

Method for estimating the position and speed errors of an inertial platform with the aid of an SAR imaging radar

The field of the invention relates to:

- the precise guidance of a missile onto a fixed target on the basis of an SAR "Synthetic Aperture Radar" seeker.
- the resetting of the inertial platform of an aircraft carrying an SAR "Synthetic Aperture Radar" radar.

The present patent application describes the invention within the framework of guidance of a missile onto a fixed target.

Generally, synthetic antenna radars (SAR: Synthetic Aperture Radar) are radars which utilize by processing, the displacement of the carrier to improve the resolution linked to the width of the antenna beam. The spatial resolution, which characterizes the fineness of the SAR image acquired, is therefore obtained:

- in the direction of the antenna axis through the distance resolution of the radar;
- in the direction perpendicular to the antenna axis through the SAR processing.

This remark demonstrates that the carrier must have a motion in the direction perpendicular to the antenna axis, thereby preventing SAR imaging being performed at the front of the trajectory, that is to say in the direction of the speed vector.

Processing of the SAR type is notably described in the work by J. Darricau: *Physique et Théorie du Radar* [Physics and Theory of Radar] – Vol. 3, 3rd edition – Chapter 21, page 483 – Published by Sodipec, Paris 1994.

Thus, the acquisition of an SAR image on a target and its environment makes it possible to reconstitute the scene with a fine resolution (of the metric or even submetric class). The utilization of this image on the basis of the information formulated during the preparation of a mission makes it possible to precisely locate the target in the image, notably when the mission consists, for a missile, in locating a target with precision and in steering towards it with precision.

The geographical positioning of the SAR image is obtained on the basis of the information delivered by the inertial platform of the missile. Now, this information may be very noisy, thus giving rise to considerable lack of precision with regard to the geographical positioning of the SAR image. It is therefore apparent that because

of this phenomenon although the target is precisely located in the SAR image, it is not possible to deduce a precise geographical location therefrom.

It is recalled that the activation of the SAR mode imposes an angular squint between the speed vector and the missile-target direction and therefore a non-direct trajectory, as illustrated in Figure 1.

A first step corresponds to a trajectory portion with repeated mappings on the target. As the target is approached, the angular squint decreases until it becomes too small to be compatible with the SAR mode corresponding to a second step. Thus from a certain distance onwards, the seeker stops operating and the missile homes in on the target in purely inertial mode on the basis of the information reset by the seeker.

During this homing, the errors in the speed vector estimated by the inertial platform generate a drift in the missile trajectory, and this may lead to a significant shift in the impact point with respect to the target, as illustrated in Figure 2.

To avoid this phenomenon, the invention proposes a method for estimating the errors of the inertial platform at the position and speed level which for small durations may be regarded as a constant bias.

More precisely, the subject of the invention is a method for determining the position error and the speed bias of the onboard inertial platform of a carrier pursuing a trajectory towards a target, characterized in that it comprises the following steps:

- acquiring aboard the carrier a series of SAR-type images of the target;
- utilizing the said images to perform on the basis of each of the images a measurement of the distance and a measurement of the Doppler of the target;
- estimating the precise position of the carrier with respect to the target and the speed bias, on the basis of the measurements of distance and of the Doppler of the target with an estimator filter.

Advantageously, the filter is a discrete Extended Kalman filter.

Advantageously, the method of the invention comprises a prior step of trajectory optimization to optimize the performance of the estimator filter.

Advantageously, the method comprises the use of a tool comprising a constrained cost function.

Advantageously, the constrained cost function is the Cramer Rao bound associated with the estimation of the position of the carrier with respect to the target and of the speed bias.

Advantageously, the method comprises the following steps:

- segmenting the trajectory into N pieces;
- determining the 2N components of the transverse acceleration of the carrier along the trajectory by optimizing the Cramer Rao bound for the last image acquired.

Additionally, the constraints taken into account by the cost function are such as the maximum load factor of the carrier and/or minimum angular squint between the speed vector and the carrier-target direction and/or the minimum impact angle in the vertical plane.

Thus according to the method of the invention, the information estimated by the carrier is the position of the target and the speed errors of the inertial platform. This information is evaluated on the basis of the distance and of the Doppler of the target, estimated on completion of the processing of each SAR image.

The filter provides an estimation of the real position of the carrier and an estimation of the speed bias on the basis of the distance and Doppler measurements arising from the utilization of the SAR images.

These measurements are referenced in a reference frame whose origin is the target, thus allowing precise determination of the position of the carrier with respect to the target.

The invention will be better understood and other advantages will become apparent on reading the description which follows and by virtue of the appended figures among which:

- Figure 1 illustrates the non-direct trajectory followed by a missile performing SAR images on the target, with an angular squint between the speed vector and the carrier-target direction;
- Figure 2 illustrates the error in the speed engendered by the inertial platform at the level of the real impact point with respect to a target during a mission without error correction;
- Figure 3 illustrates the shift between the SAR image location estimated on the basis of the inertial information and the exact location of the SAR image;
- Figure 4 illustrates the position of the target at the point of intersection of the iso distance and iso Doppler circles associated with the whole set of measurements obtained along a non-optimized trajectory
- Figure 5 illustrates the optimization of the trajectory in relation to the

location of the target.

The invention will be described within the framework of a mission for which a seeker missile tries to locate a target precisely. The invention could equally well be applied to any aircraft carrying a radar which effects repeated SAR images on an object on the ground.

Generally by defining the following parameters:

- X_e : current position of the missile
- X_d : initial position of the missile (at $t = 0$)
- X_m : displacement of the missile since $t=0$ estimated by the inertial platform
- t : current time
- b : inertial platform speed errors modelled by biases

the following evolution equation holds:

$$X_e = X_m - t \cdot b + X_d$$

The positions are expressed in a reference frame whose origin is the target. The current position of the target therefore characterizes the missile-target relative position which is the information utilized by the guidance of the missile.

And it may be considered as a first approximation that:

$$\begin{cases} \dot{X}_d = 0 \\ \dot{b} = 0 \end{cases}$$

The symbol $\dot{}$ signifies differentiation with respect to time.

Additionally, the measurement equations are the following:

- The displacement of the missile estimated by the inertial platform X_m which is falsified by a constant bias in the speed
- The Missile-Target distance

$$d = \|X_m - t \cdot b + X_d\| + \omega_d$$

The symbol $\|a\|$ corresponds to the norm of the vector a

- The Doppler frequency

$$f = \frac{2}{\lambda} \cdot \frac{\langle X_m - t \cdot b + X_d; V_m - b \rangle}{\|X_m - t \cdot b + X_d\|} + \omega_f$$

with V_m speed estimated by the inertial platform.

The symbol $\langle a b \rangle$ corresponds to the scalar product of the vectors a and b

where ω_d and ω_f are the measurement noise, with variance σ_d^2 and σ_f^2 .

5 The precision in the position of the SAR image with respect to the missile depends on the precision in the speed of the inertial platform, as illustrated in Figure 3 which shows diagrammatically the shift between the exact location of the SAR image and the estimated location of the SAR image, as a function of the shift between the true speed and the estimated speed.

10 To circumvent the errors of the inertial platform in the speed, the expedient adopted is as follows:

 The processing of each SAR image provides the Distance and the Doppler of the target, and the target is situated on a circle corresponding to the intersection of an iso Distance sphere and of an Iso Doppler cone, as illustrated in Figure 4. The
15 axes correspond to the geographical reference frame centred on the target, the unit being proportional to metres. Utilizing the repetition of the images provides an estimate of the position of the target and the speed bias.

 The paragraph hereinafter details the equations of the filter which provides a position estimation X_d and a value of the speed bias b on the basis of the distance
20 and Doppler measurements arising from the utilization of the SAR images.

 The filter can advantageously be a discrete Extended Kalman filter.

 In accordance with the previous paragraph the state vector X comprises six components which are the coordinates of the point X_d and the speed biases b along three orthogonal axes.

25 The following notation is used in this paragraph:

- $X(k|k-1)$ corresponds to the prediction of the state at the instant k estimated at the instant $k-1$
- $X(k|k)$ is the estimation of the state at the instant k performed at the instant k
- 30 - $Y(k)$ is the measurement (distance, Doppler) performed at the instant k .

 This index notation also applies to the covariance matrix of the state R_x .

The temporal discretization step size corresponds to the renewal rate of the SAR images.

Initial conditions

- 5 The state $X(0)$ and the covariance matrix $R_x(0 | 0)$ are initialized on the basis of the knowledge of the theoretical trajectory and the expected performance of the inertial platform.

Prediction

- 10 - Prediction of the state: $X(k | k-1) = X(k-1 | k-1)$
 - Prediction of the covariance of the state: $R_x(k | k-1) = R_x(k-1 | k-1)$
 - Prediction of the measurement: $Y_p(k | k-1) = c(X(k | k-1))$

The function c is detailed hereinafter.

15 Innovation

- Calculation of the innovation: $e(k) = Y(k) - Y_p(k | k-1)$
 - Covariance of the innovation: $R_e(k) = H(k) R_x(k | k-1) H(k)^T + R_m(k)$

- The matrix $H(k)$ corresponds to the Jacobian of the measurements, and this matrix is detailed hereinafter. The matrix $R_m(k)$ is the covariance of the
 20 measurement noise.

Kalman gain

- Calculation of the Kalman gain: $K(k) = R_x(k | k-1) H(k)^T R_e(k)^{-1}$

25 Final estimations

- Correction of the prediction: $X(k | k) = X(k | k-1) + K(k) e(k)$
 - Correction of the covariance: $R_x(k | k) = [I - K(k) H(k)] R_x(k | k-1)$

The function c corresponds to the expression for the measurement expected as a function of the estimated state:

$$c(X(k|k-1)) = \left[\begin{array}{c} \|X_m - t \cdot b + X_d(k|k-1)\| \\ \frac{2}{\lambda} \cdot \frac{\langle X_m - t \cdot b(k|k-1) + X_d(k|k-1); V_m - b(k|k-1) \rangle}{\|X_m - t \cdot b(k|k-1) + X_d(k|k-1)\|} \end{array} \right]$$

The expression for the Jacobian of the measurements is as follows. To simplify the notation, in this expression X_d and b correspond respectively to $X_d(k | k-1)$ and to $b(k | k-1)$:

$$H_k = \left(\begin{array}{c} \left(\frac{X_m - t \cdot b + X_d}{\|X_m - t \cdot b + X_d\|} \right)^T \\ \frac{2}{\lambda} \left(\frac{V_m - b}{\|X_m - t \cdot b + X_d\|} - \frac{(X_m - t \cdot b + X_d) \cdot (X_m - t \cdot b + X_d; V_m - b)}{\|X_m - t \cdot b + X_d\|^3} \right)^T \end{array} \right) \left(\begin{array}{c} \left(-t \cdot \frac{X_m - t \cdot b + X_d}{\|X_m - t \cdot b + X_d\|} \right)^T \\ \frac{2}{\lambda} \left(-t \cdot \left(\frac{V_m - b}{\|X_m - t \cdot b + X_d\|} - \frac{(X_m - t \cdot b + X_d) \cdot (X_m - t \cdot b + X_d; V_m - b)}{\|X_m - t \cdot b + X_d\|^3} \right) \frac{X_m - t \cdot b + X_d}{\|X_m - t \cdot b + X_d\|} \right)^T \end{array} \right)$$

According to the invention, the method proposes to use an estimator filter.

To enhance the effectiveness of such a filter, provision may advantageously be made upstream to optimize the trajectory of the carrier towards the target.

Indeed, the shape of the trajectory has a direct impact on the performance of this estimator filter. This coupling can be comprehended by considering the relative positions of the set of circles associated with the distance and Doppler of the target that are obtained by processing each SAR image. It is apparent for example that a trajectory performed in a plane at constant incidence leads to circles which are all tangential to a straight line passing through the target. Figure 4 illustrates this phenomenon. The estimated position of the target therefore has a tendency to slide along this straight line thereby resulting in significant errors. On the other hand in the case of a trajectory having some movement along the three axes, the Doppler distance circles are all in different planes, thereby making it possible to more precisely locate the intersection point which is the target, as illustrated in Figure 5.

So as to optimize the performance of the estimator filter, it is proposed to use a tool making it possible to search for an optimal trajectory in relation to the particulars of the relative position of the target and of the speed errors estimated at the end of the activation of the seeker, that is to say upon engagement of the inertial homing onto the target. This tool must make it possible to determine the trajectory by solving a problem of the optimization of a constrained cost function type.

The cost function considered is the Cramer Rao bound evaluated for the

last SAR image acquired. This bound corresponds to the best precision obtainable by an unbiased estimator filter. The constraints taken into account pertain to the seeker (range, minimum squint angle compatible with the SAR mode, etc.) and to the missile (load factor, etc.).

- 5 The trajectory is split up into N pieces. For each of these trajectory portions, the transverse acceleration is assumed constant. The tool determines the 2*N components of the accelerations along the trajectory, which after integration provides the shape of the trajectory knowing that the starting point of the trajectory is fixed by the user and that the target is at the origin of the reference frame. The
10 determination of the 2*N components of the acceleration is obtained in such a way as to optimize the Cramer Rao bound for the last image acquired.

The Cramer Rao bound is the inverse of the Fisher matrix J_k , which for the model adopted is obtained through the following recursive equation:

$$15 \quad J_{k+1} = J_k + \tilde{H}_{k+1}^T \cdot R_{k+1}^{-1} \cdot \tilde{H}_{k+1}$$

where

$\tilde{H}_{k+1}$ is the Jacobian of the measurement function, evaluated at the true value of the state with the Jacobian of f which is the matrix of partial derivatives $\left(\frac{\partial f_i}{\partial x_j} \right)_{i,j}$

- R_{k+1} is the covariance matrix of the measurement noise
 - J_0 is the inverse of the covariance of the initial state.
- 20

For the model the expression for the Jacobian of the measurements is

$$\tilde{H}_k = \begin{pmatrix} \left(\frac{X_m - t \cdot b + X_d}{\|X_m - t \cdot b + X_d\|} \right)^T & \left(-t \cdot \frac{X_m - t \cdot b + X_d}{\|X_m - t \cdot b + X_d\|} \right)^T \\ \frac{2}{\lambda} \left(\frac{V_m - b}{\|X_m - t \cdot b + X_d\|} \frac{(X_m - t \cdot b + X_d) \cdot (X_m - t \cdot b + X_d; V_m - b)}{\|X_m - t \cdot b + X_d\|^3} \right)^T & \frac{2}{\lambda} \left(-t \cdot \left(\frac{V_m - b}{\|X_m - t \cdot b + X_d\|} \frac{(X_m - t \cdot b + X_d) \cdot (X_m - t \cdot b + X_d; V_m - b)}{\|X_m - t \cdot b + X_d\|^3} \right) \frac{X_m - t \cdot b + X_d}{\|X_m - t \cdot b + X_d\|} \right)^T \end{pmatrix}$$

- 25 where the various functions are evaluated at the time $t = t_k$ corresponding to the instant of the k-th measurement.

The recursive calculation of J_k for a trajectory will make it possible to evaluate the quality of the latter by calculating the trace of the Cramer-Rao bound for the last image acquired, thereby characterizing the global precision of the impact point in the

case of a perfect missile. The smaller this trace, the more the information provided by the seeker allows precise guidance onto the target.

The optimization of the trajectory is performed by considering the following
5 constraints:

- the maximum load factor of the missile during the SAR mapping phase and during the inertial homing,
- the minimum angular squint between the speed vector and the missile-target direction,
- 10 - the minimum impact angle in the vertical plane.

Claims

1. Method for determining the position error and the speed bias of an inertial platform aboard a carrier pursuing a trajectory towards a fixed target, characterized in that it furthermore comprises the following steps:

- acquiring aboard the carrier a series of SAR-type images of the target;
- utilizing the said images to measure on the basis of each of the images a measurement of the distance and a measurement of the Doppler of the target;
- estimating the precise position of the carrier with respect to the target and the speed bias, on the basis of the measurements of distance and of the Doppler of the target with an estimator filter of the extended Kalman filter type, said method furthermore comprising a prior step of trajectory optimization to optimize the performance of the estimator filter.;
- using a tool comprising a constrained cost function associated with the estimation of the position of the carrier with respect to the target and of the speed bias.

2. Method for determining the position error and the speed bias of an inertial platform aboard a carrier pursuing a trajectory towards a target according to Claim 1, characterized in that the constrained cost function is the Cramer Rao bound.

3. Method for determining the position error and the speed bias of an inertial platform aboard a carrier pursuing a trajectory towards a target according to Claim 2, characterized in that it comprises the following steps:

- segmenting the trajectory into N pieces;
- determining the 2N components of the transverse acceleration of the carrier along the trajectory by optimizing the Cramer Rao bound for the last image acquired.

4. Method for determining the position error and the speed bias of an inertial platform of a carrier pursuing a trajectory towards a target according to one of Claims 1 to 3, characterized in that the constraints taken into account during the optimization of the trajectory are maximum load factor of the carrier and/or minimum angular squint between the speed vector and the carrier-target direction and/or the minimum impact angle in the vertical plane.