



REPLACED SHEETS	SG
☒ Description : paged 1... to paged 16....	
☒ Claim(s) : paged 17... to paged 20....	
☐ Drawing(s) : paged to paged	
☒ Abstract : paged 21... to paged-....	
DATE OF RECEIPT	

1

SZ 9-87-011

A METHOD OF GENERATING A COMPRESSED REPRESENTATION OF A SOURCE DATA STRING

FIELD OF THE INVENTION

This invention relates to a method of generating a compressed representation of a source data string. By "compression" is meant reducing the amount of data associated with the source data string without reducing its information content, so that output data may be constructed having a lower data content than the original source data, whilst still permitting the source data in its entirety to be reconstructed. In particular, the invention relates to a method for high efficiency, multiplication-free arithmetic coding. Arithmetic coding is a technique which is commonly employed in order to compress data in data processing systems.

BACKGROUND OF THE INVENTION

Such compression is rendered possible by the high degree of inter-correlation and non-uniform statistical distribution associated with the symbols usually characterizing large databases. When the source data is represented in digital form, the object of compression is to produce an output string of data having fewer data bits than the original source data string. When a statistically significant sample of source data strings are compressed, the expected length of the compressed output data string corresponds to the "entropy" of the source data string.



One way to achieve compression of source data is by means of the arithmetic coder developed by Rissanen and first published in an article entitled "Generalized Kraft Inequality Arithmetic Coding", IBM Journal of Research and Development, Volume 20, No. 3, May 1976. The Arithmetic coding procedure introduced by Rissanen permits the compression of multi-alphabet data, i.e. data each of whose symbols may be found within a multi-symbol alphabet.

In order to employ an arithmetic coding procedure, it is first necessary to determine the probability of occurrence for each symbol within the source alphabet. Typically, the probabilities will vary from one source data string to another so that if, for example, an arithmetic coding procedure is being used to compress the data corresponding to a T.V. image, the probabilities of occurrence for each picture element must first be determined. This may be determined in real time or, alternatively, may be predetermined statistically. However, the actual method for determining these probabilities is not a feature of the present invention which is equally applicable regardless of how the probabilities are determined.

Having determined the probabilities of occurrence for each different symbol within the source alphabet, the cumulative probability for each symbol may then be determined. Thus, the cumulative probability, $S(1)$ for the first symbol of the data string will be equal to zero and, in general, the cumulative probability, $S(n)$ for the n^{th} symbol will be equal to the sum of the probabilities of occurrence for each of the preceding $n-1$ symbols.

Arithmetic coding procedures normally represent the output data string as a binary fraction within the unit interval $[0,1)$. As is explained in "An Introduction to Arithmetic Coding" by Langdon, Jr. in the IBM Journal of Research and Development, Volume 28, No. 2, March 1984, arithmetic coding is related to the process of sub-dividing the unit interval. This sub-division is achieved by marking along the unit interval code points C_n for each symbol within the source alphabet, each code point being equal to the sum of the probabilities of occurrence of the preceding symbols. The width or size A_n



of the sub-interval to the right of each code point represents the probability of occurrence of the source data string up to the corresponding symbol (cf. Fig. 1).

Consider, for example, a source data string whose alphabet comprises symbols a_0 to a_m , having probabilities of occurrence equal to $p(0)$ to $p(m)$, respectively. If the source data string is $a_0 a_5 a_3 \dots$ then the first symbol a_0 will be encoded within the sub-interval $[0, p(0))$. This represents a first sub-interval within the original unit interval whose width A_1 is equal to $p(0)$ corresponding simply to the probability of occurrence of symbol a_0 . In order to encode the second symbol a_5 of the source data string, its probability of occurrence conditional on the probability of symbol a_0 occurring must be determined. Furthermore, the cumulative probability $S(5)$ associated with the second symbol a_5 must be calculated. Thus, the sub-interval corresponding to the second symbol a_5 is a second sub-interval within the first sub-interval corresponding to a_0 . Mathematically, the width A_2 of the second sub-interval is equal to $p(0) \cdot p(5)$, i.e. the product of the probabilities of occurrence of both symbols a_0 and a_5 . The starting point of the second sub-interval within the unit interval depends on the width A_1 of the first sub-interval and the cumulative probability $S(5)$ associated with the second symbol a_5 , being equal to their product $A_1 \cdot S(5)$.

Thus, as each symbol of the source data string is successively encoded within the unit interval, a succession of sub-intervals is generated each of which may be specified in terms of a specific code point and width. The code point for the current sub-interval corresponds to the start of the current sub-interval within the previous interval or sub-interval. As explained above, this is equal to the cumulative probability associated with the current symbol. Thus, the code point associated with the n^{th} sub-interval will be equal to the width of the $n-1^{\text{th}}$ sub-interval multiplied by the cumulative probability of the preceding $n-1$ symbols, i.e. $A_n S(n)$. The width of the new sub-interval itself will be equal to the product of the probabilities of all symbols (including the current one) so far encoded, i.e. $p(0) \cdot p(5) \cdot p(3) \dots$ for the above source data string. The data corresponding to



the width A_n and code points C_n of the n^{th} sub-interval thus encode the first $n+1$ symbols in the source data string. Arithmetic coders therefore require two memory registers, usually called the A and C registers, for storing these data.

Since the width of the sub-interval is equal to a product of probabilities, two factors emerge. First, as more symbols of the source data string are encoded, the width of the sub-interval defining the arithmetic code representation will decrease (since each individual probability must necessarily be smaller than 1); and, furthermore, the process of arithmetic coding requires a succession of multiplication operations for its effective implementation.

Although arithmetic coders produce optimum compression, corresponding to the entropy of the source data string, when based on the exact probabilities of occurrence of the symbols constituting the data string, in fact prior implementations of arithmetic coding procedures have tended to introduce approximations on account of the difficulty in determining the exact probabilities. Such approximations reduce the efficiency of the arithmetic coding operation and result in an output data string being generated which has more symbols than the theoretical minimum, or entropy. Moreover, further approximations have been introduced in order to eliminate the multiplication operation, which is required for determining the width of each successive sub-interval.

Arithmetic coders are implemented on computers whose memory registers contain a finite number of bits. One problem associated with multiplication results from the fact that, since the successive multiplication of probabilities always produces smaller and smaller intervals, after only a few such multiplications the resulting sub-interval may be too small to be satisfactorily stored in the computer register. For example, if each register has 16 bits, and the multiplication of successive probabilities results in a number smaller than 2^{-16} , this number will underflow the register. In other words, the register will be full of zeros, the significant bits of the probability



product being lost. A further problem associated with successive multiplication operations is the time taken for these to be implemented.

The first of the above drawbacks has been solved using a technique called normalization, whereby the probability product is stored in floating point notation. In order to do this, a further bit register is employed for storing the exponent (to base 2) for the width of the sub-interval, when the most significant 1 of the binary fraction is shifted to the extreme left-most position. Thus, the binary fraction 1.0101×2^{-20} which clearly cannot be stored in a 16-bit register, may satisfactorily be stored as $1.0101E-20$, wherein the mantissa and the exponent are stored in separate registers. Since the most significant bit of the mantissa is thus arranged always to be 1, the actual number stored in the mantissa register will always be greater than 1.0.

The second problem described above and related to the time taken to perform multiplication operations, has to some extent been solved by the implementation of so-called "multiplication-free" arithmetic coders. In computers which employ binary arithmetic, multiplication is implemented as a series of shift and add operations. The term "multiplication-free" has variously been used in the prior art to imply either a single shift or a single shift and add operation for each coding step. Strictly speaking, mathematically even a single shift operation constitutes multiplication. However, this is many orders of magnitude less time-consuming than the multiplication operations associated with precise implementations of arithmetic coders involving multiple shift and add operations. Thus, the term "multiplication-free" is often used to denote that multiplication operations are significantly simplified or reduced. It is in this context that the term "multiplication-free" has been used in the prior art and is used in the present invention.

U.S. Patent No. 4,286,256 (Langdon, Jr. et al.) discloses a method and means for arithmetic coding using a reduced number of operations. Langdon simplifies the multiplication operation by truncating one of the



inner products corresponding to the width of the sub-interval prior to encoding the current code-point. However, Langdon's method is suitable only for binary sources (i.e. alphabets containing only two symbols) wherein it is possible to encode each symbol of the source data string either as a more probable or less probable event. This procedure is unsuitable for multi-alphabet codes.

U.S. Patent No. 4,652,856 (Mohiuddin et al.) discloses a multiplication-free multi-alphabet arithmetic code in which each sub-interval is stored in floating point format, as explained above, such that the mantissa stored within the A register is a binary fraction greater than 0.1. In accordance with the approximation proposed by Mohiuddin, a variable criterion is adopted which either truncates the mantissa of the sub-interval to exactly 0.1 (binary) or, alternatively, rounds it up to 1. Such an approximation still achieves the desired compression, but at a loss of efficiency. In other words, more bits than the minimum are required for representing the compressed data string. The inefficiency associated with Mohiuddin's procedure depends on the nature of the source data being compressed.

The major drawback with Mohiuddin results from the fact that rounding up the contents of the A register approximates the probability of the corresponding symbol to a value greater than its actual value. In order to ensure that the sum of the probabilities for all symbols in the alphabet cannot exceed 1.0, Mohiuddin approximates the last sub-interval to:

$$A(m) = 1 - \sum_{i=0}^{m-1} p(i)$$

Whilst this approximation ensures that the sum of the probabilities of all symbols in the alphabet is equal to 1, it can achieve this at the expense of rendering the last sub-interval so small that the coding is highly inefficient.



SUMMARY OF THE INVENTION

It is an object of the invention to provide a method for high efficiency, multiplication-free arithmetic coding wherein the disadvantages associated with prior art methods are significantly reduced or eliminated.

According to the invention, there is provided a method of generating a compressed representation $R(\text{comp})$ of a source data string STR, as defined in Claim 1, and a method of decoding a compressed representation $R(\text{comp})$ of a source data string STR, as defined in Claim 6.

Thus, in accordance with the invention, each symbol within the source data string STR is successively coded, the representative compressed code thereof being shifted out of the C register for post processing as required. Typically, the compressed data which is successively read from the C register is transmitted to a suitable decoder which decodes the compressed string so as to reconstruct the original source data string STR. In common with the prior art arithmetic encoders referred to above, the method according to the invention is a first-in-first-out (FIFO) routine, whereby the first symbol in the source data string STR is also the first symbol to be encoded.

In a preferred embodiment, the A register is initialized to a value of 1.000, the compressed arithmetic code then being a value between 0 and 1.

Since each symbol within the source data string STR is encoded successively, it may occur that the code developed for subsequent symbols of the source data string STR will affect the code already generated for a previous symbol, owing to the effect of "carry over". The effect of carry over is well known in the literature, several solutions to the problem having been published, any one of which may suitably be used in conjunction with the proposed invention.



BRIEF DESCRIPTION OF THE DRAWINGS

The invention will now be described with regard to a method for generating a compressed representation $R(\text{comp})$ of a source data string STR, each symbol of which is taken from a finite set of $m+1$ symbols, and with reference to the accompanying drawings, in which:

Fig. 1 illustrates, for an example of an ordered symbol set comprising five symbols, the probabilities and the resulting cumulative probabilities;

Fig. 2 is a flow diagram showing a coding procedure according to the invention; and

Fig. 3 relates to Fig. 2 and illustrates the flow of data between registers when performing the coding procedure according to Fig. 2.

DETAILED DESCRIPTION OF PREFERRED EMBODIMENT

Fig. 1 shows a table of probabilities $p(i)$ and resulting cumulative probabilities $S(i)$ for an ordered symbol set comprising five symbols a_0 to a_4 , together with the corresponding code points C_n plotted on the unit interval $[0,1)$. As has been explained above, the process of recursively encoding successive symbols of a symbol set as a sub-interval within the unit interval, from the probabilities $p(i)$ and the cumulative probabilities $S(i)$ for the symbol a_i to be encoded, is a characteristic feature of arithmetic coding procedures per se.

Referring now to Fig. 2, there is shown a flow diagram of an encoding procedure according to the invention. The four main steps of the procedure are numbered 1 to 4, inclusive, for reasons which will become apparent below with reference to Fig. 3 of the drawings.

It is seen that at the start of the procedure, the C register is set to 0.000 and the A register is set to 1.000. In step (1) the probability $p(i)$ and



cumulative probability $S(i)$ are determined for each symbol a_i in the source data string STR. This can be done using any one of several known procedures. Alternatively, a fixed, predetermined set of probability values may be employed. A scaling factor K is then determined, as shown in step (2), by taking the contents of the A register and eliminating all binary digits less significant than the second most significant binary 1 digit. The A register represents the width of the current sub-interval, the scaling factor K being, in effect, an approximation of the width of the current sub-interval. This width is normalized between 1.0000 (binary) and 1.1111... and the scaling factor K is merely an approximation for the value of the A register eliminating all 1's less significant than the second.

In the description which follows, the symbols of the finite set of $m+1$ symbols are designated as $a_0, a_1, \dots, a_m$. The probability and cumulative probability for the symbol a_i is designated $p(i)$ and $S(i)$, respectively. Thus, if, for example, the eighth symbol of the multi-symbol alphabet is "h", then $a_8 = h$ and the respective probabilities are $p(8)$ and $S(8)$.

Step (3) shows that for each symbol in the source data string STR the product of K and $S(i)$ is added to the C register, and the A register is set to the product $K \cdot p(i)$, if $i < m$ and is reduced by the product $K \cdot S(m)$ if $i = m$. Thus, where the current symbol to be encoded does not correspond to the last symbol a_m within the multi-symbol alphabet, the A register is set simply to the probability of the current symbol times K , being an approximation for the width of the current sub-interval. On the other hand, when the current symbol to be encoded corresponds to the last symbol of the alphabet, the value of the A register is reduced by the product of K times the cumulative probability of the last symbol of the alphabet. This compensates for the fact that the probabilities $p(i)$ are only approximate and, in effect, ensures that the sum of the probabilities of all the symbols in the multi-symbol alphabet is equal to 1. This follows since for the m^{th} symbol of the alphabet:

$$A \leftarrow A - K \cdot S(m)$$

and since K is an approximation for A ,



$$A \leftarrow A - A \cdot S(m)$$

$$\text{Thus, } A \leftarrow A(1 - S(m))$$

$$\text{i.e. } A \leftarrow A(1 - p(0) + p(1) + \dots + p(m-1))$$

$$\text{i.e. } A \leftarrow A \cdot p(m),$$

which is exactly what is required.

The next step (4) in the algorithm is to normalize the A register so that its contents lie within a predetermined range. In order to do this, the contents of the A register are multiplied by an appropriate multiple of 2. This is achieved by a simple shift operation by means of which the contents of the A register are shifted left by a specified number of bits depending on the present contents of the A register. The C register is then shifted left by the same number of bits, the most significant bits thereof being read from the C register and constituting part of the arithmetic code for the source data string STR. This procedure is repeated for all symbols in the source data string STR and, on completion, the complete contents of the C register are shifted out of the C register and concatenated to the previously generated code string, the resulting code string representing the compressed output data string.

It will thus be seen that the method according to the invention generates a compressed output data string recursively for each symbol within the source data string STR. As the code is generated, it is shifted out of the C register whilst, at the same time, normalizing the A register.

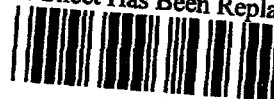
The method according to the invention will perhaps best be understood with reference to the following example.

EXAMPLE

A source data string is to be derived from a 5 symbol alphabet having symbols a_0 to a_4 equal to a,b,c,d,e, respectively, with probabilities (in binary) as follows:

$$p(0) = 0.000001$$

$$p(1) = 0.000011$$



$$p(2) = 0.000110$$

$$p(3) = 0.001010$$

$$\text{and } p(4) = 0.101100$$

Thus, the number of symbols, $m+1$, in the multi-system alphabet is equal to 5 and it may be verified that the sum of the probabilities is equal to one, i.e:

$$\sum_{i=0}^4 p(i) = 1$$

The cumulative probabilities for each symbol are as follows:

$$S(0) = 0.000000$$

$$S(1) = 0.000001$$

$$S(2) = 0.000100$$

$$S(3) = 0.001010$$

$$\text{and } S(4) = 0.101100$$

Using the method in accordance with the preferred embodiment as described above, encode the source data string given by:

$$\text{STR} = b,c,e,d,a,c,b$$

in conjunction with 6-bit A and C registers.

Proceed as follows:

Initialize the A register to 1.00000

and the C register to .000000

It will be appreciated that the binary point is understood, the actual contents of the A and C registers being 100000 and 000000, respectively.

Encode first symbol b ($= a_1$):

$$K = 1, \quad i = 1 \neq m$$

$$p(1) = 000011 \quad S(1) = 000001$$

$$C = 0.0 + 1.0 \times 0.000001 = 0.000001$$

$$\begin{array}{cc} \text{giving:} & A & C \\ & .000011 & .000001 \end{array}$$

Normalize A and shift C:

$$\begin{array}{cc} A & C \\ 1.10000 & 00000.100000 \end{array}$$



where the binary point denotes the boundary of the C register. Thus, the five binary bits 00000 to the left of the binary point denote the first five bits of the resultant arithmetic code shifted out of the C register, whilst the six binary bits 100000 to the right of the binary point denote the new contents of the C register. Note that the contents of both the A and C registers are right-packed with zeros after the corresponding left shifts. Since the symbol is not the end of the string STR, processing continues. Encode second symbol $c (= a_2)$:

$$\begin{aligned} K &= 1.1 & i &= 2 \neq m \\ p(2) &= 000110 & S(2) &= 000100 \\ C &= 0.100000 + 1.1 \times 0.0001 = 0.10011 \end{aligned}$$

giving: A C
 .001001 00000.100110

Normalize A and shift C:

$$\begin{aligned} A & & C \\ 1.00100 & & 00000100.110000 \end{aligned}$$

Since this symbol is not the end of the string STR, processing continues.

Encode third symbol $e (= a_4)$:

$$\begin{aligned} K &= 1.001 & i &= 4 = m \\ p(4) &= 101100 & S(4) &= 010100 \end{aligned}$$

Since $i = m$, $A = 1.00100 - 1.001 \times 0.01010 = 0.110010$

$$C = 0.110000 + 1.001 \times 0.01010 = 1.000110$$

Since $C > 1$, the leading 1 generates carry-over, giving:

$$\begin{aligned} A & & C \\ .110010 & & 00000101.000110 \end{aligned}$$

Normalize A and shift C:

$$\begin{aligned} A & & C \\ 1.10010 & & 000001010.001100 \end{aligned}$$

Since this symbol is not the end of the string STR, processing continues.

Encode fourth symbol $d (= a_3)$:

$$\begin{aligned} K &= 1.1 & i &= 3 \neq m \\ p(3) &= 001010 & S(3) &= 001010 \\ C &= 0.001100 + 1.1 \times 0.00101 = 0.011011 \end{aligned}$$

giving: A C



.001111 000001010.011011

Normalize A and shift C:

A C
1.11100 000001010011.011000

Since this symbol is not the end of the string STR, processing continues.

Encode fifth symbol a ($= a_0$):

$K = 1.1$ $i = 0 \neq m$
 $p(0) = 000001$ $S(0) = 000000$
 $C = 0.011000 + 0.0 = 0.011000$
giving: A C
 .000001 000001010011.011000

Normalize A and shift C:

A C
1.00000 000001010011011000.000000

Since this symbol is not the end of the string STR, processing continues.

Encode sixth symbol c ($= a_2$):

$K = 1.0$ $i = 2 \neq m$
 $p(2) = 000110$ $S(2) = 000100$
 $C = 0.0 + 1.0 \times 0.000100 = 0.000100$
giving: A C
 .000110 000001010011011000.000100

Normalize A and shift C:

A C
1.10000 0000010100110110000001.000000

Since this symbol is not the end of the string STR, processing continues.

Encode seventh symbol b ($= a_1$):

$K = 1.1$ $i = 1 \neq m$
 $p(1) = 000011$ $S(1) = 000001$
 $C = 0.0 + 1.1 \times 0.000001 = 0.000001$
giving: A C
 .000100 0000010100110110000001.000001

Normalize A and shift C:

A C



1.00000 00000101001101100000010000.010000

Since this is the last symbol in the string STR, now shift the contents of the C register left six places to produce the complete output string:

C = 00000101001101100000010000010000

Fig. 3 shows the flow of data between the registers for each of the steps (1) to (4) of the procedure described above with reference to Fig. 2.

Thus, during step (1) of the encoding procedure, each symbol a_i in turn is extracted from the INPUT string, a flag "END" being generated to indicate whether or not the symbol a_i is the last symbol in the INPUT string. The symbol a_i is input to a TABLE OF PROBABILITIES which may constantly be updated, so as to derive therefrom the probabilities $p(i)$ and $S(i)$. A flag " $i=m$ " is also generated and indicates whether or not the symbol a_i corresponds to the last symbol of the source alphabet.

During step (2), the contents of the A register are truncated in order to provide the scaling factor K.

During step (3) the values of $p(i)$ and $S(i)$ are multiplied by the value of K derived in step (2) using a single SHIFT and ADD operation, generating the values $K \cdot p(i)$ and $K \cdot S(i)$, respectively, which are stored in temporary buffers as shown. The values of A and $K \cdot S(i)$ are fed to a SUBTRACTOR whose output is equal to $A - K \cdot S(i)$. Likewise, the values of C and $K \cdot S(i)$ are fed to an ADDER whose output is equal to $C + K \cdot S(i)$. The output $A - K \cdot S(i)$ from the SUBTRACTOR and the value of $K \cdot p(i)$ are fed to a SELECT logic unit together with the value of the flag " $i=m$ ". The output from the SELECT unit is equal to $K \cdot p(i)$ if $i \neq m$ and to $A - K \cdot S(i)$ if $i = m$.

Finally, during step (4) a SHIFT CONTROL unit shifts the A and C registers until the contents of the A register lie within a specified range, when a STOP flag is generated, thereby stopping the shift operations. The "END" flag derived from step (1) is fed to the SHIFT CONTROL unit, so that at the end of the INPUT string, the SHIFT CONTROL unit shifts the C register six



places (6-bit A and C registers being employed), thereby emptying the C register. As the C register is shifted by the SHIFT CONTROL unit in response to the STOP or END flags, the data shifted out of the C register are concatenated in order to produce the OUTPUT STRING. Where required, a CARRY bit is also concatenated to the OUTPUT STRING, as explained with reference to Fig. 2 and the detailed example above.

Note that in the example just described, the A register is normalized such that it always lies within the range 1.0 to 2.0 i.e. $A \in B [1,2)$. It will readily be understood that the A register could be normalized between 0.1 and 1.0 (binary), or any multiple of 2 thereof, by employing exactly the same algorithm but dividing the probabilities by appropriate multiples of 2 (or multiplying them by appropriate multiples of the binary number 0.1). Furthermore, in the particular example shown, carry-over occurs when encoding the third symbol of the source data string STR. In the example shown, it is assumed that the resultant code is transmitted to the decoder only when all the symbols of the source data string STR have been encoded. Obviously, in practice it is desired to transmit the compressed code in real time as it is generated, rather than storing the concatenated string as it is shifted out of the C register and transmitting it only when the complete source data string STR has been encoded.

If desired, the invention may be employed within a modified method wherein the approximation effected to K is slightly more refined. Thus, if instead of truncating the A register after the second binary 1, it is truncated after the third binary 1, the method can still be employed at the price of a second shift and add operation. Obviously, this refinement can be extended even further such that in the limit the scaling factor K is exactly equal to the contents of the A register. This represents un-approximated arithmetic coding. In practice, however, the method described with reference to the figure achieves greater than 98% efficiency whilst approximating the contents of the A register as explained.



The process of decoding a compressed output data string as derived in accordance with the principles of the invention is essentially the reverse of encoding. Thus, initially the A register is filled with one (1.000...) and the C register is filled with the first 6 symbols of the compressed string $R(\text{comp})$.

Each symbol of the source data string STR is now decoded successively by determining the largest value of j such that $C \geq K \cdot S(j)$, where K and $S(j)$ are the same as defined above with respect to the encoder. Having thus determined j , the current symbol is a_j , where $a_0 \dots a_m$ is the multi-symbol alphabet defined above.

The value of $K \cdot S(j)$ is next subtracted from the contents of the C register, whilst the A register is modified according to whether or not $j = m$. If $j \neq m$, then the value of $K \cdot p(j)$ is inserted into the A register. On the other hand, if $j = m$, then the value of $K \cdot S(j)$ is subtracted from the contents of the A register.

Finally, the A and C registers are shifted in a predetermined direction until the contents of the A register lie with the range $A \in [1,2)$. The vacated positions in the A register are filled with zeros, and the those in the C register are filled with the corresponding number of most significant digits remaining in the compressed data string $R(\text{comp})$, the complete process being repeated until the compressed data string $R(\text{comp})$ is empty.



CLAIMS

1. A method of generating a compressed representation $R(\text{comp})$ of a source data string STR , wherein:

each symbol is taken from a finite set of $m+1$ symbols $a_0 \dots a_m$,

said compressed representation being a numeric value within a given range and being generated recursively in successive cycles for successive symbols in the string,

a probability value $p(i)$ and a cumulative probability value $S(i)$ being available for each symbol a_i ,

said cumulative probability value being

$$S(i) = p(0) + p(1) + \dots + p(i-1) \text{ for } i = 1 \dots m,$$

$$S(0) = 0,$$

and using a pair of shift registers A and C each comprising ω binary digit positions,

characterized by following steps:

(a) initially setting the contents of the C register to all zeros and the contents of the A register to a predetermined value; and

for each successive symbol a_i in the string:

(b) determining a scaling factor K by taking the contents of the A register and eliminating all binary digits less significant than a predetermined binary one digit;

(c) adding the value $K \cdot S(i)$ to the contents of the C register;

(d) determining the contents of the A register

(d1) if the symbol a_i is not the symbol a_m by inserting the value

$K \cdot p(i)$ into the A register, or

(d2) if the symbol a_i is the symbol a_m , by subtracting the value

$K \cdot S(i)$ from the contents of the A register;

(e) shifting both the A register and the C register in a predetermined direction until the contents of the A register lie within a predetermined range, filling the vacated positions with zeros; and

(f) if the symbol a_i were not the last symbol in string STR , repeating steps (b) to (e), or, if the symbol a_i were the last symbol in string STR , shifting the contents of the C register by ω positions in said predetermined direction.



whereby the sequence of binary digits shifted out of the C register constitutes said compressed representation $R(\text{comp})$ of the symbol string STR.

2. Method according to Claim 1, wherein said predetermined binary one digit is the second most significant binary one digit in the A register, such that the scaling factor K comprises at most two binary one digits separated by z zero digits.

3. Method according to Claim 1 or Claim 2, wherein said predetermined value is 1.000.

4. Method according to Claim 2 including the step of:

providing the value $K \cdot p(i)$ or $K \cdot S(i)$, respectively, by shifting the value $p(i)$ or $S(i)$, respectively, through $z + 1$ positions towards less significant positions, and adding the shifted quantity to the available value $p(i)$ or $S(i)$, respectively

5. Method according to any one of the preceding claims, wherein the predetermined range is from 1.0 (binary) to 1.111... (binary) and the predetermined direction is such as to increase the magnitude of the contents of the A register.

6. A method of decoding a compressed representation $R(\text{comp})$ of a source data string STR, wherein:

each symbol is taken from a finite set of $m+1$ symbols $a_0, \dots, a_m$,

said compressed representation being a numeric value within a given range,

a probability value $p(i)$ and a cumulative probability value $S(i)$ being available for each symbol a_i ,

said cumulative probability value being

$S(i) = p(0) + p(1) + \dots + p(i-1)$ for $i = 1 \dots m$,

$S(0) = 0$.



and using a pair of shift registers A and C each comprising ω binary digit positions,

characterized by following steps:

(a) initially setting the contents of the C register to the ω most significant digits of the compressed representation $R(\text{comp})$, and the contents of the A register to a predetermined value; and

decoding each successive symbol a_i in the source data string STR by:

(b) determining a scaling factor K by taking the contents of the A register and eliminating all binary digits less significant than a predetermined binary one digit;

(c) determining the largest value of j such that $C \geq K \cdot S(j)$, where:

C is interpreted as a binary fraction with a binary point outside the C register adjacent to the most significant binary digit therein, and a_i is the symbol output by the decoder at the current step;

(d) determining the contents of the A register

(d1) if the symbol a_i is not the symbol a_m by inserting the value $K \cdot p(j)$ into the A register, or

(d2) if the symbol a_i is the symbol a_m , by subtracting the value $K \cdot S(j)$ from the contents of the A register;

(e) subtracting the value $K \cdot S(j)$ from the contents of the C register;

(f) shifting both the A register and the C register in a predetermined direction until the contents of the A register lie within a predetermined range, filling the vacated positions in the A register with zeros and filling the vacated positions in the C register with the corresponding number of most significant digits remaining in the compressed representation $R(\text{comp})$, and repeating steps (b) to (f) until $R(\text{comp})$ is empty.

7. Method according to Claim 6, wherein said predetermined binary one digit is the second most significant binary one digit in the A register, such that the scaling factor K comprises at most two binary one digits separated by z zero digits.

8. Method according to Claim 6 or Claim 7, wherein the predetermined value is 1.0000... (binary).



9. Method according to Claim 7, further including the step of:
providing the value $K \cdot p(i)$ or $K \cdot S(i)$, respectively, by shifting the value $p(i)$ or $S(i)$, respectively, through $z + 1$ positions towards less significant positions, and adding the shifted quantity to the available value $p(i)$ or $S(i)$, respectively.
10. Method according to any one of Claims 6 to 9, wherein the predetermined range is from 1.0 (binary) to 1.111... (binary) and the predetermined direction is such as to increase the magnitude of the contents of the A register.



ABSTRACT

An improved method is disclosed for generating a compressed representation of a source data string each symbol of which is taken from a finite set of $m + 1$ symbols, a_0 to a_m . The method is based on an arithmetic coding procedure wherein the compressed representation is recursively generated as successive sub-intervals within a predetermined interval. The width of each sub-interval is theoretically equal to the width of the previous sub-interval multiplied by the probability of the current symbol. The improvement is achieved by approximating the width of the previous sub-interval so that the approximation can be obtained by a single shift and add operation using a suitable shift register.

~~THIS SHEET HAS BEEN
REPLACED~~

	$p(i)$	$S(i)$
a_0	0.3	0.0
a_1	0.1	0.3
a_2	0.4	0.4
a_3	0.1	0.8
a_4	0.1	0.9

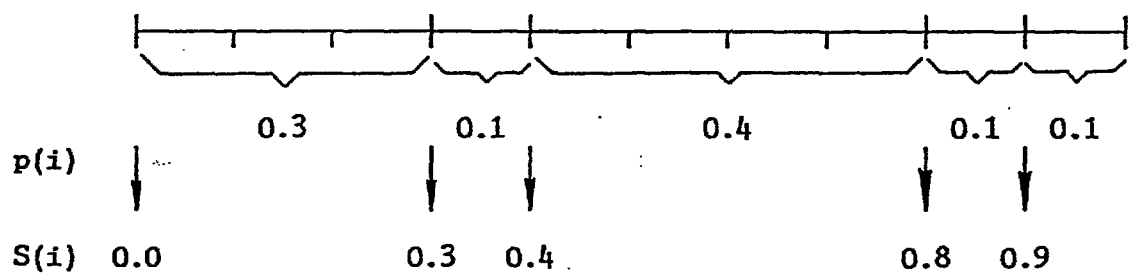


Fig. 1

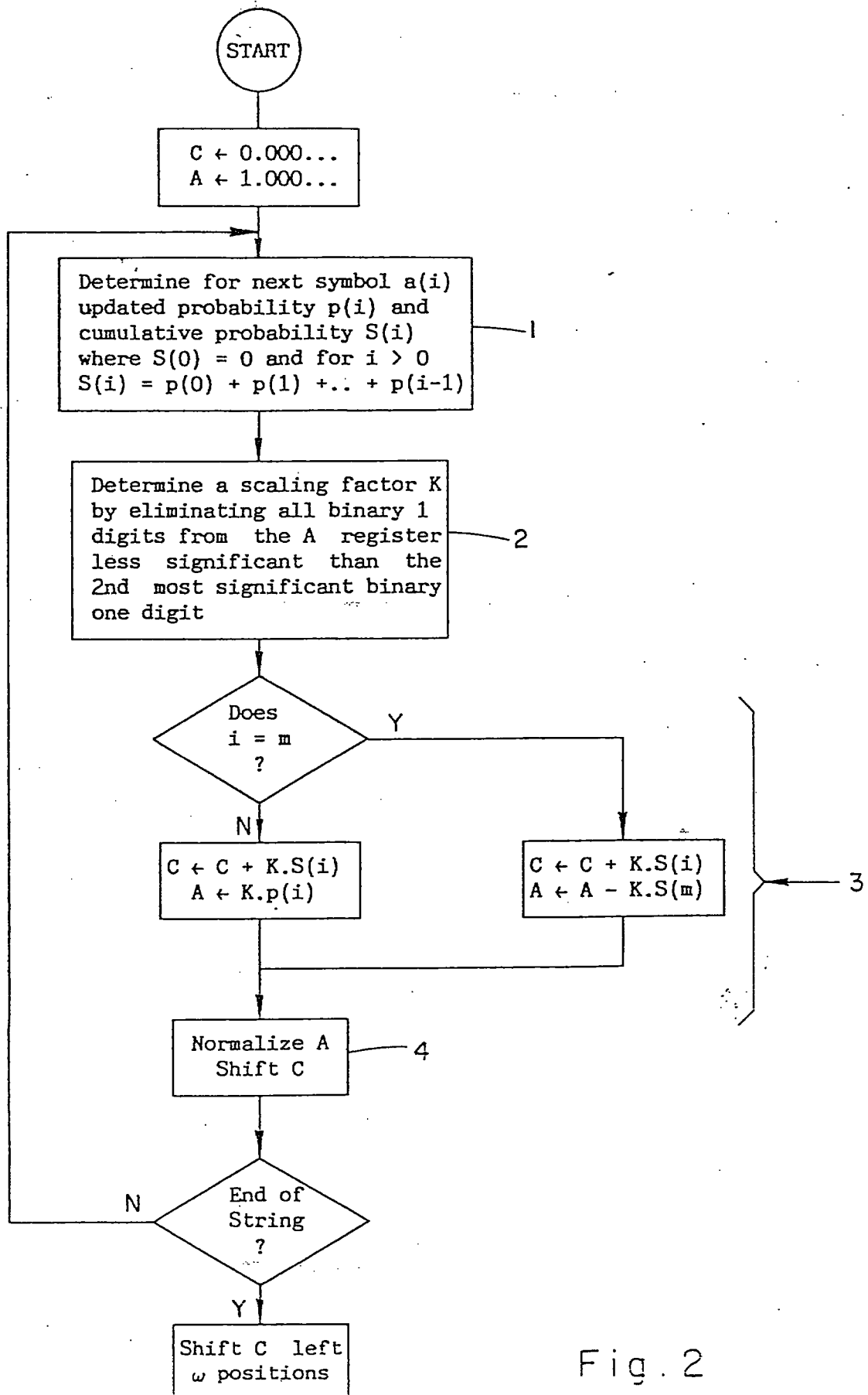
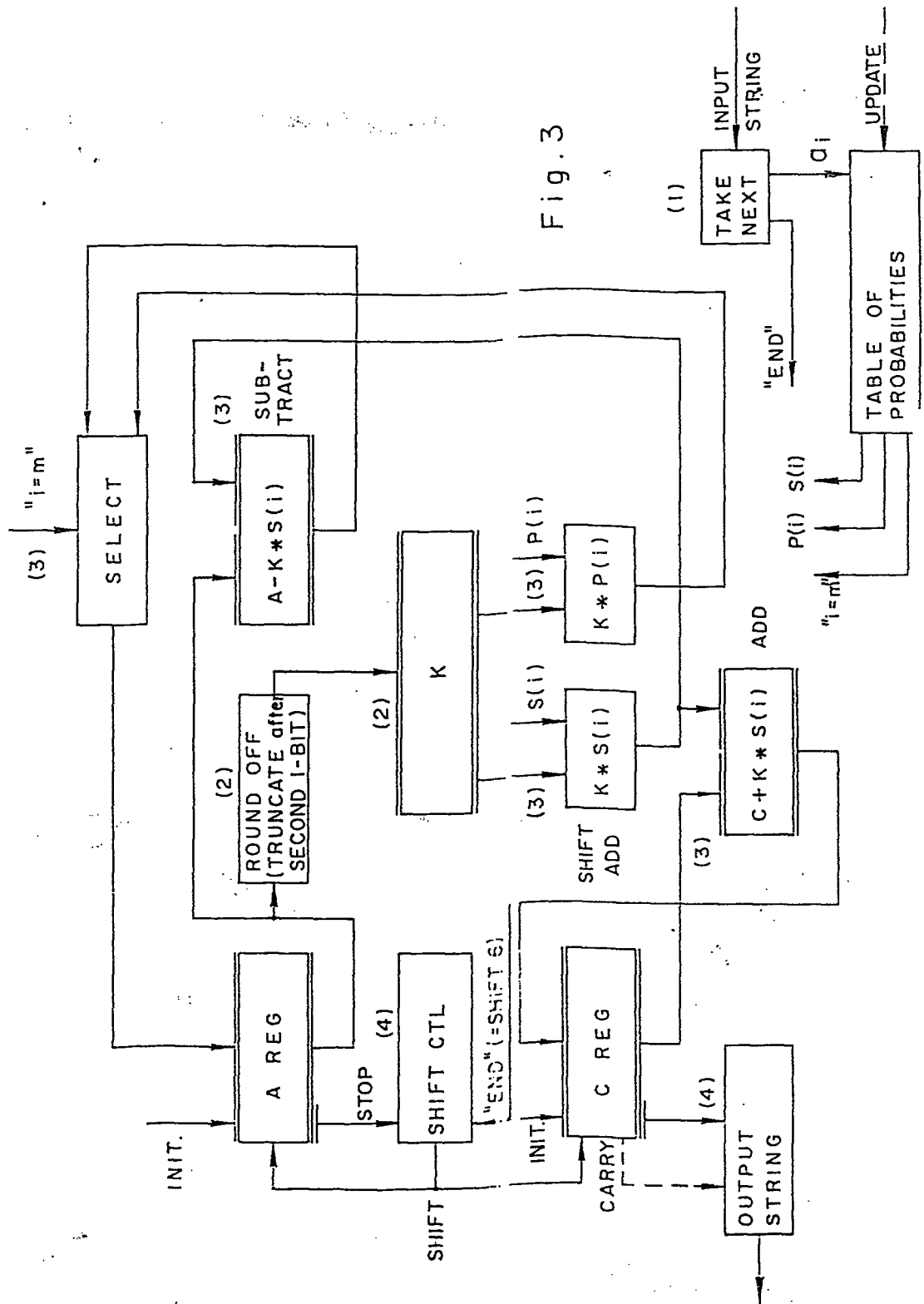


Fig. 2

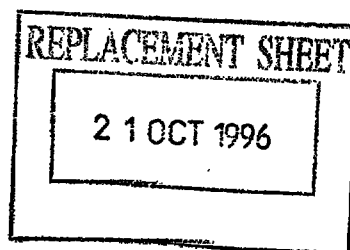


A METHOD OF GENERATING A COMPRESSED REPRESENTATION OF A SOURCE DATA STRING

ABSTRACT

An improved method is disclosed for generating a compressed representation of a source data string each symbol of which is taken from a finite set of $m + 1$ symbols, a_0 to a_m . The method is based on an arithmetic coding procedure wherein the compressed representation is recursively generated (1) as successive sub-intervals within a predetermined interval. The width of each sub-interval multiplied by the probability of the current symbol. The improvement is achieved by approximating the width of the previous sub-interval so that the approximation can be obtained by a single shift and add operation (4) using a suitable shift register.

Fig. 2



22 FEB 1997

1

REPLACED SHEETS		SG
☒	Description	: paged ...7... to paged ...7...
☒	Claim(s)	: paged ...7... to paged ...20...
☐	Drawing(s)	: paged to paged
☐	Abstract	: paged to paged
DATE OF RECEIPT		

REPLACEMENT SHEETS		SG
☒	Description	: paged ...1... to paged ...14...
☒	Claim(s)	: paged ...17... to paged ...22...
☐	Drawing(s)	: paged to paged
☒	Abstract	: paged ...7... to paged ...7...
DATE OF RECEIPT ...22/2/97.....		

A METHOD OF GENERATING A COMPRESSED REPRESENTATION OF A SOURCE DATA STRING

FIELD OF THE INVENTION

This invention relates to a method of generating a compressed representation of a source data string. By "compression" is meant reducing the amount of data associated with the source data string without reducing its information content, so that output data may be constructed having a lower data content than the original source data, whilst still permitting the source data in its entirety to be reconstructed. In particular, the invention relates to a method for high efficiency, multiplication-free arithmetic coding. Arithmetic coding is a technique which is commonly employed in order to compress data in data processing systems.

BACKGROUND OF THE INVENTION

Such compression is rendered possible by the high degree of inter-correlation and non-uniform statistical distribution associated with the symbols usually characterizing large databases. When the source data is represented in digital form, the object of compression is to produce an output string of data having fewer data bits than the original source data string. When a statistically significant sample of source data strings are compressed, the expected length of the compressed output data string corresponds to the "entropy" of the source data string.

22 FEB 1997

One way to achieve compression of source data is by means of the arithmetic coder developed by Rissanen and first published in an article entitled "Generalized Kraft Inequality Arithmetic Coding", IBM Journal of Research and Development, Volume 20, No. 3, May 1976. The Arithmetic coding procedure introduced by Rissanen permits the compression of multi-alphabet data, i.e. data each of whose symbols may be found within a multi-symbol alphabet.

In order to employ an arithmetic coding procedure, it is first necessary to determine the probability of occurrence for each symbol within the source alphabet. Typically, the probabilities will vary from one source data string to another so that if, for example, an arithmetic coding procedure is being used to compress the data corresponding to a T.V. image, the probabilities of occurrence for each picture element must first be determined. This may be determined in real time or, alternatively, may be predetermined statistically. However, the actual method for determining these probabilities is not a feature of the present invention which is equally applicable regardless of how the probabilities are determined.

Having determined the probabilities of occurrence for each different symbol within the source alphabet, the cumulative probability for each symbol may then be determined. Thus, the cumulative probability, $S(1)$ for the first symbol of the data string will be equal to zero and, in general, the cumulative probability, $S(n)$ for the n^{th} symbol will be equal to the sum of the probabilities of occurrence for each of the preceding $n-1$ symbols.

Arithmetic coding procedures normally represent the output data string as a binary fraction within the unit interval $[0,1)$. As is explained in "An Introduction to Arithmetic Coding" by Langdon, Jr. in the IBM Journal of Research and Development, Volume 28, No. 2, March 1984, arithmetic coding is related to the process of sub-dividing the unit interval. This sub-division is achieved by marking along the unit interval code points C_n for each symbol within the source alphabet, each code point being equal to the sum of the probabilities of occurrence of the preceding symbols. The width or size A_n

22 FEB 1997

of the sub-interval to the right of each code point represents the probability of occurrence of the source data string up to the corresponding symbol (cf. Fig. 1).

Consider, for example, a source data string whose alphabet comprises symbols a_0 to a_m , having probabilities of occurrence equal to $p(0)$ to $p(m)$, respectively. If the source data string is $a_0 a_5 a_3 \dots$ then the first symbol a_0 will be encoded within the sub-interval $[0, p(0))$. This represents a first sub-interval within the original unit interval whose width A_1 is equal to $p(0)$ corresponding simply to the probability of occurrence of symbol a_0 . In order to encode the second symbol a_5 of the source data string, its probability of occurrence conditional on the probability of symbol a_0 occurring must be determined. Furthermore, the cumulative probability $S(5)$ associated with the second symbol a_5 must be calculated. Thus, the sub-interval corresponding to the second symbol a_5 is a second sub-interval within the first sub-interval corresponding to a_0 . Mathematically, the width A_2 of the second sub-interval is equal to $p(0) \cdot p(5)$, i.e. the product of the probabilities of occurrence of both symbols a_0 and a_5 . The starting point of the second sub-interval within the unit interval depends on the width A_1 of the first sub-interval and the cumulative probability $S(5)$ associated with the second symbol a_5 , being equal to their product $A_1 \cdot S(5)$.

Thus, as each symbol of the source data string is successively encoded within the unit interval, a succession of sub-intervals is generated each of which may be specified in terms of a specific code point and width. The code point for the current sub-interval corresponds to the start of the current sub-interval within the previous interval or sub-interval. As explained above, this is equal to the cumulative probability associated with the current symbol. Thus, the code point associated with the n^{th} sub-interval will be equal to the width of the $n-1^{\text{th}}$ sub-interval multiplied by the cumulative probability of the preceding $n-1$ symbols, i.e. $A_n S(n)$. The width of the new sub-interval itself will be equal to the product of the probabilities of all symbols (including the current one) so far encoded, i.e. $p(0) \cdot p(5) \cdot p(3) \dots$ for the above source data string. The data corresponding to

22 FEB 1997

the width A_n and code points C_n of the n^{th} sub-interval thus encode the first $n+1$ symbols in the source data string. Arithmetic coders therefore require two memory registers, usually called the A and C registers, for storing these data.

Since the width of the sub-interval is equal to a product of probabilities, two factors emerge. First, as more symbols of the source data string are encoded, the width of the sub-interval defining the arithmetic code representation will decrease (since each individual probability must necessarily be smaller than 1); and, furthermore, the process of arithmetic coding requires a succession of multiplication operations for its effective implementation.

Although arithmetic coders produce optimum compression, corresponding to the entropy of the source data string, when based on the exact probabilities of occurrence of the symbols constituting the data string, in fact prior implementations of arithmetic coding procedures have tended to introduce approximations on account of the difficulty in determining the exact probabilities. Such approximations reduce the efficiency of the arithmetic coding operation and result in an output data string being generated which has more symbols than the theoretical minimum, or entropy. Moreover, further approximations have been introduced in order to eliminate the multiplication operation, which is required for determining the width of each successive sub-interval.

Arithmetic coders are implemented on computers whose memory registers contain a finite number of bits. One problem associated with multiplication results from the fact that, since the successive multiplication of probabilities always produces smaller and smaller intervals, after only a few such multiplications the resulting sub-interval may be too small to be satisfactorily stored in the computer register. For example, if each register has 16 bits, and the multiplication of successive probabilities results in a number smaller than 2^{-16} , this number will underflow the register. In other words, the register will be full of zeros, the significant bits of the probability

22 FEB 1997

product being lost. A further problem associated with successive multiplication operations is the time taken for these to be implemented.

The first of the above drawbacks has been solved using a technique called normalization, whereby the probability product is stored in floating point notation. In order to do this, a further bit register is employed for storing the exponent (to base 2) for the width of the sub-interval, when the most significant 1 of the binary fraction is shifted to the extreme left-most position. Thus, the binary fraction 1.0101×2^{-20} which clearly cannot be stored in a 16-bit register, may satisfactorily be stored as 1.0101E-20, wherein the mantissa and the exponent are stored in separate registers. Since the most significant bit of the mantissa is thus arranged always to be 1, the actual number stored in the mantissa register will always be greater than 1.0.

The second problem described above and related to the time taken to perform multiplication operations, has to some extent been solved by the implementation of so-called "multiplication-free" arithmetic coders. In computers which employ binary arithmetic, multiplication is implemented as a series of shift and add operations. The term "multiplication-free" has variously been used in the prior art to imply either a single shift or a single shift and add operation for each coding step. Strictly speaking, mathematically even a single shift operation constitutes multiplication. However, this is many orders of magnitude less time-consuming than the multiplication operations associated with precise implementations of arithmetic coders involving multiple shift and add operations. Thus, the term "multiplication-free" is often used to denote that multiplication operations are significantly simplified or reduced. It is in this context that the term "multiplication-free" has been used in the prior art and is used in the present invention.

U.S. Patent No. 4,286,256 (Langdon, Jr. et al.) discloses a method and means for arithmetic coding using a reduced number of operations. Langdon simplifies the multiplication operation by truncating one of the

22 FEB 1997

inner products corresponding to the width of the sub-interval prior to encoding the current code-point. However, Langdon's method is suitable only for binary sources (i.e. alphabets containing only two symbols) wherein it is possible to encode each symbol of the source data string either as a more probable or less probable event. This procedure is unsuitable for multi-alphabet codes.

U.S. Patent No. 4,652,856 (Mohiuddin et al.) discloses a multiplication-free multi-alphabet arithmetic code in which each sub-interval is stored in floating point format, as explained above, such that the mantissa stored within the A register is a binary fraction greater than 0.1. In accordance with the approximation proposed by Mohiuddin, a variable criterion is adopted which either truncates the mantissa of the sub-interval to exactly 0.1 (binary) or, alternatively, rounds it up to 1. Such an approximation still achieves the desired compression, but at a loss of efficiency. In other words, more bits than the minimum are required for representing the compressed data string. The inefficiency associated with Mohiuddin's procedure depends on the nature of the source data being compressed.

The major drawback with Mohiuddin results from the fact that rounding up the contents of the A register approximates the probability of the corresponding symbol to a value greater than its actual value. In order to ensure that the sum of the probabilities for all symbols in the alphabet cannot exceed 1.0, Mohiuddin approximates the last sub-interval to:

$$A(m) = 1 - \sum_{i=0}^{m-1} p(i) .$$

Whilst this approximation ensures that the sum of the probabilities of all symbols in the alphabet is equal to 1, it can achieve this at the expense of rendering the last sub-interval so small that the coding is highly inefficient.

This Sheet Has Been Replaced



7

REPLACEMENT SHEET

22 FEB 1997

SUMMARY OF THE INVENTION

It is an object of the invention to provide a method for high efficiency, multiplication-free arithmetic coding wherein the disadvantages associated with prior art methods are significantly reduced or eliminated.

According to the invention, there is provided a method of generating a compressed representation $R(\text{comp})$ of a source data string STR, as defined in Claim 1, and a method of decoding a compressed representation $R(\text{comp})$ of a source data string STR, as defined in Claim 6.

Thus, in accordance with the invention, each symbol within the source data string STR is successively coded, the representative compressed code thereof being shifted out of the C register for post processing as required. Typically, the compressed data which is successively read from the C register is transmitted to a suitable decoder which decodes the compressed string so as to reconstruct the original source data string STR. In common with the prior art arithmetic encoders referred to above, the method according to the invention is a first-in-first-out (FIFO) routine, whereby the first symbol in the source data string STR is also the first symbol to be encoded.

In a preferred embodiment, the A register is initialized to a value of 1.000, the compressed arithmetic code then being a value between 0 and 1.

Since each symbol within the source data string STR is encoded successively, it may occur that the code developed for subsequent symbols of the source data string STR will affect the code already generated for a previous symbol, owing to the effect of "carry over". The effect of carry over is well known in the literature, several solutions to the problem having been published, any one of which may suitably be used in conjunction with the proposed invention.

22 FEB 1997

BRIEF DESCRIPTION OF THE DRAWINGS

The invention will now be described with regard to a method for generating a compressed representation $R(\text{comp})$ of a source data string STR, each symbol of which is taken from a finite set of $m+1$ symbols, and with reference to the accompanying drawings, in which:

Fig. 1 illustrates, for an example of an ordered symbol set comprising five symbols, the probabilities and the resulting cumulative probabilities;

Fig. 2 is a flow diagram showing a coding procedure according to the invention; and

Fig. 3 relates to Fig. 2 and illustrates the flow of data between registers when performing the coding procedure according to Fig. 2.

DETAILED DESCRIPTION OF PREFERRED EMBODIMENT

Fig. 1 shows a table of probabilities $p(i)$ and resulting cumulative probabilities $S(i)$ for an ordered symbol set comprising five symbols a_0 to a_4 , together with the corresponding code points C_n plotted on the unit interval $[0,1)$. As has been explained above, the process of recursively encoding successive symbols of a symbol set as a sub-interval within the unit interval, from the probabilities $p(i)$ and the cumulative probabilities $S(i)$ for the symbol a_i to be encoded, is a characteristic feature of arithmetic coding procedures per se.

Referring now to Fig. 2, there is shown a flow diagram of an encoding procedure according to the invention. The four main steps of the procedure are numbered 1 to 4, inclusive, for reasons which will become apparent below with reference to Fig. 3 of the drawings.

It is seen that at the start of the procedure, the C register is set to 0.000 and the A register is set to 1.000. In step (1) the probability $p(i)$ and

22 FEB 1997

cumulative probability $S(i)$ are determined for each symbol a_i in the source data string STR. This can be done using any one of several known procedures. Alternatively, a fixed, predetermined set of probability values may be employed. A scaling factor K is then determined, as shown in step (2), by taking the contents of the A register and eliminating all binary digits less significant than the second most significant binary 1 digit. The A register represents the width of the current sub-interval, the scaling factor K being, in effect, an approximation of the width of the current sub-interval. This width is normalized between 1.0000 (binary) and 1.1111... and the scaling factor K is merely an approximation for the value of the A register eliminating all 1's less significant than the second.

In the description which follows, the symbols of the finite set of $m+1$ symbols are designated as $a_0, a_1, \dots, a_m$. The probability and cumulative probability for the symbol a_i is designated $p(i)$ and $S(i)$, respectively. Thus, if, for example, the eighth symbol of the multi-symbol alphabet is "h", then $a_8 = h$ and the respective probabilities are $p(8)$ and $S(8)$.

Step (3) shows that for each symbol in the source data string STR the product of K and $S(i)$ is added to the C register, and the A register is set to the product $K \cdot p(i)$, if $i < m$ and is reduced by the product $K \cdot S(m)$ if $i = m$. Thus, where the current symbol to be encoded does not correspond to the last symbol a_m within the multi-symbol alphabet, the A register is set simply to the probability of the current symbol times K , being an approximation for the width of the current sub-interval. On the other hand, when the current symbol to be encoded corresponds to the last symbol of the alphabet, the value of the A register is reduced by the product of K times the cumulative probability of the last symbol of the alphabet. This compensates for the fact that the probabilities $p(i)$ are only approximate and, in effect, ensures that the sum of the probabilities of all the symbols in the multi-symbol alphabet is equal to 1. This follows since for the m^{th} symbol of the alphabet:

$$A \leftarrow A - K \cdot S(m)$$

and since K is an approximation for A ,

22 FEB 1997

$$A \leftarrow A - A \cdot S(m)$$

$$\text{Thus, } A \leftarrow A(1 - S(m))$$

$$\text{i.e. } A \leftarrow A(1 - p(0) + p(1) + \dots + p(m-1))$$

$$\text{i.e. } A \leftarrow A \cdot p(m),$$

which is exactly what is required.

The next step (4) in the algorithm is to normalize the A register so that its contents lie within a predetermined range. In order to do this, the contents of the A register are multiplied by an appropriate multiple of 2. This is achieved by a simple shift operation by means of which the contents of the A register are shifted left by a specified number of bits depending on the present contents of the A register. The C register is then shifted left by the same number of bits, the most significant bits thereof being read from the C register and constituting part of the arithmetic code for the source data string STR. This procedure is repeated for all symbols in the source data string STR and, on completion, the complete contents of the C register are shifted out of the C register and concatenated to the previously generated code string, the resulting code string representing the compressed output data string.

It will thus be seen that the method according to the invention generates a compressed output data string recursively for each symbol within the source data string STR. As the code is generated, it is shifted out of the C register whilst, at the same time, normalizing the A register.

The method according to the invention will perhaps best be understood with reference to the following example.

EXAMPLE

A source data string is to be derived from a 5 symbol alphabet having symbols a_0 to a_4 equal to a,b,c,d,e, respectively, with probabilities (in binary) as follows:

$$p(0) = 0.000001$$

$$p(1) = 0.000011$$

22 FEB 1997

$$p(2) = 0.000110$$

$$p(3) = 0.001010$$

$$\text{and } p(4) = 0.101100$$

Thus, the number of symbols, $m+1$, in the multi-system alphabet is equal to 5 and it may be verified that the sum of the probabilities is equal to one, i.e:

$$\sum_{i=0}^4 p(i) = 1$$

The cumulative probabilities for each symbol are as follows:

$$S(0) = 0.000000$$

$$S(1) = 0.000001$$

$$S(2) = 0.000100$$

$$S(3) = 0.001010$$

$$\text{and } S(4) = 0.010100$$

Using the method in accordance with the preferred embodiment as described above, encode the source data string given by:

$$\text{STR} = b, c, e, d, a, c, b$$

In conjunction with 6-bit A and C registers.

Proceed as follows:

Initialize the A register to 1.000000

and the C register to .000000

It will be appreciated that the binary point is understood, the actual contents of the A and C registers being 100000 and 000000, respectively.

Encode first symbol $b (= a_1)$:

$$K = 1, \quad i = 1 \neq m$$

$$p(1) = 000011 \quad S(1) = 000001$$

$$C = 0.0 + 1.0 \times 0.000001 = 0.000001$$

$$\begin{array}{cc} \text{giving:} & A & C \\ & .000011 & .000001 \end{array}$$

Normalize A and shift C:

$$\begin{array}{cc} A & C \\ 1.10000 & 00000.100000 \end{array}$$

22 FEB 1997

where the binary point denotes the boundary of the C register. Thus, the five binary bits 00000 to the left of the binary point denote the first five bits of the resultant arithmetic code shifted out of the C register, whilst the six binary bits 100000 to the right of the binary point denote the new contents of the C register. Note that the contents of both the A and C registers are right-packed with zeros after the corresponding left shifts. Since the symbol is not the end of the string STR, processing continues. Encode second symbol $c (= a_2)$:

$$K = 1.1$$

$$i = 2 \neq m$$

$$p(2) = 000110$$

$$S(2) = 000100$$

$$C = 0.100000 + 1.1 \times 0.0001 = 0.10011$$

giving:

$$\begin{array}{cc} A & C \\ .001001 & 00000.100110 \end{array}$$

Normalize A and shift C:

$$\begin{array}{cc} A & C \\ 1.00100 & 00000100.110000 \end{array}$$

Since this symbol is not the end of the string STR, processing continues.

Encode third symbol $e (= a_4)$:

$$K = 1.001$$

$$i = 4 = m$$

$$p(4) = 101100$$

$$S(4) = 010100$$

Since $i = m$, $A = 1.00100 - 1.001 \times 0.01010 = 0.110010$

$$C = 0.110000 + 1.001 \times 0.01010 = 1.000110$$

Since $C > 1$, the leading 1 generates carry-over, giving:

$$\begin{array}{cc} A & C \\ .110010 & 00000101.000110 \end{array}$$

Normalize A and shift C:

$$\begin{array}{cc} A & C \\ 1.10010 & 000001010.001100 \end{array}$$

Since this symbol is not the end of the string STR, processing continues.

Encode fourth symbol $d (= a_3)$:

$$K = 1.1$$

$$i = 3 \neq m$$

$$p(3) = 001010$$

$$S(3) = 001010$$

$$C = 0.001100 + 1.1 \times 0.00101 = 0.011011$$

giving:

$$\begin{array}{cc} A & C \end{array}$$

22 FEB 1997

.001111 000001010.011011

Normalize A and shift C:

A C
1.11100 000001010011.011000

Since this symbol is not the end of the string STR, processing continues.

Encode fifth symbol a ($= a_0$):

$K = 1.1$ $i = 0 \neq m$
 $p(0) = 000001$ $S(0) = 000000$
 $C = 0.011000 + 0.0 = 0.011000$

giving: A C
.000001 000001010011.011000

Normalize A and shift C:

A C
1.00000 000001010011011000.000000

Since this symbol is not the end of the string STR, processing continues.

Encode sixth symbol c ($= a_2$):

$K = 1.0$ $i = 2 \neq m$
 $p(2) = 000110$ $S(2) = 000100$
 $C = 0.0 + 1.0 \times 0.000100 = 0.000100$

giving: A C
.000110 000001010011011000.000100

Normalize A and shift C:

A C
1.10000 0000010100110110000001.000000

Since this symbol is not the end of the string STR, processing continues.

Encode seventh symbol b ($= a_1$):

$K = 1.1$ $i = 1 \neq m$
 $p(1) = 000011$ $S(1) = 000001$
 $C = 0.0 + 1.1 \times 0.000001 = 0.000001$

giving: A C
.000100 0000010100110110000001.000001

Normalize A and shift C:

A C

22 FEB 1997

1.00000 00000101001101100000010000.010000

Since this is the last symbol in the string STR, now shift the contents of the C register left six places to produce the complete output string:

C = 00000101001101100000010000010000

Fig. 3 shows the flow of data between the registers for each of the steps (1) to (4) of the procedure described above with reference to Fig. 2.

Thus, during step (1) of the encoding procedure, each symbol a_i in turn is extracted from the INPUT string, a flag "END" being generated to indicate whether or not the symbol a_i is the last symbol in the INPUT string. The symbol a_i is input to a TABLE OF PROBABILITIES which may constantly be updated, so as to derive therefrom the probabilities $p(i)$ and $S(i)$. A flag " $i=m$ " is also generated and indicates whether or not the symbol a_i corresponds to the last symbol of the source alphabet.

During step (2), the contents of the A register are truncated in order to provide the scaling factor K.

During step (3) the values of $p(i)$ and $S(i)$ are multiplied by the value of K derived in step (2) using a single SHIFT and ADD operation, generating the values $K \cdot p(i)$ and $K \cdot S(i)$, respectively, which are stored in temporary buffers as shown. The values of A and $K \cdot S(i)$ are fed to a SUBTRACTOR whose output is equal to $A - K \cdot S(i)$. Likewise, the values of C and $K \cdot S(i)$ are fed to an ADDER whose output is equal to $C + K \cdot S(i)$. The output $A - K \cdot S(i)$ from the SUBTRACTOR and the value of $K \cdot p(i)$ are fed to a SELECT logic unit together with the value of the flag " $i=m$ ". The output from the SELECT unit is equal to $K \cdot p(i)$ if $i \neq m$ and to $A - K \cdot S(i)$ if $i = m$.

Finally, during step (4) a SHIFT CONTROL unit shifts the A and C registers until the contents of the A register lie within a specified range, when a STOP flag is generated, thereby stopping the shift operations. The "END" flag derived from step (1) is fed to the SHIFT CONTROL unit, so that at the end of the INPUT string, the SHIFT CONTROL unit shifts the C register six

22 FEB 1997

places (6-bit A and C registers being employed), thereby emptying the C register. As the C register is shifted by the SHIFT CONTROL unit in response to the STOP or END flags, the data shifted out of the C register are concatenated in order to produce the OUTPUT STRING. Where required, a CARRY bit is also concatenated to the OUTPUT STRING, as explained with reference to Fig. 2 and the detailed example above.

Note that in the example just described, the A register is normalized such that it always lies within the range 1.0 to 2.0 i.e. $A \in B [1,2)$. It will readily be understood that the A register could be normalized between 0.1 and 1.0 (binary), or any multiple of 2 thereof, by employing exactly the same algorithm but dividing the probabilities by appropriate multiples of 2 (or multiplying them by appropriate multiples of the binary number 0.1). Furthermore, in the particular example shown, carry-over occurs when encoding the third symbol of the source data string STR. In the example shown, it is assumed that the resultant code is transmitted to the decoder only when all the symbols of the source data string STR have been encoded. Obviously, in practice it is desired to transmit the compressed code in real time as it is generated, rather than storing the concatenated string as it is shifted out of the C register and transmitting it only when the complete source data string STR has been encoded.

If desired, the invention may be employed within a modified method wherein the approximation effected to K is slightly more refined. Thus, if instead of truncating the A register after the second binary 1, it is truncated after the third binary 1, the method can still be employed at the price of a second shift and add operation. Obviously, this refinement can be extended even further such that in the limit the scaling factor K is exactly equal to the contents of the A register. This represents un-approximated arithmetic coding. In practice, however, the method described with reference to the figure achieves greater than 98% efficiency whilst approximating the contents of the A register as explained.

22 FEB 1997

The process of decoding a compressed output data string as derived in accordance with the principles of the invention is essentially the reverse of encoding. Thus, initially the A register is filled with one (1.000...) and the C register is filled with the first 6 symbols of the compressed string R(comp).

Each symbol of the source data string STR is now decoded successively by determining the largest value of j such that $C \geq K \cdot S(j)$, where K and $S(j)$ are the same as defined above with respect to the encoder. Having thus determined j , the current symbol is a_j , where $a_0 \dots a_m$ is the multi-symbol alphabet defined above.

The value of $K \cdot S(j)$ is next subtracted from the contents of the C register, whilst the A register is modified according to whether or not $j = m$. If $j \neq m$, then the value of $K \cdot p(j)$ is inserted into the A register. On the other hand, if $j = m$, then the value of $K \cdot S(j)$ is subtracted from the contents of the A register.

Finally, the A and C registers are shifted in a predetermined direction until the contents of the A register lie with the range $A \in [1,2)$. The vacated positions in the A register are filled with zeros, and the those in the C register are filled with the corresponding number of most significant digits remaining in the compressed data string R(comp), the complete process being repeated until the compressed data string R(comp) is empty.



REPLACEMENT SHEET

22 FEB 1997

This Sheet Has Been Replaced

17



CLAIMS

1. A method of generating a compressed representation $R(\text{comp})$ of a source data string STR , wherein:

each symbol is taken from a finite set of $m+1$ symbols $a_0 \dots a_m$,

said compressed representation being a numeric value within a given range and being generated recursively in successive cycles for successive symbols in the string,

a probability value $p(i)$ and a cumulative probability value $S(i)$ being available for each symbol a_i ,

said cumulative probability value being

$$S(i) = p(0) + p(1) + \dots + p(i-1) \text{ for } i = 1 \dots m,$$

$$S(0) = 0,$$

and using a pair of shift registers A and C each comprising ω binary digit positions,

characterized by following steps:

(a) initially setting the contents of the C register to all zeros and the contents of the A register to a predetermined value; and

for each successive symbol a_i in the string:

(b) determining a scaling factor K by taking the contents of the A register and eliminating all binary digits less significant than a predetermined binary one digit;

(c) adding the value $K \cdot S(i)$ to the contents of the C register;

(d) determining the contents of the A register

(d1) if the symbol a_i is not the symbol a_m by inserting the value

$K \cdot p(i)$ into the A register, or

(d2) if the symbol a_i is the symbol a_m , by subtracting the value

$K \cdot S(i)$ from the contents of the A register;

(e) shifting both the A register and the C register in a predetermined direction until the contents of the A register lie within a predetermined range, filling the vacated positions with zeros; and

(f) if the symbol a_i were not the last symbol in string STR , repeating steps (b) to (e), or, if the symbol a_i were the last symbol in string STR , shifting the contents of the C register by ω positions in said predetermined direction.



REPLACEMENT SHEET

22 FEB 1997

This Sheet Has Been Replaced



18

whereby the sequence of binary digits shifted out of the C register constitutes said compressed representation $R(\text{comp})$ of the symbol string STR.

2. Method according to Claim 1, wherein said predetermined binary one digit is the second most significant binary one digit in the A register, such that the scaling factor K comprises at most two binary one digits separated by z zero digits.

3. Method according to Claim 1 or Claim 2, wherein said predetermined value is 1.000.

4. Method according to Claim 2 including the step of:

providing the value $K \cdot p(i)$ or $K \cdot S(i)$, respectively, by shifting the value $p(i)$ or $S(i)$, respectively, through $z + 1$ positions towards less significant positions, and adding the shifted quantity to the available value $p(i)$ or $S(i)$, respectively

5. Method according to any one of the preceding claims, wherein the predetermined range is from 1.0 (binary) to 1.111... (binary) and the predetermined direction is such as to increase the magnitude of the contents of the A register.

6. A method of decoding a compressed representation $R(\text{comp})$ of a source data string STR, wherein:

each symbol is taken from a finite set of $m+1$ symbols $a_0 \dots a_m$,

said compressed representation being a numeric value within a given range,

a probability value $p(i)$ and a cumulative probability value $S(i)$ being available for each symbol a_i ,

said cumulative probability value being

$S(i) = p(0) + p(1) + \dots + p(i-1)$ for $i = 1 \dots m$,

$S(0) = 0$,



This Sheet Has Been Replaced



19

REPLACEMENT SHEET

22 FEB 1997

and using a pair of shift registers A and C each comprising ω binary digit positions,

characterized by following steps:

(a) initially setting the contents of the C register to the ω most significant digits of the compressed representation R(comp), and the contents of the A register to a predetermined value; and

decoding each successive symbol a_i in the source data string STR by:

(b) determining a scaling factor K by taking the contents of the A register and eliminating all binary digits less significant than a predetermined binary one digit;

(c) determining the largest value of j such that $C \geq K \cdot S(j)$, where:

C is interpreted as a binary fraction with a binary point outside the C register adjacent to the most significant binary digit therein, and a_i is the symbol output by the decoder at the current step;

(d) determining the contents of the A register

(d1) if the symbol a_i is not the symbol a_m by inserting the value

$K \cdot p(j)$ into the A register, or

(d2) if the symbol a_i is the symbol a_m , by subtracting the value

$K \cdot S(j)$ from the contents of the A register;

(e) subtracting the value $K \cdot S(j)$ from the contents of the C register;

(f) shifting both the A register and the C register in a predetermined direction until the contents of the A register lie within a predetermined range, filling the vacated positions in the A register with zeros and filling the vacated positions in the C register with the corresponding number of most significant digits remaining in the compressed representation R(comp), and repeating steps (b) to (f) until R(comp) is empty.

7. Method according to Claim 6, wherein said predetermined binary one digit is the second most significant binary one digit in the A register, such that the scaling factor K comprises at most two binary one digits separated by z zero digits.

8. Method according to Claim 6 or Claim 7, wherein the predetermined value is 1.0000... (binary).



REPLACEMENT SHEET

22 FEB 1997

This Sheet Has Been Replaced

20



9. Method according to Claim 7, further including the step of:
providing the value $K \cdot p(i)$ or $K \cdot S(i)$, respectively, by shifting the value $p(i)$ or $S(i)$, respectively, through $z + 1$ positions towards less significant positions, and adding the shifted quantity to the available value $p(i)$ or $S(i)$, respectively.
10. Method according to any one of Claims 6 to 9, wherein the predetermined range is from 1.0 (binary) to 1.111... (binary) and the predetermined direction is such as to increase the magnitude of the contents of the A register.

SUMMARY OF THE INVENTION

It is an object of the present invention to ameliorate one or more disadvantages of the prior art.

According to one aspect of the present invention there is provided a method for generating a compressed representation of a source data string, each symbol of said string being selected from a finite set of symbols, comprising the steps of: (a) generating said compressed representation by Rissanen arithmetic encoding of said source symbols, said Rissanen encoding including generating for each symbol an arithmetic representation of a successive subinterval within a predetermined interval, the width of each subinterval being equal to the width of the previous subinterval weighted by the currently encoded symbol's frequency of occurrence; and (b) approximating the width of said previous subinterval with a single SHIFT and ADD operation.

According to another aspect of the present invention there is provided a method of generating a compressed representation $R(\text{comp})$ of a source symbol data string STR, wherein: each source symbol is taken from a finite set of $m+1$ symbols $a_0 \dots a_m$, said compressed representation being a numeric value within a given range and being generated recursively in successive cycles for successive symbols in the string, a probability value $p(i)$ and a cumulative probability value $S(i)$ being available for each symbol a_i , said cumulative probability value being

$$S(i) = p(0) + p(1) + \dots + p(i-1) \text{ for } i = 1 \dots m,$$

$$S(0) = 0,$$

and using a pair of shift registers A and C each comprising ω binary digit positions, the improvement comprising the steps of: (a) initially setting the contents of the C register to all zeros and the contents of the A register to a predetermined value; and for each successive symbol a_1 in the string: (b) determining a scaling factor K by taking the contents of the A register and eliminating all binary digits less significant than a

REPLACEMENT SHEETS		SG
☒	Description	: paged 7777 to paged 7777
☒	Claim(s)	: paged 77 to paged 77
☐	Drawing(s)	: paged 7777 to paged 7777
☐	Abstract	: paged 7777 to paged 7777
DATE OF RECEIPT 10/12/97		

predetermined binary one digit; (c) adding the value $K*S(i)$ to the contents of the C register; (d) determining the contents of the A register

(d1) if the symbol a is not the symbol a_m by inserting the value $K*p(i)$ into the A register, or

(d2) if the symbol a is the symbol a_m , by subtracting the value $K*S(i)$ from the contents of the A register;

(e) shifting both the A register and the C register in a predetermined direction until the contents of the A register lie within a predetermined range, filling the vacated positions with zeros; and (f) if the symbol a_i is not the last symbol in string STR, repeating steps (b)-(e) or, if the symbol a_i is the last symbol in string STR, shifting the contents of the C register by ω positions in said predetermined direction; whereby the sequence of binary digits shifted out of the C register constitutes said compressed representation $R(\text{comp})$ of the symbol string STR.

According to still another aspect of the present invention there is provided a method of decoding a compressed representation $R(\text{comp})$ of a source data string STR, wherein: each symbol is taken from a finite set of $m+1$ symbols $a_0 \dots a_m$, said compressed representation being a numeric value within a given range, a probability value $p(i)$ and a cumulative probability value $S(i)$ being available for each symbol a_i , said cumulative probability value being

$$S(i) = p(0) + p(1) + \dots + p(i-1) \text{ for } i = 1 \dots m,$$

$$S(0) = 0,$$

and using a pair of shift registers A and C each comprising ω binary digit positions, the improvement comprising the steps of: (a) initially setting the contents of the C register to the ω most significant digits of the compressed representation $R(\text{comp})$, and the contents of the A register to a predetermined value; and decoding each successive symbol a_i in the source data string STR by: (b) determining a scaling factor K by taking the contents of the A register and eliminating all binary digits less significant than

a predetermined binary one digit; (c) determining the largest value of j such that $C \geq K \cdot S(j)$, where C is interpreted as a binary fraction with a binary point outside the C register adjacent to the most significant binary digit therein, and a_j is the symbol output by the decoder at the current step; (d) determining the contents of the A register

(d1) if the symbol a_j is not the symbol a_m by inserting the value $K \cdot p(j)$ into the A register, or

(d2) if the symbol a_j is the symbol a_m , by subtracting the value $K \cdot S(j)$ from the contents of the A register;

(e) subtracting the value $K \cdot S(j)$ from the contents of the C register; (f) shifting both the A register and the C register in a predetermined direction until the contents of the A register lie within a predetermined range, filling the vacated positions in the A register with zeros and filling the vacated positions in the C register with the corresponding number of most significant digits remaining in the compressed representation $R(\text{comp})$, and repeating steps (b)-(f) until $R(\text{comp})$ is empty.



CLAIMS

1. A method for generating a compressed representation of a source data string, each symbol of said string being selected from a finite set of symbols, comprising the steps of:

(a) generating said compressed representation by Rissanen arithmetic encoding of said source symbols, said Rissanen encoding including generating for each symbol an arithmetic representation of a successive subinterval within a predetermined interval, the width of each subinterval being equal to the width of the previous subinterval weighted by the currently encoded symbol's frequency of occurrence; and

(b) approximating the width of said previous subinterval with a single SHIFT and ADD operation.

2. In a method of generating a compressed representation $R(\text{comp})$ of a source symbol data string STR, wherein:

each source symbol is taken from a finite set of $m+1$ symbols $a_0 \dots a_m$,

said compressed representation being a numeric value within a given range and being generated recursively in successive cycles for successive symbols in the string,

a probability value $p(i)$ and a cumulative probability value $S(i)$ being available for each symbol a_i ,

said cumulative probability value being

$$S(i) = p(0) + p(1) + \dots + p(i-1) \text{ for } i = 1 \dots m,$$

$$S(0) = 0,$$

and using a pair of shift registers A and C each comprising ω binary digit positions,

the improvement comprising the steps of:

(a) initially setting the contents of the C register to all zeros and the contents of the A register to a predetermined value; and
for each successive symbol a_1 in the string:



- (b) determining a scaling factor K by taking the contents of the A register and eliminating all binary digits less significant than a predetermined binary one digit;
 - (c) adding the value $K*S(i)$ to the contents of the C register;
 - (d) determining the contents of the A register
 - (d1) if the symbol a is not the symbol a_m by inserting the value $K*p(i)$ into the A register, or
 - (d2) if the symbol a is the symbol a_m , by subtracting the value $K*S(i)$ from the contents of the A register;
 - (e) shifting both the A register and the C register in a predetermined direction until the contents of the A register lie within a predetermined range, filling the vacated positions with zeros; and
 - (f) if the symbol a_i is not the last symbol in string STR, repeating steps (b)-(e) or, if the symbol a_i is the last symbol in string STR, shifting the contents of the C register by ω positions in said predetermined direction;
- whereby the sequence of binary digits shifted out of the C register constitutes said compressed representation $R(\text{comp})$ of the symbol string STR.

3. The method according to claim 2, wherein said predetermined binary one digit is the second most significant binary one digit in the A register, such that the scaling factor K comprises at most two binary one digits separated by z zero digits.

4. The method according to claim 2, wherein said predetermined value is 1.000.

5. The method according to claim 3, wherein said method further includes the step of:

providing the value $K*p(i)$ or $K*S(i)$, respectively, by shifting the value $p(i)$ or $S(i)$, respectively, through $z+1$ positions towards less significant positions, and adding the shifted quantity to the available value $p(i)$ or $S(i)$, respectively.



6. The method according to claim 2, wherein the predetermined range is from 1.0 (binary) to 1.111 . . . (binary) and the predetermined direction is such as to increase the magnitude of the contents of the A register.

7. In a method of decoding a compressed representation R(comp) of a source data string STR, wherein:

each symbol is taken from a finite set of $m+1$ symbols $a_0 \dots a_m$,

said compressed representation being a numeric value within a given range,

a probability value $p(i)$ and a cumulative probability value $S(i)$ being available for each symbol a_i ,

said cumulative probability value being

$$S(i) = p(0) + p(1) + \dots + p(i-1) \text{ for } i = 1 \dots m,$$

$$S(0) = 0,$$

and using a pair of shift registers A and C each comprising ω binary digit positions,

the improvement comprising the steps of:

(a) initially setting the contents of the C register to the ω most significant digits of the compressed representation R(comp), and the contents of the A register to a predetermined value; and

decoding each successive symbol a_i in the source data string STR by:

(b) determining a scaling factor K by taking the contents of the A register and eliminating all binary digits less significant than a predetermined binary one digit;

(c) determining the largest value of j such that $C \geq K * S(j)$, where C is interpreted as a binary fraction with a binary point outside the C register adjacent to the most significant binary digit therein, and a_j is the symbol output by the decoder at the current step;

(d) determining the contents of the A register



- (d1) if the symbol a_j is not the symbol a_m by inserting the value $K \cdot p(j)$ into the A register, or
- (d2) if the symbol a_j is the symbol a_m , by subtracting the value $K \cdot S(j)$ from the contents of the A register;
- (e) subtracting the value $K \cdot S(j)$ from the contents of the C register;
- (f) shifting both the A register and the C register in a predetermined direction until the contents of the A register lie within a predetermined range, filling the vacated positions in the A register with zeros and filling the vacated positions in the C register with the corresponding number of most significant digits remaining in the compressed representation $R(\text{comp})$, and repeating steps (b)-(f) until $R(\text{comp})$ is empty.

8. The method according to claim 7, wherein said predetermined binary one digit is the second most significant binary one digit in the A register, such that the scaling factor K comprises at most two binary one digits separated by z zero digits.

9. The method according to claim 8, further including the step of:
providing the value $K \cdot p(i)$ or $K \cdot S(i)$, respectively, by shifting the value $p(i)$ or $S(i)$, respectively, through $z+1$ positions towards less significant positions, and adding the shifted quantity to the available value $p(i)$ or $S(i)$, respectively.

10. The method according to claim 7, wherein the predetermined value is $1.0000 \dots$ (binary).

11. The method according to claim 7, wherein the predetermined range is from 1.0 (binary) to $1.111 \dots$ (binary) and the predetermined direction is such as to increase the magnitude of the contents of the A register.