Method for Computing and Reducing Frequency "Lock-In" for a Vibrating Beam Accelerometer and Optional Linear Enhancement

RELATED APPLICATIONS

This application claims priority from IL 315720 for "*Vibrating Beam Accelerometer with Reduced Locking Effect*" filed 18 September 2024 and IL 320445 for "*Method for Computing and Reducing Frequency "Lock-In" for a Vibrating Beam Accelerometer and Optional Linear Enhancement*" filed 22 April 2025, the whole contents of both of which are incorporated herein by reference.

FIELD OF THE INVENTION

This invention relates to Vibrating Beam Accelerometers and more particularly to reduction of the "blind zone" where the sensor does not produce a useful output in the presence of an external acceleration.

PRIOR ART

Prior art references considered to be relevant as a background to the invention are listed below and their contents are incorporated herein by reference. Acknowledgement of the references herein is not to be inferred as meaning that these are in any way relevant to the patentability of the invention disclosed herein. Each reference is identified by a number enclosed in square brackets and accordingly the prior art will be referred to throughout the specification by numbers enclosed in square brackets.

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[3] T. Loret *et al*., "Navigation grade accelerometer with quartz vibrating beam," *2014 DGON Inertial Sensors and Systems (ISS)*, Karlsruhe, Germany, 2014, pp. 1-14.

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[12] J. R. Wilkinson, "Ring Lasers," Prog. Quant. Electr., 1987, vol. 11, pp. 1-103.

[13] Y. Arazi and J. L. Gruver, "A new operational mode for axisymmetric Coriolis vibrating gyroscopes: Force Angle Tracking (FAT)," *2023 DGON Inertial Sensors and Systems (ISS)*, Braunschweig, Germany, 2023, pp. 1-13.

[14] Zhen Zhou, Jie Shi, Bao-yin Yao and Li-shuang Feng, "Theoretical investigation of the mechanical coupling in a differential vibrating beam accelerometer,"

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BACKGROUND OF THE INVENTION

Vibrating Beam Accelerometers are based on the fact that the resonant frequency of a vibrating beam attached to a seismic-mass changes when subjected to an external acceleration. The beam that is pulled vibrates at a higher frequency while the beam that is pushed vibrates at a lower frequency. This action is similar to that of a violin string, which increases pitch with an increase in tension, and decreases pitch with a decrease in tension (or relative compression). Typically, two vibrating beams are used in a differential arrangement in order to reduce common parasitic sensitivities e.g. temperature, pressure or aging. The two beams operate in a push-pull mode whereby when one beam is pushed and is subjected to compression, the other beam is pulled and subjected to tension. During operation, the two beams constantly change roles and, as is described in the literature, for each beam there is a measured vibrating frequency and it can be shown that they vibrate at two frequencies: one slightly higher corresponding to pull-mode and the other slightly lower corresponding to push-mode.

However, the use of two beams coupled by a seismic mass may lead to the presence of a “blind zone” where the sensor does not produce a useful output in the presence of an external acceleration. It is widely believed that this blind zone is caused by the “lock-in” phenomenon whereby the resonant frequency of both beams becomes “locked” (i.e. both beams vibrate with the same frequency) and therefore the output acceleration (which is proportional to the frequency difference) becomes zero even in the presence of an external acceleration. This effect limits the range of accelerations for which the sensor can be used.

SUMMARY OF THE INVENTION

The invention is based on the unexpected finding that, contrary to widespread belief, the lock-in of frequencies does not exist in the rigorous sense. This is because both beams vibrate at their two Eigen-frequencies for the whole range of external accelerations, excepting for zero acceleration. The presence of two vibration frequencies is the

consequence of the split of the resonant frequency caused by the coupling of both resonators through the seismic mass.

Specifically, for each of the beams there are corresponding first and second frequencies (f–,f+) at which the respective amplitude is dominant. The lock-in zone is bound by respective crossover points for each beam where the respective amplitudes (A–, A+) at both the first and second frequencies (f–, f+) are the same and which define a crossover region within which the respective dominant amplitudes of the first and second frequencies cross over such that the frequency for which amplitude is higher outside the crossover region has a smaller amplitude within the crossover region and *vice versa*.

This is distinguished over conventional approaches that measure only a single frequency, corresponding to the frequency whose amplitude is dominant.

The present inventor has found that a principal cause of lock-in is that conventional approaches to measuring the vibration frequency of the beams measure only the dominant frequency, i.e. the frequency whose amplitude is higher and, as a result, ignore the less dominant frequency whose amplitude is lower. Throughout the present disclosure, we will refer to these frequencies as the dominant and sub-dominant frequencies. In contrast, the present invention detects both the dominant and sub-dominant frequencies, which when properly taken into account can determine the lock-in boundary within which the amplitudes of the dominant and sub-dominant frequencies cross over. The frequencies within the lock-in boundary can then be selected to avoid the effect of lock-in.

It may appear to be a contradiction to suggest that, on the one hand, the present invention is directed to the reduction or elimination of lock-in while, on the other hand, to state that the frequency lock-in does not exist in the rigorous sense. Therefore, for the sake of abundant clarity and to avoid doubt, lock-in will occur if only the dominant frequencies of both beams are recorded across the complete range of accelerations to which the accelerometer is subjected. This is because, as will be shown analytically below, the dominant frequencies for both beams are identical in the lock-in range and therefore in an accelerometer whose output is a function of differential frequency, no signal will be output for any acceleration in the lock-in zone. The invention effectively circumvents this problem by taking into account the non-dominant frequencies in the lock-in zone.

BRIEF DESCRIPTION OF THE DRAWINGS

In order to understand the invention and to see how it may be carried out in practice, embodiments will now be described, by way of non-limiting example only, with reference to the accompanying drawings, in which:

Fig. 1 shows schematically a simple differential vibrating beam accelerometer;

Fig. 2 is the equivalent spring-mass model of the device shown in Fig. 1;

Fig. 3 is a graphical representation of the amplitude-acceleration characteristic for the device as derived using the model of Fig. 2;

Fig. 4 is a graphical representation of the frequency-acceleration characteristic for conventional accelerometers as derived using the model of Fig. 2;

Figs. 5 and **6** show the dominant frequencies of Beam 1 and Beam 2, respectively which are measured by conventional accelerometers;

Fig. 7 is a graphical representation showing frequency-difference ($\Delta f(t)$) versus acceleration for the device as derived using the model of Fig. 2 when measuring frequency according to conventional methods;

Figs. 8 and **9** show the dominant and sub-dominant frequencies of Beam 1 and Beam 2, respectively both of which may be measured according to a first embodiment of the invention;

Fig. 10 is a graphical representation of the amplitude-acceleration characteristic for a device having beams of different masses as derived using a suitably modified model to the model of Fig. 2;

Fig. 11 shows graphically the relationship between the vibration amplitudes of both beams inside and outside of the lock-in zone;

Figs. 12 and **13** are graphical representations showing corrected frequency-difference ($\Delta f(t)$) versus acceleration over different ranges of acceleration according to a first embodiment of the invention;

Fig. 14 shows graphically dominant and sub-dominant frequency versus acceleration for a single beam showing that the curves veer toward each at zero acceleration;

Figs. 15 and **16** show graphically dominant frequency versus acceleration for both beams when vibrated in-phase and anti-phase, respectively;

Fig 17 shows frequency-difference versus acceleration using the frequency-difference, $\Delta f(\Gamma)$, of Fig. 12 as compared with an enhanced linear relationship using a corrected frequency-difference, $\hat{\Delta} f(\Gamma)$, according to a second embodiment;

Fig. 18 shows the same comparison of Fig. 17 drawn to a larger scale;

Fig. 19 shows deviation from linearity for the first and second embodiments of the invention; and

Figs 20 and 21 are flow charts showing alternative methods for determining the boundary of the lock-in zone during calibration of the accelerometer.

DETAILED DESCRIPTION OF EMBODIMENTS

A proper understanding of the invention requires a detailed analysis of the vibrating beam accelerometer, much of which will be familiar to those skilled in the art. For the sake of completeness, we present a model of a vibrating beam accelerometer and derive equations that define the boundaries of the lock-in zone.

Fig. 1 shows schematically a simple vibrating beam accelerometer that can be analyzed mathematically using the equivalent spring-mass model shown in Fig. 2. For illustrative purposes and sake of simplicity, in the following explanation the model shown in Fig 2 considers the case of equal beams, it being understood that the invention defined by the appended claims encompasses any practical implementation of the vibrating beam accelerometer that can be reduced to the same or similar mathematical model including the case where the beams are not equal. Thus, Fig. 1 shows an accelerometer 10 comprising a pair of collinear or parallel beams 11, 11′ each anchored at one end to a respective support 12, 12′ and commonly coupled at their opposite ends to a seismic mass 13. The beams 11, 11′ may be formed of piezoelectric material such as quartz that generates an electric charge in response to mechanical stress. When the beams are compressed or stretched, they produce an electric voltage across their surfaces. Conversely, an electric field applied across the beams will induce vibration. To this end, an AC voltage 14 is commonly applied across a respective pair of electrodes 15, 15′ deposited on opposing surfaces of each beam, thereby inducing a steady-state vibration of the beams whose amplitudes are measured as analog signals across respective electrodes 16, 16′ deposited on opposing surfaces of both beams.

When the structure is subjected to an external acceleration, a force component parallel to the beams' longitudinal axes shifts the seismic mass axially to and fro, thereby

pushing one of the beams while pulling the other, thereby giving rise to slight differences in their respective resonant frequencies. Specifically, when using conventional methods for measuring the frequency, the shorter compressed beam will vibrate at a lower frequency, which we shall denote as *f*– compared to the longer stretched beam, whose resonant frequency *f*+ is higher. In a practical implementation, the beams are used in flexural mode as sensitive elements, and vibration is maintained by means of an oscillator circuit denoted schematically by the AC voltage 14. For so long as the beams are subjected to axial vibration, they will continually elongate and foreshorten alternately, and their resonant frequencies will likewise decrease and increase alternately.

The applied acceleration induces tensile or compressive stresses into each beam which modify the resonant frequency, measurement of which is thus indicative of the applied acceleration.

The equations of motion that describe the dynamics of the model of Fig. 2 are:

$$\begin{aligned} m_0\ddot{x}_1 + k^+(x_1 - X) &= m_0\Gamma \\ M\ddot{X} + k^+(X - x_1) + k^-(X - x_2) &= M\Gamma \\ m_0\ddot{x}_2 + k^-(x_2 - X) &= m_0\Gamma \end{aligned} \tag{1}$$

where Γ is the external applied acceleration.

$$\begin{aligned} k^+ &= k_0(1 + \lambda\Gamma) \\ k^- &= k_0(1 - \lambda\Gamma) \end{aligned} \tag{2}$$

It is clear from Eq. (2) that k^+ and k^- are functions of Γ. However, in all the following equations, we shall omit the dependency $k^+(\Gamma)$ and $k^-(\Gamma)$ on Γ in order to avoid the equations becoming unwieldy and refer simply to k^+ and k^-.

Dividing by m_0 and M, respectively, leads to:

$$\begin{aligned} \ddot{x}_1 + \frac{k^+}{m_0}(x_1 - X) &= \Gamma \\ \ddot{X} + \frac{k^+}{M}(X - x_1) + \frac{k^-}{M}(X - x_2) &= \Gamma \\ \ddot{x}_2 + \frac{k^-}{m_0}(x_2 - X) &= \Gamma \end{aligned} \tag{3}$$

Subtracting the first and third equations from the second leads to:

$$\begin{aligned}\ddot{X} - \ddot{x}_1 + \left(\frac{k^+}{M} + \frac{k^+}{m_0}\right)(X - x_1) + \frac{k^-}{M}(X - x_2) = 0 \\ \ddot{X} - \ddot{x}_2 + \frac{k^+}{M}(X - x_1) + \left(\frac{k^-}{M} + \frac{k^-}{m_0}\right)(X - x_2) = 0\end{aligned} \tag{4}$$

We define new variables y_1 and y_2 as follows:

$$\begin{aligned}y_1 &\equiv X - x_1 \\ y_2 &\equiv X - x_2\end{aligned}$$

Substituting in Eq. (4) yields:

$$\begin{aligned}\ddot{y}_1 + \left(\frac{k^+}{M} + \frac{k^+}{m_0}\right)y_1 + \frac{k^-}{M}y_2 = 0 \\ \ddot{y}_2 + \frac{k^+}{M}y_1 + \left(\frac{k^-}{M} + \frac{k^-}{m_0}\right)y_2 = 0\end{aligned} \tag{5}$$

We define new variables y_3 and y_4 as follows:

$$\begin{aligned}y_3 &\equiv \dot{y}_1 \\ y_4 &\equiv \dot{y}_2\end{aligned} \tag{6}$$

Substituting in Eq. (5) gives:

$$\begin{aligned}\dot{y}_1 &= y_3 \\ \dot{y}_2 &= y_4 \\ \dot{y}_3 &= -\left(\frac{k^+}{M} + \frac{k^+}{m_0}\right)y_1 - \frac{k^-}{M}y_2 \\ \dot{y}_4 &= -\frac{k^+}{M}y_1 - \left(\frac{k^-}{M} + \frac{k^-}{m_0}\right)y_2\end{aligned} \tag{7}$$

Equation (7) can be more conveniently written in matrix form:

$$\begin{pmatrix}\dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4\end{pmatrix} = \begin{pmatrix}0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\left(\frac{k^+}{M} + \frac{k^+}{m_0}\right) & -\frac{k^-}{M} & 0 & 0 \\ -\frac{k^+}{M} & -\left(\frac{k^-}{M} + \frac{k^-}{m_0}\right) & 0 & 0\end{pmatrix}\begin{pmatrix}y_1 \\ y_2 \\ y_3 \\ y_4\end{pmatrix} \tag{8}$$

We define:

$$M_0 \equiv \frac{m_0 M}{M + m_0}$$

We can re-write Eq. (8) as follows:

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k^+}{M_0} & -\frac{k^-}{M} & 0 & 0 \\ -\frac{k^+}{M} & -\frac{k^-}{M_0} & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} \tag{9}$$

Eq. (9) has a solution of the form:

$$\begin{aligned} y_1(t) &= 2c_1 \cos \omega_+ t - 2e_1 \sin \omega_+ t + 2c_2 \cos \omega_- t - 2e_2 \sin \omega_- t \\ y_2(t) &= -(2c_1 \cos \omega_+ t - 2e_1 \sin \omega_+ t)\frac{M}{k^-}\left(\frac{k^+}{M_0} - \omega_+^2\right) \\ &\quad - (2c_2 \cos \omega_- t - 2e_2 \sin \omega_- t)\frac{M}{k^-}\left(\frac{k^+}{M_0} - \omega_-^2\right) \\ y_3(t) &= -(2c_1 \sin \omega_+ t + 2e_1 \cos \omega_+ t)\omega_+ \\ &\quad - (2c_2 \sin \omega_- t + 2e_2 \cos \omega_- t)\omega_- \\ y_4(t) &= (2c_1 \sin \omega_+ t + 2e_1 \cos \omega_+ t)\frac{\omega_+ M}{k^-}\left(\frac{k^+}{M_0} - \omega_+^2\right) \\ &\quad + (2c_2 \sin \omega_- t + 2e_2 \cos \omega_- t)\frac{\omega_- M}{k^-}\left(\frac{k^+}{M_0} - \omega_-^2\right) \end{aligned} \tag{10}$$

and

$$\begin{aligned} c_1 &= \frac{y_1(0)}{2} - \frac{y_1(0)}{2}\left(\frac{\frac{k^+}{M_0} - \omega_+^2}{\omega_-^2 - \omega_+^2}\right) - \frac{y_2(0)k^-}{2M(\omega_-^2 - \omega_+^2)} \\ e_1 &= -\frac{y_3(0)}{2\omega_+} + \frac{y_3(0)}{2\omega_+}\left(\frac{\frac{k^+}{M_0} - \omega_+^2}{\omega_-^2 - \omega_+^2}\right) + \frac{y_4(0)k^-}{2M\omega_+(\omega_-^2 - \omega_+^2)} \\ c_2 &= \frac{y_1(0)}{2}\left(\frac{\frac{k^+}{M_0} - \omega_+^2}{\omega_-^2 - \omega_+^2}\right) + \frac{y_2(0)k^-}{2M(\omega_-^2 - \omega_+^2)} \\ e_2 &= -\frac{y_3(0)}{2\omega_-}\left(\frac{\frac{k^+}{M_0} - \omega_+^2}{\omega_-^2 - \omega_+^2}\right) - \frac{y_4(0)k^-}{2M\omega_-(\omega_-^2 - \omega_+^2)} \end{aligned} \tag{11}$$

where:

$$\omega_- \equiv \sqrt{k_0\left[\frac{1}{M_0} - \sqrt{\frac{1}{M^2} + \lambda^2\Gamma^2\left(\frac{1}{M_0^2} - \frac{1}{M^2}\right)}\right]}$$
$$\omega_+ \equiv \sqrt{k_0\left[\frac{1}{M_0} + \sqrt{\frac{1}{M^2} + \lambda^2\Gamma^2\left(\frac{1}{M_0^2} - \frac{1}{M^2}\right)}\right]} \quad (12)$$

It is reiterated that the dependency of ω_- and ω_+ on Γ is omitted for the sake of simplicity, The differential frequency $\Delta\omega = \omega_+ - \omega_-$ is given by:

$$\Delta\omega = \sqrt{2\frac{k_0}{m_0}}\sqrt{1 + \frac{m_0}{M} - \sqrt{(1 - \lambda^2\Gamma^2)\left(1 + 2\frac{m_0}{M}\right)}} \quad (13)$$

For the case that $m_0 \ll M$ and $\lambda\Gamma \ll 1$, we can neglect $2\frac{m_0}{M}\lambda^2\Gamma^2$ and Eq. (13) can be approximated to:

$$\Delta\omega \approx \sqrt{2\frac{k_0}{m_0}}\sqrt{1 + \frac{m_0}{M} - \sqrt{1 + 2\frac{m_0}{M} - \lambda^2\Gamma^2}} \quad (14)$$

Using the partial Taylor expansion for $\sqrt{1+x} \approx 1 + \frac{1}{2}x - \frac{1}{8}x^2$ where $x = 2\frac{m_0}{M} - \lambda^2\Gamma^2$and neglecting terms in $4\frac{m_0}{M}\lambda^2\Gamma^2$ and $\lambda^4\Gamma^4$:

$$\Delta\omega \approx \sqrt{2\frac{k_0}{m_0}}\sqrt{1 + \frac{m_0}{M} - 1 - \frac{1}{2}\left(2\frac{m_0}{M} - \lambda^2\Gamma^2\right) + \frac{1}{8}\cdot 4\left(\frac{m_0}{M}\right)^2}$$
$$\approx \sqrt{2\frac{k_0}{m_0}}\sqrt{1 + \frac{m_0}{M} - 1 - \frac{m_0}{M} + \frac{1}{2}\lambda^2\Gamma^2 + \frac{1}{2}\left(\frac{m_0}{M}\right)^2} \quad (15)$$

This leads to:

$$\Delta\omega \approx \sqrt{\frac{k_0}{m_0}}\sqrt{\lambda^2\Gamma^2 + \frac{m_0^2}{M^2}} \quad (16)$$

and for $\Gamma = 0$:

$$\Delta\omega(0) \approx \sqrt{\frac{k_0}{m_0}}\frac{m_0}{M} \quad (17)$$

and for $\Gamma \gg 0$:

$$\Delta\omega(\Gamma) \approx \lambda \sqrt{\frac{k_0}{m_0}}\Gamma = \eta\Gamma \tag{18}$$

where $\eta \equiv \lambda\sqrt{\frac{k_0}{m_0}} = \lambda\omega_0$ is the scale factor.

Example I:

Accelerations are commonly represented in units of the local value of the gravity constant g, which was approximated to 10 ms^{-2} for the purpose of simulation. By way of example, consider a concrete example having a seismic mass $M = 2 \times 10^{-5}kg$, beams of mass $m_0 = 2 \times 10^{-8}kg$, spring constant $k_0 = 3850\, kgs^{-2}$, and $\lambda = \frac{0.0005}{g}$. The initial conditions are $y_1(0) = y_2(0) = 1\mu m$ and $y_3(0) = y_4(0) = 0$. We use Eq. (10) to calculate the output signals $y_1(t)\ and\ y_2(t)$.

A frequency counter based on signal zero-cross in a given time interval is used in order to measure the vibrating frequency of each beam. Note that a frequency counter gives a single output frequency. The frequency difference Δf is proportional to the applied external acceleration. The vibration amplitudes of each beam can be calculated from Eq. (10) as:

$$\begin{aligned} A_{1+} &= 2\sqrt{c_1^2 + e_1^2} \\ A_{1-} &= 2\sqrt{c_2^2 + e_2^2} \\ A_{2+} &= 2\sqrt{(c_1^2 + e_1^2)\left(\frac{M}{k^-}\right)^2\left(\frac{k^+}{M_0} - \omega_+^2\right)^2} \\ A_{2-} &= 2\sqrt{(c_2^2 + e_2^2)\left(\frac{M}{k^-}\right)^2\left(\frac{k^+}{M_0} - \omega_-^2\right)^2} \end{aligned} \tag{19}$$

Again, for the sake of abundant clarity, we reiterate that in Eq. (19) the dependency of k^- and k^+ as well ω_- and ω_+ on Γ is omitted for the sake of simplicity.

Fig. 3 depicts the vibration amplitudes for each beam. The numbers 1 and 2 refer to the beams 1 and 2 also denoted 11 and 11′, respectively in Fig. 1. In order to distinguish between the four frequencies (two for each of the two beams), the amplitude-acceleration curves are shown using different line types as follows:

CLAIMS:

1. A method for calibrating a vibrating beam accelerometer that can be modelled mathematically as a system having two beams operating in a differential push-pull mode, such that the two beams vibrate at frequencies that are indicative of acceleration, said method comprising:

(a) subjecting the accelerometer to a range of known positive and negative accelerations such that for each beam there are corresponding first and second frequencies (f–, f+) such that for any given acceleration one is dominant and one is sub-dominant;

(b) for each known acceleration measuring the respective dominant frequency of each beam;

(c) determining the respective crossover points at which the dominant vibration frequencies of the two beams are equal and the absolute values of positive and negative accelerations for both beams are maximum; and

(d) obtaining a frequency-acceleration characteristic for the accelerometer by subjecting the accelerometer to a range of accelerations and measuring a corresponding differential frequency Δf equal to the difference in frequencies for each beam such that in a region bound by said crossover points the differential frequency is set to a difference in the respective frequency for one beam where the amplitude is higher and the respective frequency for the other beam where the amplitude is lower and for all points outside the region bound by the crossover points the differential frequency is set to a difference in the respective frequency for each beam for which the amplitude is higher for both beams or lower for both beams.

2. The method according to claim **1**, further including approximating at least part of the pre-calibrated differential frequency-acceleration characteristic to a linear function within a specified range of differential frequencies.

3. A method for determining acceleration using a pre-calibrated vibrating beam accelerometer that can be modelled mathematically as a system having two beams operating in a differential push-pull mode, such that each of the two beams vibrates at

corresponding first and second frequencies (f–, f+) depending on whether the respective beam is in compression or tension, said method comprising:

(a) subjecting the accelerometer to an acceleration and measuring the frequency of vibration for each beam;

(b) if the frequency exceeds a crossover frequency corresponding to the dominant frequency of the two beams at the crossover points plus a predetermined margin of error, measuring a corresponding differential frequency Δf equal to a difference in the respective frequency for each beam for which the amplitude is higher for both beams or lower for both beams;

(c) if the frequency is smaller than the crossover frequency, then measuring a corresponding differential frequency Δf equal to a difference in the respective frequency for one beam where the amplitude is higher and the respective frequency for the other beam where the amplitude is lower; and

(d) determining acceleration from the measured differential frequency Δf and a pre-calibrated frequency-acceleration characteristic.

4. A method for determining acceleration using a vibrating beam accelerometer that can be modelled mathematically as a system having two beams operating in a differential push-pull mode, such that each of the two beams vibrates at corresponding first and second frequencies (f–, f+) depending on whether the respective beam is in compression or tension, said method comprising:

(a) subjecting the accelerometer to an acceleration and measuring the respective dominant frequency of vibration for each beam;

(b) if the measured dominant frequencies of the two beams are equal within a predetermined margin of error:

i) determining a corresponding sub-dominant frequency of vibration for each beam;

ii) measuring a differential frequency Δf equal to a difference in the respective frequency for one beam where the frequency is dominant and the respective frequency for the other beam where the frequency is sub-dominant; and

(c) if the measured dominant frequencies of the two beams are unequal within a predetermined margin of error:

i) measuring differential frequency *Δf* equal to a difference in the respective frequency for each beam where the frequency is dominant for both beams or sub-dominant for both beams; and

(d) determining acceleration from the measured differential frequency *Δf* and a pre-calibrated frequency-acceleration characteristic.

5. The method for determining acceleration according to claim **3** or **4**, wherein determining acceleration includes:

i) approximating at least part of the pre-calibrated differential frequency-acceleration characteristic to a linear function within a specified range of differential frequencies; and

ii) if the differential frequency falls within the specified range, computing the acceleration using the linear function.

6. The method according to any one of claims **2** to **5**, wherein the frequency-acceleration characteristic is:

$$\Delta f \approx \lambda \cdot \frac{\mathbf{1}}{\mathbf{2}\pi}\sqrt{\frac{k_0}{m_0}}\Gamma$$

where: *Δf* = measured frequency difference,

Γ = acceleration,

$m_{\mathbf{0}}$= mass of each beam,

$k_{\mathbf{0}}$= stiffness of each beam at zero acceleration,

λ = a known scale factor.

7. The method according to any one of claims **3** to **6**, wherein the differential-frequency-acceleration characteristic is a function $\hat{\Delta}f(\Gamma)$ of differential frequency *Δf* given by:

$$\hat{\Delta}f(\Gamma) = \begin{cases} -\sqrt{(\,\Delta^{\mathbf{2}}f(\Gamma) - \Delta^{\mathbf{2}}f_v\,)} & \text{for } \Delta f < \mathbf{0} \\ \mathbf{0} & \text{for } \Delta f = \mathbf{0} \\ \sqrt{(\,\Delta^{\mathbf{2}}f(\Gamma) - \Delta^{\mathbf{2}}f_v\,)} & \text{for } \Delta f > \mathbf{0} \end{cases}$$

where: Δf_v is the minimal absolute difference between the dominant and sub-dominant frequency of each of the two beams.

8. The method according to claim **7**, where Δf_v is estimated by determining from differential-frequency (*Δf*) versus acceleration (Γ) when a measured rate of change of differential-frequency (*Δf*) with acceleration (Γ) is minimal.

9. The method according to claim **8**, wherein respective measurements are made for both negative and positive accelerations and Δf_v is set to an average of the absolute values of the two measurements.

10. The method according to claim **7**, where Δf_v is estimated by measuring the dominant frequencies of both beams when the accelerometer is operated both in-phase and anti-phase, and computing Δf_v as:

$$\Delta f_v = f_{in-phase}(\Gamma_v) - f_{anti-phase}(\Gamma_v)$$

11. The method according to claim **7**, where Δf_v is computed analytically as:

$$\Delta f_v = \frac{\mathbf{1}}{\mathbf{2}\pi}\sqrt{\frac{k_\mathbf{0}}{m_\mathbf{0}}} \cdot \frac{m_\mathbf{0}}{M}$$

where: where: *Δf* = measured frequency difference,

$m_\mathbf{0}$= mass of each beam,

$k_\mathbf{0}$= stiffness of each beam at zero acceleration,

λ = a known scale factor, and

M is the mass of the seismic mass.