signal is real valued only if the frequency-domain signal has Hermitian symmetry. Therefore, only $(N-2)/2$ "lower" frequency bins are perturbed, with the "upper" image frequency bins chosen as complex conjugate images. Thus, there are a total of $4(N-2)/2=2(N-2)$ valid non-zero perturbation vectors. Since the zero perturbation vector (i.e., no perturbation of the complex frequency components) is also allowed, there are $2(N-2)+1$ valid perturbation vectors.

25 As is illustrated in FIG. 5, the block of time-domain symbols $x, (x_0-x_{N-1})$, is added to each of the valid perturbation vectors $v_i, (v_0$ through $v_{2(N-2)})$, to produce vectors $z_i, (z_0$ through $z_{2(N-2)})$. The frequency-domain perturbation vector V_0 is the all zero perturbation vector. The frequency domain perturbation vector V_1 is shown, for example, to include a $+M_k$ perturbation of real component of the $k=1$ frequency bin and the same perturbation of the complex conjugate frequency bin $k=N-1$. And, frequency-domain perturbation vector V_2 is shown to include, for example, a $+jM_k$ perturbation of imaginary component of the $k=1$ frequency bin and a $-jM_k$ perturbation of imaginary component of the complex conjugate frequency bin $k=N-1$. Of course, the frequency-domain perturbation vectors

must be transformed into a time-domain perturbation vector v_i before being added to the time-domain symbols x .

The Inverse Discrete Fourier transform, v_i , of these $2(N-2)$ vectors (and the all zero vector) V_i may, for example, be stored in memory and used to perturb the time-domain symbols x as illustrated in flow diagram 70, FIG. 6. In step 72 the next block of time-domain symbols x is obtained from IDFT device 14, FIG. 1, and in step 74 x is added to vector v_i to form z_i . In step 76, z_i is provided to unused frequency bin perturbation device 62, FIG. 4. In step 78, i becomes $i+1$ and in step 80 it is determined if i is greater than $2(N-2)+1$, i.e., have all $2(N-2)+1$ vectors (including the all zero vector) been added to the present block of symbols x . If not, then the flow proceeds to step 74 where x is added to the next perturbation vector v_i . If i is greater than $2(N-2)+1$, then in step 82 i is set to zero and flow returns to step 72 where the next block of symbols is obtained. It must be noted that the each block of time-domain symbols z_i is provided to the unused frequency bin perturbation device 62, FIG. 4.

PAR reduction performance is expected to increase as the search space is enlarged. For example, where up to two non-zero components are allowed in the perturbation vector V_i . This requires searching over an additional $(N-2) * (N-3)$ possible vectors. Although searching over this larger space provides a greater time-domain peak reduction, performance improvement may not be significant enough to warrant the tremendous increase in complexity. For larger values of N , (e.g. 256), it may be useful to allow perturbations of $\pm 2M_k$ or larger multiples of M_k as well.

Unused Frequency Bin Perturbation

The idea of reducing the peak by transmitting energy in unused bins is not new, however, novel low complexity methods to use certain unused frequency bins to optimally minimize the peak power are described herein. The methods apply to the following unused frequency bins: the DC bin; the DC and Nyquist bins; the Nyquist bin; and the DC, Nyquist and $N/4$ bins. Typically during a training sequence, the transmitter will determine which of the frequency bins will be the unused bins. Then, using one of the techniques described below corresponding to the unused bins, each of the vectors z_i from modulo M_k perturbation device 60 is modified by unused frequency bin perturbation device 62 and the modified vectors z_i' are provided to modified

symbol selection device 64 where the modified vector z_i' with the smallest peak is chosen and transmitted as symbols y_0 - y_{N-1} .

DC bin

- 5 This technique can be used when the DC frequency bin is unused. Shifting each symbol z_0 - z_{N-1} in each time-domain vector z_i (z_0 - z_{N-1}) by an equal amount is equivalent to changing only the DC component of the corresponding frequency-domain vector Z_i . The peak of each z_i (z_0 - z_{N-1}) is reduced by adding to each component z_0 - z_{N-1} :

$$10 \quad Dc_{perturbation} = - \left(\frac{\max(z_i) + \min(z_i)}{2} \right) \quad (3)$$

where the max and the min are the maximum and minimum values of the symbols z_0 - z_{N-1} in the time-domain vector z_i . The resulting vector is denoted z_i' .

DC and Nyquist Bins

- 15 This technique can be used when the DC and Nyquist bins are unused. Shifting the even symbols of z_i by one value and shifting the odd symbols of z_i by another value is equivalent to changing the values of both the DC and Nyquist frequency components of corresponding frequency-domain vector Z_i . The even symbols are defined as any component z_i of the vector z for which i is even and the odd symbols are defined as any component z_i of the vector z for which i is odd. The time-domain peak of z_i is reduced by adding to the odd symbols of z_i :

$$20 \quad \text{Odd perturbation} = - \frac{(\max_{odd}(z_i) + \min_{odd}(z_i))}{2} \quad (4)$$

- 25 and adding to the even symbols of z_i :

$$\text{Even perturbation} = - \frac{(\max_{even}(z_i) + \min_{even}(z_i))}{2} \quad (5)$$

- where $\max_{odd}$ and $\min_{odd}$ are the maximum and minimum values of the odd symbols of time-domain vector z_i and $\max_{even}$ and $\min_{even}$ are the maximum and minimum values of the even symbols of time-domain vector z_i . The resulting vector is denoted z_i' .

Nyquist bin

In some systems, energy can not be transmitted on the DC frequency because the transformer in the transmitter is DC blocking. A shift in DC level prior to the DAC in the transmitter can still be beneficial to avoid clipping at the DAC, however, this level is then "filtered out" through the transformer. Thus, if we are interested in reducing PAR at points beyond the transformer, the DC frequency will not be effective. Fortunately, however, using the Nyquist frequency can provide the same level of improvement as the DC frequency, albeit with a slightly more complicated algorithm:

First the odd time symbols $z_1, z_3, \dots, z_{N-1}$, of z_i are multiplied by -1 . The even time symbols are not changed. Denote this vector u_i . These resulting symbols, both even and odd time symbols, are shifted by

$$\text{DC shift} = -\left(\frac{\max(u_i) + \min(u_i)}{2}\right), \quad (6)$$

where max and min are the maximum and minimum values of u_i . The odd symbols of these shifted symbols are again multiplied by -1 . The resulting vector is denoted z_i'

DC, Nyquist and N/4 bins

If the transmitter is also not using the N/4 frequency bins in addition to the DC and Nyquist frequency bins, then every fourth symbol z_n ($n \bmod 4 \equiv k$, for each $k=0,1,2,3$) can be modified independently. Symbols z_0, z_4, z_8 etc. of vector z_i are shifted by

$$k=0 \text{ perturbation} = -\frac{(\max_{n \bmod 4=0}(z_i) + \min_{n \bmod 4=0}(z_i))}{2}, \quad (7)$$

where the min and max are the minimum and maximum of symbols z_n of vector z_i such that $n \bmod 4 = 0$. Symbols z_1, z_5, z_9 etc. of vector z_i are shifted by

$$k=1 \text{ perturbation} = -\frac{(\max_{n \bmod 4=1}(z_i) + \min_{n \bmod 4=1}(z_i))}{2}, \quad (8)$$

where the min and max are the minimum and maximum of symbols z_n of vector z_i such that $n \bmod 4 = 1$. Symbols z_2, z_6, z_{10} etc. of vector z_i are shifted by

$$k=2 \text{ perturbation} = -\frac{(\max_{n \bmod 4=2}(z_i) + \min_{n \bmod 4=2}(z_i))}{2}, \quad (9)$$

where the min and max are the minimum and maximum of symbols z_n of vector z_i such that $n \bmod 4 = 2$. Symbols z_3, z_7, z_{11} etc. of vector z_i are shifted by

$$k=3 \text{ perturbation} = -\frac{(\max_{n \bmod 4=3}(z_i) + \min_{n \bmod 4=3}(z_i))}{2}, \quad (10)$$

where the min and max are the minimum and maximum of symbols z_n of vector z_i such that $n \bmod 4 = 3$.

The above approaches are now discussed in the frequency-domain.

- 5 The frequency-domain vectors Z_i are modified to produce Z_i' .

DC bin

First consider using only the DC frequency bin. Z_i' is equal to Z_i modified by Z_0' . Setting $Z_0' = V_0$ in the frequency-domain causes the constant vector $(V_0, V_0, \dots, V_0)$

- 10 $/\sqrt{N}$ to be added to the N-point time-domain signal, which has the effect of raising or lowering equally each time-domain symbol. Thus, to minimize the peak sample power, the DC value is chosen to balance the maximum and minimum values of the time-domain symbol, i.e.,

$$Z_0' = -\sqrt{N}/2 * (\max(z_i) + \min(z_i)) \quad (11)$$

15

where max and min are the maximum and minimum values of the time-domain vector z_i .

DC and Nyquist bin

- 20 This algorithm uses both the DC and Nyquist bins. Z_i' is equal to Z_i modified by Z_0' and $Z_{N/2}'$. Setting the Nyquist frequency bin $Z_{N/2}' = V_{N/2}$ in the frequency-domain causes the alternating vector $(V_{N/2}, -V_{N/2}, \dots, V_{N/2}, -V_{N/2})/\sqrt{N}$ to be added to the N-point time-domain vector, z_i . If, in addition, the DC bin $Z_0' = V_0$, then the overall perturbation in the nth time-domain symbol is $z_n' = z_n$
- 25 $+ (V_0 + (-1)^n V_{N/2})/\sqrt{N}$. Since V_0 and $V_{N/2}$ can be chosen arbitrarily, this freedom allows us to shift the even and odd time-domain samples independently, by choosing

$$Z_0' + Z_{N/2}' = -(\max_{\text{even}}(z_i) + \min_{\text{even}}(z_i)) * \sqrt{N} \quad (12)$$

30 and

$$Z_0' - Z_{N/2}' = -(\max_{\text{odd}}(z_i) + \min_{\text{odd}}(z_i)) * \sqrt{N}, \quad (13)$$

where $\max_{\text{odd}}$ and $\min_{\text{odd}}$ are the maximum and minimum values of the odd symbols of time-domain vector z_i and $\max_{\text{even}}$ and $\min_{\text{even}}$ are the maximum and minimum values of the even symbols of time-domain vector z_i .

5 Nyquist bin

If the DC bin can not be used, but the Nyquist bin is available, then Z_i' should be equal to Z_i modified by $Z_{N/2}$ as described below:

1. Determine the location, l , of the peak ($l = \arg \max_n (|z_n|)$) of the time-domain symbol z_i , (z_0 - z_{N-1}). Denote the peak value by $p=z_l$ and its sign by s ($s=1$ if $p > 0$, $s=-1$ if $p < 0$); and

2. Let

$$Z_{N/2}' = (-1)^{l+1} * (p - s * \max[\max_{n \equiv l \bmod 2} (-s * z_n), \max_{n \equiv (l+1) \bmod 2} (s * z_n)]) / 2 * \sqrt{N}, \quad (14)$$

DC, Nyquist and N/4 bins

- If the DC, Nyquist and N/4 bins can all be used to reduce PAR, the corresponding frequency-domain perturbation of 0, N/2 and N/4 can be determined following the idea used to determine DC and Nyquist bin algorithm described above.

Alternative Search Methods

- The search method described above with regard to FIG. 6 involves searching all $2(N-2)+1$ perturbation vectors v_i in the modulo M_k perturbation device 60 to determine which most reduces PAR or most improves the defined property of the transform-domain signal. Described below, there are two alternative search methods which reduce the complexity of the search for the perturbation vector which most reduces PAR or most improves the defined property of the transform-domain signal. Other search methods will be apparent to those skilled in the art.

- With the first method, illustrated in flow diagram 90, FIG. 7, the next block or vector x of time-domain symbols is obtained, step 92. The peak value for the symbols x_0 - x_{N-1} , $\text{peak}(x)$, in time-domain symbol vector x is determined and the peak time sample location, l , is also determined, step 94. Then, a limited set of all nonzero $2(N-2)$ perturbation vectors v_i is established. That set of vectors, v_i' , includes each of the $N-2$ perturbation vectors corresponding to $+M_k$, $+jM_k$. The

remaining vectors N-2 perturbation vectors are simply negatives of these N-2 perturbation vectors v_i' . The vector $z_0 = x$ corresponding to the zero perturbation is provided to the unused frequency bin perturbation device 62, FIG 4, in step 96 and in step 98 j becomes j+1. The vector v_i' is compared to x and it is determined if the sign of the vector v_i' at location l is equal to the sign of the peak(x), step 100. If it is, then at step 102 that vector is subtracted from x, thereby forming z_j which is provided to unused frequency bin perturbation device 62, FIG. 4. In step 104, j becomes j+1 and in step 106 i becomes i+1. In step 108 it is determined if i is greater than N-2. If it is, this indicates all N-2 nonzero vectors (and the all zero vector) have been considered and flow proceeds to step 110 where i and j are set to zero and then the next block or vector x of time-domain symbols is obtained.

If in step 100 it is determined that the sign of the vector at location l is not equal to the sign of the peak(x), then in step 112 it is determined if the sign of the vector v_i at location l is the opposite sign of the sign of the peak(x). If it is, then that vector is added to x in step 114, thereby forming z_j which is provided to unused frequency bin perturbation device 62, FIG. 4. In step 104 j becomes j+1 and in step 106 i becomes i+1. And, as described above, in step 108 it is determined if i is greater than N-2. If it is, this indicates all N-2 nonzero vectors (and the all zero vector) have been considered and flow proceeds to step 110 where i is reset to zero and then the next block or vector x of time-domain symbols is obtained. If in step 112 it is determined that the sign of vector v_i at location l is not the opposite sign as the sign of the peak (x), then the system moves to step 106 where i becomes i + 1 and flow proceeds as described above.

Therefore, with this search method less than half of the perturbation vectors must be used as compared to the method described above with regard to FIG. 6. Another alternative search method is depicted in FIGS. 8-10. This method successively reduces the search space by eliminating perturbation vectors which do not sufficiently reduce the peak at each time sample.

As shown in FIG. 8, we establish and store in memory a set 120, FIG. 9, of perturbation vectors. The set includes $2(N-2)+1$ time-domain perturbation vectors if we allow one real or imaginary component of one frequency bin to be perturbed by $\pm M_k$ and include the all zero perturbation vector. The vectors each include N components corresponding to the number of time symbols. Each component in the vectors may be represented as $v_{i,j}$, where i indicates the

vector and j is the component in the vector. If all the M_k are the same, each component of the vectors takes on one of $N/2+1$ possible values from $-2M/\sqrt{N}$ to $2M/\sqrt{N}$. These values are ordered from smallest to largest and are referred to as $\text{vals}(l)$, where $l=0,1,2,\dots, N/2$.

5 Then, a three-dimensional table 130, FIG. 9, is generated and stored. The time index, j ($j=0$ through $N-1$), is the first dimension in the table and $l=0$ through $N/2$ is the second dimension in the table. The third dimension is a set of vectors one corresponding to each time index j and each of the $N/2+1$ l 's, where the i th component of the corresponding $2(N-2)+1$ point vector is a one if
 10 v_{ij} in set 120 is greater than $\text{vals}(l)$. Otherwise, the value of the i th component of the $2(N-2)+1$ point vector is a zero. This three dimensional table is used in the perturbation vector search algorithm, described below.

First a threshold, T , is defined which is the maximum allowable peak after the modulo M_k perturbation. Since the unused frequency bin perturbation
 15 device further reduces the peak, this is not necessarily the maximum allowable peak of the system. The threshold, T , is system dependent and is chosen to be some value to which it is desired to reduce the peak. Another value, A , which is the maximum perturbation, is set equal to $2M/\sqrt{N}$. With a $\pm M_k$ perturbation in one real or imaginary component of one frequency bin, this is the largest
 20 perturbation in a time-domain sample generated. Then, a vector goodT is defined. This vector initially has each of its $2(N-2)+1$ components set to one, $[1,1,\dots,1]$. The components correspond to the $2(N-2)+1$ perturbation vectors of the set of vectors 120, FIG. 8, and a one indicates that the corresponding vector is a "good" vector and should be considered for reducing PAR, while a
 25 zero indicates that the vector is not a good candidate and should not be considered.

The flow diagram 140, FIG. 10, describes how the vector goodT is established for each block of time-domain symbols $\mathbf{x}$ (x_0-x_{N-1}) obtained, step
 142. In step 144, the time index, j , is set to zero. In step 146 it is determined if
 30 $|x_j| \leq |T-A|$, where x_j is the j th component of the time-domain symbol block $\mathbf{x}$. If it is, indicating that none of the perturbation vectors will cause $|x_j|$ to exceed the threshold, the system proceeds to step 148. In step 148, the time index, j , is incremented to the next time sample and in step 150 it is determined if $j > N-1$. If it is, this indicates that all of the symbols in block $\mathbf{x}$ have been considered. In
 35 step 152, from the vector goodT the "good" vectors, or some subset thereof, are used to produce the blocks of modified symbols $\mathbf{z}_i$ which are provided to

unused frequency bin perturbation device 62, FIG. 4. That is, block x is added to each perturbation vector that is a good candidate, as indicated by vector goodT, to form blocks of modified symbols z_i . Then, the flow proceeds to step 142 where the next block of symbols x is obtained. If it is determined that j is not greater than $N-1$ in step 150, flow loops back to step 146.

If it is determined in step 146 that $|x_j|$ is not less than or equal to $|T-A|$, then in step 154 it is determined if $|x_j - T| \leq A$. If it is, then in step 156 the vector goodT becomes goodT NAND table[j][pertindex1], where table[j][pertindex1] is an index to one of the $2(N-2)+1$ point vectors in three dimensional table 130, FIG. 9. The index j indicates time, e.g., 0, 1... $N-1$, and pertindex1, calculated as described below, indicates which of the set of $N/2+1$ vectors corresponding to time index j the vector is to be chosen. A value for pertindex1 is calculated by first determining a value for maxp as follows:

$$\text{maxp} = T - x_j \quad (15)$$

where maxp is the maximum allowable perturbation at time j . Then, pertindex1 is calculated as follows:

$$\text{pertindex1} = \text{ceil} \left\{ \frac{N}{2\pi} \cos^{-1} \left(\frac{\sqrt{N}}{2M} (\text{max } p) \right) \right\} \quad (16)$$

where the inverse cosine may be determined by using a stored lookup table and "ceil" corresponds to rounding up to the nearest integer. Pertindex1 could also alternatively be determined from maxp directly using a stored lookup table. After goodT has been set in step 156, the time index j is incremented in step 148 and flow proceeds as described above.

If in step 154 it is determined that $|x_j - T|$ is not less than or equal to A , then in step 158 it is determined if $|x_j + T|$ is less than or equal to A . If it is, then in step 160 the vector goodT becomes goodT AND table[j][pertindex2], where table[j][pertindex2] is an index to one of the $2(N-2)+1$ point vectors in three dimensional table 130, FIG. 9. The index j indicates time, e.g., 0, 1... $N-1$, and pertindex2, calculated as described below, indicates which of the set of $N/2+1$ vectors corresponding to time index j the vector is to be chosen. A value for pertindex2 is calculated by first determining a value for minp as follows:

$$\text{minp} = -T - x(j) \quad (17)$$

where minp is the minimum allowable perturbation at time j. Then, pertindex2 is calculated as follows:

$$\text{pertindex2} = \text{ceil} \left\{ \frac{N}{2\pi} \cos^{-1} \left(\frac{\sqrt{N}}{2M} (\min p) \right) \right\} \quad (18)$$

where the inverse cosine may be determined by using a stored lookup table and "ceil" corresponds to rounding up to the nearest integer. Pertindex2 could also alternatively be determined from minp directly by using a stored lookup table. After goodT has been set in step 160, the time index j is incremented in step 148 and flow proceeds as described above.

If in step 158 it is determined that $|x_j + T|$ is not less than or equal to A, then in step 162 each of the $2(N-2)+1$ components of goodT is set to zero, $[0, 0, \dots, 0]$, indicating that none of the perturbation vectors can sufficiently reduce the peak. Then, the system loops back to step 142 where the next block of time-domain symbols x is obtained. Or, the threshold, T, may be raised and the system may restart at step 144.

After all x_j values in each block of time-domain symbols x have been considered, the vector goodT is evaluated. A one in any component of that vector indicates that the corresponding vector in the set of vectors 120, FIG. 8, is a good candidate for a perturbation vector. Then, there are several options. The first involves choosing any one of the good vectors, adding it to the present block of symbols x to form a block of modified symbols and providing the block of modified symbols to the unused frequency bin perturbation device. Another option involves adding each of the good vectors to the present block of symbols x to form blocks of modified symbols and providing the blocks of modified symbols to the unused frequency bin perturbation device. Yet another option involves adding some specified number of the good vectors to the present block of symbols x to form blocks of modified symbols and providing the blocks of modified symbols to the unused frequency bin perturbation device.

The first option reduces complexity but also reduces performance to a certain degree. With the second option and an appropriately chosen threshold, T, performance loss should be minimal, but complexity reduction is not as significant as option 1. With option 3, the balance between performance and complexity reduction falls between options 1 and 2.

It should be noted that table 130, FIG. 9, must be modified if all the M_k 's are not equal. Two options for modifying table 130 are described below.

In the first option, define $M = \max_k M_k$, i.e. the maximum of all the M_k 's. Then assume each base constellation is expanded modulo-M. Table 130 will remain the same except that it will be based on the value of M defined above. In other words, A and vals(l) where $l = 0, 1, 2, \dots, N/2$ are determined using this value of M.

In the second option, table 130 is generated assuming that the kth base constellation is expanded modulo- M_k and that different values of M_k are possible for different frequency bins k. This implies that the time sample values of the valid perturbation vectors can take on more than $N/2+1$ values. Let maxval and minval denote the maximum and minimum values that the components of the valid perturbation vectors take on. Equivalently, maxval will be equal to

$$\text{maxval} = 2 \times \max_k \frac{M_k}{\sqrt{N}} \quad (17)$$

and minval = -maxval. Then define vals(l) as the values minval to maxval spaced by a desired granularity, where l goes from 0 to L, where $L = (\text{maxval} + \text{minval}) / \text{granularity} + 1$. Note the first dimension of the table now has $L+1$ entries instead of $N/2+1$. Therefore, the table size may be larger. Pertindex1 is now calculated as $\text{Pertindex1} = \text{floor}((\text{maxp} - \text{minval}) / \text{granularity})$ where floor corresponds to rounding down to the nearest integer and Pertindex2 is calculated as $\text{Pertindex2} = \text{ceil}((\text{minp} - \text{minval}) / \text{granularity})$ where ceil corresponds to rounding up to the nearest integer. The rest of the table generation and the use of the table is the same.

Receiver

In FIG. 11 there is shown a schematic block diagram of receiver 19 depicted in FIGS. 1A and 1B. The modified symbols y, after going through the channel, are received as symbols w at receiver 19. Receiver 19 includes a serial to parallel converter 170 which receives the time-domain symbols w in serial form and converts them to blocks of received time-domain symbols w, w_0 - w_{N-1} . The blocks of received time-domain symbols w, w_0 - w_{N-1} , are provided to Discrete Fourier Transform device 172 which converts the time domain symbols

into blocks of received frequency-domain symbols W, W_0-W_{N-1} . The blocks of received frequency-domain symbols W, W_0-W_{N-1} are provided to Frequency domain equalizer device 174 which takes into account the effect of the channel on the transmitted modified frequency domain symbols Y, Y_0-Y_{N-1} , and scales the received symbols W, W_0-W_{N-1} , to produce symbols $Y', Y'_0-Y'_{N-1}$ which are estimates of the transmitted symbols Y, Y_0-Y_{N-1} . The estimates of the transmitted symbols are provided to inverse signal mapper 176 which converts the estimates of the transmitted modified frequency domain symbols $Y', Y'_0-Y'_{N-1}$, into output bits 178 corresponding to the input bits provided to transmitters 10 and 10' of FIGS. 1A and 1B, respectively.

Inverse signal mapper 176 is designed to detect equivalence classes (not individual constellation points or symbols) or it is said to operate modulo equivalence classes. The symbols $Y', Y'_0-Y'_{N-1}$, correspond to estimates of the frequency-domain points selected from e.g., expanded constellation 20, FIG. 2, however, they may not be equal to those constellation points due to noise on the channel. Thus, inverse signal mapper 176 must account for this channel noise when inverse mapping the symbols to output bits 178. For example, the inverse signal mapper may first map each of the symbols $Y'_0-Y'_{N-1}$ to the nearest point in the expanded constellation and then map this expanded constellation point to the equivalent point in the base constellation. Other possible implementations of the inverse signal mapper will be clear to those skilled in the art.

Alternative Perturbation Selectors

In alternative perturbation selector 16a, Fig 12, the modulo- M_k perturbation is applied to the input symbols b times. The number of times that the modulo- M_k perturbation should be applied will be dependent on block size N and desired system complexity. Each iteration (i.e. each time the modulo- M_k perturbation is applied), the peak of the time domain symbol is reduced. The reduction will decrease to zero after several iterations.

Perturbation selector 16a is provided time-domain symbol x (x_0-x_{N-1}). In the j th iteration, modulo- M_k perturbation device 180 receives time domain symbol x_{j-1} , where $x_0 = x$. Modulo- M_k perturbation device modifies the block of time-domain symbols x_j by adding to it all valid perturbation vectors v_i , forming modified blocks $z_{j,i}$. Modified symbols selection device 182 chooses the modified block $z_{j,i}$ that has minimum peak value and outputs this to the next

stage modulo- M_k perturbation device 180 where the valid perturbation vectors v_i are added to it.

Perturbation selector 16b, Fig 13, is very similar to perturbation Selector 16a. The only difference is that before being passed to the first stage modulo- M_k perturbation device 180, the input symbol x is provided to unused frequency bin perturbation device 190 which chooses the unused frequency bin perturbations to minimize the peak value of x . Oftentimes, applying the unused frequency bin perturbation device prior to the modulo- M_k perturbations will allow the peak value to be reduced more quickly in the first few stages.

However, it may also lead to a slightly higher peak value after several stages. In other words, after a sufficient number of stages perturbation Selector 16a may produce a symbol with lower peak value than perturbation Selector 16b.

A number of alternative perturbation selectors are shown in FIGS. 15-18. These alternative perturbation selectors apply several iterations of perturbations derived from a reduced complexity perturbation vector search which is illustrated in flow diagram 200, FIG 14. The operation of flow diagram 200, steps 202-220, is essentially the same as that of flow diagram 140, FIG. 10, and is therefore not described again. At a given iteration, if the reduced complexity perturbation search achieves its threshold T , the threshold T is reduced. If the threshold is not achieved, the threshold is raised. If the number of iterations allocated, numloops, have not been completed, the reduced complexity perturbation search is applied to the modified symbols and another perturbation is determined. The thresholds T and the number of iterations, numloops, are system dependent. Note that the amount that T is raised or lowered does not have to be the same at each iteration.

The operation of one alternative perturbation selector is illustrated in flow diagram 230, FIG. 15. In step 232, the next blocks of time-domain symbols x and an initial threshold T are obtained and in step 234 an index k is initialized to zero. In step 236, using the symbol x and threshold T the reduced complexity perturbation search (flow diagram 200, FIG. 14) which determines a set of "good" perturbation vectors, described by goodT, is performed. In step 238 if goodT is non-zero, then in step 240 any one of the "good" vectors, denoted v , is chosen and used to modify x , i.e. $x = x + v$. Typically, the first good vector is chosen first, i.e. the perturbation vector corresponding to the first "one" in the vector goodT. In step 242 the threshold T is lowered and in step 244 the index k is incremented. It is then determined in step 246 if $k < \text{numloops}$. If it is then the reduced complexity perturbation search is performed again with the

modified x . If it is not, the modified symbol $y=x$ is output in step 248 to parallel to serial converter 18, FIG. 1A. Alternatively, if in step 238 goodT is not non-zero, the threshold T is raised in step 250 and index k is incremented in step 244. It is then determined in step 246 if $k < \text{numloops}$ and flow continues as described above.

The operation of another alternative perturbation selector is illustrated in flow diagram 260, steps 262-280. The operation of flow diagram 260 is essentially the same as that of flow diagram 230, FIG. 15 and only differs in the following way. If in step 268 it is determined that goodT is not non-zero, the threshold T is raised in step 280, but the index k is not incremented. Instead, flow proceeds to step 266 where the reduced complexity perturbation search is performed without incrementing the index k . In this implementation the time-domain symbol's peak value is always reduced numloops times. This may lead to a lower peak value.

The operation of two additional alternative perturbation selectors are illustrated in flow diagram 290, FIG. 17, steps 292-312 and flow diagram 320, FIG. 18, steps 322-342. The operation of these perturbation selectors is similar to the operation of the selectors of FIGS. 15 and 16, respectively. The only difference is that the input time-domain symbols x , prior to being used in the reduced complexity perturbation search (step 298, FIG. 17 and step 328, FIG. 18) for the first time, are modified by performing an unused frequency bin perturbation (step 294, FIG. 17 and step 324, FIG. 18) which uses the unused frequency bin perturbations to reduce the peak value of x . Then a reduced complexity perturbation search is performed on the modified symbol.

It should be noted that this invention may be embodied in software and/or firmware, which may be stored on a computer useable medium, such as a computer disk or memory chip. The invention may also take the form of a computer data signal embodied in a carrier wave, such as when the invention is embodied in software/firmware, which is electrically transmitted, for example, over the Internet.

The present invention may be embodied in other specific forms without departing from the spirit or essential characteristics. The described embodiments are to be considered in all respects only as illustrative and not restrictive. The scope of the invention is, therefore, indicated by the appended claims rather than by the foregoing description. All changes, which come within the meaning and range within the equivalency of the claims, are to be embraced within their scope.

What is claimed is:



Claims

- 5 1. In a transmitter for transmitting data in blocks over a channel to a receiver, a device for improving a defined property of transform-domain symbols, the device, comprising:
- 10 a signal mapper which maps the input data into blocks of symbols in a first domain; wherein each of the symbols is chosen from a base constellation contained in an expanded constellation having expansion symbols, and wherein at least some of the symbols in the base constellation have one or more corresponding expansion symbols; each base symbol and its corresponding expansion symbols defining an equivalence class of symbols which represent the same data to the receiver; and
- 15 a perturbation transform device, responsive to the blocks of symbols, which produces for each block of symbols in the first domain a block of perturbed transform-domain symbols in order to improve a defined property of the transform-domain symbols; wherein each symbol in the block of perturbed transform-domain symbols, in the first domain, is in the same equivalence
- 20 class as each corresponding symbol in the block of symbols.