

QUANTUM PROCESSING SYSTEMS

TECHNICAL FIELD

[0001] Aspects of the present disclosure are related to quantum processing systems and methods for operating the same, and more particularly, to quantum processing systems which include nuclear spins.

BACKGROUND

[0002] Large-scale quantum processing systems hold the promise of a technological revolution, with the prospect of solving problems that are out of reach with classical machines. To date, several different structures, materials, and architectures have been proposed to implement quantum processing systems and fabricate their basic information units (quantum bits or qubits).

SUMMARY

[0003] Embodiments of the present disclosure are directed towards scalable quantum computing architectures comprising nuclear spins. This includes any quantum processing systems that include nuclear spin registers, such as Nitrogen-Vacancy (NV) centres in diamond, solid-state quantum systems based on donor atoms in semiconductor substrates, etc.. Aspects of the present disclosure focus on investigating error sources arising from dipolar coupling within nuclear spin registers. Furthermore, the disclosure provides solutions to minimise these error sources to provide a more efficient and/or reliable initialisation, operation and/or read-out of such quantum processing systems.

[0004] Characterising and minimising dipolar-induced errors induced by nuclear spins is an important tool for improving and scaling up the quantum computing architectures comprising nuclear spin registers. Inventors of the present disclosure have found that nuclear-nuclear dipolar coupling errors can be minimised by decreasing the tunnel time which may be achieved through optimal atomic-scale engineering of the quantum computing devices. Such engineering may reliably minimise nuclear spin errors and hence provide reliable, efficient, and scalable quantum computers.

[0005] According to a first aspect of the present disclosure, there is provided a quantum processing device comprising: a nuclear spin register comprising at least two nuclear spin qubits; at least one unpaired electron coupled to the nuclear spin register; a control system configured to: control and/or readout the electron spin qubit; control and/or readout a nuclear spin qubit; and reduce errors arising from dipolar coupling between the nuclear spin qubits and/or between the electron spin and nuclear spins.

[0006] In some embodiments, the errors in the quantum processing device arise from direct dipolar coupling between at least one pair of nuclear spins in the nuclear spin register. Reducing these errors comprises performing electron spin readout at a rate faster than the fastest nuclear spin flip-flop oscillation in the nuclear spin register. Further, the rate of the electron spin readout is dependent on a tunnel rate of the electron moving on and off the nuclear spin register. In addition, the tunnel rate of the electron moving on and off the nuclear spin register is engineered by controlling a distance between a reservoir and the nuclear spin register during a fabrication process.

[0007] In other embodiments, the errors in the quantum processing device arise from an anisotropic coupling between the at least one unpaired electron and at least one of the nuclear spins in the nuclear spin register. In some embodiments, reducing these errors comprises: engineering a hyperfine coupling between the at least one unpaired electron and at least one of the nuclear spins of the nuclear spin register by precision fabrication of the at least one nuclear spin in the nuclear spin register. In particular, the precision fabrication of the plurality of nuclear spins in the nuclear spin register comprises engineering the location of each nuclear spin in the plurality of nuclear spins. In some other embodiments, reducing these errors comprises tuning a static magnetic field B_0 .

[0008] In some embodiments, the control system comprises a single electron transistor (SET) that acts as a charge sensor and an electron reservoir to load the at least one unpaired electron onto the nuclear spin register. Further, the control system may include one or more control gates to control the electrochemical potential of the two or more donor atoms in the nuclear spin register.

[0009] According to another aspect of the present disclosure there is provided a method for reducing readout errors in a nuclear spin register, wherein the nuclear spin register comprises at least two nuclear spins and an unpaired electron confined to the nuclear spin register, the method comprising: performing electron spin readout at a rate faster than the

fastest nuclear spin flip-flop oscillation in the nuclear spin register for errors arising from direct nuclear-nuclear dipolar coupling; and/or engineering a hyperfine coupling between the at least one unpaired electron and at least one of the nuclear spins of the plurality of nuclear spins by precision fabrication of the plurality of nuclear spins wherein the errors arise from an anisotropic hyperfine coupling.

[0010] The rate of the electron spin readout is dependent on a tunnel rate of the electron moving on and off the nuclear spin register and the method further includes controlling a distance between a reservoir and the nuclear spin register during a fabrication process to engineer the tunnel rate of the electron moving on and off the nuclear spin register.

[0011] In some embodiments, the precision fabrication of the plurality of nuclear spins in the nuclear spin register comprises engineering the location of each nuclear spin in the plurality of nuclear spins. In some embodiments, the precision fabrication of the plurality of nuclear spins comprises: determining the strength of the static magnetic field at which a quantum device comprising the nuclear spin register is to be operated; determining a plurality of nuclear spin register layouts; for each of the plurality of nuclear spin register layouts: computing hyperfine strengths between each donor atom in the nuclear spin register and the unpaired electron; computing error rate based on the computed hyperfine strengths; and discarding one or more of the plurality of nuclear spin register layouts if the computed error rate for the one or more of the plurality of nuclear spin register layouts exceeds a threshold error rate; and fabricating the quantum device based on any one of the remaining layout of the plurality of nuclear spin register layouts.

[0012] In some embodiments, reducing the errors arising from the anisotropic coupling comprises tuning a static magnetic field B_0 . Tuning the static magnetic field comprises: determining strength of hyperfine interaction between each nuclear spin in the nuclear spin register and the unpaired electron confined to the nuclear spin register; calculating predicted error value for the error arising from the anisotropic coupling based on the computed strengths of the hyperfine interactions; and changing the static magnetic field to reduce the predicted error value if the predicted error value exceeds a threshold value.

BRIEF DESCRIPTION OF DRAWINGS

[0013] While the invention is amenable to various modifications and alternative forms, specific embodiments are shown by way of example in the drawings and are described in detail. It should be understood, however, that the drawings and detailed description are not intended to limit the invention to the particular form disclosed. The intention is to cover all modifications, equivalents, and alternatives falling within the spirit and scope of the present invention as defined by the appended claims.

[0014] Fig. 1A is an example multi-donor quantum dot device illustrating a nuclear spin register.

[0015] Fig. 1B is a Scanning Tunnelling Microscopy (STM) micrograph of a nuclear spin register with a control system according to aspects of the present disclosure.

[0016] Fig. 2 is a schematic of three different multi-donor quantum dots.

[0017] Fig. 3 shows an example 1D architecture with five multi-donor quantum dots.

[0018] Fig. 4A and 4B show further example 1D chains with varying inter-dot distances and angles.

[0019] Fig. 5A shows an example of a 3P nuclear spin register in a silicon crystal.

[0020] Fig. 5B and 5C shows ESR transitions and frequencies of the multi-nuclear spin register shown in Fig. 5A, respectively.

[0021] Fig. 6A shows a schematic protocol for implementing quantum non-demolition (QND) measurement.

[0022] Figs. 6B-6D show Random Telegraph Signals (RTS) showing the evolution of the three nuclear spins $\hat{f}^{(1)}$, $\hat{f}^{(2)}$, $\hat{f}^{(3)}$, respectively.

[0023] Figs. 6E-6G show histograms of the raw signal in Figs. 6B-6D for nuclear spins $\hat{f}^{(1)}$, $\hat{f}^{(2)}$, $\hat{f}^{(3)}$ respectively.

[0024] Figs. 6H-M show histograms of the nuclear spin lifetimes of nuclear spins $\hat{f}^{(1)}$, $\hat{f}^{(2)}$, $\hat{f}^{(3)}$ for the $|\uparrow\rangle$ (left) and $|\downarrow\rangle$ (right) states.

[0025] Fig. 7A is a schematic protocol for a Hahn echo circuit implementing a Hahn echo sequence to measure nuclear-nuclear dipole oscillations.

[0026] Fig. 7B is a plot of the probability between nuclear spin flips $\hat{I}^{(1)}$ & $\hat{I}^{(2)}$ as a function of the total Hahn echo time.

[0027] Fig. 7C is a plot of the probability between nuclear spin flips $\hat{I}^{(2)}$ & $\hat{I}^{(3)}$ as a function of the total Hahn echo time.

[0028] Fig. 7D is a plot of the probability between nuclear spin flips $\hat{I}^{(1)}$ & $\hat{I}^{(3)}$ as a function of the total Hahn echo time.

[0029] Fig. 7E shows the fitted dipolar coupling strengths between each pair of nuclear spins in the example 3P nuclear spin register.

[0030] Fig. 7F shows the error incurred per electron $|\uparrow\rangle$ measurement due to nuclear-nuclear dipole coupling, as a function of the electron $|\downarrow\rangle_{in}$ tunnel time, for three dipole strengths.

[0031] Fig. 7G is a plot depicting the strength of nuclear-nuclear dipolar coupling as a function of the distance between a pair of donor nuclei.

[0032] Fig. 7H is a plot depicting dipolar coupling error for all the nuclear-nuclear dipolar couplings depicted in Fig. 7G.

[0033] Fig. 7I depicts the relationship between tunnel rates and distance between donor atoms and a charge sensor for a 2P and 3P system.

[0034] Figs. 8A-8C show the energy levels in the subspace of the electron spin and nuclear spins $\hat{I}^{(1)}$, $\hat{I}^{(2)}$, $\hat{I}^{(3)}$ respectively.

[0035] Fig. 8D shows a fitted ratio $|R|$ of the Zeeman splitting and anisotropic hyperfine coupling as a function of a static external magnetic field.

[0036] Fig. 8E is a plot of the theoretical error induced by an electron $|\uparrow\rangle$ tunnel event as a function of a static magnetic field.

[0037] Fig. 8F is a plot of the theoretical error induced by an electron $|\downarrow\rangle$ tunnel event as a function of the static magnetic field.

[0038] Fig. 8G shows theoretical errors due to the double spin transitions during ESR driving, compared to the intended one spin ESR transitions, expressed as a ratio of the transition dipole moments.

[0039] Fig. 9 is a flowchart illustrating an example method according to aspects of the present disclosure.

[0040] Fig. 10 illustrates an example method for fabricating the quantum processor according to aspects of the present disclosure.

DETAILED DESCRIPTION

[0041] Although quantum processors and quantum dots described herein refer to donor atoms and unpaired electrons, it will be appreciated that these are merely examples and that the quantum processors and quantum dots of the present disclosure can be formed of donor or acceptor atoms (commonly referred to as dopant atoms) and unpaired electrons or holes can be confined in such quantum dots without departing from the scope of the present disclosure.

[0042] To date, several quantum processing architectures in silicon have been disclosed. Nuclear spins represent one of the earliest incarnations of qubits (the basic units of quantum information). By virtue of their weak coupling to the environment, nuclear spins have the longest coherence times in solid-state systems. That is why individual phosphorus nuclear spins in silicon have been considered promising as qubits for quantum computing. The architecture proposed in 1998 by B. E. Kane included an array of nuclear spins located in donor atoms in silicon. Following that, other architectures including electron spins, nuclear spins or a combination of electron and nuclear spins have also been proposed.

[0043] In general, systems that include nuclear spins and in particular nuclear spin registers provide a promising future for developing scalable quantum processing systems. A nuclear spin register is a system in which the quantum states of nuclear spins are used to store and process quantum information. A nuclear spin register includes at least two nuclear spins, where each nuclear spin can act as a qubit. A nuclear spin register comprises at least one electron confined to the nuclear spin register. Nuclear spin registers take advantage of the long coherence times of nuclear spins to maintain quantum information over extended periods.

[0044] The present disclosure is directed to a solid-state system including at least one nuclear spin register. For example, solid-state quantum systems based on donor atoms in semiconductor substrates is an example where nuclear spin registers are present.

[0045] In solid-state quantum systems, closely spaced dopant atoms in a semiconductor substrate can be used as a nuclear spin register, in which an unpaired electron can interact with multiple nuclear qubits which are addressable by their unique hyperfine couplings to the electron. In these nuclear spin registers, an unpaired electron can be used to address multiple nuclei, allowing for powerful multi-qubit gates including the multi-controlled-Z gate. A multi-qubit gate is a fundamental operation in quantum computing that acts on two or more qubits simultaneously, altering their quantum states in a correlated manner.

[0046] Whilst nuclear spin registers are a promising candidate for quantum computation, to scale and improve nuclear spin registers, it is important to understand and reduce the error sources present in such nuclear spin registers.

[0047] The present disclosure focuses on investigating error sources arising from dipolar coupling within such nuclear spin registers. In a nuclear spin register with at least one electron there are two sources of dipolar coupling: direct nuclear spin dipole coupling and anisotropic hyperfine coupling.

[0048] Direct nuclear spin dipole coupling (or nuclear-nuclear dipolar coupling/interaction) refers to the interaction between magnetic dipole moments of nearby nuclear spins. Nuclear spins have an intrinsic magnetic moment due to their angular momentum. These magnetic moments generate local magnetic fields, and when nuclear spins are close to each other, they can interact through their respective magnetic fields. This interaction is distance-dependent, meaning the strength of the coupling depends on the spatial separation between the nuclear spins (typically scaling as $1/r^3$, where r is the distance between the pair of nuclear spins). Further, the direct nuclear spin dipole coupling is an interaction between pairs of nuclear spins. For example, in a system with two nuclear spins, there is one direct nuclear spin dipole interaction to consider. However, in a system with three nuclear spins, there are three pairs of nuclear spins and therefore three direct nuclear spin dipole interactions to consider. In general, for a system with n nuclear spins there are $n(n-1)/2$ direct nuclear spin dipole interactions to consider.

[0049] The nuclear-nuclear dipolar interaction can cause pairs of nuclear spins to swap with each other while the (single unpaired) electron is not present on the nuclear spin register. During normal operation, the unpaired electron is removed during readout of the electron. Hence, whenever the electron is measured (i.e., readout), there is a chance that a pair of nuclear spins will erroneously flip with each other, causing an error to the nuclear spin qubits.

The chance of error generally increases with increasing time spent without the unpaired electron and generally decreases if the nuclei are placed further apart from each other.

[0050] The second type of dipolar coupling present is anisotropic hyperfine coupling which refers to the interaction between the magnetic moments of a nuclear spin and an electron spin. Like nuclear spins, electrons also have an intrinsic magnetic moment due to their angular momentum. Anisotropic hyperfine coupling is direction-dependent and occurs when the electron's magnetic field interacts with the nuclear spin in an anisotropic manner, depending on their relative orientation and distance between. Therefore, the strength of the hyperfine coupling between an electron and different nuclear spins in the nuclear spin register may vary based on the orientation and distance between the two magnetic moments.

[0051] The anisotropic hyperfine interaction (AHF) causes nuclear spin states to be tilted slightly. Formally, the AHF introduces off-diagonal terms in the Hamiltonian – a function that describes the total energy of a system. In particular, AHF causes mixing between the nuclear spin-up $|\uparrow\rangle$ and spin-down $|\downarrow\rangle$ states, with more tilting/mixing present when the electron is in the spin-up state. This is because the amount of mixing that occurs is related to how strong the AHF is compared to the energy difference the nuclear spin-up $|\uparrow\rangle$ and spin-down $|\downarrow\rangle$ states. This energy difference is smaller when the electron is up, allowing the AHF to mix the states more. When the electron is removed (again, during readout of the electron), this tilting/mixing switches off; when the electron is loaded, the tilting/mixing turns on again. The sudden switching on/off of the tilting/mixing may cause the nuclear spins to flip which may result in nuclear spin errors during electron readout. The chance of error increases with the number of readouts performed, and also increases if the nuclear spin-up $|\uparrow\rangle$ and spin-down $|\downarrow\rangle$ states are closer in energy (which is determined by the strength of the contact hyperfine interaction and the magnetic field).

[0052] Characterising and minimising dipolar-induced errors is an important tool for scaling up quantum computing architectures comprising nuclear spin registers. This disclosure describes systems and methods to reduce these error sources and to provide a more efficient and/or reliable system and/or method for initialisation, operation and/or readout of the quantum processing systems that include nuclear spin registers.

[0053] More specifically, two forms of dipolar coupling are demonstrated 1) direct nuclear-nuclear spin dipole coupling which creates errors during electron readout, and 2) the anisotropic hyperfine coupling between the electron and nuclear spins which also leads to

error “hotspots” during electron readout and control. In the present disclosure, both sources of error are quantified to provide solutions for better initialisation and/or control and/or readout of the quantum processing system.

[0054] In particular, inventors of the present disclosure found that with control over the magnetic field or contact hyperfine strength, along with engineering of electron tunnel rates, these errors can be minimised to obtain >99.9 % fidelity in long-lived nuclear spin qubits.

[0055] In particular, nuclear-nuclear dipolar coupling errors can be avoided by decreasing tunnel time, which can be achieved through atomic-scale engineering of the distance between a nuclear spin register and a reservoir. Anisotropic hyperfine coupling errors can also be minimised, either through engineering of the donor locations and/or through adjustment of stark shift to control nuclear hyperfine couplings, or by tuning of the global magnetic field B_0 to avoid error hot spots. The present disclosure shows that such engineering reliably produces nuclear spin errors below 0.1% and hence provides more reliable and/or efficient quantum computing systems including nuclear spin registers.

Solid-state Quantum Systems

[0056] Figs. 1-4 show examples of solid-state quantum processing systems comprising nuclear spin registers. These example systems comprise multi-donor dots in a silicon substrate.

[0057] Fig. 1 illustrates an example multi-donor quantum dot device 100 comprising a nuclear spin register. The quantum dot device 100 includes a quantum dot 101 located in a semiconductor substrate 104. In this example, the semiconductor substrate 104 is 28-Silicon. The silicon substrate 104 is topped by a barrier material/dielectric 105 such as silicon dioxide.

[0058] The multi-donor quantum dot 101 includes a plurality of dopant atoms 110 embedded in the semiconductor substrate 104. In this example, the quantum dot 101 includes three dopant atoms, 110A, 110B, and 110C. The dopant atoms may also be referred to as donor atoms. In some examples, the dopant/donor atoms are phosphorus atoms. In the case where the quantum dot 101 comprises m phosphorus atoms, the device may be generally referred to as a ‘mP’ quantum dot device. In the example shown in Fig. 1, the quantum dot system may generally be referred to as a 3P quantum dot system or a 3P system. The nuclear spins provided by the donor atoms 110 act as a nuclear spin register.

[0059] Further, a gate 111 may be located on top of the dielectric 105 in a region substantially above the cluster of donor atoms 110A, 110B and 110C. It will be appreciated

that although gate 111 is shown as a surface gate, it can (in some implementations) be an in-plane gate that is fabricated within the substrate 104. In some examples, the gate may be in the same plane as the quantum dot 101. In some embodiments, the quantum dot device 100 may include additional gates that may be located on top of the dielectric 105 and/or embedded within the substrate 104.

[0060] Voltages may be applied to gate 111 to create a potential well (confinement potential) in which one or more electrons can be trapped/confined in the quantum dot 101. The one or more electrons are confined by the Coulombic potential of the donor atoms 110. If there are an odd number of electrons, then the outermost unpaired electron interacts with the nuclear spins provided by the donor atoms 110 in the same way that a single electron interacts with the nuclear spins (i.e., nuclear spin register). The electron is loaded through quantum tunnelling from a nearby electron reservoir (not shown). Once loaded, an unpaired electron's spin state can serve as a qubit, and this state can be manipulated and measured for quantum computing applications. In the example shown in Fig. 1, one electron 112 is confined in the quantum dot 101. However, the 3P quantum dot 101 shown in Fig. 1 can confine more electrons. In some examples, physical constraints may limit the number of electrons in an mP system to be $m+1$. That is, if an attempt is made to add another electron to the register, the extra electron will have so much energy it will escape the register and not be bound to it. In a preferred embodiment, the number of donor atoms is less than 10. The number of donor atoms may be limited by a practical constraint – i.e., the number of ESR peaks scales as 2^m , so if you have $m = 10$ nuclear spins, there will be $2^{10} = 1024$ ESR peaks, which will likely make such high donor numbers infeasible to control, especially if they are inside a larger device with multiple registers. Other limitations may also exist, but these are some examples.

[0061] A static external magnetic field, denoted B_0 is applied to the quantum dot system 100 to generate well-defined spin polarised states (i.e., a separation of the spin-up state and the spin-down state) split by the Zeeman energy. This external magnetic B_0 field splits both the nuclear spin states and the electron spin states by the Zeeman interaction (i.e., the degenerate energy levels of the spins are split in the presence of a magnetic field). In some examples, the external static magnetic field B_0 may be generated by one or more permanent magnets, which are positioned around the qubit device 100 to create a relatively stable magnetic field. In some examples, the external magnetic field B_0 is generated by one or more electromagnets or superconducting magnets or solenoid magnets.

[0062] Fig. 1B is a Scanning Tunnelling Microscopy (STM) micrograph of a quantum processing system or a quantum computing device 120. The quantum processing system 120 comprises a nuclear spin register, at least one unpaired electron that is coupled to the nuclear spin register and a control system (not shown) according to aspects of the present disclosure. Further, the nuclear spin register comprises at least two nuclear spins, which may each act as a qubit in the quantum processing system. As such, the quantum computing device 120 comprises a quantum dot 101.

[0063] The control system is configured to perform operations on the electron spin qubit and/or one or more nuclear spin qubits in the quantum processing system 120. For example, the control system is configured to control and/or readout the electron spin qubit. Further, the control system is configured to control and/or readout one or more of the nuclear spins of the plurality of nuclear spin qubits in the system.

[0064] In particular, the control system may include or control a single-electron transistor (SET) 122, and one or more gates (124, 126, 128). The qubits may be tunnel coupled to the SET 122 that acts as a charge sensor and an electron reservoir to load electrons onto the quantum dot 101. Further, the qubits may be controlled by one or more of the gates 124-128. Fig. 1B illustrates three gates – left gate 124, middle gate 126, and right gate 128, which can be used to control the electrochemical potentials of the donor dot 101, whereas the SET gate 130 is predominately used to control the electrochemical potential of the SET 122. In one implementation, the gates may be metal contacts on the surface. In another implementation, the gates may be phosphorus-doped silicon (SiP) gates fabricated epitaxially within the semiconductor substrate. In either case, the control system may apply signals to the various gates for full electrostatic control of the qubits.

[0065] Although an SET 122 is depicted in Fig. 1B, qubit readout can be performed using other mechanisms. For instance, it may be performed dispersively using the left, right, or middle gates 124-128 mentioned previously.

[0066] Additionally, the control system may include or control a global or local nuclear magnetic resonance (NMR) antenna, which allows control of the nuclear spins of the donors via radio frequency (RF) magnetic fields in the range of between 1 MHz and 100 MHz. The NMR antenna (not shown) can be manufactured on chip, or off-chip (cavity or coil). The control of the nuclear spins is necessary for initialization and gate operations. Additionally, the

control system may further include or control an on-chip and/or off-chip resonator (not shown) to read out the state of the unpaired electron.

[0067] Electronic structures for readout and control can be placed on chip, or on the printed circuit board (PCB) which holds the silicon chip. They include waveguides, resonators, bias tees, amplifiers, filters, mixers circulators, etc. Any of these structures can be implemented using on chip lithographic structures or on the PCB using commercially available surface mount devices (SMD).

[0068] Although quantum device 120 shows a single quantum dot 101, a quantum processor may be formed of a plurality of such multi-donor quantum dots 101 arranged in a suitable array or pattern. In addition to multi-donor quantum dots, such quantum processors may also include one or more single donor atom quantum dots without departing from the scope of the present disclosure.

[0069] It will be appreciated that the quantum dot device 100 may be part of a quantum processor capable of performing quantum operations on one or more qubits in any given quantum dot. Further, the 3P quantum dot shown in Fig. 1 is an example nuclear spin register with three nuclear spins (provided by the three donor atoms 110) and one electron confined to the nuclear spin register.

[0070] Fig. 2 shows a schematic of three different multi-donor quantum dots 101A, 101B and 101C. The large circles 202 represent the wavefunction of an unpaired electron 112 confined to each multi-donor quantum dot 101A-C. The small filled-in circles represent the donor atoms 208-212. For example, in multi-donor quantum dot 101A, there are two donor atoms 208 and an unpaired electron confined to the quantum dot 101A, where the electron wavefunction is represented by 202. In quantum dot 101B, there are three donor atoms 210 and an unpaired electron confined to the quantum dot 101B, where the electron wavefunction is represented by 202, and in quantum dot 101C, there are four donor atoms 212 and an unpaired electron confined to the quantum dot 101C, where the electron wavefunction is represented by 202. The multi-donor quantum dots 101A, 101B and 101C may be denoted as 2P, 3P, and 4P quantum dots, respectively.

[0071] The quantum dot architecture may include a one-dimensional (1D) array of quantum dots. Fig. 3 shows an example architecture 300 including a 1D array of five quantum dots QD1-QD5. Each quantum dot may have one or more donor atoms. The inter-donor distance in any given quantum dot or the size of a quantum dot (r) is less than the Bohr radius.

In some examples, this inter-donor distance or size of the quantum dot is $r \leq 3$ nanometers. The inter-dot distance (d) – the distance between adjacent quantum dots may be in the range of 5-20 nanometers. It will be appreciated that the inter-dot distances (d) between quantum dots (e.g., QD1-QD2 and QD2-QD3) may not be uniform, but may vary within the range 5-20 nanometres. Further, the size of a quantum dot (r) may be dictated by the number of donor atoms present in the quantum dot – the greater the number of donor atoms in a quantum dot, the larger its size or inter-donor distance r and the fewer the number of donor atoms in a quantum dot, the smaller its size. For example, the size of a 1P quantum dot may be about 0.7 nanometres, the size of a 2P quantum dot may be about 1 nanometre and the size of a 3P quantum dot may be about 1.5 nanometres.

[0072] Figs. 4A and 4B each show an example of other quantum processor architectures 410, 420 that include a 1D array of quantum dots with varying inter-dot distances and angles. The quantum dots are shown as open circles positioned along the 1D array. For example, array 410 has a staggered geometry, where the quantum dots are not aligned along a single axis. Instead, odd-numbered quantum dots are positioned along a first line axis and even-numbered multi-donor quantum dots are positioned along a second line axis, where the first axis is parallel to the second axis.

[0073] The array 420 is similar to array 410, but in this case, pairs of quantum dots are positioned along the first and second line axes.

[0074] In yet other examples, a quantum processor that includes a plurality of multi-donor quantum dots described with respect to Fig. 1 may be formed of two-dimensional (2D) or three-dimensional (3D) patterns – not shown.

[0075] It will be appreciated that the present disclosure is directed to various types of operational modes of the quantum processing systems shown in Figs. 1-4. For example, in the disclosed quantum processors, the electron spins and/or nuclear spins of the quantum dots can be used as qubits for various different types of quantum operations. In one operational mode, unpaired electrons of a quantum dot may be used as qubits. In another operational mode, nuclear spins of the donors may be used as qubits. In yet another example, both electron and nuclear spins may be used as qubits. In this operational mode, the electron spin may be used as a data qubit, while the nuclear spins of a quantum dot are used as atomic magnets. Similarly, the electron spin may be used as a data qubit while the nuclear spins of the quantum dot are used for error correction such that the quantum dot can function as an error

corrected logical qubit. In another example, the nuclear spins in a quantum dot can be used as data qubits, whereas the electron spin in the quantum dot is used to address or measure the nuclear spin qubits. In another example, the nuclear spins can be used as data qubits, whereas the electron spin in the quantum dot is used to couple the quantum dot to adjacent quantum dots via electron shuttling or exchange coupling. Finally, both the electron spin and one or more nuclear spins can be used in combination as data qubits.

Triple-donor dot readout

[0076] As an example, initialisation, control and readout of both electron and nuclear spins in a quantum device with three closely spaced phosphorus donors in silicon is demonstrated in Figs. 5A-C.

[0077] Initialisation, control and readout are all examples of operations that are performed on one or more qubits. For example, initialisation refers to the process of preparing a qubit in a well-defined quantum state at the start of a quantum computation. Many quantum algorithms start with qubits initialised in the ground state, which is typically denoted as the $|0\rangle$ state. Control operations involve applying quantum gates (e.g., single or multi-gates) to manipulate the qubits into superposition states, entangled states, or other complex configurations needed for the computation. Lastly, readout refers to the process of measuring the state of one or more qubits after a quantum computation has been performed – e.g., after one or more control operations. The readout measurement collapses the quantum state of the qubits, which may be in a superposition of states, into one of the classical basis states – typically $|0\rangle$ or $|1\rangle$ for each qubit.

[0078] Fig. 5A shows an example nuclear spin register comprising three phosphorus atoms providing three nuclear spins denoted by $\hat{I}^{(1)}$, $\hat{I}^{(2)}$ and $\hat{I}^{(3)}$. The nuclear spins of the three donor atoms provide the nuclear spin register. These three donor atoms have been precision placed in silicon crystal using STM lithography to form a nuclear spin register. The silicon atoms 502 represent the position of the atoms within the crystal structure. Crystallographic axis labels 504 are also provided for reference in Fig. 5A. Loaded onto the 3P nuclear spin register is a single electron – the wavefunction of the electron is illustrated by oval 506 (dashed line).

[0079] By applying a static magnetic field, e.g., $B_0 \sim 1.4\text{T}$, the electron and nuclear spin states are split by the Zeeman interaction on the order of $2\pi \cdot 39\text{ GHz}$ and $2\pi \cdot 24\text{ MHz}$, respectively. The standard Hamiltonian used to describe this 3P nuclear spin register when the electron is loaded onto the system, H_{load} , is

$$\hat{H}_{load}(B_0) = \left(\gamma_e \hat{S}_z + \sum_{i=1}^3 \gamma_n \hat{I}_z^{(i)} \right) B_0 + \sum_{i=1}^3 A^{(i)} \hat{\mathbf{S}} \cdot \hat{\mathbf{I}}^{(i)} \quad \text{Eq. (1)}$$

where $\gamma_e/2\pi \approx 27.96$ GHz/T ($\gamma_n/2\pi \approx -17.24$ 99 MHz/T) is the electron (phosphorus nuclear) gyromagnetic ratio, $\mathbf{S}$ ($\hat{\mathbf{I}}^{(i)}$) is the electron (i^{th} phosphorus nuclear spin) spin operator, and $A^{(i)}$ is the contact hyperfine interaction between the electron and the i^{th} nuclear spin.

[0080] The energy levels given by Eq. 1 are illustrated in Fig. 5B, based on the hyperfine couplings observed on the device. Fig. 5B is annotated with the eight electron spin resonance (ESR) transitions (denoted ν_i^e , where $i=1, \dots, 8$) that can be driven by applying an oscillating magnetic field B_1 . The oscillating magnetic field B_1 is typically generated using microwave signals at specific frequencies. In some examples, the oscillating magnetic field B_1 is generated using microwave sources or on-chip microwave components. In other examples, the oscillating magnetic field B_1 is generated using an antenna. Typically, the oscillating magnetic field B_1 is applied perpendicular to B_0 at a frequency matching the respective energy splitting of the electron spin states, when controlling the electron qubit. The oscillating magnetic field B_1 is applied perpendicular to B_0 at a frequency matching the respective energy splitting of the nuclear spin states, when controlling a nuclear spin qubit. An ESR transition effectively flips the electron spin while the nuclear spins remain unchanged. For example, ν_1^e flips the electron spin while the nuclear spin configuration remains $|\Downarrow\Downarrow\Downarrow\rangle$. Where the position of the nuclear spin in the state $|\Downarrow\Downarrow\Downarrow\rangle$ corresponds to the state of the first, second and third nuclear spins, respectively.

[0081] One option to control the nuclear spins of the donor atoms in the nuclear spin register is using nuclear magnetic resonance (NMR). In particular, NMR allows for control over the nuclear spins, as they are typically individually addressable. Therefore, NMR can be used for initialisation of the nuclear spins to any desired configuration. Typically, NMR operates at a very different frequency band than ESR (10-100 MHz rather than 20-40 GHz for ESR). Accordingly, all spins (both electron and nuclear) of the nuclear spin register can be controlled using magnetic driving.

[0082] Fig. 5C shows the ESR spectrum showing the frequencies (in GHz) required to perform an ESR operation on the 3P system shown in Fig. 5A. Further, Fig. 5C includes the corresponding nuclear spin states shown above indicating which transitions are used for readout of any particular nuclear spin configuration. Based on the measured ESR frequencies, shown in Fig. 5C, the three hyperfine coupling strengths can be extracted. In particular, the

hyperfine coupling strengths (A_i) between the i^{th} nuclear spin and the single confined electron are as follows: $A_1 = 197.5 \pm 1$ MHz, $A_2 = 45 \pm 1$ MHz and $A_3 = 15 \pm 1$ MHz.

[0083] Physically, the values of the hyperfine couplings are determined by the shape of the electron wavefunction, which depends on the exact locations of the nuclei in a quantum dot. The donor atom locations within the nuclear spin register can be simulated using the Nano Electronic Modelling tool (NEMO) and compared to the measured hyperfine coupling values. NEMO-3D is a software which computes the shape of the electron wavefunction, and hence can predict the values of the hyperfine couplings. Since experimentally, the hyperfine couplings are directly measured, the positions of the nuclei can be reverse-engineered by trying different configurations in NEMO-3D until a close match with the experimental values is found.

[0084] The donor atom configuration shown in Fig. 5A closely matches the observed hyperfine strengths with inter-nuclear distances of $|r_{12}| = 1.2$ nm, $|r_{13}| = 1.4$ nm and $|r_{23}| = 2.5$ nm.

[0085] Each ESR transition (ν_1^e through ν_8^e) corresponds to a unique nuclear configuration ($|\downarrow\downarrow\downarrow\rangle, |\downarrow\downarrow\uparrow\rangle, \dots, |\uparrow\uparrow\downarrow\rangle, |\uparrow\uparrow\uparrow\rangle$), and hence can be used to flip the electron spin conditional on the nuclear spin configuration. Driving one of the ESR transitions followed by an electron spin readout operation constitutes a quantum non-demolition (QND) measurement of the nuclear spin configuration. By repeating this process for all 8 ESR transitions, and then repeating the entire process N times, full nuclear spin readout of the nuclear spin register can be performed.

[0086] Fig. 6A shows an example circuit to implement a QND measurement of all three nuclear spin qubits via the electron spin qubit. The nuclear spin configuration can then be determined by examining which ESR transition resulted in the electron being inverted (flipped from the spin up/spin down state to the spin down/spin up state). The evolution of the three nuclear spins in time is found by repeatedly applying the circuit depicted in Fig. 6A, which uses QND readout to observe nuclear spin states by inverting the electron spin for each nuclear spin configuration frequency in turn. In one specific example, readout of all 8 ESR transitions took 8 ms, which is then repeated $N = 30$ times so that the full nuclear spin configuration is read out every 240 ms.

[0087] To determine the state of a particular nuclear spin i within the nuclear spin register, it is examined whether it's likely to observe electron- $|\uparrow\rangle$ on ESR transitions corresponding to i being $|\uparrow\rangle$ as compared to ESR transitions where i is $|\downarrow\rangle$. For example, as

per the annotations above the peaks in Fig. 5C, when measuring the state of nuclear spin 1 ($\hat{f}^{(1)}$) the probability of measuring electron- $|\uparrow\rangle$ after driving peaks $\{\nu_5^e, \nu_6^e, \nu_7^e, \nu_8^e\}$ (which would correspond to nuclear spin 1 being in the $|\uparrow\rangle$ state) is compared to the electron- $|\uparrow\rangle$ after driving peaks $\{\nu_1^e, \nu_2^e, \nu_3^e, \nu_4^e\}$ (where nuclear spin 1 is $|\downarrow\rangle$). Quantitatively, the following probabilities are measured:

$$R_{\uparrow}^{(i)}(t) \equiv P(|\uparrow\rangle | \hat{f}_z^{(i)}(t) = \uparrow) \quad \text{Eq. (2)}$$

$$R_{\downarrow}^{(i)}(t) \equiv P(|\uparrow\rangle | \hat{f}_z^{(i)}(t) = \downarrow) \quad \text{Eq. (3)}$$

which correspond to the probabilities of recording electron- $|\uparrow\rangle$ after driving the peaks corresponding to $\hat{f}_z^{(i)}(t) = \uparrow$ (Eq. 2) or $\hat{f}_z^{(i)}(t) = \downarrow$ (Eq. 3).

[0088] To construct a Random Telegraph Signal (RTS) of the nuclear spin state $R^{(i)}(t)$, the following formula is applied:

$$R_A^{(i)}(t) = \frac{[R_{\uparrow}^{(i)}(t) + 1 - R_{\downarrow}^{(i)}(t)]}{2} \quad \text{Eq. (4)}$$

where a RTS is a type of noise or signal that switches randomly between two discrete levels (states) over time. It resembles the output of a binary telegraph system, where the signal can take on one of two values.

[0089] By averaging $R_{\uparrow}^{(i)}(t)$ and $1 - R_{\downarrow}^{(i)}(t)$ together in Eq. 4, the signal-to-noise ratio for each spin is improved. The averaging is only possible because each nuclear spin has to be in one of two states ($|\uparrow\rangle$ or $|\downarrow\rangle$), which means that if an $\hat{f}_z^{(i)}(t) = \uparrow$ ESR peak is occupied then the $\hat{f}_z^{(i)}(t) = \downarrow$ peaks will not be occupied, making $R_{\uparrow}^{(i)}(t)$ and $R_{\downarrow}^{(i)}(t)$ anti-correlated.

[0090] Fig. 6B is a plot of the evolution of the RTS trace of nuclear spin 1 ($\hat{f}_z^{(1)}$) with $N=30$ averages. The raw signal 602 is shown along with the upper and lower thresholds (604, 606) as well as the threshold signal 608 produced from double thresholding of the raw signal 602. Where the thresholded signal 608 only changes state when the raw signal exceeds the opposite threshold.

[0091] Fig. 6C is a plot of the evolution of the RTS trace of nuclear spin 2 ($\hat{f}_z^{(2)}$) with $N=30$ averages. The raw signal 612 is shown along with the upper and lower thresholds (614, 616) as well as the threshold signal 618 produced from double thresholding of the raw signal 612.

[0092] Fig. 6D is a plot of the evolution of the RTS trace of nuclear spin 3 ($\hat{f}_z^{(3)}$) with $N=30$ averages. The raw signal 622 is shown along with the upper and lower thresholds (624,

626) as well as the threshold signal 628 produced from double thresholding of the raw signal 622.

[0093] The state of individual nuclei is discriminated according to a double threshold procedure (as shown by the histograms in Fig. 6E-G), where the nuclear spin will only be recorded as $\uparrow$ if $R_A^{(i)}(t)$ exceeds an upper threshold, and as $\downarrow$ only if $R_A^{(i)}(t)$ falls below a lower threshold, otherwise the recorded nuclear spin state will remain unchanged. Measurement fidelities of 98.2 %, 80.7 % and 99.8 % for nuclear spins 1, 2 and 3, respectively, are obtained. The measurement fidelity varies significantly between the three nuclear spins despite their nanometre-scale proximity, with the nuclear spin readout error (i.e., the chance of reading out the nuclear spin incorrectly) varying over two orders of magnitude (0.2 % to 19.3 %). As described previously, this disclosure describes a detailed examination of the physical mechanisms which would give rise to such varied error and provides a solution to reduce or mitigate such errors.

[0094] By recording statistics of the lifetimes for each nuclear spin state, the nuclear spin relaxation (T_1) times can be determined, as per the histograms in Figs. 6H-M. Nuclear spin relaxation time refers to the time it takes for nuclear spins to return to thermal equilibrium after being perturbed by an external influence. T_1 is the time it takes for the nuclear spin system to return to its equilibrium alignment along the direction of the external magnetic field (often called the z-axis or longitudinal direction) after being perturbed.

[0095] Figs. 6H and 6I are plots 630 and 632 showing histograms of the nuclear spin lifetime of nuclear spin 1 in the spin-up state $|\uparrow\rangle$ and spin-down $|\downarrow\rangle$ state, respectively. On each plot, the x-axis shows the lifetime of the nuclear spin in seconds and the y-axis shows the count. Plot 630 also shows the exponential decay fit 634 used to determine the nuclear spin relaxation time $T_1^{(1)}$. Similarly, plot 632 also shows the exponential decay fit 636 used to determine the nuclear spin relaxation time $T_1^{(1)}$.

[0096] Figs. 6J and 6K are plots 640 and 642 showing histograms of the nuclear spin lifetime of nuclear spin 2 in the spin-up state $|\uparrow\rangle$ and spin-down $|\downarrow\rangle$ state, respectively. On each plot, the x-axis shows the lifetime of the nuclear spin in seconds and the y-axis shows the count. Plot 620 also shows the exponential decay fit 624 used to determine the nuclear spin relaxation time $T_1^{(1)}$. Similarly, plot 622 also shows the exponential decay fit 626 used to determine the nuclear spin relaxation time $T_1^{(1)}$. The observed histograms are fit to

$A \exp\left(-\left(\frac{t}{T_1}\right)\right)$, with the fitting done by a standard least square fitting algorithm. The fit is

not performed for nuclear spin 3 ($\hat{I}_z^{(3)}$) because it is so long lived that it did not flip sufficiently often to fit such an exponential decay. Instead, an average was taken of the few observed flip times (which is a good approximation in such cases of low amounts of data).

[0097] Figs. 6L and 6M are plots 650 and 652 showing histograms of the nuclear spin lifetime of nuclear spin 3 in the spin-up state $|\uparrow\rangle$ and spin-down $|\downarrow\rangle$ state, respectively. On each plot, the x-axis shows the lifetime of the nuclear spin in seconds and the y-axis shows the count. $T_1^{(3)}$ was estimated by taking the average of all observed lifetimes, giving values of $T_{\uparrow}^{(3)} \sim 280 \pm 100$ s and $T_{\downarrow}^{(3)} \sim 68 \pm 23$ s with uncertainties $\propto 1/\sqrt{n}$ for the n lifetimes observed.

[0098] The resulting T_1 times, annotated in Fig. 6H-M, show a notable difference between the $T_{\uparrow/\downarrow}^{(i)}$ times for each nuclear spin, with a few seconds for $\hat{I}^{(1)}$ and $\lesssim 1$ second for $\hat{I}^{(2)}$, in contrast to minutes for nuclear spin $\hat{I}^{(3)}$ (much closer to the value expected T_1 times of isolated single donors in silicon). The readout fidelity of nuclear spin $\hat{I}^{(1)}$ (98.2 %) is higher than that of nuclear spin $\hat{I}^{(2)}$ (80.7 %), despite $\hat{I}^{(1)}$ having the largest hyperfine coupling strength (i.e., $A_1 = 197.5 \pm 1$ MHz), indicating that measurement errors are not proportional to the hyperfine coupling strength. The sudden loss of hyperfine coupling $A_{(i)}$ when the electron is removed from the dot during electron readout, a process known as ionisation shock, was thought to be a dominating source of error limiting nuclear spin lifetimes in silicon. However, this effect is known to become stronger with increasing hyperfine coupling strength $A_{(i)}$, the opposite of what is seen between nuclear spins $\hat{I}^{(1)}$ and $\hat{I}^{(2)}$. Hence, a more careful consideration of nuclear spin errors is required to understand the nuclear spin lifetimes and measurement fidelities in nuclear spin registers (shown in Fig 6H-J).

[0099] Understanding these nuclear spin errors and implementing solutions to reduce/mitigate these errors may allow for more efficient and/or reliable initialisation, operation and readout of the quantum processing systems. The experimental data shows that fidelities above 99.9 % can be reliably achieved by reducing/mitigating nuclear spin errors in Phosphorous-Silicon quantum devices.

Direct measurement of direct dipolar-coupled oscillations

[0100] To start, error sources which may be limiting the lifetime of nuclear spin 1 are investigated. Motivated by the fact that $\hat{I}^{(2)}$ has the highest error observed, and that $\hat{I}^{(1)}$ is in

closer proximity to $\hat{I}^{(2)}$ as compared to $\hat{I}^{(3)}$ – see Fig. 5A, the interactions between $\hat{I}^{(1)}$ and $\hat{I}^{(2)}$ which could cause some errors on $\hat{I}^{(2)}$ to be passed to $\hat{I}^{(1)}$ are investigated. One such mechanism is the direct through-space nuclear-nuclear dipolar coupling. In a strong magnetic field B_0 , nuclear-nuclear dipolar coupling $b_D^{(i,j)}$ introduces an additional term H_D to the Hamiltonian:

$$H_D = \sum_{i < j} b_D^{(i,j)} \left(3 \hat{I}_z^{(i)} \hat{I}_z^{(j)} - \hat{\mathbf{I}}^{(i)} \cdot \hat{\mathbf{I}}^{(j)} \right) \quad \text{Eq. (5)}$$

[0101] The dipolar coupling strength $b_D^{(i,j)}$ is defined by:

$$b_D^{(i,j)} = \frac{1}{\hbar} \frac{\mu_0 (\hbar \gamma_N)^2}{4 \pi |r_{i,j}|^3} \left(\frac{3 \cos^2 \theta_{i,j} - 1}{2} \right) \quad \text{Eq. (6)}$$

where μ_0 is the vacuum permeability, $\hbar$ is the reduced Planck constant, $|r_{i,j}|$ is the length of the vector joining nuclear spins i and j , and $\theta_{i,j}$ is the angle between $r_{i,j}$ and B_0 (i.e. computational z-axis). All terms are in SI units, with the exception of $b_D^{(i,j)}$ and γ_N which are in units of $\hbar = 1$ as per the rest of this disclosure.

[0102] Evaluating these constants for phosphorus nuclei in silicon yields:

$$\frac{1}{\hbar} \frac{\mu_0 (\hbar \gamma_N)^2}{4 \pi} = 2\pi \cdot 122.886 \text{ Hz } \text{\AA}^3 \quad \text{Eq. (7)}$$

where $a = 0.543095 \text{ nm}$ is the silicon lattice constant. Typically, dipolar coupling strengths are on the order of 100 Hz (from Eq. 7). This means that when an electron is present, nuclear-nuclear dipolar coupling is effectively suppressed, since the hyperfine interaction energy scale is $\sim 100 \text{ MHz} \gg 100 \text{ Hz}$. In theory, even electron cases are equivalent to zero electrons, and odd electron cases are equivalent to one electron. Hence, when the unpaired electron is loaded onto the multi-donor quantum dot, the nuclear-nuclear dipolar coupling does not significantly affect the nuclear spin dynamics.

[0103] However, when the nuclear spin register is ionised, which occurs during the readout of an $|\uparrow\rangle$ electron, all nuclear spin energy splittings become almost degenerate with each other, having a common splitting of $\gamma_N B_0$. This degeneracy has the result that the nuclear-nuclear dipolar coupling can now have a significant effect on the nuclear spin dynamics. The effect still happens when a $|\downarrow\rangle$ electron tunnels off, but with less error. The reason the error is stronger for electron spin-up is that when the electron is spin-up, the nuclear spin states are closer in energy, allowing the terms provided by the AHF to cause more nuclear state mixing, and hence cause more error.

[0104] Specifically, the nuclear-nuclear dipolar interaction leads to pairs of dipolar-coupled nuclear spins swapping with each other (i.e. $|\uparrow\downarrow\rangle \leftrightarrow |\downarrow\uparrow\rangle$). In the case that a pair of nuclei i, j start antiparallel, the probability that nuclei i and j will swap is given by:

$$P(|\uparrow\downarrow\rangle \leftrightarrow |\downarrow\uparrow\rangle) = \frac{1}{2} - \frac{1}{2} \cos(b_D^{(i,j)} t) \quad \text{Eq. 8}$$

[0105] To experimentally measure this nuclear-nuclear dipolar coupling, a Hahn echo sequence was employed (as illustrated in Fig. 7A). Traditionally, a Hahn echo sequence is used to refocus slow-varying noise in the frequency of a qubit. However, when all nuclei are controlled simultaneously during ionised NMR control (i.e., NMR control when there is no electron confined to the nuclear spin register), the dipolar coupling Hamiltonian commutes with the refocusing pulse of the Hahn echo. This commutation causes dipolar coupling between nuclear spins to produce oscillations in the Hahn echo experiment as a function of the delay time – i.e., the time allowed for the qubit(s) to idle.

[0106] Since nuclear spin 2 has larger errors (having a readout error of 80.7%, found from the overlap of the peaks in Fig. 6F), a different procedure was used to readout the nuclear spins compared to Fig. 6A. Instead of driving a single ESR frequency followed by an immediate electron measurement (Fig. 6A) which results in low-fidelity readout of nuclear spin 2, all 4 ESR frequencies corresponding to a particular nuclear spin i being $|\uparrow\rangle$ are driven sequentially before measuring the electron spin state. For example, nuclear spin $\hat{f}^{(1)}$ is measured by inverting the transitions ν_5^e , ν_6^e , ν_7^e , and ν_8^e followed by an electron spin measurement. A total of $N = 3$ QND measurements of each nuclear spin were applied for each readout of the register, in the order $\hat{f}^{(1)}, \hat{f}^{(2)}, \hat{f}^{(3)}, \hat{f}^{(1)}, \hat{f}^{(2)}, \hat{f}^{(3)}, \hat{f}^{(1)}, \hat{f}^{(2)}, \hat{f}^{(3)}$.

[0107] Such a readout scheme allows for each nuclear spin to be measured without performing a large number of ESR pulses and electron spin readouts, with $4 \cdot 3 \cdot 3 = 36$ ESR pulses and 9 electron readouts used, as opposed to the $8 \cdot 30 = 240$ ESR pulses and electron readouts for Fig. 6. This reduction in the number of ESR and readout pulses performed reduces the impact of any errors which accumulate during ESR and electron readout.

[0108] To overcome any residual errors (i.e., any errors associated with readout of nuclear spin 2, which were not mitigated by reducing the number of readout shots taken of nuclear spin 2) affecting nuclear spin 2, readouts of nuclear spin 2 were also postselected – meaning that data is only kept when either 0 or 3 of the three measurements performed on

nuclear spin 2 registering electron- $|\uparrow\rangle$, since these measurements have a lower probability of $\hat{f}^{(2)}$ flipping during readout. Under the assumption that nuclear spins 1 and 2 have slightly different ionised NMR frequencies, the resulting Hahn echo data was analysed by binning pairs of before/after measurements, surrounding the Hahn echo pulse as per Fig. 7A, to analyse which spin(s) flipped between the two measurements.

[0109] Fig. 7A shows a Hahn echo sequence used to measure nuclear-nuclear dipole interactions. A Hahn echo sequence is a technique used in quantum computing and nuclear magnetic resonance (NMR) to mitigate the effects of decoherence (loss of quantum information) caused by environmental noise. It is designed to refocus the dephasing of quantum states and extend the coherence time of qubits or spins. For a nuclear spin measurement, each spin is read out using $N = 3$ electron measurements, each consisting of driving all ESR transitions where that nuclear spin is $|\uparrow\rangle$ (see Fig. 6B-6D), followed by an electron measurement. Measurements are taken in the order $\hat{f}^1, \hat{f}^2, \hat{f}^3, \hat{f}^1, \hat{f}^2, \hat{f}^3, \hat{f}^1, \hat{f}^2, \hat{f}^3$. The measurement for spin 2 is post-selected to improve measurement fidelity, only keeping measurements with either none or all 3 of the measurements resulting in an electron- $|\uparrow\rangle$ measurement.

[0110] It will be appreciated that the Hahn echo sequence is performed without the electron (so the electron is ionised/removed after the initial readout), as this sequence is performed to probe the dynamics of the nuclear spins while ionised. The measurement of nuclear spins involves multiple single-shot measurements of the nuclear spin state. Each single shot measurement either indicates that the nuclear spin is in spin-up or spin-down state. For nuclear spin 2, which has particularly high readout errors, these single shots have a noticeably high chance of reporting the incorrect spin state. In addition, nuclear spin 2 is measured 3 times per circuit. To mitigate these errors affecting the measurement of the dipolar interaction, only circuit runs where all 3 measurements of nuclear spin 2 agree are kept (either all spin-up or all spin-down); circuit runs without unanimity are discarded as the accuracy of the measurement result is less. When ionised, all the nuclear spins have the same resonance frequency, so any gate that is applied will be applied to all nuclear spins simultaneously.

[0111] Before the Hahn echo sequence starts, nuclear non-demolition readout is performed, to establish the state of the nuclear spins. The Hahn echo sequence consists of a first $\pi/2$ pulse (e.g., a $\sqrt{X}$ gate) that rotates the nuclear spin state (qubit) into the superposition state on the equatorial plane of the Bloch sphere. After the initial pulse, the qubit is allowed

to evolve freely for a time, during which the state begins to dephase due to noise or imperfections. After the free evolution, a π pulse (e.g., a Y gate) is applied, which effectively "flips" the state. This reverses the direction of the phase evolution, making the system refocus. Again, the qubit is allowed to evolve freely for a time, during which the state begins to dephase due to noise or imperfections. Next, a second $\pi/2$ pulse (e.g., a $\sqrt{X}$ gate) is applied that rotates the nuclear spin state (qubit) into the superposition state on the equatorial plane of the Bloch sphere. Lastly, the nuclear spin states are readout. This is repeated three times.

[0112] For example, if the initial measurement registered nuclear spin configuration $|\uparrow\uparrow\downarrow\rangle$ and the final measurement registered the nuclear spin configuration $|\downarrow\downarrow\uparrow\rangle$, then this run would be binned along with a run initially measuring $|\downarrow\downarrow\downarrow\rangle$ and finally measuring $|\uparrow\downarrow\uparrow\rangle$. This is because in both cases, spins 1 and 3 flip between the initial and final measurements but spin 2 does not. The resulting correlations between the flipping of pairs of nuclei as a function of the total Hahn echo wait time can be seen in Fig. 7B-7D.

[0113] Fig. 7B shows a correlation between nuclear spins 1 and 2. On the x-axis is time in ms and the y-axis shows the probability for a spin or spins to flip. Data 702 shows the probability of both nuclei flipping (labelled $\uparrow_1\uparrow_2$), data 704 shows the probability of neither nuclei flipping (labelled $\downarrow_1\downarrow_2$), data 706 shows the probability of only nuclei 1 flipping (labelled $\uparrow_1\downarrow_2$) and data 708 shows the probability of only nuclei 2 flipping (labelled $\downarrow_1\uparrow_2$). The population of the two cases $\uparrow_1\uparrow_2$ and $\downarrow_1\downarrow_2$ oscillate out of phase, which indicates correlated oscillations between these nuclear spins. This correlation shows that there is a direct dipolar interaction between nuclear spin 1 and 2.

[0114] Fig. 7C shows a correlation between nuclear spins 2 and 3. On the x-axis is time in ms and the y-axis shown the probability for a spin or spins to flip. Data 712 shows the probability of both nuclei flipping (labelled $\uparrow_2\uparrow_3$), data 714 shows the probability of neither nuclei flipping (labelled $\downarrow_2\downarrow_3$), data 716 shows the probability of only nuclei 2 flipping (labelled $\uparrow_2\downarrow_3$) and data 718 shows the probability of only nuclei 3 flipping (labelled $\downarrow_2\uparrow_3$).

[0115] Fig. 7D shows the correlation between nuclear spins 1 and 3. On the x-axis is time in ms and the y-axis shown the probability for a spin or spins to flip. Data 722 shows the probability of both nuclei flipping (labelled $\uparrow_1\uparrow_3$), data 724 shows the probability of neither nuclei flipping (labelled $\downarrow_1\downarrow_3$), data 726 shows the probability of only nuclei 1 flipping (labelled $\uparrow_1\downarrow_3$) and data 728 shows the probability of only nuclei 3 flipping (labelled $\downarrow_1\uparrow_3$).

[0116] In contrast to Fig. 7B, Fig. 7C shows the flipping of nuclear spin 2 mostly independent of nuclear spin 3 ($\uparrow_2\uparrow_3$ and $\downarrow_2\uparrow_3$ show clear oscillations), and similarly Fig. 7D

shows nuclear spin 1 oscillates mostly independent of nuclear spin 3, which therefore indicates substantially lower coupling between nuclear spins 1 and 3, and nuclear spins 2 and 3. The dipolar oscillations shown in Figs. 7B-7D were fit to a numerical model. The numerical fits are shown as the solid lines in Figs. 7B-7D, and provide an estimate for the strengths of the nuclear-nuclear dipolar couplings of $b_D^{(1,2)}/2\pi = 33$ Hz, $b_D^{(1,3)}/2\pi = 6$ Hz, $b_D^{(2,3)}/2\pi = 0$ Hz, as summarised in Fig. 7E.

[0117] Fig. 7G depicts a plot of the strength of nuclear-nuclear dipolar coupling as a function of the distance between a pair of donor nuclei. A dipolar coupling strength close to the strongest dipolar strength measured experimentally herein is highlighted by reference 740. This plot only looks at possible donor separations for donors which are in the same plane, and is limited to donors which are closer than ~ 3.5 nm (donors further apart will have weaker dipolar couplings than those shown). The plot in Fig. 7G shows that typical dipolar coupling strengths are around 1-10 Hz, with only the closest nuclei feeling a dipolar coupling strength of 10-300 Hz.

[0118] Knowledge of the dipolar coupling strength $b_D^{(1,2)}$ between nuclear spins 1 and 2 can be used to predict the amount of error experienced by nuclear spin 1 during an electron spin readout event. During an electron readout event, if an $|\uparrow\rangle$ electron tunnels out, the dot will remain ionised for an exponentially-distributed stochastic amount of time, with characteristic time $T_{\downarrow, \text{in}} \sim 200$ μs . Using this distribution, and the dipolar oscillation frequency as per Eq. 8, the expectation value for nuclear spins 1 and 2 to flip-flop during a readout ionisation event can be calculated as follows:

$$\begin{aligned} & P(|\uparrow_1 \downarrow_2\rangle \leftrightarrow |\downarrow_1 \uparrow_2\rangle) \\ &= \int_0^{T_m} \frac{1}{T_{\downarrow, \text{in}}} e^{-\frac{t}{T_{\downarrow, \text{in}}}} \left(\frac{1}{2} - \frac{1}{2} \cos(b_D^{(1,2)} t) \right) dt \\ &\approx \frac{(b_D^{(1,2)})^2}{2} T_{\downarrow, \text{in}}^2 \\ &\approx 8.6 \times 10^{-4} \quad \text{Eq. (9, 10)} \end{aligned}$$

[0119] Here, it was assumed that $b_D^{(1,2)} t \ll 1$ (i.e., that the tunnel-in time is sufficiently fast such that there is much less than 1 flip-flop oscillation) for the expansion, and $T_{\downarrow, \text{in}} \ll T_m$ (i.e., the electron tunnel-in time is much shorter than the full readout duration) for integration over the measurement time T_m . For the device studied here, $b_D^{(1,2)} T_{\downarrow, \text{in}} \approx 0.0066$, $T_{\downarrow, \text{in}} \approx 200$ μs and $T_m = 1.1$ ms, justifying these assumptions. Under the assumption that each set of 8 readouts in Fig. 6A had on average 1 electron- $|\uparrow\rangle$ ionisation event, assuming that only one

nuclear configuration is populated, then the error in Eq. 10 predicts that nuclear spins 1 and 2 should pick up an average error of 4.3×10^{-4} per ionisation event, due to an assumed 50% chance for nuclei 1 and 2 to be in the states $|\uparrow_1 \downarrow_2\rangle$ or $|\downarrow_1 \uparrow_2\rangle$.

[0120] In Figs. 6H-6I, the average relaxation time for nuclear spin 1 $T_1^{(1)}$ is 12.5 s (average of $T_{\uparrow}^1 = 17$ s and $T_{\downarrow}^1 = 8$ s). This can be used to estimate the error per nuclear spin readout using $1 - e^{-\tau_R/T_1}$ where $\tau_R = 8$ ms is the time taken for a single shot of all 8 ESR peaks, which gives an error of $6.4 * 10^{-4}$ per ionisation event. This is similar to the observed error of $4.3 * 10^{-4}$ per ionisation event, with excess errors (i.e., any errors remaining that affect nuclear spin 1) likely arising due to anisotropic hyperfine-mediated ionisation shock.

[0121] As is evident in Eq. 9, the error due to nuclear dipolar coupling is strongly dependent on the electron tunnel rates during readout. Namely, the electron- $|\downarrow\rangle$ tunnelling time onto the dot, $T_{\downarrow, \text{in}}$, determines the duration of ionisation.

[0122] Fig. 7F is a plot of the calculated errors for each of the three dipolar coupling strengths. On the x-axis is the electron spin down tunnelling time onto the nuclear spin register in μs and on the y-axis is the error per electron spin up measurement. Data 732, 734 and 736 show the calculated errors expected as a function of $T_{\downarrow, \text{in}}$ for each of the three dipolar coupling strengths.

[0123] Fig. 7H is a plot of the dipolar error with respect to tunnel time. This plot is similar to the plot depicted in Fig. 7F, but includes dipolar couplings calculated in in Fig. 7G. The line 742 shows a dipolar coupling strength close to that measured experimentally. The lines fade out to the bottom-right, intended to illustrate that these lines will continue towards the bottom-right for greater and greater donor separations. The vertical black line 744 indicates a tunnel-in time of 10 μs , which causes all but the strongest dipolar error to fall below 10^{-5} ; this tunnel-in time corresponds to a tunnelling rate of 100 kHz. The dashed vertical black line 746 indicates when all the errors are below 10^{-4} , corresponding to a tunnelling rate of 350 kHz. The plot depicted in Fig. 7H illustrates that tunnelling rates of above ~ 100 -350 kHz are sufficient to mitigate errors coming from nuclear-nuclear dipolar errors, since other error sources are expected to dominate below 10^{-5} (for example, ionisation shock).

[0124] One option to minimise $b_D^{(i,j)}$ and hence the error in Eq. 9 is to fabricate a quantum dot device 100 with an engineered donor atom separation r_{ij} and/or the angle θ_{ij} between donor atom pairs. Another option to minimise $b_D^{(i,j)}$ and hence the error in Eq. 9 is to control the direction of the static magnetic field B_0 direction. Yet another option is to

engineer the tunnel time (i.e., the time it takes for an electron to tunnel from a reservoir to the nuclear spin register) to reduce errors due to nuclear-nuclear dipolar coupling. Tunnel times can be shortened by decreasing the distance between the donors and the sensor. With shorter tunnel times being previously demonstrated as low as $T_{1,\text{in}} \sim 1 \mu\text{s}$ in silicon, which would give a negligible dipolar error ($\sim 10^{-8}$, as indicated by the circular black point on Fig. 7F). The nuclear-nuclear dipolar coupling studied here further reinforces that electron tunnel rates should be engineered sufficiently fast to facilitate high-fidelity quantum operations.

[0125] In some examples, a tunnel rate above a threshold will lead to a reduction in errors arising from direct nuclear spin coupling. In some examples, a tunnel rate above approximately 100 kHz will reduce errors. It will be appreciated, that although the experimental data shown herein is for a 3P, the requirement for tunnel rates to be above a threshold value of approximately 100 kHz holds for nuclear spin registers of any sizes. Since for a given distance, the tunnel rate depends on the donor number, this may require different distances depending on donor number.

[0126] A theoretical model of how the tunnel rate depends on distance for 2P and 3P nuclear registers is depicted in Fig. 7I. In particular, the trace 750 depicts the tunnel rate for a 2P system and trace 752 depicts the tunnel rate for a 3P system. These simulations roughly include two steps. In the first step, the wavefunctions of the quantum dot and the SET are obtained. The SET may be approximated as an elongated dot consisting of 25 donors and 25 electrons (for simplicity, so that the simulation is tractable, with the 25 donors positioned along where the edge of the SET closest to the dot would be in the actual device). Any suitable known techniques may be utilized to obtain the wavefunctions.

[0127] In the second step, an overlap between the wavefunctions is calculated. The overlap is then used to calculate a tunnel rate. Any suitable technique may be utilized for these calculations. Due to the approximation in the shape of the SET, this tunnel rate may not be accurate, and so it is normalised by comparing it with a known tunnel rates and distances. The tunnel rate and distance used in the plot of Fig. 7I is a 2P1e dot with an SET-dot distance of 16.7 nm. From this plot, it can be determined that an SET-dot separation of $< \sim 15$ nm should be enough to ensure that a simulated 3P and 2P have tunnel rates > 100 kHz; and $< \sim 14$ nm should ensure tunnel rates > 350 kHz. Note that these trends may change for different donor layouts within the quantum dot and that similar simulations could be performed specific to any donor atom layout in a nuclear register.

Anisotropic hyperfine coupling

[0128] Having established the dominant error source for nuclear spin 1, consideration is turned to the error sources which limit the lifetime of nuclear spin 2. The spin relaxation mechanisms giving rise to T_1 lifetimes of single phosphorus donor nuclei in silicon seem to indicate that nuclear spin 2 should not exhibit such high errors. Firstly, spontaneous electron-nuclear flip-flops have been studied, in which a phonon is emitted as the system undergoes the relaxation pathway $|\uparrow\downarrow\rangle \rightarrow |\downarrow\uparrow\rangle$, where single line arrow indicates the state of an electron and the double line arrow indicates the state of the nuclear spin. Ionisation shock has also been previously considered, in which nuclei are projected onto different eigenbases during a readout operation because the contact hyperfine interaction $A^{(i)}$ vanishes when the electron is removed from the nuclear spin register. Both of these mechanisms, however, are known to exhibit lower nuclear spin T_1 times for higher contact hyperfine strengths. This is the opposite of what we observe with nuclear spins 1 and 2, where $T_{\uparrow/\downarrow}^{(1)} > T_{\uparrow/\downarrow}^{(2)}$ even though $A^{(1)} > A^{(2)}$. The errors exhibited by nuclear spin 2 must therefore be due to a different mechanism which creates significant errors for nuclear spin 2 but not for nuclear spins 1 or 3.

[0129] Figs. 8A-C illustrate the energy levels in the subspace of the electron spin along with nuclear spins 1, 2 and 3, respectively. To first order the energy splitting $\Delta E_Z^{(i)}$ between $|\uparrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$ is:

$$\Delta E_Z^{(i)} \equiv \gamma_n B_0 + \frac{A^{(i)}}{2} \quad \text{Eq. 11}$$

[0130] Using Eq. 11, experimental values of $\Delta E_Z^{(1)}/2\pi \approx 74.8 \pm 0.5$ MHz, $\Delta E_Z^{(2)}/2\pi \approx -1.5 \pm 0.5$ MHz, and $\Delta E_Z^{(3)}/2\pi \approx -16.5 \pm 0.5$ MHz are found. Here, negative values indicate that $|\uparrow\downarrow\rangle$ has a higher energy than $|\uparrow\uparrow\rangle$. The main notable feature of nuclear spin 2 which distinguishes it from the other two nuclei is the small energy splitting between $|\uparrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$, indicated by $\Delta E_Z^{(2)}$ in Fig. 8B. This is due to the contact hyperfine strength of nuclear spin 2 ($A^{(2)} = 45 \pm 1$ MHz) in the particular sample studied here being comparable in magnitude to twice the nuclear Zeeman splitting ($2\gamma_n B_0 = -47.9$ MHz). That is, for nuclear spin 2, $A^{(2)}/2 \approx |\gamma_n|B_0$, which is equivalent to $|\Delta E_Z^{(2)}| \approx 0$, which causes the $|\uparrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$ states to be near-degenerate.

[0131] In such a regime, any weak interactions between $|\uparrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$ will become relevant due to the near-degeneracy of these states. Now, considering the anisotropic hyperfine (AHF) interaction, the Hamiltonian in Eq. 1 is modified as follows:

$$\hat{H}'_{load}(B_0) = \hat{H}_{load}(B_0) + \sum_{i=1}^3 \hat{\mathbf{S}} \cdot \vec{\hat{A}}_B^{(i)} \cdot \hat{\mathbf{I}}^{(i)} \quad Eq. (12)$$

where $\hat{H}_{load}(B_0)$ is the Hamiltonian from Eq. 1, but now the contribution of the anisotropic hyperfine tensor $\vec{\hat{A}}_B^{(i)}$ which arises from the magnetic dipolar interaction between the electron and the nuclear spins is additionally considered. This tensor introduces terms proportional to $\hat{S}_x \hat{I}_x$, $\hat{S}_y \hat{I}_y$ and $\hat{S}_z \hat{I}_z$ similar to the contact hyperfine, but it also introduces numerous additional terms proportional to $\hat{S}_x \hat{I}_y$, $\hat{S}_x \hat{I}_z$, $\hat{S}_y \hat{I}_z$ and so on.

[0132] The effect of the anisotropic hyperfine interaction has been typically ignored in silicon systems since it is predicted to be small (~ 100 kHz) compared to the contact hyperfine A which is typically ~ 100 MHz (around 2-3 orders of magnitude larger). However, since the anisotropic hyperfine tensor contains z, x and z, y terms which are off-diagonal in the $\{|\uparrow\downarrow\rangle, |\uparrow\uparrow\rangle\}$ basis, when $\Delta E_z \sim 0$ (as is the case for nuclear spin 2), there can be significant mixing between $|\uparrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$. Indeed, this mixing becomes maximised when $\Delta E_z = 0$. The strength of this off-diagonal nuclear mixing term $[A_{AHF}^{(i)}]_{z,x} \pm i[A_{AHF}^{(i)}]_{z,y}$ in the AHF tensor can be characterised by its magnitude:

$$A_{AHF,M}^{(i)} \equiv \sqrt{[A_{AHF}^{(i)}]_{z,x}^2 \pm i[A_{AHF}^{(i)}]_{z,y}^2} \quad Eq. (13)$$

which, produces mixing in the $|\uparrow\uparrow\rangle$ and $|\uparrow\downarrow\rangle$ subspace:

$$\hat{H} \begin{bmatrix} \frac{\Delta E_z^{(i)}}{2} & \frac{A_{AHF,M}^{(i)}}{4} \\ \frac{A_{AHF,M}^{(i)}}{4} & -\frac{\Delta E_z^{(i)}}{2} \end{bmatrix}$$

$$\hat{H}'_{load}(B_0) \approx \begin{bmatrix} \frac{\Delta E_z^{(i)}}{2} & \frac{A_{AHF,M}^{(i)}}{4} \\ \frac{A_{AHF,M}^{(i)}}{4} & -\frac{\Delta E_z^{(i)}}{2} \end{bmatrix} \quad Eq. (14)$$

[0133] There are two main sources of error that can then arise due to the mixing of $|\uparrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$ when $\Delta E_z \sim 0$ during the operation of a nuclear spin register. Firstly, the chirped ESR inversion pulse used for nuclear spin readout has a chance to not only flip the electron spin but also flip the nuclear spin, as illustrated by dashed lines 802, 804 in Fig. 8B, due to mixing between $|\uparrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$. This means that ESR can change the total spin of the register by either 0 or 2 (i.e. $|\uparrow\downarrow\rangle \leftrightarrow |\downarrow\uparrow\rangle$ or $|\downarrow\downarrow\rangle \leftrightarrow |\uparrow\uparrow\rangle$). This is different compared to the usual ESR transitions which only flip the electron and hence change the total spin of the register by

1 (i.e. $|\uparrow\downarrow\rangle \leftrightarrow |\downarrow\downarrow\rangle$ or $|\downarrow\uparrow\rangle \leftrightarrow |\uparrow\uparrow\rangle$). The unusual ESR transitions are referred to as spin-0/2 errors. As $\Delta E_z \rightarrow 0$, these spin-0/2 errors approach the ESR inversion rate, which can be arbitrarily close to 100 %.

[0134] Secondly, the process of ionisation shock gains additional error in the nuclear spin state preservation, particularly when an $|\uparrow\rangle$ electron tunnels on or off the quantum dot/nuclear spin register. This arises again due to mixing between $|\uparrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$. The mixing between nuclear spin states $|\downarrow\rangle$ and $|\uparrow\rangle$ due to the AHF is suddenly activated (deactivated) when an electron- $|\uparrow\rangle$ tunnels onto (off) the dot, causing error as mixed states are projected onto non-mixed states or vice versa.

[0135] The AHF has minimal effect for electron- $|\downarrow\rangle$ tunnel events, as the $|\downarrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ are not close in energy, but the AHF will still cause a small amount of mixing for $|\downarrow\rangle$. As $\Delta E_z \rightarrow 0$, ionisation shock error (during electron- $|\uparrow\rangle$ events) is dominated by the AHF, and approaches 50% due to maximal mixing between $|\uparrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$. Importantly, mixing caused by the AHF interaction is only significant when $\Delta E_z^{(2)}$ is a similar magnitude to the AHF mixing term $A_{AHF,M}^{(2)}$. Due to the small magnitude of $A_{AHF,M}$, both of the above error sources only occur over a very small range or “hotspot” of B_0 and $A^{(i)}$. To quantify this effect, the ratio

$$R \equiv \frac{\Delta E_z^{(i)}}{A_{AHF,M}^{(i)}} \quad \text{Eq. (15)}$$

Is introduced, which must be small for AHF errors to be relevant.

[0136] Whilst estimates for $\Delta E_z^{(i)} \equiv \gamma_n B_0 + A^{(2)}/2$ can be made from the applied magnetic field $B_0 = 1.39T$ and the measured value of $A^{(2)}/2\pi = 45 \pm 1$ MHz from Fig. 5c, the anisotropic hyperfine mixing strength $A_{AHF,M}^{(2)}$ is not directly measurable here. Instead, the anisotropic hyperfine strength can be determined indirectly through the nuclear spin error. Specifically, non demolition readout of both nuclear spin 1 and 2 using $N = 49$ repetitions for a series of different B_0 strengths can be performed, ensuring that ESR frequency chirps are performed in increasing frequency for $\Delta E_z^{(2)} > 0$ and decreasing frequency for $\Delta E_z^{(2)} < 0$ to avoid errors that would be caused by electron dipole spin resonance (EDSR). Where EDSR in donor quantum dot systems is due to the modulation of the hyperfine coupling of the electron spin and the nuclear spins of the donor atoms in the system. EDSR is mediated by an electric field that simultaneously flips the electron spin and one of the nuclear spins in the multi-donor system.

[0137] By examining histograms similar to Figs. 6E-G for the first n readouts ($n \in [1, N]$) as a function of n , it can be determined how nuclear spin errors accumulate as more ESR and electron readouts are applied. This reshaping allows the ratio $|R|$ to be fit at each B_0 field which are plotted in Fig. 8D. A linear fit for $|R|$ according to the equation

$$|R(B_0)| \equiv \left| \frac{\gamma_n B_0 + \frac{A^{(2)}}{2}}{A_{AHF,M}^{(i)}} \right| \quad \text{Eq. (16)}$$

gives the value of the contact hyperfine directly as $A^{(2)}/2\pi = 44.47 \pm 0.13$ MHz, and, more importantly, a value can now be extracted for the anisotropic hyperfine of $A_{AHF,M}^{(2)}/2\pi = 875 \pm 63$ kHz. This value is significantly higher than that predicted by NEMO-3D atomistic modelling where $A_{AHF,M}^{(2)}/2\pi = 181$ kHz.

[0138] This difference suggests that the electron wavefunction is more anisotropic in the device studied than was previously expected from simulation, as the AHF strength is dependent on the anisotropy of the electron wavefunction. The predicted errors induced by ionisation shock (due to both contact and anisotropic hyperfine interactions) on nuclear spin 2 are shown in Figs. 8E-F over a wide range of B_0 values.

[0139] Fig. 8E is a plot of the error of reading out the spin up electron as a function of the magnetic field strength B_0 . In particular, Fig. 8E shows narrow error regions as high as 50 % for electron- $|\uparrow\rangle$ tunnel events. It is important to note that such high errors only occur when the tunnel event involves an $|\uparrow\rangle$ electron, since it is these states that are nearly degenerate (see Fig. 8B).

[0140] This error hotspot is not observed for $|\downarrow\rangle$ -electron tunnel events in Fig. 8F. The error for spin-0/2 transitions compared to the intended spin-1 transitions for nuclear spin 2 are shown in Fig. 8H, as a function of B_0 , again showing a narrow region of very high error. Note that since spin-0/2 errors require the ESR chirp to pass over their transition frequencies, if ESR chirp widths are kept to within 10 MHz, these errors will not extend further than ~ 580 mT from a hotspot. In Fig. 8E, the hotspot where significant anisotropic hyperfine induced ionisation shock errors occur is very narrow. Specifically, readout errors are only $> 1\%$ over a range of ~ 300 mT in B_0 , or equivalently a range of ~ 10 MHz in $A^{(2)}/2\pi$. This implies errors are only $> 1\%$ when $|\Delta E_z| \lesssim 3A_{AHF,M}^{(i)}$ or equivalently $|R| \lesssim 3$, which quantifies the width of the AHF error hotspot at 1% error. Similarly, $|R| \lesssim 8$ gives $> 0.1\%$ error during electron readout.

[0141] Contact hyperfine mediated ionisation shock is even lower ($\sim 10^{-8} - 10^{-4}$ per ionisation event) meaning that anisotropic hyperfine errors dominate over a wide range of B_0 values. Nuclear spin errors arising from the anisotropic hyperfine interactions can be mitigated in two main ways. Firstly, for devices that only contain a few nuclear spins, one can model the hotspot locations and choose a magnetic field where nuclear spin error is minimised (e.g. setting $B_0 = 2T$ according to Figs. 8 E-G).

[0142] As the number of nuclear spins is increased this becomes more challenging, and hence donor locations can be engineered with atomic precision to create well-defined contact hyperfine strengths with error hotspots far from the desired operating B_0 field strength. In addition, stark shift can be used to further tune contact hyperfine strengths. By ensuring these methods give $\gamma_n B_0 + A^{(2)}/2 \gtrsim 3A_{AHF,M}^{(i)} \sim 2\pi \cdot 2.6 \text{ MHz}$, nuclear spin errors from the anisotropic hyperfine can be reduced below 1%, or if $\gamma_n B_0 + A^{(2)}/2 \gtrsim 8A_{AHF,M}^{(i)} \sim 2\pi \cdot 7.0 \text{ MHz}$, errors drop below 0.1%.

[0143] Nuclear spin registers in silicon represent a promising platform for the realisation of a quantum computer. This scaling of any qubit platform requires a detailed understanding of the platform's error sources. The experimental examples provided in the present disclosure characterises errors from dipolar-coupling in a 3P multi-nuclear spin register in purified silicon-28. Firstly, undesired magnetic dipolar interactions between each pair of phosphorus nuclear spins are considered, and a dipolar interaction is observed between nuclear spins 1 and 2 of $b_D^{(1,2)}/2\pi = 33\text{Hz}$. Secondly, dipolar interactions between the phosphorus nuclear spin and the bound single electron spin are examined, in the form of the anisotropic hyperfine interaction, and mixing of nuclear spin 2 due to the anisotropic hyperfine is observed with a strength of $A_{AHF,M}^{(2)} = 875 \pm 63 \text{ kHz}$. Importantly, to reduce both these errors from dipolar coupling below 0.1% error, the following may be required: (a) The inter-nuclear spacing $|r_{ij}|$, the angle between pairs of nuclei and the magnetic field θ_{ij} , or the tunnel rate of electron- $|\downarrow\rangle$ onto the dot ($T_{\downarrow,\text{in}}$) during measurement must be controlled so that $[b_D^{(i,j)}]^2 T_{\downarrow,\text{in}}^2 < 0.001$; and (b) The magnetic field strength B_0 and/or contact hyperfine strength $A^{(i)}$ must be controlled so as to satisfy $\gamma_n B_0 + A^{(2)}/2 \gtrsim 8A_{AHF,M}^{(i)} \sim 2\pi \cdot 7.0 \text{ MHz}$.

[0144] Understanding and characterising dipolar-induced errors is important for scaling up quantum computing architectures comprising multi-nuclear spin register. Nuclear-nuclear dipolar coupling errors can be avoided by decreasing the tunnel time $T_{\downarrow,\text{in}}$, which can be achieved through atomic-scale engineering of the distance between the register and reservoir.

Anisotropic errors can also be minimised, either through engineering of the donor locations and/or stark shift to control nuclear hyperfine couplings, or by tuning of the global magnetic field B_0 to avoid error hot spots. The experimental examples provided in the present disclosure show that such engineering will reliably produce nuclear spin errors below 0.1% and hence will provide very reliable, efficient quantum computing systems comprising nuclear spin registers.

Method for reducing errors

[0145] An example method 900 for reducing AHF errors via magnetic field tuning is depicted in Fig. 9. The method commences at step 902, where the strength of the contact hyperfine interaction between each nuclear spin in a register and an unpaired electron in the register is determined. In one example, this may be determined based on an ESR spectrum, for example, as shown in Fig. 5C.

[0146] Tables A and B below illustrate example contact hyperfine values for different configurations of nuclear spins in a 2P (table A) and 3P (table B) registers. The positions depicted in the tables are in units of the silicon lattice constant ($a_0 = 0.543$ nm), given in X/Y/Z coordinates in the crystal plane. Corresponding hyperfine strengths are shown for each example configuration. The example hyperfine values in bold highlight examples where at least one of the hyperfine strengths is close to an error hotspot at $B_0 = 1.4$ T of 48 MHz. The hyperfine frequencies here are given in units of linear frequency. These can be multiplied by 2π to convert them into angular frequency.

Donor 1 position (a_0)	Donor 2 position (a_0)	Hyperfine 1 (MHz)	Hyperfine 2 (MHz)
(0.0, 0.0, 0.0)	(1.0, 1.0, 0.0)	249	249
(0.0, 0.0, 0.0)	(1.5, 0.5, 0.0)	138	138
(0.0, 0.0, 0.0)	(2.5, 1.0, 0.5)	118	50
(0.0, 0.0, 0.0)	(3.0, 2.5, 1.5)	79	44

Table A: Example hyperfine interactions for a 2P nuclear register

Donor 1 pos. (a_0)	Donor 2 pos. (a_0)	Donor 3 pos. (a_0)	Hyperfine 1 (MHz)	Hyperfine 2 (MHz)	Hyperfine 3 (MHz)
(0.0, 0.0, 0.0)	(0.0, 2.0, 0.0)	(2.0, 4.0, 0.0)	95	174	20
(0.0, 0.0, 0.0)	(0.0, 6.0, 0.0)	(2.0, 2.0, 0.0)	63	13	102
(0.0, 0.0, 0.0)	(2.0, -2.0, 0.0)	(2.5, 1.5, 0.0)	108	44	66
(0.0, 0.0, 0.0)	(-0.5, 1.5, 0.0)	(1.5, 2.5, 0.0)	91	156	70

(0.0, 0.0, 0.0)	(1.5, -2.5, 0.0)	(3.5, -0.5, 0.0)	66	108	44
(0.0, 0.0, 0.0)	(-0.5, 4.5, 0.0)	(0.5, 1.5, 0.0)	49	14	215

Table B: Example hyperfine interactions for a 3P nuclear register

[0147] In some examples, the hyperfine values can be predicted by NEMO-3D using the same procedure as described in “High Precision Quantum Control of Single Donor Spins in Silicon” Rajib Rahman, or in “Characterizing Si:P quantum dot qubits with spin resonance techniques” Yu Wang. Once the hyperfine interaction values are determined, a predicted error rate is calculated for the current magnetic field B_0 at step 904. In some examples, this predicted error rate may be computed using equation 14. Further, an estimated AHF of ~500-1000 kHz (example calculation results shown in Fig. 8E-G) may be utilized for this computation. The predicted error rate is calculated for all nuclear spins in the register and the calculated error rates are combined.

[0148] Next, at step 906, a determination is made whether the combined calculated error rate is higher than a threshold error rate. The threshold error rate may be determined based on what is tolerable. If the calculated error rate is lower than the threshold error rate, the method 900 ends. Otherwise, the method proceeds to step 908, where the applied magnetic field B_0 is changed such that the total error rate is sufficiently low. For example, in the case of Fig. 8E, the magnetic field may be reduced to $B_0 \approx 1.2$ T such that the error rate is below 10^{-3} . It will be appreciated that the range of available values for B_0 may be limited by other unrelated factors. For example, one may have the restriction that $B_0 \geq 1.0$ T when using Elzermann readout, or that $B_0 \leq 1.8$ T due to limitations in generating the high-frequency ESR signal to control the electron.

[0149] If even after execution of step 908, the error rate is not below the threshold error rate, the voltages applied to the electrostatic control gates can be changed (which due to the Stark shift will change the strength of the contact hyperfine interactions). Thereafter method steps 902-908 are repeated to determine a new error rate. This process can be repeated, exploring the space of possible voltages until the error rate is below the threshold rate.

Fabricating a device to mitigate and/or reduce errors

[0150] Fig. 10 illustrates an example method for fabricating the quantum processor according to aspects of the present disclosure.

[0151] Fig. 10 outlines the individual processing steps (steps a-k) for fabricating nuclear spin register according to aspects of the present disclosure.

[0152] In one example, if the nuclear spin register were to be fabricated such that dipolar coupling errors are reduced, a number of process steps may be performed before starting the fabrication process depicted in Fig. 10. These include, for example, initially determining the magnetic field strength B_0 at which the quantum device is to be operated (e.g., 1.4T).

[0153] The process steps also include determining a number of register layouts in which donor atoms can be positioned relative to each other in the nuclear registers. Then, for each register layout, the corresponding hyperfine strengths for each donor atom in the nuclear register layouts are calculated. These hyperfine strengths may be determined in a similar way as those computed for table A and table B. For example, the hyperfine values may be predicted by NEMO-3D.

[0154] Next, the corresponding error rates at the target magnetic field B_0 are computed for each of the nuclear register layouts. These error rates may be computed as done in step 904. Depending on the error rates, all nuclear register layouts for which the error rate is unacceptably high may be discarded.

[0155] The device may then be fabricated using method 1000 in any one of the remaining nuclear register layouts. A particular selected nuclear register layout may depend on other factors, such as certain layouts being more reliable to fabricate, or the need for hyperfine values to be sufficiently different to ensure frequency addressability.

[0156] In method 1000, initially, a clean Si 2×1 surface is formed in an ultra-high-vacuum (UHV) by heating to near the melting point. This surface has a 2×1 unit cell and consists of rows of σ -bonded Si dimers with the remaining dangling bond on each Si atom forming a weak π -bond with the other Si atom of the dimer of which it comprises.

[0157] Processing step (a) (i.e., monohydride deposition) involves exposing the clean Si 2×1 surface to atomic H to break the weak Si π -bonds, allowing H atoms to bond to the Si dangling bonds. Under controlled conditions a monolayer of H can be formed with one H atom bonded to each Si atom, satisfying the reactive dangling bonds, effectively passivating the surface; see step (a).

[0158] Next, at processing step (b) (i.e., hydrogen desorption), an STM tip is used to selectively desorb H atoms from the passivated surface by the application of appropriate voltages and tunnelling currents, forming a pattern in the H resist; see step (b).

[0159] It will be appreciated that H atoms are desorbed from precise locations where donor atoms are to be placed. For example, if the quantum processor includes a 2D square lattice of exchange-coupled quantum dots, H atoms are desorbed in such a manner as to create multiple lithographic patches in a square lattice formation, where the distance between adjacent patches is between 5-20 nanometres. Further, the size of each of the lithographic patches created by the hydrogen desorption may depend on the number of donor atoms that are required to be placed in the quantum dots. In one example, if 1 donor atom is to be positioned in one of the lithographic patches (to form a 1P quantum dot) and two donor atoms are to be positioned in an adjacent lithographic patch (to form a 2P quantum dot), the STM tip may be utilized to desorb 6 hydrogen atoms in a first location to create a first patch and 15 hydrogen atoms may be desorbed in a second location 5-20 nanometres apart to create a second larger patch. Similarly, if larger number of donor atoms are to be placed in the patches, more hydrogen atoms can be desorbed to create lithographic patches of larger sizes. In other examples, the sizes of the patches may be smaller or larger than those described in the example above. Further still, in some examples, machine learning techniques may be utilized to control the number of donor atoms placed in any lithographic patch.

[0160] The position of the patches may also be controlled such that the distance between the patches and corresponding charge sensors is within a predetermined distance to reduce dipolar coupling errors.

[0161] This process is repeated to create positions for other nuclear registers and/or quantum dots based on the selected layout. In this way regions of bare, reactive Si atoms are exposed along dimer rows, allowing the subsequent adsorption of reactive species directly to the Si surface.

[0162] Returning to Fig. 10, at step (c) (i.e., PH₃ dosing), phosphine (PH₃) gas is introduced into the vacuum system via a controlled leak valve connected to a specially designed phosphine micro-dosing system. The phosphine molecule bonds strongly to the exposed Si surface, through the holes in the hydrogen resist; see step(c). As noted previously, at a particular donor site, a phosphine molecule may bond with any one of the exposed silicon dimers.

[0163] Subsequent heating of the STM patterned surface for crystal growth causes the dissociation of the phosphine molecules and results in the incorporation of P into the first layer of Si; see step (d). It is therefore the exposure of an STM patterned H passivated surface to PH₃ that is used to produce the required donor molecules.

[0164] The hydrogen may then be desorbed, at step (e), before overgrowing the surface with silicon at room temperature, at step (f). An alternative is to grow the silicon directly through the hydrogen layer, as shown in step (g).

[0165] At step (h), the surface is rapidly annealed.

[0166] Silicon is then grown on the surface at elevated temperature, shown in step (i). In one example, approximately 50 ± 10 nm of epitaxial silicon is grown at a temperature of 250°C . In some cases, a barrier, also known as a locking layer, may be grown as shown in step (j). Finally, conductive gates may be aligned on the surface, as shown in step (k) using electron beam lithography. Using registration markers, such as evaporated metal markers, the gates may be aligned at a lateral distance of 300 ± 50 nm from the buried quantum dots. Further, an antenna may also be aligned on the surface to produce an oscillating magnetic field B_1 perpendicular to the substrate at the position of the quantum dots.

[0167] The manner in which the quantum dots 500 are fabricated dictates how the donor nuclei and/or electrons within a quantum dot can be used as qubits. In particular, specific geometries and placement of donors within a lithographic patch or within a quantum dot enable reliable control of hyperfine coupling, tunnel coupling, and tunnel rate for controlling the quantum operations for a single-, two- or multi-qubit gates as described above.

[0168] Donor atoms incorporated within a given site form a collective confinement potential to bind electrons. The number of donors and their spatial configuration within each quantum dot determines the confinement strength. The confinement strength, in turn, determines the hyperfine couplings between the electron spin and each of the nuclear spins.

[0169] As described above, the lithographic openings can be patterned with approximately 5-20 nanometre separation such that the tunnel coupling between the electron spins bound to two neighbouring dots allows for high-fidelity two-qubit gates between electron spin qubits.

[0170] In the presently disclosed quantum processors, quantum information is encoded within the electron spin and/ or the nuclear spin. For electron spin, readout may be achieved using a process called spin-to-charge conversion. In this process, a Single Electron Transistor (SET) charge sensor is used to determine the state of the electron spin qubit. The qubit-reservoir distance determines the electron tunnel rate – i.e. how quickly the electron spin can be measured.

[0171] It should be appreciated that the errors arising because of the presence of nuclear spins and the related solution as described in the embodiments of the present disclosure are

not limited to solid-state systems but also occur in systems directed to nitrogen-vacancy in diamonds. Therefore, the present disclosure is directed to all systems comprising nuclear spin registers.

[0172] The term "comprising" (and its grammatical variations) as used herein are used in the inclusive sense of "having" or "including" and not in the sense of "consisting only of".

[0173] It will be appreciated by persons skilled in the art that numerous variations and/or modifications may be made to the invention as shown in the specific embodiments without departing from the spirit or scope of the invention as broadly described. The present embodiments are, therefore, to be considered in all respects as illustrative and not restrictive.

CLAIMS

1. A quantum processing device comprising:
a nuclear spin register comprising at least two nuclear spin qubits;
at least one unpaired electron coupled to the nuclear spin register;
a control system configured to:
control and/or readout the electron spin qubit;
control and/or readout a nuclear spin qubit; and
reduce errors arising from dipolar coupling between the nuclear spin qubits
and/or between the electron spin and nuclear spins.
2. The quantum processing device of claim 1, wherein the errors arise from
direct dipolar coupling between at least one pair of nuclear spins in the nuclear spin register.
3. The quantum processing device of claim 2, wherein reducing the errors arising
from direct dipolar coupling comprises performing electron spin readout at a rate faster than
the fastest nuclear spin flip-flop oscillation in the nuclear spin register.
4. The quantum processing device of claim 3, wherein the rate of the electron
spin readout is dependent on a tunnel rate of the electron moving on and off the nuclear spin
register.
5. The quantum processing device of claim 4, wherein the tunnel rate of the
electron moving on and off the nuclear spin register is engineered by controlling a distance
between a reservoir and the nuclear spin register during a fabrication process.

6. The quantum processing device of claim 1, wherein the errors arise from an anisotropic coupling between the at least one unpaired electron and at least one of the nuclear spins in the nuclear spin register.

7. The quantum processing device of claim 6, wherein reducing the errors arising from the anisotropic coupling comprises:

engineering a hyperfine coupling between the at least one unpaired electron and at least one of the nuclear spins of the nuclear spin register by precision fabrication of the at least one nuclear spin in the nuclear spin register.

8. The quantum processing device of claim 7, wherein precision fabrication of the plurality of nuclear spins in the nuclear spin register comprises engineering the location of each nuclear spin in the plurality of nuclear spins.

9. The quantum processing device of claim 6, wherein reducing the errors arising from the anisotropic coupling comprises tuning a static magnetic field B_0 .

10. The quantum processing device of any one of claims 1-9 wherein the control system comprises a single electron transistor that acts as a charge sensor and an electron reservoir to load the at least one unpaired electron onto the nuclear spin register.

11. The quantum processing device of any one of claims 1-9, wherein the control system comprises one or more control gates to control the electrochemical potential of the two or more donor atoms in the nuclear spin register.

12. A method for reducing readout errors in a nuclear spin register, wherein the nuclear spin register comprises at least two nuclear spins and an unpaired electron confined to the nuclear spin register, the method comprising:

performing electron spin readout at a rate faster than the fastest nuclear spin flip-flop oscillation in the nuclear spin register for errors arising from direct nuclear-nuclear dipolar coupling; or

engineering a hyperfine coupling between the at least one unpaired electron and at least one of the nuclear spins of the plurality of nuclear spins by precision fabrication of the plurality of nuclear spins wherein the errors arise from an anisotropic hyperfine coupling.

13. The method of claim 12, wherein the rate of the electron spin readout is dependent on a tunnel rate of the electron moving on and off the nuclear spin register.

14. The method of claim 13, further comprising controlling a distance between a reservoir and the nuclear spin register during a fabrication process to engineer the tunnel rate of the electron moving on and off the nuclear spin register.

15. The method of claim 12, wherein precision fabrication of the plurality of nuclear spins in the nuclear spin register comprises engineering the location of each nuclear spin in the plurality of nuclear spins.

16. The method of claim 15, wherein precision fabrication of the plurality of nuclear spins comprises:

determining the strength of the static magnetic field at which a quantum device comprising the nuclear spin register is to be operated;

determining a plurality of nuclear spin register layouts;

for each of the plurality of nuclear spin register layouts:

computing hyperfine strengths between each donor atom in the nuclear spin register and the unpaired electron;

computing error rate based on the computed hyperfine strengths;

discarding one or more of the plurality of nuclear spin register layouts if the computed error rate for the one or more of the plurality of nuclear spin register layouts exceeds a threshold error rate; and

fabricating the quantum device based on any one of the remaining layout of the plurality of nuclear spin register layouts.

17. The method of claim 12, wherein reducing the errors arising from the anisotropic coupling comprises tuning a static magnetic field B_0 .

18. The method of claim 17, wherein reducing the errors arising from the anisotropic coupling comprises:

determining strength of hyperfine interaction between each nuclear spin in the nuclear spin register and the unpaired electron confined to the nuclear spin register;

calculate predicted error value for the error arising from the anisotropic coupling based on the computed strengths of the hyperfine interactions; and

changing the static magnetic field to reduce the predicted error value if the predicted error value exceeds a threshold value.