

GPS - AIDED INERTIAL NAVIGATION METHOD AND SYSTEM

FIELD OF THE INVENTION

The present invention concerns a multi-sensor (multi-IMU and multi-GPS) GPS-aided inertial navigation system and method.

BACKGROUND OF THE INVENTION

The subject of federated filtering, and specifically federated navigation with multiple IMUs, has received considerable attention over the past three decades. Reference [1] contains a broad systematic review of distributed Kalman filtering, not necessarily in the context of navigation. The seminal works of Carlson, references [2]-[6], have introduced decentralized/federated filtering ideas and techniques into the realm of navigation. These navigation approaches were thoroughly investigated at the USAF institute of technology, references [7]-[8]. Some more recent works on this subject are exemplified in references [9]-[11]. References [12]-[16] are books and papers on inertial navigation, and references [17]-[19] are the cornerstone paper on Kalman filtering and two well-known practical books on the subject. The basic kinematic and dynamic relationships between points within a rigid-body appear in numerous sources, e.g. reference [20].

However, regarding navigation, most if not all of the past research on multi-sensor federated navigation focused solely on its *filtering aspects*: central versus distributed/cascaded/federated filtering; the depth of information-sharing between the ‘master’ filter and the ‘local’ filters; states sizes and content (at ‘local’ and ‘master’ levels); with or without filters’ resets (‘local’/‘master’); and the correlation incurred between local filters due to common dynamics and common updates, etc. An important virtue of multi-sensor federated navigation, emphasized in most if not all previous research on the subject, is redundancy (fault tolerance) and fault detection/isolation capability, regarding both ‘hard’ and ‘soft’ failures.

SUMMARY OF THE INVENTION

The present system includes a mobile rigid body (‘vehicle’, e.g. aircraft) on which a plurality of inertial measurement units (IMUs) and one or more GPS antennas and receivers are fixedly

installed. A navigation computer (NC) completes the navigation hardware, and interfaces all the above hardware elements. The present GPS-aided inertial navigation system and method fuse the data from all available sensors to produce the navigation solution (position, velocity, attitude). It is assumed that one has already developed a single-IMU/single-GPS INS, and one wishes to modify it and create a multi-GPS/multi-IMU INS with minimal software changes and with minimal additional computational burden. These ‘minimality requirements’ practically exclude prior art federated navigation. Compared to a ‘baseline’ single-IMU/single-GPS inertial navigation system (INS), the present system and method provide (with IMUs and GPS receivers similar to the IMU and receiver of the baseline INS) better navigation accuracy with minimal additional algorithmic and computational complexity. The present method only minimally deviates from a single-IMU/single-GPS GPS-aided inertial navigation scheme, and results in performance and cost benefits associated with a multi-agent navigation system. The cost benefit stems from the fact that several relatively low-accuracy and inexpensive IMUs and GPS receivers can deliver navigation accuracy that otherwise (with a single-IMU/single-GPS INS) requires high-end and very expensive IMU and GPS receiver.

The present system and method provide a navigation approach that differs from past research and practice in several ways. First, there is only one navigation filter, and it practically is a basic ‘standard’ single-IMU/single-GPS navigation filter, except for minute filter-tuning changes. This filter does not have more states than a single-IMU/single-GPS navigation filter, and accommodates only one (fused) IN solution and one (fused) GPS solution, as if there were one IMU and one GPS receiver in the system. Further, despite including only a single ‘standard’ filter, the invention includes individual corrections and resets to all the IMUs and INs, and individual aidings to all the GPS receivers, similarly to the prior art, multi-filter federated navigation. And, the above is accomplished by intensive application of well-known rigid-body static, kinematic and dynamic internal relationships, rather than by yet another complex filtering scheme. Key to the individual sensor corrections is an innovative algorithm that computes a real-time least-squares solution for the body’s *angular acceleration*. Fault detection/isolation, not discussed here, can also be supported within the scope of the invention.

The computational burden and software complexity here are just a little greater than in a single-IMU/single-GPS INS, however both the burden and complexity are significantly smaller than with federated navigation, which is the prior art of navigating with multiple IMUs, where there are multiple filters, usually a dedicated filter per each IMU plus a central filter.

In accordance with embodiments of one aspect of the present invention there is provided a multi-sensor GPS-aided inertial navigation method. The method includes providing multiple sensors, including multiple inertial measurement units (IMUs) and one or more global positioning system receivers and antennas (GPSs); and providing a navigation computer (NC) with embedded navigation software. The below activities of the NC may be performed by a computer which is not navigation-dedicated, however has the required hardware interfaces and navigation software.

The NC performs the following activities: interfacing with all IMUs and all GPS receivers; running, in parallel, multiple standard inertial navigation (IN) schemes, one per each IMU; computing the mean of all the INs, thus obtaining a fused IN solution for the IMUs' mean location ('mean IMU' location); computing the mean of all the GPS solutions, thus obtaining a fused GPS solution for the GPS antennas' mean location ('mean antenna' location); applying standard lever-arm corrections, with the vector from the 'mean IMU' location to the 'mean antenna' location, to the fused GPS solution; feeding the fused IN solution and the lever-arm corrected fused GPS solution to a single navigation filter, as if there were a single IMU and a single GPS; and running an IMU/IN/GPS correction module. The correction module is based on rigid-body physics and computes appropriate individual: (i) acceleration and angular rate corrections for each IMU; (ii) position, velocity and attitude corrections for each IN, and (iii) position, velocity, attitude, acceleration and angular rate aiding-message for each GPS receiver.

In some embodiments, the method uses an inertial navigation (IN) scheme per each IMU.

In some embodiments, the method uses only one navigation filter to accommodate one fused IN solution and one fused GPS solution, as if there were a single IMU and a single GPS.

In some embodiments, all corrections and aiding-messages are performed individually for each sensor and IN-output.

In some embodiments, all the corrections and aiding-messages are computed by algorithms

based on rigid-body physics.

In some embodiments, the algorithms compute a real-time least-squares solution for the body's angular acceleration.

In some embodiments, individual IMU-corrections are computed as increments to prior corrections, and integrated.

In some embodiments, individual IMU-corrections are computed wholly, after each filter-update anew, without integrators.

In some embodiments, the computation of individual corrections to the sensors uses angular rate and angular acceleration lever-arm corrections, and filter-updated/lever-corrected linear accelerations at the individual IMUs' locations, to define either local IMU correction-increments (that undergo integration) or whole local IMU corrections.

In accordance with embodiments of another aspect of the present invention there is provided a GPS-aided inertial navigation system (INS). The system includes multiple IMUs, a NC with embedded navigation software, and at least one GPS receiver and antenna. The NC is configured to: (a) interface all IMUs and all GPS receivers; (b) run, in parallel, multiple standard inertial navigation (IN) schemes, one per each IMU; (c) compute the mean of all INs, thus obtaining a fused IN solution for the IMUs' mean location ('mean IMU' location); (d) compute the mean of all GPS solutions, thus obtaining a fused GPS solution for the GPS antennas' mean location ('mean antenna' location); (e) apply lever-arm corrections, with the vector from the 'mean IMU' location to the 'mean antenna' location, to the fused GPS solution; (f) feed the fused IN solution and the lever-arm corrected fused GPS solution to a single navigation filter, as if there were a single IMU and a single GPS; and (g) run an IMU/IN/GPS correction module. The correction module is based on rigid-body physic and computes appropriate individual (i) acceleration and angular rate corrections for each IMU, (ii) position, velocity and attitude corrections for each IN, and (iii) position, velocity, attitude, acceleration and angular rate aiding-messages for each GPS receiver.

ACRONYMS AND DEFINITIONS

LLN or *L* - the true Local-Level Local-North (LLN) frame, where *XYZ* is North-East-Down (NED)

ECEF or *E* - Earth-Centered Earth-Fixed (ECEF) frame

BODY or *B* - Body-fixed frame of the rigid body, where *XYZ* is forward-right-down

I - the Inertial frame

CG - Center of Gravity of the rigid body, and origin of *B*

IN - (standard) Inertial Navigation scheme

INS - (standard) Inertial Navigation System

IMU - Inertial Measurement Unit

GPS - Global Positioning System

NC - Navigation Computer

KF - Kalman filter

HW - Hardware

MC - Monte Carlo simulation

CEP - Circular Error Probable, the radius of the circle in which 50% of the results reside

R95 - The radius of the circle in which 95% of the results reside

DCM - Direction Cosines Matrix

1PPS - One Pulse Per Second

$\sim \mathcal{N}(\mu, R)$ - Normally (Gaussianly) distributed with mean μ and variance R

L_{IN} - the (erroneous) LLLN computed by the *IN*

C_M^N - DCM for transforming vectors from frame *M* to *N*

$C_L^B(\psi, \theta, \phi)$ - DCM for transforming vectors from some LLLN frame *L* (true or computed) to the body frame *B*, where the attitude of the body frame with respect to the LLLN frame is defined by the western standard Euler angles triplet ψ, θ, ϕ

$\Psi = [\Psi^R, \Psi^P, \Psi^Y]^T$ - the (minute) angles vector relating *L_{IN}* to *LLN*; the RPY (Roll-Pitch-

Yaw) attitude-errors of the IN

C_B^E - DCM for transforming vectors from the body frame B to ECEF frame E

PVT - Position (3-vectors), Velocity (3-vectors), Time

$PV\Theta A\Omega$ - Position, Velocity, attitude (Θ), Acceleration, and angular rate (Ω) (all 3-vectors)

$\Delta\theta, \Delta v$ - attitude and velocity increments (IMU outputs)

$()_f$ - $()$ fused

$()_X^Z$ - $()$ of entity X , in frame Z

$()_{YX}^Z$ - $()$ of entity X , relative to frame Y , in frame Z

Ω_{IB}^B - angular velocity (Ω) of frame B relative to frame I , expressed in frame B (the superscript)

$\mathbf{r}_{I_j}$ - the position-vector of IMU No. j ($j = 1, \dots, M$), in B (r_{IMU_j} in Fig. 3)

$\bar{\mathbf{r}}_I$ - the mean of $\mathbf{r}_{I_j}$, $j = 1, \dots, M$ ('mean IMU' location), in B ($r_{IMU^{mean}}$ in Fig. 3)

$\mathbf{r}_{A_i}$ - the position-vector of Antenna No. i ($i = 1, \dots, N$), in B (r_{ANT_i} in Fig. 3)

$\bar{\mathbf{r}}_A$ - the mean of $\mathbf{r}_{A_i}$, $i = 1, \dots, N$ ('mean antenna' location), in B ($r_{ANT^{mean}}$ in Fig. 3)

$\mathbf{r}_{IA}$ - vector in B from the 'mean IMU' location $\bar{\mathbf{r}}_I$ to the 'mean antenna' location $\bar{\mathbf{r}}_A$ ($r_{IMU^{mean}ANT^{mean}}$ in Fig. 3)

$\mathbf{r}_{IA_i}$ - vector in B from the 'mean IMU' location to the i^{th} antenna, $i = 1, \dots, N$ ($r_{IMU^{mean}ANT_i}$ in Fig. 3)

$\mathbf{r}_{II_j}$ - vector in B from the 'mean IMU' location to the j^{th} IMU, $j = 1, \dots, M$ ($r_{IMU^{mean}IMU_j}$, Fig. 3)

DESCRIPTION OF EMBODIMENTS OF THE INVENTION

THE BASELINE INS

The INS assumed to exist (Fig. 1), and to be at hand and well-recognized, includes a single standard IMU, which provides coning and sculling -corrected $\{\Delta\theta, \Delta v\}$; a single GPS receiver (with a single antenna), which provides $\{P, V, T\}$ fixes; and a navigation computer which runs both an inertial navigation (IN) scheme and a standard navigation filter with the 15 standard error states $\{PV\Theta A\Omega\}$. The filter customarily is a Kalman filter [KF], but may be some other filter, e.g. an unscented KF, an H_∞ filter, a particle filter, etc. The position and velocity GPS fixes are corrected for the lever-arm effect of the body-vector between the IMU and antenna locations. These lever-arm corrections are performed, in the standard fashion, before passing the fixes to the filter. The computer uses the 1PPS signal from the GPS receiver, together with the receiver's time message, to adapt its own clock to GPS time and identify the fixes' timings. The computer also sends sync signals to the IMU, to ensure timely IMU-output arrival. Filter outputs are used to correct (reset) both IN-inputs and IN-outputs. All navigation outputs (navigation filter-corrected full $PV\Theta A\Omega$ solution) are sent as 'aiding' to the GPS receiver, to facilitate satellites tracking and re-acquisition.

MAIN ELEMENTS OF THE INVENTION

The invention (Fig. 2) includes the following computation and inter-communication activities, which are fully defined in this document (except for the first two, hardware interfaces and inertial navigation computations, which are performed as in the baseline 'standard' INS):

- The navigation computer (NC) interfaces all IMUs in the same way the baseline NC interfaces its IMU, and all GPS receivers in the same way the baseline NC interfaces its GPS receiver;
- The NC runs (in parallel) M standard inertial navigation (IN) schemes, one per each IMU;
- The NC computes the mean of all INs, thus obtaining a *fused* IN solution for the IMUs' mean location;

- The NC computes the mean of all GPS solutions, thus obtaining a *fused* GPS solution for the GPS antennas' mean location;
- The NC feeds the fused IN and the fused GPS solutions (after correcting the fused GPS solution for the lever-arm between the 'mean IMU' and the 'mean antenna' locations) to a 'standard' (single-IMU/single-GPS) navigation filter (only re-tuned because of the smaller 'mean sensors' errors); and,
- The NC runs an individual IMU/IN/GPS correction module which, based on rigid-body physics, computes appropriate individual
 - $\{A, \Omega\}$ -correction for each IMU,
 - $\{P, V, \Theta\}$ -correction for each IN, and
 - $\{P, V, \Theta, A, \Omega\}$ -aiding message for each GPS receiver,

and applies these individual corrections/aidings to all IMUs, INs and GPS receivers.

GPS FUSION

With reference to Fig. 3, the mean location of the GPS antennas ('mean IMU' location, or 'IMUs center') in body axes is

$$\bar{\mathbf{r}}_A \triangleq \frac{1}{N} \sum_{i=1}^N \mathbf{r}_{A_i}. \quad (1)$$

POSITION-FIX FUSION

The true ECEF position of the i^{th} GPS receiver is $\mathbf{P}_{CG}^E(t) + C_B^E(t) \mathbf{r}_{A_i}$, where $\mathbf{P}_{CG}^E$ is the body's CG position in *ECEF*. Hence, a simple model of the i^{th} receiver position-fix in *ECEF* at time t , denoted $\mathbf{P}_{GPS_i}^E(t)$, is

$$\mathbf{P}_{GPS_i}^E(t) = \mathbf{P}_{CG}^E(t) + C_B^E(t) \mathbf{r}_{A_i} + \mathbf{v}_{P_i}^E(t) \quad (2)$$

where $\mathbf{v}_{P_i}^E$ is the position-fix error in *ECEF* of the i^{th} receiver, assumed here to be a zero-mean white Gaussian noise with covariance $R_{P_i}(t)$. All receivers are assumed to have the same position error statistics: $R_{P_i}(t) = R_P(t) \forall i$.

The *fused* GPS position-fix is obtained by summing equation (2) over the $i = 1, \dots, N$ available fixes and dividing by N :

$$\begin{aligned}
\bar{\mathbf{P}}_{GPS}^E &= \frac{1}{N} \sum_{i=1}^N \mathbf{P}_{GPS_i}^E(t) = \\
&\mathbf{P}_{CG}^E(t) + C_B^E(t) \frac{1}{N} \sum_{i=1}^N \mathbf{r}_{A_i} + \frac{1}{N} \sum_{i=1}^N \mathbf{v}_{P_i}^E(t) = \\
&\mathbf{P}_{CG}^E(t) + C_B^E(t) \bar{\mathbf{r}}_A + \bar{\mathbf{v}}_P^E(t)
\end{aligned} \tag{3}$$

where $\bar{\mathbf{v}}_P^E$ is the ‘mean GPS’ position-noise, whose covariance is $\bar{R}_P(t) = \frac{1}{N} R_P(t)$. Thus, for equal receivers, the mean of the receivers ECEF position-fixes represents a (fused) ECEF position-fix $\bar{\mathbf{P}}_{GPS}^E$ of the mean body location of the GPS antennas ($\bar{\mathbf{r}}_A$), and $\bar{\mathbf{P}}_{GPS}^E$ has an error covariance N times smaller than that of a single receiver. The latter fact affects the proposed navigation filter tuning. If the receivers are different, $\bar{R}_P^{-1} = \sum_{i=1}^N R_{P_i}^{-1}$ and $\bar{\mathbf{P}}_{GPS}^E = \bar{R}_P \sum_{i=1}^N (R_{P_i}^{-1} \mathbf{P}_{GPS_i}^E)$ should be used; $\bar{\mathbf{P}}_{GPS}^E$ then represents a covariance-weighted ‘antenna location’. Position-fix bias errors, e.g. ionospheric (residual) errors, are common to all the receivers and thus shift $\bar{\mathbf{P}}_{GPS}^E$.

VELOCITY-FIX FUSION

The i^{th} GPS receiver velocity-fix in *ECEF* at time t , denoted $\mathbf{V}_{GPS_i}^E(t)$, is

$$\begin{aligned}
\mathbf{V}_{GPS_i}^E(t) &= \mathbf{V}_{CG}^E(t) + C_B^E(t) (\boldsymbol{\Omega}_{EB}^B \times \mathbf{r}_{A_i}) + \mathbf{v}_{V_i}^E(t) \\
&= \mathbf{V}_{CG}^E(t) + C_B^E(t) [\boldsymbol{\Omega}_{EB}^B \times] \mathbf{r}_{A_i} + \mathbf{v}_{V_i}^E(t)
\end{aligned} \tag{4}$$

where $\mathbf{V}_{CG}^E$ is the body’s CG velocity in *ECEF* and $\mathbf{v}_{V_i}^E$ is the velocity-fix error (noise) in *ECEF* of the i^{th} receiver, assumed $\sim \mathcal{N}(0, R_{V_i})$. All receivers have the same velocity error statistics: $R_{V_i}(t) = R_V(t) \forall i$. Note that $\boldsymbol{\Omega}_{EB}^B = \boldsymbol{\Omega}_{IB}^B - \boldsymbol{\Omega}_{IE}^B$, where $\boldsymbol{\Omega}_{IE}$ is (the constant) earth rotation-rate, $\boldsymbol{\Omega}_{IE}^B$ is $\boldsymbol{\Omega}_{IE}$ in B , and $\boldsymbol{\Omega}_{IB}^B$ (the body’s inertial angular rate, in B) is measured (with errors) by the rate-gyros in the IMUs. The matrix $[\boldsymbol{\Omega} \times]$ is the skew-symmetric (also known as cross-product) form of the vector’s $\boldsymbol{\Omega} = [\Omega_X \ \Omega_Y \ \Omega_Z]^T$ cross product, and defined as

$$[\boldsymbol{\Omega} \times] \triangleq \begin{bmatrix} 0 & -\Omega_Z & \Omega_Y \\ \Omega_Z & 0 & -\Omega_X \\ -\Omega_Y & \Omega_X & 0 \end{bmatrix}. \tag{5}$$

The *fused* GPS velocity-fix is obtained by summing equation (4) over $i = 1, \dots, N$ and dividing by N :

$$\begin{aligned}
\bar{\mathbf{V}}_{GPS}^E &= \frac{1}{N} \sum_{i=1}^N \mathbf{V}_{GPS_i}^E(t) = \\
&\mathbf{V}_{CG}^E(t) + C_B^E(t) \boldsymbol{\Omega}_{EB}^B \times \left(\frac{1}{N} \sum_{i=1}^N \mathbf{r}_{Ai} \right) + \frac{1}{N} \sum_{i=1}^N \mathbf{v}_{V_i}^E(t) \\
&= \mathbf{V}_{CG}^E(t) + C_B^E(t) \boldsymbol{\Omega}_{EB}^B \times \bar{\mathbf{r}}_A + \bar{\mathbf{v}}_V^E(t) \\
&= \mathbf{V}_{CG}^E(t) + C_B^E(t) [\boldsymbol{\Omega}_{EB}^B \times] \bar{\mathbf{r}}_A + \bar{\mathbf{v}}_V^E(t)
\end{aligned} \tag{6}$$

where $\bar{\mathbf{v}}_V^E$ is the ‘mean GPS’ velocity-noise, of covariance $\bar{R}_V(t) = \frac{1}{N} R_V(t)$. Thus, the mean of the (equal) receivers ECEF velocity-fixes represents a (fused) ECEF velocity-fix $\bar{\mathbf{V}}_{GPS}^E$ for the mean antennas body location $\bar{\mathbf{r}}_A$, and this velocity-fix has an error covariance N times smaller than that of a single receiver (this affects the navigation filter tuning). If the receivers are different, $\bar{R}_V^{-1} = \sum_{i=1}^N R_{V_i}^{-1}$ and $\bar{\mathbf{V}}_{GPS}^E = \bar{R}_V \sum_{i=1}^N (R_{V_i}^{-1} \mathbf{V}_{GPS_i}^E)$ should be used.

IN FUSION

Before looking into IN fusion, note that each IN scheme is identical to the IN scheme of the baseline INS but is individually initialized, with its own best initially-known position and velocity, which may be significant for large bodies and non-rest initializations. If the body is ‘small’ and initially at rest, it may be practically acceptable to initialize all INs with the same position (and zero velocity). Attitude initialization is the same for all INs.

The $\{\Delta\theta, \Delta v\}$ output of each IMU is routed (after individual $\{A, \Omega\}$ -corrections, as in the baseline INS; the computation of the corrections will be described later) to a dedicated standard IN scheme. The $\{P, V, \Theta\}$ output of each IN is routed (after individual $\{P, V, \Theta\}$ -corrections, as in the baseline INS) to the IN fusion module; the latter also receives the corrected $\{\Delta\theta, \Delta v\}$. See Fig. 2.

Note that though individual errors of each IN will be addressed in this sub-section, and though each IN obviously ‘has’ an error covariance matrix, *no ‘local’ filters operate in the navigation method of this invention*. IN errors are discussed here just to show how the IMUs’-output noise parameters (that drive IN $\{P, V, \Theta\}$ -errors) should be used as process-noise parameters in the re-tuning of the (single, standard) navigation filter.

IN POSITION-OUTPUT FUSION

IN position-output fusion is similar to GPS position-fix fusion described above. Denote by $\mathbf{P}_{IN_j}^E$ the j^{th} IN position-output in *ECEF*, and denote by $\mathbf{e}_{\mathbf{P}_{IN_j}}^E$ its error. Then,

$$\mathbf{P}_{IN_j}^E(t) = \mathbf{P}_{CG}^E(t) + C_B^E(t) \mathbf{r}_{I_j} + \mathbf{e}_{\mathbf{P}_{IN_j}}^E(t). \quad (7)$$

The *fused* IN position-output $\bar{\mathbf{P}}_{IN}^E$ is obtained by summing equation (7) over the M INs and dividing by M :

$$\begin{aligned} \bar{\mathbf{P}}_{IN}^E &= \frac{1}{M} \sum_{j=1}^M \mathbf{P}_{IN_j}^E(t) = \\ &\mathbf{P}_{CG}^E(t) + C_B^E(t) \frac{1}{M} \sum_{j=1}^M \mathbf{r}_{I_j} + \frac{1}{M} \sum_{j=1}^M \mathbf{e}_{\mathbf{P}_{IN_j}}^E(t) \\ &= \mathbf{P}_{CG}^E(t) + C_B^E(t) \bar{\mathbf{r}}_I + \bar{\mathbf{e}}_{\mathbf{P}_{IN}}^E(t) \end{aligned} \quad (8)$$

where $\bar{\mathbf{e}}_{\mathbf{P}_{IN}}^E \triangleq \frac{1}{M} \sum_{j=1}^M \mathbf{e}_{\mathbf{P}_{IN_j}}^E$ is the ‘mean IN’ position-error. Thus, $\bar{\mathbf{P}}_{IN}^E$ represents a (fused) ECEF position-solution for the mean body location of the IMUs ($\bar{\mathbf{r}}_I$), and its (un-named) error covariance is about M times smaller than that of a single IN. See reference [10] for a detailed discussion.

IN VELOCITY-OUTPUT FUSION

Similarly,

$$\mathbf{V}_{IN_j}^E(t) = \mathbf{V}_{CG}^E(t) + C_B^E(t) [\boldsymbol{\Omega}_{EB}^B \times] \mathbf{r}_{I_j} + \mathbf{e}_{\mathbf{V}_{IN_j}}^E(t) \quad (9)$$

$$\begin{aligned} \bar{\mathbf{V}}_{IN}^E &= \frac{1}{M} \sum_{j=1}^M \mathbf{e}_{\mathbf{V}_{IN_j}}^E(t) = \\ &\mathbf{V}_{CG}^E(t) + C_B^E(t) \boldsymbol{\Omega}_{EB}^B \times \left(\frac{1}{M} \sum_{j=1}^M \mathbf{r}_{I_j} \right) + \frac{1}{M} \sum_{j=1}^M \mathbf{e}_{\mathbf{V}_{IN_j}}^E(t) \\ &= \mathbf{V}_{CG}^E(t) + C_B^E(t) [\boldsymbol{\Omega}_{EB}^B \times] \bar{\mathbf{r}}_I + \bar{\mathbf{e}}_{\mathbf{V}_{IN}}^E(t) \end{aligned} \quad (10)$$

Here $\bar{\mathbf{e}}_{\mathbf{V}_{IN}}^E \triangleq \frac{1}{M} \sum_{j=1}^M \mathbf{e}_{\mathbf{V}_{IN_j}}^E$ is the ‘mean IN’ velocity-error and $\bar{\mathbf{V}}_{IN}^E$ represents a (fused) ECEF velocity-solution for $\bar{\mathbf{r}}_I$. The fused IN velocity error covariance is about M times smaller than that of a single IN.

IN ATTITUDE-OUTPUT FUSION

The *true* $C_{LLN}^B(\psi, \theta, \phi)$ applies over the whole body, however each IN outputs a slightly different *computed* C_{LIN}^B . For the j^{th} IN, $j = 1, \dots, M$, the following relation between the IN’s attitude-outputs $\{\psi_j, \theta_j, \phi_j\}$, its attitude-errors $\{\Psi^{R_j}, \Psi^{P_j}, \Psi^{Y_j}\}$ and the true Euler angles $\{\psi, \theta, \phi\}$ holds (see Fig. 4):

$$\begin{aligned}
C_{LLLN}^B(\psi, \theta, \phi) &= \\
C_{LIN_j}^B(\psi_j, \theta_j, \phi_j) C_{LLLN}^{LIN_j}(\Psi^{R_j}, \Psi^{P_j}, \Psi^{Y_j}) &\implies \\
C_B^{LIN_j} C_{LLLN}^B &= C_{LLLN}^{LIN_j}(\Psi^{R_j}, \Psi^{P_j}, \Psi^{Y_j}). \tag{11}
\end{aligned}$$

By approximating the well-known full Euler-angles DCM formulation to minute angles, C_{LLLN}^{LIN} can be expressed using $[\Psi^R, \Psi^P, \Psi^Y]^T$ (each normally a few milli-radians) as

$$C_{LLLN}^{LIN} \cong \begin{bmatrix} 1 & \Psi^Y & -\Psi^P \\ -\Psi^Y & 1 & \Psi^R \\ \Psi^P & -\Psi^R & 1 \end{bmatrix}. \tag{12}$$

Definition (5) and the latter approximation lead to the well-known result

$$C_{LLLN}^{LIN} \cong I - [\Psi \times]. \tag{13}$$

Hence, equation (11) can be well approximated as

$$C_B^{LIN_j}(\psi_j, \theta_j, \phi_j) C_{LLLN}^B(\psi, \theta, \phi) \cong I - [\Psi_j \times]. \tag{14}$$

Summing over j and dividing by M :

$$\begin{aligned}
\frac{1}{M} \sum_{j=1}^M C_B^{LIN_j}(\psi_j, \theta_j, \phi_j) C_{LLLN}^B(\psi, \theta, \phi) &\cong \\
\frac{1}{M} \sum_{j=1}^M (I - [\Psi_j \times]) &= I - \frac{1}{M} \sum_{j=1}^M [\Psi_j \times] = \\
I - [(\frac{1}{M} \sum_{j=1}^M \Psi_j) \times] &\implies
\end{aligned}$$

$$\bar{C}_B^{LIN} C_{LLLN}^B \cong I - [\bar{\Psi} \times]. \tag{15}$$

$\bar{C}_B^{LIN} \triangleq \frac{1}{M} \sum_{j=1}^M C_B^{LIN_j}$ is the mean of the DCM-outputs of all the INs and $\bar{\Psi} \triangleq \frac{1}{M} \sum_{j=1}^M \Psi_j$ is the INs' mean attitude-error (small-angles vector). Therefore, the fused attitude-output of the INs, $\bar{C}_B^{LIN}$, has an error covariance about M times smaller than that of a single IN. A standard ortho-normalization (see reference [15] section 7.1.1.3) on $\bar{C}_B^{LIN}$ is required.

IN ANGULAR RATE FUSION

The *true* Ω_{IB}^B applies over the whole body, however each IMU outputs a slightly different *measured* Ω_j , or $\Delta\theta_j$. The measurement model is

$$\Omega_j(t) = \Omega_{IB}^B(t) + \mathbf{e}_{\Omega_{IN_j}}(t). \tag{16}$$

The *fused* IN angular-rate output is the average of all Ω_j s:

$$\begin{aligned}\bar{\Omega}_{IN} &= \frac{1}{M} \sum_{j=1}^M \Omega_j(t) = \\ \Omega_{IB}^B(t) + \frac{1}{M} \sum_{j=1}^M \mathbf{e}_{\Omega_{IN_j}}(t) &= \Omega_{IB}^B(t) + \bar{\mathbf{e}}_{\Omega_{IN}}(t).\end{aligned}\tag{17}$$

$\bar{\mathbf{e}}_{\Omega_{IN}}$ is the ‘mean IN’ angular-rate error; it has a covariance M times smaller than that of a single IN.

IN ACCELERATION FUSION

A fundamental physical fact about rigid bodies (see e.g. reference [20]) is that the inertial linear acceleration expressed in the body frame of a *fixed* arbitrary point D in the body, $\mathbf{a}_{ID}^B$, defined by the fixed position-vector $\mathbf{r}_D = [x, y, z]^T$ in frame B , where the origin O of B is the body’s CG, is related to the inertial linear acceleration of the CG, $\mathbf{a}_{IO}^B$, through the body’s inertial angular velocity Ω_{IB}^B and the body’s inertial angular acceleration $\dot{\Omega}_{IB}^B$:

$$\begin{aligned}\mathbf{a}_{ID}^B &= \mathbf{a}_{IO}^B + \Omega_{IB}^B \times \Omega_{IB}^B \times \mathbf{r}_D + \dot{\Omega}_{IB}^B \times \mathbf{r}_D \\ &= \mathbf{a}_{IO}^B + ([\Omega_{IB}^B \times][\Omega_{IB}^B \times] + [\dot{\Omega}_{IB}^B \times]) \mathbf{r}_D.\end{aligned}\tag{18}$$

Accelerometers measure inertial *sensed* acceleration ($\mathbf{sa}$), not true inertial acceleration ($\mathbf{a}$); these two types of acceleration are related by (see reference [15] section 4.3)

$$\mathbf{sa} = \mathbf{a} - \frac{\mathbf{G}}{m}\tag{19}$$

where $\mathbf{G}$ is the gravitational force acting on the body’s total mass m . By subtracting $\mathbf{G}/m$ from both sides of equation (18) it is evident that equation (18) holds also for the *sensed* accelerations $\mathbf{sa}_{ID}^B$ and $\mathbf{sa}_{IO}^B$.

By the above observations, the j^{th} IMU (or IN) sensed acceleration *measurement* model is

$$\begin{aligned}\mathbf{sa}_j &= (\mathbf{sa}_{IB}^B)_j + \mathbf{e}_{\mathbf{sa}_{IN_j}} = \\ \mathbf{sa}_{IO}^B + ([\Omega_{IB}^B \times][\Omega_{IB}^B \times] + [\dot{\Omega}_{IB}^B \times]) \mathbf{r}_{I_j} &+ \mathbf{e}_{\mathbf{sa}_{IN_j}}\end{aligned}\tag{20}$$

where both Ω_{IB}^B and $\dot{\Omega}_{IB}^B$ apply over the whole body. Thus, when equation (20) is averaged over the M IMUs the following result is obtained for the fused acceleration $\bar{\mathbf{sa}}_{IN}$ valid at $\bar{\mathbf{r}}_I$:

$$\begin{aligned}\bar{\mathbf{sa}}_{IN} &= \frac{1}{M} \sum_{j=1}^M \mathbf{sa}_j = \\ \mathbf{sa}_{IO}^B + ([\Omega_{IB}^B \times][\Omega_{IB}^B \times] + [\dot{\Omega}_{IB}^B \times]) \bar{\mathbf{r}}_I &+ \bar{\mathbf{e}}_{\mathbf{sa}_{IN}}\end{aligned}\tag{21}$$

where $\bar{\mathbf{e}}_{sa/IN} = \frac{1}{M} \sum_{j=1}^M \mathbf{e}_{sa/IN_j}$ is the ‘mean IN’ sensed acceleration error, and its covariance is M times smaller than that of $\mathbf{e}_{sa/IN_j}$, the sensed acceleration error of IN_j . In practice, IMUs output (for navigation) velocity and attitude *increments*, and results similar to eqs. (21) and (17) hold.

NAVIGATION FILTER TUNING AND OPERATION

The IN fusion discussion above leads to the following conclusions regarding filter tuning (determination of the matrices $P0$, Q and R , used in the filter) in the proposed scheme:

- $P0$: The $\{P, V, \Theta\}$ terms should reflect the (fused) navigation initialization accuracy. Each IN is individually initialized, but all position initializations are practically derived from one given position, with lever corrections to the IMUs, and all attitude initializations are equal to a given attitude. (If the body is not at rest, there will be $\Omega \times \mathbf{r}$ individual velocity ‘corrections’ added to one given velocity). Thus, the P, V, Θ terms of the ‘fused $P0$ ’ should equal those of the baseline INS. The $\{A, \Omega\}$ terms, however, should be $\frac{1}{M}$ of the respective parameters of a single IMU.
- Q : Since for all IN terms it was found that their error covariance is about M times smaller than that of the baseline IN, and since Q represents the covariances of the random processes that *drive* the IN’s error covariance, the ‘fused Q ’ should be $\frac{1}{M}$ of the baseline Q .
- R : The ‘fused R ’ should be $\frac{1}{N}$ of the baseline R .

The fused IN represents $\bar{\mathbf{r}}_I$ and the fused GPS represents $\bar{\mathbf{r}}_A$. Thus, the GPS data should, before being input to the navigation filter, undergo standard position and velocity lever-corrections using $\bar{\mathbf{r}}_{IA}$ (instead of the lever from the single IMU to the single GPS antenna, used in the baseline INS).

Both the a posteriori (after GPS-update) filter-state and the full-state navigation output apply *only* to the body-point defined by $\bar{\mathbf{r}}_I$ (except attitude, which applies throughout the body). Since all the sensors are located *elsewhere*, computation of *individual* IMU/IN corrections and *individual* GPS-aiding requires special attention (see below).

Details regarding the *standard* computation of the $\{P, V, \Theta\}$ -reset in the navigation filter will not be described here. Note that in the present method (similarly to the baseline navigation

scheme), after each GPS-update the integrators for the ‘fused $\{A, \Omega\}$ -corrections’ are incremented, and then deducted from the current *a priori* fused $\{A, \Omega\}$, which are sums of the current *individual corrected* $\{A, \Omega\}$, to obtain ‘fused filter-corrected’ (*aposteriori*) $\{A, \Omega\}$.

INDIVIDUAL IMU/IN CORRECTIONS COMPUTATION

The inertial navigation (IN) module within a *standard* INS is traditionally associated with two kinds of ‘corrections’, both performed at the GPS-fix frequency, which is the filter-update rate (and often also the filter-propagation rate), and both are performed *after each filter-update*.

Note that the $\{\Delta\theta, \Delta v\}$ output of the IMU is *always* corrected before being delivered to the IN; the correction is applied at the IN computation-rate (= IMU-output rate). The $\{\Delta\theta, \Delta v\}$ correction terms are *integrals* of the filter-states regarding $\{A, \Omega\}$ (Fig. 1). The filter-state usually ‘exists’ only at filter-updates: the state is computed at the filter-update stage, used for IN-corrections, and immediately reset to zero since the IN-corrections carry all the filter’s state information.

So, in a *standard* INS, after each filter-update:

- The IN’s $\{P, V, \Theta\}$ output is being ‘reset’, that is overridden, by the corresponding full-state $\{P, V, \Theta\}$ -output of the whole INS (which, at that instant, already optimally incorporates the GPS-update information).
- The above-mentioned $\{\Delta\theta, \Delta v\}$ IMU-rate correction terms are integrated, that is incremented by the $\{A, \Omega\}$ parts of the *aposteriori* filter-state.

In the *present* method, all the above items are *individually* performed for each IN. See Fig. 2 and the below description.

INDIVIDUAL IN OUTPUT RESET

- P : Since the filter-updated full-state position-output of the navigation scheme ($\mathbf{P}_{\mathbf{r}_I}^E$) applies at $\bar{\mathbf{r}}_I$, and the IMUs are located at $\mathbf{r}_{I_j}$, the ‘reset position’ for the j^{th} IMU is

$$\mathbf{P}_{IN_j}^{E(+)} = \mathbf{P}_{\mathbf{r}_I}^E + \hat{C}_B^E \mathbf{r}_{II_j} \quad (22)$$

where $\hat{C}_B^E$ is the filter-updated DCM-output of the navigation system and (+) indicates (individual) after-reset data.

- V :

$$\mathbf{V}_{IN_j}^{E(+)} = \mathbf{V}_{r_I}^E + \hat{C}_B^E [\hat{\boldsymbol{\Omega}} \times] \mathbf{r}_{II_j} \quad (23)$$

where $\mathbf{V}_{r_I}^E$ and $\hat{\boldsymbol{\Omega}}$ are the velocity and rate filter-updated full-state outputs of the navigation system.

- Θ :

$$C_B^{L_{IN_j}(+)} = \hat{C}_B^E, \quad \forall j. \quad (24)$$

INDIVIDUAL IMU OUTPUT CORRECTION

Correcting $\Delta\theta$ is based on the fact that $\hat{\boldsymbol{\Omega}}$ is the best-known current angular rate at *all* body-points, thus we wish to attain

$$\boldsymbol{\Omega}_j^{(+)} = \hat{\boldsymbol{\Omega}}, \quad \forall j \quad (25)$$

by manipulating the individual $\Delta\theta$ (or $\boldsymbol{\Omega}$) correction terms. There are two ways to attain this: either directly define the *aposteriori* ‘full’ individual $\boldsymbol{\Omega}$ -correction as the difference between the current individual *raw* rate-measurement and $\hat{\boldsymbol{\Omega}}$, thus dispensing with the integrators regarding the rates (more precisely, the rate-error estimation); or maintain rate integrators and define the individual rate-error estimation *increment* as the difference between the *apriori* local *corrected* rate and $\hat{\boldsymbol{\Omega}}$ (this is the option applied in the example). Note that both ways *differ from the standard practice* (for a single IMU) of incrementing integrators with the appropriate filter-update states, and both lead to ‘noisier’ (however individual) IMU error-estimations.

Individual Δv (or $\mathbf{A}$) corrections are trickier, since according to equation (18) computation of the linear acceleration at a given body-point, given the linear acceleration of the CG, requires knowing the *angular acceleration* $\dot{\boldsymbol{\Omega}}_{IB}^B$, which is neither measured nor calculated in INSs. Up to here, it was sufficient to just know that $\dot{\boldsymbol{\Omega}}_{IB}^B$ is the same for the whole body.

One may try to somehow estimate $\dot{\boldsymbol{\Omega}}_{IB}^B$, for example by a filter based on $\hat{\boldsymbol{\Omega}}$. This approach incurs a detrimental time-lag. Instead, the present method applies a novel version of a known technique (see e.g. references [22],[23]), that will now be presented.

First, re-write equation (18) by switching sides for $\mathbf{a}_{ID}^B$ and $\mathbf{a}_{IO}^B$:

$$\mathbf{a}_{IO}^B = \mathbf{a}_{ID}^B - ([\boldsymbol{\Omega}_{IB}^B \times][\boldsymbol{\Omega}_{IB}^B \times] + [\dot{\boldsymbol{\Omega}}_{IB}^B \times]) \mathbf{r}_D.$$

Since the latter holds for *any* $\mathbf{r}_D$, we have for $\mathbf{r}_1^B$ and $\mathbf{r}_2^B$

$$\begin{aligned} (\mathbf{a}_{IB}^B)_1 - ([\boldsymbol{\Omega}_{IB}^B \times][\boldsymbol{\Omega}_{IB}^B \times] + [\dot{\boldsymbol{\Omega}}_{IB}^B \times]) \mathbf{r}_1^B &= \\ (\mathbf{a}_{IB}^B)_2 - ([\boldsymbol{\Omega}_{IB}^B \times][\boldsymbol{\Omega}_{IB}^B \times] + [\dot{\boldsymbol{\Omega}}_{IB}^B \times]) \mathbf{r}_2^B, \end{aligned}$$

whence

$$(\mathbf{a}_1 - \mathbf{a}_2)_{IB}^B - ([\boldsymbol{\Omega}_{IB}^B \times][\boldsymbol{\Omega}_{IB}^B \times] + [\dot{\boldsymbol{\Omega}}_{IB}^B \times]) (\mathbf{r}_1^B - \mathbf{r}_2^B) = 0. \quad (26)$$

Examining equation (26) in our context, all IMU-locations are known and the IMUs measure accelerations and angular rates. Thus, (26) *applied to any two IMUs* is a set of three linear equations from which the angular acceleration 3-vector $\dot{\boldsymbol{\Omega}}_{IB}^B$ can be extracted, as follows: (26) $\implies$

$$[\dot{\boldsymbol{\Omega}}_{IB}^B \times](\mathbf{r}_1^B - \mathbf{r}_2^B) = (\mathbf{a}_1 - \mathbf{a}_2)_{IB}^B - [\boldsymbol{\Omega}_{IB}^B \times][\boldsymbol{\Omega}_{IB}^B \times](\mathbf{r}_1^B - \mathbf{r}_2^B).$$

Note that the RHS above is completely known. The LHS can be simplified by noting that for any two vectors $\mathbf{v}_1, \mathbf{v}_2$

$$\mathbf{v}_1 \times \mathbf{v}_2 = [\mathbf{v}_1 \times] \mathbf{v}_2 = -\mathbf{v}_2 \times \mathbf{v}_1 = -[\mathbf{v}_2 \times] \mathbf{v}_1.$$

Hence,

$$\begin{aligned} [\dot{\boldsymbol{\Omega}}_{IB}^B \times] (\mathbf{r}_1^B - \mathbf{r}_2^B) &= -[(\mathbf{r}_1^B - \mathbf{r}_2^B) \times] \dot{\boldsymbol{\Omega}}_{IB}^B \implies \\ [(\mathbf{r}_1^B - \mathbf{r}_2^B) \times] \dot{\boldsymbol{\Omega}}_{IB}^B &= [\boldsymbol{\Omega}_{IB}^B \times][\boldsymbol{\Omega}_{IB}^B \times](\mathbf{r}_1^B - \mathbf{r}_2^B) - (\mathbf{a}_1 - \mathbf{a}_2)_{IB}^B. \end{aligned} \quad (27)$$

Now, equation (27) in shorthand is $C_{(1,2)} \dot{\boldsymbol{\Omega}}_{IB}^B = D_{(1,2)}$ (the meaning of $C_{(1,2)}$ and $D_{(1,2)}$ is obvious), recognized as the familiar form $C\mathbf{x} = D$ where the vector $\mathbf{x}$ is the unknown and the matrix C and vector D are given.

In order to actually apply equation (27), use any two IMU locations $\mathbf{r}_{I_k}$ and $\mathbf{r}_{I_l}$, where $k, l \in \{1, \dots, M\}$; replace $\mathbf{a}_1$ and $\mathbf{a}_2$ by the *sensed* (measured) accelerations $s\mathbf{a}_k$ and $s\mathbf{a}_l$; and replace $\boldsymbol{\Omega}_{IB}^B$ by the best-known current body-rate estimation $\hat{\boldsymbol{\Omega}}$:

$$[(\mathbf{r}_{I_k} - \mathbf{r}_{I_l}) \times] \dot{\boldsymbol{\Omega}}_{IB}^B = [\hat{\boldsymbol{\Omega}} \times][\hat{\boldsymbol{\Omega}} \times](\mathbf{r}_{I_k} - \mathbf{r}_{I_l}) - (s\mathbf{a}_k - s\mathbf{a}_l). \quad (28)$$

Since $s\mathbf{a}_k$ and $s\mathbf{a}_l$ and even $\hat{\boldsymbol{\Omega}}$ contain biases and noises, the solution of equation (28) for $\dot{\boldsymbol{\Omega}}_{IB}^B$ is erroneous to some extent. In order to minimize this error it is proposed to gather the available data from *all* the IMUs and obtain a minimum-variance *least-squares* solution for $\dot{\boldsymbol{\Omega}}_{IB}^B$, as follows.

Equation (28) can be written with *any two* IMUs. Apply eq. (28) to *all* possible IMU-pairs, concatenate the $q = M(M-1)/2$ resulting equations into one over-determined equation, and solve it by a least-squares procedure. The details: shorthand (28) as $\tilde{C}_{(k,l)} \dot{\mathbf{\Omega}}_{IB}^B = \tilde{D}_{(k,l)}$, or $\tilde{C}_p \dot{\mathbf{\Omega}}_{IB}^B = \tilde{D}_p$, where $p \in \{1, \dots, q\}$ is one *pair* of IMUs. The concatenated equation is then

$$\tilde{C} \dot{\mathbf{\Omega}}_{IB}^B = \tilde{D}, \quad \tilde{C}_{3q \times 3} = \begin{bmatrix} \tilde{C}_1 \\ \vdots \\ \tilde{C}_q \end{bmatrix}, \quad \tilde{D}_{3q \times 1} = \begin{bmatrix} \tilde{D}_1 \\ \vdots \\ \tilde{D}_q \end{bmatrix}. \quad (29)$$

The least-squares solution of equation (29) is $\hat{\dot{\mathbf{\Omega}}} = (\tilde{C}^T \tilde{C})^{-1} \tilde{C}^T \tilde{D}$. Note that the matrix inversion only involves a 3×3 matrix. Since there is a simple closed-form analytic way to invert a 3×3 matrix, the matrix inversion and the whole least-squares solution are *not* computation-intensive.

To finalize, now that the body's angular acceleration is known it is possible to compute the individual IMU-output *A*-correction. Since equation (28) applies to *any* two body-points, choose point l to be the 'mean IMU', choose point k to be the j^{th} IMU, and solve for $\mathbf{sa}_j$:

$$\begin{aligned} \mathbf{sa}_j &= [\hat{\mathbf{\Omega}} \times] [\hat{\mathbf{\Omega}} \times] (\mathbf{r}_{I_j} - \bar{\mathbf{r}}_I) + \mathbf{sa}_I - [(\mathbf{r}_{I_j} - \bar{\mathbf{r}}_I) \times] \hat{\mathbf{\Omega}} \\ &= [\hat{\mathbf{\Omega}} \times] [\hat{\mathbf{\Omega}} \times] (\mathbf{r}_{II_j}) + \mathbf{sa}_I - [(\mathbf{r}_{II_j}) \times] \hat{\mathbf{\Omega}} \\ &= \mathbf{sa}_I + ([\hat{\mathbf{\Omega}} \times] [\hat{\mathbf{\Omega}} \times] + [\hat{\mathbf{\Omega}} \times]) \mathbf{r}_{II_j}. \end{aligned} \quad (30)$$

Here, $\mathbf{sa}_I$ is the fused filter-corrected (*a posteriori*) sensed acceleration at $\bar{\mathbf{r}}_I$ (see the last paragraph in subsection "NAVIGATION FILTER TUNING AND OPERATION" above).

Individual acceleration-corrections can now be computed in two ways (see the two individual *rate*-correction ways in the paragraph following equation (25)): with or without integrators.

An individual error-estimation *A-increment* applicable to the IMU at $\mathbf{r}_{I_j}$ can be defined as the difference between the *a priori* local *corrected* acceleration and the *a posteriori* 'full' $\mathbf{sa}_j$ of equation (30). Each individual *A*-integral is advanced, after a GPS-update, by its applicable individual *A*-increment. This is the approach applied in the example below. Alternatively, define the *a posteriori* 'full' individual *A*-correction as the difference between the current individual *raw* acceleration-measurement (=IMU-output) and $\mathbf{sa}_j$, and dispense with the integrators regarding the acceleration-errors estimation.

Both ways *differ on two counts from the standard practice* (for a single IMU): because of the angular rate/acceleration lever-arm correction terms, and because of the use of local ‘best-known’ aposteriori acceleration ($\mathbf{sa}_j$) to define the local increments (for the integrators), rather than using filter-update states. The resulting A -error estimations are ‘noisier’ than customary for a single IMU, however truly individual.

INDIVIDUAL GPS-AIDING COMPUTATION

The $\{P, V, \Theta, \Omega\}$ individual aiding items to the GPS receivers, which can be sent at IN-rate, are computed similarly to equations (22)-(25) except with $\mathbf{r}_{IA_i}$ in lieu of $\mathbf{r}_{II_j}$. The A -aiding individual items are computed by (see equation (30))

$$\mathbf{sa}_i = \mathbf{sa}_I + ([\hat{\mathbf{\Omega}} \times][\hat{\mathbf{\Omega}} \times] + [\hat{\mathbf{\dot{\Omega}}} \times]) \mathbf{r}_{IA_i}. \quad (31)$$

DURING GPS OUTAGE

When GPS signals become unavailable, after having been available for some time during the navigation, the system continues navigating like any standard INS, that is with pure-inertial navigation using the latest (and ‘frozen’) IMU-corrections. In fact, each IN within the system individually does that, and IN fusion goes on regardless of GPS status.

With GPS data, navigation performance regarding the ‘mean IMU’ location (or ‘IMUs center’) $\bar{\mathbf{r}}_I$ is much better than that of the baseline INS (regarding its single IMU location). In fact, it is up to about $\sqrt{MN}$ times better due just to the reduced noises; and the individual sensor-corrections significantly improve upon that. During GPS *outage*, navigation is better than that of the baseline INS both because residual IMUs’ errors are smaller and because the ‘initial conditions’ of the pure-inertial navigation are more accurate.

However, the invention has an additional useful feature that is applicable during GPS outages: though navigation accuracy unavoidably deteriorates during the outage, the *fused* navigation solution is always significantly *better* than that of any *individual* IN within the system. Thus, *individual IN-output reset* (see equations (22)-(24)) *can still be performed*, for improved $\{P, V, \Theta\}$

navigation data regarding *all* $\mathbf{r}_{I_j}$. ($\mathbf{P}_{\mathbf{r}_I}^E$, $\mathbf{V}_{\mathbf{r}_I}^E$, $\hat{C}_B^E$ and $\hat{\Omega}$ are in such a case just the fused respective variables, not filter-updated.) This may be practically important, for example when an optical sensor is located near an IMU. Needless to say, individual IN-output resets during GPS outage *do not improve the ensemble navigation* regarding $\bar{\mathbf{r}}_I$.

Note that $\mathbf{r}_{II_j}$ multiplies $\hat{C}_B^E$, *which is not perfectly accurate*, so that a ‘very large’ $\mathbf{r}_{II_j}$ is not advisable (even *with* GPS data).

SIMULATION EXAMPLE

The numerical simulation example described below demonstrates the main features of the invention, with several IMUs and a single GPS receiver, and compares the invention’s performance to the performance of an (almost) identical navigation scheme (with the same hardware), however lacking the individual IMU corrections.

The main reason for using one GPS receiver in the example, rather than several receivers, is the wish *to highlight the contribution of the individual IMUs corrections*. The approach to which the invention is compared applies (like the invention) full inertial navigation for each IMU, including individual initializations and individual $PV\Theta$ filter-resets. That is, *the compared approach is significantly more involved than the basic multi-IMU approach*, which consists in averaging the IMUs’ outputs and running a single IN. The compared approach differs from the invention only in that it applies the *same* navigation filter acceleration and rate corrections to *all* the IMUs (rather than individual corrections to each IMU, in the invention).

The (simulated) hardware configuration used in all the navigation simulations includes *five* ADIS16448 low-grade IMUs, manufactured by ANALOG DEVICES. Four of these IMUs are ‘placed’ at the four vertices of a tetrahedron whose edge length is 0.2 meter; one IMU is ‘placed’ at the tetrahedron’s center. See reference [23] for a discussion of IMU-arrangement effects. The simulation includes modelling of *individual* IMU locations and misalignments, and of *individual* bias/scale-factor/random-walk errors (based on the ADIS16448’s specification) for each of the 30 sensors (an IMU includes an accelerometer triad and a rate-gyro triad). A second hardware configuration tested was a collapsed version of the first, with all IMUs co-located. For fairness,

the same random sequences were used in the simulation runs with the compared approach.

A 3-minute trajectory of a rocket was used, which includes the boost phase (about 20 seconds) and a portion of the ballistic phase. GPS reception is available for the first 2 minutes. Note that *the attitude accuracy of navigation in a high-altitude ballistic trajectory is usually poor*.

Fig. 5 presents position, velocity and attitude *errors* in a *single simulation run* for the two compared navigation schemes, and for two hardware (HW) configurations: tetrahedron+center and co-located IMUs. In the first 120 seconds the GPS dominates navigation accuracy and all four runs exhibit practically identical performance. During the GPS outage (from 120 to 180 seconds; see Fig. 6), a clear advantage of the invention is apparent, with about *half* the errors of the compared approach (with a small advantage for the *non* co-located configuration, but not in attitude). Hence, in this example individual IMU corrections are equivalent to using (in the compared approach) about a *four-fold* number of IMUs (*20 IMUs*). Since the compared approach is significantly more advanced than mere IMUs-averaging, the advantage of the invention over IMUs-averaging is even greater.

The reason for this success can be seen in Fig. 7, which addresses one of the off-center IMUs in the simulation runs made for Fig. 5, and presents for each of this IMU's six sensors the total *true* error (excluding random walk, for clarity) and the error *estimations* made by the two compared methods (note: after $t = 120$ the estimations are 'freezed'). The variables presented are velocity and angle increments; the plots pertaining to the X and Z velocity-increments are zoomed-in. Though the invention does *not* provide flawless error estimations, it still does much better than the compared approach, which applies the *same* filter-corrections to *all* IMUs. Note the distinctly better estimations regarding the Y accelerometer and X and Z rate-gyros. The Z accelerometer estimation is also significantly better. The X accelerometer estimation captures the true error dynamics, while the compared approach misses the latter altogether.

The invention and the compared approach have also been *statistically* compared by *Monte-Carlo simulation runs*. The resulting *horizontal position navigation accuracies*, in terms of CEP and R95, are presented in Fig. 8, which zooms in on the GPS outage period. In this MC simulation, the invention is *twice as accurate* as the compared approach, regarding both CEP and R95.

Fig. 9 shows the respective 50% and 95% 3-dimensional *attitude accuracy* measures, and also zooms in on the GPS outage period. The shown attitude accuracy is poor because of the ballistic high-altitude flight condition. Regarding attitude accuracy, the 50% measures of the invention and the compared approaches are quite similar; however, the 95% attitude accuracy measure of the invention is far better (2.7 times better at $t=180$ seconds) than that of the compared approach. This means that the attitude error-distribution of the present method is much narrower than that of the compared method. Note again that the compared approach would require about *20 IMUs* (rather than 5 IMUs) to attain the position accuracy of the invention, and that IMUs-averaging (the basic approach to multi-IMU navigation) would require *even more* IMUs.

Obviously, one can run both an inertial navigation scheme *and* an individual navigation filter *per IMU*, and fuse their outputs (see references [10], [2]-[6]) to attain performance *similar* to that of the invention, perhaps even better. However, the associated real-time code complexity, the extent of deviation from an existing single-IMU/single-GPS real-time code, and the resulting computational load would all be *much greater* than with the present invention.

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CLAIMS

1. A GPS-aided inertial navigation method comprising:

providing multiple sensors including multiple inertial measurement
5 units (IMUs) fixedly installed on a rigid body; and at least one global
positioning system receiver and antenna (GPS), said at least one GPS
providing position and velocity solutions; and
providing a navigation computer with embedded navigation software

or

10 a computer, which is not navigation-dedicated, however has
the required hardware interfaces and navigation software;

wherein the computer is configured to perform the following activities:

(a) interfacing with all IMUs and all GPS receivers;

15 (b) running, in parallel, multiple standard inertial navigation (IN)
schemes, one per each IMU;

(c) computing the mean of all the INs, thus obtaining a fused IN solution
for the IMUs' mean location ('mean IMU' location);

(d) computing the mean of all the GPS solutions, thus obtaining a fused
20 GPS solution for the GPS antennas' mean location ('mean antenna' location);

(e) applying lever-arm corrections, using the vector from the 'mean IMU'
location to the 'mean antenna' location, on the fused GPS solution;

(f) feeding the fused IN solution and the lever-arm corrected fused
GPS solution to a single standard navigation filter, so that said standard
25 navigation filter is capable of operating with a single GPS and also
capable of operating with a plurality of GPSs of said at least one GPS, as
if there were a single IMU and a single GPS, and said standard
navigation filter retains the same number of standard navigation filter
error states; and

(g) running an IMU/IN/GPS correction module which, based on rigid-body physics, which is configured to compute appropriate individual:

- (i) acceleration and angular rate corrections for each IMU,
- (ii) position, velocity and attitude corrections for each IN, and
- (iii) position, velocity, attitude, acceleration and angular rate-aiding messages for each GPS receiver.

2. The method of claim 1, using an inertial navigation (IN) scheme per each IMU.

3. The method of any one of claims 1 and 2, using only one navigation filter to accommodate one fused IN solution and one fused GPS solution, as if there were a single IMU and a single GPS.

4. The method of claim 1, wherein all corrections and aiding-messages are performed individually for each sensor and IN-output.

5. The method of claim 4, wherein all the corrections and aiding-messages are computed by algorithms based on rigid-body physics.

6. The method of claim 5, wherein the algorithms compute a real-time least-squares solution for the body's angular acceleration.

7. The method of any one of claims 1 to 6, wherein individual IMU-corrections are computed as increments to prior corrections, and integrated.

8. The method of any one of claims 1 to 6, wherein individual IMU-

corrections are computed wholly, after each navigation filter update anew, without integrators.

9. The method of claim 4, wherein the computation of individual corrections
5 to all the sensors uses:

- a. angular rate and angular acceleration lever-arm correction terms, and
- b. filter-updated and lever-corrected linear accelerations at the individual IMUs' locations, to define either local IMU correction-increments; or to define whole local IMU corrections.

10. A GPS-aided inertial navigation system (INS) comprising multiple IMUs fixedly installed on a rigid body and a navigation computer with embedded navigation software; or a computer that is not navigation-dedicated, however has the required hardware interfaces and navigation software, characterized in that the
15 system comprises at least one global positioning system receiver and antenna (GPS), said at least one GPSs providing position and velocity solutions, wherein the computer is configured to:

- (a) interface all IMUs and all GPS receivers;
- (b) run, in parallel, multiple standard inertial navigation (IN) schemes, one
20 per each IMU;
- (c) compute the mean of all INs, thus obtaining a fused IN solution for the IMUs' mean location ('mean IMU' location);
- (d) compute the mean of all GPS solutions, thus obtaining a fused GPS solution for the GPS antennas' mean location ('mean antenna' location);
- 25 (e) apply lever-arm corrections, using the vector from the 'mean IMU' location to the 'mean antenna' location, on the fused GPS solution;
- (f) feed the fused IN solution and the lever-arm corrected fused GPS solution to a single standard navigation filter, so that said standard navigation filter

is capable of operating with a single GPS and also capable of operating with a plurality of GPSs of said at least one GPS, as if there were a single IMU and a single GPS, and said standard navigation filter retains the same number of standard navigation filter error states; and

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(g) run an IMU/IN/GPS correction module which, based on rigid-body physics, which is configured to compute appropriate individual:

(i) acceleration and angular rate corrections for each IMU,

(ii) position, velocity and attitude corrections for each IN, and

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(iii) position, velocity, attitude, acceleration and angular rate-aiding messages for each GPS receiver.