

METHOD AND SYSTEM FOR LOCATING A TRANSMITTER

The invention relates to a method and a system for locating one or more radioelectric or acoustic transmitters.

5 The principle of hyperbolic procedures based on arrival time difference or TDOA (Time Differential Of Arrival) consists in measuring the difference of the propagation lag of a signal on N reception systems which are at different positions and which are synchronized by the same reference signal (PPS of GPS – Global Positioning Satellite - for example). The
10 transmitter will then be situated at the intersection of the hyperbolas having the N receivers as focus. The procedures known from the prior art rely notably on the calculation of the arrival time differences (TDOA) by mutual inter-correlation of the signals or by arrival differences (TOA or Time Of Arrival) by implementing hyperbolic location algorithms using a unique
15 combination of TDOA measurements; it is possible to cite in particular the method of Torrieri known to the person skilled in the art, which uses an approximate way of solving a nonlinear least squares problem to estimate the position of the transmitter on the basis of these TDOAs.

 Despite the advantages afforded by the prior art procedures, the
20 latter exhibit certain drawbacks. For example, to calculate the position of a radio transmitter in a system with N receiving stations, the Torrieri algorithm considers only (N-1) TDOAs between two stations by systematically using a station numbering of type 1-2, 2-3, and so on and so forth up to (N-1)-N. This approach does not utilize all of the information available, only N-1 out of the
25 $N(N-1)/2$ possible TDOAs, and the choice of the unique combination of the pairs of stations may not be relevant. The approach using a unique combination of N-1 pairs of stations is not optimal in TDOA location for various reasons: since the TDOA measurements are carried out by inter correlation of the signals received on a pair of stations, they are not
30 redundant and on the contrary the $N(N-1)/2$ TDOA measurements according to the invention comprise information utilizable for location. The fact of being

restricted to N-1 measurements does not make it possible to utilize all of the information available at the level of the measured data. Moreover, the choice in the use of a unique combination of pairs of stations may not be relevant as a function of the geometry of deployment, of the levels of the signals received or else of date-stamping errors on certain stations.

The method and the system according to the invention rely notably on the use of several combinations of TDOA measurements utilizing a set of $N(N-1)/2$ measurements, where N is the number of stations present in a network, so as to minimize the influence of a particular choice as in the prior art and to discard combinations which would lead to aberrant locations.

The invention relates to a method for locating a transmitter device on the basis of N stations in a communications network, each of the stations being equipped with at least one receiver, by using at least one common benchmark for the date-stamping, at all the N stations, the network comprising a master station suitable for managing the measurements of instant of arrival, characterized in that it comprises at least the following steps:

- triggering one or more synchronized acquisitions on N stations for a given frequency, a bandwidth and a given instant t, the said values being chosen as a function of a transmitter device of interest characterized by its frequency, its bandwidth,
- for an acquisition:
 - calculating the $N(N-1)/2$ measurements of TDOA arrival time delay for the $N(N-1)/2$ possible pairs of stations,
 - choosing a number P of measurements out of the $M=N(N-1)/2$ measurements, with $3 < P < N(N-1)/2$, and forming K vectors of measurements associated with the K possible combinations of P out of M with:

$$K = \frac{M!}{P!(M-P)!}$$

- for each of the K vectors, estimating an elementary value of location of the transmitter device by minimizing a quadratic criterion taking into account a matrix corresponding to the covariance of the P measurements of arrival time delay, and by iteration starting from a first value of a vector $\bar{X}_0$ of initial position of the transmitter, the position is situated as near as possible to the sought-after minimum and then by iterating on the first estimation by successively estimating locations making it possible to decrease the criterion to be minimized until a threshold value is satisfied,
- calculating a fused location value on the basis of the elementary estimated location values.

The initialization step consists, by example, in estimating a first coarse location of the transmitter, either by virtue of a priori knowledge of the zone of positioning of the transmitter, or, failing that, by a direct TDOA location algorithm (intersection of hyperbolas).

According to a variant embodiment, in the stationary case, when the transmitter device hardly changes position, the method comprises a step of estimating the location value on the basis of all the location values estimated in the course of several acquisitions.

According to a variant embodiment, the method will minimize, for the location calculation, a quadratic criterion defined as:

$$(\bar{T} - \bar{T}(\bar{X}))^T \Sigma^{-1} (\bar{T} - \bar{T}(\bar{X})) = \left(\bar{T} - H \frac{\bar{D}}{c} \right)^T \Sigma^{-1} \left(\bar{T} - H \frac{\bar{D}}{c} \right), \text{ where}$$

c is the speed of the electromagnetic waves,

$\bar{T}(\bar{X})$ is the model vector (size P) of the observation, making it possible to analytically link the theoretical vector of TDOA measurements to the position of the transmitter $\bar{X}$,

$\bar{D}$ is the vector of size N, whose components are the distances of the location point $\bar{X}$ from the stations,

H is a deterministic $P \times N$ rectangular matrix, consisting of 1, of -1 and of 0, making it possible to pass from the measurements of instants of arrival to measurements of differences of arrival times with the following analytical relation $\bar{T}(\bar{X}) = H \cdot \frac{\bar{D}}{c}$,

- 5 Σ : corresponds to the $P \times P$ weighting matrix of the criterion corresponding to the covariance of the P measurements of TDOA.

The covariance matrix associated with P measurements of TDOA can then be written in the following general form:

$$\Sigma \approx \Lambda_P(\sigma_{TDOA_1}^2, \dots, \sigma_{TDOA_P}^2) + H \Lambda_N(\sigma_{TOA_1}^2, \dots, \sigma_{TOA_N}^2) H^T$$

- 10 where:

$\Lambda_P(\sigma_{TDOA_1}^2, \dots, \sigma_{TDOA_P}^2)$ designates a diagonal matrix of size $(P \times P)$ whose diagonal terms are written $\sigma_{TDOA_i}^2$ with i ranging from 1 to P ,

$\sigma_{TDOA_i}^2$ is the variance of the i th measurement of TDOA,

- 15 $\Lambda_N(\sigma_{TOA_1}^2, \dots, \sigma_{TOA_N}^2)$ designates a diagonal matrix of size $(N \times N)$ whose diagonal terms are written $\sigma_{TOA_i}^2$ with i ranging from 1 to N ,

$\sigma_{TOA_i}^2$ is the variance of the date-stamping measurement on the i th station.

The covariance matrix can also be determined using arrival delay measurements and by defining the diagonal terms of the covariance matrix Σ corresponding to the variances of each of the TDOA measurements:

$$V_i = \frac{1}{N_{acqui}} * \sum_{n=1}^{N_{acqui}} (TDOA_{i,n} - \overline{TDOA_i})^2$$

- 20 with

N_{acqui} being the number of acquisitions taken into account for the calculation of the variance

$TDOA_{i,n}$ corresponds to the n^{th} value obtained at the n th acquisition of the i th TDOA corresponding to the i th pair of stations,

- 25 $\overline{TDOA_i}$ corresponds to the average, over the N_{acqui} acquisitions of the measurements of the i th TDOA.

According to a variant of the method, during the step of initializing the first location, the initialization value $\bar{X}_0$ is calculated on the basis of a priori information about the location of the transmitter device or on the basis of TDOA measurements making it possible to determine an initialization value automatically.

The initialization value $\bar{X}_0$ is, for example, determined on the basis of the vector of TDOA measurements, with a pseudo-linear algorithm PSL, by executing the following steps:

- a reference station j is chosen, typically that of highest SNR or deemed the most reliable,
- the unknown state vector to be estimated becomes $Y=(x_{sj},y_{sj},R_{sj})^T$ where R_{sj} is the distance of the transmitter and (x_{sj},y_{sj}) its coordinates, referenced with respect to the reference station j ,
- the vector of pseudo-measurements M consists of terms that are written: $\frac{1}{2}[D_{ij}^2 - (cTDOA_{ij})^2]$ where D_{ij} is the distance separating the stations i,j and $TDOA_{ij}$ is the measured delay between the signals received at these stations i and j , so that we have the linear relation: $M = A Y$, where A is a matrix, consisting of the coordinates of the stations in relation to the reference station j , and of the terms $(cTDOA_{ij})$ arising from the delay measurements,
- the estimation of the unknown vector Y is then done by conventional solution of a linear least squares problem,
- the relevance of the components of the solution Y is verified, by verifying the consistency of the estimated distance R_{sj} with the estimated coordinates (x_{sj},y_{sj}) ; by verifying that R_{sj} is positive and is not too far from $\sqrt{x_{sj}^2 + y_{sj}^2}$,
- in case of relevance, the estimation of the position of the transmitter is deduced directly from the vector Y ,

- in case of non-relevance, the algorithm can be relaunched with another reference station.

The method can use as iterative algorithm an algorithm for which each iteration makes it possible to calculate, on the basis of a vector $\vec{X}_k$, a

5 new vector $\vec{X}_{k+1}$ in the following manner:

$$\vec{x}_{k+1} = \vec{x}_k + c (F^T H^T \Sigma^{-1} H F)^{-1} F^T H^T \Sigma^{-1} (\vec{T} - \frac{H \vec{D}}{c})$$

where F is a jacobian matrix, calculated at the point $\vec{X}_k$, of the journey times between the transmitter device to be located and the N receiving stations, with

$$10 \quad F = \begin{pmatrix} \frac{(\vec{x}_k - \vec{S}_1)^T}{D_1} \\ \vdots \\ \frac{(\vec{x}_k - \vec{S}_N)^T}{D_N} \end{pmatrix}$$

The row i of the matrix is the unit vector directed from station i towards the current estimation, this equation is calculated at the point $\vec{X}_0$ thereby
15 providing a new location point $\vec{X}_1$ which will be used at the following iteration to obtain a new point $\vec{X}_2$ which will itself be used to obtain a new point until convergence or divergence is detected.

According to an implementation variant, the transmitter device is located on the basis of a set of elementary locations by executing the
20 following steps:

- at each acquisition on the basis of the K elementary locations,
- filter values of aberrant locations, isolated locations or ones which are far from the average of the locations obtained,

- calculate a barycentre by averaging the elementary locations retained, corresponding to a value of location of the transmitter device to be located,
- associate with this value a probability of error CEP determined in the following manner: consider a set of elementary locations $Loc_{x,y}$ taken for an acquisition, B the barycentre of the locations $Loc_{x,y}$ retained, CEP 50% of $Loc_{x,y}$ is the radius of the circle, centred on B, which contains half the locations of $Loc_{x,y}$.

It is also possible to locate the transmitter device on the basis of a set of elementary locations by executing the following steps:

- after N_{acqui} acquisitions and in the stationary case,
- filter values of aberrant locations, isolated locations or ones which are far from the average of the locations obtained,
- calculate a barycentre by averaging the elementary locations retained, corresponding to a value of location of the transmitter device to be located,
- associate with this value a probability of error CEP determined in the following manner: consider a set of elementary locations $Loc_{x,y}$ taken for an acquisition, B the barycentre of the locations $Loc_{x,y}$ retained, CEP 50% of $Loc_{x,y}$ is the radius of the circle, centred on B, which contains half the locations of $Loc_{x,y}$.

Other characteristics and advantages of the method and of the system according to the invention will be more clearly apparent on reading the description which follows given by way of wholly nonlimiting illustration together with the appended figures which represent:

- Figure 1, an exemplary system for the implementation of the method according to the invention,
- Figure 2, an illustration of a calculation by barycentre.

Figure 1 represents an exemplary system in which the method according to the invention can be implemented. A master station 1 is equipped with communication means, a radio transmitter 2 and a radio

receiver 3 as well as a processor 4 adapted for executing the steps of the method according to the invention. A memory area 5 allows the storage of the data and of the measurements performed. The master station is linked (by radioelectric link) to several so-called slave stations 10i each equipped with a radioelectric receiver, 11, 12, in this example. The N reception stations are synchronized by virtue of a precise date-stamping benchmark common to all the stations, such as a precise clock, common to all the stations, typically a GPS reference signal. The master station 1 will sequence and parametrize the acquisitions on each slave station: the acquisitions being done on a list of discrete central frequencies representing the transmitters of interest (transmitter 20) to be located.

The master station 1 transmits an order for acquisition of the signals on a frequency F_c , a bandwidth BW, a duration T (characterization of the transmitter of interest). The master station receives the N signals acquired by each of the slave stations 10n used in the TDOA system, the stations being in radio link for example with the master station. In the course of time, it is possible to have several successive acquisitions on F_c/BW at t_1 , t_2 , ..., $t_{N_{acqui}}$. If the transmitter that it is sought to detect is mobile, or in the "non-stationary" case, the acquisitions will be processed independently of one another. In the converse case, the "stationary" case, the method will be able to utilize the history of the acquisitions so as to improve the detection and the location of the targeted transmitter. These two examples will be detailed in the subsequent description.

The method according to the invention will execute at least the three steps explained hereinafter for each acquisition of N signals which is commanded by the master device, at a given instant t, for a given frequency, these values being chosen as a function of the transmitter that one wishes to locate, the characteristics of the transmitter (F_c , BW) being known a priori, or else resulting from a prior step of measurements of signals present in a given zone.

Step 1: Calculation of the TDOA delay times and filtering

In the course of a first step, the master station will calculate all the TDOA delay times and optionally apply a filtering. The master station calculates all the TDOA delay times between all the $N(N-1)/2$ possible pairs of stations. For example, in the case where $N=5$ stations, the set of the 10 station pairs is grouped together in the following table I:

TDOA number	Pair of associated stations	TDOA number	Pair of associated stations
1	(1,2)	6	(2,4)
2	(1,3)	7	(2,5)
3	(1,4)	8	(3,4)
4	(1,5)	9	(3,5)
5	(2,3)	10	(4,5)

For each couple of signals, corresponding to a couple of stations, the estimation of the delay time τ is performed by calculating an inter correlation (generally normalized) between the signals and uses an interpolation to refine the search for the maximum.

At the end of this calculation, the method has a maximum of $N(N-1)/2$ measurements of TDOA, that is to say as many as there are possible pairs of stations out of the N stations.

Each TDOA measurement, τ , is accompanied by a correlation score and by a value of the associated signal-to-noise ratio. Certain measurements will be eliminated if the correlation score or if the value of the associated signal-to-noise ratio is too low for the TDOA measurement to be utilizable. This filtering is carried out by the master station which stores values of utilization thresholds and applies a thresholding algorithm known to the person skilled in the art.

Step 2: Choice of combinations of TDOA measurements

The second step consists in choosing a number P of TDOA measurements out of the M available ($M \leq N(N-1)/2$).

The method can operate with a number P lying between 3 and M , but the value usually chosen lies in the interval defined by these two extremes and results from a compromise between precision of an elementary location and statistical representativity, aimed at judicious utilization of all the

available information. Once the number P has been chosen, the method establishes all the possible combinations of P measurements of TDOA out of the M available, i.e. a number K of combinations equal to:

$$K = C_M^P = \frac{M!}{P!(M-P)!}$$

5

By way of illustration, in the case of $N=5$ stations and of $M=10$ TDOA measurements, this corresponding to the availability of all the pairs of stations available, table II hereinafter groups together the number of K combinations obtained as a function of the number P chosen, P ranging from 3 to $M=10$:

P	Number of combinations K
3	120
4	210
5	252
6	210
7	120
8	45
9	10
10	1

10

By way of example, in the case where one chooses to retain all the combinations each comprising $P=7$ measurements of TDOA, the 120 possible combinations are retrieved, table III:

Combination number	TDOA									
	1	2	3	4	5	6	7	8	9	10
1	X	X	X	X	X	X	X			
2			X	X	X		X	X	X	X
3		X		X	X		X	X	X	X
4	X	X	X			X	X	X		X
...										
120				X	X	X	X	X	X	X

Once the number P of measurements has been chosen, the K vectors of measurements $\vec{T}$ associated with the K possible combinations of

P out of M are formed. Each vector $\vec{T}$ contains the P measurements of TDOA associated with the considered combination of pairs of stations and is therefore of dimension $P \times 1$. In the example given hereinabove, $P=7$.

Step 3: Calculation of the K elementary locations

5 The third step consists in calculating a location of the transmitter, separately for each of the K combinations therefore for each of the K vectors of measurements $\vec{T}$ arising from the previous step 2, doing so for a given acquisition of signals. Each location is for example obtained by executing a nonlinear weighted least squares algorithm, an example of which is given
10 hereinafter.

For each of the K vectors $\vec{T}$ formed:

Let $\vec{T}$ be the vector of P measurements of TDOA corresponding to a considered combination.

15 The location calculation is carried out by minimizing a quadratic criterion, consisting of the Mahalanobis distance known to the person skilled in the art regarding measurements: we seek the position vector $\vec{X}$ of the transmitter minimizing the following quadratic criterion:

$$(\vec{T} - \vec{T}(\vec{X}))^T \Sigma^{-1} (\vec{T} - \vec{T}(\vec{X})) = \left(\vec{T} - H \frac{\vec{D}}{c} \right)^T \Sigma^{-1} \left(\vec{T} - H \frac{\vec{D}}{c} \right)$$

where:

20 c is the speed of the electromagnetic waves,

$\vec{T}(\vec{X})$ is the model vector (size P) of the observation, making it possible to analytically link the theoretical vector of TDOA measurements to the position of the transmitter $\vec{X}$,

25 $\vec{D}$ is the vector of size N, whose components are the distances of the location point $\vec{X}$ from the stations,

H : is a deterministic $P \times N$ rectangular matrix (consisting of 1, of -1 and of 0) making it possible to pass from the measurements of instants of arrival to

measurements of differences of arrival times: the following analytical relation

is satisfied $\bar{T}(\bar{X}) = H \cdot \frac{\bar{D}}{c}$,

Σ : corresponds to the $P \times P$ weighting matrix of the criterion. To optimize the location calculation in the sense of the likelihood (under the Gaussian assumption), this matrix must correspond to the covariance of the P measurements of TDOA.

This leads to a Dimension of the matrix Σ :

Matrix	size
Σ	$P \times P$

The location calculation therefore requires the prior calculation (step 3a) of the matrix Σ associated with the vector $\bar{T}$ of P measurements of TDOA. Once this calculation has been performed, the location point (step 3b) will be calculated by minimizing the criterion presented above. These two sub-steps 3a and 3b are described hereinafter.

Step 3a: Calculation of the covariance matrix Σ

The calculation of the matrix Σ is performed in various ways, depending on the case treated (stationary or non-stationary), and in the stationary case depending on whether or not a sufficient history is available:

Non-stationary case or stationary case with insufficient history:

In the case where the transmitter to be located is not fixed or where a sufficient history of prior acquisitions is not available, the covariance matrix is calculated in a theoretical manner as described hereinafter.

The theoretical covariance matrix $P \times P$, associated with P measurements of TDOA is written in the following general form:

$$\Sigma \approx \Lambda_P(\sigma_{TDOA_1}^2, \dots, \sigma_{TDOA_P}^2) + H \Lambda_N(\sigma_{TOA_1}^2, \dots, \sigma_{TOA_N}^2) H^T$$

where:

$$\Lambda_P(\sigma_{TDOA_1}^2, \dots, \sigma_{TDOA_P}^2)$$

designates a diagonal matrix of size (PxP) whose diagonal terms are written $\sigma_{TDOA_i}^2$ with i ranging from 1 to P, with $\sigma_{TDOA_i}^2$ the variance of the ith measurement of TDOA (arising from the inter-correlation)

5 $\Lambda_N(\sigma_{TOA_1}^2, \dots, \sigma_{TOA_N}^2)$ designates a diagonal matrix of size (NxN) whose diagonal terms are written $\sigma_{TOA_i}^2$ with i ranging from 1 to N.

$\sigma_{TOA_i}^2$ is the variance of the date-stamping measurement on the ith station.

Provided that $\sigma_{TDOA_i}^2$ is non-zero, this matrix is of rank P and is invertible.

10 In practice, the values of $\sigma_{TOA_i}^2$ are a priori values, dependent on the date-stamping means used (typically, heel error due to the precision of the 1 PPS or more generally of the common reference clock used, or else precision of the acquisition chain of each station).

15 The values of $\sigma_{TDOA_i}^2$ are either values fixed a priori and based on experiment, or theoretical values (for example the Cramer-Rao bounds) arising from a calculation and dependent notably on the signal-to-noise ratio received on the stations (this being aimed at favouring the measurements performed with the stations of larger signal-to-noise ratio).

20 In the stationary case where the covariance matrix is estimated on the basis of measurements acquired over time, the "aberrant" TDOA measurements are filtered on the basis of statistical calculation.

Stationary case where a sufficient number of successive acquisitions is available

25 In the stationary case where sufficient prior acquisitions (at least 3) are available, the covariance matrix can be refined by an empirical covariance calculation using the TDOA measurements arising from the previous successive acquisitions.

The calculation of the empirical covariance matrix is carried out in the manner described hereinafter.

$$Vi = \frac{1}{Nacqui} * \sum_{n=1}^{Nacqui} (TDOA_{i,n} - \overline{TDOA_i})^2$$

with

- $\text{TDOA}_{i,n}$ = corresponds to the n^{th} value (i.e. obtained at the n^{th} acquisition) of the i^{th} TDOA (corresponding to the i^{th} pair of stations out of the M possible),

A similar equation provides the cross terms of the covariance matrix (correlation between TDOA measurements carried out on 2 distinct pairs of stations).

To return to the example of $N=10$ stations, when after *Nacqui* acquisitions (≥ 3), *Nacqui* samples TDOA1, TDOA2, ..., TDOA10 of the 10 TDOAs measured are available (1 TDOA for each of the 10 pairs of station), it is possible to create the covariance matrix Σ of the TDOAs (which is obtained here with $P=10$), table IV:

Covariance matrix									
V_{TDOA1}	$C_{TDOA1,TDOA2}$	$C_{TDOA1,TDOA3}$	$C_{TDOA1,TDOA4}$	$C_{TDOA1,TDOA5}$	$C_{TDOA1,TDOA6}$	$C_{TDOA1,TDOA7}$	$C_{TDOA1,TDOA8}$	$C_{TDOA1,TDOA9}$	$C_{TDOA1,TDOA10}$
	V_{TDOA2}	$C_{TDOA2,TDOA3}$	$C_{TDOA2,TDOA4}$	$C_{TDOA2,TDOA5}$	$C_{TDOA2,TDOA6}$	$C_{TDOA2,TDOA7}$	$C_{TDOA2,TDOA8}$	$C_{TDOA2,TDOA9}$	$C_{TDOA2,TDOA10}$
		V_{TDOA3}	$C_{TDOA3,TDOA4}$	$C_{TDOA3,TDOA5}$	$C_{TDOA3,TDOA6}$	$C_{TDOA3,TDOA7}$	$C_{TDOA3,TDOA8}$	$C_{TDOA3,TDOA9}$	$C_{TDOA3,TDOA10}$
			V_{TDOA4}	$C_{TDOA4,TDOA5}$	$C_{TDOA4,TDOA6}$	$C_{TDOA4,TDOA7}$	$C_{TDOA4,TDOA8}$	$C_{TDOA4,TDOA9}$	$C_{TDOA4,TDOA10}$
				V_{TDOA5}	$C_{TDOA5,TDOA6}$	$C_{TDOA5,TDOA7}$	$C_{TDOA5,TDOA8}$	$C_{TDOA5,TDOA9}$	$C_{TDOA5,TDOA10}$
					V_{TDOA6}	$C_{TDOA6,TDOA7}$	$C_{TDOA6,TDOA8}$	$C_{TDOA6,TDOA9}$	$C_{TDOA6,TDOA10}$
Symmetry with respect to the variance of the covariance results						V_{TDOA7}	$C_{TDOA7,TDOA8}$	$C_{TDOA7,TDOA9}$	$C_{TDOA7,TDOA10}$
							V_{TDOA8}	$C_{TDOA8,TDOA9}$	$C_{TDOA8,TDOA10}$
								V_{TDOA9}	$C_{TDOA9,TDOA10}$
									V_{TDOA10}

Step 3b: location calculation by minimizing the quadratic criterion

Since the criterion to be minimized involves a nonlinear function $T(X)$, the problem becomes one of "nonlinear least squares", and the minimization is performed by an iterative process which requires an initialization point $\bar{X}_0$ situated as near as possible to the sought-after minimum.

The algorithm for minimizing the criterion therefore comprises two major steps:

- 10 • An initialization step aimed at estimating a first location of the transmitter: $\bar{X}_0$, initial position vector of the transmitter, situated, for example, as near as possible to the sought-after minimum,
- An iteration step aimed at refining this first "coarse" estimation by firstly using the vector $\bar{X}_0$, and then by successively estimating locations $\bar{X}_k$ making it possible to decrease the criterion to be minimized.

Initialization step: calculation of an initial vector $\bar{X}_0$

Several cases are possible depending on context. If we are in the stationary case and a location arising from previous acquisitions is available beforehand, or else if reliable a priori information is available about the location of the transmitter (restricted probable zone of presence, previous location for a weakly mobile transmitter of interest), the initial position vector $\bar{X}_0$ is fixed at this prior value known a priori. The initialization consists, for example, in estimating a first coarse location of the transmitter, either by virtue of a priori knowledge of the zone of positioning of the transmitter, or, by default, through a direct TDOA location algorithm (intersection of hyperbolas).

In any other case, the initial position vector is calculated on the basis of the vector of TDOA measurements, by a pseudo-linear algorithm

PSL (Pseudo-Linear Estimator) making it possible to determine an initialization value automatically.

This large family of algorithms consists in applying a transformation to the measurements and/or in transforming the state vector to be estimated so that the problem reduces to the direct solution of a linear least squares problem.

In the particular case of TDOA measurements and of at least four stations:

- a reference station j is chosen (typically, that of highest SNR or deemed the most reliable),
- the unknown state vector to be estimated becomes $Y = (x_{sj}, y_{sj}, R_{sj})^T$ where R_{sj} is the distance of the transmitter and (x_{sj}, y_{sj}) its coordinates, referenced with respect to the reference station j ,
- the vector of pseudo-measurements M consists of terms that are written: $\frac{1}{2} [D_{ij}^2 - (cTDOA_{ij})^2]$ where D_{ij} is the distance separating the stations i, j and $TDOA_{ij}$ is the measured delay between the signals received at these stations i and j , so that we have the linear relation: $M = A Y$, where A is a matrix, consisting of the coordinates of the stations in relation to the reference station j , and of the terms $(cTDOA_{ij})$ arising from the delay measurements,
- the estimation of the unknown vector Y is then done by conventional solution of a linear least squares problem (A and M being known),
- the relevance of the components of the solution Y is verified, by verifying the consistency of the estimated distance R_{sj} with the estimated coordinates (x_{sj}, y_{sj}) ; this verification is typically done by verifying that R_{sj} is positive and is not too far from $\sqrt{x_{sj}^2 + y_{sj}^2}$,
- in case of relevance, the estimation of the position of the transmitter is deduced directly from the vector Y ,

- in case of non-relevance, the algorithm can be relaunched with another reference station for example.

This pseudo-linear algorithm is sub-optimal in the sense of the minimization of the aforementioned criterion but it presents the advantage of direct solution based on the TDOA measurements. This first location serves to initialize the iterative nonlinear least squares minimization algorithm described hereinafter. In the absence of precise a priori information or of a reliable previous location of the transmitter of interest, the use of this search algorithm is preferable (solution point closer to the position of the transmitter) to a random or arbitrary initialization.

At the end of this initialization step, a first location point $\bar{X}_0$ is available which will serve as starting point in the step of minimizing the criterion presented above.

Step of minimizing the quadratic criterion by successive iterations:

In this step, we seek to calculate a location point that minimizes the quadratic criterion defined above, by starting from the initial location point $\bar{X}_0$.

Numerous algorithms for solving nonlinear least squares problems are usable, provided that they succeed in minimizing the criterion described above.

The method used implements for example a second-order iterative algorithm (of Gauss-Newton type, corresponding to successive iterations of an algorithm of Torrieri type), for which each iteration makes it possible to calculate, on the basis of a vector $\bar{X}_k$ a new vector $\bar{X}_{k+1}$ in the following manner:

$$\vec{x}_{k+1} = \vec{x}_k + c (F^T H^T \Sigma^{-1} H F)^{-1} F^T H^T \Sigma^{-1} (\vec{T} - \frac{H \vec{D}}{c})$$

where F is a jacobian matrix, calculated at the point $\bar{X}_k$, of the journey times between the source and the N receiving stations. The dimension of this matrix is N x 2, and it may be written:

$$F = \begin{pmatrix} \frac{(\vec{x}_k - \vec{s}_1)^T}{D_1} \\ \vdots \\ \frac{(\vec{x}_k - \vec{s}_N)^T}{D_N} \end{pmatrix}$$

5

The row i of the matrix is the unit vector directed from station i towards the current estimation.

At the first iteration, this equation is calculated at the point $\bar{X}_0$ and provides a new location point $\bar{X}_1$ which will be used at the following iteration to obtain a new point $\bar{X}_2$ etc. This calculation is thus iterated for each new location $\bar{X}_k$ calculated, until convergence or divergence is detected.

The method can use various stopping criteria. For example, it is possible to use a threshold, below which the iterations are stopped, in terms of distance between several successive estimations of the position. Other stopping criteria are also usable (such as relative decrease of the criterion below a threshold, maximum number of iterations, etc).

The calculations described in step 3 are applied to each of the K TDOA combinations, that is to say for each of the K vectors $\bar{T}$ of TDOA measurements.

On completion of this step 3, K location points per acquisition are therefore available.

In case of failure of the location algorithm (for example, non-convergence of the minimization algorithm for a given combination of measurements), it may happen that fewer than K locations are available, this number K therefore constitutes a maximum, but this is the number generally obtained in the "nominal" regime.

Step 4: Calculation of a fused location on the basis of the elementary locations

On entry to the fourth step, K location points (at most) are therefore available per acquisition, therefore potentially $K \times N_{\text{acqui}}$ location points.

Returning to the example presented above ($N=5$, $M=10$, $P=7$), if a series of $N_{\text{acqui}}=40$ acquisitions is available for the calculation of the position of the transmitter, we then have $40 \text{ acquisitions} \times 120 \text{ combinaisons} = 4800$ elementary loc at the maximum.

Combination Acquisition	1	2	3	...	120
1	$\text{LOC}_{1,1}$	$\text{LOC}_{1,2}$	$\text{LOC}_{1,3}$	...	$\text{LOC}_{1,120}$
2	$\text{LOC}_{2,1}$	$\text{LOC}_{2,2}$	$\text{LOC}_{2,3}$	...	$\text{LOC}_{2,120}$
3	$\text{LOC}_{3,1}$	$\text{LOC}_{3,2}$	$\text{LOC}_{3,3}$	...	$\text{LOC}_{3,120}$
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
40	$\text{LOC}_{40,1}$	$\text{LOC}_{40,2}$	$\text{LOC}_{40,3}$	...	$\text{LOC}_{40,120}$

Step 4 consists in calculating:

- for each acquisition: a single location point on the basis of the K locations available, supplemented with an uncertainty zone (translated into CEP form),
- in the stationary case (fixed transmitter) and when N_{acqui} acquisitions are available, a location point resulting from the fusion of the N_{acqui} locations obtained over time.

At each acquisition, we have at most K elementary locations (one for each of the K combinations which culminated in a location) and the locations are fused to construct a summary location per acquisition: after N_{acqui} acquisitions, and in the stationary case, further refinement is possible by using the location results obtained over time, i.e. the fusion of the N_{acqui} summary locations. This step 4 presents the fusion of the locations at each acquisition to obtain a summary location, and then if appropriate the fusion of the N_{acqui} summary locations obtained after N_{acqui} acquisitions.

Calculation of the location of the transmitter on the basis of the K (at most) elementary locations:

For each acquisition, the following processing is performed on the basis of the K locations:

- 5 • Filtering of the "aberrant" locations by processing of "clustering" type, the principle of which is to identify and to discard – if appropriate- the isolated locations that deviate too far from the group of locations obtained,
- calculation of a barycentre by taking the average (optionally weighted)
- 10 of the elementary locations retained.

This barycentre constitutes the location ultimately obtained for a given acquisition.

A 50% CEP (Circular Error Probability) is associated with this location result. The calculation of the CEP50% is done, for example, in the following manner:

- 15 Consider a set of elementary locations $Loc_{x,y}$ taken for an acquisition. Denoting by B the barycentre of the locations $Loc_{x,y}$ retained. The radius of the circle, centred on B, which contains half the locations of $Loc_{x,y}$, is called CEP 50% of $Loc_{x,y}$.

20 Calculation of the location of the transmitter on the basis of the Nacqui locations (stationary case):

In the stationary case where the transmitter to be located is fixed and several acquisitions are available, the same type of calculation is applied on the basis of the locations obtained at each acquisition, furnished with their respective CEP:

- 25 • Filtering of the "aberrant" locations by processing of "clustering" type, the principle of which is to identify and to discard – if appropriate- the isolated locations that deviate too far from the group of locations obtained,
- Calculation of a new, optionally weighted, barycentre ("improved"
- 30 barycentre) on the basis of the locations retained and calculation of a new CEP (so-called "improved" CEP)

Returning to the previous example ($N=5$, $M=10$, $P=7$, $N_{acqui}=40$), the following type of results is obtained, table V:

Combination Acquisition	1	2	3	...	120	Barycentre	CEP 50%
1	$LOC_{1,1}$	$LOC_{1,2}$	$LOC_{1,3}$	...	$LOC_{1,120}$	B_1	CEP_1
2	$LOC_{2,1}$	$LOC_{2,2}$	$LOC_{2,3}$	...	$LOC_{2,120}$	B_2	CEP_2
3	$LOC_{3,1}$	$LOC_{3,2}$	$LOC_{3,3}$	...	$LOC_{3,120}$	B_3	CEP_3
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
40	$LOC_{40,1}$	$LOC_{40,2}$	$LOC_{40,3}$	...	$LOC_{40,120}$	B_{40}	CEP_{40}
						Raw barycentre	raw CEP 50%
						Improved barycentre	improved CEP 50%

In this table:

- 5 - The raw barycentre corresponds to the weighted average of all the barycentres calculated at each acquisition
- The raw CEP 50% is calculated on the basis of the raw barycentre.
- The improved barycentre is calculated on the basis of the barycentres having an associated $CEP_{50\%} < X$ m.
- 10 - The CEP 50% is calculated on the basis of the improved barycentre by taking only the $CEP_{50\%} < X$.

X is a threshold value parametrizable according to context.

The use of all the available information, on account of the calculation of all the TDOAs by inter-correlation, and of several combinations of measurements to yield a statistic makes it possible to improve the location and notably to filter the aberrant locations. It multiplies the number of location results, each location being able to be considered to be an elementary location, contributing to a more reliable and more precise summary location result by virtue of this statistical approach.

CLAIMS

1 – Method for locating a transmitter device (20i) on the basis of N stations (10j) in a communications network, each of the stations being equipped with at least one receiver (12), by using at least one date-stamping benchmark common to all the N stations, the network comprising at least one master station (1) suitable for managing measurements of instant of arrival, characterized in that it comprises at least the following steps:

- triggering one or more synchronized acquisitions on N stations for a given frequency, a bandwidth and a given instant t, values of the given frequency, the bandwidth and the given instant t being chosen as a function of a transmitter device of interest characterized by its frequency, its bandwidth,
- for an acquisition:
 - calculating the $N(N-1)/2$ measurements of arrival time delay TDOA between all the $N(N-1)/2$ pairs of stations,
 - choosing a number P of measurements out of the $M=N(N-1)/2$ measurements, with $3 < P < N(N-1)/2$ and forming K vectors of measurements associated with the K possible combinations of P out of M with:

$$K = \frac{M!}{P!(M-P)!}$$
 - for each of the K vectors, estimating an elementary value of location of the transmitter device by minimizing a quadratic criterion taking into account a matrix corresponding to the covariance of the P measurements of arrival time delay and by iteration starting from a first value of a vector $\vec{X}_0$ of initial position of the transmitter, the position is situated as near as possible to the sought-after minimum and then by iterating on the first estimation by successively estimating locations making it possible to decrease the criterion to be minimized until a threshold value is satisfied,

- calculating a fused location value on the basis of the estimated values of elementary location.

2 – Method according to Claim 1, characterized in that in a stationary case
5 for quasi-motionless stations, the method moreover comprises a step of calculating a location on the basis of all the elementary locations estimated for all the acquisitions.

3 – Method according to one of Claims 1 and 2, characterized in that to
10 estimate a location value, we minimize a quadratic criterion defined as:

$$(\vec{T} - \vec{T}(\vec{X}))^T \Sigma^{-1} (\vec{T} - \vec{T}(\vec{X})) = \left(\vec{T} - H \frac{\vec{D}}{c} \right)^T \Sigma^{-1} \left(\vec{T} - H \frac{\vec{D}}{c} \right), \text{ where}$$

c is the speed of the electromagnetic waves,

$\vec{T}(\vec{X})$ is the model vector (size P) of the observation, making it possible to
analytically link the theoretical vector of TDOA measurements to the position
15 of the transmitter $\vec{X}$,

$\vec{D}$ is the vector of size N, whose components are the distances of the location
point $\vec{X}$ from the stations,

H : is a deterministic P x N rectangular matrix, consisting of 1, of -1 and of 0,
making it possible to pass from the measurements of instants of arrival to
20 measurements of differences of arrival times with the following analytical

$$\text{relation } \vec{T}(\vec{X}) = H \cdot \frac{\vec{D}}{c},$$

Σ : corresponds to the PxP weighting matrix of the criterion corresponding to
the covariance of the P measurements of TDOA.

25 4 – Method according to Claim 3, characterized in that the covariance matrix Σ associated with P measurements of TDOA is determined, written in the following general form:

$$\Sigma \approx \Lambda_P(\sigma_{TDOA_1}^2, \dots, \sigma_{TDOA_P}^2) + H \Lambda_N(\sigma_{TOA_1}^2, \dots, \sigma_{TOA_N}^2) H^T$$

where:

$\Lambda_P(\sigma_{TDOA_1}^2, \dots, \sigma_{TDOA_P}^2)$ designates a diagonal matrix of size (PxP) whose diagonal terms are written $\sigma_{TDOA_i}^2$ with i ranging from 1 to P,

$\sigma_{TDOA_i}^2$ is the variance of the ith measurement of TDOA,

5 $\Lambda_N(\sigma_{TOA_1}^2, \dots, \sigma_{TOA_N}^2)$ designates a diagonal matrix of size (NxN) whose diagonal terms are written $\sigma_{TOA_i}^2$ with i ranging from 1 to N,

$\sigma_{TOA_i}^2$ is the variance of the date-stamping measurement on the ith station.

10 5 – Method according to Claim 3, characterized in that the covariance matrix is determined using arrival delay measurements and by defining diagonal terms of the covariance matrix Σ corresponding to the variances of each of the TDOA measurements with:

$$Vi = \frac{1}{N_{acqui}} * \sum_{n=1}^{N_{acqui}} (TDOA_{i,n} - \overline{TDOA_i})^2$$

with

15 N_{acqui} being the number of acquisitions taken into account for the calculation of the variance

$TDOA_{i,n}$ corresponds to the n_{th} value obtained at the nth acquisition of the ith TDOA corresponding to the ith pair of stations

20 $\overline{TDOA_i}$ corresponds to the average, over the N_{acqui} acquisitions of the measurements of the ith TDOA.

6 – Method according to one of the preceding claims, characterized in that during the step of initializing the first location, the initialization value $\vec{X}_0$ is calculated on the basis of a priori information about the location of the transmitter device or on the basis of TDOA measurements making it possible to
25 determine an initialization value automatically.

7 – Method according to Claim 6, characterized in that the initialization value $\bar{X}_0$ is determined on the basis of the vector of TDOA measurements, with a pseudo-linear algorithm PSL, by executing the following steps:

- 5 • a reference station j, that of largest signal-to-noise ratio SNR or deemed the most reliable, is chosen,
- the unknown state vector to be estimated becomes $Y=(x_{sj},y_{sj},R_{sj})^T$ where R_{sj} is the distance of the transmitter and (x_{sj},y_{sj}) its coordinates, referenced with respect to the reference station j,
- the vector of pseudo-measurements M consists of terms that may be
 10 written: $\frac{1}{2}[D_{ij}^2 - (cTDOA_{ij})^2]$ where D_{ij} is the distance separating the stations i,j and $TDOA_{ij}$ is the measured delay between the signals received at these stations i and j, with $M = A Y$, where A is a matrix, consisting of the coordinates of the stations in relation to the reference station j, and of the terms $(cTDOA_{ij})$ arising from the delay
 15 measurements,
- the unknown vector Y is estimated by conventional solution of a linear least squares problem,
- the relevance of the components of the solution Y is verified, by verifying the consistency of the estimated distance R_{sj} with the
 20 estimated coordinates (x_{sj},y_{sj}) , by verifying that R_{sj} is positive and is not too far from $\sqrt{x_{sj}^2 + y_{sj}^2}$,
- in case of relevance, the estimation of the position of the transmitter is deduced directly from the vector Y,
- in case of non-relevance, the algorithm is relaunched while using
 25 another reference station.

8 – Method according to one of the preceding claims, characterized in that, as iterative algorithm, use is made of an algorithm for which each iteration

makes it possible to calculate, on the basis of a vector $\bar{X}_k$, a new vector $\bar{X}_{k+1}$ in the following manner:

$$\vec{x}_{k+1} = \vec{x}_k + c (F^T H^T \Sigma^{-1} H F)^{-1} F^T H^T \Sigma^{-1} (\vec{T} - \frac{H \vec{D}}{c})$$

where F is a jacobian matrix, calculated at the point $\bar{X}_k$, of the journey times between the transmitter device to be located and the N receiving stations, with:

$$F = \begin{pmatrix} \frac{(\vec{x}_k - \vec{S}_1)^T}{D_1} \\ \vdots \\ \frac{(\vec{x}_k - \vec{S}_N)^T}{D_N} \end{pmatrix}$$

row i of the matrix is the unit vector directed from station i towards the current estimation,

this equation is calculated at the point $\bar{X}_0$ thereby providing a new location point $\bar{X}_1$ which will be used at the following iteration to obtain a new point

$\bar{X}_2$ which will itself be used to obtain a new point until convergence or divergence is detected.

9 – Method according to one of the preceding claims, characterized in that the transmitter device is located on the basis of a set of elementary locations by executing the following steps:

- at each acquisition on the basis of the K elementary locations,
- filter values of aberrant locations, isolated locations or ones which are far from the average of the locations obtained,

- calculate a barycentre by averaging the elementary locations retained, corresponding to a value of location of the transmitter device to be located,
 - associate with this value a probability of error CEP determined in the following manner: consider a set of elementary locations $Loc_{x,y}$ taken for an acquisition, B the barycentre of the locations $Loc_{x,y}$ retained, CEP 50% of $Loc_{x,y}$ is the radius of the circle, centred on B, which contains half the locations of $Loc_{x,y}$.
- 10 – Method according to one of Claims 1 to 8, characterized in that the transmitter device is located on the basis of a set of elementary locations by executing the following steps:
- after N_{acqui} acquisitions and in the stationary case,
 - filter values of aberrant locations, isolated locations or ones which are far from the average of the locations obtained,
 - calculate a barycentre by averaging the elementary locations retained, corresponding to a value of location of the transmitter device to be located,
 - associate with this value a probability of error CEP determined in the following manner: consider a set of elementary locations $Loc_{x,y}$ taken for an acquisition, B the barycentre of the locations $Loc_{x,y}$ retained, CEP 50% of $Loc_{x,y}$ is the radius of the circle, centred on B, which contains half the locations of $Loc_{x,y}$.
- 11 – System for locating a transmitter device (20i) on the basis of N stations (10j) in a communications network, each of the stations being equipped with at least one receiver (12), by using at least one date-stamping benchmark common to all the N stations, characterized in that the network comprises at least one master station (1) suitable for executing the steps of the method according to one of Claims 1 to 9.