

A METHOD AND SYSTEM FOR PREDICTING BALLISTIC TIME-OF-FLIGHT AND RANGE OF AN UNGUIDED WEAPON

Field of the Invention

The present invention relates to unguided ballistic weapons, and in particular
5 relates to a method and system for predicting the ballistic time-of-flight and range of an
aerially delivered unguided weapon.

Background of the Invention

When a pilot is preparing to release an unguided weapon, such as a bomb, from
an aircraft, it is imperative that he be provided with accurate and timely data to enable
10 him to assess when to release the bomb so as to maximise the chance of impacting the
selected target. Once the pilot has aligned the aircraft with the target, he requires
information to enable him to determine when he should release the bomb to ensure that
the range of the bomb from the point of release will ensure that the target is hit.

One currently available method for predicting the range and time-of-flight
15 utilises look up tables. Bomb manufacturers routinely provide tables of data providing
the range and time-of-flight for an extensive array of combinations of bomb true air
speed, dive angle and altitude at time of release. When utilising this method, the weapon
delivery and navigation system (WDNS) of the aircraft utilise the look up data tables and
interpolation techniques to predict the range and time-of-flight of the bomb based on the
20 true speed, dive angle and altitude of the aircraft (and hence the unreleased bomb) at any
point in time. Once the range of the bomb equals the range of the selected target, the pilot
is provided with an indication that the bomb is ready for release. Alternatively, the
WDNS may be configured to automatically release the bomb at the appropriate time once
the target and weapon have been selected. This approach in calculating the range and
25 time-of-flight is both rapid and direct, however it requires a tremendous amount of
memory storage to store the ballistic tables for the bomb, rendering the method unsuitable
where minimum memory storage and aircraft weight are desired.

An alternate approach uses numerical integration methods to numerically
integrate a set of equations of motion that describe the kinematic behaviour of the bomb
30 to iteratively plot the trajectory of the bomb, and accordingly determine the range and
time-of-flight. Whilst this method is dynamically attractive, it is potentially slow and
difficult to implement. This method also requires a compromise between accuracy and
speed. An example of such a numerical integration method is disclosed in US Patent No.

3748440. This patent discloses a real-time system for calculating the ballistic time-of-flight and range of a weapon from a known altitude above a target using weighted clock pulses for digital integrators in multi-loop function generators, providing a result in less than about 40 msec of actual time. A delay of 40 msec, however, is an entirely unacceptable delay in calculating the range and time-of-flight data for a high speed aircraft, where the aircraft will have flown several tens of feet during the calculation period, effecting the accuracy of delivery of the bomb.

Object of the Invention

It is an object of the present invention to overcome or substantially ameliorate at least one of the above disadvantages, or at least to provide a useful alternative.

Summary of the Invention

In a first aspect, the present invention provides a method of predicting range and time-of-flight of an aerially delivered unguided weapon comprising the steps of:

providing theoretical equations approximating the theoretical range and time-of-flight of ballistic trajectory motion in a vertical plane of a point-mass moving over a flat non-rotating earth as a function of weapon release altitude, climb angle and true air speed and weapon mass;

modifying said theoretical equations utilising a plurality of correction factors to provide empirical equations approximating the range and time-of-flight of said weapon, each said correction factor comprising a function of a correction constant and a parameter selected from the group of weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight;

calculating substantially optimal values of said correction constants for said weapon using a computational parameter optimization process correlating solutions of said empirical equations with known ballistic results of range and time-of-flight for said weapon over an array of combinations of weapon release altitude, climb angle and true air speed;

determining altitude, climb angle and true air speed of said aircraft prior to release of said weapon; and

calculating predicted range and time-of-flight of said weapon by applying said aircraft climb angle, true air speed and altitude and said optimal correction constant values to said empirical equations.

In a second aspect, the present invention provides a method of aeri-ally delivering an unguided weapon from an aircraft comprising the steps of:

providing theoretical equations approximating the theoretical range and time-of-flight of ballistic trajectory motion in a vertical plane of a point-mass moving over a flat non-rotating earth as a function of weapon release altitude, climb angle and true air speed and weapon mass;

modifying said theoretical equations utilising a plurality of correction factors to provide empirical equations approximating the range and time-of-flight of said weapon, each said correction factor comprising a function of a correction constant and a parameter selected from the group of weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight;

calculating substantially optimal values of said correction constants for said weapon using a computational parameter optimization process correlating solutions of said empirical equations with known ballistic results of range and time-of-flight for said weapon over an array of combinations of weapon release altitude, climb angle and true air speed;

selecting a target;

aligning said aircraft with said target;

determining the range of said target from said aircraft;

determining altitude, climb angle and true air speed of said aircraft;

calculating predicted range and time-of-flight of said weapon by applying said aircraft climb angle, true air speed and altitude and said optimal correction constant values to said empirical equations; and

releasing said weapon when said predicted range of said weapon is substantially equal to said range of said target.

In a third aspect, the present invention provides a system for predicting range and time-of-flight of an aeri-ally delivered unguided weapon comprising:

data storage means storing a look-up table of ballistic results of range and time-of-flight for said weapon over an array of combinations of weapon release altitude, climb angle and true air speed;

aircraft mounted data acquisition means for determining altitude, climb angle and true air speed of said aircraft prior to release of said weapon; and

a first processor module operating on codified empirical equations approximating the range and time-of-flight of said weapon, said empirical equations being derived by modifying theoretical equations approximating the theoretical range and

time-of-flight of ballistic trajectory motion in a vertical plane of a point-mass moving over a flat non-rotating earth as a function of weapon release altitude, climb angle and true air speed and weapon mass, said empirical equations being derived from said theoretical equations utilising a plurality of correction factors each comprising a function of a correction constant and a parameter selected from the group of weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight;

a second processor module operating on a parameter optimization code to calculate substantially optimal values of said correction constants for said weapon by correlating solutions of said empirical equations with said look-up table ballistic results; and

an aircraft mounted third processor module operating on said empirical equations to calculate predicted range and time-of-flight of said weapon by applying said aircraft climb angle, true air speed and altitude and said optimal correction constant values to said empirical equations.

In a fourth aspect, the present invention provides a system for aurally delivering an unguided weapon from an aircraft to a selected target comprising:

processor means for calculating predicted range and time-of-flight of said weapon, said processor means including codified empirical equations approximating range and time-of-flight of said weapon, said empirical equations being derived by modifying theoretical equations approximating the theoretical range and time-of-flight of ballistic trajectory motion in a vertical plane of a point-mass moving over a flat non-rotating earth as a function of weapon release altitude, climb angle and true air speed and weapon mass, said empirical equations being derived from said theoretical equations utilising a plurality of correction factors each comprising a function of a correction constant and a parameter selected from the group of weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight, said processor means including substantially optimal values of said correction constants for said weapon derived using a computational parameter optimization process correlating solutions of said empirical equations with known ballistic results of range and time-of-flight for said weapon over an array of combinations of weapon release altitude, climb angle and true air speed;

flight control means for aligning said aircraft with said target;

first data acquisition means for determining the range of said target from said aircraft;

second data acquisition means for determining altitude, climb angle and true air speed of said aircraft for use by said processor means in calculating said predicted weapon range and time-of-flight; and

means for releasing said weapon when said predicted range of said weapon is substantially equal to said range of said target.

Typically, said theoretical time-of-flight equation is:

$$t_f = \left[\frac{h\eta + \sqrt{l^2\eta^2 + \frac{4mg}{b}z_0}}{\frac{2mg}{b}} \right]^2$$

where:

t_f = weapon time-of-flight,

m = weapon mass,

z_0 = weapon release altitude,

b = unknown drag factor constant,

η = unknown constant,

$$l = \frac{mV_{xinit}b + m^2g}{b^2}$$

g = acceleration due to earth's gravity, and

V_{xinit} = vertical component of weapon release true air speed

Typically, said theoretical range equation is:

$$r = \frac{mV_{xinit}}{b} \left(1 - e^{-\frac{b}{m}t_f} \right)$$

where:

r = weapon range,

m = weapon mass,

b = unknown drag factor constant,

t_f = weapon time-of-flight,

V_{xinit} = horizontal component of weapon release true air speed

Preferably, said empirical equations include a modification of the vertical component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

Preferably, said empirical equations step include a modification of said drag factor using correction factors based on weapon release true air speed and air density.

Preferably, said empirical equations include a modification of the time-of-flight calculated with said theoretical time-of-flight equation using correction factors based on weapon release altitude, the vertical component of the weapon release true air speed and air density.

Preferably, said empirical equations includes a modification of the horizontal component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

Preferably, said empirical equations include a modification of the weapon range using correction factors based on weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight.

Brief Description of the Drawings

A preferred form of the present invention will now be described by way of example with reference to the accompanying drawings, wherein:

Figure 1 is a schematic side elevation view of an aircraft carrying an unguided bomb and flying above a target.

Figure 2 is a schematic representation of the forces acting on a bomb.

Figure 3 is a schematic representation of the horizontal component of the forces acting on the bomb of Figure 2.

Figure 4 is a schematic representation of the vertical component of the vertical forces acting on the bomb of Figure 2.

Figure 5 is a flow diagram of a method of predicting time-of-flight and range of a bomb.

Detailed Description of the Preferred Embodiments

Figure 1 depicts an aircraft 1 carrying an unguided weapon in the form of a bomb 2 and flying at an altitude z above ground level (AGL). The bomb 2 is carried underneath the aircraft wing or fuselage on a weapon pylon 3. The weapon pylon 3 includes means 4 for releasing the bomb, typically in the well known form of a releasable detent means and ejectors which force the bomb away from the pylon on release of the detent means in response to a command from the pilot relayed through the WDNS computer 6 of the aircraft. A target 5 is located a range r from the aircraft. The range r can be readily determined by the aircraft's data acquisition systems, such as by a laser

target designator and range finder (LTDR) 7 usually used to designate targets for laser guided weapons.

Once the pilot has selected the target 5, he needs to align the aircraft 1 with the target 5 such that the target 5 is in the same vertical plane as the plane of motion of the aircraft 1. This is achieved in the usual manner with control inputs from the pilot being processed through the flight control computer (FCC) 8 to the various flight control surfaces 9 of the aircraft. To decide when to release the bomb 2, the pilot needs to know how far the bomb 2 will travel (it's range) if it is released at any point in time, and ideally release the bomb 2 when the bomb range is equal to the range of the target 5.

To assess the range of the bomb, the ballistic motion of an aerially delivered unguided weapon can first be greatly simplified by considering the weapon as a fixed weight point-mass moving over a flat non-rotating earth. Such a point-mass is represented in Figure 2. The forces acting on the point-mass are a combination of gravity and aerodynamic drag, and the general system of equations of motion acting on the mass can be represented as follows:

$$m \frac{dV}{dt} = -D - mg \sin \gamma \quad (1)$$

$$mV \frac{d\gamma}{dt} = -mg \cos \gamma \quad (2)$$

$$\frac{dz}{dt} = V \sin \gamma \quad (3)$$

$$\frac{dx}{dt} = V \cos \gamma \quad (4)$$

where:

V = true airspeed,

m = bomb mass,

D = drag,

γ = flight path climb angle,

z = altitude above ground level,

x = horizontal distance,

g = acceleration due to earth's gravity, and

t = time.

The aircraft true airspeed, flight path climb angle and altitude are available from the aircraft's flight data acquisition systems 10, including, for example pitot-static probes, global positioning system (GPS), gyroscopes and processors processing data output from these data sources.

When a fairly accurate aerodynamic drag model of the bomb is included in the motion, it renders the above equations extremely nonlinear, and no general solution in closed form has been found that is satisfactory for an airborne flight-control computer.

To simplify these equations, drag (D) is assumed to be a function of true air speed, giving:

$$D = bV \quad (5)$$

where:

b = unknown drag factor constant.

Applying the assumed drag function of Eqn (5) to Eqn (1) above, and considering motion in the horizontal plane, as depicted in Figure 3, we get:

$$m \frac{d^2x}{dt^2} = -b \frac{dx}{dt} \quad (6)$$

The solution to this Eqn (6), for the forward travel of the bomb at any point in time, can be approximated with the following trial solution:

$$x = a(1 - e^{-kt}) \quad (7)$$

where:

a, k = unknown constants.

$$\frac{dx}{dt} = ake^{-kt} = V_x \quad (8)$$

$$\frac{d^2x}{dt^2} = -ak^2e^{-kt} \quad (9)$$

Substituting Eqns (9) and (8) into (6), we get:

$$\begin{aligned} -mak^2e^{-kt} &= -bake^{-kt} \\ \Rightarrow k &= \frac{b}{m} \end{aligned} \quad (10)$$

From Eqn (9), at $t = 0$, we get:

$$\begin{aligned}
 V_{xinit} &= ak \\
 &= a \frac{b}{m} \\
 \rightarrow a &= \frac{mV_{xinit}}{b}
 \end{aligned} \tag{11}$$

Substituting Eqns (10) and (11) into Eqn (7), we get the horizontal distance travelled by the bomb (known as the forward-throw) at any point in time as:

$$x = \frac{mV_{xinit}}{b} \left(1 - e^{-\frac{b}{m}t}\right) \tag{12}$$

If we now consider Eqn (1) for the component of motion in the vertical plane, as depicted in Figure 4, we get:

$$m \frac{d^2 z}{dt^2} = -b \frac{dz}{dt} - mg \tag{13}$$

The solution to this Eqn (13), for the altitude of the bomb at any point in time, can be approximated with the following trial solution:

$$z = c \left(1 - e^{-\frac{b}{m}t}\right) + dt + f \tag{14}$$

where:

c, d, f = unknown constants.

$$\frac{dz}{dt} = \frac{bc}{m} e^{-\frac{b}{m}t} + d = V_z \tag{15}$$

$$\frac{d^2 z}{dt^2} = -\frac{b^2}{m^2} c e^{-\frac{b}{m}t} \tag{16}$$

Substituting Eqns (16) and (15) into Eqn (13), we get:

$$m \left(-\frac{b^2}{m^2} c e^{-\frac{b}{m}t} \right) + b \left(\frac{bc}{m} e^{-\frac{b}{m}t} + d \right) + mg = 0 \tag{17}$$

From Eqn (17) at $t = 0$, we get:

$$-\frac{b^2}{m} c + \frac{b^2}{m} c + bd + mg = 0 \tag{18}$$

$$\rightarrow d = -\frac{mg}{b} \quad (19)$$

From Eqn (9), at $t = 0$, we get:

$$V_{zinit} = \frac{bc}{m} + \left(-\frac{mg}{b}\right) \quad (20)$$

$$\rightarrow c = \frac{m}{b} \left(V_{zinit} + \frac{mg}{b}\right) \quad (21)$$

Also, at $t = 0$, $z = z_0$, from Eqn (14), we get:

$$f = z_0 \quad (22)$$

Substituting Eqns (19), (21), and (22) into Eqn (14), we get:

$$z = z_0 + \frac{m}{b} \left(V_{zinit} + \frac{mg}{b}\right) \left(1 - e^{-\frac{b}{m}t}\right) - \frac{mg}{b}t$$

or

$$z = z_0 + \left(\frac{mV_{zinit}}{b} + \frac{m^2g}{b^2}\right) \left(1 - e^{-\frac{b}{m}t}\right) - \frac{mg}{b}t \quad (23)$$

Eqn (23) represents the height of the bomb above the ground at t seconds after it is released from the aircraft. Clearly, to obtain an explicit solution for the time-of-flight taken by the bomb before it hits the ground (i.e. $z=0$) using Eqn (23) is nearly impossible.

Accordingly, it is necessary to derive an approximate solution for the ballistic time-of-flight (t_f).

Firstly, let:

$$l = \frac{mV_{zinit}b + m^2g}{b^2} \quad (24)$$

From Eqn (23), we get:

$$z = z_0 + l - le^{-\frac{b}{m}t} - \frac{mg}{b}t \quad (25)$$

At $t = t_f$, $z = 0$. Eqn (25) gives:

$$0 = z_0 + l - le^{-\frac{b}{m}t_f} - \frac{mg}{b}t_f \quad (26)$$

or

$$\frac{mg}{b}t_f + le^{-\frac{b}{m}t_f} - z_0 - l = 0 \quad (27)$$

Eqn (27), can be simplified by making the following approximation:

$$e^{-\frac{b}{m}t_f} = 1 - \eta\sqrt{t_f} \quad (28)$$

where:

5 η = unknown constant.

Substituting Eqn (28) into Eqn (22), we obtain:

$$\frac{mg}{b}t_f - h\eta\sqrt{t_f} - z_0 = 0 \quad (29)$$

Eqn (29) represents a readily solvable quadratic equation in $\sqrt{t_f}$. Solving this equation, we get:

$$10 \quad t_f = \left[\frac{h\eta + \sqrt{l^2\eta^2 + \frac{4mg}{b}z_0}}{\frac{2mg}{b}} \right]^2 \quad (30)$$

Eqn (30) thus provides the predicted ballistic time-of-flight (t_f).

Accordingly, Eqns (12) and (30) are theoretical equations approximating the theoretical range and time-of-flight of ballistic trajectory motion in a vertical plane of the bomb when it is considered to be a point-mass moving over a flat non-rotating earth.

15 As these equations are theoretical, and do not account for a complex aerodynamic drag model, the results utilising these approximate theoretical solutions do not provide sufficient accuracy to be relied upon in practice. Accordingly, Eqns (12) and (30) are modified utilising various correction factors in an attempt to provide empirical solutions which more closely match known actual results.

20 At any point in time prior to release of the bomb 2 from the aircraft 1, the aircraft true air speed (V_T), altitude ($z=h$) and climb angle (γ) are known from the flight control computer (FCC) 8, determined from the various flight data acquisition systems 10 of the aircraft. With the bomb being fixed to the aircraft prior to release, the true air speed, altitude and climb angle of the bomb will clearly equal that of the aircraft.

25 The true air speed (V_T) can be resolved into vertical and horizontal components as follows:

$$V_{xinit} = V_T \cos \gamma \quad - \text{horizontal component} \quad (31)$$

$$V_{zinit} = V_T \sin \gamma \quad - \text{vertical component} \quad (32)$$

The air density (ρ) at a given altitude (in metres) can be approximated by the following:

$$\rho = \rho_0 e^{-\frac{h}{10000}} \quad (33)$$

where:

ρ_0 = air density at sea level.

In modifying the approximate theoretical results of Eqns (12) and (30), correction factors are applied based on parameters which are expected to effect the theoretical result. These parameters are expected to include altitude, climb angle (or dive angle, being equal to $-\text{climb angle}$), true air speed, air density and time-of-flight (for the range calculation).

A first group of correction factors are applied to the vertical velocity component (V_{zinit}) as follows:

$$V_z = V_{zinit} \left(1 + \beta_1 \cos \gamma + \frac{\beta_2}{\sqrt{\rho}} + \beta_3 \frac{V_T}{250} + \beta_4 \frac{h}{1500} \right) \quad (34)$$

Where β_1 , β_2 , β_3 and β_4 are the first through fourth correction constants.

The velocity correction factor has a division of the true air speed (V_T) by 250 m/s in an attempt to reduce the order of magnitude of the velocity expression to single digits. Similarly, the altitude correction constant has a division by 1500 m.

A second group of correction factors are applied to calculate the drag factor (b) as follows:

$$b = 50 + \beta_5 \rho + \beta_6 \rho \frac{V_T}{250} + \beta_7 \left(\frac{V_T}{250} \right)^{\beta_8} \quad (35)$$

Where β_5 , β_6 , β_7 and β_8 are the fifth through eighth correction constants.

From Eqn (28) we have an unknown constant η , which we relabel as the ninth correction constant (β_9).

To simplify expression of Eqn (30), we then let:

$$p_1 = \beta_9 l \quad (36)$$

$$p_2 = \sqrt{p_1^2 + \beta_{10} \frac{4mgh}{b}} \quad (37)$$

Where β_{10} is the tenth correction constant.

Eqn (30) can thus be rewritten as:

$$t_f = \left(\frac{p_1 + p_2}{2mg/b} \right)^2 \quad (38)$$

5 Further correction factors are then added to provide an empirical equation approximating the time-of-flight of the bomb as follows:

$$t_f = \left(\frac{p_1 + p_2}{2mg/b} \right)^2 + \beta_{11} \frac{h}{1500} + \beta_{12} \left(\frac{h}{1500} \right)^{\beta_{13}} + \beta_{14} \rho + \beta_{15} V_z \sqrt{\rho} + \beta_{16} V_z \quad (39)$$

Where β_{11} , β_{12} , β_{13} , β_{14} , β_{15} and β_{16} are the eleventh through sixteenth correction constants.

10 A further group of correction factors is applied to the horizontal velocity component (V_{xinit}):

$$V_x = V_{xinit} \left(1 + \beta_{17} \sin \gamma + \frac{\beta_{18}}{\sqrt{\rho}} + \beta_{19} \frac{V_T}{250} + \beta_{20} \frac{h}{1500} \right) \quad (40)$$

Where β_{17} , β_{18} , β_{19} and β_{20} are the seventh through twentieth correction constants.

15 To simplify Eqn (12), and to add a further correction constant, we let:

$$p_3 = 1 - e^{-\frac{b}{m} t_f} + \beta_{21} \quad (41)$$

Where β_{21} is a twenty-first correction factor.

Substituting this into Eqn (12), and adding further correction factors, we arrive at an empirical equation approximately the range or forward-throw of the bomb as follows:

$$r = \left\{ \frac{mV_x}{b} p_3 + \beta_{22} V_x t_f + \beta_{23} t_f^2 + \beta_{24} V_x \sqrt{\rho} + \beta_{25} \frac{V_T h}{1500} + \beta_{26} \frac{h}{1500} + \right. \\ \left. \beta_{27} \left(\frac{h}{1500} \right)^2 + \beta_{28} \sqrt{\rho} + \beta_{29} (V_x \rho)^2 + \beta_{30} V_x t_f^2 + \beta_{31} V_T (\cos \gamma)^{\beta_{32}} + \beta_{33} V_x \right\} \quad (42)$$

Where $\beta_{22}, \beta_{23}, \beta_{24}, \beta_{25}, \beta_{26}, \beta_{27}, \beta_{28}, \beta_{29}, \beta_{30}, \beta_{31}, \beta_{32}, \beta_{33}$ are the twenty-second through thirty-third correction constants.

All of the above equations are expressed in SI units, however it will be usual to convert the range result of Eqn 42 into British units of feet for use by the pilot.

The thirty-three correction constants β_1 to β_{33} now need to be determined to provide the best fit of results utilising the above Eqns to fit the tabulating results provided by the manufacturer for a given weapon. These correction constants can be selected by solving the following parameter optimization problem utilising any standard optimization technique:

$$J = w_1 \sum_1^n \left(\frac{t_{\text{empirical}} - t_{\text{table}}}{20} \right)^2 + w_2 \sum_1^n \left(\frac{r_{\text{empirical}} - r_{\text{table}}}{1500} \right)^2 \quad (43)$$

where:

J = performance index to be minimised with the optimization process,

w_1, w_2 = weightage factors to be selected dependent upon the relative importance of range accuracy versus time-of-flight accuracy,

$t_{\text{empirical}} - t_{\text{table}}$ = time-of-flight error between empirical fit values utilising the above Eqns and the tabulated values,

$r_{\text{empirical}} - r_{\text{table}}$ = range error between empirical fit values utilising the above Eqns and tabulated values,

n = number of data points compared.

The result from solving this optimization problem, which has the time-of-flight and range errors normalised by 20 seconds and 1500 metres respectively, is the set of optimal correction constants β_1 to β_{33} . These optimal correction constants are bomb specific.

As accuracy of the range calculation is much more critical than the accuracy of the time-of-flight calculation, the second weightage factor (w_2) should typically be selected to be much greater than the first weightage factor (w_1). These weightage factors can be changed through trial and error if desired in a further attempt to minimise the range and time-of-flight errors in calculating the optimal correction constants.

With the empirical equations (39) and (42) approximating the time-of-flight and range of an unguided weapon 2 having been derived manually, these empirical equations are next implemented to predict time-of-flight and range by first codifying the empirical equations into a first process module 11 as depicted in the flow diagram of Figure 5. The person skilled in the art will be well versed in standard methods to code such an equation

into the first processor module 11. A look-up table of ballistic results of range and time-of-flight data for the weapon of interest is stored in a data storage means 12. Again, the person skilled in the art will be well versed in standard data storage devices. A second processor module 13 operates on a standard parameter optimization code utilizing standard optimization methods such as a least square regression method, Newton's method or a Taylor series reduction method. Again the person skilled in the art will be well versed in such numerical optimization methods. The parameter optimization code is used to calculate substantially optimal values of the correction constants β_1 to β_{33} using the parameter optimization Eqn (43).

The first and second processor modules 11, 13 can be a single processor module, with these operations to be being carried out in a processor, such as in a PC, on the ground, providing substantially optimal values of the correction constants for loading into the aircraft. When the aircraft 1 is loaded with a given unguided weapon 2, the optimal correction constants for that weapon will be loaded into an aircraft mounted third processor module, which would typically consist of the WDNS computer 6. Alternatively, the optimal correction constants for various unguided weapons to be used with a given aircraft type might already be preloaded into the WDNS computer 6. Loading of a particular weapon will then activate retrieval of the preloaded correction constants for that weapon. The WDNS computer 6 also operates on the empirical equations to calculate predicted range and time-of-flight of the weapon by applying the aircraft climb angle, true air speed and altitude, available from the aircraft flight data acquisition systems 10, and the optimal correction constant values for the weapon to the empirical equations.

With the correction constants having already calculated, the calculation of range and time-of-flight is almost instantaneously calculated directly from the empirical equations, enabling instant feedback to the pilot.

Once the pilot has selected a target 5, aligned the aircraft 1 with the target 5 and armed the unguided weapon 2, the WDNS computer 6 will continually update the predicted range and time-of-flight of the weapon and compare the same with the continually updated range of the target provided by the laser target designator and range finder (LTDR). When the predicted range of the weapon equates with that of the selected target, a signal will be provided to the pilot to indicate that the weapon 2 should be released so as to maximize the chance of hitting the target. The WDNS computer 6 could be configured such that the predicted time prior to release is continually calculated based on the difference between the target range and the predicted weapon range, and the

horizontal component of the aircraft true air speed (V_x) so as that the pilot effectively gets a count down to when the weapon should be released. This will minimise reaction delay times of the pilot. In another alternative configuration, the WDNS computer 6 will be configured to automatically release the weapon 2 at the appropriate time when the predicted weapon range and target range coincide once the weapon 2 has been armed by the pilot.

Whilst the calculation of the optimal values of the correction constants will typically be carried out in separate processors on the ground, it is envisaged that the optimal correction constants calculations might be carried out within the WDNS computer, prior to the commencement of any relevant bombing missions.

Example

The empirical equations (39) and (42) derived above were applied to a MK 82 bomb having a weight of 500 pounds (mass of 227.27 kg). Table 1 provides a look-up-table data of known ballistic range and time-of-flight for an array of combinations of altitude, dive angle and true air speed (all expressed in British units) as supplied by the original equipment manufacturer. Solution of the optimization problem of Eqn (43), utilising weightage factors of $w_1 = 8000$ and $w_2 = 213000$, provided the following table of correction constants (in row then column order):

20	$\beta_{1-33} =$	100*[0.00291082347602	0.00066850452991	-0.00138340193913
		-0.00638158812863	1.67519490029754	0.14592396993844
		-1.97903705273431	-0.00090418306503	0.00256277380455
		0.00436642397023	-0.03447759688978	0.14735863202910
		0.00719707351935	0.00231809175132	-0.00303710633834
25		0.00349536751854	0.00135259750495	0.00062295741717
		-0.00202951535514	0.00107244877289	0.00028695252947
		0.00202603830698	0.00144499816415	-0.02368766941289
		0.00208232078074	-1.08872607704417	-2.41869503298574
		0.27630999987104	0.00002857107493	-0.00007086918037
30		-0.00521386015853	0.03964192332155	0.02113397094226]

Utilising these correction factors provided an average error for the bomb range of 390.48 mm (1.2811 ft), a root mean square (RMS) error of 531.82 mm (1.7448 ft) and a maximum error of 1893.5 mm (6.2124 ft). The average error for the time-of-flight was 0.0362 secs, with an RMS error of 0.0465 secs and a maximum error of 0.1571 secs.

It can thus be seen that the empirical calculations provide an extremely accurate prediction of range and time-of-flight over the full spectrum of typical combinations of altitude, dive angle and true air speed as set out in Table 1.

The methods described have also been applied to the tabulated ballistic data of four other bombs, with results as follows:

	Time-of-flight Error(s)			Range Error		
	Avg	RMS	Max	Avg	RMS	Max
mk82hd1	0.0341	0.0480	0.1983	500.79mm (1.6430ft)	673.27mm (2.2089ft)	2967.1mm (9.7347ft)
mk83	0.0654	0.0840	0.4017	703.81mm (2.3091ft)	3950.37mm (3.1180ft)	4419.6mm (14.5001ft)
mk831	0.0559	0.0744	0.4375	563.64mm (1.8492ft)	730.06mm (2.3952ft)	2916.2mm (9.5676ft)
mk84	0.0974	0.1233	0.4124	1006.1mm (3.3009ft)	1016.5mm (4.3349ft)	4402.5mm (14.4440ft)

CLAIMS:

1. A method of predicting range and time-of-flight of an aerially delivered unguided weapon comprising the steps of:

providing theoretical equations approximating the theoretical range and time-of-flight of ballistic trajectory motion in a vertical plane of a point-mass moving over a flat non-rotating earth as a function of weapon release altitude, climb angle and true air speed and weapon mass;

modifying said theoretical equations utilising a plurality of correction factors to provide empirical equations approximating the range and time-of-flight of said weapon, each said correction factor comprising a function of a correction constant and a parameter selected from the group of weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight;

calculating substantially optimal values of said correction constants for said weapon using a computational parameter optimization process correlating solutions of said empirical equations with known ballistic results of range and time-of-flight for said weapon over an array of combinations of weapon release altitude, climb angle and true air speed;

determining altitude, climb angle and true air speed of said aircraft prior to release of said weapon; and

calculating predicted range and time-of-flight of said weapon by applying said aircraft climb angle, true air speed and altitude and said optimal correction constant values to said empirical equations.

2. The method of claim 1 wherein said theoretical time-of-flight equation is:

$$t_f = \left[\frac{h\eta + \sqrt{l^2\eta^2 + \frac{4mg}{b}z_0}}{\frac{2mg}{b}} \right]^2$$

where:

t_f = weapon time-of-flight,

m = weapon mass,

z_0 = weapon release altitude,

b = unknown drag factor constant,

η = unknown constant,

$$l = \frac{mV_{zinit}b + m^2g}{b^2}$$

g = acceleration due to earth's gravity, and

V_{zinit} = vertical component of weapon release true air speed

- 5 3. The method of either of claims 1 and 2 wherein said theoretical range equation is:

$$r = \frac{mV_{xinit}}{b} \left(1 - e^{-\frac{b}{m}t_f} \right)$$

where:

r = weapon range,

10 m = weapon mass,

b = unknown drag factor constant,

t_f = weapon time-of-flight,

V_{xinit} = horizontal component of weapon release true air speed

- 15 4. The method of any one of claims 1 to 3 wherein said modifying step includes modifying the vertical component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

- 20 5. The method of either of claims 2 and 3 wherein said modifying step includes modifying said drag factor using correction factors based on weapon release true air speed and air density.

6. The method of any one of claims 1 to 5 wherein said modifying step includes modifying the time-of-flight calculated with said theoretical time-of-flight equation using correction factors based on weapon release altitude, the vertical component of the weapon release true air speed and air density.

- 25 7. The method of any one of claims 1 to 6 wherein said modifying step includes modifying the horizontal component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

- 30 8. The method of any one of claims 1 to 7 wherein said modifying step includes modifying the weapon range using correction factors based on weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight.

9. A method of aerially delivering an unguided weapon from an aircraft comprising the steps of:

providing theoretical equations approximating the theoretical range and time-of-flight of ballistic trajectory motion in a vertical plane of a point-mass moving over a flat non-rotating earth as a function of weapon release altitude, climb angle and true air speed and weapon mass;

5 modifying said theoretical equations utilising a plurality of correction factors to provide empirical equations approximating the range and time-of-flight of said weapon, each said correction factor comprising a function of a correction constant and a parameter selected from the group of weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight;

10 calculating substantially optimal values of said correction constants for said weapon using a computational parameter optimization process correlating solutions of said empirical equations with known ballistic results of range and time-of-flight for said weapon over an array of combinations of weapon release altitude, climb angle and true air speed;

15 selecting a target;

aligning said aircraft with said target;

determining the range of said target from said aircraft;

determining altitude, climb angle and true air speed of said aircraft;

20 calculating predicted range and time-of-flight of said weapon by applying said aircraft climb angle, true air speed and altitude and said optimal correction constant values to said empirical equations; and

releasing said weapon when said predicted range of said weapon is substantially equal to said range of said target.

25 10. The method of claim 9 wherein said theoretical time-of-flight equation is:

$$t_f = \left[\frac{h\eta + \sqrt{l^2\eta^2 + \frac{4mg}{b}z_0}}{\frac{2mg}{b}} \right]^2$$

where:

t_f = weapon time-of-flight,

30 m = weapon mass,

z_0 = weapon release altitude,

b = unknown drag factor constant,

η = unknown constant,

$$l = \frac{mV_{zinit}b + m^2g}{b^2}$$

g = acceleration due to earth's gravity, and

V_{zinit} = vertical component of weapon release true air speed

- 5 11. The method of either of claims 9 and 10 wherein said theoretical range equation is:

$$r = \frac{mV_{xinit}}{b} \left(1 - e^{-\frac{b}{m}t_f} \right)$$

where:

r = weapon range,

10 m = weapon mass,

b = unknown drag factor constant,

t_f = weapon time-of-flight,

V_{xinit} = horizontal component of weapon release true air speed

- 15 12. The method of any one of claims 9 to 11 wherein said modifying step includes modifying the vertical component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

- 20 13. The method of either of claims 10 and 11 wherein said modifying step includes modifying said drag factor using correction factors based on weapon release true air speed and air density.

14. The method of any one of claims 9 to 13 wherein said modifying step includes modifying the time-of-flight calculated with said theoretical time-of-flight equation using correction factors based on weapon release altitude, the vertical component of the weapon release true air speed and air density.

- 25 15. The method of any one of claims 9 to 14 wherein said modifying step includes modifying the horizontal component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

- 30 16. The method of any one of claims 9 to 15 wherein said modifying step includes modifying the weapon range using correction factors based on weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight.

17. A system for predicting range and time-of-flight of an aerially delivered unguided weapon comprising:

data storage means storing a look-up table of ballistic results of range and time-of-flight for said weapon over an array of combinations of weapon release altitude, climb angle and true air speed;

aircraft mounted data acquisition means for determining altitude, climb angle and true air speed of said aircraft prior to release of said weapon; and

a first processor module operating on codified empirical equations approximating the range and time-of-flight of said weapon, said empirical equations being derived by modifying theoretical equations approximating the theoretical range and time-of-flight of ballistic trajectory motion in a vertical plane of a point-mass moving over a flat non-rotating earth as a function of weapon release altitude, climb angle and true air speed and weapon mass, said empirical equations being derived from said theoretical equations utilising a plurality of correction factors each comprising a function of a correction constant and a parameter selected from the group of weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight;

a second processor module operating on a parameter optimization code to calculate substantially optimal values of said correction constants for said weapon by correlating solutions of said empirical equations with said look-up table ballistic results; and

an aircraft mounted third processor module operating on said empirical equations to calculate predicted range and time-of-flight of said weapon by applying said aircraft climb angle, true air speed and altitude and said optimal correction constant values to said empirical equations.

18. The system of claim 17 wherein said first and third processor modules comprise a single processor module.

19. The system of claim 17 wherein said first and second processor modules comprise a single processor module.

20. The method of any one of claims 17 to 19 wherein said theoretical time-of-flight equation is:

$$t_f = \left[\frac{h_1 + \sqrt{l^2 \eta^2 + \frac{4mg}{b} z_0}}{\frac{2mg}{b}} \right]^2$$

where:

t_f = weapon time-of-flight,

m = weapon mass,

z_0 = weapon release altitude,

5 b = unknown drag factor constant,

η = unknown constant,

$$l = \frac{mV_{zinit}b + m^2g}{b^2}$$

g = acceleration due to earth's gravity, and

V_{zinit} = vertical component of weapon release true air speed

10 21. The system of any one of claims 17 to 20 wherein said theoretical range equation is:

$$r = \frac{mV_{xinit}}{b} \left(1 - e^{-\frac{b}{m}t_f} \right)$$

where:

r = weapon range,

15 m = weapon mass,

b = unknown drag factor constant,

t_f = weapon time-of-flight,

V_{xinit} = horizontal component of weapon release true air speed

20 22. The system of any one of claims 17 to 21 wherein said empirical equations include a modification of the vertical component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

23. The system of either of claims 20 and 21 wherein said empirical equations include a modification of said drag factor using correction factors based on
25 weapon release true air speed and air density.

24. The system of any one of claims 17 to 23 wherein said empirical equations include a modification of the time-of-flight calculated with said theoretical time-of-flight equation using correction factors based on weapon release altitude, the vertical component of the weapon release true air speed and air density.

30 25. The system of any one of claims 17 to 24 wherein said empirical equations include a modification of the horizontal component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

26. The system of any one of claims 17 to 24 wherein said empirical equations include a modification of the weapon range using correction factors based on weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight.

27. A system for aerially delivering an unguided weapon from an aircraft to a selected target comprising:

processor means for calculating predicted range and time-of-flight of said weapon, said processor means including codified empirical equations approximating range and time-of-flight of said weapon, said empirical equations being derived by modifying theoretical equations approximating the theoretical range and time-of-flight of ballistic trajectory motion in a vertical plane of a point-mass moving over a flat non-rotating earth as a function of weapon release altitude, climb angle and true air speed and weapon mass, said empirical equations being derived from said theoretical equations utilising a plurality of correction factors each comprising a function of a correction constant and a parameter selected from the group of weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight, said processor means including substantially optimal values of said correction constants for said weapon derived using a computational parameter optimization process correlating solutions of said empirical equations with known ballistic results of range and time-of-flight for said weapon over an array of combinations of weapon release altitude, climb angle and true air speed;

flight control means for aligning said aircraft with said target;

first data acquisition means for determining the range of said target from said aircraft;

second data acquisition means for determining altitude, climb angle and true air speed of said aircraft for use by said processor means in calculating said predicted weapon range and time-of-flight; and

means for releasing said weapon when said predicted range of said weapon is substantially equal to said range of said target.

28. The system of claim 27 wherein said theoretical time-of-flight equation is:

$$t_f = \left[\frac{l\eta + \sqrt{l^2\eta^2 + \frac{4mg}{b}z_0}}{\frac{2mg}{b}} \right]^2$$

where:

t_f = weapon time-of-flight,

m = weapon mass,

5 z_0 = weapon release altitude,

b = unknown drag factor constant,

η = unknown constant,

$$l = \frac{mV_{zinit}b + m^2g}{b^2}$$

g = acceleration due to earth's gravity, and

10 V_{zinit} = vertical component of weapon release true air speed

29. The system of either of claims 27 and 28 wherein said theoretical range equation is:

$$r = \frac{mV_{xinit}}{b} \left(1 - e^{-\frac{b}{m}t_f} \right)$$

15 where:

r = weapon range,

m = weapon mass,

b = unknown drag factor constant,

t_f = weapon time-of-flight,

20 V_{xinit} = horizontal component of weapon release true air speed

30. The system of any one of claims 27 to 29 wherein said empirical equations include a modification of the vertical component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

25 31. The system of either of claims 28 and 29 wherein said empirical equations step include a modification of said drag factor using correction factors based on weapon release true air speed and air density.

32. The system of any one of claims 27 to 31 wherein said empirical equations include a modification of the time-of-flight calculated with said theoretical time-of-flight equation using correction factors based on weapon release altitude, the vertical component of the weapon release true air speed and air density.

33. The system of any one of claims 27 to 32 wherein said empirical equations includes a modification of the horizontal component of weapon release true air

speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

34. The system of any one of claims 27 to 33 wherein said empirical equations include a modification of the weapon range using correction factors based on
5 weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight.

Table 1

%{data for mk82 Ballistics} 500 lbs
 %{TAS, dive, Alt, Range, TOF, Relative air, density}
 %{kts deg ft ft sec}

$V(kts) : \gamma : h(ft) : r(ft) : t_f(s) : \rho/\rho_0$					
360	0	500	2199.00	6.41	0.985450572
400	0	500	2368.00	6.48	0.985450572
440	0	500	2527.00	6.55	0.985450572
480	0	500	2678.00	6.62	0.985450572
520	0	500	2822.00	6.66	0.985450572
400	0	600	2506.00	7.21	0.982560227
440	0	600	2670.00	7.29	0.982560227
480	0	600	2825.00	7.36	0.982560227
520	0	600	2972.00	7.42	0.982560227
480	0	700	2950.00	8.05	0.979676373
520	0	700	3100.00	8.11	0.979676373
560	0	700	3242.00	8.17	0.979676373
360	-10	400	1390.00	3.19	0.988347417
400	-10	400	1464.00	3.06	0.988347417
440	-10	400	1530.00	2.93	0.988347417
480	-10	400	1590.00	2.81	0.988347417
360	-10	500	1597.00	3.93	0.985450572
400	-10	500	1688.00	3.80	0.985450572
440	-10	500	1770.00	3.67	0.985450572
480	-10	500	1845.00	3.54	0.985450572
360	-10	600	1773.00	4.63	0.982560227
400	-10	600	1878.00	4.50	0.982560227
440	-10	600	1973.00	4.38	0.982560227
480	-10	600	2061.00	4.26	0.982560227
520	-10	600	2142.00	4.14	0.982560227
360	-10	700	1924.00	5.30	0.979676373
400	-10	700	2040.00	5.18	0.979676373
440	-10	700	2147.00	5.07	0.979676373
480	-10	700	2245.00	4.95	0.979676373
520	-10	700	2337.00	4.84	0.979676373
560	-10	700	2422.00	4.73	0.979676373
360	-10	800	2057.00	5.94	0.976798998
400	-10	800	2182.00	5.83	0.976798998
440	-10	800	2298.00	5.72	0.976798998
480	-10	800	2405.00	5.61	0.976798998
520	-10	800	2505.00	5.51	0.976798998
560	-10	800	2598.00	5.40	0.976798998
360	-10	900	2174.00	6.56	0.973928094
400	-10	900	2307.00	6.46	0.973928094
440	-10	900	2430.00	6.35	0.973928094
480	-10	900	2545.00	6.25	0.973928094
520	-10	900	2652.00	6.15	0.973928094
560	-10	900	2752.00	6.05	0.973928094
360	-10	1000	2278.00	7.15	0.971063649
400	-10	1000	2418.00	7.06	0.971063649
440	-10	1000	2548.00	6.96	0.971063649
480	-10	1000	2668.00	6.87	0.971063649
520	-10	1000	2782.00	6.77	0.971063649
560	-10	1000	2888.00	6.68	0.971063649
360	-10	1100	2372.00	7.73	0.968205654
400	-10	1100	2518.00	7.64	0.968205654
440	-10	1100	2653.00	7.55	0.968205654
480	-10	1100	2779.00	7.46	0.968205654
520	-10	1100	2897.00	7.37	0.968205654
560	-10	1100	3008.00	7.29	0.968205654
400	-10	1200	2608.00	8.20	0.965354098
440	-10	1200	2748.00	8.12	0.965354098
480	-10	1200	2879.00	8.04	0.965354098

V (kts)	γ	h (ft)	r (ft)	t_f (s)	ρ/ρ_0
520	-10	1200	3001.00	7.95	0.965354098
560	-10	1200	3117.00	7.87	0.965354098
400	-10	1300	2690.00	8.75	0.962508972
440	-10	1300	2834.00	8.67	0.962508972
480	-10	1300	2969.00	8.59	0.962508972
520	-10	1300	3095.00	8.52	0.962508972
560	-10	1300	3214.00	8.44	0.962508972
440	-10	1400	2913.00	9.21	0.959670266
480	-10	1400	3052.00	9.14	0.959670266
520	-10	1400	3181.00	9.06	0.959670266
560	-10	1400	3303.00	8.99	0.959670266
520	-10	1500	3260.00	9.60	0.956837969
560	-10	1500	3385.00	9.53	0.956837969
360	-20	700	1415.00	3.54	0.979676373
400	-20	700	1472.00	3.33	0.979676373
440	-20	700	1521.00	3.14	0.979676373
360	-20	800	1555.00	4.09	0.976798998
400	-20	800	1622.00	3.87	0.976798998
440	-20	800	1680.00	3.66	0.976798998
480	-20	800	1731.00	3.47	0.976798998
360	-20	900	1681.00	4.63	0.973928094
400	-20	900	1758.00	4.41	0.973928094
440	-20	900	1826.00	4.20	0.973928094
480	-20	900	1886.00	4.00	0.973928094
520	-20	900	1939.00	3.81	0.973928094
360	-20	1000	1797.00	5.17	0.971063649
400	-20	1000	1883.00	4.95	0.971063649
440	-20	1000	1959.00	4.73	0.971063649
480	-20	1000	2027.00	4.53	0.971063649
520	-20	1000	2088.00	4.33	0.971063649
560	-20	1000	2143.00	4.15	0.971063649
360	-20	1100	1902.00	5.71	0.968205654
400	-20	1100	1996.00	5.48	0.968205654
440	-20	1100	2081.00	5.26	0.968205654
480	-20	1100	2156.00	5.06	0.968205654
520	-20	1100	2225.00	4.86	0.968205654
560	-20	1100	2286.00	4.67	0.968205654
360	-20	1200	1998.00	6.23	0.965354098
400	-20	1200	2100.00	6.01	0.965354098
440	-20	1200	2192.00	5.79	0.965354098
480	-20	1200	2275.00	5.58	0.965354098
520	-20	1200	2350.00	5.38	0.965354098
560	-20	1200	2418.00	5.19	0.965354098
360	-20	1300	2087.00	6.75	0.962508972
400	-20	1300	2198.00	6.53	0.962508972
440	-20	1300	2294.00	6.31	0.962508972
480	-20	1300	2384.00	6.10	0.962508972
520	-20	1300	2465.00	5.91	0.962508972
560	-20	1300	2539.00	5.72	0.962508972
360	-20	1400	2169.00	7.26	0.959670266
400	-20	1400	2284.00	7.04	0.959670266
440	-20	1400	2388.00	6.83	0.959670266
480	-20	1400	2484.00	6.62	0.959670266
520	-20	1400	2571.00	6.42	0.959670266
560	-20	1400	2651.00	6.23	0.959670266
360	-20	1500	2245.00	7.76	0.956837969
400	-20	1500	2365.00	7.55	0.956837969
440	-20	1500	2475.00	7.34	0.956837969
480	-20	1500	2576.00	7.13	0.956837969
520	-20	1500	2668.00	6.94	0.956837969
560	-20	1500	2753.00	6.75	0.956837969
360	-20	1600	2315.00	8.26	0.954012072
400	-20	1600	2441.00	8.14	0.954012072
440	-20	1600	2556.00	7.84	0.954012072
480	-20	1600	2661.00	7.64	0.954012072
520	-20	1600	2758.00	7.45	0.954012072
560	-20	1600	2848.00	7.26	0.954012072
360	-20	1700	2380.00	8.75	0.951192564
400	-20	1700	2511.00	8.54	0.951192564

V (kts) :	γ :	h (ft) :	r (ft) :	t_f (s) :	ρ/ρ_0
440	-20	1700	2630.00	8.33	0.951192564
480	-20	1700	2740.00	8.14	0.951192564
520	-20	1700	2841.00	7.95	0.951192564
560	-20	1700	2936.00	7.76	0.951192564
360	-20	1800	2441.00	9.23	0.948379435
400	-20	1800	2576.00	9.02	0.948379435
440	-20	1800	2699.00	8.82	0.948379435
480	-20	1800	2813.00	8.62	0.948379435
520	-20	1800	2919.00	8.44	0.948379435
560	-20	1800	3017.00	8.26	0.948379435
360	-20	1900	2497.00	9.70	0.945572676
400	-20	1900	2637.00	9.50	0.945572676
440	-20	1900	2764.00	9.30	0.945572676
480	-20	1900	2882.00	9.11	0.945572676
520	-20	1900	2991.00	8.93	0.945572676
560	-20	1900	3092.00	8.75	0.945572676
360	-20	2000	2551.00	10.17	0.942772276
400	-20	2000	2693.00	9.97	0.942772276
440	-20	2000	2824.00	9.78	0.942772276
480	-20	2000	2945.00	9.59	0.942772276
520	-20	2000	3058.00	9.41	0.942772276
560	-20	2000	3162.00	9.23	0.942772276
360	-30	1200	1559.00	4.73	0.965354098
400	-30	1200	1619.00	4.45	0.965354098
360	-30	1300	1646.00	5.19	0.962508972
400	-30	1300	1713.00	4.89	0.962508972
440	-30	1300	1771.00	4.62	0.962508972
360	-30	1400	1729.00	5.65	0.959670266
400	-30	1400	1802.00	5.34	0.959670266
440	-30	1400	1866.00	5.06	0.959670266
360	-30	1500	1806.00	6.11	0.956837969
400	-30	1500	1885.00	5.80	0.956837969
440	-30	1500	1955.00	5.51	0.956837969
480	-30	1500	2016.00	5.24	0.956837969
360	-30	1600	1878.00	6.56	0.954012072
400	-30	1600	1963.00	6.25	0.954012072
440	-30	1600	2038.00	5.96	0.954012072
480	-30	1600	2105.00	5.68	0.954012072
360	-30	1700	1946.00	7.02	0.951192564
400	-30	1700	2036.00	6.71	0.951192564
440	-30	1700	2117.00	6.41	0.951192564
480	-30	1700	2188.00	6.13	0.951192564
520	-30	1700	2252.00	5.87	0.951192564
360	-30	1800	2009.00	7.47	0.948379435
400	-30	1800	2105.00	7.16	0.948379435
440	-30	1800	2190.00	6.86	0.948379435
480	-30	1800	2267.00	6.58	0.948379435
520	-30	1800	2336.00	6.31	0.948379435
360	-30	1900	2069.00	7.93	0.945572676
400	-30	1900	2170.00	7.61	0.945572676
440	-30	1900	2260.00	7.31	0.945572676
480	-30	1900	2341.00	7.03	0.945572676
520	-30	1900	2414.00	6.76	0.945572676
560	-30	1900	2480.00	6.50	0.945572676
360	-30	2000	2125.00	8.37	0.942772276
400	-30	2000	2230.00	8.06	0.942772276
440	-30	2000	2325.00	7.76	0.942772276
480	-30	2000	2410.00	7.48	0.942772276
520	-30	2000	2488.00	7.21	0.942772276
560	-30	2000	2558.00	6.95	0.942772276
360	-30	2100	2178.00	8.82	0.939978226
400	-30	2100	2288.00	8.51	0.939978226
440	-30	2100	2386.00	8.21	0.939978226
480	-30	2100	2476.00	7.92	0.939978226
520	-30	2100	2557.00	7.65	0.939978226
560	-30	2100	2631.00	7.39	0.939978226
360	-30	2200	2228.00	9.27	0.937190515
400	-30	2200	2341.00	8.95	0.937190515
440	-30	2200	2444.00	8.65	0.937190515

V (kts)	$-\gamma$	h (ft)	r (ft)	t_f (s)	ρ/ρ_0
480	-30	2200	2537.00	8.37	0.937190515
520	-30	2200	2622.00	8.10	0.937190515
560	-30	2200	2700.00	7.84	0.937190515
360	-30	2300	2275.00	9.71	0.934409133
400	-30	2300	2392.00	9.39	0.934409133
440	-30	2300	2499.00	9.10	0.934409133
480	-30	2300	2595.00	8.81	0.934409133
520	-30	2300	2684.00	8.54	0.934409133
560	-30	2300	2765.00	8.28	0.934409133
360	-30	2400	2319.00	10.15	0.931634071
400	-30	2400	2440.00	9.83	0.931634071
440	-30	2400	2550.00	9.54	0.931634071
480	-30	2400	2650.00	9.25	0.931634071
520	-30	2400	2742.00	8.98	0.931634071
560	-30	2400	2827.00	8.72	0.931634071
360	-30	2500	2361.00	10.58	0.928865319
400	-30	2500	2486.00	10.27	0.928865319
440	-30	2500	2599.00	9.98	0.928865319
480	-30	2500	2702.00	9.69	0.928865319
520	-30	2500	2797.00	9.42	0.928865319
560	-30	2500	2885.00	9.17	0.928865319
360	-30	2600	2401.00	11.02	0.926102866
400	-30	2600	2529.00	10.71	0.926102866
440	-30	2600	2645.00	10.41	0.926102866
480	-30	2600	2751.00	10.13	0.926102866
520	-30	2600	2849.00	9.86	0.926102866
560	-30	2600	2939.00	9.60	0.926102866
360	-30	2700	2439.00	11.45	0.923346703
400	-30	2700	2569.00	11.14	0.923346703
440	-30	2700	2688.00	10.85	0.923346703
480	-30	2700	2797.00	10.57	0.923346703
520	-30	2700	2898.00	10.30	0.923346703
560	-30	2700	2991.00	10.04	0.923346703
360	-30	2800	2475.00	11.87	0.920596819
400	-30	2800	2608.00	11.57	0.920596819
440	-30	2800	2730.00	11.28	0.920596819
480	-30	2800	2842.00	11.00	0.920596819
520	-30	2800	2945.00	10.73	0.920596819
560	-30	2800	3040.00	10.48	0.920596819
360	-30	2900	2509.00	12.30	0.917853206
400	-30	2900	2645.00	12.00	0.917853206
440	-30	2900	2769.00	11.71	0.917853206
480	-30	2900	2883.00	11.43	0.917853206
520	-30	2900	2989.00	11.16	0.917853206
560	-30	2900	3087.00	10.91	0.917853206
360	-30	3000	2541.00	12.73	0.915115852
400	-30	3000	2680.00	12.42	0.915115852
440	-30	3000	2806.00	12.13	0.915115852
480	-30	3000	2923.00	11.86	0.915115852
520	-30	3000	3031.00	11.59	0.915115852
560	-30	3000	3131.00	11.34	0.915115852