

CLAIMS

1. A method for determining a discrete physical properties model of a subsurface region, which may be referred to as the model, by iteratively inverting measured geophysical data acquired from the subsurface region, comprising:
 - 5 using a Hessian matrix of an objective function, then times a vector, called "Hessian times vector," to determine an update for the model, wherein the Hessian times vector is approximated, using a computer, with a single forward-wave simulated propagation and a single computation of gradient of the objective function, in a modified subsurface model.
- 10 2. The method of claim 1, wherein the modified subsurface model is a linear combination of a current subsurface model and the vector.
3. The method of claim 2, wherein the linear combination = the current subsurface model + {(a scalar constant) x (the vector)}, wherein $0 < \text{the scalar constant} \leq 1$.
- 15 4. The method of claim 3, wherein the scalar constant is chosen according to whether the Hessian matrix will be computed exactly, i.e. taking multiple scattering into account, or will be approximated by the Gauss-Newton Hessian matrix.
- 20 5. The method of claim 4, wherein if the Gauss-Newton approximation is to be used, the scalar constant is chosen such that {(a scalar constant) x (the vector)} is approximately 1 % in magnitude of the current subsurface model; but if the Hessian matrix is to be computed exactly, then the scalar constant is chosen so that the {(a scalar constant) x (the vector)} is approximately 10 % in magnitude of the current subsurface model.
- 25 6. The method of claim 3, wherein the vector in a first iteration of the method is gradient of the objective function with respect to parameters of the model, wherein the objective function measures misfit between the measured geophysical data and corresponding synthetic geophysical data simulated using the model, said misfit being a selected norm of a data residual, wherein the data residual is the difference between the measured geophysical data and corresponding synthetic geophysical data simulated using the model; **provided that** for
30 said single computation of gradient of the objective function, the measured geophysical data in the data residual is replaced by synthetic data simulated using the modified subsurface model, and the data residual is divided by the scalar constant.

7. The method of claim 1, where the measured geophysical data being inverted is a full wavefield of seismic data.

8. The method of claim 7, where the physical properties model is a model of at least one of a group consisting of 21 elastic constants including bulk modulus; density; or any combination of two or more of them.

9. A method for determining a discrete physical properties model of a subsurface region by iteratively inverting measured geophysical data acquired from the subsurface region, comprising using a Hessian matrix operating on an objective function, then times a vector, called "Hessian times vector," to determine an update for an initial model, wherein the Hessian times vector is approximated, using a computer, with a single forward wave simulated propagation and a single gradient of the objective function computation in a modified subsurface model, comprising:

15 forming a scaled vector by multiplying the vector by a number ε , where $0 < \varepsilon \leq 1$, and adding the scaled vector to the initial model;

simulating a first synthetic data set using the model with the scaled vector added to it;

forming the objective function to measure misfit between a second synthetic data set, simulated using the initial model, and said first synthetic data set;

20 computing a gradient of the formed objective function with respect to the physical property parameters, dividing the gradient by ε , and using that as an estimate of the Hessian times vector.

10. The method of claim 9, wherein the vector in a first iteration of the iterative inversion is gradient in model space of the objective function, wherein the objective function measures misfit between the measured geophysical data and corresponding model-simulated geophysical data.

11. A computer program product, comprising a non-transitory computer usable medium having a computer readable program code embodied therein, said computer readable program code adapted to be executed to implement a method for determining a discrete physical properties model of a subsurface region, which may be referred to as the model, by iteratively

inverting measured geophysical data acquired from the subsurface region, said method comprising:

5 using a Hessian matrix of an objective function, then times a vector, called "Hessian times vector," to determine an update for the model, wherein the Hessian times vector is approximated with a single forward-wave simulated propagation and a single computation of gradient of the objective function, in a modified subsurface model.

12. The method of claim 11, wherein the vector in a first iteration of the iterative inversion is gradient in model space of the objective function, wherein the objective function
10 measures misfit between the measured geophysical data and corresponding model-simulated geophysical data.

CLAIMS

1. A method for determining a discrete physical properties model of a subsurface region, referred to herein as the model or the subsurface model, by iteratively inverting measured geophysical data acquired from the subsurface region, comprising:

approximating a Hessian matrix of an objective function, then times a vector, called Hessian times vector, using a computer, with a single forward-wave simulated propagation and a single computation of gradient of the objective function, in a modified subsurface model, thereby requiring only three forward-wave or reverse-wave propagations;

wherein said approximation is based on an approximate equation for a Born scattered pressure field $\tilde{p}_{d,F}$ where the Hessian times vector is approximated by a gradient computation using $\tilde{p}_{d,F}$ as an artificial residual;

then computing a direction in model parameter space for an update to a current model by multiplying inverse of the Hessian matrix times a gradient of the objective function, wherein the inverse of the Hessian matrix is computed iteratively using a conjugate gradient method in which said approximate Hessian times vector is used to evaluate the Hessian matrix times a model perturbation vector;

performing a line search to determine magnitude of the model update using said computed direction; and

adding the model update to the current model to form an updated model, and using the updated model for geophysical prospecting.

2. The method of claim 1, wherein the modified subsurface model is a linear combination of a current subsurface model and the vector.

3. The method of claim 2, wherein the linear combination = the current subsurface model + {(a scalar constant) x (the vector)}, wherein $0 < \text{the scalar constant} \leq 1$.

4. The method of claim 3, wherein the scalar constant is chosen according to whether the Hessian matrix will be computed exactly, which means taking multiple scattering into account, or will be approximated by the Gauss-Newton Hessian matrix.

5. The method of claim 4, wherein if the Gauss-Newton approximation is to be used, the scalar constant is chosen such that $\{(a \text{ scalar constant}) \times (\text{the vector})\}$ is approximately 1 % in magnitude of the current subsurface model; but if the Hessian matrix is to be computed exactly, then the scalar constant is chosen so that the $\{(a \text{ scalar constant}) \times (\text{the vector})\}$ is approximately 10 % in magnitude of the current subsurface model.

6. The method of claim 3, wherein the vector in a first iteration of the method is gradient of the objective function with respect to parameters of the model, wherein the objective function measures misfit between the measured geophysical data and corresponding synthetic geophysical data simulated using the model, said misfit being a selected norm of a data residual, wherein the data residual is the difference between the measured geophysical data and corresponding synthetic geophysical data simulated using the model; provided that for said single computation of gradient of the objective function, the measured geophysical data in the data residual is replaced by synthetic data simulated using the modified subsurface model, and the data residual is divided by the scalar constant.

7. The method of claim 1, where the measured geophysical data being inverted is a full wavefield of seismic data.

8. The method of claim 7, where the physical properties model is a model of at least one of a group consisting of 21 elastic constants including bulk modulus; density; or any combination of two or more of them.

9. A method for determining a discrete physical properties model of a subsurface region by iteratively inverting measured geophysical data acquired from the subsurface region, comprising using a Hessian matrix operating on an objective function, then times a vector, called Hessian times vector, to determine an update for an initial model, wherein the Hessian times vector is approximated, using a computer, with a single forward wave simulated propagation and a single gradient of the objective function computation in a modified subsurface model thereby requiring only three forward-wave or reverse-wave propagations, comprising:

forming a scaled vector by multiplying the vector by a number ϵ , where $0 < \epsilon \leq 1$, and adding the scaled vector to the initial model;

simulating a first synthetic data set using the model with the scaled vector added to it;

forming the objective function to measure misfit between a second synthetic data set, simulated using the initial model, and said first synthetic data set;

computing a gradient of the formed objective function with respect to the physical property parameters, dividing the gradient by ϵ , and using that as an estimate of the Hessian times vector;

computing a direction in model parameter space for an update to the initial model by multiplying inverse of the Hessian matrix times a gradient of the objective function, wherein the inverse of the Hessian matrix is computed iteratively using a conjugate gradient method in which said estimate of the Hessian times vector is used to evaluate the Hessian matrix times a model perturbation vector;

performing a line search to determine magnitude of the model update using said computed direction; and

adding the model update to the initial model to form an updated model, and using the updated model for geophysical prospecting.

10. The method of claim 9, wherein the vector in a first iteration of the iterative inversion is gradient in model space of the objective function, wherein the objective function measures misfit between the measured geophysical data and corresponding model-simulated geophysical data.

11. A computer program product, comprising a non-transitory computer usable medium having a computer readable program code embodied therein, said computer readable program code adapted to be executed to implement a method for determining a discrete physical properties model of a subsurface region, referred to herein as the model or the subsurface model, by iteratively inverting measured geophysical data acquired from the subsurface region, said method comprising:

using a Hessian matrix of an objective function, then times a vector, called "Hessian times vector," to determine an update for the model, wherein the Hessian times vector is approximated with a single forward-wave simulated propagation and a single computation of gradient of the objective function, in a modified subsurface model, thereby requiring only three forward-wave or reverse-wave propagations;

wherein said approximation is based on an approximate equation for a Born scattered pressure field $\tilde{P}_{d,F}$ where the Hessian times vector is approximated by a gradient

computation using $\tilde{p}_{d,F}$ as an artificial residual;

then computing a direction in model parameter space for an update to a current model by multiplying inverse of the Hessian matrix times a gradient of the objective function, wherein the inverse of the Hessian matrix is computed iteratively using a conjugate gradient method in which said approximate Hessian times vector is used to evaluate the Hessian matrix times a model perturbation vector;

performing a line search to determine magnitude of the model update using said computed direction; and

adding the model update to the current model to form an updated model.

12. The method of claim 11, wherein the vector in a first iteration of the iterative inversion is gradient in model space of the objective function, wherein the objective function measures misfit between the measured geophysical data and corresponding model-simulated geophysical data.

13. The method of claim 1, wherein $\tilde{p}_{d,F}$ is approximated as

$$\tilde{p}_{d,F} = \frac{1}{\epsilon} (\tilde{p}_F - p_b),$$

where ϵ is a number smaller than a pre-determined value, p_b is a pressure modeled using a current subsurface model, and $\tilde{p}_F$ is the solution to a wave equation

$$\nabla^2 \tilde{p}_F(\mathbf{r}) - \frac{\rho}{K_b(\mathbf{r}) + \epsilon \tilde{K}(\mathbf{r})} \ddot{\tilde{p}}_F(\mathbf{r}) = -q(t) \delta(\mathbf{r} - \mathbf{r}_s).$$

14. The method of claim 13, wherein said approximation further includes taking $\tilde{p}_{d,F}$ to be equal to $\tilde{p}_d$, where $\tilde{p}_d$ is difference between measured pressure and the modeled pressure p_b .