

[DESCRIPTION]

[Title of Invention] IMAGING DEVICE AND IMAGING METHOD

[Technical Field]

[0001]

5           The present disclosure relates to an imaging device, etc., that images an object in a measurement area using measurement data of scattered waves.

[Background Art]

[0002]

10           As techniques related to an imaging device or the like for imaging an object in a measurement area using measurement data of scattered waves, there are techniques described in Patent Literatures (PTLs) 1 to 5.

[0003]

15           For example, according to the technique described in PTL 1, beams sent out from a microwave sender are incident on an object to be inspected, and the amplitudes and phases of scattered beams are detected by a microwave detector. Then, the distribution of dielectric constants is calculated from output signals output from the  
20 microwave detector to display a tomogram of the object to be inspected.

[Citation List]

[Patent Literature]

[0004]

25 [PTL 1] Japanese Unexamined Patent Application Publication No. S62-66145

[PTL 2] WO 2014/125815

[PTL 3] WO 2015/136936

[PTL 4] WO 2021/020387

30 [PTL 5] WO 2021/053971

[Summary of Invention]

[Technical Problem]

[0005]

35           However, it is not easy to image an object in a measurement area using measurement data of scattered waves. More specifically,

estimating measurement data on scattered waves radiated from the measurement area in relation to waves that are incident on the measurement area when the condition in the measurement area is known is called a forward problem and easy. However, obtaining the condition in the measurement area when measurement data on scattered waves is known is called an inverse problem and is not easy.

[0006]

When an object moves in the measurement area, the Doppler effect causes a difference between the properties of waves transmitted to the measurement area and the properties of scattered waves received from the measurement area. Accordingly, it becomes difficult to analyze these relationships. That is, when an object moves in the measurement area, it becomes difficult to image the object with high accuracy using measurement data of scattered waves.

[0007]

In view of this, the present disclosure provides an imaging device or the like that is capable of imaging a moving object in the measurement area with high accuracy.

[Solution to Problem]

[0008]

An imaging device according to one aspect of the present disclosure includes: a plurality of transmitters that each transmit a wave to a measurement area; a plurality of receivers that each receive a scattered wave of the wave from the measurement area; and an information processing circuit that images an object in the measurement area using measurement data of the scattered wave. In imaging the object, the information processing circuit: derives, using the measurement data and a velocity vector of the object, a scattering field function that receives a transmission position of the wave and a reception position of the scattered wave as input and outputs an amount of the scattered wave at the reception position; derives an imaging function that receives an imaging target position as input and outputs an image intensity at the imaging target

position, and is defined using an amount output from the scattering field function in response to inputting the imaging target position into the scattering field function as the transmission position and the reception position; and images the object in the measurement area using the imaging function. In deriving the scattering field function, the information processing circuit reflects in the scattering field function that a wavenumber of the scattered wave changes from a wavenumber of the wave due to a Doppler effect corresponding to the velocity vector.

[0009]

These general or specific aspects may be implemented as a system, a device or apparatus, a method, an integrated circuit, a computer program, or a non-transitory computer-readable recording medium such as a CD-ROM, or any combination thereof.

[Advantageous Effects of Invention]

[0010]

According to the present disclosure, it is possible to image a moving object in the measurement area with high accuracy.

[Brief Description of Drawings]

[0011]

[FIG. 1]

FIG. 1 is a conceptual diagram showing a one-dimensional multistatic array antenna.

[FIG. 2]

FIG. 2 is a conceptual diagram showing the relationship between a transmission point and a reception point in a plane.

[FIG. 3]

FIG. 3 is a conceptual diagram showing an example of coordinates relating to a semi-two-dimensional array antenna.

[FIG. 4]

FIG. 4 is a conceptual diagram illustrating the relationship between a plurality of transmitters, a plurality of receivers, and a flying object.

[FIG. 5]

FIG. 5 is a conceptual diagram showing a coordinate system

including a one-dimensional array antenna and a flying object.

[FIG. 6]

FIG. 6 is a block diagram showing a basic configuration of an imaging device.

5 [FIG. 7]

FIG. 7 is a flowchart showing basic operations of an imaging device.

[Description of Embodiments]

[0012]

10 An imaging device according to one aspect of the present disclosure includes: a plurality of transmitters that each transmit a wave to a measurement area; a plurality of receivers that each receive a scattered wave of the wave from the measurement area; and an information processing circuit that images an object in the  
15 measurement area using measurement data of the scattered wave. In imaging the object, the information processing circuit: derives, using the measurement data and a velocity vector of the object, a scattering field function that receives a transmission position of the wave and a reception position of the scattered wave as input and  
20 outputs an amount of the scattered wave at the reception position; derives an imaging function that receives an imaging target position as input and outputs an image intensity at the imaging target position, and is defined using an amount output from the scattering field function in response to inputting the imaging target position  
25 into the scattering field function as the transmission position and the reception position; and images the object in the measurement area using the imaging function. In deriving the scattering field function, the information processing circuit reflects in the scattering field function that a wavenumber of the scattered wave changes from a  
30 wavenumber of the wave due to a Doppler effect corresponding to the velocity vector.

[0013]

This allows the imaging device to derive the scattering field function and the imaging function in which the Doppler effect  
35 resulting from the movement of the object is reflected. Accordingly,

the imaging device is capable of imaging a moving object in the measurement area with high accuracy.

[0014]

For example, the plurality of transmitters and the plurality of  
5 receivers are arranged along a straight line parallel to a y-axis, and  
the information processing circuit derives, as the scattering  
field function, a solution for an item expressed by Math. 2 of the  
following equation expressed by Math. 1:

[Math. 1]

$$\left[\frac{1}{4}\{\square_4 - 2(ik)^2\}^2 - D_{y_1}^2 D_{y_2}^2 + (ik)^2(D_{y_1}^2 + D_{y_2}^2) - (ik)^4\right]\varphi = 0$$

$$\square_4 = D_x^2 + D_{y_1}^2 + D_{y_2}^2 + D_z^2$$

$$D_x = \partial_x - ik\alpha$$

$$D_{y_1} = \partial_{y_1} - ik\beta$$

$$D_{y_2} = \partial_{y_2} - ik\beta$$

$$10 \quad D_z = \partial_z$$

[Math. 2]

$\varphi$

where x and z of the equation respectively represent an  
x-coordinate and a z-coordinate of the transmission position and the  
15 reception position,  $y_1$  of the equation represents a y-coordinate of  
the transmission position,  $y_2$  of the equation represents a  
y-coordinate of the reception position, k represents the wavenumber  
of the wave,  $\alpha$  is defined by  $\alpha = 2v_x/c$ ,  $\beta$  is defined by  $\beta = v_y/c$ ,  $v_x$  and  
 $v_y$  respectively represent an x-component and a y-component of the  
20 velocity vector, and c represents a speed of the wave.

[0015]

This allows the imaging device to derive the scattering field  
function from an equation that reflects the Doppler effect, and to  
image a moving object in the measurement area with high accuracy  
25 by using the derived scattering field function.

[0016]

For example, the scattering field function is expressed as:

[Math. 3]

$$\begin{aligned} \varphi(x, y_1, y_2, z, k) \\ = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x + k_{y1} y_1 + k_{y2} y_2)} \tilde{\Phi}(k_x, k_{y1}, k_{y2}, k) \\ \cdot e^{iz \sqrt{\left\{ \sqrt{k^2 - (k_{y1} + k\beta)^2} + \sqrt{k^2 - (k_{y2} + k\beta)^2} \right\}^2 - (k_x + k\alpha)^2}} dk_x dk_{y1} dk_{y2} \end{aligned}$$

where x and z of the scattering field function respectively represent an x-coordinate and a z-coordinate of the transmission position and the reception position,  $y_1$  of the scattering field function represents a y-coordinate of the transmission position,  $y_2$  of the scattering field function represents a y-coordinate of the reception position, and  $k_x$ ,  $k_{y1}$ , and  $k_{y2}$  respectively represent variables corresponding to wavenumbers with respect to x,  $y_1$ , and  $y_2$  of the scattering field function, and

10 [Math. 4]

$\tilde{\Phi}$

represents the measurement data that has been Fourier transformed.

[0017]

15 This allows the imaging device to image a moving object in the measurement area with high accuracy by using the scattering field function derived from the measurement data obtained by the plurality of transmitters and the plurality of receivers arranged in a straight line.

20 [0018]

For example, the imaging function is expressed as:

[Math. 5]

$$\rho(x, y, z) = \int_0^{\infty} \lim_{y_2 \rightarrow y_1 = y} [\varphi(x, y_1, y_2, z, k)] dk$$

25 where x, y, and z of the imaging function respectively represent an x-coordinate, a y-coordinate, and a z-coordinate of the imaging target position.

[0019]

This allows the imaging device to image a moving object in the measurement area with high accuracy by using the imaging function

derived from the measurement data obtained by the plurality of transmitters and the plurality of receivers arranged in a straight line.  
[0020]

For example, using a function defined by:

5 [Math. 6]

$$\begin{aligned} & \tilde{\Psi}(k_{\xi}, k_{\eta_1}, k_{\eta_2}, k) \\ &= \int_{-\infty}^{\infty} e^{ickt} dt \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(k_{\xi}x + k_{\eta_1}y_1 + k_{\eta_2}y_2)} \left\{ e^{-ik\alpha x - ik\beta y_1 - ik\beta y_2} \Phi(x, y_1, y_2, t) \right\} dx dy_1 dy_2 \end{aligned}$$

the imaging function is expressed as:

[Math. 7]

$$\begin{aligned} & \rho(x, y, z) \\ &= \int_0^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{i(k\alpha x + k\beta y + k\beta y)}}{(2\pi)^3} e^{-i(k_{\xi}x + k_{\eta_1}y + k_{\eta_2}y)} \\ & \quad \cdot e^{ik_{\xi}z} \tilde{\Psi}(k_{\xi}, k_{\eta_1}, k_{\eta_2}, k) \left( \frac{dk}{dk_{\xi}} \right) dk_{\xi} dk_{\eta_1} dk_{\eta_2} dk_{\xi} \end{aligned}$$

10 where x, y, and z of the imaging function respectively represent an x-coordinate, a y-coordinate, and a z-coordinate of the imaging target position,  $\Phi$  represents the measurement data, and  $k_{\xi}$ ,  $k_{\eta_1}$ ,  $k_{\eta_2}$ ,  $k_z$ , and  $dk/dk_z$  are defined as:

[Math. 8]

$$k_{\xi} = k_x + k\alpha$$

$$k_{\eta_1} = k_{y_1} + k\beta$$

$$k_{\eta_2} = k_{y_2} + k\beta$$

$$k_{\xi} = \sqrt{(\sqrt{k^2 - k_{\eta_1}^2} + \sqrt{k^2 - k_{\eta_2}^2})^2 - k_{\xi}^2}$$

$$\frac{dk}{dk_{\xi}} = \frac{k_{\xi} \sqrt{k^2 - k_{\eta_1}^2} \sqrt{k^2 - k_{\eta_2}^2}}{k(k_{\xi}^2 + k_{\xi}^2)}$$

15

.

[0021]

This allows the imaging device to image a moving object in the measurement area with high accuracy by using the imaging function  
20 derived from the measurement data obtained by the plurality of

transmitters and the plurality of receivers arranged in a straight line, the imaging function having inhibited complexity.

[0022]

5 For example, the information processing circuit: derives a provisional scattering field function using the measurement data without using the velocity vector; derives a provisional imaging function using the provisional scattering field function; and derives the velocity vector using the provisional imaging function.

[0023]

10 This allows the imaging device to derive the velocity vector of the object using the provisional scattering field function and the provisional imaging function derived without using the velocity vector of the object. The imaging device is capable of imaging an object that moves in the measurement area with high accuracy by  
15 using the scattering field function and the imaging function derived using the velocity vector.

[0024]

For example, in deriving the velocity vector, the information processing circuit:

20 derives the velocity vector using an arithmetic expression represented as:

[Math. 9]

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = - \begin{pmatrix} 2a+2\varepsilon & e & g \\ e & 2b+2\varepsilon & f \\ g & f & 2c+2\varepsilon \end{pmatrix}^{-1} \begin{pmatrix} h \\ m \\ n \end{pmatrix}$$

25 where a, b, c, e, f, g, h, m, and n in the arithmetic expression are defined as:

[Math. 10]

$$a = \frac{1}{(2\pi)^3} \iiint k_x^2 |\rho_k|^2 d\mathbf{k}^3$$

$$b = \frac{1}{(2\pi)^3} \iiint k_y^2 |\rho_k|^2 d\mathbf{k}^3$$

$$c = \frac{1}{(2\pi)^3} \iiint k_z^2 |\rho_k|^2 d\mathbf{k}^3$$

$$e = \frac{1}{(2\pi)^3} \iiint k_x k_y |\rho_k|^2 d\mathbf{k}^3 + cc.$$

$$f = \frac{1}{(2\pi)^3} \iiint k_y k_z |\rho_k|^2 d\mathbf{k}^3 + cc.$$

$$g = \frac{1}{(2\pi)^3} \iiint k_z k_x |\rho_k|^2 d\mathbf{k}^3 + cc.$$

$$h = \frac{-i}{(2\pi)^3} \iiint k_x \rho_k \cdot \partial_\tau \bar{\rho}_k d\mathbf{k}^3 + cc.$$

$$m = \frac{-i}{(2\pi)^3} \iiint k_y \rho_k \cdot \partial_\tau \bar{\rho}_k d\mathbf{k}^3 + cc.$$

$$n = \frac{-i}{(2\pi)^3} \iiint k_z \rho_k \cdot \partial_\tau \bar{\rho}_k d\mathbf{k}^3 + cc.$$

where  $v_x$ ,  $v_y$ , and  $v_z$  respectively represent an x-component, a y-component, and a z-component of the velocity vector,  $\varepsilon$  represents a positive value,  $\rho_k$  represents the provisional imaging function that includes  $\tau$  as one of a plurality of input variables and is Fourier transformed with respect to  $x$ ,  $y$ , and  $z$ , and  $k_x$ ,  $k_y$ , and  $k_z$  in the arithmetic expression respectively represent wavenumbers with respect to  $x$ ,  $y$ , and  $z$  of the provisional imaging function,

[Math. 11]

10 **k**

represents a wavenumber vector of  $k_x$ ,  $k_y$ , and  $k_z$  in the arithmetic expression,  $\tau$  represents a time unit corresponding to a number of data collections, and  $cc.$  represents a complex conjugate of a term immediately preceding  $cc.$ , and

15 [Math. 12]

$\bar{\rho}_k$

represents the complex conjugate of  $\rho_k$ .

[0025]

This allows the imaging device to derive the velocity vector of the object from the provisional imaging function using an arithmetic expression.

5 [0026]

An imaging method according to one aspect of the present disclosure includes: transmitting, by each of a plurality of transmitters, a wave to a measurement area; receiving, by each of a plurality of receivers, a scattered wave of the wave from the measurement area; and imaging an object in the measurement area using measurement data of the scattered wave. The imaging of the object includes: deriving, using the measurement data and a velocity vector of the object, a scattering field function that receives a transmission position of the wave and a reception position of the scattered wave as input and outputs an amount of the scattered wave at the reception position; deriving an imaging function that receives an imaging target position as input and outputs an image intensity at the imaging target position, and is defined using an amount output from the scattering field function in response to inputting the imaging target position into the scattering field function as the transmission position and the reception position; and imaging the object in the measurement area using the imaging function. The deriving of the scattering field function includes reflecting in the scattering field function that a wavenumber of the scattered wave changes from a wavenumber of the wave due to a Doppler effect corresponding to the velocity vector.

[0027]

This allows deriving the scattering field function and the imaging function that reflects the Doppler effect resulting from the movement of the object. Accordingly, it is possible to image a moving object in the measurement area with high accuracy.

[0028]

Hereinafter, embodiments will be described with reference to the drawings. Each of the following embodiments describes a general or specific example. The numerical values, shapes,

materials, elements, the arrangement and connection of the elements, steps, the order of the steps, etc., presented in the following embodiments are mere examples, and do not limit the scope of the claims.

5 [0029]

In the following description, in particular the techniques or the like described in PTLs 2 to 5 given above may be referenced to as existing techniques. Although radio waves or electromagnetic waves such as microwaves are primarily assumed as the waves in the following description, the waves are not limited to radio waves or electromagnetic waves such as microwaves, and may be elastic waves or the like. Imaging based on scattering may be expressed as scattering tomography. Thus, the imaging device and imaging method described below may also be expressed as a scattering tomographic device and a scattering tomographic method, respectively.

[0030]

[Embodiment]

An imaging device according to the present embodiment images an object in a measurement area using measurement data of scattered waves. Hereinafter, the imaging device according to the present embodiment, including techniques and theories serving as the basis of the imaging device, will be described in detail.

[0031]

25 <1. Overview>

Scattering field theory is a theory used for imaging an object in a measurement area. For example, waves are transmitted from each of a plurality of transmission positions to the measurement area, and scattered waves from the measurement area are received at each of a plurality of reception positions. Measurement data of the scattered waves is obtained for each combination of transmission position and reception position. The object in the measurement area is then imaged using the measurement data. At this time, using the scattering field theory, the condition of scattering in the measurement area is calculated from the measurement data, and the

object is imaged with high accuracy based on the condition of scattering.

[0032]

5 However, when an object moves in the measurement area, the Doppler effect causes a difference between the properties of waves transmitted to the measurement area and the properties of scattered waves received from the measurement area. Accordingly, it becomes difficult to analyze these relationships. That is, when an object moves in the measurement area, it becomes difficult to image  
10 the object with high accuracy using measurement data of scattered waves.

[0033]

In view of this, the present disclosure describes a scattering field theory for imaging a moving object in the measurement area  
15 with high accuracy. More specifically, first, a scattering field theory that does not take the movement of the object into consideration will be described, and then, a scattering field theory that does take the movement of the object into consideration will be described.

[0034]

20 <2. Scattering Field Theory Without Consideration of Object Movement>

In this chapter, a scattering field theory that does not take the movement of the object into consideration will be described.

[0035]

25 <2-1. One-dimensional Array>

FIG. 1 is a conceptual diagram showing a one-dimensional multistatic array antenna. As illustrated in FIG. 1, by using two arbitrary elements from among  $n$  elements as a transmitting element and a receiving element, it is possible to receive signals with a high  
30 S/N ratio at a range of distances from short distance to long distance compared to a monostatic environment where the pair of transmitting and receiving elements is fixed. This considerably improves the quality of an ultimate image. Whereas, as a matter of course, the amount of data is increased to  $n$  times, and the time  
35 required for reconstruction is also shortened dramatically according

to the theory described below.

[0036]

Here, a situation is examined in which a radio wave radiated from point  $P_1(x, y_1, z)$  is reflected at point  $P(\xi, \eta, \zeta)$  and received at point  $P_2(x, y_2, z)$  as illustrated in FIG. 1. In the case where point P is assumed to move in the entire region D, the signal received at point  $P_2$  is represented by expression (2-1-1) below.

[0037]

[Math. 13]

$$\phi(x, y_1, y_2, z) = \iint_D \frac{e^{ik\rho_1}}{\rho_1} \frac{e^{ik\rho_2}}{\rho_2} \varepsilon(\xi, \eta, \zeta) d\xi d\eta d\zeta$$

$$\rho_1 = \sqrt{(x - \xi)^2 + (y_1 - \eta)^2 + (z - \zeta)^2}$$

$$\rho_2 = \sqrt{(x - \xi)^2 + (y_2 - \eta)^2 + (z - \zeta)^2}$$

$$\dots (2-1-1)$$

[0038]

Here,  $\varepsilon(\xi, \eta, \zeta)$  represents the function of the dielectric constant at point  $P(\xi, \eta, \zeta)$  and corresponds to the reflectance at point  $P(\xi, \eta, \zeta)$ . Point  $P(\xi, \eta, \zeta)$  corresponds to the reflection point. Note that  $\varepsilon(\xi, \eta, \zeta)$  is unknown. It is assumed that the time factor is proportional to  $\exp(-i\omega t)$ . The kernel function in the integrand term of the above equation is represented as  $\phi$  in expression (2-1-2) below.

[0039]

[Math. 14]

$$\phi = \frac{e^{ik\rho_1}}{\rho_1} \frac{e^{ik\rho_2}}{\rho_2}$$

$$\dots (2-1-2)$$

[0040]

Next, a partial differential equation that has expression (2-1-2) as an asymptotical solution is examined. Thus, calculation is performed while ignoring a high-order term with respect to  $1/p$  obtained as a result of differentiation. Here, an abridged notation for differentiation is defined by expression (2-1-3).

[0041]

[Math. 15]

$$\frac{\partial}{\partial t} \rightarrow \partial_t, \frac{\partial}{\partial x} \rightarrow \partial_x, \frac{\partial}{\partial y_1} \rightarrow \partial_{y_1}, \frac{\partial}{\partial y_2} \rightarrow \partial_{y_2}, \frac{\partial}{\partial z} \rightarrow \partial_z$$

$$\cdot \cdot \cdot (2 - 1 - 3)$$

[0042]

- 5 Here, a partial differential equation that has expression (2-1-2) as an asymptotical solution at short wavelengths (at high frequency or when k is large) is examined. This solution to the partial differential equation may be regarded as almost an exact solution in imaging using microwaves. First, the result of
- 10 differentiation of each order of  $\phi$  is represented by expression (2-1-4) below.

[0043]

[Math. 16]

$$\partial_x \phi = ik(x - \xi) \left( \frac{1}{\rho_1} + \frac{1}{\rho_2} \right) \phi + o(\rho^{-3}) \quad \partial_{y_1} \phi = ik \frac{y_1 - \eta}{\rho_1} \phi + o(\rho^{-3})$$

$$\partial_{y_2} \phi = ik \frac{y_2 - \eta}{\rho_2} \phi + o(\rho^{-3}) \quad \partial_z \phi = ik(z - \zeta) \left( \frac{1}{\rho_1} + \frac{1}{\rho_2} \right) \phi + o(\rho^{-3})$$

$$\partial_x \partial_x \phi = (ik)^2 (x - \xi)^2 \left( \frac{1}{\rho_1} + \frac{1}{\rho_2} \right)^2 \phi + o(\rho^{-3})$$

$$\partial_z \partial_z \phi = (ik)^2 (z - \zeta)^2 \left( \frac{1}{\rho_1} + \frac{1}{\rho_2} \right)^2 \phi + o(\rho^{-3})$$

$$\partial_{y_1} \partial_{y_1} \phi = (ik)^2 \left( \frac{y_1 - \eta}{\rho_1} \right)^2 \phi + o(\rho^{-3})$$

$$\partial_{y_2} \partial_{y_2} \phi = (ik)^2 \left( \frac{y_2 - \eta}{\rho_2} \right)^2 \phi + o(\rho^{-3})$$

$$\cdot \cdot \cdot (2 - 1 - 4)$$

- 15 [0044]

Hereinafter, the complexity  $o(*)$  is omitted. In accordance with the sum of four differential equations of the second order, expression (2-1-5) below is obtained.

[0045]

[Math. 17]

$$\begin{aligned}\Delta_4\phi &= (\partial_x^2 + \partial_{y_1}^2 + \partial_{y_2}^2 + \partial_z^2)\phi \\ &= (ik)^2 \left\{ 2 + 2 \frac{(x-\xi)^2 + (z-\zeta)^2}{\rho_1\rho_2} \right\} \phi\end{aligned}$$

• • • (2 - 1 - 5)

[0046]

5 Accordingly, expression (2-1-6) below is obtained from expression (2-1-5).

[0047]

[Math. 18]

$$\begin{aligned}\{\Delta_4 - 2(ik)^2\}\phi &= 2(ik)^2 \frac{\rho_1^2 - (y_1 - \eta)^2}{\rho_1\rho_2} \phi \\ &= 2(ik)^2 \frac{\rho_2^2 - (y_2 - \eta)^2}{\rho_1\rho_2} \phi\end{aligned}$$

• • • (2 - 1 - 6)

10 [0048]

By applying the operation of expression (2-1-6) two times, expression (2-1-7) below is obtained.

[0049]

[Math. 19]

$$\begin{aligned}\{\Delta_4 - 2(ik)^2\}^2\phi &= 4(ik)^4 \frac{\{\rho_1^2 - (y_1 - \eta)^2\}\{\rho_2^2 - (y_2 - \eta)^2\}}{\rho_1^2\rho_2^2} \phi \\ &= 4(ik)^4 \{1 - (ik)^{-2}\partial_{y_1}^2\}\{1 - (ik)^{-2}\partial_{y_2}^2\}\phi\end{aligned}$$

15 • • • (2 - 1 - 7)

[0050]

Expression (2-1-7) is summarized to obtain expression (2-1-8) below.

[0051]

20 [Math. 20]

$$\left[\frac{1}{4}\{\Delta_4 - 2(ik)^2\}^2 - \partial_{y_1}^2 \partial_{y_2}^2 + (ik)^2(\partial_{y_1}^2 + \partial_{y_2}^2) - (ik)^4\right]\phi = 0$$

• • • (2 - 1 - 8)

[0052]

Although expression (2-1-8) is derived assuming a steady state, it is easy to extend expression (2-1-8) to a non-steady state.

5 Thus, variables are substituted as given by expression (2-1-9) below, using partial differential  $\partial_t$  with respect to time  $t$  and using propagation velocity  $c$  of radio waves.

[0053]

[Math. 21]

$$-ik \rightarrow \frac{1}{c} \partial_t$$

10 • • • (2 - 1 - 9)

[0054]

Through the process described above, an equation represented by expression (2-1-10) below is ultimately obtained.

[0055]

15 [Math. 22]

$$\left\{\Delta_4^2 - \frac{4}{c^2}(\partial_t^2 \partial_x^2 + \partial_t^2 \partial_z^2) - 4\partial_{y_1}^2 \partial_{y_2}^2\right\}\phi = 0$$

$$\Delta_4 = \partial_x^2 + \partial_{y_1}^2 + \partial_{y_2}^2 + \partial_z^2$$

• • • (2 - 1 - 10)

[0056]

Expression (2-1-10) described above is a partial differential equation that has  $\phi$  in expression (2-1-2) as a solution. By applying

20 differentiation to the kernel of expression (2-1-1),

[Math. 23]

$\phi$

of expression (2-1-1) also satisfies the partial differential equation described above. This equation is a four-dimensional pseudo wave

25 equation configured by five variables ( $t, x, y_1, y_2, z$ ).

[0057]

Next, this equation is solved by Fourier transform. First,

[Math. 24]

$\phi$

is subjected to multiplex Fourier transform with respect to  $t, x, y_1, y_2$  as given by expression (2-1-11) below.

[0058]

5 [Math. 25]

$$\tilde{\phi}(k_x, k_{y_1}, k_{y_2}, z, \omega) = \int_{-\infty}^{\infty} e^{i\omega t} dt \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(k_x x + k_{y_1} y_1 + k_{y_2} y_2)} \phi(x, y_1, y_2, z, t) dx dy_1 dy_2$$

• • • (2 - 1 - 1 1)

[0059]

When the differential with respect to  $z$  is expressed as  $D_z$ , expression (2-1-12) below is obtained from expressions (2-1-10) and (2-1-11).

10

[0060]

[Math. 26]

$$\{(D_z^2 - k_x^2 - k_{y_1}^2 - k_{y_2}^2)^2 + 4k^2(D_z^2 - k_x^2) - 4k_{y_1}^2 k_{y_2}^2\} \tilde{\phi} = 0$$

• • • (2 - 1 - 1 2)

[0061]

15 Here, the relationship of  $\omega = ck$  is used. Four basic solutions to this equation are expressed as given by expression (2-1-13) below.

[0062]

[Math. 27]

$$E_1 = e^{i\{\sqrt{(\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2})^2 - k_x^2}\}z}$$

$$E_2 = e^{-i\{\sqrt{(\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2})^2 - k_x^2}\}z}$$

$$E_3 = e^{i\{\sqrt{(\sqrt{k^2 - k_{y_1}^2} - \sqrt{k^2 - k_{y_2}^2})^2 - k_x^2}\}z}$$

$$E_4 = e^{-i\{\sqrt{(\sqrt{k^2 - k_{y_1}^2} - \sqrt{k^2 - k_{y_2}^2})^2 - k_x^2}\}z}$$

20 • • • (2 - 1 - 1 3)

[0063]

Considering the facts that the time factor is  $e^{-i\omega t}$ , the phase is added using the path of radiated radio waves, and radio waves reflected off the object are bounced off toward a measurement

surface (measurement plane),  $E_1$  is the unique meaningful solution. Accordingly, expression (2-1-14) below is obtained.

[0064]

[Math. 28]

$$\tilde{\varphi}(k_x, k_{y_1}, k_{y_2}, z, k) = a(k_x, k_{y_1}, k_{y_2}, k) e^{i\{\sqrt{(k^2 - k_{y_1}^2) + \sqrt{(k^2 - k_{y_2}^2)^2 - k_x^2}}\}z}$$

5     • • • (2 – 1 – 1 4)

[0065]

By substituting  $z = 0$  in expression (2-1-14),  $a(k_x, k_{y_1}, k_{y_2}, k)$  is obtained as given by expression (2-1-15) below.

[0066]

10 [Math. 29]

$$a(k_x, k_{y_1}, k_{y_2}, k) = \tilde{\varphi}(k_x, k_{y_1}, k_{y_2}, 0, k)$$

• • • (2 – 1 – 1 5)

[0067]

Ultimately,

[Math. 30]

15      $\varphi$

is obtained as given by expression (2-1-16) below.

[0068]

[Math. 31]

$$\varphi(x, y_1, y_2, z, k)$$

$$= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x + k_{y_1} y_1 + k_{y_2} y_2)} a(k_x, k_{y_1}, k_{y_2}, k)$$

$$\cdot e^{i\{\sqrt{(k^2 - k_{y_1}^2) + \sqrt{(k^2 - k_{y_2}^2)^2 - k_x^2}}\}z} dk_x dk_{y_1} dk_{y_2}$$

• • • (2 – 1 – 1 6)

20 [0069]

By applying a limit operation ( $y_2 \rightarrow y_1 = y$ ) to expression (2-1-16) on condition that  $k$  and  $z$  are fixed and integrating the result with respect to  $k$ , the imaging function is obtained as given by expression (2-1-17) below.

25 [0070]

[Math. 32]

$$\begin{aligned}
\phi(x, y, y, z, k) &= \lim_{y_2 \rightarrow y_1 = y} [\phi(x, y, y_2, z, k)] \\
&= \lim_{y_2 \rightarrow y_1 = y} \left[ \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x + k_{y_1} y + k_{y_2} y_2)} a(k_x, k_{y_1}, k_{y_2}, k) \right. \\
&\quad \left. \cdot e^{i\{\sqrt{(k^2 - k_{y_1}^2) + \sqrt{(k^2 - k_{y_2}^2)}^2 - k_x^2\} z} dk_x dk_{y_1} dk_{y_2} \right] \\
\rho(x, y, z) &= \int_0^{\infty} \phi(x, y, y, z, k) dk
\end{aligned}$$

• • • (2 - 1 - 1 7)

[0071]

As described above, it becomes possible to analytically solve a multistatic inverse scattering problem with a one-dimensional array.

5 [0072]

<2-2. Two-dimensional Array>

This section describes the theory for a case pertaining to a two-dimensional array.

[0073]

10 FIG. 2 is a conceptual diagram showing the relationship between a transmission point and a reception point in a plane. As illustrated in FIG. 2, a microwave radiated from point P<sub>1</sub> is reflected at point P on a target and received at point P<sub>2</sub>. Points P<sub>1</sub> and P<sub>2</sub> move to arbitrary points on a grid (two-dimensional array antenna) in a plane. Under this assumption, there are n<sup>4</sup> different microwave paths passing through point P on the target. This large number of paths considerably contributes to an improvement in the quality of an ultimate image. A method for processing such complex data to obtain an image will be described below.

20 [0074]

For example, as illustrated in FIG. 2, the radio wave radiated from point P<sub>1</sub>(x<sub>1</sub>, y<sub>1</sub>, z) is reflected at point P(ξ, η, ζ) and received at point P<sub>2</sub>(x<sub>2</sub>, y<sub>2</sub>, z). When point P is assumed to move in the entire region D, a signal received at P<sub>2</sub> is expressed by the following expression.

25 [0075]

[Math. 33]

$$\varphi(x_1, y_1, x_2, y_2, z) = \iiint_D \frac{e^{ik\rho_1}}{\rho_1} \frac{e^{ik\rho_2}}{\rho_2} \varepsilon(\xi, \eta, \zeta) d\xi d\eta d\zeta$$

$$\rho_1 = \sqrt{(x_1 - \xi)^2 + (y_1 - \eta)^2 + (z - \zeta)^2}$$

$$\rho_2 = \sqrt{(x_2 - \xi)^2 + (y_2 - \eta)^2 + (z - \zeta)^2}$$

$$\cdot \cdot \cdot (2 - 2 - 1)$$

[0076]

It is assumed here that the time factor is proportional to  $\exp(-i\omega t)$ . The kernel function in the integrand term of the above expression is expressed as given by expression (2-2-2) below.

[0077]

[Math. 34]

$$\phi = \frac{e^{ik\rho_1}}{\rho_1} \frac{e^{ik\rho_2}}{\rho_2}$$

$$\cdot \cdot \cdot (2 - 2 - 2)$$

[0078]

Next, a partial differential equation that has expression (2-2-2) as an asymptotical solution at short wavelengths is examined. Thus, calculation is performed while ignoring a high-order term with respect to  $1/\rho$  obtained as a result of differentiation. Here, an abridged notation for differentiation is defined by expression (2-2-3).

[0079]

[Math. 35]

$$\frac{\partial}{\partial t} \rightarrow \partial_t, \frac{\partial}{\partial x_1} \rightarrow \partial_{x_1}, \frac{\partial}{\partial x_2} \rightarrow \partial_{x_2}, \frac{\partial}{\partial y_1} \rightarrow \partial_{y_1}, \frac{\partial}{\partial y_2} \rightarrow \partial_{y_2}, \frac{\partial}{\partial z} \rightarrow \partial_z$$

$$\cdot \cdot \cdot (2 - 2 - 3)$$

[0080]

Using expression (2-2-3), differentiation of each order of the kernel function is expressed as given by expression (2-2-4) below.

[0081]

[Math. 36]

$$\partial_{x_1}\phi = ik \frac{x_1 - \xi}{\rho_1} \phi + o(\rho^{-3}) \quad \partial_{x_2}\phi = ik \frac{x_2 - \xi}{\rho_2} \phi + o(\rho^{-3})$$

$$\partial_{y_1}\phi = ik \frac{y_1 - \eta}{\rho_1} \phi + o(\rho^{-3}) \quad \partial_{y_2}\phi = ik \frac{y_2 - \eta}{\rho_2} \phi + o(\rho^{-3})$$

$$\partial_z\phi = ik(z - \zeta)\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)\phi + o(\rho^{-3})$$

$$\partial_{x_1}\partial_{x_1}\phi = (ik)^2 \left(\frac{x_1 - \xi}{\rho_1}\right)^2 \phi + o(\rho^{-3})$$

$$\partial_{y_1}\partial_{y_1}\phi = (ik)^2 \left(\frac{y_1 - \eta}{\rho_1}\right)^2 \phi + o(\rho^{-3})$$

$$\partial_{x_2}\partial_{x_2}\phi = (ik)^2 \left(\frac{x_2 - \xi}{\rho_2}\right)^2 \phi + o(\rho^{-3})$$

$$\partial_{y_2}\partial_{y_2}\phi = (ik)^2 \left(\frac{y_2 - \eta}{\rho_2}\right)^2 \phi + o(\rho^{-3})$$

$$\partial_z\partial_z\phi = (ik)^2 (z - \zeta)^2 \left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)^2 \phi + o(\rho^{-3})$$

• • • (2 - 2 - 4)

[0082]

Hereinafter, the complexity  $o(*)$  is omitted. In accordance with the sum of five differential equations of the second order, expression (2-2-5) below is obtained.

[0083]

[Math. 37]

$$\begin{aligned} \Delta_5\phi &= (\partial_{x_1}^2 + \partial_{y_1}^2 + \partial_{x_2}^2 + \partial_{y_2}^2 + \partial_z^2)\phi \\ &= (ik)^2 \left\{ 2 + 2 \frac{(z - \zeta)^2}{\rho_1 \rho_2} \right\} \phi \end{aligned}$$

• • • (2 - 2 - 5)

[0084]

Accordingly, expression (2-2-6) below is obtained from expression (2-2-5).

[0085]

[Math. 38]

$$\begin{aligned}
\{\Delta_5 - 2(ik)^2\}\phi &= 2(ik)^2 \frac{(z - \xi)^2}{\rho_1 \rho_2} \phi \\
&= 2(ik)^2 \frac{\rho_1^2 - (x_1 - \xi)^2 - (y_1 - \eta)^2}{\rho_1 \rho_2} \phi \\
&= 2(ik)^2 \frac{\rho_2^2 - (x_2 - \xi)^2 - (y_2 - \eta)^2}{\rho_1 \rho_2} \phi
\end{aligned}$$

• • • (2 - 2 - 6)

[0086]

By applying the operation of expression (2-2-6) two times, expression (2-2-7) below is obtained.

5 [0087]

[Math. 39]

$$\begin{aligned}
&\{\Delta_5 - 2(ik)^2\}^2 \phi \\
&= 4(ik)^4 \frac{\{\rho_1^2 - (x_1 - \xi)^2 - (y_1 - \eta)^2\} \{\rho_2^2 - (x_2 - \xi)^2 - (y_2 - \eta)^2\}}{\rho_1^2 \rho_2^2} \phi \\
&= 4(ik)^4 \{1 - (ik)^{-2} \partial_{x_1}^2 - (ik)^{-2} \partial_{y_1}^2\} \{1 - (ik)^{-2} \partial_{x_2}^2 - (ik)^{-2} \partial_{y_2}^2\} \phi \\
&\cdot \cdot \cdot (2 - 2 - 7)
\end{aligned}$$

[0088]

10 Expression (2-2-7) is summarized to obtain expression (2-2-8) below.

[0089]

[Math. 40]

$$\begin{aligned}
&\left[ \frac{1}{4} \{\Delta_5 - 2(ik)^2\}^2 - (\partial_{x_1}^2 + \partial_{y_1}^2)(\partial_{x_2}^2 + \partial_{y_2}^2) \right. \\
&\quad \left. + (ik)^2 (\partial_{x_1}^2 + \partial_{y_1}^2 + \partial_{x_2}^2 + \partial_{y_2}^2) - (ik)^4 \right] \phi = 0 \\
&\cdot \cdot \cdot (2 - 2 - 8)
\end{aligned}$$

[0090]

15 Although expression (2-2-8) is derived assuming a steady state, it is easy to extend expression (2-2-8) to a non-steady state. Thus, variables are substituted as given by expression (2-2-9) below.

[0091]

[Math. 41]

$$-ik \rightarrow \frac{1}{c} \partial_t$$

• • • (2 – 2 – 9)

[0092]

By this substitution, expression (2-2-8) is converted into expression (2-2-10) below that includes time.

5 [0093]

[Math. 42]

$$\{\Delta_5^2 - \frac{4}{c^2} \partial_t^2 \partial_z^2 - 4(\partial_{x_1}^2 + \partial_{y_1}^2)(\partial_{x_2}^2 + \partial_{y_2}^2)\} \phi = 0$$

$$\Delta_5 = \partial_{x_1}^2 + \partial_{y_1}^2 + \partial_{x_2}^2 + \partial_{y_2}^2 + \partial_z^2$$

• • • (2 – 2 – 1 0)

[0094]

10 Expression (2-2-10) described above is a partial differential equation that has the kernel function given by expression (2-2-2) as a solution, and by applying differentiation to the kernel of expression (2-2-1),

[Math. 43]

$\phi$

15 also satisfies the partial differential equation described above. This equation is a five-dimensional pseudo wave equation configured by six variables (t, x<sub>1</sub>, y<sub>1</sub>, x<sub>2</sub>, y<sub>2</sub>, z).

[0095]

Next, this equation is solved by Fourier transform. First,

20 [Math. 44]

$\phi$

is subjected to multiplex Fourier transform with respect to t, x<sub>1</sub>, y<sub>1</sub>, x<sub>2</sub>, and y<sub>2</sub> as given by expression (2-2-11) below.

[0096]

25 [Math. 45]

$$\begin{aligned} & \tilde{\varphi}(k_{x_1}, k_{y_1}, k_{x_2}, k_{y_2}, z, \omega) \\ &= \int_{-\infty}^{\infty} e^{i\omega t} dt \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(k_{x_1}x_1 + k_{y_1}y_1)} e^{i(k_{x_2}x_2 + k_{y_2}y_2)} \varphi(x_1, y_1, x_2, y_2, z, t) dx_1 dy_1 dx_2 dy_2 \\ & \cdot \cdot \cdot (2 - 2 - 1 \ 1) \end{aligned}$$

[0097]

When the differential with respect to z is expressed as  $D_z$ , expression (2-2-12) below is obtained from expressions (2-2-10) and (2-2-11).

[0098]

[Math. 46]

$$\begin{aligned} & \{(D_z^2 - k_{x_1}^2 - k_{y_1}^2 - k_{x_2}^2 - k_{y_2}^2)^2 + 4k^2 D_z^2 - 4(k_{x_1}^2 + k_{y_1}^2)(k_{x_2}^2 + k_{y_2}^2)\} \tilde{\varphi} = 0 \\ & \cdot \cdot \cdot (2 - 2 - 1 \ 2) \end{aligned}$$

[0099]

Here, the relationship of  $\omega = ck$  is used. Four basic solutions to this equation are expressed as given by expression (2-2-13) below.

[0100]

[Math. 47]

$$\begin{aligned} E_1 &= e^{i\left\{\sqrt{k^2 - k_{x_1}^2 - k_{y_1}^2} + \sqrt{k^2 - k_{x_2}^2 - k_{y_2}^2}\right\}z} \\ E_2 &= e^{-i\left\{\sqrt{k^2 - k_{x_1}^2 - k_{y_1}^2} + \sqrt{k^2 - k_{x_2}^2 - k_{y_2}^2}\right\}z} \\ E_3 &= e^{i\left\{\sqrt{k^2 - k_{x_1}^2 - k_{y_1}^2} - \sqrt{k^2 - k_{x_2}^2 - k_{y_2}^2}\right\}z} \\ E_4 &= e^{-i\left\{\sqrt{k^2 - k_{x_1}^2 - k_{y_1}^2} - \sqrt{k^2 - k_{x_2}^2 - k_{y_2}^2}\right\}z} \end{aligned}$$

• • • (2 - 2 - 1 3)

[0101]

Considering the facts that the time factor is  $e^{-i\omega t}$ , the phase is added using the path of radiated radio waves, and radio waves reflected off the object are bounced off toward a measurement plane,  $E_1$  is the unique meaningful solution. Accordingly, expression (2-2-14) below is obtained.

[0102]

[Math. 48]

$$\tilde{\varphi}(k_{x_1}, k_{y_1}, k_{x_2}, k_{y_2}, z, k) = a(k_{x_1}, k_{y_1}, k_{x_2}, k_{y_2}, k) e^{i \left\{ \sqrt{k^2 - k_{x_1}^2 - k_{y_1}^2} + \sqrt{k^2 - k_{x_2}^2 - k_{y_2}^2} \right\} z}$$

• • • (2 - 2 - 1 4)

[0103]

By substituting  $z = 0$  in expression (2-2-14),  $a(k_{x_1}, k_{y_1}, k_{x_2}, k_{y_2}, k)$  is obtained as given by expression (2-2-15) below.

[0104]

[Math. 49]

$$a(k_{x_1}, k_{y_1}, k_{x_2}, k_{y_2}, k) = \tilde{\varphi}(k_{x_1}, k_{y_1}, k_{x_2}, k_{y_2}, 0, k)$$

• • • (2 - 2 - 1 5)

[0105]

From the above,

[Math. 50]

$\varphi$

is obtained as given by expression (2-2-16) below.

[0106]

[Math. 51]

$$\begin{aligned} \varphi(x_1, y_1, x_2, y_2, z, k) \\ = \frac{1}{(2\pi)^4} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_{x_1}x_1 + k_{y_1}y_1)} e^{-i(k_{x_2}x_2 + k_{y_2}y_2)} e^{i \left\{ \sqrt{k^2 - k_{x_1}^2 - k_{y_1}^2} + \sqrt{k^2 - k_{x_2}^2 - k_{y_2}^2} \right\} z} \\ \cdot a(k_{x_1}, k_{y_1}, k_{x_2}, k_{y_2}, k) dk_{x_1} dk_{y_1} dk_{x_2} dk_{y_2} \end{aligned}$$

• • • (2 - 2 - 1 6)

[0107]

Next, under the condition where  $k$  and  $z$  are fixed, by applying a limit operation ( $y_1 \rightarrow y$  and  $y_2 \rightarrow y$ ) to expression (2-2-16), we obtain the expression (2-2-17).

[0108]

[Math. 52]

$$\begin{aligned}
\Phi(x, y, z, k) &= \varphi(x, y, x, y, z, k) \\
&= \lim_{\substack{x_2 \rightarrow x_1 = x \\ y_2 \rightarrow y_1 = y}} [\varphi(x_1, y_1, x_2, y_2, z, k)] \\
&= \lim_{\substack{x_2 \rightarrow x_1 = x \\ y_2 \rightarrow y_1 = y}} \left[ \frac{1}{(2\pi)^4} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_{x_1}x_1 + k_{y_1}y_1)} e^{-i(k_{x_2}x_2 + k_{y_2}y_2)} e^{iz(\sqrt{k^2 - k_{x_1}^2 - k_{y_1}^2} + \sqrt{k^2 - k_{x_2}^2 - k_{y_2}^2})} \right. \\
&\quad \cdot a(k_{x_1}, k_{y_1}, k_{x_2}, k_{y_2}, k) dk_{x_1} dk_{y_1} dk_{x_2} dk_{y_2} \left. \right] \\
&\quad \cdot \cdot \cdot (2 - 2 - 1 \ 7)
\end{aligned}$$

[0109]

Next, expression (2-2-17) is integrated with respect to k to obtain expression (2-2-18) below as an imaging function.

5 [0110]

[Math. 53]

$$\begin{aligned}
\rho(x, y, z) &= \int_0^{\infty} \Phi(x, y, z, k) dk = \int_0^{\infty} \varphi(x, y, x, y, z, k) dk \\
&= \lim_{\substack{x_2 \rightarrow x_1 = x \\ y_2 \rightarrow y_1 = y}} \left[ \int_0^{\infty} \varphi(x_1, y_1, x_2, y_2, z, k) dk \right] \\
&= \lim_{\substack{x_2 \rightarrow x_1 = x \\ y_2 \rightarrow y_1 = y}} \left[ \frac{1}{(2\pi)^4} \int_0^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_{x_1}x_1 + k_{y_1}y_1)} e^{-i(k_{x_2}x_2 + k_{y_2}y_2)} e^{iz(\sqrt{k^2 - k_{x_1}^2 - k_{y_1}^2} + \sqrt{k^2 - k_{x_2}^2 - k_{y_2}^2})} \right. \\
&\quad \cdot a(k_{x_1}, k_{y_1}, k_{x_2}, k_{y_2}, k) dk_{x_1} dk_{y_1} dk_{x_2} dk_{y_2} dk \left. \right] \\
&\quad \cdot \cdot \cdot (2 - 2 - 1 \ 8)
\end{aligned}$$

[0111]

10 In expression (2-2-18), the integration with respect to  $k_{x1}$ ,  $k_{y1}$ ,  $k_{x2}$ , and  $k_{y2}$  is in the form of Fourier transform and suitable for processing performed by a calculator. On the other hand, the term  $\exp(iz\dots)$  of the integrand is not in the form of Fourier transform. Thus, ordinary integration is performed with respect to k while specifying, for example, the value of z. Alternatively, in order to  
15 reduce the calculation time, expression (2-2-18) may be modified so as to express the whole using only a Fourier transform.

[0112]

For example, the coefficient of iz in the term  $\exp(iz\dots)$  of expression (2-2-17) is expressed by expression (2-2-19) below using

a new variable u.

[0113]

[Math. 54]

$$u = \sqrt{k^2 - k_{x_1}^2 - k_{y_1}^2} + \sqrt{k^2 - k_{x_2}^2 - k_{y_2}^2}$$

• • • (2 - 2 - 1 9)

5 [0114]

By rationalizing the right-hand side of expression (2-2-19), expression (2-2-20) below is obtained.

[0115]

[Math. 55]

$$\frac{k_{x_2}^2 + k_{y_2}^2 - k_{x_1}^2 - k_{y_1}^2}{u} = \sqrt{k^2 - k_{x_1}^2 - k_{y_1}^2} - \sqrt{k^2 - k_{x_2}^2 - k_{y_2}^2}$$

10 • • • (2 - 2 - 2 0)

[0116]

By solving each square root from the two expressions including expressions (2-2-19) and (2-2-20), expression (2-2-21) below is obtained.

15 [0117]

[Math. 56]

$$2\sqrt{k^2 - k_{x_1}^2 - k_{y_1}^2} = u + \frac{k_{x_2}^2 + k_{y_2}^2 - k_{x_1}^2 - k_{y_1}^2}{u}$$

$$2\sqrt{k^2 - k_{x_2}^2 - k_{y_2}^2} = u - \frac{k_{x_2}^2 + k_{y_2}^2 - k_{x_1}^2 - k_{y_1}^2}{u}$$

• • • (2 - 2 - 2 1)

[0118]

Accordingly, k is expressed as given by expression (2-2-22)

20 below.

[0119]

[Math. 57]

$$k = \frac{1}{2} \sqrt{u^2 + \frac{(k_{x_2}^2 + k_{y_2}^2 - k_{x_1}^2 - k_{y_1}^2)^2}{u^2} + 2(k_{x_2}^2 + k_{y_2}^2 + k_{x_1}^2 + k_{y_1}^2)}$$

• • • (2 - 2 - 2 2)

[0120]

Next, expression (2-2-23) below is obtained by differentiation of both sides of expression (2-2-19) with respect to k and u.

[0121]

5 [Math. 58]

$$du = kdk \left( \frac{1}{\sqrt{k^2 - k_{x_1}^2 - k_{y_1}^2}} + \frac{1}{\sqrt{k^2 - k_{x_2}^2 - k_{y_2}^2}} \right) \\ \cdot \cdot \cdot (2 - 2 - 2 \ 3)$$

[0122]

By solving dk from expression (2-2-23), expression (2-2-24) below is obtained.

10 [0123]

[Math. 59]

$$dk = \frac{1}{ku} \sqrt{k^2 - k_{x_1}^2 - k_{y_1}^2} \sqrt{k^2 - k_{x_2}^2 - k_{y_2}^2} du \\ \cdot \cdot \cdot (2 - 2 - 2 \ 4)$$

[0124]

15 In summary, expression (2-2-18) is converted as given by expression (2-2-25) below.

[0125]

[Math. 60]

$$\rho(x, y, z) = \int_0^\infty \Phi(x, y, z, k) dk = \int_0^\infty \varphi(x, y, x, y, z, k) dk \\ = \lim_{\substack{x_2 \rightarrow x_1 = x \\ y_2 \rightarrow y_1 = y}} \left[ \int_0^\infty \varphi(x_1, y_1, x_2, y_2, z, k) dk \right] \\ = \lim_{\substack{x_2 \rightarrow x_1 = x \\ y_2 \rightarrow y_1 = y}} \left[ \frac{1}{(2\pi)^4} \int_0^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty e^{-i(k_{x_1} x_1 + k_{y_1} y_1)} e^{-i(k_{x_2} x_2 + k_{y_2} y_2)} e^{izu} \right. \\ \left. \cdot a(k_{x_1}, k_{y_1}, k_{x_2}, k_{y_2}, k) dk_{x_1} dk_{y_1} dk_{x_2} dk_{y_2} dk \right] \\ \cdot \cdot \cdot (2 - 2 - 2 \ 5)$$

[0126]

20 <2-3. Semi-two-dimensional Array>

FIG. 3 is a conceptual diagram showing an example of

coordinates relating to a semi-two-dimensional array antenna. In this example, the semi-two-dimensional array antenna is configured by two linear array antennas including single-row transmitting array antenna TA and single-row receiving array antenna RA. Such a

[0127]

Transmitting array antenna TA includes n transmitting antenna elements T. Receiving array antenna RA includes n receiving antenna elements R. The x coordinate of transmitting array antenna TA is expressed as  $x_1$ , the x coordinate of receiving array antenna RA is expressed as  $x_2$ , and the distance in the x-axis direction between transmitting array antenna TA and receiving array antenna RA is expressed as d. This configuration is capable of

[0128]

This section describes the theory for imaging an object from data obtained by a semi-two-dimensional array antenna as illustrated in FIG. 3. First, expression (2-2-10) relating to a two-dimensional array is used as the starting point for examination. Expression (2-3-1) below is the same operator as expression (2-2-10).

[0129]

[Math. 61]

$$\{\Delta_5^2 - \frac{4}{c^2} \partial_t^2 \partial_z^2 - 4(\partial_{x_1}^2 + \partial_{y_1}^2)(\partial_{x_2}^2 + \partial_{y_2}^2)\} \phi = 0$$

$$\Delta_5 = \partial_{x_1}^2 + \partial_{y_1}^2 + \partial_{x_2}^2 + \partial_{y_2}^2 + \partial_z^2$$

$$\cdot \cdot \cdot (2 - 3 - 1)$$

[0130]

Also,

[Math. 62]

$\phi$

with respect to t,  $x_1$ ,  $y_1$ , and  $y_2$  is expressed as given by expression

(2-3-2) below.

[0131]

[Math. 63]

$$\begin{aligned} & \tilde{\varphi}(k_{x_1}, k_{y_1}, x_2, k_{y_2}, z, k) \\ &= \int_{-\infty}^{\infty} e^{ickt} dt \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(k_{x_1}x_1 + k_{y_1}y_1 + k_{y_2}y_2)} \varphi(x_1, y_1, x_2, y_2, z, t) dx_1 dy_1 dy_2 \\ & \cdot \cdot \cdot (2-3-2) \end{aligned}$$

5 [0132]

In the following description, variable  $x_2$  is expressed as  $u$ . Expression (2-3-3) below is obtained by Fourier transform of both sides of expression (2-3-1) with respect to  $t$ ,  $x_1$ ,  $y_1$ , and  $y_2$ .

[0133]

10 [Math. 64]

$$\begin{aligned} & \left\{ \left( \partial_u^2 + \partial_z^2 - k_{x_1}^2 - k_{y_1}^2 - k_{y_2}^2 \right)^2 + 4k^2 \partial_z^2 + 4(k_{x_1}^2 + k_{y_1}^2)(\partial_u^2 - k_{y_2}^2) \right\} \tilde{\varphi} = 0 \\ & \cdot \cdot \cdot (2-3-3) \end{aligned}$$

[0134]

A solution to expression (2-3-3) described above, which is the two-dimensional partial differential equation with respect to  $u$  and  $z$ ,

15 is assumed as given by expression (2-3-4) below.

[0135]

[Math. 65]

$$\begin{aligned} & \tilde{\varphi} \propto e^{s_3 u} e^{s_4 z} \\ & \cdot \cdot \cdot (2-3-4) \end{aligned}$$

[0136]

20 Here,  $s_3$  and  $s_4$  are functions with respect to  $k_{x_1}$ ,  $k_{y_1}$ ,  $k_{y_2}$ , and  $k$  as given by expression (2-3-5) below. Stated differently,  $s_3$  and  $s_4$  are constants defined by  $k_{x_1}$ ,  $k_{y_1}$ ,  $k_{y_2}$ , and  $k$ .

[0137]

[Math. 66]

$$\begin{aligned} & s_3 = s_3(k_{x_1}, k_{y_1}, k_{y_2}, k) \\ & s_4 = s_4(k_{x_1}, k_{y_1}, k_{y_2}, k) \end{aligned}$$

25  $\cdot \cdot \cdot (2-3-5)$

[0138]

By substituting expression (2-3-4) in expression (2-3-3), expression (2-3-6) below is obtained.

[0139]

5 [Math. 67]

$$\left(s_3^2 + s_4^2 - k_{x_1}^2 - k_{y_1}^2 - k_{y_2}^2\right)^2 + 4k^2 s_4^2 + 4(k_{x_1}^2 + k_{y_1}^2)(s_3^2 - k_{y_2}^2) = 0$$

• • • (2-3-6)

[0140]

10 However,  $s_3$  and  $s_4$  cannot be determined from only this algebraic equation. Next, expression (2-3-4) is changed into expression (2-3-7) below.

[0141]

[Math. 68]

$$\tilde{\varphi}(k_{x_1}, k_{y_1}, u, k_{y_2}, z, k) = b(k_{x_1}, k_{y_1}, k_{y_2}, k) e^{s_3 u} e^{s_4 z}$$

• • • (2-3-7)

[0142]

15 By inverse Fourier transform of expression (2-3-7) with respect to  $k_{x_1}$ ,  $k_{y_1}$ , and  $k_{y_2}$  and application of the result to  $u \rightarrow x_2$ , expression (2-3-8) below is obtained.

[0143]

[Math. 69]

$$\varphi(x_1, y_1, x_2, y_2, z, k)$$
$$= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_{x_1} x_1 + k_{y_1} y_1 + k_{y_2} y_2)} b(k_{x_1}, k_{y_1}, k_{y_2}, k) e^{s_3 x_2} e^{s_4 z} dk_{x_1} dk_{y_1} dk_{y_2}$$

20 • • • (2-3-8)

[0144]

By applying  $x_2 = x_1 = x$  to expression (2-3-8), expression (2-3-9) below is obtained.

[0145]

25 [Math. 70]

$$\begin{aligned}
& \varphi(x, y_1, x, y_2, z, k) \\
&= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_{x_1}x + k_{y_1}y_1 + k_{y_2}y_2)} b(k_{x_1}, k_{y_1}, k_{y_2}, k) e^{s_3x} e^{s_4z} dk_{x_1} dk_{y_1} dk_{y_2} \\
&= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i\{(k_{x_1} + is_3)x + k_{y_1}y_1 + k_{y_2}y_2\}} b(k_{x_1}, k_{y_1}, k_{y_2}, k) e^{s_4z} dk_{x_1} dk_{y_1} dk_{y_2} \\
&= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x + k_{y_1}y_1 + k_{y_2}y_2)} b(k_{x_1}, k_{y_1}, k_{y_2}, k) e^{s_4z} \left( \frac{dk_{x_1}}{dk_x} \right) dk_x dk_{y_1} dk_{y_2} \\
&\cdot \cdot \cdot (2 - 3 - 9)
\end{aligned}$$

[0146]

Here,  $k_x$  is expressed as given by expression (2-3-10) below.

[0147]

5 [Math. 71]

$$\begin{aligned}
& k_x = k_{x_1} + is_3 \\
& \cdot \cdot \cdot (2 - 3 - 1 \ 0)
\end{aligned}$$

[0148]

Expression (2-3-9) described above is supposed to agree with expression (2-1-16) because it agrees with the solution to the scattering field equation for the one-dimensional array. Expression (2-3-11) below is the same as expression (2-1-16).

[0149]

[Math. 72]

$$\begin{aligned}
& \varphi(x, y_1, y_2, z, k) \\
&= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x + k_{y_1}y_1 + k_{y_2}y_2)} a(k_x, k_{y_1}, k_{y_2}, k) \\
&\quad \cdot e^{i\{\sqrt{(\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2})^2 - k_x^2}\}z} dk_x dk_{y_1} dk_{y_2} \\
&\cdot \cdot \cdot (2 - 3 - 1 \ 1)
\end{aligned}$$

15 [0150]

By comparing expressions (2-3-9) and (2-3-11), expression (2-3-12) below is obtained.

[0151]

[Math. 73]

$$b(k_{x_1}, k_{y_1}, k_{y_2}, k) \left( \frac{dk_{x_1}}{d(k_{x_1} + is_3)} \right) = a(k_{x_1} + is_3, k_{y_1}, k_{y_2}, k)$$

$$s_4 = i \sqrt{(\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2})^2 - (k_{x_1} + is_3)^2}$$

• • • (2 - 3 - 1 2)

[0152]

The second equation given by expression (2-3-12) is raised to the second power to obtain expression (2-3-13) below.

5 [0153]

[Math. 74]

$$s_4^2 + s_3^2 = 2ik_{x_1}s_3 + k_{x_1}^2 - \left( \sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2} \right)^2$$

• • • (2 - 3 - 1 3)

[0154]

10 By substituting expression (2-3-13) in expression (2-3-6), expression (2-3-14) below is obtained.

[0155]

[Math. 75]

$$\left\{ 2ik_{x_1}s_3 - k_{y_1}^2 - k_{y_2}^2 - \left( \sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2} \right)^2 \right\}^2$$

$$+ 4k^2 \left\{ - \left( \sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2} \right)^2 + (k_{x_1} + is_3)^2 \right\}$$

$$+ 4(k_{x_1}^2 + k_{y_1}^2)(s_3^2 - k_{y_2}^2) = 0$$

• • • (2 - 3 - 1 4)

[0156]

15 Expression (2-3-14) is summarized to obtain expression (2-3-15) below.

[0157]

[Math. 76]

$$(k^2 - k_{y_1}^2)s_3^2 + 2 \left( ik_{x_1} \sqrt{k^2 - k_{y_1}^2} \sqrt{k^2 - k_{y_2}^2} \right) s_3 - k_{x_1}^2 (k^2 - k_{y_2}^2) = 0$$

• • • (2 - 3 - 1 5)

20 [0158]

Since the solution to this equation is a multiple root, the

solution expressed as given by expression (2-3-16) is uniquely obtained.

[0159]

[Math. 77]

$$s_3 = \frac{-i k_{x_1} \sqrt{k^2 - k_{y_2}^2}}{\sqrt{k^2 - k_{y_1}^2}}$$

5     • • • ( 2 – 3 – 1 6 )

[0160]

In accordance with expressions (2-3-12) and (2-3-16) obtained through the above-described process,  $s_3$  and  $s_4$  are obtained analytically. Then, the scattering field function is obtained from expression (2-3-8) as expressed by expression (2-3-17) below.

[0161]

[Math. 78]

$$\begin{aligned} & \varphi(x_1, y_1, x_2, y_2, z, k) \\ &= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_{x_1} x_1 + k_{y_1} y_1 + k_{y_2} y_2)} b(k_{x_1}, k_{y_1}, k_{y_2}, k) e^{s_3 x_2} e^{s_4 z} dk_{x_1} dk_{y_1} dk_{y_2} \\ &= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_{x_1} x_1 + k_{y_1} y_1 + k_{y_2} y_2)} a(k_{x_1} + i s_3, k_{y_1}, k_{y_2}, k) \\ & \quad \cdot \left( \frac{d(k_{x_1} + i s_3)}{dk_{x_1}} \right) e^{s_3 x_2} e^{s_4 z} dk_{x_1} dk_{y_1} dk_{y_2} \end{aligned}$$

$$s_3 = \frac{-i k_{x_1} \sqrt{k^2 - k_{y_2}^2}}{\sqrt{k^2 - k_{y_1}^2}}$$

$$s_4 = i \sqrt{(\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2})^2 - (k_{x_1} + i s_3)^2}$$

• • • ( 2 – 3 – 1 7 )

[0162]

15     Next, connecting measurement data  $\Phi(x_1, y_1, \text{ and } y_2, k)$  with  $a(k_{x_1}, k_{y_1}, k_{y_2}, k)$  is examined. By defining  $k_x = k_{x_1} + i s_3$  and substituting  $z = 0$  and  $x_2 = x_1 + d$  in expression (2-3-17), an equation as given by expression (2-3-18) holds true. Here,  $\Phi(x_1, y_1, \text{ and } y_2, k)$  represents measurement data on transmission point  $(x_1, y_1, 0)$ ,

reception point  $(x_1+d, y_2, 0)$ , and wavenumber  $k$ .

[0163]

[Math. 79]

$$\begin{aligned}
 & \Phi(x_1, y_1, y_2, k) \\
 &= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_{x_1}x_1 + k_{y_1}y_1 + k_{y_2}y_2)} a(k_{x_1} + is_3, k_{y_1}, k_{y_2}, k) \\
 & \quad \cdot \left( \frac{d(k_{x_1} + is_3)}{dk_{x_1}} \right) e^{s_3(x_1+d)} dk_{x_1} dk_{y_1} dk_{y_2} \\
 &= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i\{(k_{x_1} + is_3)x_1 + k_{y_1}y_1 + k_{y_2}y_2\}} a(k_{x_1} + is_3, k_{y_1}, k_{y_2}, k) \\
 & \quad \cdot \left( \frac{d(k_{x_1} + is_3)}{dk_{x_1}} \right) e^{s_3 d} dk_{x_1} dk_{y_1} dk_{y_2} \\
 &= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x_1 + k_{y_1} y_1 + k_{y_2} y_2)} a(k_x, k_{y_1}, k_{y_2}, k) e^{s_3 d} dk_x dk_{y_1} dk_{y_2} \\
 & \quad \cdot \cdot \cdot (2-3-18)
 \end{aligned}$$

5 [0164]

Hereinafter,  $k_x$  and  $s_3$  defined by expression (2-3-19) below are used.

[0165]

[Math. 80]

$$\begin{aligned}
 & k_x = k_{x_1} + is_3 \\
 & s_3 = \frac{-i k_{x_1} \sqrt{k^2 - k_{y_2}^2}}{\sqrt{k^2 - k_{y_1}^2}} = \frac{-i k_x \sqrt{k^2 - k_{y_2}^2}}{\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2}}
 \end{aligned}$$

10  $\cdot \cdot \cdot (2-3-19)$

[0166]

Expression (2-3-20) below is obtained by Fourier transform of both sides of expression (2-3-18) with respect to  $x_1$ ,  $y_1$ , and  $y_2$ .

[0167]

15 [Math. 81]

$$\begin{aligned}
& \tilde{\Phi}(k'_x, k'_{y_1}, k'_{y_2}, k) \\
&= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(k'_x x_1 + k'_{y_1} y_1 + k'_{y_2} y_2)} \Phi(x_1, y_1, y_2, k) dx_1 dy_1 dy_2 \\
&= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(k'_x x_1 + k'_{y_1} y_1 + k'_{y_2} y_2)} \\
&\quad \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i\{(k_x x_1 + k_{y_1} y_1 + k_{y_2} y_2)\}} a(k_x, k_{y_1}, k_{y_2}, k) e^{s_3 d} dk_x dk_{y_1} dk_{y_2} dx_1 dy_1 dy_2 \\
&= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(k'_x - k_x) \delta(k'_{y_1} - k_{y_1}) \delta(k'_{y_2} - k_{y_2}) a(k_x, k_{y_1}, k_{y_2}, k) e^{s_3 d} dk_x dk_{y_1} dk_{y_2} \\
&= a(k'_x, k'_{y_1}, k'_{y_2}, k) e^{s_3 d} \\
&\quad \cdot \cdot \cdot (2 - 3 - 2 \ 0)
\end{aligned}$$

[0168]

Function  $a(k_x, k_{y_1}, k_{y_2}, k)$  is obtained from expression (2-3-20) as given by expression (2-3-21).

5 [0169]

[Math. 82]

$$\begin{aligned}
a(k_x, k_{y_1}, k_{y_2}, k) &= e^{-s_3 d} \tilde{\Phi}(k_x, k_{y_1}, k_{y_2}, k) \\
&= e^{\frac{ik_x d \sqrt{k^2 - k_{y_2}^2}}{\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2}}} \tilde{\Phi}(k_x, k_{y_1}, k_{y_2}, k)
\end{aligned}$$

$\cdot \cdot \cdot (2 - 3 - 2 \ 1)$

[0170]

10 Therefore, expression (2-3-17) that represents the scattering field function is obtained in a complete form as given by expression (2-3-22) below.

[0171]

[Math. 83]

$$\begin{aligned}
& \varphi(x_1, y_1, x_2, y_2, z, k) \\
&= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_{x_1} x_1 + k_{y_1} y_1 + k_{y_2} y_2)} a(k_{x_1} + is_3, k_{y_1}, k_{y_2}, k) \\
&\quad \cdot \left( \frac{d(k_{x_1} + is_3)}{dk_{x_1}} \right) e^{s_3 x_2} e^{s_4 z} dk_{x_1} dk_{y_1} dk_{y_2} \\
&= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_{x_1} x_1 + k_{y_1} y_1 + k_{y_2} y_2)} e^{s_3 x_2} e^{s_4 z} e^{\frac{id(k_{x_1} + is_3)\sqrt{k^2 - k_{y_2}^2}}{\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2}}} \\
&\quad \cdot \tilde{\Phi}(k_x, k_{y_1}, k_{y_2}, k) \left( \frac{d(k_{x_1} + is_3)}{dk_{x_1}} \right) dk_{x_1} dk_{y_1} dk_{y_2} \\
& s_3 = \frac{-i k_{x_1} \sqrt{k^2 - k_{y_2}^2}}{\sqrt{k^2 - k_{y_1}^2}} = \frac{-i k_x \sqrt{k^2 - k_{y_2}^2}}{\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2}} \\
& s_4 = i k_z = i \sqrt{(\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2})^2 - (k_{x_1} + is_3)^2} \\
& \cdot \cdot \cdot (2 - 3 - 2 \ 2)
\end{aligned}$$

[0172]

Then, the imaging function is obtained as given by expression (2-3-23) below.

5 [0173]

[Math. 84]

$$\begin{aligned}
& \rho(x, y, z) \\
&= \int_0^\infty \lim_{\substack{x_2 \rightarrow x_1 = x \\ y_2 \rightarrow y_1 = y}} \varphi(x_1, y_1, x_2, y_2, z, k) dk \\
&= \frac{1}{(2\pi)^3} \int_0^\infty dk \int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty e^{-i(k_x x + k_{y_1} y + k_{y_2} y)} e^{s_4 z} e^{\frac{i d k_x \sqrt{k^2 - k_{y_2}^2}}{\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2}}} \\
&\quad \cdot \tilde{\Phi}(k_x, k_{y_1}, k_{y_2}, k) dk_x dk_{y_1} dk_{y_2} \\
&= \frac{1}{(2\pi)^3} \int_0^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty e^{-i(k_x x + k_{y_1} y + k_{y_2} y)} e^{i k_z z} e^{\frac{i d k_x \sqrt{k^2 - k_{y_2}^2}}{\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2}}} \\
&\quad \cdot \tilde{\Phi}(k_x, k_{y_1}, k_{y_2}, k) \left( \frac{dk}{dk_z} \right) dk_x dk_{y_1} dk_{y_2} dk_z \\
& k_z = \sqrt{(\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2})^2 - k_x^2} \\
& k = \frac{1}{2} \sqrt{k_x^2 + k_z^2 + 2(k_{y_1}^2 + k_{y_2}^2) + \frac{(k_{y_1}^2 - k_{y_2}^2)^2}{k_x^2 + k_z^2}} \\
& \frac{dk}{dk_z} = \frac{k_z \sqrt{k^2 - k_{y_1}^2} \sqrt{k^2 - k_{y_2}^2}}{k(k_x^2 + k_z^2)} \\
& \cdot \cdot \cdot (2 - 3 - 2 \ 3)
\end{aligned}$$

[0174]

Moreover, as compared with the one-dimensional array, the semi-two-dimensional array is capable of including a larger number of transmitting elements and a larger number of receiving elements. Accordingly, it is possible to more efficiently acquire information.

[0175]

<3. Scattering Field Theory With Consideration of Object Movement>

In this chapter, a scattering field theory that takes the movement of the object into consideration will be described. In particular, a scattering field theory for imaging a flying object moving in the atmosphere is described.

[0176]

For example, first, based on the scattering field theory that

does not take the movement of the object into consideration, provisional imaging functions are derived at constant time intervals. The velocity of the object is derived based on the provisional imaging functions derived at constant time intervals. The imaging function is derived based on the scattering field theory that takes into consideration the movement of the object, specifically the velocity of the object.

[0177]

### <3-1. Velocity Analysis of Flying Object>

FIG. 4 is a conceptual diagram illustrating the relationship between a plurality of transmitters, a plurality of receivers, and a flying object. The x-axis direction and y-axis direction correspond to the horizontal direction, and the z-axis direction corresponds to the vertical direction. The measurement data obtained using a plurality of transmitters ( $T_0, \dots, T_N$ ) and a plurality of receivers ( $R_0, \dots, R_N$ ) is expressed as follows.

[Math. 85]

$$\varphi(\tau, x, y_1, y_2, t)$$

[0178]

Here,  $t$  represents the real time.  $\tau$  is a time unit corresponding to the number of data collections. More specifically, when the transmission-reception time of the wave is expressed as  $\Delta t$ ,  $t = \tau \Delta t$  holds true. Accordingly,  $\tau$  can represent time. In this section, distance  $d$  between the transmission and reception arrays is assumed to be 0 ( $d=0$ ). The transmitting array antenna and the receiving array antenna are arranged along the y-axis and do not move or scan, so the measurement data does not depend on  $x$ .

[0179]

Therefore, the measurement data is newly expressed as follows.

[Math. 86]

$$\varphi(\tau, y_1, y_2, t)$$

[0180]

By inserting the above  $\tau$  into the imaging function of expression (2-1-17), the imaging function of expression (3-1-1)

below is obtained.

[0181]

[Math. 87]

$$\rho(\tau, x, y, z) = \frac{1}{(2\pi)^3} \int_0^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty e^{-i(k_x x + k_{y_1} y + k_{y_2} y)} a(\tau, k_x, k_{y_1}, k_{y_2}, k) \cdot e^{ik_z z} \left( \frac{dk}{dk_z} \right) dk_x dk_{y_1} dk_{y_2} dk_z$$

$$a(\tau, k_x, k_{y_1}, k_{y_2}, k) = \tilde{\varphi}(\tau, k_{y_1}, k_{y_2}, k)$$

$$\frac{dk}{dk_z} = \frac{k_z \sqrt{k^2 - k_{y_1}^2} \sqrt{k^2 - k_{y_2}^2}}{k(k_x^2 + k_z^2)}$$

• • • (3 - 1 - 1)

5 [0182]

The imaging function of expression (3-1-1) above is an imaging function obtained by a scattering field theory that does not take the movement of the object into consideration, and depends on  $\tau$ .

10 [0183]

To calculate the velocity of the object from  $\rho(\tau, x, y, z)$ , a Lagrangian derivative is used. The Lagrangian derivative is defined by expression (3-1-2) below.

[0184]

15 [Math. 88]

$$\frac{D}{D\tau} = \frac{\partial}{\partial \tau} + \mathbf{V} \cdot \nabla = \frac{\partial}{\partial \tau} + v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z}$$

• • • (3 - 1 - 2)

[0185]

In expression (3-1-2),  $\mathbf{V}$  represents the velocity vector ( $v_x, v_y, v_z$ ) of the incompressible flow field in  $(x, y, z)$  space.  $\mathbf{V}$  can be considered a function of  $(x, y, z)$ . However, since only the flying object has velocity, when there is only one flying object,  $\mathbf{V}$  is assumed to be constant in space and assumed to depend on time  $\tau$ . When there are a plurality of flying objects,  $\mathbf{V}$  is not constant in space,

but is constant in the vicinity of each flying object. From this condition, the imaging function is subject to the condition given by expression (3-1-3) below.

[0186]

5 [Math. 89]

$$\frac{D}{D\tau} \rho(\tau, x, y, z) = 0$$

• • • (3-1-3)

[0187]

When  $V = (v_x, v_y, v_z)$  is constant in space, the velocity of the flying object is derived as  $(v_x, v_y, v_z)$  satisfying expression (3-1-3).

10 However, when  $\rho(\tau, x, y, z)$  is 0,  $(v_x, v_y, v_z)$  cannot be determined from expression (3-1-3). Therefore, the velocity of the flying object is determined based on expression (3-1-4).

[0188]

[Math. 90]

$$\begin{aligned} \underset{(v_x, v_y, v_z)}{Min} \left[ \iiint \left( \frac{\partial}{\partial \tau} + v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z} \right) \rho(\tau, x, y, z) \right]^2 dx dy dz \\ + \varepsilon (v_x^2 + v_y^2 + v_z^2) \end{aligned}$$

15 • • • (3-1-4)

[0189]

Here,  $\varepsilon$  is a positive number. Expression (3-1-4) is a quadratic equation with respect to  $(v_x, v_y, v_z)$ , and since the coefficients are positive, a minimum value necessarily exists.

20 Furthermore, term  $\varepsilon$  is provided to prevent  $(v_x, v_y, v_z)$  from reaching a minimum value at an arbitrary value when  $\rho(\tau, x, y, z)$  is 0. When  $\rho(\tau, x, y, z)$  is 0,  $(v_x, v_y, v_z)$  is derived as (0, 0, 0) due to the term  $\varepsilon$ . To avoid errors based on the  $\varepsilon$  term, it is preferable that the value of  $\varepsilon$  be small.

25 [0190]

From expression (3-1-4), in order to derive  $(v_x, v_y, v_z)$ , first, a Fourier transform is performed on  $\rho(\tau, x, y, z)$ , and expression (3-1-5) below is obtained.

[0191]

[Math. 91]

$$\rho(\tau, x, y, z) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x + k_y y - k_z z)} \rho_k(\tau, k_x, k_y, k_z) dk_x dk_y dk_z$$

. . . (3 - 1 - 5)

[0192]

By substituting expression (3-1-5) in expression (3-1-3),  
5 expression (3-1-6) below is obtained.

[0193]

[Math. 92]

$$\left( \frac{\partial}{\partial \tau} + v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z} \right) \rho(\tau, x, y, z)$$

$$= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x + k_y y - k_z z)} \left\{ \partial_\tau - ik_x v_x - ik_y v_y + ik_z v_z \right\}$$

$$\cdot \rho_k(\tau, k_x, k_y, k_z) dk_x dk_y dk_z$$

. . . (3 - 1 - 6)

[0194]

10 The function appearing on the right-hand side of expression (3-1-6) will be written as follows for convenience.

[0195]

[Math. 93]

$$A(\tau, x, y, z) = \frac{-i}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x + k_y y - k_z z)} k_x \rho_k(\tau, k_x, k_y, k_z) dk_x dk_y dk_z$$

$$B(\tau, x, y, z) = \frac{-i}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x + k_y y - k_z z)} k_y \rho_k(\tau, k_x, k_y, k_z) dk_x dk_y dk_z$$

$$C(\tau, x, y, z) = \frac{i}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x + k_y y - k_z z)} k_z \rho_k(\tau, k_x, k_y, k_z) dk_x dk_y dk_z$$

$$D(\tau, x, y, z) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x + k_y y - k_z z)} \partial_\tau \rho_k(\tau, k_x, k_y, k_z) dk_x dk_y dk_z$$

. . . (3 - 1 - 7)

15 [0196]

Using the symbols in expression (3-1-7), expression (3-1-6) can be expressed as given by expression (3-1-8) below.

[0197]

[Math. 94]

$$\begin{aligned} & \left( \frac{\partial}{\partial \tau} + v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z} \right) \rho(\tau, x, y, z) \\ &= D(\tau, x, y, z) + v_x A(\tau, x, y, z) + v_y B(\tau, x, y, z) + v_z C(\tau, x, y, z) \\ & \cdot \cdot \cdot (3 - 1 - 8) \end{aligned}$$

[0198]

5 By evaluating expression (3-1-8) as the expression inside the absolute value symbols of expression (3-1-4) in order to actually calculate expression (3-1-4), expression (3-1-9) below is obtained.

[0199]

[Math. 95]

$$\begin{aligned} & \iiint \left| \left( \frac{\partial}{\partial \tau} + v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z} \right) \rho(\tau, x, y, z) \right|^2 dx dy dz \\ &= \iiint \left| D(\tau, x, y, z) + v_x A(\tau, x, y, z) + v_y B(\tau, x, y, z) + v_z C(\tau, x, y, z) \right|^2 dx dy dz \\ &= v_x^2 \iiint A(\tau, x, y, z) \bar{A}(\tau, x, y, z) d\mathbf{r}^3 \\ & \quad + v_y^2 \iiint B(\tau, x, y, z) \bar{B}(\tau, x, y, z) d\mathbf{r}^3 \\ & \quad + v_z^2 \iiint C(\tau, x, y, z) \bar{C}(\tau, x, y, z) d\mathbf{r}^3 \\ & \quad + \iiint D(\tau, x, y, z) \bar{D}(\tau, x, y, z) d\mathbf{r}^3 \\ & \quad + v_x v_y \iiint \{ A(\tau, x, y, z) \bar{B}(\tau, x, y, z) + cc. \} d\mathbf{r}^3 \\ & \quad + v_y v_z \iiint \{ B(\tau, x, y, z) \bar{C}(\tau, x, y, z) + cc. \} d\mathbf{r}^3 \\ & \quad + v_z v_x \iiint \{ C(\tau, x, y, z) \bar{A}(\tau, x, y, z) + cc. \} d\mathbf{r}^3 \\ & \quad + v_x \iiint \{ A(\tau, x, y, z) \bar{D}(\tau, x, y, z) + cc. \} d\mathbf{r}^3 \\ & \quad + v_y \iiint \{ B(\tau, x, y, z) \bar{D}(\tau, x, y, z) + cc. \} d\mathbf{r}^3 \\ & \quad + v_z \iiint \{ C(\tau, x, y, z) \bar{D}(\tau, x, y, z) + cc. \} d\mathbf{r}^3 \\ & \cdot \cdot \cdot (3 - 1 - 9) \end{aligned}$$

10 [0200]

Here, cc. represents the complex conjugate of the immediately

preceding term. Variables with a bar above them indicate the complex conjugate of that variable. The integral calculation appearing on the right side of this expression (3-1-9) can be considerably simplified as given by expression (3-1-10) below.

5 [0201]

[Math. 96]

$$a = \iiint A(\tau, x, y, z) \bar{A}(\tau, x, y, z) = \frac{1}{(2\pi)^3} \iiint k_x^2 |\rho_k|^2 d\mathbf{k}^3$$

$$b = \iiint B(\tau, x, y, z) \bar{B}(\tau, x, y, z) = \frac{1}{(2\pi)^3} \iiint k_y^2 |\rho_k|^2 d\mathbf{k}^3$$

$$c = \iiint C(\tau, x, y, z) \bar{C}(\tau, x, y, z) = \frac{1}{(2\pi)^3} \iiint k_z^2 |\rho_k|^2 d\mathbf{k}^3$$

$$d = \iiint D(\tau, x, y, z) \bar{D}(\tau, x, y, z) = \frac{1}{(2\pi)^3} \iiint |\partial_\tau \rho_k|^2 d\mathbf{k}^3$$

$$e = \iiint A(\tau, x, y, z) \bar{B}(\tau, x, y, z) + cc. = \frac{1}{(2\pi)^3} \iiint k_x k_y |\rho_k|^2 d\mathbf{k}^3 + cc.$$

$$f = \iiint B(\tau, x, y, z) \bar{C}(\tau, x, y, z) + cc. = \frac{1}{(2\pi)^3} \iiint k_y k_z |\rho_k|^2 d\mathbf{k}^3 + cc.$$

$$g = \iiint C(\tau, x, y, z) \bar{A}(\tau, x, y, z) + cc. = \frac{1}{(2\pi)^3} \iiint k_z k_x |\rho_k|^2 d\mathbf{k}^3 + cc.$$

$$h = \iiint A(\tau, x, y, z) \bar{D}(\tau, x, y, z) + cc. = \frac{-i}{(2\pi)^3} \iiint k_x \rho_k \cdot \partial_\tau \bar{\rho}_k d\mathbf{k}^3 + cc.$$

$$m = \iiint B(\tau, x, y, z) \bar{D}(\tau, x, y, z) + cc. = \frac{-i}{(2\pi)^3} \iiint k_y \rho_k \cdot \partial_\tau \bar{\rho}_k d\mathbf{k}^3 + cc.$$

$$n = \iiint C(\tau, x, y, z) \bar{D}(\tau, x, y, z) + cc. = \frac{-i}{(2\pi)^3} \iiint k_z \rho_k \cdot \partial_\tau \bar{\rho}_k d\mathbf{k}^3 + cc.$$

• • • (3 - 1 - 1 0)

[0202]

10 Expression (3-1-9) is expressed, using expression (3-1-10), as given by expression (3-1-11) below.

[0203]

[Math. 97]

$$\iiint \left| \left( \frac{\partial}{\partial \tau} + v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z} \right) \rho(\tau, x, y, z) \right|^2 dx dy dz$$

$$= av_x^2 + bv_y^2 + cv_z^2 + ev_x v_y + fv_y v_z + gv_z v_x + hv_x + mv_y + nv_z + d$$

• • • (3 - 1 - 1 1)

[0204]

Expression (3-1-4) is expressed as a problem as given by expression (3-1-12) below.

5 [0205]

[Math. 98]

$$\underset{(v_x, v_y, v_z)}{Min} \cdot [\Psi(v_x, v_y, v_z)]$$

$$\Psi(v_x, v_y, v_z)$$

$$= av_x^2 + bv_y^2 + cv_z^2 + ev_x v_y + fv_y v_z + gv_z v_x + hv_x + mv_y + nv_z + d$$

$$+ \varepsilon(v_x^2 + v_y^2 + v_z^2)$$

• • • (3 - 1 - 1 2)

[0206]

10 Expression (3-1-13) below is derived from expression (3-1-12).

[0207]

[Math. 99]

$$\begin{pmatrix} \frac{\partial}{\partial v_x} \Psi(v_x, v_y, v_z) \\ \frac{\partial}{\partial v_y} \Psi(v_x, v_y, v_z) \\ \frac{\partial}{\partial v_z} \Psi(v_x, v_y, v_z) \end{pmatrix} = \begin{pmatrix} 2a+2\varepsilon & e & g \\ e & 2b+2\varepsilon & f \\ g & f & 2c+2\varepsilon \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} + \begin{pmatrix} h \\ m \\ n \end{pmatrix} = 0$$

• • • (3 - 1 - 1 3)

[0208]

15 Furthermore, expression (3-1-14) below is derived from expression (3-1-13).

[0209]

[Math. 100]

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = - \begin{pmatrix} 2a+2\varepsilon & e & g \\ e & 2b+2\varepsilon & f \\ g & f & 2c+2\varepsilon \end{pmatrix}^{-1} \begin{pmatrix} h \\ m \\ n \end{pmatrix}$$

• • • (3 - 1 - 1 4)

[0210]

The elements of the matrix can be easily calculated from expression (3-1-10). Accordingly, the velocity vector can be calculated.

[0211]

Here, the integration region of expression (3-1-10) will be explained. For example, by performing integration in a region near a single flying object, the velocity of that flying object is obtained.

For example, by performing integration in a region including a plurality of flying objects moving in formation, the overall average velocity of the plurality of flying objects is obtained. When no flying object is present, a, b, c, ..., m, and n approach infinitely close to 0. In such cases, as evident from expression (3-1-13),  $v_x$ ,  $v_y$ , and  $v_z$  are derived as 0 due to the introduced  $\varepsilon$ .

[0212]

### <3-2. Doppler Effect of Electromagnetic Waves>

In this section, the influence that the Doppler effect has on scattered waves, in particular the influence on the wavenumber of scattered waves, will be examined.

[0213]

The wave in scattering field theory may be a high-speed wave such as an electromagnetic wave. The Doppler effect of such high-speed waves also involves the relationship between time and space. The square of the length of the radial vector ( $x^0, x^1, x^2, x^3$ ) = (ct, x, y, z) in the four-dimensional space corresponding to Minkowski space is expressed as given by expression (3-2-1) below.

[0214]

[Math. 101]

$$(x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$$

$$\cdot \cdot \cdot (3 - 2 - 1)$$

[0215]

Note that a in  $x^a$  represents a subscript, not an exponent. A four-vector having components that are transformed in the same manner as the components of the radial vector having the characteristics described above can be defined as  $(A^0, A^1, A^2, A^3)$ . Based on the Lorentz transformation,  $(A^0, A^1, A^2, A^3)$  is transformed as given by expression (3-2-2) below.

[0216]

10 [Math. 102]

$$A^0 = \frac{A'^0 + \frac{V}{c} A'^1}{\sqrt{1 - \frac{V^2}{c^2}}}, \quad A^1 = \frac{A'^1 + \frac{V}{c} A'^0}{\sqrt{1 - \frac{V^2}{c^2}}}, \quad A^2 = A'^2, \quad A^3 = A'^3$$

$$\cdot \cdot \cdot (3 - 2 - 2)$$

[0217]

Here, it is assumed that the coordinate system  $(x')$  is moving at velocity  $V$  relative to the reference coordinate system  $(x)$ .

15 [0218]

Next, when a flying object approaches along the  $x$ -axis direction at velocity  $-V$ , it is assumed that an electromagnetic wave having a frequency of  $\omega_0$  and a wavenumber of  $k_0$  is radiated toward the flying object in stationary coordinate system  $K_0$ . In flying object coordinate system  $K_1$ , if the wavenumber of the electromagnetic wave is  $k_1$ , then by the above transformation, expression (3-2-3) below holds true.

[0219]

[Math. 103]

$$k^{(0)0} = \frac{k^{(1)0} - \frac{V}{c} k^{(1)1}}{\sqrt{1 - \frac{V^2}{c^2}}}$$

$$k^{(0)0} = k_0 = \omega_0 / c, \quad k^{(1)0} = k_1 = \omega_1 / c, \quad k^{(1)1} = k_1 \cos \alpha$$

$$\cdot \cdot \cdot (3 - 2 - 3)$$

[0220]

Here, a in  $k^{(a)b}$  indicates the observation coordinate system and corresponds to 0 of  $K_0$  or 1 of  $K_1$ , etc. Here, b in  $k^{(a)b}$  indicates the subscript of the four-vector and corresponds to 0 of  $A^0$  or 1 of  $A^1$ , etc.  $\alpha$  indicates the angle between the wave direction and the x-axis direction of the flying object coordinate system.  $\omega_1$  indicates the frequency of the electromagnetic wave in flying object coordinate system  $K_1$ . Furthermore, expression (3-2-4) below is obtained.

10 [0221]

[Math. 104]

$$k^{(1)0} = k^{(0)0} \frac{\sqrt{1 - \frac{V^2}{c^2}}}{1 - \frac{V}{c} \cos \alpha}$$

$$\cdot \cdot \cdot (3 - 2 - 4)$$

[0222]

15 When the reflected wave is observed in reference coordinate system  $K_2$ , expression (3-2-5) below holds true from the transformation rule of the four-vector.

[0223]

[Math. 105]

$$k^{(1)0} = \frac{k^{(2)0} + \frac{V}{c} k^{(2)1}}{\sqrt{1 - \frac{V^2}{c^2}}}$$

$$k^{(1)0} = k_1 = \omega_1 / c, \quad k^{(2)0} = k_2 = \omega_2 / c, \quad k^{(2)1} = k_2 \cos \beta'$$

$$\cdot \cdot \cdot (3 - 2 - 5)$$

20 [0224]

Here,  $\beta'$  is the angle between the reflected wave direction and

the x-axis direction of the flying object coordinate system. Since reference coordinate system  $K_2$  is moving at  $V$  in the positive x-axis direction relative to flying object coordinate system  $K_1$ , the vector inner product between the wave direction vector and the x-axis direction of the flying object coordinate system has a negative value. Furthermore, expression (3-2-6) below holds true.

[0225]

[Math. 106]

$$k_2 = k^{(1)0} \frac{\sqrt{1 - \frac{V^2}{c^2}}}{1 + \frac{V}{c} \cos \beta'} = k^{(0)0} \frac{\sqrt{1 - \frac{V^2}{c^2}}}{1 - \frac{V}{c} \cos \alpha} \cdot \frac{\sqrt{1 - \frac{V^2}{c^2}}}{1 + \frac{V}{c} \cos \beta'}$$

$$\approx k_0 \left( 1 + \frac{V}{c} \cos \alpha \right) \left( 1 + \frac{V}{c} \cos \beta \right)$$

• • • (3-2-6)

10 [0226]

Here,  $\beta$  and  $\beta'$  have the relationship  $\beta' = \beta + \pi$ . In expression (3-2-6), approximation is performed using the fact that  $V$  is sufficiently small compared to  $c$ .

[0227]

15 Stated differently, according to expression (3-2-6), wavenumber  $k_0$  of the electromagnetic wave radiated toward the flying object changes to wavenumber  $k_2$  of the electromagnetic wave reflected by the flying object.

[0228]

20 <3-3. Scattering Field Equation with Consideration of Doppler Effect>

In this section, the Doppler effect is reflected in the scattering field theory for a one-dimensional array.

[0229]

25 FIG. 5 is a conceptual diagram showing a coordinate system including a one-dimensional array antenna and a flying object. Here, a situation is examined in which a radio wave radiated from point  $P_1(x, y_1, z)$  is reflected at point  $P(\xi, \eta, \zeta)$  and received at point  $P_2(x, y_2, z)$

as shown in FIG. 5. In the case where point P is assumed to move in the entire region D, the signal received at point P<sub>2</sub> is represented by expression (3-3-1) below.

[0230]

5 [Math. 107]

$$\varphi(x, y_1, y_2, z) = \iint_D \frac{e^{ik\rho_1}}{\rho_1} \frac{e^{ik_s\rho_2}}{\rho_2} \varepsilon(\xi, \eta, \zeta) d\xi d\eta d\zeta$$

$$\rho_1 = \sqrt{(x - \xi)^2 + (y_1 - \eta)^2 + (z - \zeta)^2}$$

$$\rho_2 = \sqrt{(x - \xi)^2 + (y_2 - \eta)^2 + (z - \zeta)^2}$$

$$\cdot \cdot \cdot (3 - 3 - 1)$$

[0231]

Here,  $\varepsilon(\xi, \eta, \zeta)$  represents the function of the dielectric constant at point P( $\xi, \eta, \zeta$ ) and corresponds to the reflectance at point P( $\xi, \eta, \zeta$ ). Point P( $\xi, \eta, \zeta$ ) corresponds to the reflection point. Note that  $\varepsilon(\xi, \eta, \zeta)$  is unknown. It is assumed that the time factor is proportional to  $\exp(-i\omega t)$ . Wavenumber  $k_s$  of the radio wave after reflection may differ from wavenumber  $k$  of the radio wave before reflection. The kernel function in the integrand term of the above equation is represented as  $\phi$  in expression (3-3-2) below.

[0232]

[Math. 108]

$$\phi = \frac{e^{ik\rho_1}}{\rho_1} \frac{e^{ik_s\rho_2}}{\rho_2}$$

$$\cdot \cdot \cdot (3 - 3 - 2)$$

[0233]

20 When the velocity of the flying object is assumed to be  $(u, v, 0)$ , wavenumbers  $k = k_0$  and  $k_s = k_2$  with consideration of the Doppler effect are expressed as given by expression (3-3-3) below. Note that  $u$  and  $v$  are also expressed as  $v_x$  and  $v_y$ .

[0234]

25 [Math. 109]

$$k = \frac{\omega}{c}$$

$$\begin{aligned} k_s &= k \left( 1 + \frac{V}{c} \cos \alpha \right) \left( 1 + \frac{V}{c} \cos \beta \right) \\ &= k \left( 1 + \frac{u}{c} \frac{x - \xi}{\rho_1} + \frac{v}{c} \frac{y_1 - \eta}{\rho_1} \right) \left( 1 + \frac{u}{c} \frac{x - \xi}{\rho_2} + \frac{v}{c} \frac{y_2 - \eta}{\rho_2} \right) \\ &\approx k \left( 1 + \frac{u}{c} \frac{x - \xi}{\rho_1} + \frac{v}{c} \frac{y_1 - \eta}{\rho_1} + \frac{u}{c} \frac{x - \xi}{\rho_2} + \frac{v}{c} \frac{y_2 - \eta}{\rho_2} \right) \end{aligned}$$

• • • (3 - 3 - 3)

[0235]

Using expression (3-3-3),  $k_s \rho_2$  in the kernel function is expressed as given by expression (3-3-4) below. Here, it is assumed that the flying object is positioned sufficiently far from the array antenna, and the distance from the array antenna to the flying object is sufficiently large compared to the total length of the array. Accordingly, in the product of the extremely small  $u/c$  or  $v/c$  and  $\rho_2/\rho_1$ ,  $\rho_1$  and  $\rho_2$  can be regarded as the same.

10 [0236]

[Math. 110]

$$\begin{aligned} k_s \rho_2 &= k \left( \rho_2 + \frac{u}{c} (x - \xi) \frac{\rho_2}{\rho_1} + \frac{v}{c} (y_1 - \eta) \frac{\rho_2}{\rho_1} + \frac{u}{c} (x - \xi) + \frac{v}{c} (y_2 - \eta) \right) \\ &= k \left( \rho_2 + \frac{2u}{c} (x - \xi) + \frac{v}{c} (y_1 - \eta) + \frac{v}{c} (y_2 - \eta) \right) \end{aligned}$$

• • • (3 - 3 - 4)

[0237]

15 Accordingly, the kernel function considering the Doppler effect is expressed as given by expression (3-3-5) below.

[0238]

[Math. 111]

$$\phi = \frac{e^{ik\rho_1}}{\rho_1} \frac{e^{ik \left\{ \rho_2 + \frac{2u}{c} (x - \xi) + \frac{v}{c} (y_1 - \eta) + \frac{v}{c} (y_2 - \eta) \right\}}}{\rho_2}$$

• • • (3 - 3 - 5)

[0239]

Furthermore, for convenience, by introducing constants  $\alpha = 2u/c$  and  $\beta = v/c$ ,  $\phi$  is expressed as given by expression (3-3-6) below.

5 [0240]

[Math. 112]

$$\phi = \frac{e^{ik\rho_1}}{\rho_1} \frac{e^{ik\{\rho_2 + \alpha(x-\xi) + \beta(y_1-\eta) + \beta(y_2-\eta)\}}}{\rho_2}$$

• • • ( 3 - 3 - 6 )

[0241]

10 A partial differential equation that has expression (3-3-6) as an asymptotical solution at short wavelengths is examined. The result of differentiation of each order of  $\phi$  is represented by expression (3-3-7). Note that high-order terms with respect to  $1/\rho$  may be ignored.

[0242]

15 [Math. 113]

$$\partial_x \phi = ik(x - \xi) \left( \frac{1}{\rho_1} + \frac{1}{\rho_2} \right) \phi + ik\alpha \phi + o(\rho^{-3})$$

$$\partial_{y_1} \phi = ik \frac{y_1 - \eta}{\rho_1} \phi + ik\beta \phi + o(\rho^{-3})$$

$$\partial_{y_2} \phi = ik \frac{y_2 - \eta}{\rho_2} \phi + ik\beta \phi + o(\rho^{-3})$$

$$\partial_z \phi = ik(z - \zeta) \left( \frac{1}{\rho_1} + \frac{1}{\rho_2} \right) \phi + o(\rho^{-3})$$

$$(\partial_x - ik\alpha)(\partial_x - ik\alpha)\phi = (ik)^2 (x - \xi)^2 \left( \frac{1}{\rho_1} + \frac{1}{\rho_2} \right)^2 \phi + o(\rho^{-3})$$

$$\partial_z \partial_z \phi = (ik)^2 (z - \zeta)^2 \left( \frac{1}{\rho_1} + \frac{1}{\rho_2} \right)^2 \phi + o(\rho^{-3})$$

$$(\partial_{y_1} - ik\beta)(\partial_{y_1} - ik\beta)\phi = (ik)^2 \left( \frac{y_1 - \eta}{\rho_1} \right)^2 \phi + o(\rho^{-3})$$

$$(\partial_{y_2} - ik\beta)(\partial_{y_2} - ik\beta)\phi = (ik)^2 \left( \frac{y_2 - \eta}{\rho_2} \right)^2 \phi + o(\rho^{-3})$$

• • • (3 - 3 - 7)

[0243]

Subsequently, the differential operator of expression (3-3-8) below is introduced.

5 [0244]

[Math. 114]

$$D_x = \partial_x - ik\alpha$$

$$D_{y_1} = \partial_{y_1} - ik\beta$$

$$D_{y_2} = \partial_{y_2} - ik\beta$$

$$D_z = \partial_z$$

• • • (3 - 3 - 8)

[0245]

By performing calculations similar to those in Chapter 2, Section 1 using the new operator of expression (3-3-8), expression (3-3-9) below is obtained.

10

[0246]

[Math. 115]

$$\left[\frac{1}{4}\{\square_4 - 2(ik)^2\}^2 - D_{y_1}^2 D_{y_2}^2 + (ik)^2(D_{y_1}^2 + D_{y_2}^2) - (ik)^4\right]\phi = 0$$

$$\square_4 = D_x^2 + D_{y_1}^2 + D_{y_2}^2 + D_z^2$$

• • • (3 - 3 - 9)

[0247]

5 Expression (3-3-9) above is a scattering field equation that takes the Doppler effect into consideration, and is suitable for detecting flying objects moving at high speed. Furthermore, variable substitution is performed as given by expression (3-3-10) below.

10 [0248]

[Math. 116]

$$-ik \rightarrow \frac{1}{c} \partial_t$$

• • • (3 - 3 - 1 0)

[0249]

15 Through the process described above, an equation represented by expression (3-3-11) below is ultimately obtained.

[0250]

[Math. 117]

$$\left\{\square_4^2 - \frac{4}{c^2}(\partial_t^2 D_x^2 + \partial_t^2 D_z^2) - 4D_{y_1}^2 D_{y_2}^2\right\}\phi = 0$$

$$\square_4 = D_x^2 + D_{y_1}^2 + D_{y_2}^2 + D_z^2$$

• • • (3 - 3 - 1 1)

[0251]

20 Expression (3-3-11) described above is a partial differential equation that has  $\phi$  in expression (3-3-2) as a solution. By applying differentiation to the kernel of expression (3-3-1),

[Math. 118]

$\phi$

25 of expression (3-3-1) also satisfies the partial differential equation described above. This equation is a four-dimensional pseudo wave

equation configured by five variables (t, x, y<sub>1</sub>, y<sub>2</sub>, z).

[0252]

Next, this equation is solved by Fourier transform. First,

[Math. 119]

5  $\varphi$

is subjected to multiplex Fourier transform with respect to t, x, y<sub>1</sub>, y<sub>2</sub> as given by expression (3-3-12) below.

[0253]

[Math. 120]

$$\tilde{\varphi}(k_x, k_{y_1}, k_{y_2}, z, \omega) = \int_{-\infty}^{\infty} e^{i\omega t} dt \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(k_x x + k_{y_1} y_1 + k_{y_2} y_2)} \varphi(x, y_1, y_2, z, t) dx dy_1 dy_2$$

10  $\cdot \cdot \cdot (3 - 3 - 1 \ 2)$

[0254]

Expression (3-3-13) below is obtained from expressions (3-3-11) and (3-3-12).

[0255]

15 [Math. 121]

$$\left[ \left\{ D_z^2 - (k_x + k\alpha)^2 - (k_{y_1} + k\beta)^2 - (k_{y_2} + k\beta)^2 \right\}^2 + 4k^2 \left\{ D_z^2 - (k_x + k\alpha)^2 \right\} - 4(k_{y_1} + k\beta)^2 (k_{y_2} + k\beta)^2 \right] \tilde{\varphi} = 0$$

$\cdot \cdot \cdot (3 - 3 - 1 \ 3)$

[0256]

Here, the relationship of  $\omega = ck$  is used. Four basic solutions to this equation are expressed as given by expression (3-3-14)

20 below.

[0257]

[Math. 122]

$$E_1 = e^{iz\sqrt{\left\{\sqrt{k^2-(k_{y1}+k\beta)^2}+\sqrt{k^2-(k_{y2}+k\beta)^2}\right\}^2-(k_x+k\alpha)^2}}$$

$$E_2 = e^{-iz\sqrt{\left\{\sqrt{k^2-(k_{y1}+k\beta)^2}+\sqrt{k^2-(k_{y2}+k\beta)^2}\right\}^2-(k_x+k\alpha)^2}}$$

$$E_3 = e^{iz\sqrt{\left\{\sqrt{k^2-(k_{y1}+k\beta)^2}-\sqrt{k^2-(k_{y2}+k\beta)^2}\right\}^2-(k_x+k\alpha)^2}}$$

$$E_4 = e^{-iz\sqrt{\left\{\sqrt{k^2-(k_{y1}+k\beta)^2}-\sqrt{k^2-(k_{y2}+k\beta)^2}\right\}^2-(k_x+k\alpha)^2}}$$

• • • ( 3 – 3 – 1 4 )

[0258]

Considering the facts that the time factor is  $e^{-i\omega t}$ , the phase is added using the path of radiated radio waves, and radio waves reflected off the object are bounced off toward a measurement plane,  $E_1$  is the unique meaningful solution. Accordingly, expression (3-3-15) below is obtained.

[0259]

[Math. 123]

$$\begin{aligned} \varphi(x, y_1, y_2, z, k) \\ = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x + k_{y1} y_1 + k_{y2} y_2)} a(k_x, k_{y1}, k_{y2}, k) \\ \cdot e^{iz\sqrt{\left\{\sqrt{k^2-(k_{y1}+k\beta)^2}+\sqrt{k^2-(k_{y2}+k\beta)^2}\right\}^2-(k_x+k\alpha)^2}} dk_x dk_{y1} dk_{y2} \end{aligned}$$

10 • • • ( 3 – 3 – 1 5 )

[0260]

By substituting  $z = 0$  in expression (3-3-15),  $a(k_x, k_{y1}, k_{y2}, k)$  is obtained as the Fourier transform result of measurement data  $\Phi$  at  $z = 0$ , as given by expression (3-3-16) below.

15 [0261]

[Math. 124]

$$\begin{aligned} a(k_x, k_{y1}, k_{y2}, k) = \tilde{\varphi}(k_x, k_{y1}, k_{y2}, 0, k) = \tilde{\Phi}(k_x, k_{y1}, k_{y2}, k) \\ \cdot \cdot \cdot ( 3 – 3 – 1 6 ) \end{aligned}$$

[0262]

Ultimately, the scattering field function reflecting the Doppler

effect is obtained as given by expression (3-3-17) below.

[0263]

[Math. 125]

$$\begin{aligned} \varphi(x, y_1, y_2, z, k) \\ = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x + k_{y_1} y_1 + k_{y_2} y_2)} \tilde{\Phi}(k_x, k_{y_1}, k_{y_2}, k) \\ \cdot e^{iz \sqrt{\left\{ \sqrt{k^2 - (k_{y_1} + k\beta)^2} + \sqrt{k^2 - (k_{y_2} + k\beta)^2} \right\}^2 - (k_x + k\alpha)^2}} dk_x dk_{y_1} dk_{y_2} \end{aligned}$$

• • • (3 – 3 – 1 7)

5 [0264]

By applying a limit operation ( $y_2 \rightarrow y_1 = y$ ) to expression (3-3-17) on condition that  $k$  and  $z$  are fixed and integrating the result with respect to  $k$ , an imaging function that reflects the Doppler effect is obtained as given by expression (3-3-18) below.

10 [0265]

[Math. 126]

$$\begin{aligned} \varphi(x, y, y, z, k) \\ = \lim_{y_2 \rightarrow y_1 = y} [\varphi(x, y_1, y_2, z, k)] \\ = \lim_{y_2 \rightarrow y_1 = y} \left[ \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x + k_{y_1} y_1 + k_{y_2} y_2)} \tilde{\Phi}(k_x, k_{y_1}, k_{y_2}, k) \right. \\ \left. \cdot e^{iz \sqrt{\left\{ \sqrt{k^2 - (k_{y_1} + k\beta)^2} + \sqrt{k^2 - (k_{y_2} + k\beta)^2} \right\}^2 - (k_x + k\alpha)^2}} dk_x dk_{y_1} dk_{y_2} \right] \end{aligned}$$

$$\rho(x, y, z) = \int_0^{\infty} \varphi(x, y, y, z, k) dk$$

• • • (3 – 3 – 1 8)

[0266]

15 For example, to simplify the integral on the right side, the variables of expression (3-3-19) below are introduced.

[0267]

[Math. 127]

$$k_{\xi} = k_x + k\alpha$$

$$k_{\eta_1} = k_{y_1} + k\beta$$

$$k_{\eta_2} = k_{y_2} + k\beta$$

• • • ( 3 – 3 – 1 9 )

[0268]

In such cases, the first expression of expression (3-3-18) is expressed as given by expression (3-3-20) below.

5 [0269]

[Math. 128]

$$\varphi(x, y, y, z, k)$$

$$= \lim_{y_2 \rightarrow y_1 = y} [\varphi(x, y_1, y_2, z, k)]$$

$$= \lim_{y_2 \rightarrow y_1 = y} \left[ \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i\{(k_{\xi} - k\alpha)x + (k_{\eta_1} - k\beta)y_1 + (k_{\eta_2} - k\beta)y_2\}} \cdot \tilde{\Phi}(k_{\xi} - k\alpha, k_{\eta_1} - k\beta, k_{\eta_2} - k\beta, k) e^{iz\sqrt{\{k^2 - (k_{\eta_1})^2 + k^2 - (k_{\eta_2})^2\}^2 - (k_{\xi})^2}} dk_{\xi} dk_{\eta_1} dk_{\eta_2} \right]$$

$$= \lim_{y_2 \rightarrow y_1 = y} \left[ \frac{1}{(2\pi)^3} e^{i(k\alpha x + k\beta y_1 + k\beta y_2)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_{\xi}x + k_{\eta_1}y + k_{\eta_2}y_2)} \cdot \tilde{\Phi}(k_{\xi} - k\alpha, k_{\eta_1} - k\beta, k_{\eta_2} - k\beta, k) e^{iz\sqrt{\{k^2 - (k_{\eta_1})^2 + k^2 - (k_{\eta_2})^2\}^2 - (k_{\xi})^2}} dk_{\xi} dk_{\eta_1} dk_{\eta_2} \right]$$

• • • ( 3 – 3 – 2 0 )

[0270]

In expression (3-3-20) above,

10 [Math. 129]

$$\tilde{\Phi}$$

is the Fourier transform result of measurement data  $\Phi$  and is defined by expression (3-3-21) below.

[0271]

15 [Math. 130]

$$\tilde{\Phi}(k_x, k_{y_1}, k_{y_2}, k) = \int_{-\infty}^{\infty} e^{ickt} dt \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(k_x x + k_{y_1} y_1 + k_{y_2} y_2)} \Phi(x, y_1, y_2, t) dx dy_1 dy_2$$

• • • ( 3 – 3 – 2 1 )

[0272]

Furthermore, the function of expression (3-3-22) below is introduced.

[0273]

[Math. 131]

$$\begin{aligned}
 & \tilde{\Psi}(k_{\xi}, k_{\eta_1}, k_{\eta_2}, k) \\
 &= \tilde{\Phi}(k_{\xi} - k\alpha, k_{\eta_1} - k\beta, k_{\eta_2} - k\beta, k) \\
 &= \int_{-\infty}^{\infty} e^{ickt} dt \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i\{(k_{\xi} - k\alpha)x + (k_{\eta_1} - k\beta)y_1 + (k_{\eta_2} - k\beta)y_2\}} \Phi(x, y_1, y_2, t) dx dy_1 dy_2 \\
 &= \int_{-\infty}^{\infty} e^{ickt} dt \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(k_{\xi}x + k_{\eta_1}y_1 + k_{\eta_2}y_2)} \left\{ e^{-ik\alpha x - ik\beta y_1 - ik\beta y_2} \Phi(x, y_1, y_2, t) \right\} dx dy_1 dy_2
 \end{aligned}$$

5     · · · ( 3 – 3 – 2 2 )

[0274]

Using expression (3-3-22), the imaging function is expressed as given by expression (3-3-23) below.

[0275]

10   [Math. 132]

$$\begin{aligned}
 & \rho(x, y, z) \\
 &= \int_0^{\infty} \varphi(x, y, y, z, k) \left( \frac{dk}{dk_{\xi}} \right) dk_{\xi} \\
 &= \int_0^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{i(k\alpha x + k\beta y + k\beta y)}}{(2\pi)^3} e^{-i(k_{\xi}x + k_{\eta_1}y + k_{\eta_2}y)} \\
 & \quad \cdot e^{ik_{\xi}z} \tilde{\Psi}(k_{\xi}, k_{\eta_1}, k_{\eta_2}, k) \left( \frac{dk}{dk_{\xi}} \right) dk_{\xi} dk_{\eta_1} dk_{\eta_2} dk_{\xi}
 \end{aligned}$$

· · · ( 3 – 3 – 2 3 )

[0276]

Here, each variable is defined as given by expression (3-3-24) below.

15   [0277]

[Math. 133]

$$k_{\xi} = \sqrt{(\sqrt{k^2 - k_{\eta_1}^2} + \sqrt{k^2 - k_{\eta_2}^2})^2 - k_{\xi}^2}$$

$$k = \frac{1}{2} \sqrt{k_{\xi}^2 + k_{\xi}^2 + 2(k_{\eta_1}^2 + k_{\eta_2}^2) + \frac{(k_{\eta_1}^2 - k_{\eta_2}^2)^2}{k_{\xi}^2 + k_{\xi}^2}}$$

$$\frac{dk}{dk_{\xi}} = \frac{k_{\xi} \sqrt{k^2 - k_{\eta_1}^2} \sqrt{k^2 - k_{\eta_2}^2}}{k(k_{\xi}^2 + k_{\xi}^2)}$$

• • • (3 – 3 – 2 4)

[0278]

Through the above, the imaging function reflecting the Doppler effect is simplified.

5 [0279]

<3-4. Error Caused by Doppler Effect>

In this section, the error caused by the Doppler effect that is improved by the scattering field theory of this chapter will be examined assuming one dimension. Here, the transmitting and receiving antenna is positioned at the origin  $x=0$ . When the transmitted wave is expressed as  $e^{kx-i\omega t}$  and the Doppler shift of the wavenumber is  $\Delta k = (2v/c)k$ , the reflected wave is expressed, using  $k_d = k + \Delta k$ , as given by expression (3-4-1) below.

10 [0280]

15 [Math. 134]

$$e^{-k_d x - i\omega t}$$

• • • (3 – 4 – 1)

[0281]

In this case, expression (3-4-2) below holds true.

[0282]

20 [Math. 135]

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-2ikx} \{e^{ikx_0} e^{i(k+\alpha k)x_0}\} dk = \delta(2x - 2x_0 - \alpha x_0)$$

• • • (3 – 4 – 2)

[0283]

Here, the content within the curly braces (“{ }”) represents the

scattering field function. Baseband transmission is assumed.

[0284]

When there is no Doppler shift, based on  $\alpha=0$ , the flying object is estimated to be positioned at  $x=x_0$ . When there is a Doppler shift,

5 based on  $\alpha=2v/c$ , the flying object is estimated to be positioned at  $x$  in expression (3-4-3) below.

[0285]

[Math. 136]

$$x = x_0 + \frac{\alpha x_0}{2} = x_0 + \frac{v}{c} x_0$$

• • • (3-4-3)

10 [0286]

Here,  $c$  represents the velocity of the wave, and  $v$  represents the velocity of the flying object. Accordingly, the error caused by the Doppler shift is  $(v/c) \times x_0 = 0.5$  m at  $x_0 = 100$  km ahead and  $(v/c) \times x_0 = 5$  m at  $x_0 = 1000$  km ahead, where  $c = 3 \times 10^8$  m/sec and  $v = 5 \times 300$  m/sec.

15

[0287]

Velocity  $v$  of the flying object is obtained as follows. When the flying object moves from  $x = x_1$  to  $x = x_2$  during time  $\Delta t$ ,  $x_1$  and  $x_2$  are expressed as given by expression (3-4-4) below.

20 [0288]

[Math. 137]

$$x_1 = x_0 + \frac{v}{c} x_0$$

$$x_2 = (x_0 - v\Delta t) + \frac{v}{c}(x_0 - v\Delta t)$$

• • • (3-4-4)

[0289]

Expression (3-4-5) below is obtained from expression (3-4-4), and velocity  $v$  is obtained.

25

[0290]

[Math. 138]

$$\frac{x_1 - x_2}{\Delta t} = v + v \frac{v}{c} \approx v$$

• • • ( 3 - 4 - 5 )

[0291]

Stated differently, even with Doppler shift, velocity can be obtained substantially accurately by the method described in Section 1 of this chapter, and thus errors caused by Doppler shift can be easily removed.

[0292]

<3-5. Variations>

In this chapter, a velocity derived using a provisional imaging function is used. However, a velocity determined by other methods may be used. For example, when the velocity of an object is known, the known velocity may be used. Alternatively, a velocity measured using other velocity sensors such as Doppler sensors may be used. A velocity estimated as the velocity of the object based on properties and the like of the object may be used.

[0293]

In this chapter, the scattering field theory that takes the movement of the object into consideration is applied to an example using a one-dimensional array. However, the scattering field theory that takes the movement of the object into consideration may be further applied to an example using a two-dimensional array and a semi-two-dimensional array.

[0294]

<4. Configuration and Operations of Imaging Device>

Based on the contents described above, the configuration and operations of an imaging device for imaging an object in a measurement area using measurement data of scattered waves will be described hereinafter.

[0295]

The waves used for measuring scattered waves as used herein may, for example, be radio waves or may be other waves such as microwaves, millimeter waves, or terahertz waves. The waves may also be light or sound. The measurement area may be a region in

the air, and the object may be a flying object. The object in the measurement area has a physical characteristic that is different from those of the surrounding media. Specifically, the physical characteristic is a physical characteristic that corresponds to the reflectance of the waves. In the case where radio waves are used as the waves, the physical characteristic may be the dielectric constant.  
[0296]

FIG. 6 is a basic schematic diagram of the imaging device according to the present embodiment. Imaging device 100 shown in FIG. 6 includes a plurality of transmitters 101, a plurality of receivers 102, and information processing circuit 103. Imaging device 100 may further include display 104.

[0297]

Each transmitter 101 is a circuit that transmits waves. More specifically, each transmitter 101 transmits waves to the measurement area. Each transmitter 101 may be a transmitting antenna or may include a plurality of transmitting elements such as a plurality of transmitting antenna elements.

[0298]

Each receiver 102 is a circuit that receives scattered waves. More specifically, each receiver 102 receives scattered waves from the measurement area. Each receiver 102 may be a receiving antenna or may include a plurality of receiving elements such as a plurality of receiving antenna elements.

[0299]

The plurality of transmitters 101 and the plurality of receivers 102 may be arranged in a one-dimensional array, may be arranged in a two-dimensional array, and may be arranged in a semi-two-dimensional array.

[0300]

Information processing circuit 103 is a circuit that performs information processing. More specifically, information processing circuit 103 obtains measurement data of scattered waves, and images an object in the measurement area using the measurement data of the scattered waves. For example, information processing

circuit 103 may perform computational processing indicated by the theory described above when imaging the object using the measurement data.

[0301]

5           Information processing circuit 103 may also be a computer or a processor of a computer. Information processing circuit 103 may perform information processing by reading out a program from memory and executing the program. Alternatively, information processing circuit 103 may be a dedicated circuit that images an  
10   object in the measurement area in accordance with measurement data.

[0302]

          Information processing circuit 103 may be capable of communicating with the plurality of transmitters 101 and the  
15   plurality of receivers 102. Information processing circuit 103 may control the operation of the plurality of transmitters 101 and the plurality of receivers 102. Information processing circuit 103 may obtain position information and measurement data from the plurality of transmitters 101 and the plurality of receivers 102.

20   [0303]

          In order to image the object, information processing circuit 103 may generate an image that indicates the object. Information processing circuit 103 may output the image indicating the object on display 104 or the like. Alternatively, information processing circuit  
25   103 may output the image indicating the object to a printer (not illustrated in the drawings). As another alternative, information processing circuit 103 may transmit the image as electronic data to a different device (not illustrated in the drawings) via wired or wireless communication.

30   [0304]

          Display 104 is a display device such as a liquid crystal display. Note that display 104 is merely an arbitrary element and is not an essential element. Display 104 may be an external device that is not included in the configuration of imaging device 100.

35   [0305]

FIG. 7 is a flowchart showing basic operations of imaging device 100 shown in FIG. 6. More specifically, the operations shown in FIG. 7 are performed by the elements of imaging device 100 shown in FIG. 6 such as the plurality of transmitters 101, the plurality of receivers 102, and information processing circuit 103.

[0306]

First, each transmitter 101 transmits waves to the measurement area (S101). Each receiver 102 receives scattered waves from the measurement area (S102). Then, information processing circuit 103 images the object in the measurement area using measurement data of the scattered waves (S103, S104, and S105).

[0307]

More specifically, in imaging the object, information processing circuit 103 first derives the scattering field function using the measurement data and the velocity vector of the object (S103). Here, the scattering field function is a function that receives the transmission position of the wave and the reception position of the scattered wave as input and outputs the amount of the scattered wave at the reception position. In deriving the scattering field function, information processing circuit 103 reflects in the scattering field function that the wavenumber of the scattered wave changes from the wavenumber of the wave due to the Doppler effect corresponding to the velocity vector.

[0308]

Next, information processing circuit 103 derives the imaging function using the scattering field function (S104). Here, the imaging function is a function that receives an imaging target position as input and outputs an image intensity at the imaging target position, and is a function defined using an amount output from the scattering field function in response to inputting the imaging target position into the scattering field function as the transmission and reception positions.

[0309]

Finally, information processing circuit 103 images the object in

the measurement area using the imaging function (S105).

[0310]

This allows imaging device 100 to derive the scattering field function and the imaging function in which the Doppler effect resulting from the movement of the object is reflected. Accordingly, imaging device 100 is capable of imaging a moving object in the measurement area with high accuracy.

[0311]

For example, the plurality of transmitters 101 and the plurality of receivers 102 may be arranged along a straight line parallel to the y-axis.

[0312]

Information processing circuit 103 may derive, as the scattering field function, a solution for an item expressed by Math. 140 of the following equation expressed by Math. 139.

[Math. 139]

$$\left[\frac{1}{4}\{\square_4 - 2(ik)^2\}^2 - D_{y_1}^2 D_{y_2}^2 + (ik)^2(D_{y_1}^2 + D_{y_2}^2) - (ik)^4\right]\varphi = 0$$

$$\square_4 = D_x^2 + D_{y_1}^2 + D_{y_2}^2 + D_z^2$$

$$D_x = \partial_x - ik\alpha$$

$$D_{y_1} = \partial_{y_1} - ik\beta$$

$$D_{y_2} = \partial_{y_2} - ik\beta$$

$$D_z = \partial_z$$

[Math. 140]

$\varphi$

[0313]

Here, x and z of the equation respectively represent the x-coordinate and z-coordinate of the transmission position and reception position.  $y_1$  of the equation represents the y-coordinate of the transmission position, and  $y_2$  of the equation represents the y-coordinate of the reception position. k represents the wavenumber of the wave.  $\alpha$  is defined by  $\alpha = 2v_x/c$ .  $\beta$  is defined by  $\beta = v_y/c$ .  $v_x$  and  $v_y$  respectively represent the x-component and y-component of the velocity vector. c represents the speed of the

wave.

[0314]

This allows imaging device 100 to derive the scattering field function from an equation that reflects the Doppler effect, and to  
5 image a moving object in the measurement area with high accuracy by using the derived scattering field function.

[0315]

For example, the scattering field function may be expressed as follows.

10 [Math. 141]

$$\begin{aligned} \varphi(x, y_1, y_2, z, k) \\ = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x + k_{y_1} y_1 + k_{y_2} y_2)} \tilde{\Phi}(k_x, k_{y_1}, k_{y_2}, k) \\ \cdot e^{iz \sqrt{\left\{ \sqrt{k^2 - (k_{y_1} + k\beta)^2} + \sqrt{k^2 - (k_{y_2} + k\beta)^2} \right\}^2 - (k_x + k\alpha)^2}} dk_x dk_{y_1} dk_{y_2} \end{aligned}$$

[0316]

Here, x and z of the scattering field function respectively represent the x-coordinate and z-coordinate of the transmission  
15 position and reception position.  $y_1$  of the scattering field function represents the y-coordinate of the transmission position, and  $y_2$  of the scattering field function represents the y-coordinate of the reception position.  $k_x$ ,  $k_{y_1}$ , and  $k_{y_2}$  respectively represent variables corresponding to the wavenumbers with respect to x,  $y_1$ , and  $y_2$  of  
20 the scattering field function.

[0317]

Also,

[Math. 142]

$\tilde{\Phi}$

25 represents the Fourier transformed measurement data.

[0318]

This allows imaging device 100 to image a moving object in the measurement area with high accuracy by using the scattering field function derived from the measurement data obtained by the  
30 plurality of transmitters 101 and the plurality of receivers 102

arranged in a straight line.

[0319]

For example, the imaging function may be expressed as shown below.

5 [Math. 143]

$$\rho(x, y, z) = \int_0^{\infty} \lim_{y_2 \rightarrow y_1 = y} [\varphi(x, y_1, y_2, z, k)] dk$$

[0320]

Here, x, y, and z of the imaging function respectively represent the x-coordinate, y-coordinate, and z-coordinate of the imaging target position.

[0321]

This allows imaging device 100 to image a moving object in the measurement area with high accuracy by using the imaging function derived from the measurement data obtained by the plurality of transmitters 101 and the plurality of receivers 102 arranged in a straight line.

[0322]

For example, using a function defined by:

[Math. 144]

$$\begin{aligned} & \tilde{\Psi}(k_{\xi}, k_{\eta_1}, k_{\eta_2}, k) \\ &= \int_{-\infty}^{\infty} e^{ickt} dt \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(k_{\xi}x + k_{\eta_1}y_1 + k_{\eta_2}y_2)} \left\{ e^{-ik_{\alpha}x - ik_{\beta}y_1 - ik_{\beta}y_2} \Phi(x, y_1, y_2, t) \right\} dx dy_1 dy_2 \end{aligned}$$

the imaging function may be expressed as:

[Math. 145]

$$\begin{aligned} & \rho(x, y, z) \\ &= \int_0^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{i(k_{\alpha}x + k_{\beta}y + k_{\beta}y)}}{(2\pi)^3} e^{-i(k_{\xi}x + k_{\eta_1}y + k_{\eta_2}y)} \\ & \quad \cdot e^{ik_{\xi}z} \tilde{\Psi}(k_{\xi}, k_{\eta_1}, k_{\eta_2}, k) \left( \frac{dk}{dk_{\xi}} \right) dk_{\xi} dk_{\eta_1} dk_{\eta_2} dk_{\xi} \end{aligned}$$

.

25 [0323]

Here, x, y, and z of the imaging function respectively represent

the x-coordinate, y-coordinate, and z-coordinate of the imaging target position.  $\Phi$  represents the measurement data.

[0324]

$k_\xi$ ,  $k_{\eta_1}$ ,  $k_{\eta_2}$ ,  $k_\zeta$ , and  $dk/dk_\zeta$  are defined as follows.

5 [Math. 146]

$$k_\xi = k_x + k\alpha$$

$$k_{\eta_1} = k_{y_1} + k\beta$$

$$k_{\eta_2} = k_{y_2} + k\beta$$

$$k_\zeta = \sqrt{(\sqrt{k^2 - k_{\eta_1}^2} + \sqrt{k^2 - k_{\eta_2}^2})^2 - k_\xi^2}$$

$$\frac{dk}{dk_\zeta} = \frac{k_\zeta \sqrt{k^2 - k_{\eta_1}^2} \sqrt{k^2 - k_{\eta_2}^2}}{k(k_\xi^2 + k_\zeta^2)}$$

[0325]

This allows imaging device 100 to image a moving object in the measurement area with high accuracy by using the imaging function with inhibited complexity derived from the measurement data obtained by the plurality of transmitters 101 and the plurality of receivers 102 arranged in a straight line.

[0326]

For example, information processing circuit 103 may derive a provisional scattering field function using the measurement data without using the velocity vector. Information processing circuit 103 may derive a provisional imaging function using the provisional scattering field function. Information processing circuit 103 may derive the velocity vector using the provisional imaging function.

20 [0327]

This allows imaging device 100 to derive the velocity vector of the object using the provisional scattering field function and the provisional imaging function derived without using the velocity vector of the object. Imaging device 100 is capable of imaging an object that moves in the measurement area with high accuracy by using the scattering field function and the imaging function derived using the velocity vector.

[0328]

For example, in the deriving of the velocity vector, information processing circuit 103 may derive the velocity vector using an arithmetic expression represented as follows.

[Math. 147]

$$5 \quad \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = - \begin{pmatrix} 2a+2\varepsilon & e & g \\ e & 2b+2\varepsilon & f \\ g & f & 2c+2\varepsilon \end{pmatrix}^{-1} \begin{pmatrix} h \\ m \\ n \end{pmatrix}$$

[0329]

Here, a, b, c, e, f, g, h, m, and n in the arithmetic expression are defined as follows.

[Math. 148]

$$a = \frac{1}{(2\pi)^3} \iiint k_x^2 |\rho_k|^2 d\mathbf{k}^3$$

$$b = \frac{1}{(2\pi)^3} \iiint k_y^2 |\rho_k|^2 d\mathbf{k}^3$$

$$c = \frac{1}{(2\pi)^3} \iiint k_z^2 |\rho_k|^2 d\mathbf{k}^3$$

$$e = \frac{1}{(2\pi)^3} \iiint k_x k_y |\rho_k|^2 d\mathbf{k}^3 + cc.$$

$$f = \frac{1}{(2\pi)^3} \iiint k_y k_z |\rho_k|^2 d\mathbf{k}^3 + cc.$$

$$g = \frac{1}{(2\pi)^3} \iiint k_z k_x |\rho_k|^2 d\mathbf{k}^3 + cc.$$

$$h = \frac{-i}{(2\pi)^3} \iiint k_x \rho_k \cdot \partial_\tau \bar{\rho}_k d\mathbf{k}^3 + cc.$$

$$m = \frac{-i}{(2\pi)^3} \iiint k_y \rho_k \cdot \partial_\tau \bar{\rho}_k d\mathbf{k}^3 + cc.$$

$$n = \frac{-i}{(2\pi)^3} \iiint k_z \rho_k \cdot \partial_\tau \bar{\rho}_k d\mathbf{k}^3 + cc.$$

10

[0330]

$v_x$ ,  $v_y$ , and  $v_z$  respectively represent the x-component, y-component, and z-component of the velocity vector.  $\varepsilon$  represents a positive value.  $\rho_k$  represents a provisional imaging function that

includes  $\tau$  as one of a plurality of input variables and is Fourier transformed with respect to  $x$ ,  $y$ , and  $z$ .  $k_x$ ,  $k_y$ , and  $k_z$  in the arithmetic expression respectively represent the wavenumbers with respect to  $x$ ,  $y$ , and  $z$  of the provisional imaging function.

5 [0331]

Also,

[Math. 149]

**k**

represents a wavenumber vector of  $k_x$ ,  $k_y$ , and  $k_z$  in the arithmetic  
10 expression.  $\tau$  represents a time unit corresponding to the number of data collections.  $cc.$  represents the complex conjugate of the term immediately preceding  $cc.$ .

[0332]

Also,

15 [Math. 150]

$\bar{\rho}_k$

represents the complex conjugate of  $\rho_k$ .

[0333]

This allows imaging device 100 to derive the velocity vector of  
20 the object from the provisional imaging function using an arithmetic expression.

[0334]

For example, the wave may be a microwave. This allows  
imaging device 100 to inhibit attenuation of the wave by moisture in  
25 the measurement area compared to electromagnetic waves with shorter wavelengths.

[0335]

For example, other elements, expressions, variables, and so  
on described in the present embodiment are applicable as  
30 appropriate to the plurality of transmitters 101, the plurality of receivers 102, information processing circuit 103, the scattering field function, the imaging function, and so on described above as to the basic configuration and the basic operations. The scattering field function, the imaging function, and so on given in the present  
35 embodiment may be modified and applied as appropriate. For

example, it is possible to use a mathematical expression that represents substantially the same content as that given by the mathematical expression described above, or to use any other mathematical expression derived based on the theory described  
5 above.

[0336]

(Additional Comments)

While some aspects of the imaging device have been described thus far with reference to the embodiment, the modes of the imaging  
10 device are not limited to this embodiment. Any modification conceivable by those skilled in the art may be made to the embodiment, and a plurality of elements according to the embodiment may be combined arbitrarily. For example, processing that is executed by a specific element according to the embodiment  
15 may be executed by a different element, instead of the specific element. Moreover, a sequence of a plurality of processes may be changed, or a plurality of processes may be executed in parallel.

[0337]

The imaging method including steps executed by each element  
20 of the imaging device may be executed by any arbitrary device or system. For example, part or all of the imaging method may be executed by a computer that includes, for example, a processor, memory, and an input/output circuit. At this time, a program for causing the computer to execute the imaging method may be  
25 executed by the computer to execute the imaging method.

[0338]

The above-described program may be recorded on a non-transitory computer-readable recording medium.

[0339]

Each element of the imaging device may be configured by  
30 dedicated hardware or by general-purpose hardware that executes the above-described program or the like, or may be configured by a combination of these. The general-purpose hardware may be configured by, for example, memory that records the program and a  
35 general-purpose processor that reads out and executes the program

from the memory. The memory as used herein may, for example, be semiconductor memory or a hard disk, and the general-purpose processor may, for example, be a CPU.

[0340]

5           The dedicated hardware may be configured by, for example, memory and a dedicated processor. For example, the dedicated processor may execute the imaging method described above with reference to the memory for recording measurement data.

[0341]

10           Each element of the imaging device may be an electric circuit. These electric circuits may be configured as a single electric circuit as a whole, or each may be a different electric circuit. These electric circuits may correspond to dedicated hardware or general-purpose hardware that executes the above-described program or the like.

15           [0342]

          The imaging device is not limited to being a physically integrated device, and may include a plurality of sub-devices arranged in a distributed manner. The imaging device may also be referred to as an imaging system.

20           [Industrial Applicability]

[0343]

          One aspect of the present disclosure is useful in an imaging device that images an object in a measurement area using measurement data of scattered waves, and is applicable to an  
25   exploration system or the like that explores an object that moves in the measurement area.

[Reference Signs List]

[0344]

|          |                                |
|----------|--------------------------------|
| 100      | imaging device                 |
| 30   101 | transmitter                    |
| 102      | receiver                       |
| 103      | information processing circuit |
| 104      | display                        |

[CLAIMS]

[Claim 1]

An imaging device comprising:

a plurality of transmitters that each transmit a wave to a  
5 measurement area;

a plurality of receivers that each receive a scattered wave of  
the wave from the measurement area; and

an information processing circuit that images an object in the  
measurement area using measurement data of the scattered wave,

10 wherein

in imaging the object, the information processing circuit:

derives, using the measurement data and a velocity  
vector of the object, a scattering field function that receives a  
transmission position of the wave and a reception position of the  
15 scattered wave as input and outputs an amount of the scattered wave  
at the reception position;

derives an imaging function that receives an imaging  
target position as input and outputs an image intensity at the  
imaging target position, and is defined using an amount output from  
20 the scattering field function in response to inputting the imaging  
target position into the scattering field function as the transmission  
position and the reception position; and

images the object in the measurement area using the  
imaging function, and

25 in deriving the scattering field function, the information  
processing circuit reflects in the scattering field function that a  
wavenumber of the scattered wave changes from a wavenumber of  
the wave due to a Doppler effect corresponding to the velocity vector.

30 [Claim 2]

The imaging device according to claim 1, wherein

the plurality of transmitters and the plurality of receivers are  
arranged along a straight line parallel to a y-axis, and

the information processing circuit derives, as the scattering  
35 field function, a solution for an item expressed by Math. 2 of the

following equation expressed by Math. 1:

[Math. 1]

$$\left[\frac{1}{4}\{\square_4 - 2(ik)^2\}^2 - D_{y_1}^2 D_{y_2}^2 + (ik)^2(D_{y_1}^2 + D_{y_2}^2) - (ik)^4\right]\varphi = 0$$

$$\square_4 = D_x^2 + D_{y_1}^2 + D_{y_2}^2 + D_z^2$$

$$D_x = \partial_x - ik\alpha$$

$$D_{y_1} = \partial_{y_1} - ik\beta$$

$$D_{y_2} = \partial_{y_2} - ik\beta$$

$$D_z = \partial_z$$

[Math. 2]

5  $\varphi$

where x and z of the equation respectively represent an x-coordinate and a z-coordinate of the transmission position and the reception position,  $y_1$  of the equation represents a y-coordinate of the transmission position,  $y_2$  of the equation represents a y-coordinate of the reception position, k represents the wavenumber of the wave,  $\alpha$  is defined by  $\alpha = 2v_x/c$ ,  $\beta$  is defined by  $\beta = v_y/c$ ,  $v_x$  and  $v_y$  respectively represent an x-component and a y-component of the velocity vector, and c represents a speed of the wave.

15 [Claim 3]

The imaging device according to claim 2, wherein the scattering field function is expressed as:

[Math. 3]

$$\begin{aligned} & \varphi(x, y_1, y_2, z, k) \\ &= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x + k_{y_1} y_1 + k_{y_2} y_2)} \tilde{\Phi}(k_x, k_{y_1}, k_{y_2}, k) \\ & \quad \cdot e^{iz \sqrt{\left\{ \sqrt{k^2 - (k_{y_1} + k\beta)^2} + \sqrt{k^2 - (k_{y_2} + k\beta)^2} \right\}^2 - (k_x + k\alpha)^2}} dk_x dk_{y_1} dk_{y_2} \end{aligned}$$

20 where x and z of the scattering field function respectively represent an x-coordinate and a z-coordinate of the transmission position and the reception position,  $y_1$  of the scattering field function represents a y-coordinate of the transmission position,  $y_2$  of the

scattering field function represents a y-coordinate of the reception position, and  $k_x$ ,  $k_{y1}$ , and  $k_{y2}$  respectively represent variables corresponding to wavenumbers with respect to  $x$ ,  $y_1$ , and  $y_2$  of the scattering field function, and

5 [Math. 4]

$$\tilde{\Phi}$$

represents the measurement data that has been Fourier transformed.

10 [Claim 4]

The imaging device according to claim 3, wherein the imaging function is expressed as:

[Math. 5]

$$\rho(x, y, z) = \int_0^{\infty} \lim_{y_2 \rightarrow y_1 = y} [\varphi(x, y_1, y_2, z, k)] dk$$

15 where  $x$ ,  $y$ , and  $z$  of the imaging function respectively represent an x-coordinate, a y-coordinate, and a z-coordinate of the imaging target position.

[Claim 5]

20 The imaging device according to claim 3, wherein using a function defined by:

[Math. 6]

$$\begin{aligned} & \tilde{\Psi}(k_{\xi}, k_{\eta_1}, k_{\eta_2}, k) \\ &= \int_{-\infty}^{\infty} e^{ickt} dt \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(k_{\xi}x + k_{\eta_1}y_1 + k_{\eta_2}y_2)} \left\{ e^{-ik\alpha x - ik\beta y_1 - ik\beta y_2} \Phi(x, y_1, y_2, t) \right\} dx dy_1 dy_2 \end{aligned}$$

the imaging function is expressed as:

25 [Math. 7]

$$\begin{aligned} & \rho(x, y, z) \\ &= \int_0^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{i(k\alpha x + k\beta y + k\beta y)}}{(2\pi)^3} e^{-i(k_{\xi}x + k_{\eta_1}y + k_{\eta_2}y)} \\ & \quad \cdot e^{ik_{\xi}z} \tilde{\Psi}(k_{\xi}, k_{\eta_1}, k_{\eta_2}, k) \left( \frac{dk}{dk_{\xi}} \right) dk_{\xi} dk_{\eta_1} dk_{\eta_2} dk_{\xi} \end{aligned}$$

where  $x$ ,  $y$ , and  $z$  of the imaging function respectively represent an x-coordinate, a y-coordinate, and a z-coordinate of the imaging target position,  $\Phi$  represents the measurement data, and  $k_\xi$ ,  $k_{\eta_1}$ ,  $k_{\eta_2}$ ,  $k_\zeta$ , and  $dk/dk_\zeta$  are defined as:

5 [Math. 8]

$$k_\xi = k_x + k\alpha$$

$$k_{\eta_1} = k_{y_1} + k\beta$$

$$k_{\eta_2} = k_{y_2} + k\beta$$

$$k_\zeta = \sqrt{(\sqrt{k^2 - k_{\eta_1}^2} + \sqrt{k^2 - k_{\eta_2}^2})^2 - k_\xi^2}$$

$$\frac{dk}{dk_\zeta} = \frac{k_\zeta \sqrt{k^2 - k_{\eta_1}^2} \sqrt{k^2 - k_{\eta_2}^2}}{k(k_\xi^2 + k_\zeta^2)}$$

.

[Claim 6]

10 The imaging device according to any one of claims 1 to 5, wherein

the information processing circuit:

derives a provisional scattering field function using the measurement data without using the velocity vector;

15 derives a provisional imaging function using the provisional scattering field function; and

derives the velocity vector using the provisional imaging function.

20 [Claim 7]

The imaging device according to claim 6, wherein

in deriving the velocity vector, the information processing circuit:

derives the velocity vector using an arithmetic expression

25 represented as:

[Math. 9]

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = - \begin{pmatrix} 2a+2\varepsilon & e & g \\ e & 2b+2\varepsilon & f \\ g & f & 2c+2\varepsilon \end{pmatrix}^{-1} \begin{pmatrix} h \\ m \\ n \end{pmatrix}$$

where a, b, c, e, f, g, h, m, and n in the arithmetic expression are defined as:

[Math. 10]

$$a = \frac{1}{(2\pi)^3} \iiint k_x^2 |\rho_k|^2 d\mathbf{k}^3$$

$$b = \frac{1}{(2\pi)^3} \iiint k_y^2 |\rho_k|^2 d\mathbf{k}^3$$

$$c = \frac{1}{(2\pi)^3} \iiint k_z^2 |\rho_k|^2 d\mathbf{k}^3$$

$$e = \frac{1}{(2\pi)^3} \iiint k_x k_y |\rho_k|^2 d\mathbf{k}^3 + cc.$$

$$f = \frac{1}{(2\pi)^3} \iiint k_y k_z |\rho_k|^2 d\mathbf{k}^3 + cc.$$

$$g = \frac{1}{(2\pi)^3} \iiint k_z k_x |\rho_k|^2 d\mathbf{k}^3 + cc.$$

$$h = \frac{-i}{(2\pi)^3} \iiint k_x \rho_k \cdot \partial_\tau \bar{\rho}_k d\mathbf{k}^3 + cc.$$

$$m = \frac{-i}{(2\pi)^3} \iiint k_y \rho_k \cdot \partial_\tau \bar{\rho}_k d\mathbf{k}^3 + cc.$$

$$n = \frac{-i}{(2\pi)^3} \iiint k_z \rho_k \cdot \partial_\tau \bar{\rho}_k d\mathbf{k}^3 + cc.$$

5

where  $v_x$ ,  $v_y$ , and  $v_z$  respectively represent an x-component, a y-component, and a z-component of the velocity vector,  $\varepsilon$  represents a positive value,  $\rho_k$  represents the provisional imaging function that includes  $\tau$  as one of a plurality of input variables and is Fourier transformed with respect to x, y, and z, and  $k_x$ ,  $k_y$ , and  $k_z$  in the arithmetic expression respectively represent wavenumbers with respect to x, y, and z of the provisional imaging function,

10

[Math. 11]

**k**

represents a wavenumber vector of  $k_x$ ,  $k_y$ , and  $k_z$  in the arithmetic expression,  $\tau$  represents a time unit corresponding to a number of data collections, and  $cc.$  represents a complex conjugate of a term immediately preceding  $cc.$ , and

5 [Math. 12]

$\bar{\rho}_k$

represents the complex conjugate of  $\rho_k$ .

[Claim 8]

10 An imaging method comprising:

transmitting, by each of a plurality of transmitters, a wave to a measurement area;

receiving, by each of a plurality of receivers, a scattered wave of the wave from the measurement area; and

15 imaging an object in the measurement area using measurement data of the scattered wave, wherein

the imaging of the object includes:

deriving, using the measurement data and a velocity vector of the object, a scattering field function that receives a transmission position of the wave and a reception position of the scattered wave as input and outputs an amount of the scattered wave at the reception position;

20 deriving an imaging function that receives an imaging target position as input and outputs an image intensity at the imaging target position, and is defined using an amount output from the scattering field function in response to inputting the imaging target position into the scattering field function as the transmission position and the reception position; and

25 imaging the object in the measurement area using the imaging function, and

30 the deriving of the scattering field function includes reflecting in the scattering field function that a wavenumber of the scattered wave changes from a wavenumber of the wave due to a Doppler effect corresponding to the velocity vector.