

FIG. 1

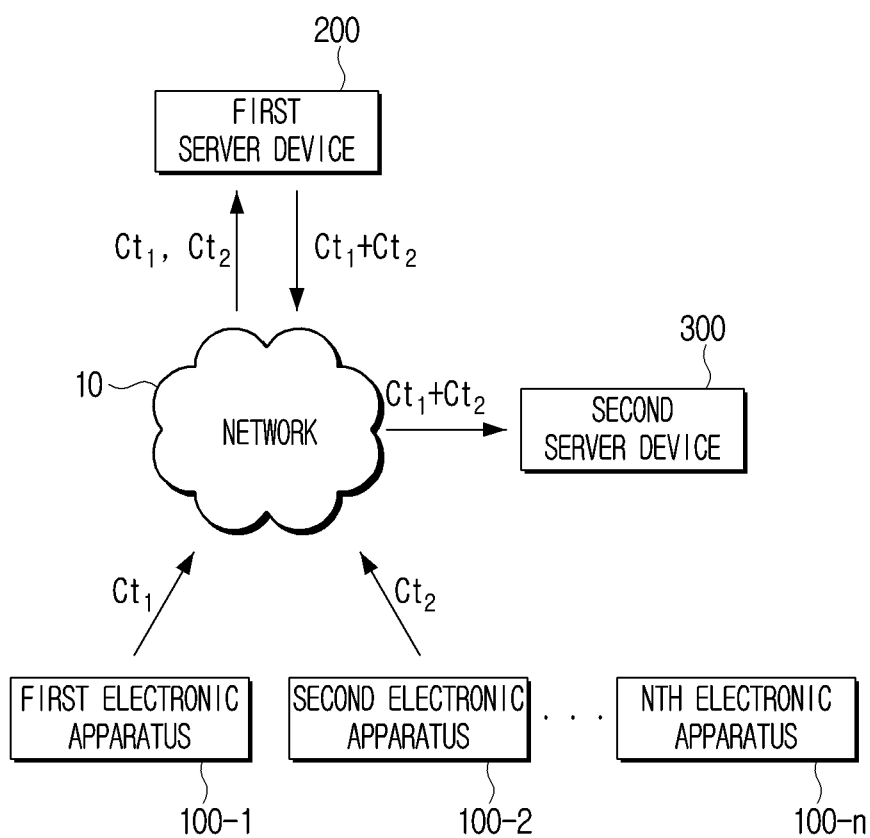


FIG. 2

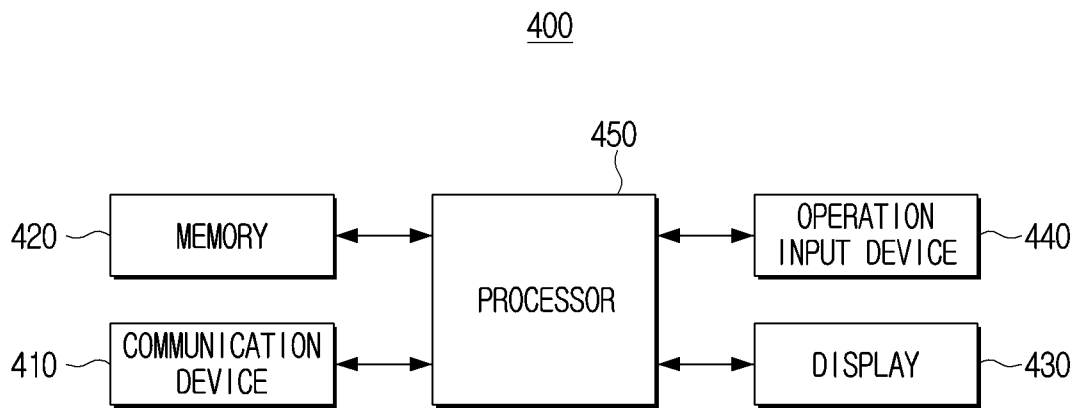


FIG. 3

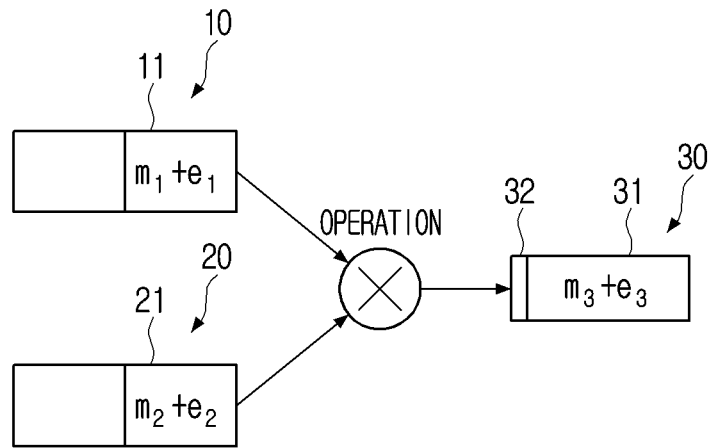


FIG. 4

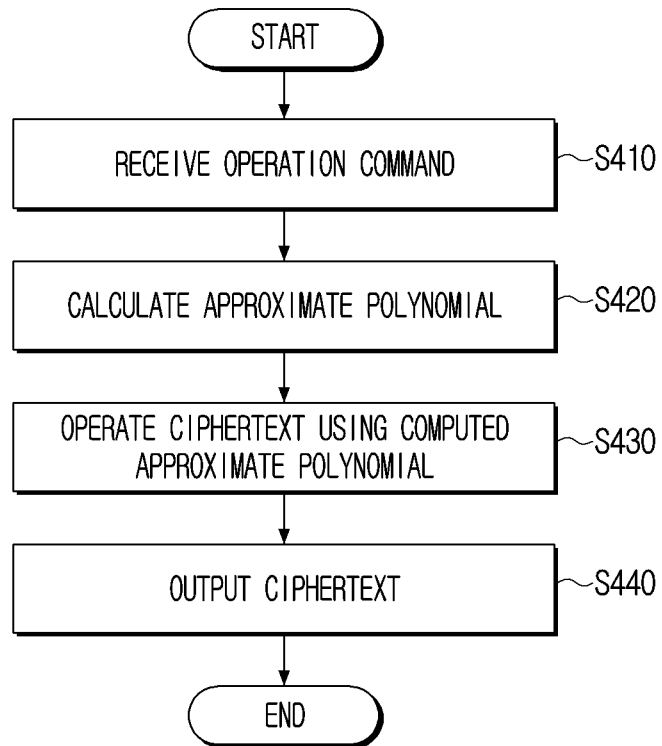


FIG. 5

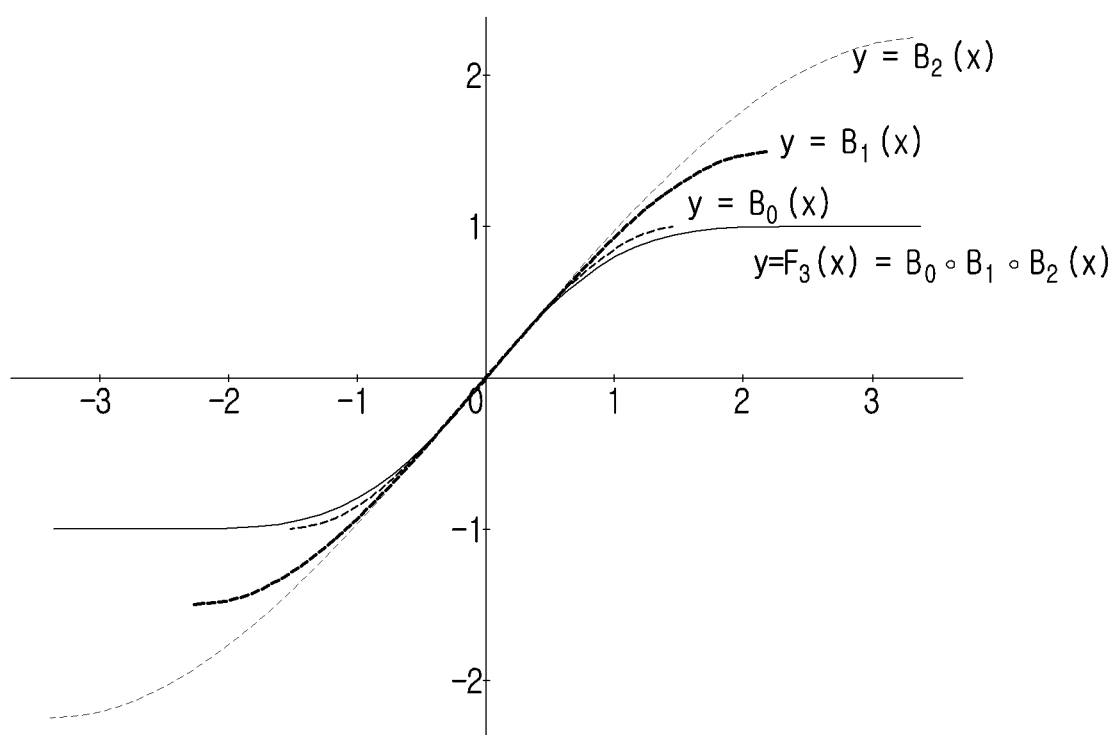


FIG. 6

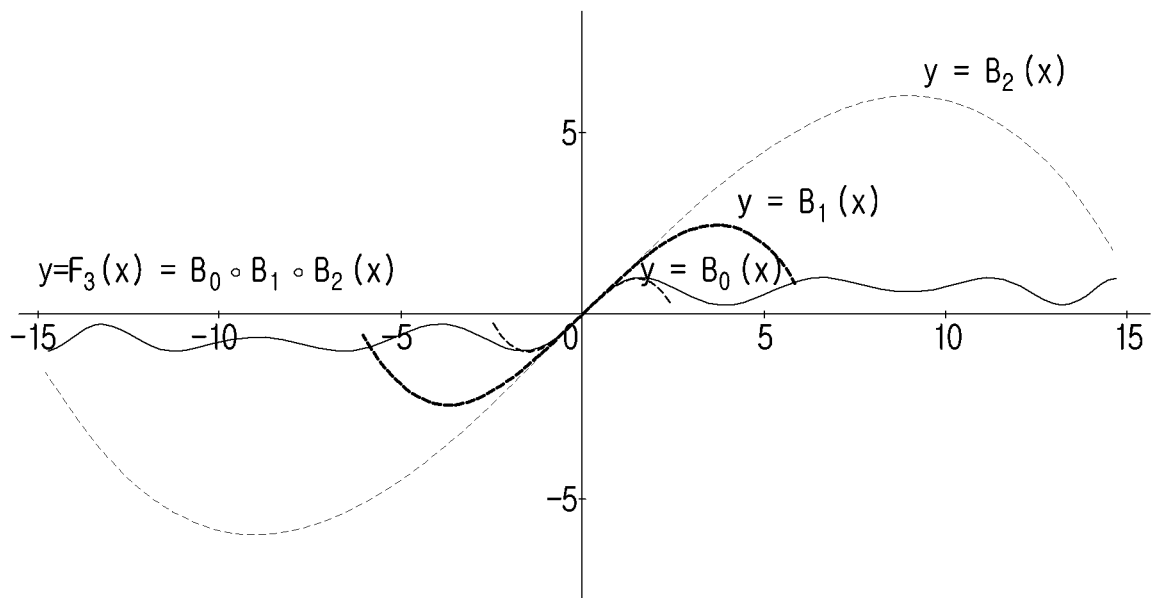


FIG. 7

Algorithm 1 polyDEF $F_{n,r}$ in Example 1

Input: $x_0 \in [-1.5^n r, 1.5^n r]$

Output: Approximate r -DEF at x .

```

1:  $y \leftarrow \frac{x_0}{r}$ 
2: for  $i \leftarrow n - 1$  to 0 do
3:    $y \leftarrow y - \frac{4}{27} \frac{1}{2.25^n} y^3$ 
4: end for
5: return  $ry$ 

```

FIG. 8

Algorithm 2 Approximate sigmoid function in Example 2

Input: Approximation $P(x)$ of sigmoid function on $[-r, r]$
and $x_0 \in [-1.5^n r, 1.5^n r]$.

Output: Approximate sigmoid function at x_0 .

```

1:  $y \leftarrow \frac{x_0}{r}$ 
2: for  $i \leftarrow n - 1$  to 0 do
3:    $y \leftarrow y - \frac{4}{27} \frac{1}{2.25^n} y^3$ 
4: end for
5: return  $P(ry)$ 
```

FIG. 9

Algorithm 3 Approximate logistic function with wavy poly-DEF

Input: $x \in [-2.45^n \cdot 14.5, 2.45^n \cdot 14.5]$.

Output: Approximate logistic function at x .

// $P(x)$ is the minimax polynomial of the logistic function on $[-14.5, 14.5]$.

```

1:  $y \leftarrow \frac{x}{14.5}$ 
2: for  $i \leftarrow n - 1$  to 0 do
3:    $y \leftarrow y - \frac{4}{27} \frac{1}{6.0025^n} y^3$ 
4: end for
5: return  $P(14.5y)$ 

```

FIG. 10

Algorithm 4 Approximate Logistic Regression

Input: Data $D_i \in [0, 1]^d$ with label $\ell_i \in \{-1, 1\} \forall i \in [1, M]$, learning rate α , number of iterations T , mini-batch size N .

Output: Trained logistic regression weight W and bias b .

```

1: for  $i \leftarrow 1$  to  $M$  do
2:    $z_i \leftarrow \ell_i \cdot (D_i, 1) \in [-1, 1]^{d+1}$ 
3: end for
4:  $\beta \leftarrow 0^{d+1}$ 
5: for iteration  $\leftarrow 1$  to  $T$  do
6:    $G \leftarrow 0^{d+1}$ 
7:   for  $i \leftarrow 1$  to  $N$  do
8:     Sample  $z$  from  $z_i$ 
9:      $x \leftarrow -z \cdot \beta$ 
10:    for  $j \leftarrow 1$  to  $\frac{\log(\sqrt{t(\alpha^2 d^2 + 0.6\alpha d)})}{\log 2.45}$  do
11:       $x \leftarrow x - \frac{4}{6.0025^n 27} x^3$ 
12:    end for
13:     $G \leftarrow G + P(x)z$  //  $P(x)$  is the degree 9 minimax polynomial of the logistic function on  $[-14.5, 14.5]$ 
14:  end for
15:   $\beta \leftarrow \beta + \frac{\alpha}{N} G$ 
16: end for
17:  $(b, W) \leftarrow \beta$ 
18: return  $(W, b)$ 

```
