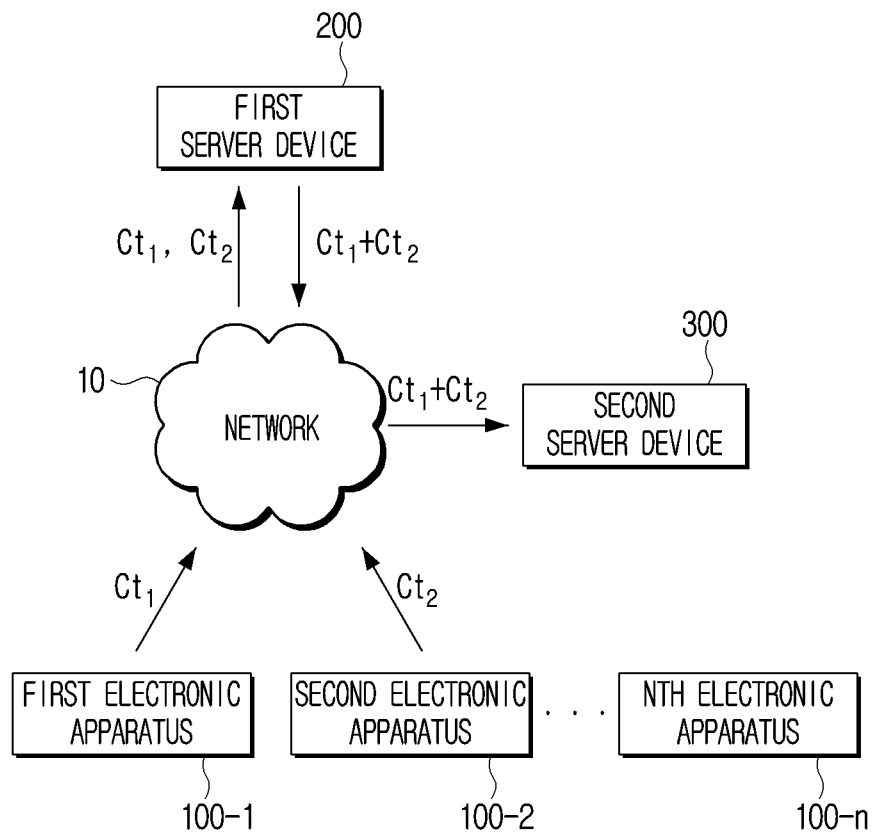


FIG. 1



## FIG. 2

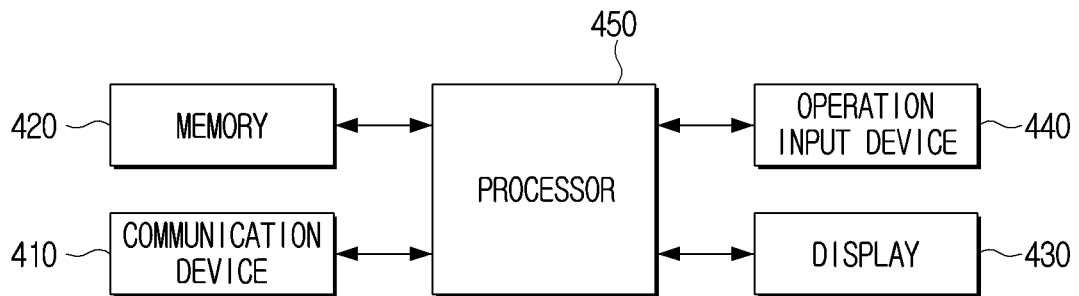
400

FIG. 3

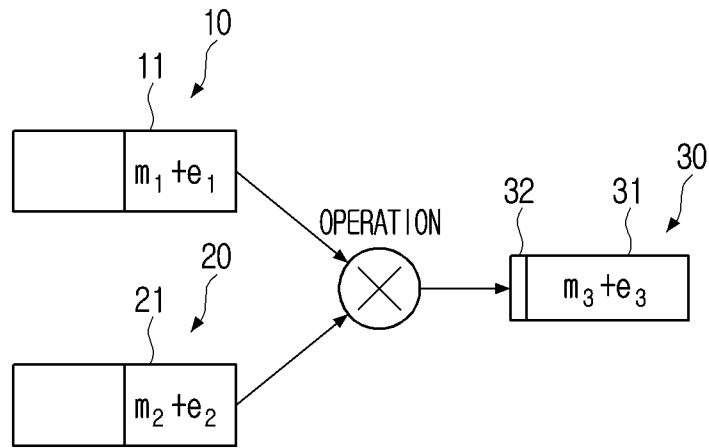


FIG. 4

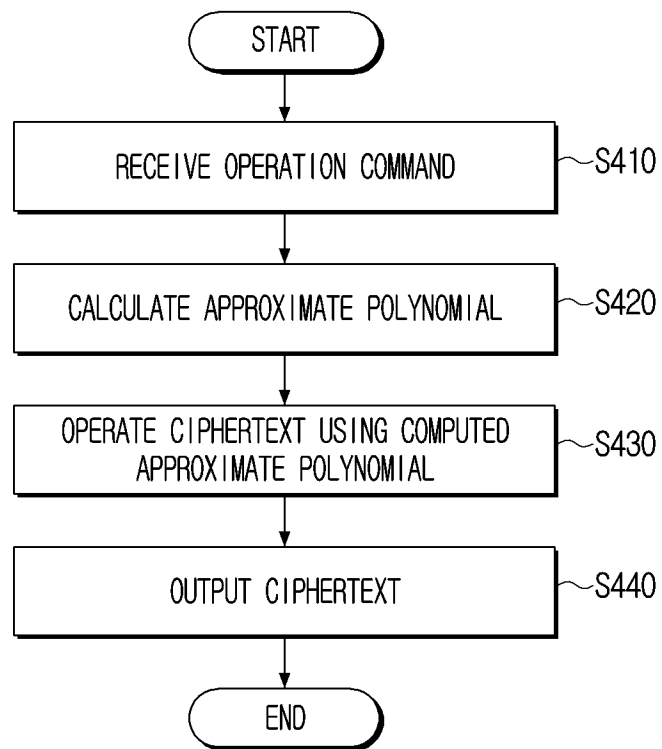


FIG. 5

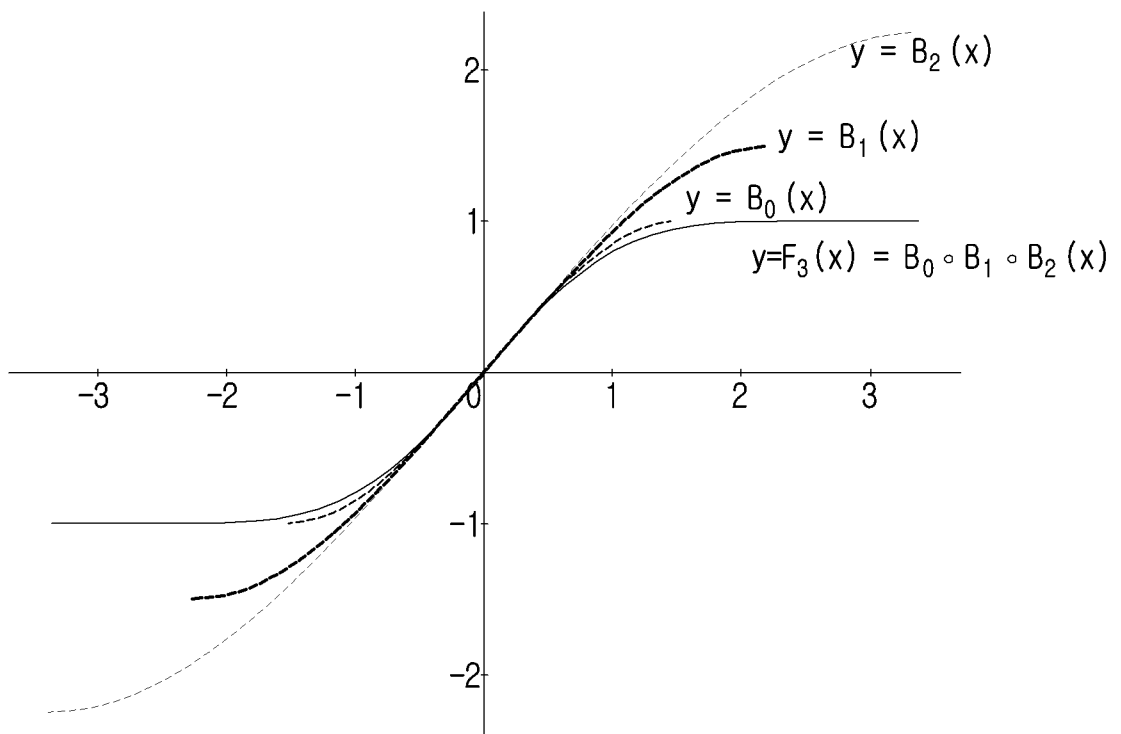
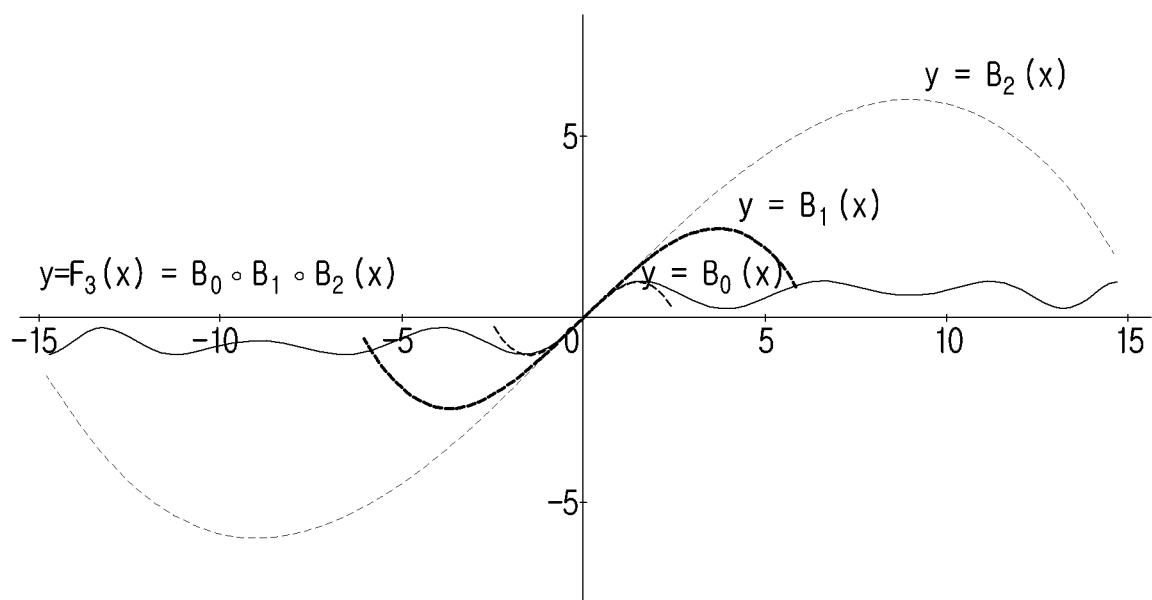


FIG. 6



# FIG. 7

---

**Algorithm 1** polyDEF  $F_{n,r}$  in Example 1

---

**Input:**  $x_0 \in [-1.5^n r, 1.5^n r]$

**Output:** Approximate  $r$ -DEF at  $x$ .

```

1:  $y \leftarrow \frac{x_0}{r}$ 
2: for  $i \leftarrow n - 1$  to 0 do
3:    $y \leftarrow y - \frac{4}{27} \frac{1}{2.25^n} y^3$ 
4: end for
5: return  $ry$ 

```

---

# FIG. 8

---

**Algorithm 2** Approximate sigmoid function in Example 2

---

**Input:** Approximation  $P(x)$  of sigmoid function on  $[-r, r]$   
and  $x_0 \in [-1.5^n r, 1.5^n r]$ .

**Output:** Approximate sigmoid function at  $x_0$ .

```

1:  $y \leftarrow \frac{x_0}{r}$ 
2: for  $i \leftarrow n - 1$  to 0 do
3:    $y \leftarrow y - \frac{4}{27} \frac{1}{2.25^n} y^3$ 
4: end for
5: return  $P(ry)$ 
```

---

# FIG. 9

---

**Algorithm 3** Approximate logistic function with wavy poly-DEF

---

**Input:**  $x \in [-2.45^n \cdot 14.5, 2.45^n \cdot 14.5]$ .

**Output:** Approximate logistic function at  $x$ .

//  $P(x)$  is the minimax polynomial of the logistic function on  $[-14.5, 14.5]$ .

1:  $y \leftarrow \frac{x}{14.5}$

2: **for**  $i \leftarrow n - 1$  **to** 0 **do**

3:    $y \leftarrow y - \frac{4}{27} \frac{1}{6.0025^n} y^3$

4: **end for**

5: **return**  $P(14.5y)$

---

# FIG. 10

---

**Algorithm 4** Approximate Logistic Regression
 

---

**Input:** Data  $D_i \in [0, 1]^d$  with label  $\ell_i \in \{-1, 1\} \forall i \in [1, M]$ , learning rate  $\alpha$ , number of iterations  $T$ , mini-batch size  $N$ .

**Output:** Trained logistic regression weight  $W$  and bias  $b$ .

```

1: for  $i \leftarrow 1$  to  $M$  do
2:    $z_i \leftarrow \ell_i \cdot (D_i, 1) \in [-1, 1]^{d+1}$ 
3: end for
4:  $\beta \leftarrow 0^{d+1}$ 
5: for iteration  $\leftarrow 1$  to  $T$  do
6:    $G \leftarrow 0^{d+1}$ 
7:   for  $i \leftarrow 1$  to  $N$  do
8:     Sample  $z$  from  $z_i$ 
9:      $x \leftarrow -z \cdot \beta$ 
10:    for  $j \leftarrow 1$  to  $\frac{\log(\sqrt{t(\alpha^2 d^2 + 0.6\alpha d)})}{\log 2.45}$  do
11:       $x \leftarrow x - \frac{4}{6.0025^n 27} x^3$ 
12:    end for
13:     $G \leftarrow G + P(x)z$  //  $P(x)$  is the degree 9 minimax polynomial of the logistic function on  $[-14.5, 14.5]$ 
14:  end for
15:   $\beta \leftarrow \beta + \frac{\alpha}{N} G$ 
16: end for
17:  $(b, W) \leftarrow \beta$ 
18: return  $(W, b)$ 

```

---