METHOD AND DATA SYSTEM FOR DETERMINING FINANCIAL INSTRUMENTS FOR, AND TERM TO MATURITY OF, A LOAN

INTRODUCTION

This invention concerns a method and a data processing system/computer system for calculation of the type, the number and the volume of financial instruments for funding of a loan with equivalent proceeds to a debtor, the loan being designed to be at least partially refinanced during the remaining term to maturity of the loan. In the method according to the invention the term to maturity of the loan is also determined at the beginning of each period of time so that debtor's payments on the loan during the entire term to maturity of the loan is within a band defined by a set of maximum and minimum limits which can be fixed for each period of time, and so that the term to maturity of the loan also is within a band defined by a maximum and a minimum limit. To the extent necessary, a rule for prioritization of the limits regarding the payments on the loan and the term to maturity is established. The results of the method according to the invention may be used by the creditor, e.g. a financial institution such as a mortgage credit institution, in order to secure that such a loan is funded in such a way that interest rate risks as well as imbalances in the payment flow are avoided or minimized. Thus, by the use of the results of the method according to the invention the creditor has the possibility of hedging lending and funding.

In the refinancing of a loan other financial instruments than the instruments which have formed the basis of the principal of the original loan may be used, which is the reason why, in connection with refinancing, an adjustment of the interest

rate on the loan may be necessary in relation to the interest rate level applicable at the time of the refinancing. Loans which are fully or partially refinanced during the term to maturity of the loan are thus termed Loan with Adjustable Interest Rates (LAIR). An example of the financial instruments is non-callable bullet bonds. In the following, financial instruments are also called funding instruments, just as funding volume is also used as a term for the financial instruments constituting the principal.

BACKGROUND FOR THE INVENTION AND INTRODUCTION TO THE INVENTION.

In the Danish mortgage credit market callable loans used to be far the dominating type of loans, and, therefore, callable bonds in a pure "pass through" form were also as dominating as bonds. For a number of years, up to the withdrawal of the permission to grant cash loans in 1985 by the Danish Ministry of Housing, mortgage credit institutions also offered the so-called loans with adjustable interest rates. The previous loans with adjustable interest rates were characterized as follows:

1) Long-term credit commitment.
2) Funding by the issue of bonds with a term to maturity of 1 to 5 years every fifth year.
3) Fixed interest rate in successive periods of 5 years.
4) The underlying bonds with a term to maturity of 1 to 5 years are non-callable. This gives the debtor the possibility to terminate the loan at par prior to the first occurring interest rate adjustment.

The Danish loans with adjustable interest rates did not turn out very successfully, thus only per milles of the total lending made by mortgage credit institutions was granted as loans with adjustable interest rates. The reasons were,

probably, that the call premium was insignificant at that time due to a very large difference between market interest rates and coupon rates and in addition, the investor market did not pay as much attention to the problem as today. Therefore, the difference in yields between callable and non-callable bonds was not sufficiently large in itself to make loans with adjustable interest rates attractive. Furthermore, the product was not transparent seen from the debtor's side. An aspect which might also have had some influence at times was that a continued rise in the Danish interest rate level was expected so that the debtor would not expect an interest rate adjusted loan to be advantageous in the long run. Finally, the previous structure of loans with adjustable interest rates involved an arbitrary and unpredictable interest rate risk every fifth year. Most likely, these conditions explain the poor success in these years.

In June 1993 certain Danish tax laws were changed so that the mortgage credit institutions were, in reality, once again given the opportunity to offer loans with adjustable interest rates.

This offers the possibility of changing the long-term mortgage market so that, in the future, the funding products will be attractive to foreign investors. A precondition is, probably, that bonds are offered in conformity with international practice, e.g. as non-callable bullet bonds. Thus, it has been of interest to examine whether variants of loans with adjustable interest rates can be made attractive to the debtors.

The traditional loans with adjustable interest rates involve a risk for, in principle, unlimited upward jumps in the interest rate. To many debtors, especially in the segment comprising private customers, this risk is assumed to be unacceptable, in particular with regard to the consequences

as to liquidity for debtor of an increase in the interest rate to a very high level. Therefore, it is of interest to examine whether the design behind loans with adjustable interest rates may be combined with an adjustable term to maturity, where a raising or falling, respectively, interest rate only affects the payments on the loan to a predetermined extent defined by a set of maximum and minimum limits, whereas the term to maturity on the loan is varied in accordance with the interest rate on the loan.

Typically there will be a maximum as well as a minimum limit for the range wherein the term to maturity may vary that may be determined by debtor, lender, public authorities or legislation, respectively. Hence, a particular problem will be to prioritize the maximum as well as the minimum limit for the payments on the loan and the term to maturity of the loan, respectively, and to establish a rule for determining the payments on the loan and the term to maturity in cases where the limits are incompatible with the given interest rate on the loan.

Characteristic of the traditional loans with adjustable interest rates was a match between the term to maturity of the funding instrument having the longest term to maturity and the period between interest rate adjustments, viz. 5 years. If this precondition is abolished a much wider range of possibilities with respect to funding and interest rate adjustment will, in principle, become available.

Thus, it becomes possible to secure a gradual adaptation of the borrowing costs to the market interest rate with an adjustment time depending on the maximum term to maturity of the bonds applied and on the weight with which the individual bond applied is included. This principle will, just as the above-mentioned opportunity for adjustable term to maturity, reduce the risk of substantial upward jumps in the payments

on the loan which characterizes the traditional loans with adjustable interest rates.

If the short-term interest rate is systematically lower than the long-term interest rate it will be possible to reduce debtor's long-term borrowing costs. Furthermore, borrowing costs may, as mentioned above, be reduced compared to callable bonds due to the absence of the call right and through increased liquidity and internationalization of sales.

Whether it is possible to counter a change in the interest rates by varying the term to maturity of the loan depends, apart from the determined maximum and minimum limits for payments on the loan and the term to maturity, on the extent to which the outstanding debts are adjusted to the market interest rates at the time of interest rate adjustment. Characteristic of the traditional loans with adjustable interest rates was the 100 per cent adjustment of the outstanding debts of the loan to the market interest rates every 5 years. Partly by allowing other frequencies of adjustment of the interest rate, and partly by allowing only a partial adjustment of the interest rate of the outstanding debts of the loan, larger shifts in the interest rate than in the conventional loan design would be compatible with the maximum and the minimum limits for the payments on the loan. Thus, also a partial adjustment of the interest rate of the term to maturity of the loan and other interest rate adjustment frequencies should be compatible with adjustable term to maturity.

In connection with loans with adjustable interest rates, relations to the balance principle must be mentioned. It is a leading principle in the legislative regulation of the activities of Danish mortgage credit institutions that the institutions may undertake a limited interest rate and

funding risk. On the face of it, designs of loans with adjustable interest rates are contrary to these basic principles as the funding side has a substantially shorter term to maturity than has the lending side. The traditional loans with adjustable interest rates are nevertheless regarded as lying within the balance principle seen in the perspective that debtor accepts to pay any interest rate that may occur in connection with a future refinancing. In principle, therefore, this is a "pass through" which does not incur any risk onto the creditor.

In connection with funding of a new type of loans with adjustable interest rates with a variety of e.g. non-callable bullet bonds, four conditions must be fulfilled according to Danish practice and legislation:

1. The volumes of each funding instrument must be determined in such a way that the market price of the funding instruments equals the principal of the loan.

2. The interest rate on the loan must be determined in such a way that the interest rate equals the yield to maturity of the funding portfolio, the yield being the interest rate at which the present value of a future payment flow on the funding instruments equals the remaining debt on the loan.

3. The requirement as to maximum permissible imbalances between payments from debtor and payments to creditor must be fulfilled.

4. In addition, the legislative requirements to terms to maturity and method of repayment must be met, also regarding loans with adjustable interest rate and adjustable term to maturity.

Previously, when calculating the funding of conventional loans with adjustable interest rates, the above-mentioned requirement as to the interest rate on the loan was not taken into consideration.

In the funding of conventional loans with adjustable interest rates there was an unambiguous connection between the maximum term to maturity of the funding instruments and the interest rate adjustment period. This structure may briefly be explained as follows: The funding principle was based on a 5 year period during which the interest rate on the loan was fixed. The conventional loans with adjustable interest rates were funded by debtor by issuing underlying bonds with a term to maturity of 1 to 5 years.

However, this funding principle is not compatible with the preferences for issuing a range of e.g. 10 non-callable bullet bonds with terms to maturity of 1 to 10 years and at the same time keeping the duration of the interest rate adjustment period down at e.g. 1-2 years.

Thus, in Denmark there is a demand for a general funding principle comprising funding with the above-mentioned range of non-callable bullet bonds or other financial instruments suitable for that purpose. In international financial markets there is, at the moment, no tradition of a close connection between lending and funding of loans. In spite of this, however, the broad applicability of a principle that links the loan to a range of financial instruments must be presumed to give rise to also an international interest in a general funding principle of the type described herein.

Thus, the funding principle may, *inter alia*, be used for a mark-to-market pricing of otherwise non-traded loans and debts. By applying the principle it will be possible to determine a portfolio of traded financial instruments with an

equivalent cash flow based on which the non-traded loan or debt may be priced in accordance with observed market prices.

Accordingly, the funding principle may find possible applications in the risk management of loans and debts, since the principle may be used for determination of a hedge consisting of a portfolio of financial instruments and pricing of such a hedge. During recent years the development has focused on financial risks, including the possibility of hedging these risks, so particularly in this field the international attention towards the funding principle is expected.

However, a technical problem in connection with such a general funding principle has been that there was no knowledge of an efficient general calculation method for a computerized calculation of the volume of financial instruments or funding volume for the funding of a loan where at least a partial refinancing of the loan during the remaining term to maturity of the loan is performed under the condition that the calculation result must partly fulfil the requirement that loan issuing institutions must not undertake interest rate or funding risk or at least they must or will not undertake such risks above a certain maximum, and must partly be able to contribute to minimize costs of the debtor so that the loan with adjustable interest rates gets as inexpensive as possible within the given preconditions.

In Danish Patent Application No. 0165/96 and International Patent Application No. PCT/DK97/00044 such a suitable computerized method for calculation of the volume of financial instruments or funding capital sums for financing of the above-described type of loans is described.

BRIEF DESCRIPTION OF THE INVENTION

The present invention is a further development of the invention described in the above-mentioned patent applications.

In connection with loans with adjustable interest rate it is sometimes considered a problem that even short term increases in the interest rate lead to substantial upsurges in the payments on the loan, which puts a strain on lender's economy. It would be desirable to provide the possibility for performing the calculations of the loans in such a way that instead of a raise in the payments on the loan, or in combination with a small raise in the payments on the loan, the term to maturity is prolonged so that the lender would be able to manage the payments on the loan from his current economy.

The invention relates to a method by which not only the above-mentioned parameters may be determined, but in which also conditions concerning maximum (or minimum) payments on the loan for the lender through one or more periods of the term to maturity of the loan may be laid down, the term to maturity of the loan then, if required, being calculated according to these conditions. On the other hand, it will be possible in the method according to the invention to lay down conditions for the maximum (or minimum) term to maturity of the loan and then calculate adjusted payments on the loan.

As it appears from the following, the invention is a technical enrichment of the field for data processing: because of the number of variables to be calculated, the financial conditions to be fulfilled and their mutual relations, the problem to be solved is a complex simultaneous problem. By the technical solution provided by the invention, the complex simultaneous calculation problem is divided into a sequence of processing steps, which makes it processable by

sequential calculation in a data processing system/computer system within realistic and effective computation times.

By the method according to the invention calculation results of a high value may be obtained, which, among other things, comprises a high level of stability of the size of the payments on the loan calculated despite input of relatively large fluctuations of the interest rate through the individual periods of financing.

Thus, the invention relates to a method for determining the type, the number, and the volume of financial instruments for the funding of a loan with equivalent proceeds to a debtor as well as the term to maturity and the payment profile on the loan by means of a first computer system, the loan being designed to be at least partially refinanced during the remaining term to maturity,

- requirements having been made to the effect that
  - the term to maturity of the loan is not longer than a predetermined maximum limit or shorter than a predetermined minimum limit,
  - debtor's payments on the loan are within predetermined limits,
    - rules having been made as to how the two above-mentioned requirements are mutually prioritized,
- requirements having been made as to a maximum permissible difference in balance between, on the one hand, payments on the loan and refinancing amounts and, on the other hand, net payments to the owner of the financial instruments applied for the funding,
- requirements having been made as to a maximum permissible difference in proceeds between, on the one hand, the sum of the market price of the volume of the financial instruments applied for the funding of the loan and, on the other hand, the principal of the loan,

- and requirements having been made as to a maximum permissible difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding,

which method comprises

(a) inputting and storing, in a memory or a storage medium of the computer system, a first set of data indicating the parameters: principal of the loan and repayment profile of the loan,

(b) inputting and storing, in a memory or a storage medium of the computer, a second set of data indicating

(i) a maximum and a minimum limit for debtor's payments on the loan for each of a number of periods which together cover the term to maturity of the loan,

(ii) a maximum and a minimum limit for the term to maturity of the loan, and

(iii) rules for the mutual prioritization of, on the one hand, the limits for debtor's payments on the loan input under (i) and, on the other hand, the limits for the term to maturity of the loan input under (ii)

(iv) and optionally a desired/intended payment on the loan or a desired/intended term to maturity when there is not equivalence between the maximum and the minimum limit for the payment on the loan during the first period (i) or when there is not equivalence between the maximum and the minimum limit for the term to maturity (ii),

(c) inputting and storing, in a memory or a storage medium of the computer system, a third set of data indicating a desired/intended refinancing profile, such as one or more point(s) in time at which refinancing is to take place, and

indicating the amount of the remaining debt to be refinanced at said point(s) in time,

and/or said set of data indicating a desired/intended funding profile such as the desired/intended number of financial instruments applied for the funding, with their type and volume,

(d) inputting and storing, in a memory or a storage medium of the computer system, a fourth set of data indicating a maximum permissible difference in balance within a predetermined period, a maximum permissible difference in proceeds and optionally a maximum permissible difference in interest rates equivalent to the difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding,

(e) determining and storing, in a memory or a storage medium of the computer system, a fifth set of data indicating a selected number of financial instruments with inherent characteristics such as type, price/market price, and date of the price/market price,

(f) determining and storing, in a memory or a storage medium of the computer system, a sixth set of data representing a first interest rate profile and either a first term to maturity profile or a first payment profile on the loan,

(g) calculating and storing, in a memory or a storage medium of the computer system, a seventh set of data representing

- a first term to maturity profile or a first payment profile (depending on what was determined under (f)) corresponding to interest and repayment for debtor
- as well as a first remaining debt profile,

the term to maturity profile or payment profile and the remaining debt profile being calculated on the basis of

- the principal of the loan and the repayment profile input under (a),
- the set of data input under (b),
- the refinancing profile and/or the funding profile input under (c),
- and the interest rate profile and either the payment profile or the term to maturity profile determined under (f),

(h) selecting a number of financial instruments among the financial instruments stored under (e), and calculating and storing an eight set of data indicating said selected financial instruments with their volumes to be applied in the funding of the loan, which eight set of data is calculated on the basis of

- the payment profile determined under (f) or calculated under (g) and
- the remaining debt profile calculated under (g),
- the refinancing profile input under (c) and/or the funding profile input under (c),
- the set of data input under (b),
- the requirements input under (d), and
- when the calculation is for a refinancing where financial instruments from an earlier funding have not matured yet, the type, the number and the volume of these instruments,

if necessary performing one or more recalculations, including, if necessary, selection of a new number of the financial instruments stored under (e),

after each recalculation storing, in a memory or a storage medium of the computer system

- the recalculated interest rate profile,
- the recalculated term to maturity profile,
- the recalculated payment profile,
- the recalculated remaining debt profile, and
- the selected financial instruments with their calculated volumes,

until all conditions stated under (b) and (d) have been fulfilled,

after which, if desired, the thus determined combination of the type, the number and the volume of financial instruments for funding the loan

- together with the calculated term to maturity,
- together with the calculated payment profile,
- preferably together with the calculated interest rate, and
- preferably together with the calculated remaining debt profile,

is read out, transferred to a storage medium or sent to another computer system.

In addition to the input, determined and/or calculated data being stored in a memory or on a storage medium, they may be, e.g., output to a display or a printer. The memories applied may, e.g., be electronic memories such as ROM, PROM, EEPROM or RAM and the storage media may e.g. be tapes, discs or CD-ROM.

It will also be possible to input data for use in or resulting from the data processing according to the invention in one set of memories or storage media which may be part of the first or a second computer system and to transfer these data to another set of memories or storage media which may be part of the second or first computer system, these data being

transferred, e.g., via a data transmission line or net connecting the first and the second computer systems, or in a wireless manner, e.g., electro-magnetically or optically.

Among other things, the method according to the invention calculates the volumes of the individual financial instruments which are to be sold to finance the loan. Normally, these volumes will not be whole or round figures, and in certain cases they may be fairly small. The loan-issuing institution solves the divisibility problem by adding together many small loans when financial instruments are sold in the market. It is, of course, crucial that the loan-issuing institution makes an exact registration of the volume of each individual financial instrument applied for the funding of each individual loan.

When it is stated in the present description and claims that the volume, the type and the number of financial instruments for the financing of a loan are determined by the method according to the invention, this indicates that the information which is the result of the method according to the invention may, for instance, be applied as basis for the actual physical action that the lender (for instance a mortgage credit bank) issues/sells the instruments in question. The information, which is the result of the method according to the invention may, of course, also be applied for pricing of a loan in connection with an offer for a loan and/or for calculating lender's risk profile, for instance with the aim of countering the risk through a hedge, without actually issuing the said financial instruments.

It will be appreciated that the order of the above-mentioned inputs/determinations/storage operations (a)-(e) is arbitrary, and, therefore, that the sequence in letters does not indicate an equivalent compulsory sequence in the steps. Step (f) may also be carried out at an arbitrary stage in the

sequence, unless, which is often preferred, it is chosen to have the computer system calculate a first guess at the interest rate profile and either a first term to maturity profile or a first payment profile, in which case step (f) will follow at least step (e). Instead of expressing that data are input/determined and stored in the individual steps, it may simply be expressed (and should be regarded as an equivalent to the first expression form) that by means of the computer system, the method according to the invention calculates on the basis of stored input of data sets (a) - (f). It will also be appreciated that these and other inputs necessary for starting the individual calculations, for instance the first interest rate profile and either the term to maturity profile or the payment profile (f) as mentioned above may be a guess or an initial value which may, of course, also be made/determined by means of the computer system according to predefined rules, and which is then stored/used as initial value. Another example of data which may either be input or guessed/calculated is the desired/intended payment profile or the desired/intended term to maturity under (b)(iv); if an initial value therefor is not input/stored, the computer system will be adapted to "guess" or calculate a value according to a predetermined rule, for instance as an average of the values stored under (b)(i) and (b)(ii).

It will also be appreciated that the condition in (b)(i) also covers the case where no limits for the debtor payments have been set for a period: in this case, the limits in (b)(i) are zero and infinite.

A number of the inputs mentioned above apply for a corresponding period. This is, for instance, the case for the maximum permissible difference in balance and the limits on the payments on the loan. In such a case the corresponding period where the input in question applies is in itself

input, or the period is already generally entered in the computer system. For annuity loans, the period mentioned under (b)(i) is preferably a refinancing period which therefore normally will be default in the computer system, but in principle it may be any period the lender may wish, which is then normally input together with the limits mentioned.

The requirement as to the maximum permissible difference in balance is related to a period which, depending on the legislation or practice on which the calculations are to be based, may be a calendar year or any arbitrarily defined year. In Denmark a balance requirement must be fulfilled per calendar year.

In the calculation of data corresponding to the volumes of the financial instruments applied for the funding, the requirement as to a maximum difference in balance is, according to present Danish rules for mortgage loans, given by a strict balance, i.e. that no substantial difference in balance occurs, or, expressed in another way, the difference is practically zero. The method according to the invention may, however, also be used in cases where a certain difference in balance is tolerated or perhaps even intended, in which case this tolerance or positive difference in balance will be stored as a part of the data set in (d).

In connection with the calculation according to the invention both the requirement as to the difference in proceeds and the requirement as to the difference in interest rates as well as the requirement as to the difference in balance may be indicated in different ways. For example, data may be input which indicate a direct maximum permissible difference between the total amount of the market price of the volumes of the financial instruments applied for the funding and the principal of the loan, and data may be input which indicate a

direct maximum permissible difference between interest rate and the yield to maturity of the financial instruments applied for the funding, as well as data directly indicating a maximum difference in balance allowed. The requirement as to a maximum permissible difference in proceeds may also be entered as data indicating a convergence condition for the difference in proceeds and/or the requirement as to the maximum permissible difference in balance between interest rate and the yield may be given by inputting data indicating a convergence condition for the difference in interest rates and/or the requirement as to the maximum permissible difference in balance may be given by a convergence condition for the difference in balance.

When the loan is disbursed, the disbursement date and/or the maturity date on the loan on the one hand will normally not coincide with the settlement date of the financial instruments applied on the other hand. Thus, the calculations according to the method of the invention are preferably adjusted for a possible difference between on the one hand the disbursement date of the loan and/or the repayment date and on the other hand the payment date of the financial instruments in that a proportional adjustment is made for the already past part or the remaining part of the payment period and the redemption period, respectively. As an example, data may be input or calculated which indicate an adjustment factor for use in the calculation.

In practice it can be chosen to let the maturity of the loan coincide with a debtor date of payment, which will normally require that the last element in the term to maturity profile is calculated accordingly to obtain this coincidence. Here it is most suitable that the term to maturity is calculated prolonged, which then implies that the payments on the loan are reduced, for which reason the minimum limit for the payments on the loan in the last period of financing is

suspended in the calculation.

The calculation method according to the present invention may also be used in cases where the data entered indicate that more than one payment will be made by the debtor within one bond settlement period.

In order to enable calculation of the volume of financial instruments, information is input or present under (c) as to at which point(s) in time refinancing is to be performed and how much is to be refinanced at the said points in time. In one case which is important in practice, the data input indicate that full refinancing of the remaining debt is made at the end of a predetermined period which period is shorter than the term to maturity of the loan and in another important case, the data input indicate that a refinancing of the remaining debt is made by a fixed annual proportion.

The method according to the invention may be used for the determination of the number and the volume of financial instruments, the term to maturity profile and the payment profile both in the situation where the loan is to be calculated for the first time, i.e. the first funding situation, and in the situation where a refinancing is to be calculated. The expression funding thus comprises both "new funding" and "refinancing". In the refinancing situation the calculations include, in addition to the parameters mentioned under (a)-(f), information on type, number and volume of the financial instruments which have still not matured at the time of refinancing. This information will often be stored in the computer system from the previous calculation, but it is, of course, within the scope of the invention to input this information. Evidently, the parameters under (a)-(f) are parameters relating to the funding situation in question, so that in the cases where a refinancing is calculated, the parameters will, of course, relate to the remaining debt of

the loan as the principal of the loan and to the remaining term to maturity as to the term to maturity.

When the patent claims refer to "remaining term to maturity" or "term to maturity", this means - depending on the context - either the remaining term to maturity or the term to maturity which is the basis for the first calculation in the financing period for which the calculation is made, or the remaining term to maturity or the term to maturity that results from a later calculation or recalculation for the financing period to be calculated.

The result of the method according to the invention as defined above is normally at least one set of data which may be used in the first forthcoming financing situation, whether this situation is the first financing period for the loan or a later refinancing period.

To secure that the method according to the invention can check that the data entered under (b) regarding payments on the loan and term to maturity may be complied with, it is, however, normally appropriate to make calculations for all future financing periods until the maturity of the loan. This implies that both some of the data constituting input for calculations for later periods, and all the data being calculated for these later periods are simulated data. The simulations may of course be made on the basis of any desired set of rules, but are suitably made based on an assumption of an unchanged interest rate structure or based upon observed/implicit forward interest rates. The financial instruments in the simulations may for instance be defined either as being unchanged or as being already existing instruments, the term to maturity of the instruments being adjusted. The simulations will typically be of importance to, *inter alia*, the volume of the financial instruments and the payments on the loan in the forthcoming financing period,

which is why the simulations are an essential element of the invention.

It is obvious that at any time of calculation it may, instead, be chosen to have the same interest rate on the loan through the simulations of all future financing periods.

The expression "term to maturity profile" is related to the fact that in the method according to the invention an estimate for the term to maturity is normally made for each individual financing period or refinancing period, such as mentioned above. However, at the end of the calculation, a single well-defined term to maturity must of course result. Thus, term to maturity profile means the sequence of terms to maturity which in each calculation is assigned to the respective refinancing periods.

Correspondingly, the expression "interest rate profile" is related to the fact that in the method according to the invention, an estimate of interest rates on the loan is normally made for each financing period of refinancing period; however, most importantly, at the end of the calculation, a single well-defined interest rate on the loan must result which applies for the nearest financing period for which the calculations are made. Thus, interest rate profile means the sequence of interest rates on the loan which in each calculation is assigned to the respective refinancing periods. As mentioned above it may, in a simplified embodiment, be assumed that the interest rate on the loan is constant throughout all simulated future financing periods, whereby the interest rate profile will be constituted by a sequence of elements of identical value. In this case, the term to maturity profile will often be very simple and simulations will then not be necessary.

In the present description and claims, the expression "financial instruments" has the meaning normally used and thus covers, *inter alia*, all types of interest rate related debts, i.e. all types of bonds, including zero-coupon bonds and derivatives like options, interest rate swaps, CAPS and FLOORS.

When, in the method according to the invention, calculation is made with derivatives which are not directly interest bearing, it is suitable to begin by calculating the expected payment flows in order to be able to calculate an internal interest rate, whereby the payment flow or flows or the likely payment flow/flows is/are expressed in parameters corresponding to the above-mentioned parameters for interest bearing debts, first and foremost a yield to maturity. Thus, for instance, for an option which has a price of DKK 100 and which has a probability of 50 per cent of resulting in proceeds amounting to DKK 210 and a probability of 50 per cent of resulting in proceeds amounting to DKK 0 after one year, this can be done by a statistical calculation of the average proceeds amounting to DKK 105, and expression of the relevant parameters as a price of DKK 100, a quotation of DKK 100 and an interest rate of 5 per cent per year which - together with the interest rate on the other financial instruments applied - is to constitute the basis on which it is checked whether the requirement as to maximum difference in interest rates has been fulfilled. Then these parameters may be entered in the computer system. Alternatively, and often preferred, the data which are entered as characteristics for the instruments in (e) above may be data defining the said financial instruments directly, and the computer system may be adapted to perform a recalculation into parameters characterizing an interest bearing debt according to predetermined principles. In case of CAPS or FLOORS the same procedure may be used as the same payment flows may be expressed by means of equivalent interest

bearing instruments the characteristics of which may then be stored as indicated in (e), or the computer system may preferably be prepared to make a recalculation to the parameters characterising an interest bearing claim according to predetermined principles. It will be appreciated that in the individual case a calculation may also be made on the basis that in the individual funding or refinancing situation, a combination of different types of financial instruments is used, the characteristics to be applied for the calculation being indicated for each type of instrument. In this situation the fulfilment of the requirement as to a maximum permitted difference in interest rates is preferably checked on the basis of a total calculation which is based on the total payment flows from all financial instruments applied. Alternatively, a weighted average of interest rates of the individual instruments may be used.

Thus, in the method according to the invention, calculations may be made on the basis of various types of financial instruments or funding volumes, but in one case which is important in practice, calculation is made on the basis of bonds with a maximum term to maturity corresponding to the refinancing period. The bonds are usually non-callable bullet bonds, including also zero coupon bonds. However, as explained above, the method according to the invention may also advantageously be used for the calculation in connection with other types of financial instruments like e.g. bonds used for serial loans, bonds used for annuity loans, options, CAPS or FLOORS.

As conventionally, meaning of the type of a financial instrument means the combination of all basic information or basic data together defining the said financial instrument unambiguously, thus, so for mortgage bonds, the nominal principal, coupon interest rate, date of maturity, all dates fixed for settlement of interest payments and the ex-coupon

date, i.e. the deadline for the investor to receive the first occurring yield on the bond, as well as possibly the day count convention, i.e. the formula used for the calculation of the payment flow of the bond to an annual yield. The number of financial instruments indicates how many different financial instruments are to be applied. The volume indicates how many entities of the individual financial instrument or how large a nominal sum of the individual financial instruments is/are to be applied.

In accordance with normal practice, the expressions "repayment profile", "remaining debt profile" and "payment profile" indicate the development over time of repayment, remaining debt and payments on the loan, respectively.

The repayment profile may reflect the annuity principle as well as the serial principle and may also represent a bullet loan. In addition to that, any arbitrary placing in time of the repayments is of course possible. For types of loans where the repayment profile depends on the interest rate on the loan, the repayment profile may be determined either on the basis of the interest rates on the loan applying at the time in question or on the basis of the original interest rate on the loqn or on the basis of an arbitrarily determined interest.

The expressions "financing profile" and "funding profile", respectively, indicate the type, the number and the volume of the financial instruments applied for funding. In the present description and claims, the expression may be used for both the desired/intended funding profile which is entered and stored under (c) and which can perhaps not be fulfilled, and for the exact funding profile which is the result of the calculations after the application of the method according to the invention.

The expression "refinancing profile" indicates at which points in time and with which amounts the loan is to be refinanced.

It should be noted that the desired/intended refinancing profile which is stored as a second set of data under (c) above, may in some cases be rewritten as a funding profile, i.e. as a number of financial instruments with their type and volume. An indication of a desired annual interest rate adjustment percentage of 100 may, for example, be rewritten into an indication that the loan is desired to be funded - exclusively through sales of bullet bonds with a term to maturity of 1 year. It is evident that the invention also comprises the case where such a rewriting has taken place in the data stored under (c).

The method according to the invention may be carried out in many different ways; thus, a simultaneous calculation of all parameters to be calculated may be performed, or the calculations may be grouped according to various criteria. In a presently preferred embodiment, which is described in detail in the following example section, the method is divided into a "inner model" and a "outer model", where the inner model has a certain term to maturity as input and then calculates the other parameters to be calculated, including the payment profile, and the outer model determines whether the payments on the loan lie within the limits stipulated, recalculation are being made as explained in the following until all conditions are met.

It is obvious that if the limits stipulated for the payments on the loan in the outer model are 0 and infinite, the solution in the inner model will always meet the demands in the outer model. The inner model will in this case constitute an independent method for determination of, among other things, the type, the number, and the volume of financial

instruments in the situation where the loan has a fixed term to maturity. This independent method is by large described in Danish Patent Application No. 165/96, Danish Patent Application No. 233/97 and International Patent Application No. PCT/DK97/00044. The above comment is therefore of special interest in relation to the below-described preferred embodiment of the method, named Type P, where the determination of the below-discussed factors for function coefficients is made analytically, since this embodiment of the inner model is considered an independent new method according to the invention.

It is obvious that an embodiment in which the term to maturity is calculated simultaneously with the other parameters in "the inner model" will also be comprised by the invention. For instance, this may be the case if the dimension of the iteration procedure is increased by one. (In a preferred embodiment, a Gauss-Newton iteration algorithm is applied).

It is also obvious that there will be cases where a set of data indicates limits for payments on the loan and term to maturity that are not compatible. For instance, it is obvious that a wish for calculation of data for a loan with proceeds of DKK 1,000,000 and a maximum limit for the quarterly payments on the loan of DKK 100 and a maximum limit for the term to maturity of 30 years cannot be fulfilled. In this and similar cases the method according to the invention will either

- in accordance with the prioritization of the limits for the payments on the loan and term to maturity calculate data for a loan for which the highest priority limit is met, and the lowest priority limit is exceeded to the necessary extent

or

- omit to calculate data for the loan, stating which inputs are not compatible.

In the presently preferred embodiment of the method according to the invention the calculations will meet the limits for the term to maturity, and if necessary the limit for the payments on the loan is then broken.

In accordance with the above-explained demand which the present invention is to satisfy, the set of rules will most often be one which to the greatest possible extent maintains the payments on the loan within the given limits and then adjusts the term to maturity to compensate for any fluctuation in the interest rates. However, there may be limits for the duration of the term to maturity; in Denmark, there is at present, for certain market segments, a legislative limit of 30 years for mortgages, and lender may of course have set as a condition that the term to maturity may not exceed a certain value. The set of rules will suitably be structured in accordance herewith so that when a maximum term to maturity (or minimum term to maturity) is reached, a regulation of the payment on the loan to a level above a maximum limit for the payment on the loan (or below a minimum limit for the payment on the loan) will be calculated. The set of rules may be structured in such a way that the regulation of the payment on the loan is made only for the last period(s), or it may be structured in such a way that the regulation is started as soon as a change in the interest rate is observed (or simulated). In this context (and other contexts) it may be allowable for situations where the actual calculation is, as regards time, still far from an absolute term to maturity limit, to calculate, in the term to maturity profile, with terms which are longer than the absolute term to maturity limit.

In the case where the limits for the term to maturity stipulated under (b)(ii) are given a higher priority than the limits for the payment on the loan stipulated under (b)(i) and the term to maturity at which the payment profile is within the limits stipulated under (b)(i) is not within the limits for the term to maturity stipulated under (b)(ii), the term to maturity ma be determined as for instance

(1) the term to maturity which is compatible with a recalculated fixed payment on the loan until the maturity, the recalculated fixed payment on the loan being the smallest possible

(2) the limit for the term to maturity, among the limits for the term to maturity stipulated in (b)(ii), which would otherwise be exceeded,

(3) the term to maturity that is the shorter of either

the term to maturity at which the payment profile is within the limits stipulated therefor under (b)(i)

or

the limit for the term to maturity, among the limits for the term to maturity stipulated under (b)(ii), which is binding, that is, which would otherwise be exceeded.

Among these possibilities it is normally preferred to chose variant (3) as this variant implies that the calculated payment on the loan will increase after an increase in the interest rate and decrease after a decrease in the interest rate, which would not generally be the case in (2). Another advantage of variant (3) is that the calculated payment on the loan normally will normally develop in a more stable

manner at fluctuations in the yield to maturity for the financial instruments applied than in variant (1).

In combination with the maturity of a loan, an immediate result of the calculations may indicate that the maturity of the loan is not coincident with the maturity of the last maturing financial instrument considered applied. It is, of course, possible to use such a result, but in a preferred embodiment the maturity of the loan is adjusted so that it coincides with the maturity of the last maturing financial instrument applied. The adjustment comprises a determination of whether the term to maturity is to be rounded up to a settlement date on the funding side (the maturity date of a financial instrument) or rounded down to the previous settlement date on the funding side (a one period earlier maturity date of a financial instrument). In this case the adjustment of the maturity may suitably be performed as follows:

When the set of data under (c) indicates that calculation is to be performed for the case where a full refinancing of the remaining debt is to be performed periodically with a predetermined period which is shorter than the term to maturity of the loan, and the remaining term to maturity of the loan is shorter than the period of time which according to (c) passes between two consecutive interest rate adjustments, and the remaining term to maturity does not correspond to the maturity of the last maturing financial instruments selected under (h), but it is desired that the loan matures at the same time as the maturity of the last maturing financial instrument selected under (h), then the term to maturity can suitably be determined in the method according to the invention as

(i) the term to maturity of the loan prolonged as little as possible to a date of maturity of one or more of

the selected financial instruments provided the payment profile will not thereby be below the minimum limit for the payment on the loan stated under (b)(i), or

(ii) the term to maturity of the loan shortened as little as possible to a date of maturity of one or more of the selected financial instruments provided the payment profile will not thereby be above the maximum limit for the payment on the loan stated under (b)(i) and provided the condition under (i) is not met, or

(iii) the term to maturity of the loan prolonged as little as possible to a date of maturity of one or more of the selected financial instruments provided none of the conditions stated under (i) or (ii) are met.

When, on the other hand, the set of data under (c) indicates that calculation is to be made for the case where a partial refinancing of the remaining debt is to be performed periodically with a predetermined period which is shorter than the term to maturity of the loan, e.g., so that the refinancing is equivalent to a fixed fraction of the remaining debt of the loan, and the remaining term to maturity of the loan is shorter than or equal to a fixed value, and it is desired that the loan matures no later than the time of maturity indicated under (e) for one or more of the financial instruments applied for refinancing of the loan, then the term to maturity is suitably determined by the method according to the invention as

the term to maturity prolonged as little as possible to a date of maturity of one or more financial instruments.

As mentioned above, it is at present preferred to carry out the method according to the invention by the use of an "inner model" and an "outer model", such as described in detail in

the examples. Here, a more or less arbitrary term to maturity will normally be determined first, whereupon the inner model will be started (as described in the following) and as a result will calculate the payment on the loan, which is then is checked for whether it is within the desired/permissible interval. If the payment on the loan is not within the interval, the term to maturity is adjusted, and the inner model is started with the adjusted term to maturity. This process is repeated until the payment on the loan is within the desired/permissible interval. Then it is checked whether the term to maturity is within the desired/permissible interval. If this is the case, the resulting data may be used. If the term to maturity is not within the desired/permissible interval, then the term to maturity is adjusted so that it is within the interval, and the adjusted term to maturity is used as input to the inner model, which then calculates the payment on the loan for the now determined term to maturity. In this way the payment on the loan is found which is necessary for the term to maturity criterion to be fulfilled.

It will be understood that this embodiment of the method comprises a series of recalculations in the outer model, where each of these recalculations will normally result in a series of recalculations in the inner model. Every time, the recalculations in the inner model are performed until the conditions of the inner model are met.

It is a characteristic feature of the inner model of the presently preferred embodiments that it has an interest rate on a loan as a basis for its calculations. However, it will be understood that here, also any mathematical expression may be used which represents the interest rate on a loan, e.g., the remaining debt profile, the payment profile on the loan or the repayment profile on an annuity loan or the payment profile on the loan for a bullet loan or a serial loan, as

long as the other calculation parameters are adjusted thereto in accordance with current and obvious mathematical principles. It is also evident that, in principle, a first calculation of the number, the type and the volume of the financial instruments may be performed before the first value for the interest rate on a loan is determined, but even if this should be the case, subsequent calculations and if necessary recalculations of, among other things, the interest rate on the loan will have to be performed according to the principles stated above. Thus it would, for instance, be possible under (f) to replace the interest rate profile on the loan with the volume of the financial instruments and under (g) to either calculate the interest rate profile on the loan or recalculate the volume of the financial instruments.

A possible equivalent way to express steps (f) and (g) could thus in these two steps be to replace the interest rate profile with the volume of the financial instruments, however in such a way that the volume of the financial instruments meets the proceeds criterion, and then calculate the interest rate on the loan in step (h).

In the case where no instrument is selected in (e) for which payment falls due within the period till the first point in time where, according to the refinancing profile entered under (c), a refinancing is to be performed, the calculations in the inner model in the preferred embodiment concern a situation where the resulting volume for at least one of the financial instruments applied for the funding will be negative, that is, the debtor should purchase one or more financial instruments during the first to come period in order for the balance requirement to be fulfilled. As it appears from the following, it is presently preferred that precautions are taken to adjust the calculations so that they

do not result in negative volumes of the financial instruments.

In cases where it is indicated in the refinancing profile input that remaining debt is to be refinanced in full, the financial instruments applied for the funding in the inner model may, for instance, be calculated in the same way as the financial instruments used for the initial funding, in other words, a new calculation may be made according to the method of the volume of financial instruments for the funding of a new loan, where the principal of the new loan is equivalent to the amount to be refinanced.

In another embodiment of the invention for the inner model it may be indicated in data input corresponding to the refinancing profile that a partial refinancing of the remaining debt is to be performed. Here, it will be possible in the inner model to find a solution as to the volume of the financial instruments constituting the principal, for instance if the input indicates that a periodic refinancing is desired with a predetermined period which is shorter than the term to maturity of the loan. A solution may also be calculated if it is indicated that a periodical refinancing of a fraction of the remaining debt of the loan is to be made, the denominator of the fraction corresponding to the total number of years until the maturity of the last maturing financial instrument applied at the disbursement of the loan. Here, the selected period may be, e.g., 1 year, but other periods like 2, 4, 5, 6 or 10 years may be selected. Furthermore, periods corresponding to a total number of months, e.g. 2, 3, 4 and 6 months may be selected.

In connection with full or partial refinancing, it will normally be necessary in the inner model to calculate with one or more new refinancing instruments that are not comprised by the range of initial financial instruments

which, according to the data given, constituted the series of funding volumes applied when the loan was disbursed or when a previous refinancing of the loan was performed. Normally, these new refinancing instruments will have such a term to maturity that they mature on a later date than the dates on which the range of initial financial instruments mature. In case of partial refinancing, the refinancing in the inner model may also include an additional volume of the financial instruments applied remaining at the time of the refinancing. In the following, the volume of such additional funding and new refinancing instruments are also designated as the marginal funding.

The calculation method according to the present invention will also be capable of providing a solution as to the volume of the marginal funding. When calculating the volume of the marginal funding, data comprising possible new refinancing instruments within the range of selected financial instruments must be entered. In case of calculation of refinancing the requirement as to proceeds may, e.g., be given as a requirement as to the difference between, on the one hand, a funding demand based on the balance requirement and, on the other hand, the sum of the market price of the marginal funding.

As mentioned above, a refinancing may be performed by the issue of new financial instruments as well as additional issue of already applied financial instruments. However, it will also theoretically be possible to repurchase already applied financial instruments, but this would involve a number of inconveniences, among other things, an extra interest rate risk for lender and problems with the pledges, which is why repurchasing is not performed in practice.

According to a preferred embodiment of the method, the volume of the marginal funding will therefore be calculated taking

into consideration the volume of the previously applied financial instruments remaining at the time of refinancing.

In an embodiment of the inner model, in the following detailed description also called Type F, the set of data under (c) indicates that a calculation is to be made for the case where a full refinancing of the remaining debt is to be performed periodically with a predetermined period which period is shorter than the term to maturity of the loan, which method for determination of the indicated volumes of financial instruments in step (h) comprises calculating the difference in proceeds for the calculated volumes of the financial instruments applied for the funding and/or calculation of an adjustment of the interest rate on the loan, said adjustment of the interest rate preferably being calculated taking into consideration the calculated difference in proceeds, it being calculated to whether the adjustment of the interest rate is so small that it fulfils the requirement as to a maximum permissible difference in interest rates or a convergence condition of the difference in interest rates.

It should be noted that when a requirement as to a difference in proceeds of 0 or very close to 0 is met, then the difference in the interest rate will automatically be 0 or very close to 0, which means in this case, the requirement as to difference in the interest rate may suitably be left out from the starting conditions input.

This is the reason why input about maximum permissible difference in the interest rate is stated as optional, while input about maximum permissible difference in the proceeds is stated as compulsory in all cases. However, it should be noted that a compulsory input about maximum permissible difference in proceeds may be accomplished by inputting information which is fully equivalent herewith, e.g., an

interest rate input, and that such substitutions are, of course, comprised by the present invention.

In case the requirements or conditions as to the difference in proceeds or the difference in the interest rate is not fulfilled, the recalculations in the inner model of Type F include one or more interest rate iterations, each interest rate iteration including

calculating and storing, in a memory or a storage medium of the computer, data indicating a new interest rate which is preferably based on the previous interest rate on the loan and the calculated adjustment of the interest rate,

calculating and storing, in a memory or a storage medium of the computer, data indicating a new payment profile and a new remaining debt profile for debtor, which payment profile and remaining debt profile are calculated taking into consideration the new interest rate on the loan, the principal of the loan, the repayment profile entered under (a) and the refinancing profile and/or the funding profile entered under (c), and

calculating and storing, in a memory or a storage medium of the computer system, of data indicating a new set of volumes for the financial instruments applied for the funding.

The interest rate iteration is preferably made by applying a numeric optimization algorithm or by "grid search".

As examples of numeric optimization algorithms may be mentioned a Gauss-Newton algorithm, a Gauss algorithm, a Newton-Ramphson algorithm, a quadratic hill climbing algorithm, a quasi-Newton algorithm, a maximum likelihood algorithm, a method of scoring algorithm and a BHHH algorithm. As it appears from the detailed description of Type F, a Gauss-Newton algorithm has proved to be very suitable.

When the relevant requirement(s) as to a maximum permissible difference in proceeds and/or the requirement as to a maximum permissible difference in interest rates is/are fulfilled in the Type F embodiment, it is appropriate to determine whether all the calculated volumes of financial instruments are positive. In case the calculated set of volumes comprises at least one negative volume, an option is to apply the result as such, meaning that the calculation indicates that the debtor is to buy one or more financial instruments in order to fulfil the balance requirement. As mentioned above, this is normally not preferred, and therefore, in this case the method according to the invention usually further comprises either

i) selection of a new number of financial instruments among the financial instruments stored under (e), in that one or more of the instruments in the new number of instruments is/are determined in such a way that the payments on this/these instruments falls due relatively later in relation to the original number of financial instruments, whereupon a recalculation is made as indicated in connection with the description of the Type F embodiment given here and in the following, or

ii) the negative volume or the negative volumes is/are set equal to 0, after which recalculation is made as indicated in connection with the description of the Type F embodiment given here and in the following.

In case the data entered indicate that a partial refinancing is to be performed, the volumes of the financial instruments applied for the funding or the refinancing will, in a preferred embodiment for the inner model, be calculated so that they reflect the remaining debt development given by the remaining debt profile. This calculation may include the use of a first function adapted to the remaining debt profile as explained in the following. For example, in case the entered data indicate a difference between on the one hand the

payment date of the loan and/or the repayment date of the loan, and on the other hand the settlement date of the financial instruments, it is possible in the calculation according to a preferred embodiment of the invention to determine the volume or volumes of one or more financial instruments in such a manner that this instrument or these instruments does not/do not reflect the polynomial function, but rather contribute(s) to a solution to the above marginal conditions.

In a preferred embodiment of the invention, a determination is made in the inner model as to whether the so calculated volumes of financial instruments fulfil one or more predetermined convergence condition(s). If such a condition or such conditions is/are not fulfilled, one or more iterations may be calculated until the new set of data of financial instruments fulfils one or more convergence condition(s).

In a preferred embodiment of the inner model, the function coefficients will be calculated on the basis of a calculated difference in proceeds and/or a calculated difference in refinancing, preferably corresponding to the difference between on the one hand a funding demand based on the balance requirement and on the other hand a preferred refinancing. The function coefficients may be found either analytically or by iteration.

In case the calculation according to this embodiment of the inner model determines that the calculated volumes of financial instruments applied for the funding or refinancing do not fulfil requirements as to a difference in the interest rates stipulated in the input data, then, in a preferred embodiment, one or more recalculations in the form of interest rate iterations will be made to determine or calculate a new interest rate, after which a new set of

financial instruments is calculated. An interest rate iteration is made until the requirements as to a difference in the interest rates is fulfilled. The discussion which follows gives examples with a detailed explanation of this embodiment. Both the situation where the function coefficients of the function adapted to the remaining debt profile are found by iteration and the situation where the function coefficients are found analytically are described.

The following is a detailed description of the case where the set of data (c) indicates that calculation is to be made for the case that a partial refinancing of the remaining debt is made periodically with a predetermined period which is shorter than the period being shorter than the term to maturity of the loan, e.g., in such a manner that the refinancing equals a fixed fraction of the remaining debt of the loan. In this embodiment, generally termed P in the following detailed description, some of or all of the financial instruments applied for the funding are, in the inner model, calculated in the first calculation in step (g) so that they substantially reflect a shifted-level remaining debt profile, whereupon, if necessary, recalculations are made until all conditions indicated under (d) are fulfilled.

The adjustment to a shifted-level remaining debt profile is suitably made by calculating the volume of some of or all of the financial instruments in the calculation in step (h) and possibly in one or more recalculations in step (h) using a function which is adapted to a shifted-level remaining debt profile. This function is suitably a polynomial function with a maximum degree of 1 (one) less than the number of financial instruments applied.

The polynomial function is suitably calculated using a statistic curve fit method. It has been found that the least squares' method is an appropriate statistic curve fit method,

but other statistic curve fit methods like other maximum likelihood methods or cubic splines methods are also interesting methods for this use.

In a preferred embodiment of the invention a calculation is made based on an analytic derivation (as opposed to iteration) of one or more coefficients in the polynomial function. The calculation is performed so the subsequent determination of the marginal funding values, which are equal to the difference between the value of the polynomial function and the volume of the already issued financial instruments, meet the requirements for maximum difference in proceeds and, if possible, the demand as to maximum difference in balance when at the same time the actual refinancing percentage equals the intended refinancing percentage.

In cases where coefficients in the polynomial function are calculated so that one or more of the marginal volumes of the financial instruments are negative, the said marginal volumes are not applied, which is indicated by an adjustment of an indicator function. The indicator function will, in this embodiment, be an m-dimensional vector in which the elements have either the value one or the value zero, and where the value zero indicates that the said financial instrument is not applied in the financing. Based on the adjusted indicator function a new calculation of one or more coefficients in the polynomial function is performed, the resulting marginal volumes of the financial instruments are checked, and, if necessary, the indicator function is adjusted again.

The calculation of the coefficients in the polynomial function and the adjustment of the indicator function continue until either

all increases in the financial instruments are non-

negative (i.e. either positive or zero)

or

either the first element in the indicator function has the value zero, or the sum of the elements in the indicator function is strictly less than 2, in each of which cases only one coefficient in the polynomial function is calculated so that the resulting series of marginal volumes of the financial instruments meets the requirements as to maximum difference in proceeds; the resulting refinancing will be defined by a residual calculation in accordance with the demand as to maximum difference in balance.

It is also possible to adjust only one element in the indicator function at a time.

The above-discussed analytic method for determination of the function coefficients in the polynomial function is efficient with respect to calculation and hence a time-saving method. This method is described in detail in section 3.2.3 with the heading "the inner model for Type P - the analytic solution" in the following detailed part of this description.

As mentioned above, an aspect of the invention concerns an embodiment which applies the analytical determination of the function coefficients in the polynomial function in calculations which correspond to the above-explained embodiment designated type P, but where the limits stipulated for the payments on the loan in the inner model are zero and infinite. Thus, this aspect of the invention concerns a method for determining the type, the number, and the volume of financial instruments for the funding of a loan with equivalent proceeds to a debtor by means of a first computer

system, the loan being designed to be at least partially refinanced during the remaining term to maturity,

- requirements having been made as to a maximum permissible difference in balance between, on the one hand, payments on the loan and refinancing amounts and, on the other hand, net payments to the owner of the financial instruments applied for the funding,
- requirements having been made as to a maximum permissible difference in proceeds between, on the one hand, the sum of the market price of the volume of the financial instruments applied for the funding of the loan, and on the other hand, the principal of the loan,
- and requirements having been made as to a maximum permissible difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding,

which method comprises

(a) inputting and storing, in a memory or a storage medium of the computer system, a first set of data indicating the parameters: principal of the loan and repayment profile of the loan,

(b) inputting and storing, in a memory or a storage medium of the computer, a second set of data indicating the term to maturity of the loan,

(c) inputting and storing, in a memory or a storage medium of the computer system, a third set of data indicating a desired/intended refinancing profile, such as one or more point(s) in time at which refinancing is to take place, and indicating the amount of the remaining debt to be refinanced at said point(s) in time,

(d) inputting and storing, in a memory or a storage medium of the computer system, a fourth set of data indicating a maximum permissible difference in balance within a predetermined period, a maximum permissible difference in proceeds and a maximum permissible difference in interest rates equivalent to the difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding,

(e) determining and storing, in a memory or a storage medium of the computer system, a fifth set of data indicating a selected number of financial instruments with inherent characteristics such as type, price/market price, and date of the price/market price,

(f) determining and storing, in a memory or a storage medium of the computer system, a sixth set of data representing a first interest rate profile,

(g) calculating and storing, in a memory or a storage medium of the computer system, a seventh set of data representing

- a first payment profile corresponding to interest and repayment for debtor
- as well as a first remaining debt profile,

the payment profile and the remaining debt profile being calculated on the basis of

- the principal of the loan and the repayment profile input under (a),
- the set of data input under (b),
- the refinancing profile and/or the funding profile input under (c),
- and the interest rate profile determined under (f),

(h) selecting a number of financial instruments among the financial instruments stored under (e), and calculating and storing an eight set of data indicating said selected financial instruments with their volumes to be applied in the funding of the loan, which eight set of data is calculated on the basis of

- the payment profile calculated under (g) and
- the remaining debt profile calculated under (g),
- the refinancing profile input under (c) and/or the funding profile input under (c),
- the set of data input under (b),
- the requirements input under (d), and
- when the calculation is for a refinancing where financial instruments from an earlier funding have not matured yet, the type, the number and the volume of these instruments,

if necessary performing one or more recalculations, including, if necessary, selection of a new number of the financial instruments stored under (e),

after each recalculation storing, in a memory or a storage medium of the computer system

- the recalculated interest rate profile,
- the recalculated payment profile,
- the recalculated remaining debt profile, and
- the selected financial instruments with their calculated volumes,

until all conditions stated under (b) and (d) have been fulfilled,

after which, if desired, the thus determined combination of the type, the number and the volume of financial instruments for funding the loan

- together with the calculated payment profile,
- preferably together with the calculated interest rate, and
- preferably together with the calculated remaining debt profile,

is read out, transferred to a storage medium or sent to another computer system,

the set of data (c) indicating that calculation is to be performed for the case where partial refinancing of the remaining debt is performed periodically with a predetermined period, which period is shorter than the term to maturity of the loan, e.g. in such a way that the refinancing is equivalent to a fixed fraction of the remaining debt of the loan, some of or all of the financial instruments applied for the funding in the first calculation in step (h) being calculated in such a way that they substantially reflect a shifted level remaining debt profile, whereupon, if necessary, recalculations are performed until all the requirements mentioned under (d) are fulfilled,

the volume of some of or all of the financial instruments being calculated, in the calculation in step (h), by applying a function adjusted to a shifted level remaining debt profile, the determination of one or more of the function coefficients for the function adjusted to the shifted level remaining debt profile being performed analytically.

Interesting and preferred embodiments of this aspect of the invention appear from the patent claims depending on the independent patent claim corresponding to the definition

given immediately above; the more detailed explanations of these embodiments are the same as the explanations, given above and in the detailed part of this description, of the corresponding embodiments where the limits stipulated for the payments on the loan in the outer model differ from 0 and infinity.

On the other hand, the function coefficients may also be calculated by iteration as described in the in the immediately following sections and as described in detail in section 3.2.2 with the heading "The inner model for Type P - the iterative solution" in the following detailed part of this description.

In the embodiment designated Type P, recalculation of all of or some of the data mentioned in (g) and (h), and/or one or more function coefficients to the function representing the shifted-level remaining debt profile, and/or the interest rate in the inner model may be performed by iteration carried out using numeric optimization algorithms or by grid search. Also in this case, one of the optimization algorithms mentioned above in connection with Type F may be used as an optimization algorithm, and also in this case the optimization algorithm is, suitably, a Gauss-Newton algorithm.

In case the requirements as to the difference in proceeds and/or the difference in interest rates and/or the difference in balance calculated taking into consideration the refinancing profile entered under (c) are not fulfilled, then the recalculations in the inner model of the Type P embodiment may include one or more iterations, each iteration comprising

calculating and storing data indicating a new interest rate and/or

calculating and storing data indicating a new payment profile and a new remaining debt profile for debtor, which payment profile and remaining debt profile are calculated taking into consideration the new interest rate on the loan, the principal of the loan, and the repayment profile entered under (a) the refinancing profile and/or the funding profile entered under (c), and the term to maturity and/or

calculating and storing data indicating a new set of coefficients for the function which is adapted to the shifted-level remaining debt profile, and/or

calculating and storing data indicating a new set of volumes of the financial instruments applied for the funding, which new set of volumes is calculated on the basis of the financial instruments already determined for the funding, and the new payment profile, and the new remaining debt profile as well as the requirement as to the maximum difference in balance.

In the example given below in the detailed part of this description, section 3.2.2, it has been chosen to iterate as to the proceeds requirement and the difference in balance extended taking into consideration the refinancing profile entered under (c), and only when the two requirements are fulfilled, iteration is carried out as to the interest rate. It will be understood that the iteration may be performed in an arbitrary order, and that here also iteration of the function applied, the so-called trend function, may be included.

In this case, the method in step (h) in the inner model may comprise determination of whether the calculated volumes of financial instruments fulfil at least two of two or more predetermined convergence conditions, which are preferably calculated taking into consideration a calculated difference in proceeds and a difference in balance calculated taking into consideration the refinancing profile entered under (c),

and in case the calculated volumes of financial instruments do not fulfil these conditions, then the recalculations may include one or more iteration(s) of the coefficients for the function which is adjusted to a shifted-level remaining debt profile, each iteration comprising

calculating and storing data indicating two or more new function coefficients for the function representing the shifted-level remaining debt profile,

calculating and storing data indicating a new set of volumes for the financial instruments applied for the funding, which new set of volumes is calculated taking into consideration the new function representing the shifted-level remaining debt profile,

determining whether the new set of calculated volumes of financial instruments fulfils the at least two or more predetermined convergence conditions, until the new set of calculated volumes of financial instruments fulfils these conditions. The new function coefficient(s) is/are suitably calculated taking into consideration the calculated difference in proceeds and a difference in balance calculated taking into consideration the refinancing profile entered under (c).

The difference between the interest rate on the loan and the yield on the calculated volumes of the financial instruments may be calculated in the inner model, it being calculated whether the difference in interest rates is so small that it fulfils the requirement as to maximum permissible difference in interest rates or a convergence condition for the difference in interest rates, and in case the determined requirements as to the difference in interest rates are not fulfilled, then the recalculations may comprise one or more interest rate iterations, each interest rate iteration including

calculating and storing an adjustment of the interest rate, the adjustment of the interest rate preferably being

calculated taking into consideration the difference between the interest rate on the loan and the yield to maturity on the calculated volumes of the financial instruments, e.g. by use of a Gauss-Newton algorithm,

calculating and storing data indicating a new interest rate which is preferably based on the previous interest rate and the calculated adjustment of the interest rate on the loan,

calculating and storing data indicating a new payment profile and remaining debt profile for debtor, which payment profile and remaining debt profile are calculated taking into consideration the new interest rate, the principal of the loan, the term to maturity and the repayment profile entered under (a), and the refinancing profile and/or the funding profile entered under (c), and

calculating and storing data indicating a new set of coefficients for the function adapted to the shifted-level remaining debt profile, and

calculating and storing data indicating a new set of volumes for financial instruments applied for the funding.

It is also within the scope of the invention in connection with the calculations in the inner model of Type P to determine, at the same time, whether the calculated volumes of financial instruments fulfil at least three of three or more predetermined convergence conditions which are preferably calculated taking into consideration a calculated difference in proceeds, a difference in balance calculated taking into consideration the refinancing profile entered under (c) and a maximum permissible difference in interest rates, and in case the calculated volumes of financial instruments do not fulfil these conditions, then to have the recalculations comprise one or more iterations, each iteration comprising

calculating and storing an adjustment of the interest rate, said adjustment of the interest rate preferably being

calculated taking into consideration the difference between the interest rate on the loan and the yield to maturity of the calculated volumes of financial instruments,

calculating and storing data indicating a new interest rate which is preferably based on the previous interest rate and the calculated adjustment of the interest rate on the loan,

calculating and storing data indicating a new payment profile and remaining debt profile for debtor, said payment profile and remaining debt profile being calculated taking into consideration the new interest rate on the loan, the principal of the loan, the term to maturity, the repayment profile entered under (a) and the refinancing profile and/or the funding profile entered under (c),

calculating and storing data indicating a new set of coefficients for the function adapted to the shifted-level remaining debt profile, and

calculating and storing data indicating a new set of volumes for financial instruments applied for the funding, which new set of volumes is calculated taking into consideration the new function representing the shifted-level remaining debt profile,

determining whether the new set of calculated volumes of financial instruments fulfils the at least three or more predetermined convergence conditions.

Also in this connection, the iterations may be performed by the use of a numeric optimization algorithm, preferably a three-dimensional Gauss-Newton algorithm.

In case the calculated set of volumes in the calculations of the inner model of type P includes at least one negative volume, the negative volume(s) may suitably be set equal to 0 - to avoid negative volumes in the result, cf. the above comments concerning the normally undesired situation where debtor is to purchase financial instruments - after which the

calculations continue on the basis of the thus determined volumes of the financial instruments.

According to an embodiment of the invention it will also be possible to determine the volumes of the financial instruments applied for the loan in the cases where data are entered corresponding to a funding profile desired by the debtor and comprising desired financial instruments. In this case the calculations may also comprise calculation of whether the volumes of financial instruments in the funding profile indicated fulfils the requirement as to maximum permissible difference in proceeds, and in case the indicated volumes do not fulfil this requirement, then, according to a preferred embodiment of the invention, one or more adjustments of the previously indicated volumes is/are performed, adjustments being performed until the new set of financial instruments fulfils the requirement as to a maximum permissible difference in proceeds.

In addition to the calculation of the proceeds criterion, it is preferred that a calculation is also made as to whether the requirement for maximum permissible difference in balance is fulfilled, and in case the calculated volumes do not fulfil this requirement, one or more calculations of the new financial instruments which do not fulfil the requirement as to maximum permissible difference in balance is/are made.

Calculation of new financial instruments will here preferably be made for one or more financial instruments to which repayments are to be made in a period in which the requirement as to maximum difference in balance is not fulfilled. In a preferred embodiment, the calculation will be performed for one or more financial instruments to which repayments are to be made in the last period wherein the requirement as to maximum permissible difference in balance is not fulfilled. Preferably, the calculation of new

financial instruments is based on the difference in balance for the periods in which the corresponding previously found financial instruments do not fulfil the requirement as to maximum permissible difference in balance.

It applies generally to the method according to the invention that in many cases it will be possible after a result has been reached to make a new calculation on the basis of other instruments in order to assess whether a cheaper loan is thereby obtainable.

The range of financial instruments determined under (e) is selected among a number of available financial instruments. It will be appreciated that, if desired, this number of instruments may be entered into a database in the computer system or may be available via a net and that, if desired, the determination may be performed automatically or semi-automatically by means of the computer system according to predetermined criteria or functions.

The invention also relates to a computer system for determining the type, the number, and the volume of financial instruments for the funding of a loan with equivalent proceeds to a debtor as well as the term to maturity and the payment profile on the loan, the loan being designed to be at least partially refinanced during the remaining term to maturity,

- requirements having been made to the effect that
  - the term to maturity of the loan is not longer than a predetermined maximum limit or shorter than a predetermined minimum limit,
  - debtor's payments on the loan are within predetermined limits,

- rules having been made as to how the two above-mentioned requirements are mutually prioritized,

- requirements having been made as to a maximum permissible difference in balance between, on the one hand, payments on the loan and refinancing amounts and, on the other hand, net payments to the owner of the financial instruments applied for the funding,
- requirements having been made as to a maximum permissible difference in proceeds between, on the one hand, the sum of the market price of the volume of the financial instruments applied for the funding of the loan and, on the other hand, the principal of the loan,
- and requirements having been made as to a maximum permissible difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding,

which computer system comprises

(a) means, typically input means and a memory or a storage medium, for inputting and storing a first set of data indicating the parameters: principal of the loan and repayment profile of the loan,

(b) means, typically input means and a memory or a storage medium, for inputting and storing a second set of data indicating

(i) a maximum and a minimum limit for debtor's payments on the loan for each of a number of periods which together cover the term to maturity of the loan,

(ii) a maximum and a minimum limit for the term to maturity of the loan, and

(iii) rules for the mutual prioritization of, on the one hand, the limits for debtor's payments on the loan input under (i) and, on the other hand, the limits for

the term to maturity of the loan input under (ii)

(iv) and optionally a desired/intended payment on the loan or a desired/intended term to maturity when there is not equivalence between the maximum and the minimum limit for the payment on the loan during the first period (i) or when there is not equivalence between the maximum and the minimum limit for the term to maturity (ii),

(c) means, typically input means and a memory or a storage medium, for inputting and storing a third set of data indicating a desired/intended refinancing profile, such as one or more point(s) in time at which refinancing is to take place, and indicating the amount of the remaining debt to be refinanced at said point(s) in time,

and/or said set of data indicating a desired/intended funding profile such as the desired/intended number of financial instruments applied for the funding, with their type and volume,

(d) means, typically input means and a memory or a storage medium, for inputting and storing a fourth set of data indicating a maximum permissible difference in balance within a predetermined period, a maximum permissible difference in proceeds and optionally a maximum permissible difference in interest rates equivalent to the difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding,

(e) means, typically input means and a memory or a storage medium, for determining and storing a fifth set of data indicating a selected number of financial instruments with inherent characteristics such as type, price/market price, and date of the price/market price,

(f) means, typically input means and/or calculation means and a memory or a storage medium, for determining and storing a sixth set of data representing a first interest rate profile and either a first term to maturity profile or a first payment profile on the loan,

(g) means, typically calculation means and a memory or a storage medium, for calculating and storing a seventh set of data representing

- a first term to maturity profile or a first payment profile (depending on what was determined under (f)) corresponding to interest and repayment for debtor
- as well as a first remaining debt profile,

the term to maturity profile or payment profile and the remaining debt profile being calculated on the basis of

- the principal of the loan and the repayment profile input under (a),
- the set of data input under (b),
- the refinancing profile and/or the funding profile input under (c),
- and the interest rate profile and either the payment profile or the term to maturity profile determined under (f),

(h) means, typically calculation means and a memory or a storage medium, for selecting a number of financial instruments among the financial instruments stored under (e), and calculating and storing an eight set of data indicating said selected financial instruments with their volumes to be applied in the funding of the loan, which eight set of data is calculated on the basis of

- the payment profile determined under (f) or calculated under (g) and
- the remaining debt profile calculated under (g),
- the refinancing profile input under (c) and/or the funding profile input under (c),
- the set of data input under (b),
- the requirements as to maximum permissible difference in balance, maximum permissible difference in proceeds and maximum permissible difference in interest rates input under (d), and
- when the calculation is for a refinancing where financial instruments from an earlier funding have not matured yet, the type, the number and the volume of these instruments,

the means being adapted to perform, if necessary, one or more recalculations, including, if necessary, selection of a new number of the financial instruments stored under (e),

the means being further adapted to store, after each recalculation,

- the recalculated interest rate profile,
- the recalculated term to maturity profile,
- the recalculated payment profile,
- the recalculated remaining debt profile, and
- the selected financial instruments with their calculated volumes,

until all conditions stated under (b) and (d) have been fulfilled,

means for outputting the thus determined combination of the type, the number and the volume of financial instruments for funding the loan

- together with the calculated term to maturity,
- together with the calculated payment profile,
- preferably together with the calculated interest rate, and
- preferably together with the calculated remaining debt profile,

or means for transferring the combination to a storage medium or for sending it to another computer system.

A corresponding system which constitutes a separate aspect of the present invention is adapted to the above-mentioned aspect of the invention which concerns the type P-calculation for the case where the limits for the payment on the loan in the outer model is 0 and infinite and where the relevant function coefficients are found analytically.

The system according to the present invention thus comprises means for inputting and storing the necessary data for the calculations. The input means may comprise a keyboard or a mouse, a scanner, a microphone, a touch-sensitive screen or plate or the like, but may also comprise means for electronic input via a storage medium or via a data network. As mentioned above, the means for storing may be electronic memories such as ROM, PROM, EPROM, EEPROM or RAM, or erasable or non-erasable plate or tape storage media such as tapes, discs or CD-ROM.

The input means for may comprise a database or another computer system and/or a data network from which the computer system can enter data such as the necessary prices of the financial instruments used for the funding, information about lender and/or other information needed or desired for the calculations or other purposes.

The output means may comprise one or more data screens, one or more printers, one or more telefax machines, one or more voice generating devices and/or connection means which electronically connect the computer system to a data network designed to transfer data from the computer system to a data transmission system that comprise or is connected to one or more output means of the above-mentioned type. The output means will typically be connected to a data transmission system which, for instance, may comprise a computer close to or remote from the computer system and which is connected electronically to the computer system via the data network. The output means may, for instance, be placed with an adviser such as a real estate agent, a bank branch or a branch of another financial institution, who or which via the data network has access to the calculation means used for the calculations and comprised by the computer system.

The data network may constitute or be a part of a local network, which may be a part of or be connected to a wide area network. The local network is typically adapted to electronically distribute and/or collect data from a number of units. A unit may, e.g., comprise a geographic district which comprises a number of data systems/computers, and/or a local area which also may comprise a number of data systems/computers.

Furthermore the data network may comprise one or be one or more ISDN connections or further developments hereof, one or more telecommunication connections connected to the computer system by means of at least one telecommunication connection means such as a modem, the telecommunication connections comprising one or more telephone connections, wireless connections or other means of data connection. Typically, the data network may combine the above telecommunication connections and telecommunication connection means.

For example, an ISDN connection (or a further development hereof) or a telecommunication connection with modem may be used as telecommunication connection between a real estate agent and the computer system, or these connections may be used to transfer, from central financial institutions, such as stock exchanges, data for the financial instruments used for the calculations, such as price and yield to maturity of bonds.

Furthermore, the system comprises means for calculation adapted to perform the calculations which are necessary for the method. The means for calculation may typically comprise one or more microprocessors.

A system according to the present invention may be a computer system programmed in such a way that it is capable of performing the necessary for working the method according to the invention. In this connection it should be noted that there may be different embodiments of the system, corresponding to these different embodiments being designed to perform the calculations indicated in the various embodiments of the method according to the invention mentioned above and in the claims.

The means for calculations may comprise one or more electronic calculation circuits, such as a microprocessor which is an integrated part of the computer system, or which is connected to the computer system via the data network, so that one or more external processors which may, e.g., be a part of a calculation central, may perform the calculations necessary for the method based on data transferred via the data network from the data processing system.

The method according to the invention is typically performed on the computer system according to the invention.

Further embodiments and details of the method and the system according to the present invention appear from the claims and the detailed description given in connection with the drawings and the examples.

BRIEF DESCRIPTION OF THE DRAWINGS

Fig. 1 shows an example of the variation in the bond yield as a function of the term to maturity,

Fig. 2 shows an example of the determination of the interest rate on a loan which is financed by issue/sale of financial instruments in accordance with a preferred embodiment of the invention at an increase in the yield to maturity of the financial instruments,

Fig. 3 shows an example of the determination of a payment profile on the loan and the term to maturity of a loan which is financed by issue/sale of financial instruments in accordance with a preferred embodiment of the invention at an increase in the yield to maturity of the financial instruments whereby a maximum limit input for the payment on the loan becomes binding,

Fig. 4 shows a computer system which may be used in performing the methods according to the invention,

Fig. 5 shows a characteristic example of limits for the payments on an annuity loan,

Fig. 6 shows a characteristic example of limits for the payments on a serial loan,

Fig. 7 shows characteristic shifts of the payments on the loan as a function of the term to maturity,

Fig. 8 shows characteristic shifts of the payments on the loan as a function of the term to maturity of a annuity loan,

Fig. 9 shows characteristic shifts of the payments on the loan as a function of the term to maturity of a serial loan,

Fig. 10 shows characteristic jump-wise shifts of the payments on the loan as a function of the term to maturity of an annuity loan of Type F,

Fig. 11 shows characteristic jump-wise shifts of the payments on the loan as a function of the term to maturity of a serial loan of Type F,

Fig. 12 shows a flow chart describing a data processing method for the outer model for the calculation of the volumes of funding volumes of a loan in accordance with a first embodiment of the invention, called Type F,

Fig. 13 shows a graphic representation of an example of a function F, which indicates how much the payments on the loan differ from the maximum and the minimum limit; it is taken into consideration whether it is the maximum or the minimum limit that is exceeded without adjustment of the term to maturity,

Fig. 14 shows a graphic representation of a function F, which indicates how much the payments on the loan differ from the maximum and the minimum limit; it is not taken into consideration whether it is the maximum or the minimum limit that is exceeded without adjustment of the term to maturity,

Fig. 15 shows a step in the iteration routine in the Gauss-Newton algorithm used for calculation of the term to maturity of a loan,

Fig. 16 shows a flow chart describing the inner model for the data processing method for the calculation of the volumes of volumes of funding volumes for a loan according to a first embodiment of the invention, called Type F,

Fig. 17 shows a step in the iteration routine in the Gauss-Newton algorithm used for calculation of the interest rate,

Fig. 18 shows a flow chart describing the inner model for the data processing method for the calculation of the funding volumes for a loan according to an embodiment of the invention called Type F+, which is a continuation of Type F in cases where Type F leads to one or more negative funding volumes,

Fig. 19 shows a characteristic example of a payment profile on a annuity loan as a function of time,

Fig. 20 shows a characteristic example of a payment profile on an annuity loan as a function of time, the payments on the loan being close to the maximum limit for the payments on the loan at an increase in the interest rates,

Fig. 21 shows a characteristic example of a payment profile on a serial loan as a function of time,

Fig. 22 shows a characteristic example of a payment profile on a serial loan as a function of time, the payments on the loan being close to the maximum limit for the payments on the loan at an increase in the interest rates,

Fig. 23 shows a flow chart describing the outer model of a data processing method for the calculation of the funding volumes for the funding of a loan according to a first variant of the embodiment called Type P,

Fig. 24 shows a flow chart describing the iterative inner model for the embodiment called Type P for the calculation of the funding volumes of a loan,

Fig. 25 shows a characteristic example of a trend function which can be used in the embodiment called Type P,

Fig. 26 shows a flow chart describing the analytical inner model for the embodiment called Type P for the calculation of the funding volumes of a loan, and

Fig. 27 shows a characteristic example of an initial trend function and an adjusted trend function used in the embodiment called Type P.

GENERAL DESCRIPTION OF LOANS WITH ADJUSTABLE INTEREST RATES

In the following is given a brief general description of the new type of loans with adjustable interest rates where an exact calculation of the funding is made possible by an embodiment according to the present invention, the loans also termed LAIR.

The debtor receives exact calculations of various alternatives concerning the combination of a loan with respect to term to maturity and repayment profile. For example, debtor may decide for himself how often and when a LAIR is to be refinanced and the part of the debt he prefers to be refinanced.

An investor prefers, as an important element, to combine portfolios where the return is known with a reasonable certainty. Internationally the most well-known type of bond is non-callable bullet bonds. They fulfil the demand for a stable return which also is known beforehand. However, this is not the case with the Danish callable mortgage bonds.

LAIR seeks to combine the preferences of the debtor and the creditor. Debtor may want a loan with a term to maturity of 30 years, where the investor may have an maximum investment period of 5 years. When a loan with a term to maturity of 30 years is funded only by means of a bond series of 30 years, as is the case at the moment, it is difficult to combine the preferences of the two parties.

The two parties meet, as LAIR permits funding with a range of e.g. non-callable bullet bonds with maturities from 1 to 10 years, irrespective whether the debtor desires a loan with a term to maturity of 10, 20 or 30 years.

In case of a yield structure, where the short-term interest rate is lower than the long-term interest rate (see the example in Fig. 1, in which the curve 1 shows the interest rate as a function of the term to maturity of the loan), it is cheaper for the debtor to provide funding by the issue of short-term bonds instead of a long-term bond with a term to maturity of e.g. 30 years. The example in Fig. 1 shows that as of January 15 1996 the interest rate on a 1 year loan was approx. 4 points lower than the interest rate of 30 years.

LAIR may also be used in connection with loans of e.g. 12 payment periods, i.e. a loan where debtor makes monthly payments on the loan.

It is also possible to disburse bullet loans with maturities of 1 or 10 years as well as various combinations of bullet loans matching the funding need of the individual debtor.

When calculating the principals of a loan according to the principle of LAIR it is intended that the payment profile and the repayment profile reflect the profile on e.g. an annuity loan with a term to maturity of 30 years irrespective of the

refinancing percentage and the intervals between the refinancing.

In case the debtor wants a LAIR with a term to maturity of 30 years, this may thus be funded by issuing e.g. up to 10 bullet bonds with maturities of 1 to 10 years. Thus, as the term to maturity of the debtor loan is longer than the funding, the loan is to be fully or partially refinanced on its way. However, a loan with more than 10 financial instruments with maturities of more than 10 years may also be selected.

Here, the debtor has different options. The debtor may choose to refinance 10 per cent of the remaining debt each year, 50 per cent every second year, 100 per cent every fifth year etc.

In cases where loans are refinanced by means of bullet bonds, debtor's selection of refinancing percentage and refinancing time determines the number of bullet bonds with which the creditor is to refinance the loan. If, therefore, the debtor selects a refinancing of 20 per cent each year, the creditor will fund a LAIR by issuing 5 bullet bonds with maturities from 1 to 5 years, if a refinancing of 10 per cent is selected, the creditor issues 10 bullet bonds with maturities from 1 to 10 years.

Figs. 2 and 3 show an example of a LAIR of DKK 1,000,000, the debtor selecting a 10 per cent refinancing of the remaining debt each year. Thus, at the time of a disbursement, the creditor issues 10 bullet bonds with maturities of 1 to 10 years. In the example, the debtor repays a LAIR as an annuity loan with a term to maturity of 30 years. In the example the interest rate structure is presumed unchanged during all 30 years and it is equal to the interest rate structure shown in Fig. 1.

Fig. 2 shows the development in the interest rate, and Fig. 3 shows the development in the quarterly payments on the loan and the term to maturity of the same loan. The loan is a LAIR with a refinancing of 20 per cent each year and with a maximum and minimum limits for the payments on the loan of DKK 23,000 and DKK 20,000, respectively. The loan is disbursed with a term to maturity of 20 years.

An increase of 3 percentage points in the yield to maturity of the financial instruments applied after a period of 5 years has been assumed. The increase in the yield causes the payments on the loan to reach the maximum limit after 8 years, whereupon the term to maturity is adjusted. This appears from Fig. 3. When the payments on the loan 4 reach the maximum limit 3, then the term to maturity 5 increases concurrently. At the maturity of the loan, the term is prolonged to the settlement date corresponding to a term to maturity of 22 years. At the same time as the prolongation, the payment on the loan exceeds the minimum limit 3.

After the first year, the bullet bond with a term of 1 year matures, and, at the same time, 20 per cent of the remaining debt on the loan is refinanced. The refinancing of the loan is carried out by issue of a new bullet bond with a term of 5 years and additional issue in the 4 bonds with terms to maturity from 1 to 4 years. The same procedure applies to the refinancing in year 2 and onward.

Hence, the LAIR is financed in 5 bullet bonds until year 18. From year 18 till year 22 the number of bullet bonds is reduced by 1 each year so that the loan may mature after the above-mentioned 22 years. The number of bullet bonds is reduced at the same time as the loan is refinanced.

DETAILED DESCRIPTION

Below follows a detailed description of embodiments of the method and the system according to the invention.

Where, in the following, the symbol × is applied in connection with vector and matrix multiplication, the symbol is not to be regarded as the outer product. Thus, the symbol represents the inner product of vector and matrix multiplication.

1. Adjustable term to maturity - the general problem

Characteristic for loans with adjustable interest rates and adjustable term to maturity is that the payments on the loan float within a band defined by a set of maximum and minimum limits as the interest rate on the loan is adjusted to the market rate. The limits are denoted YD_J^{max} and YD_J^{min}, respectively, YD denoting the payments on the loan in general and J indexing the interest rate adjustment periods.

The fluctuations are kept within the band by adjusting the term to maturity of the loan, when, otherwise, the payments on the loan would exceed the limits of the band. Only to the extent that the adjusted term to maturity exceeds a pre-defined limit, the payments on the loan will exceed the limits of the band. The limits for the term to maturity of the loan are denoted, correspondingly, L^{max} and L^{min}, L denoting the term to maturity in general.

It is not a requirement in the model that the term to maturity is set as whole years or payment periods. Thus, the possible adjustments of the term to maturity are defined on a continuous interval limited by L^{max} and L^{min}. This is necessary if it is to be possible to fix the payments on the loan within a relatively narrow band at every interest rate adjustment.

The fact that the possible adjustments of the term to maturity are defined on a continuous interval opens up the possibility of offering the debtor the option of fixed payments on the loan as long as the term to maturity is within L^{max} and L^{min}. In the model, fixed payments on the loan correspond to equal maximum and minimum limits for the payments on the loan, hence

$$YD_J^{max}=YD_J^{min}$$

In the model, there is no need in general for a distinction between loans with fixed payment and loans with payments floating within maximum and minimum limits. The debtors, however, are likely to view loans with fixed payment and loans with floating payments as two different types of products that require different computations, especially when the loans are disbursed. Hence, a distinction can not be completely avoided in the following.

1.1 Fixing the limits for the payments on the loan

The fixing of the limits for the payments on the loan is, basically, the debtor's choice and is therefore considered an input to the model.

Fixing the limits at an unreasonable level implies that the model, at an early stage, will disregard the limits to observe the limit for the term to maturity for the loan, which will be discussed further in detail in the following. Hence, unreasonable limits will automatically become non-binding.

For annuity loans, it is reasonable to set the maximum and minimum limits at a fixed level, hence

$$YD_0^{max}=YD_1^{max}=\ldots=YD_M^{max} \text{ and } YD_0^{min}=YD_1^{min}=\ldots=YD_M^{min}$$

For serial loans, corresponding limits with fixed values will not be applicable unless the spread between the limits is very wide. By definition, the payments on a serial loan decrease over time - it will therefore be most reasonable to fix similar decreasing maximum and minimum limits e.g. by fixing the maximum and minimum limits as a spread to the expected payment profile at the disbursement of the loan.

Adjusting the term to maturity to observe the limits for the payments on the loan causes a special problem in regard to serial loans. When adjusting the term to maturity, it is not possible simultaneously to determine a level and a slope for the payment profile. Therefore, it will not be possible, in general, to observe the limits for the payments on the loan for a multi-annual period (e.g. a full refinancing period) at one time unless a very wide band is defined. In the model, this problem is solved by observing the limits YD_J^{max} and YD_J^{min} only in the first year of each refinancing period.

Yet, the question of fixing the limits after the maturity of the loan as originally expected is still to be answered. This applies to a situation where increases in the interest rate have caused the term to maturity to exceed the original term to maturity which applied at the disbursement of the loan.

For annuity loans the model simply continues to observe the defined limits which are at a fixed level and therefore well-defined. For serial loans the gradually decreasing limits can not just be applied onwards, since this would imply that the maximum limit at one stage might be negative. For debtor, it would be more appropriate if the maximum limit instead is fixed at the level of the last well-defined maximum limit, whereas the minimum limit is fixed at 0 (zero). Thus, the loan may be continued as a serial loan without the limits becoming binding per definition. An example of the limits for the payment on an annuity loan is shown in Fig. 5, while Fig.

6 correspondly shows an example of limits for a serial loan. In both figures, the vertical lines denoted L^{min} and L^{max} illustrate the minimum and maximum limits, respectively, for the term to maturity on the loan, whereas the lines denoted YD_J^{max} and YD_J^{min} illustrate the maximum and minimum limits, respectively, for the payments on the loan.

1.2 The concept of adjustable term to maturity

The financing of LAIR in more than one financial instruments implies that the future interest rate on the loan will not only shift, due to shifts in the yield to maturity of the financial instruments applied, but will also shift due to shifts in the distribution of the volume of the financial instruments applied, provided the yield curve is not horizontal.

Hence, to assume the yield curve to be constant does not correspond to assuming constant interest rates on the loan. On the contrary, at the time of each interest rate adjustment a sequence of interest rates must be determined assuming constant yields to maturity of the financial instruments applied. In particular, this is relevant for the P-product, since interest rate shifts penetrate gradually as the loan is partially refinanced.

The non-constant interest rates complicate the concept of the term to maturity for LAIR with adjustable term to maturity.

For each future refinancing period the payments on the loan are calculated as a function of the term to maturity i.e.

$$YD(J,L_J) \qquad J=0,\ldots,M$$

wherein

$YD(J,L_J)$	is the annual payments on the loan in the period from the Jth refinancing till the (J+1) refinancing.
L_J	is the term to maturity at the Jth refinancing.
J	denotes the future interest rate adjustments. J is set to 0 (zero) at each interest rate adjustment. Thus J=0 denotes the time of the computations (either the disbursement or a refinancing of the loan)
M	denotes the last refinancing of the loan before maturity. At the disbursement of a F5 loan with a 20 year term to maturity, M is set to 3 (J=0,1,2,3), since the loan is not refinanced at the maturity. L_M determines when the loan has been repaid and is, thus, the term to maturity of the loan. M will shift continuously according to the remaining term to maturity of the loan.

If the payments on the loan are fixed or either YD_J^{max} or YD_J^{min} are binding, it is required, disregarding the situation where also L^{max} or L^{min} are binding, that

$$YD(0,L_0)=YD(1,L_1)=\ldots YD(J,L_J)=\ldots=YD(M,L_M) \quad (=YD_J^{max} \vee YD_J^{min})$$

For a non-constant interest rate on the loan, this will, in general, imply that

$$L_0 \neq L_1 \neq \ldots \neq L_J \neq \ldots \neq L_M$$

Thus, the model must handle a sequence of terms to maturity.

1.3 Fixing the limits for the term to maturity

The traditional concept of the term to maturity of a loan corresponds to L_M, and this will be the term to maturity which debtor is informed of. In relation to the limits for the maturity it will, therefore, be most natural to define the limits relating to L_M and not the other elements in the sequence of the terms to maturity. Hence, it is accepted that $L_J > L^{max}$ for $J<M$. This also implies that at each computation it is necessary to simulate future refinancing periods to determine whether the limits for the term to maturity are observed.

In contrast to the limits for the payment on the loan, the limits for the term to maturity will, typically, be determined by external circumstances. Thus, the maximum limit may be determined by legislative rules or credit policy considerations, whereas the minimum limit may be determined so that the debtor is not deprived of a capital loss tax deduction. If the debtor prefers more narrow limits than required by the external factors, nothing will prevent the model from operating with these more narrow limits.

In the model the maximum and minimum limits for the term to maturity are given a higher priority than the limits for the payments on the loan. Only when the payments on the loan are at the same level as the maximum or minimum limit, the term to maturity is adjusted as mentioned above. If it is allowed that L^{max} or L^{min} is exceeded in order to observe the limits for the payments on the loan, L^{max} or L^{min} will never be binding and may, thus, be left out of the model. In addition the nature of the limits for the term to maturity does not allow that the limits are exceeded in practice.

The prioritization of the limits for the term to maturity and the payments on the loan determines the fundamental pattern

of shifts in the payments on the loan as shown in Fig. 7, in which line 10 shows the fundamental pattern of shifts in the payments on the loan.

The starting point is given by A. If the yield to maturity of the financial instruments applied shift upwards, the payments on the loan increase until YD_J^{max} is reached. When YD_J^{max} is reached, the payments on the loan are held constant at this level, whereas the term to maturity is prolonged. This continues until the term to maturity reach the level L^{max}, which must not be exceeded. Thus, the payments on the loan must be increased to a level exceeding the maximum limit, e.g. point B.

Accordingly, in case the yield to maturity of the financial instruments applied falls, the payments on the loan will drop to the level YD_J^{min}. Then the term to maturity is reduced so that the payment on the loan is shifted horizontally in the plane. The horizontal shift continues until L^{min} is reached, at which the shifts again are vertical, e.g., to the point C.

The fundamental pattern of shifts in the payments on the loan imply that, within the band, the payments on the loan shift correspondingly to a LAIR with fixed term to maturity i.e. in the vertical plane. Thus, LAIR with fixed term to maturity (the conventional LAIR) can be considered as the special case where the maximum and minimum limits for the payments on the loan are fixed at ∞ (infinity) and 0 (zero) respectively, or where the maximum and minimum limits for the term to maturity are identical, in which case the limits for the payments on the loan are disregarded.

A most obvious simplification of the model would be to make the limits for the term to maturity apply constantly. In each refinancing period it would only be necessary to check that L_J is not higher than L^{max} and not lower than L^{min} respectively.

Hence, it would not be necessary to simulate future refinancing periods.

However, the simplification would incur unnecessary deviations from debtor's intended payment profile, since an expected future fall in the interest rate on the loan can not be applied in the current period, cf. the above discussion. When the debtor himself has fixed the limits for the term to maturity more narrowly than required by the legislative rules, this model is not applied.

Yet, problems regarding the legislative requirements can not be ruled out, e.g., terms to maturity exceeding 30 years for private debtors might not be accepted at any stage in the payment profile, thus $L_J<J$ for all values of J. Correspondingly, it can not be ruled out that the debtor will be deprived of a capital loss tax deduction if just one of the elements in the sequence of terms to maturity is lower than the term to maturity of the loan previously held by the debtor. In the model it is thus necessary to introduce a further set of limits for the term to maturity denoted L^{MAX} and L^{MIN}, which limits must be observed by L_J at all times.

The simplified version of the model, in which each element in the sequence of terms to maturity is tested against the maximum and minimum limits for the term to maturity fixed by the debtor, thus corresponds to putting L^{MAX} and L^{MIN} equal to L^{max} and L^{min}, respectively. If it turns out that L^{MAX} and L^{MIN} are not necessary, the limits will be fixed at ∞ (infinity) and 0 (zero), respectively.

1.4 Deviations from the limits for the payments on the loan

According to the fundamental pattern of shifts in the payments on the loan, the term to maturity is constant if $YD(0,L_0)<YD_J^{max}$. Yet, it can not be ruled out that

$$YD(0,L_0)<YD_J^{max} \text{ and } YD(0,L_J)>YD_J^{max} \text{ for } L_J=L_0 \text{ and } 1\leq J\leq M$$

corresponding to a situation where the current payments on the loan are within the band, but it is expected that the payments on the loan will exceed YD_J^{max} at a future refinancing of the loan if the term to maturity is not prolonged compared to L_0. In this situation $YD(0,L_0)$ is maintained, whereas L_J is prolonged so that

$$YD(0,L_J)=YD_J^{max}$$

Thus, the term to maturity is prolonged without the current payments on the loan being increased to the level YD_J^{max}. Another solution will, however, result in unreasonable payment profiles for loans with a long remaining term to maturity.

The problem can be extended to a situation where L_M will exceed the maximum limit for the term to maturity even though $YD(0,L_J)=YD_J^{max}$ under the given yield curve.

The limit for the term to maturity may not be exceeded. Instead the payment on the loan is increased, if necessary to a level exceeding YD_J^{max}. Yet, it is not completely trivial when and by how much the payment on the loan should be increased. At a first glance, it would seem obvious to determine the payment on the loan as

(i) $YD(J,L^{max}) \quad \forall\ J=0,1,\ldots,M$

which ensures that the loan matures at the right point in time. However, this solution ignores the maximum limit for the payments on the loan fixed by the debtor, which is not suitable. In addition, it can not be ruled out that $L_0<L^{max}$ even though $L_M>L^{max}$. In this situation, the payment on the loan

decreases for J=0 and will later increase beyond YD_J^{max}. This also implies that the solution in (i) is not suitable.

However, a more suitable solution is to determine the payments on the loan based on a combination of $YD(J,L_J)$ and (i) so that the payments on the loan follow $YD(J,L_J)$, as long as this is larger than $YD(J.L^{max})$ and, when this does not apply, the payments on the loan are fixed at $YD(J,L^{max})$. In mathematical terms, this can be expressed as;

Let J' denote the minimum value of J where

$$YD(J,L^{max})>YD(J,L_J) \qquad \forall\ J=0,1,\ldots,M$$

For $0\leq J\leq J'$ the payment on the loan is determined as

$$YD(J,.)=YD(J,L_J)$$

and for $J'\leq J\leq M$

$$YD(J,.)=YD(J,L^{max})$$

The pattern of shifts in the payments on the loan will be as illustrated in Fig. 8 and Fig. 9. Line 12 in Fig. 8 indicates the pattern of shifts in the payments on the loan for an annuity loan when it is necessary to exceed the limit YD_J^{max} to observe the maximum limit for the term to maturity of the loan. Analogeously, line 14 in Fig. 9 indicates the pattern of shifts in the payments on the loan for a serial loan when it is necessary to exceed the limit YD_J^{max} to observe the maximum limit for the term to maturity of the loan.

In the analogous situation where the payments on the loan will fall below YD_J^{min} if the minimum limit for the term to maturity is to be observed, the payment on the loan is determined accordingly.

Let J'' denote the minimum value of J where

$$YD(J,L^{min})<YD(J,L_J) \qquad \forall\ J=0,1,\ldots,M$$

For $0 \leq J \leq J''$ the payment on the loan is determined as

$$YD(J,.)=YD(J,L_J)$$

and for $J'' \leq J \leq M$

$$YD(J,.)=YD(J,L^{min})$$

This solution is considered to be the most suitable. However, other solutions are outlined below.

1) Firstly, the payments on the loan might be fixed at a level corresponding to L^{max}. In the model, this would imply the definition of a new maximum limit at a level either higher or lower than the previous maximum limit. For serial loans it would imply the further problem that the payments on the loan are decreasing in each refinancing period. Thus, the payments on the loan would have to shift upwards at each refinancing of the loan and then, gradually, fall. Finally, the solution would imply that increases in the interest rate in the early stages of the loan would effect the payments more than observed in the preferred solution.

2) A second possibility is to fix a new maximum limit for the payments on the loan at a higher (lower) level, and, gradually, let the payments on the loan rise (fall) until the new limit is reached in the same manner as the preferred solution. The limits fixed by the debtor, are hereby ignored which is inappropriate.

3) Finally, it is possible to determine the payments on the loan based on the term to maturity L^{max} or L^{min} respectively as mentioned under (i). However, as mentioned above, this implies the risk that the payments on the loan will first decrease and then increase and vice versa.

2. Type F

2.1 The general problem

LAIR type F is characterized by the fact that all the remaining debt of the loan is refinanced at predetermined points in time with a fixed interval.

The duration of the interval is determined by the debtor, yet observing that the refinancing always takes place on 1 January. Hence, the duration of the interval will always be a whole (interger) number of years except for the first period where the duration depends on the date of the disbursement.

In the period between two consecutive interest rate adjustments, the interest rate and thereby the payments on the loan are fixed. Depending on the frequency of refinancing desired by the debtor, the payments on the loan will shift discontinously over time.

This is shown in Fig. 10 and Fig. 11. In Fig. 10, line 16 indicates the payments on an annuity loan shifting intermittently, whereas line 18 in Fig. 11 denotes the payments on a serial loan also shifting discontinously.

The model must find a solution which complies with a number of requirements. The conditions may be assined to two categories.

Firstly, the solution must observe requirements defined for the total period from the disbursement till the maturity of the loan. These intertemporal conditions were the main focus point in the above description of the general problem. However, the conditions are briefly summarized in the following.

1) The term to maturity must not be longer than the maximum limit

$$L_M \leq L^{max}$$

2) The term to maturity must not be shorter than the minimum limit

$$L_M \geq L^{min}$$

3) If $L_M < L^{max}$, the payment on the loan is determined observing the condition

$$YD(J,LJ) \leq YD_J^{max}$$

and otherwise

$$\begin{aligned} YD(J,.) &= YD(J,L_J) && \text{for } 0 \leq J \leq J' \\ YD(J,.) &= YD(J,L^{max}) && \text{for } J' \leq J \leq M \end{aligned} \qquad (A)$$

where J' denotes the minimum value of J for which $YD(J,L_J) < YD(J,L^{max})$

4) If $L_M > L^{min}$, the payment on the loan is determined observing the condition

$$YD(J,LJ) \geq YD_J^{min}$$

and otherwise

$$YD(J,.)=YD(J,L_J) \quad \text{for } 0 \le J \le J''$$

(B)

$$YD(J,.)=YD(J,L^{min}) \quad \text{for } J'' \le J \le M$$

where J'' denotes the minimum value of J for which $YD(J,L_J)>YD(J,L^{min})$

The model observes these conditions by determining a sequence of terms to maturity $L_0,L_1,\ldots,L_M$.

The additional conditions are defined for each refinancing period, hence for each value of J and L_J. Thus, it is necessary to alter the notation, since J and M are insignificant in this context. Instead, it is necessary to focus on each year in the refinancing period as denoted by j=0,1,2,…,m, where m denotes the next refinancing of the loan. Furthermore, the payment on the loan is denoted as a function of j, i.e. YD(j).

First of all, the payments on the financial instruments applied must match the debtor's payments on the loan as required by the principle of strict balance in the Danish act on mortgage credit loans. Thus,

Debtor's interest and repayments=
Bond redemptions+interest payments to the bond holders

In order to have the payments on the loan, which in addition to payments of interest also comprise repayments, correspond to payments on the bonds for each year, bullet bonds are issued having maturities up to the duration of the refinancing interval so that bonds mature each year until the next refinancing.

For each year (j') until the refinancing, the following shall thus apply

$$(C) \quad YD(j')=H(j')+\sum_{j=j'}^{m} R^{N}(j)H(j), \quad j'<m$$

where:

YD(j)	is debtor's payments on the loan in year j.
H(j)	is the jth bond principal at a given time j.
R^N (j)	is the coupon interest rate of the jth bond.
m	(Not to be mistaken for M) is the number of bonds at the beginning of the refinancing period and also the time of the next refinancing.
j	(Not to be mistaken for J) indicates years within the refinancing period and, at the same time, numbers the funding volumes. j is thus set to zero after each refinancing. In the notation there is a direct compliance between each year and the bond maturing that year.

That j at the same time may indicate years and funding principals is solely due to the fact that the bonds only have one annual settlement date on 1 January. If the number of annual settlements is changed, the notation must be changed as well.

As to the loan side, however, 1 or 4 annual payment dates may be selected. Thus, j cannot also indicate debtor payment dates. In order to facilitate the notation, debtor's repayment and interest within one year is, nevertheless, called YD(j), then summing up the payments made on payment dates within the year. Let n indicate the number of debtor's payment dates per year,

$$YD(j) = \sum_{i=1}^{n} AFD(i) + \sum_{i=1}^{n} \frac{R^K}{n} RG(i-1)$$

where

	R^K	is the interest rate on the loan
	RG(j)	is the remaining debt at the end of year j
5	AFD(i)	is debtor's repayment on the payment date i
	i	indicates the payment dates within the year, that is, i=1,2,…,n

The principle of strict balance is observed each year. However, there are two years in which (C) is to be modified - the first year and the year in which the loan is refinanced.

In the year of disbursement of the loan, the sum of debtor's interest payments and repayments must be equivalent to the sum of the principal payment on the bond having the shortest term to maturity H(1) and interest payments on all bonds applied in the financing of the loan. In addition, in year 1 an adjustment must be made so that the first settlement is only paid in full by the debtor if the loan is disbursed exactly on a settlement date.

Thus, for year 1 applies:

$$(D) \quad YD(1) = H(1) + REG \sum_{j=1}^{m} R^N H(j)$$

wherein

REG	is a regulation factor determining how much of the interest payments on the first settlement date the bond holder should receive from the debtor. REG is determined as the part of the year in which the loan has existed since 30 November on which date the bonds mature ex-coupon. Thus,

REG can assume values between 1/12 (if the loan is disbursed 30 November) and 13/12 (if the loan is disbursed 1 December, and thus, the first year last 13 months).

In the year of the refinancing of the remaining debt (year m) the total payments on the loan comprise, apart from debtor's repayment and interest, the remaining debt at the end of the year, corresponding to the refinancing amount. As to the bond side, these payments match payments on the one bond which has not yet matured. For the years in which the loan is refinanced, (C) may therefore be formulated as follows:

(E) $YD(m)+RG(m)=H(m)+R^N(m)H(m)$

Thus, the total balance conditions may be written as (seen from the point in time 0):

Year 1: $YD(1) = H(1) + REG \sum_{j=1}^{m} R^N(j)H(j)$

Year 2: $YD(2) = H(2) + \sum_{j=2}^{m} R^N(j)H(j)$

...

Year m: $YD(m) + RG(m) = [1 + R^N(m)]H(m)$

Besides observing the balance condition, the market price of the bonds issued must exactly match the principal of debtor's loan. In the following, this condition is referred to as the proceeds criterion, i.e.:

$$RG(0) = \sum_{j=1}^{m} K(j)H(j)$$

wherein

K(j) is the price of the jth funding instrument.

RG(0) is the remaining debt at the beginning of the refinancing period, which at the disbursement of the loan is equivalent to the principal of the loan, and, at the refinancing of the loan, complies to the refinancing amount since the remaining debt in full is refinanced.

The problem to be solved by the model for each refinancing period comprises the m equations that resulted from the balance condition as well as the one equation which resulted from the proceeds criterion. However, the problem is simultaneous. This results from the financing of the loan being determined both by the debtor's payments on the loan and the proceeds criterion. The problem is solved by fixing the interest rate on the loan. When starting off with an "arbitrary" interest rate, debtor's repayment and interest payments may be determined. They determine the funding with regard to the balance condition. The proceeds from the issue of the funding may then be compared to the desired proceeds of the loan. In case of a deficit more bonds must be sold, and the interest rate must be raised so that the larger payments on the bonds - more bonds are sold - are covered by the payments on the loan. If, on the other hand, there is a surplus from the issuing of the bonds, the interest rate may be lowered.

Thus, it will always be possible to determine an unambiguous, positive interest rate solving the problem for each refinancing period. The explanation hereto is that the proceeds from the sale of the funding instruments is a strictly growing function of the interest rate:

higher interest rate → larger payments on the loan → larger funding principals → higher proceeds.

The structure of the model reflects the structure of the conditions that the solution must observe. In a outer model the intertemporal conditions, i.e. the conditions defined for the total period from disbursement to maturity of the loan, are applied. The remaining conditions are applied in an inner model.

The structure in the model is that the outer model applies a iteration routine involving the inner model. When, for each refinancing period, the volume of the financing instruments applied, the interest rate and payments on the loan have been determined in the inner model, this solution is tested against the conditions in the outer model. If the conditions are not observed, the term to maturity is adjusted, and subsequently the inner model once again computes volumes of the financing instruments applied, interest rate and payments on the loan etc.

A attractive characteristic of the model is a close compliance between the interest rate on the loan and the yield to maturity of the portfolio of financing instruments applied. The compliance is obtained by the principle of balance, since the compliance between the payments on the bonds and on the loan only allows the interest rate on the loan to deviate from the yield to maturity as a result of difference in the timing of the payments within the year. Thus, it is not necessary to advance explicit conditions to the compliance between the interest rate on the loan and the yield to maturity of the portfolio of financing instruments applied.

2.2 Method

For the sake of good order the general problem is repeated below:

A sequence of terms to maturity must be determined observing that:

in each refinancing period, the payments on the loan are within the band defined by the maximum and minimum limits for the payments on the loan

stipulated that the total term to maturity is within the band defined by the maximum and minimum limits for the term to maturity and stipulated that the payments on the loan, in each refinancing period, observe:

the balance condition (equations (C) - (E)) and the proceeds criterion.

2.2.1 The outer model of type F

As mentioned above, the problem is solved in an inner and an outer model. The outer model is shown in the flow chart in Fig. 12, which will be described thoroughly in the following. It appears that the solution is obtained in 12 steps, which will be described below.

It should be noted that steps A to E are repeated for each refinancing to follow i.e. for J=0,1,…,M. Thus, the model determines a full payment profile which, period for period, observes the YD_J^{max} and YD_J^{min} conditions.

Thus, the model also determines a value of L_M. When the full payment profile has been calculated, it is therefore to be tested that $L^{min} \leq L_M \leq L^{max}$. The test is applied in steps F and I.

If $L_M \geq L^{max}$, a new payment profile must be determined according to (A). $YD(J,L_J)$ has already been calculated in the model, but $YD(J,L^{max})$ must be calculated in the inner model, which takes place in steps J and K. If, on the contrary, $L_M \leq L^{min}$, the

term to maturity is adjusted complying with (B) in steps G and H.

If $L^{min} \leq L_M \leq L^{max}$ the model continues in step K, where the calculations are finalized.

Step A - Determine initial L_J

In step A the model determines an initial term to maturity which may be considered as a first guess in the iterative routine.

First, an initial value for J=0 is determined. The following steps determine, *inter alia*, the payments and interest rate on the loan and a final value of L_0. Having determined these, the model returns to step A to determine an initial value of L_1 etc. Thus, each time step A is applied an initial term to maturity is determined.

If J>0 the initial value of L_J is relatively simply determined as L_{J-1}, L_{J-1} being, of course the final value - after the necessary iterations - of the term to maturity in the preceding refinancing period. This procedure is applied since the term to maturity is only adjusted if, otherwise, the payments on the loan would be outside the band. Thus, as a starting point it is assumed that the term to maturity needs no adjustment.

If J=0, the determination depends on whether the (physical) time of calculation coincides with the disbursement of the loan or a refinancing of the loan.

At the disbursement of the loan, the term to maturity is determined based on input from the debtor which may comprise:

1) The debtor selects an annuity loan with fixed payments (fixed payments rule out serial loans). One could imagine that the debtor prefers that the payments on the loan in the first year of each refinancing period is determined according to a particular pattern. However, this possibility would be comprised by the intended payments on the loan under 3) i.e. $YD_J^{max}=YD_J^{min}$. An initial term to maturity must be calculated. The initial guess of L_0 may be determined applying the annuity formula

$$YD(0,L_0)=\frac{RG(0)R^K}{1-(1+R^K)^{-(L_0-\varphi)}}$$

which, solved for L_0 yields

(F) $$L_0=\frac{\ln\left(1-\frac{RG(0)R^K}{YD(0,L_0)}\right)}{\ln(1+R^K)}+\varphi$$

where $YD(0,L_0)=YD_J^{max}=YD_J^{min}$,

and where

φ is the number of years since the disbursement of the loan. Thus, $L_0-\varphi$ is the remaining term to maturity to be applied in the annuity formula.

Yet, R^K has not been determined. To reduce the number of iterations, R^K is set equal to the yield to maturity of the last maturing financing instrument applied.

2) The debtor selects an intended term to maturity and maximum and minimum limits for the payments on the loan.

In this situation, L_0 is simply set equal to the intended term to maturity. After the disbursement of

the loan, the intended term to maturity is of no consequence and is, thus, not stored in the model.

3) The debtor selects an intended payment on the loan and maximum and minimum limits for the payments on the loan.

Analogously to the situation in which the debtor selects fixed payments on the loan, L_0 is determined by applying (F) so that $YD(0,L_0)$ is the intended payment on the loan. However, for serial loans a similar formula must be developed. The first year's payments on the loan are given by

$$YD(0,L_0)=\left(nRG(0)-\frac{n(n-1)}{2}\frac{RG(0)}{n(L_0-\varphi)}\right)\frac{R^K}{n}+n\frac{RG(0)}{n(L_0-\varphi)}$$

which, solved for L_0 yields

$$L_0=\frac{RG(0)-RG(0)R^K\frac{n-1}{2n}}{YD(0,L_0)-RG(0)R^K}-\varphi$$

After the disbursement of the loan, the model disregards the intended payment on the loan, which thus is not stored in the model.

Other situations can be imagined, among which that the debtor may want a loan with fixed payments and an intended term to maturity of, e.g., 25 years. This would imply that $YD_J^{max}=YD_J^{min}$ has to be determined so that $L_M=25$. However, to solve this problem a further iterative routine is required, which would complicate the model to an extent not commensurate with the possible gain.

All inputs to the model (both the inner and the outer model) are entered in step A. These inputs are:

The bond side

Number of financial instrument applied (and thus the frequency of interest rate adjustments), coupon interest rate on each financial instrument applied, number and date of settlements each year and price of each of the financial instruments applied.

The loan side

Date of disbursement, principal, principle of redemption, number and date of payments on the loan, maximum and minimum limits for the payments on the loan, maximum and minimum limits for the term to maturity on the loan, and, possibly, an intended payment on the loan or term to maturity.

In addition, values of inc must also be entered. In each iteration routine, two values of each function are calculated - a function value of x and a function value of x+inc - x denoting the variable subject to the iteration routine, e.g., the term to maturity. Hence, in the model the calculation of the adjustment of x is based on two observations. inc is thus a set of parameters indicating an increment in the iterative routine. Unless otherwise stated all elements in inc have the value 0,00001.

Finally, also ϵ must be assigned a value. ϵ is a set of accuracy parameters indicating a maximum permissible deviation from the conditions allowed.

When entering the inputs, it is tested that

$$YD_J^{max} \geq YD_J^{min} \geq 0 \text{ and } L^{max} \geq L^{min} \geq 0$$

Since the loan is to mature on a settlement date, it is required that the entered values of L^{max} and L^{min} comply with a settlement date. For instance, if the loan is disbursed at 30 June, the model requires that L^{max} and L^{min} are indicated by an integer number of years +½.

Step B - Adjust L_M, Determine M

In general, it is necessary to adjust the term to maturity in the refinancing period preceding the maturity of the loan (J=M). This is due to fact that the bonds financing LAIR mature at the end of the year. Hence, it is suitable that the loan also matures at the end of the year, amongst other considerations because of the interest rate on the loan. Thus, the term to maturity in the refinancing period preceding the maturity of the loan (L_M) is either prolonged or shortened to the nearest settlement date.

If possible, the term to maturity must be adjusted in compliance with YD_M^{max} and YD_M^{min}. However, it cannot be precluded that it may be necessary to violate one of the limits, in particular if the limits are identical. If violating one of the limits is necessary violating the minimum limit, and thus prolonging the term to maturity, is most suitable. Therefore, adjusting the term to maturity proceeds in three steps:

1) L_M is prolonged till the nearest settlement date if

$$YD(M,\bar{L}_M) > YD_M^{min}$$

$\bar{L}_M$ indicating the prolonged term to maturity. Otherwise move on to 2.

2) L_M is shortened till the nearest settlement date if

$$YD(M, L_M) < YD_M^{max}$$

L_M indicating the shortened term to maturity. Otherwise move on to 3.

3) The term to maturity is prolonged till the nearest settlement date and YD_M^{min} is suspended.

Step 3 ensures that a solution will always be found, thus, the calculations will not continue infinitely. The steps imply that for J=M two different terms to maturity, L_M and L_M respectively, must be input into the inner model.

In step B the value of M is also determined. Depending on the term to maturity, the model simply determines M as the round down value of the remaining term to maturity divided by the duration of the refinancing periods. This is particularly important when the model arrives at step B from step E with a new term to maturity.

Finally, also L^{MAX} and L^{MIN} are input into the inner model to test if these limits are binding for the payment on the loan.

Step C - Calculate adjustment of L_J

Based on the terms to maturity input the model has calculated the payments on the loan. Step C calculates an adjustment ΔL_J of L_J based on the ratio between the payments on the loan (for a full payment period) on the one hand, and the limits for the payments on the loan on the other hand. Depending on the time of the calculations, the first payment on the loan may be fractioned, in which case the first payment is calculated proportionally. Thus, the model allows the first payment after the disbursement of the loan to violate the band.

In step B, three and four terms to maturity were determined and input into the inner model. Thus, the inner model has returned with 3 or 4 different payments on the loan i.e.

$YD(J,L^{MAX})$ and $YD(J,L^{MIN})$ for all J

$YD(J,L_J)$ for all J<M and

$YD(M,\bar{L}_M)$ and $YD(M,L_M)$ for J=M

If either

$YD(J.L^{MAX}) \geq YD_M^{max}$ or

$YD(J,L^{MIN}) \leq YD_M^{min}$

L^{MAX} respectively L^{MIN} is binding, thus

$\Delta L_J=0$

i.e. the calculated adjustment to L_J is 0 (zero). On the other hand, if

$YD(J,L^{MIN})>YD_M^{min}$ and $YD(J,L^{MAX})<YD_M^{max}$

the adjustment of the term to maturity is determined by the limits for the payments on the loan.

<u>If J<M</u> a function F is first defined as the deviation of the payments on the loan from the maximum and minimum limits. The function is given by

1) For $YD(J.L_J)>YD_J^{max}$

(i) $F(L_J)=YD(J,L_J)-YD_J^{max}$

2) For $YD_J^{min} \leq YD(J,L_J) \leq YD_J^{max}$

(ii) $F(L_J)=0$

3) and for $YD(J.L_J) < YD_J^{min}$

(iii) $F(L_J)=YD(J,L_J)-YD_J^{min}$

For a serial loan, $YD(J.L_J)$ is defined as the payments on the loan the first year of the refinancing period. The function may appear as shown in Fig. 13, in which the lines (i), (ii) and (iii) refer to the corresponding formulae derived above (However the graph F is conventionalized, as a linear relation between $YD(J,L_J)$ and L_J will only appear as a special case).

At a first glance, it may seem more relevant to define F as indicated by line 20 in Fig. 14. However, this would imply that the model loses information on how large adjustment of L_J required to exactly match YD_J^{max} or YD_J^{min} as prescribed by the fundamental pattern of shifts. If an F-function according to Fig. 14 is applied, it will only be possible for the model to calculate an adjustment of L_J, which results in the payment on the loan to be within the band, but not necessarily an adjustment, which results in the payment on the loan being equal to the relevant limit. Hence, a method corresponding to the method illustrated in Fig. 14 can not be applied in the present invention.

Whether the adjustment of the term to maturity is calculated according to (i), (ii) or (iii) is determined the first time the calculations are performed in step C for each value of J. Subsequently, this is not altered. If the model were to decide, whether (i), (ii) or (iii) were relevant each time the calculations are performed, there would be a risk that the model shifts from, e.g., (i) to (iii), thus the model would aim at the wrong limit when calculating the term to

maturity. From starting off with too high payments on the loan, the model would in this case end with payments on the loan equal to the minimum limit, which is, of course, not acceptable.

Based on the value of $F(L_J)$ an adjustment of L_J is to be calculated. The adjustment is calculated applying the Gauss-Newton algorithm, which however is slightly modified.

If $F(L_J)=0$, $YD(J,L_J)$ is within the band and thus L_J is not to be adjusted i.e.

$$\Delta L_J=0$$

If $F(L_J)\neq 0$, the adjustment is calculated applying

$$\Delta L_J=\left[[D^TD]^{-1}D^Tg\right]\circ j_v^{\circ -1}$$

for $j_v=[\mathrm{Diag}J_a^TJ_a]^{0.5}$

$$D=[J_a^TJ_a]\circ[j_v^Tjv]^{\circ -1}$$

$$g=[J_a^TF(L_J)]\circ j_v^{\circ -1}$$

where $J_a=[F(L_J)-F(L_J+inc)]\circ inc^{\circ -1}$

and where

D^TD	is the Hesse matrix
g	is the gradient
J_a	is the Jacoby matrix
j_v	is the diagonal elements
inc	is a parameter with a standard value of 0,0001.

The influence of inc is elaborated in the following. At this point it should only be mentioned that the idea of inc is to

have two observations of $F(L_J)$ and $F(L_J+inc)$ from which to calculate ΔL_J.

AOB^{o-1} is the Schur product of two matrices meaning that the elements of the matrix are divided one by one.

In appendix 1 the elements of the matrix are written in full.

However, in this case the problem is one-dimensional, which is the reason why the Gauss-Newton algorithm may be reduced, as D, J and j_v are all of the dimension 1×1. Thus, it applies

$$j_v=J_A$$

as $j_v=[DiagJ^TJ]^{0.5}=[J^2]^{0.5}=J$. If $j_v=J$ is placed in the expression for D it applies

$$D = \frac{J^TJ}{j_vj_v^T} = \frac{J^2}{j_v^2} = 1$$

At the same time, the expression for g is reduced to

$$g = \frac{J^TF(L_J)}{j_v} = F(L_J)$$

If the reduced expressions are introduced in ΔL_J, the following applies

$$\Delta L^J = \frac{[D^TD]^{-1}D^Tg}{j_v} \quad \Leftrightarrow$$

$$\Delta L_J = F(L_J)\frac{inc}{F(L_J)-F(L_J+inc)}$$

The method may be illustrated graphically. In Fig. 15 the graph of F is indicated by line 24, shown for $YD(J,L_J)>YD_J^{max}$ i.e. (i) applies.

The algorithm approximates the value of L_J fulfilling the condition $YD_J^{max}=YD(J,L_J)$ if the payments on the loan start off exceeding YD_J^{max}. Correspondingly, the algorithm will approximate the term to maturity fulfilling $YD(J,L_J)=YD_J^{min}$ if the payments on the loan start off below YD_J^{min}.

The basic idea of the algorithm is to use the secant 26 through two points 30 on the graph 24 of F for $(L_J,F(L_J))$ and $(L_J+inc,F(L_J+inc))$, respectively. When the secant has been determined, the intersection 32 of the secant with the x-axis is calculated. The intersection 32 is the next guess as to the interest rate in the iterative routine. If the graph $F(L_J)$ is strictly declining, the algorithm will always reach a solution. This will be the case, since a prolongation of the term to maturity always yields lower payments on the loan and vice versa.

Fig. 15 shows that inc should not be regarded as an accuracy parameter. inc determines the step size in the iteration routine.

In this connection it is not necessary to apply the matrix apparatus, but the same method is to be applied later on. Therefore, the formula apparatus is now introduced.

<u>For J=M</u> the F-function is to be defined according to the three steps in the adjustment procedure in step B.

If $YD(M,\bar{L}_M)>YD_M^{min}$, the term to maturity is prolonged to $\bar{L}_M$ at first hand. However, it can not be excluded that the term to maturity has to be prolonged even more, corresponding to $YD(M,\bar{L}_M)>YD_M^{max}$, due to an upward shift in interest rates.

Hence, it is necessary to calculate an adjustment relative to the maximum limit. F is therfore defined by

(i) $F(\bar{L}_M)=YD(M,\bar{L}_M)-YD_M^{max}$ for $YD(M,\bar{L}_M)>YD_M^{max}$ and

(ii) $F(\bar{L}_M)=0$ otherwise.

If $YD(M,\bar{L}_M)<YD_M^{min}$ and at the same time $YD(M,\underline{L}_M)<YD_M^{max}$, the term to maturity is immediately shortened to the nearest settlement date. Analogously, the possibility that the term to maturity must be further shortened can not be excluded. Thus, an adjustment relative to the minimum limit is calculated. F is defined by

(i) $F(\underline{L}_M)=YD(M,\underline{L}_M)-YD_M^{min}$ for $YD(M,\underline{L}_M)>YD_M^{min}$ and

(ii) $F(\underline{L}_M)=0$ otherwise.

Finally, if $YD(M,\bar{L}_M)<YD_M^{min}$ and at the same time $YD(M,\underline{L}_M)>YD_M^{max}$ the term to maturity is prolonged to $\bar{L}_M$. Thereby, the payments on the loan fall below the minimum limit, which just has to be accepted since, at the term to maturity $\underline{L}_M$, the payments on the loan exceed the maximum limit, which must be regarded even worse, and a term to maturity between $\underline{L}_M$ and $\bar{L}_M$ does not comply with the condition that the loan must mature at a settlement date.

Step D - Has $F(L_J)$ converged ?

In step D it is tested whether $F(L_J)$ has converged so that the payments on the loan are within the band defined by the maximum and minimum limits. If not, the model applies the calculated adjustment.

Whether $F(L_J)$ has converged is determined by testing the mathematical convergence of the adjustment. This is done by

evaluating the size of the adjustment of the term to maturity which was determined in step C against the value of $F(L_J)$. If the adjustment is very small, cf. the conditions mentioned below, there is no reason to continue the iteration routine: The model can not get any closer.

Mathematical convergence is defined by the conditions

(i) $$\frac{|\Delta L_J|}{\epsilon |L_J|} < \epsilon$$

(ii) $$\left|\frac{[F(L_J)^2 - F(L_J+\Delta L_J)^2]}{F(L_J)^2}\right| < \epsilon$$

where ϵ is a very small figure. If just one of the conditions are fulfilled, L_J is accepted as the term to maturity at the Jth refinancing. The convergence may also be tested by applying

(iii) $$|F(L_J)| < \epsilon$$

where ϵ is fixed at 0,00001. Thus, a maximum deviation from the maximum or minimum limit of DKK 0,00001 is accepted.

The definition of $F(L_J)$ in step C implies that the convergence conditions are identical irrespective of the term to maturity for which the value of F is calculated.

Step E - Apply the adjustment of L_J

$YD(J,L_J)$ was rejected in step D.

Therefore, L_J is to be adjusted and subsequently, in the inner model the payments on the loan for $L_J+\Delta L_J$ are calculated. However, at first step B is applied so that the

term to maturity is rounded for J=M.

Step F - Is $L_M<L^{min}$?

In step F, it is checked that the full term to maturity is not shorter than the minimum limit. If

$$L_M<L^{min}$$

the model moves on to step G in which the payments on the loan are adjusted according to (B). Otherwise, the model proceeds in step F.

It should be noted that steps F and I are the only steps which are not applied for every value of J.

Step G - Calculate the payments for $L_J=L^{min}$

If $L_M<L^{min}$ the payments on the loan are adjusted according to (B) which was formulated as

$$YD(J,.)=YD(J,L_J) \quad \text{for } 0\leq J\leq J'' \qquad (B)$$
$$YD(J,.)=YD(J,L^{min}) \quad \text{for } J''\leq J\leq M$$

where J'' denotes the minimum value of J for which $YD(J,L_J)>YD(J,L^{min})$.

$YD(J,L_J)$ has already been calculated for J=0,1,2,…,M, whereas $YD(J,L^{min})$ is to be calculated in the inner model. However, the transition from step G to the inner model is not simple. At first, (B) is calculated for J=0. Depending on whether

$$YD(0,.)=YD(0,L_0) \text{ or } YD(0,.)=YD(0,L^{min})$$

the remaining debt at the end of the period may be determined. Based on the remaining debt, $YD(1,L^{min})$ is calculated and $YD(1,.)$ can be determined and thus the remaining debt at the end of the period etc. Hence, the link from step G to the inner model is successive.

However, it will never be necessary to recalculate $YD(J,L_J)$ in a corresponding manner. From the point where $YD(L^{min})$ determines the payments on the loan, this will be the case until the maturity.

Step H - Adjust the payments according to minimum condition

Having determined the value of $YD(J,.)$ for each J, it only remains to compound a payment profile. Information on funding, interest rate on the loan etc. are compounded in compliance with the payments on the loan determined, of (B).

When the payment profile in full has been determined, the model proceeds in step L.

Step I - Is $L_M>L^{max}$?

Fully analogously to step F, it is to be checked that L_M does not exceed L^{max}. If $L_M>L^{max}$, the payments on the loan are adjusted in step J. Otherwise, the calculations are completed and the model can proceed to step L.

Step J - Calculate the payments for $L_J=L^{max}$

This step is fully analogous to step G except from the fact that $L_M>L^{max}$. Thus, the payments on the loan are adjusted according to (A).

$$YD(J,.)=YD(J,L_J) \quad \text{for } 0\leq J\leq J' \qquad \text{(A)}$$

$$YD(J,.)=YD(J,L^{max}) \qquad \text{for } J' \leq J \leq M$$

where J' denotes the minimum value of J for which $YD(J,L_J)<YD(J,L^{max})$. Just as in step G the link from step J to the inner model is successive.

Step K - Adjust the payments according to maximum condition

Analogously to step H, a payment profile based on the payments on the loan for J=0,1,2,…,M determined in step J is to be compounded. Subsequently, the model continues in step L.

Step L - The calculations are complete. The result may be applied

A compounded payment profile has been determined, the term to maturity L_M has been calculated, and in the inner model the volumes of the financial instruments applied and the interest rate on the loan have been determined. Thus, the computation of the loan is complete and the result may be applied.

2.2.2 The inner model of type F

The model solves the problem in 10 steps as shown in Fig. 16. In the following, the model is discussed in detail.

The model applies an iterative routine. On the basis of a start value of the interest rate, the funding is determined in steps 2 to 4. In step 5, an adjustment of the interest rate is determined. If the proceeds criterion is not fulfilled in step 6, the interest rate will be adjusted, and step 2 is repeated. If the proceeds criterion is fulfilled, the model will make a final test that all funding principals are positive. If this condition is fulfilled also, the results of the model may be applied.

If on the other hand that one or more funding principals are negative, the model proceeds in the inner model for type F^+.

Step 1 - Determine initial interest rate

At first all inputs are entered from the outer model. In the inner model, L_J is considered an input just like the other inputs. Thus, every time the inner model is applied, the value of L_J is fixed.

In step 1, an initial value of the interest rate is determined. To minimize the number of iterations which are to be carried out later, it is expedient to set the first interest rate equal to the yield on the funding instrument having the longest term to maturity.

On the basis of the number of funding instruments entered and the remaining term to maturity, a value for m is determined as

> m=max(number of funding instruments initially; remaining term to maturity rounded up to the next integer)

so that the model does not issue bonds with maturity after the maturity of the loan.

Step 2 - Determine payments on the loan

Based on the interest rate, the principal input, and the term to maturity, the payments on the loan until the next refinancing may be calculated for both R^K and R^K+inc.

The model will return with the following information concerning debtor's payments: interest rate, repayments, payments on the loan, and the remaining debt profile.

Step 3 - Determine funding volumes

Provided that the payments on the loan YD(j) (j≤m) and the refinancing amount at the end of the period RG(m) are determined, the individual volumes of the financial instruments applied may be found applying the equation system resulting from the the balance requirements. The equation system may be written in a matrix form

(G) $\mathbf{YD+RD=A\times H}$,

where $\mathbf{YD}=(YD(1),YD(2),\ldots,YD(m))$,
$\mathbf{RG}=(0,0,\ldots,RG(m))$ and
$\mathbf{H}=(H(1),H(2),\ldots,H(m))$ are vectors with the dimension m×1

and where **A** is defined as a m×m upper triangular matrix:

$$A=\begin{bmatrix} [1+REG\cdot R^N(1)] & REG\cdot R^N(2) & REG\cdot R^N(3) & . & . & REG\cdot R^N(m) \\ 0 & 1+R^N(2) & R^N(3) & R^N(4) & . & R^N(m) \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & [1+R^N(m)] \end{bmatrix}$$

The **A**-matrix is designed in such a way that the balance equations appear immediately, if (G) is calculated.

In the **A**-matrix the number of columns corresponds to the number of financial instruments applied, whereas the number of rows corresponds to the number of years in the refinancing period. If the number of settlements within each year are increased, the dimension of the matrix will increase by the same factor.

By this rewriting, the solution as to the m volumes of the financial instruments applied in matrix form (the top sign T means that the matrix is transposed) is as follows:

(H) $H=[A^TA]^{-1}A^T[YD+RG]$

In principle, $[A^TA]^{-1}A^T$ may be replaced by A^{-1} in (H), as long as A is quadratic. Only if more funding instruments are applied, the rewriting $[A^TA]^{-1}A^T$ is necessary. Thus, the rewriting is only a method by which non-quadratic matrices may be inverted.

The funding for R^K+inc is determined by the same method.

Step 4 - Determine the proceeds function

A function F is defined as the difference between the proceeds required and the market prices of the bond principals calculated under step 3.

$$F(R^K) = RG(0) - \sum_{j=1}^{m} K(j)H(j),$$

Hence, the proceeds criterion is fulfilled for $F(R^K)=0$. The values of $F(R^K)$ and $F(R^K+inc)$ are calculated given the volumes of the financial instruments calculated in step 3.

Step 5 - Calculate adjustment of the interest rate

Then an adjustment ΔR^K of the interest rate guess is calculated. The adjustment will not be implemented until step 7 and only if the convergence condition in step 6 is not fulfilled. The adjustment is calculated applying the Gauss-Newton algorithm described in step C in the outer model. Thus

$$\Delta R^K=\left[[D^TD]^{-1}D^Tg\right]oj_v^{o-1}$$

where j_v, D and g are defined as in B. J_a is defined by

$$J_a=[F(R^K)-F(R^K+inc)]oj_v^{o-1}$$

Also in this case, the expression may be reduced which implies

$$\Delta R^K = F(R^K)\frac{inc}{F(R^K)-F(R^K+inc)}$$

The solution is illustrated in Fig. 17. Also here, the basic idea of the algorithm is to use the secant 42 through two points 44, 46, on the graph for F, $(R^K,F(R^K))$ and $(R^K+inc,F(R^K+inc))$ respectively. When the secant 42 has been determined, the intersection 48 with the x-axis is determined. The intersection 48 is the next guess as to R^K in the iteration routine. If the graph 40 for $F(R^K)$ is strictly declining, the algorithm will always reach a solution. This will be the case, as an increase of the term to maturity results in decreasing payments on the loan.

The algorithm approximates an adjustment of R^K so that $F(R^K+\Delta R^K)=0$. $F(R^K)$ is strictly declining, since an increase in the interest rate on the loan implies that (strictly) more bonds are sold, and thus the proceeds from the funding of the loan are increased (strictly) as mentioned in the introduction.

Step 6 - Is the proceeds criterion fulfilled?

In this step, it is tested whether the interest rate on the loan complies with the proceeds criterion.

Once again, this may be determined by the mathematical convergence of the calculated adjustment. Thus, one of the two conditions below must be fulfilled.

(i) $$\frac{|\Delta R^K|}{\epsilon\cdot|R^K|} < \varepsilon$$

(ii) $$\left|\frac{[F(R^K)^2 - F(R^K + \Delta R^K)^2]}{F(R^K)^2}\right| < \epsilon$$

where ϵ is a very small figure. It is also possible to test the proceeds criterion directly. This may be formulated as

(iii) $$|F(R^K)| < \epsilon$$

Where ϵ is fixed at 0,00001. Thus a maximum deviation from the proceeds criterion of DKK 0,00001 is accepted.

If (iii) is fulfilled, the requested interest rate on the loan (R^K) has been found, and the model can move on to step 8.

If none of the conditions are fulfilled, the computations proceed in step 7.

Step 7 - Adjust the interest rate

The interest rate rejected in step 6 is now adjusted with the adjustment factor ΔR^K calculated in step 5. The computations in the model proceed in step 2, where the payments on the loan are recalculated applying the adjusted interest rate $(R^K + \Delta R^K)$.

Step 8 - Are all funding volumes positive?

In certain cases, one or more of the calculated volumes of the financial instruments applied may be found to be negative. Negative funding volumes correspond to a debtor purchasing a number of the bonds applied in the financing of the loan. In principle, there is nothing which excludes this. However, a number of considerations, including tax considerations, indicate that negative funding volumes should be avoided.

The negative funding volumes may occur as a result of bond market prices above par. An obvious way to avoid negative funding volumes is thus to change funding instruments to be applied, which in this connection would mean the opening of bonds with a lower coupon rate and thus lower market price.

Alternatively, the negative funding principals may be avoided by treating the loan as a LAIR Type F^+ until the next interest rate adjustment is performed or even after that. Thus, if negative funding principals occur, the model continues in step 9.

If, on the other hand, all funding principals are positive, the model continues in step 10.

Step 9 - Shift to F^+

In step 9 the model transfers computation of the loan to the inner model for type F^+. Information concerning the loan, i.e. principal, term to maturity, amortization principle etc. is entered in the F^+ model from the F model.

Step 10 - Interest rate, funding, and payments on the loan may be applied!

An interest rate on the loan as well as a number of positive volumes for the funding instruments fulfilling the given conditions have been found and the calculations are therfore transferred for the given term to maturity. Thus, the model may return to the outer model with information of payments on the loan, the interest rate on the loan, and the volumes of the funding instruments applied.

2.3 The inner model for type F^+

First, it should be noted that the link to the outer model is identical for type F and type F^+. In general, the inner model for type F^+ may thus be considered exclusively as a replacement of type F when the volumes of the financial instruments applied would otherwise be negative. Hence, is is only to keep a clear overview that types F and F^+ are separated in the model.

If the bond with the longest term to maturity in a LAIR with 100 per cent interest rate adjustment has a price over 100, the nominal issue is less than the principal of the loan. This will affect the balance principle in the last year of the refinancing period when the bonds applied mature at price 100 at the same time as the remaining debt of the loan is to be refinanced.

If the difference in the nominal bond volume and the principal of the loan exceeds debtor's repayment in the period until the next refinancing, the balance principle can not be fulfilled when the interest rate on the loan is constant. The problem is solved in that the loan is financed solely in one bond and that a so-called minimum refinancing is introduced.

The basic idea in the minimum refinancing is to transfer payments from the previous years to the last year in the refinancing period by covering a deficit in the payments on the loans by a new bond issue with the same term to maturity at the end of each year so that the balance principle is complied with. Thereby the volume of bonds is increased and the surplus the last year is reduced. Normally, the issue will not only have the same term to maturity, but also the same bond ID code year after year. Yet, adjustments of the statutory minimum coupon interest rate and the like may imply that the coupon interest rate of the issue is changed. In the following, this possibility is disregarded, however.

A particular problem is connected to the determination of the interest rate. As a consequence of the minimum refinancing, the interest rate is not constant during the whole refinancing period, but will, on the contrary, vary from one year to another in the refinancing period.

This means that the limits for the payments on the loan might be violated in the refinancing period. Typically, however, the fluctuations in the interest rate will be moderate, since the minimum refinancing is minimal, as the term suggests. Thus, the consequences will be negligible in practice. The present method only ties up the interest rate during the last year of the refinancing period. For the other years, an arbitrary determination of the interest rate will only change the minimum refinancing. However, an unsuitable determination of the interest rate will result in a substantial change of the interest rate the last year in order to comply with the balance principle. An aim is, therefore, to choose a method for the determination of the interest rate which results in a stable course. The present method is thus a suitable method among others. For example the interest rate could be determined as the bond yield or the like.

Funding in only one bond considerably facilitates the calculation of the funding, the funding principal for the bond with the longest term to maturity being unambigously determined by the proceeds criterion. Thus, the funding may be determined already in the first step of the model.

When the funding has been determined, the interest rate may be determined by applying the principle of strict balance. The interest rate must be determined subject to the condition that the total payments on the bond side equal to the total payments on the loan side to the next ordinary refinancing. However, the remaining debt profile depends on the interest

rate on the loan, thus, it is necessary to iterate over the interest rate on the loan.

The inner model for type F^+ comprises 7 steps as shown in Fig. 18. In the following, the model will be described in detail.

In the model, it is necessary to distinguish between ordinary refinancing and the minimum refinancing. The annual minimum refinancing imply that the model is to be applied not only at the refinancing times (e.g. every fifth year for a type F5), but each year.

Step 9 of the F model

The F^+ model can only be initiated from step 9 of the inner model for type F. Therefore, input has already been entered into the F model and is transferred to the F^+ model without changes.

Step 1 - Calculate the funding volume

The funding is determined at the disbursement of the loan or immediately after an ordinary refinancing as

$$H(m) = \frac{RG(0)}{K(m)} \qquad H(J)=0 \quad for\ j<m$$

m is unchanged in relation to the inner model of the F model. If the loan is refinanced every fifth year, m has the value 5 irrespective of the issue being in one bond only.

For the other years until the ordinary refinancing the minimum refinancing is given by

$$M_{j'}(m) = \frac{Fin(j')}{K_{j'}(m)} \qquad \text{for } J'=1,2,\ldots,m$$

wherein

Fin (j')	generally designates the funding demand year j'. In this connection Fin(j') is thus the minimum refinancing.
$M_{j'}(m)$	(Not to be mistaken for M) designates the marginal funding in the mth financial instrument in year j', i.e. the part of H(m) to be issued at the end of year j'.
$K_{j'}(m)$	is the market price of the mth funding instrument in year j'.

It should be noted that j gets a slightly different meaning, as j is only set to zero in connection with ordinary interest rate adjustments. Thus, funding is issued at times j=1,2..,m etc. and not only at the time j=0 as in the F model.

The minimum refinancing is determined as the difference between the annual payments on the loan side and on the bond side.

$$Fin(j')=R^N(m)J(0,m)-YD(j') \qquad \text{for } 1\leq j'\leq m-1$$

wherein

H(0,m)	designates the already issued funding, i.e. the volume of the mth funding instrument before the minimum refinancing.

Step 2 - Determine initial interest rate

The iterative routine is initiated with an initial value for the interest rate on the loan. The initial value is

$$R^K_{j'} = \frac{H(m)-RG(j')}{(m-j')RG(j')} + \frac{H(m)R^N(m)}{RG(j')}$$

Step 3 - Determine payments on the loan

Based on the resulting interest rate on the loan, the payments on the loan, and in this context, the remaining debt profile are calculated.

Step 4 - Calculate adjustment of the interest rate

The adjustment of the interest rate is calculated according to the same principles as in the steps 4 and 5 of the inner model for type F. First, a function measuring the difference between payments on the loan side and on the bond side is defined.

$$F(R^K) = \sum_{j=j'}^{m} YD(j) + RG(m) - [H(m) + (m-j')H(m)R^N(m)]$$

After this, the adjustment may be calculated by use of the reduced version of the Gauss-Newton algorithm.

$$\Delta R^K = F(R^K)\frac{inc}{F(R^k)-F(R^K+inc)}$$

It should be noted that $F(R^K)$ is strictly increasing, ensuring that the algorithm reaches a solution.

Step 5 - Is the balance requirement fulfilled?

Step 5 determines whether the routine is to continue or whether an interest rate has been found which fulfil the balance requirement. As in step 6 of the F model, the question may be determined by the mathematical convergence of the iteration, but it is more obvious to evaluate the actual

balance condition. This means that the routine stops and the model continues in step 7 if the condition

$$\left| \sum_{j-j'}^{m} YD(j) + RG(m) - [H(m) + (m-j')H(m)R^{N}(m)] \right| < e$$

has been fulfilled for $e=0.00001$. If the condition has not been fulfilled, the model continues in step 6 instead.

Step 6 - Adjust the interest rate

The interest rate did not fulfil the balance requirement and must therefore be adjusted with the adjustment ΔR^{K}. Then step 3 is repeated.

Step 7 - The model is complete. The result may be applied

If the balance condition has been fulfilled, the funding and the interest rate fulfil all conditions and may, together with the payments on the loan, be used in the outer model. Thus, the inner model returns to the outer model with information on volumes of the financial instruments applied, interest rate and payments on the loan.

3. Type P

3.1 The general problem

In LAIR type P, a part of debtor's opening remaining debt is refinanced each year. The debtor chooses an intended annual refinancing percentage which at the same time determines the number of financial instruments to be issued and thus the duration of the funding period.

If debtor chooses, e.g., an annual refinancing percentage of 10 per cent, it will take 1/10 per cent = 10 years before the

loan is fully refinanced. The funding period is therefore 10 years and the number of funding instruments 10. At every refinancing, a funding instrument matures. In order to keep the refinancing proportion at the intended level, a new funding instrument with a term to maturity of m_0 years is, therefore, issued. This continues until the number of funding instruments is gradually reduced when the loan approaches its maturity.

The relation between the number of funding instruments and the proportion of the remaining debt that is refinanced each year implies that the latter can be expressed as $1/m_0$. When debtor chooses the value m_0, the part of remaining debt that is refinanced is chosen at the same time.

The gradual adjustment of the interest rate on the loan implies that the effects of an increase or decrease in interest rates can be noted in the payment profile for a period of m_0 years. Thus, the need for continous adjustments of the payments on the loan is intensified compared to type F.

A payment profile for a LAIR type P at increasing interest rates may be as shown in Figs. 19 to 22. Fig. 19 shows a payment profile 52 for an annuity loan at increasing interest rates where the payments on the loan 52 remain within the given limits, whereas Fig. 20 shows a payment profile 54 at increasing interest rates where the payments on the loan 54 exceed the maximum limit.

Analogeously, Fig. 21 shows a payment profile 56 for a serial loan at increasing interest rates where the payments on the loan remain within the given limits, whereas Fig. 20 shows a payment profile 58 at increasing interest rates where the payments on the loan 58 exceed the maximum limit.

If the payments on the loan are close to the maximum limit, already when the rise in interest rates occurs, the model will soon be in a situation where the payments on the loan exceed YD_J^{max} in the remaining part of the funding period. However, L_J is not adjusted for this reason, as this would imply that $YD(J,L_J)$ would be too low. On the contrary it must be accepted here and now that the future payments on the loan - also within the funding period - exceed YD_J^{max} and are not adjusted via the term to maturity until the model proceeds to later points in time. In other words, the conditions stipulated for the payments on the loan are only to be tested for 1 year at a time. Thus, the model will accept a payment profile 54, 58 as shown in Figs. 20 and 22, respectively, well aware that the term to maturity must be adjusted for the next J.

Thus, J=0,1,2,…,M should not be interpreted as funding periods, but as 1 year refinancing periods. Thus a condition stipulated on, for instance, $YD(J,L_J)$ will only be binding in one year.

The intertemporal conditions may be formulated as in the model for adjustable term to maturity for type F. As a matter of form the conditions are repeated below.

1) The term to maturity must not be longer than the maximum limit, that is,

$$L_M \leq L^{max}$$

2) The term to maturity must not be shorter than the minimum limit

$$L_M \geq L^{min}$$

3) If $L_M<L^{max}$, the payment on the loan is determined observing the condition

$$YD(J,LJ) \leq YD_J^{max}$$

and otherwise

$$\text{(A)} \quad \begin{aligned} YD(J,.) &= YD(J,L_J) && \text{for } 0 \leq J \leq J' \\ YD(J,.) &= YD(J,L^{max}) && \text{for } J' \leq J \leq M \end{aligned}$$

where J' denotes the minimum value of J for which $YD(J,L_J)<YD(J,L^{max})$

4) If $L_M>L^{min}$, it is required that the payment on the loan is above the minimum limit, that is

$$YD(J,LJ) \geq YD_J^{min}$$

and if the limit for the term to maturity is violated

$$\text{(B)} \quad \begin{aligned} YD(J,.) &= YD(J,L_J) && \text{for } 0 \leq J \leq J'' \\ YD(J,.) &= YD(J,L^{min}) && \text{for } J'' \leq J \leq M \end{aligned}$$

where J'' denotes the minimum value of J for which $YD(J,L_J)>YD(J,L^{min})$

The model observes these conditions by determining a sequence of terms to maturity $L_0,L_1,\ldots,L_M$. This is done in an outer model.

Like in the model for type F, it is required that the principle of strict balance and the proceeds criterion are observed in each refinancing period. In addition, an explicit condition is stipulated on the relation between the interest

rate on the loan and the yield to maturity of the portfolio of the bonds applied. These conditions are handled in the inner model.

However, the inner model may be solved both applying an iterative routine and analytically. The iterative solution has the advantage of very stable funding profiles. The analytical solution has the advantages of a more simple structure and shorter computation times. Both an iterative inner model as well as an analytical inner model are described in the following.

The annual refinancing implies that the balance principle plays a slightly different role. At the end of each year, the total payments on the debtor side and on the bond side are known. By determining the interest rate adjustment amount residually, the principle of strict balance has been fulfilled by definition. It is, however, not necessarily so that the intended refinancing percentage corresponds to the actual one.

Therefore, the problem to be solved in the model is to adjust the funding so that the intended and the actual proportions of the remaining debt that are refinanced correspond, at the same time as the proceeds criterion is fulfilled and at the same time as all the volumes of the financial instruments applied are positive (≥0). Upon the disbursement of the loan the problem is described by the equations

Year 1 $$YD(1)+REG^{D}\frac{RG(0)}{m_0}\cdot H(1)+REG\sum_{j=1}^{m} R^{N}(j)H(j)$$

Year 2 $$YD(2)+\frac{RG(1)}{m_0}\cdot H(2)+REG\sum_{j=2}^{m+1} R^{N}(j)H(j)$$

...

Year m $$YD(m) + \frac{RG(m-1)}{m_0} \cdot H(m) + REG \sum_{j=m}^{2m-1} R^N(j) H(j)$$

wherein

REG^D is a regulation factor for the refinancing percentage in the first year. REG^D can assume the values {¼,½,¾,1} which will be elaborated on in the following.

In the following, these equations are designated as the balance requirements.

If year 1 is considered alone, the problem seems easy to solve. The task is merely to determine the volumes of the financial instruments applied which lead to the intended payments and the intended proceeds. In principle, the distribution of the volumes of the financial instruments applied is of minor importance, so there is an infinite number of solutions to the problem.

The problems start to arise when the interest rate is adjusted by the end of the year. If an arbitrary distribution of the funding was chosen at the disbursement of the loan, it is not certain that it is possible to hit the desired refinancing profile again, if at the same time all the volumes of the financial instruments applied must be positive (≥0). If most of the funding of the loan was placed in H(2) at the disbursement of the loan, the right side of the equation will be large. Thereby the refinancing amount will be large too, if the balance requirement is to be fulfilled. Thus, it is not possible to keep the part of the remaining debt that is refinanced down at the intended level unless bonds are redeemed.

If only a very small part of the funding was placed in H(2), it may, on the other hand, be difficult to reach the intended refinancing percentage. This may, however, be partly saved by placing a large part of the proceeds from the refinancing in the bond now having a term to maturity of 1 year. The result, however, will be an unstable funding profile.

If the balance requirements with the intended refinancing percentage every year are to be fulfilled, it is necessary to formulate a dynamic strategy for the placing of the funding also taking into consideration the long term perspective.

An arbitrary strategy for the funding will also raise another, just as essential problem. At each refinancing, the model cannot merely place the entire funding in bonds maturing in a single year. As it will be remembered, the actual volume of bond redemptions corresponds to the actual refinancing amount (apart from premium or discount), which on its side is determined by the refinancing percentage and the remaining debt on the loan. When the newly issued bonds are redeemed, the remaining debt is smaller, but the refinancing amount is of fairly the same size as currently. Thus, the future refinancing percentage will be higher than intended. Therefore, the model must continuously have the possibility of placing the funding in bonds with different terms to maturity.

In the iterative inner model as wells as in the analytical model, the dynamic strategy implies that the funding is adjusted each year in such a way that the payments of the bonds correspond to a decreasing proportion of the payments from the debtor side as time is progressing. At each refinancing, issue is further performed in each financial instrument applied - thus, gradually there will be increasing accordance between the bond side and the debtor side given the part of the remaining debt that is refinanced. At the

disbursement of the loan, the problem being solved via the model can, therefore, be formulated as

Year 1 $$YD(1)+REG^{D}\frac{RG(0)}{m_0}-H(1)+REG\sum_{j-1}^{m}R^{N}(j)H(j)$$

Year 2 $$F(YD(2)+\frac{RG(1)}{m_0})-H(2)+\sum_{j-2}^{m}R^{N}(j)H(j)$$

...

Year m $$F(YD(m)+\frac{RG(m-1)}{m_0})-[1+R^{N}(m)]H(m)$$

wherein F is a declining function of j.

In principle, the expressions may be generalized so that the bonds maturing in year j may have different coupon interest rates. However, this is excluded in order not to unnecessarily complicate the notation.

In both models the funding profile follows a so-called trend function which is estimated based on the intended refinancing percentage and the funding already issued H(0,j).

The trend function is adjusted in such a way that to the extent possible, the actual refinancing percentage correspond to the intended refinancing percentage while at the same time the proceeds criterion is fulfilled and strictly negative volumes of the financial instruments applied are avoided. The adjustment is applied either by iteration or analytically, thus, at this stage the two inner models differ.

When these conditions are met, the interest rate on the loan is subject to iteration in an outer routine until the interest rate equals the yield to maturity of the portfolio of bonds applied for the financing of the loan. An adjustment

is made for differences in the number of settlements on the debtor side and the bond side. Typically, the number of settlements on the debtor side exceeds the number of settlements on the bond side, which means that the debtor pays in advance compared to the one annual settlement on the debtor side. Thus, in order for the total payments to be equal the interest rate on the loan must be a little higher than the yield to maturity of the bonds applied.

Hence, the inner model is solved in a 2-step-procedure and not simultaneously as for the F model.

There is a special problem attached to the date of disbursement of the loan. If the loan is disbursed in December, the first refinancing will be performed well over one year later. The rule set out above can thus be applied.

On all other dates during the year, the refinancing percentage at the first refinancing is, however, written down in relation to the quarter in which the loan has been disbursed. This is indicated by the regulation factor REG^D defined above. This means that the funding period is prolonged by 1 year, and the number of funding instruments is increased by 1. At the disbursement of the loan the following applies

$$m=m_0+1$$

Already at the first refinancing, the general rule with m_0 funding instruments is followed. Thus, no new instrument is issued at the first refinancing. In the model, there is a special procedure in connection with the extra funding instrument. The procedure is indicated by a variable, TILT, being given the value 1.

In general, the notation is the same as in the F model. However, the continuos issue of bonds makes it necessary to differentiate between three definitions of funding volumes.

M(j) designates the marginal funding, i.e. the volume of bonds issued in the jth year when the current issuing of bonds is performed.

H(0,j) are the bonds already issued before the present issuing of bonds.

H(j) is the total amount of bonds in the jth year.

From the definitions the following relation appears

$$M(j)+H(0,j)=H(j)$$

At the disbursement of the loan, it is obvious that $H(0,j)=0$, and thus it applies that $M(j)=H(j)$. In the notation, it is therefore not necessary to differentiate between the disbursement of the loan and the interest rate adjustment in this respect.

Correspondingly, the funding demand at the disbursement of the loan as well as at the refinancing will be indicated by Fin(j). Thus, Fin(j) may either by the principal of the loan or the current amount of refinancing. Also in this respect, it is unnecessary to differentiate between disbursement and refinancing.

As a consequence of the fact that no differentiation is made between disbursement of the loan and refinancing, REG and REG^D will be a part of expressions which also apply to refinancing. Here, REG=1 and REG^D=1 in accordance with the definition.

3.2 Type P - method

A sequence of terms to maturity is to be determined observing that in each refinancing period, the payments on the loan are within the band defined by the maximum and minimum limits for the payments on the loan, stipulated that the total term to maturity is within the band defined by the maximum and minimum limits for the term to maturity, and stipulated that the payments on the loan, in each refinancing period, fulfil:

actual refinancing percentage=intended refinancing percentage

and

the proceeds criterion

and

adjusted interest rate on the loan=yield to maturity of the portfolio of financial instruments applied.

3.2.1 The outer model of type P

Like the type F model, this model is divided into an outer and an inner model. While the differences in the inner model for the two types are substantial, the outer models differ less. The outer model is illustrated in Fig. 23. The general structure in the outer model is not altered. Furthermore, several steps are identical. To facilitate the overview of the model, the steps are repeated here, but with an indication that the content is the same as in the type F-model.

The procedure applied when the loan matures is altered.

A substantial part of the funding is issued with a term to maturity exceeding the refinancing period. When the loan approaches maturity, the term to maturity must, therefore, not be shortened without due consideration - this would imply

the risk of the loan maturing before one or more of the financial instruments already issued. Thus, in the model when computations are carried out for the last funding period, i.e., for $J \geq M-m_0$, the minimum limit for the payments on the loan is to be suspended so that the term to maturity will not be shortened.

At the same time, the maturity of LAIR type P must coincide with a bond settlement. However, it is not sufficient to take this into account in the last refinancing period for J=M. As soon as M is within the funding period, debtor payments in the last year before maturity are funded in the model. Hence, it is necessary to adjust the term to maturity at an earlier point in time. Therefore a parameter termed "closing time" is implemented designating when the model is to start adjustuding the term to maturity, i.e., for which J L_J is to be adjusted. The adjustment of the term to maturity is carried out like in step B in the type F model, but, as will be understood, not only for J=M as in the type F model.

Closing time must be assigned a value based on the type of loan. Thus, it would not make sense to assign closing time>m_0. Closing time=m_0 will lead to the most suitable maturity of the loan, but on the other hand, implies that the payment on the loan deviates from the intended pattern of shifts in the payments for an unnecessary time period. Thus, assigning the value of closing time is a trade off between considerations relating to the maturity of the loan and considerations relating to the payment profile until then.

As an alternative to implementing closing time, one could choose to fund the refinancing in the last m_0 years in the bond with the second longest term to maturity. Thereby, the full remaining debt on the loan would be refinanced one year before the maturity of the loan. Then, in the last year before maturity, the term to maturity may be prolonged to the

nearest settlement date without any problems in the last year before maturity.

Step A - Determine initial L_J

As in the type F model, the term to maturity is assigned in step A, an initial value which is to be considered a first guess in the iterative routine.

If J>0, the initial value of L_J is relatively simply determined as L_{J-1}.

If J=0 and the calculations coincide with a refinancing of the loan, L_0 is assigned an initial value based on the sequence of terms to maturity as calculated at the preceding refinancing of the loan. The point in time defined by J=0 corresponds to J=1 at the preceding refinancing of the loan since J is set to zero. Thus, L_0 is to be assigned an initial value equal to L_1 at the preceding refinancing of the loan in the current refinancing.

If J=0 and the calculations coincide whit the disbursement of the loan, the term to maturity is determined based on input from the debtor which may comprise:

1) The debtor selects an annuity loan with fixed payments, thus $YD_J^{max}=YD_J^{min}$, and an initial term to maturity is to be determined. The initial guess as to L_0 may be determined applying the annuity formula

$$YD(0,L_0)=\frac{RG(0)R^K}{1-(1+R^K)^{-(L_0-\varphi)}}$$

which, solved for L_0. yields

$$(F) \qquad L_0 = -\frac{\ln\left(1-\frac{RG(0)R^K}{YD(0,L_0)}\right)}{\ln(1+R^K)}+\varphi$$

where $YD(0,L_0)=YD_J^{max}=YD_J^{min}$,

and where

φ is the number of years since the disbursement of the loan at the point in time of the calculations. Thus, $L_0-\varphi$ is the remaining term to maturity to be applied in the annuity formula.

However, R^K has not been determined. To reduce the number of iterations, R^K is set equal to a weighted average of the yield to maturity of the financial instruments applied.

$$R^K = \frac{\sum_{t=1}^{m}\frac{1}{m}t\,r(0,t)}{\sum_{t=1}^{m}\frac{1}{m}t}$$

where

t is the term to maturity for each financial instrument. $t=\{1,\ldots,m\}$

r(0,t) is the yield to maturity of the bond which matures at t.

2) The debtor selects an intended term to maturity and maximum and minimum limits for the payments on the loan.

In this situation, L_0 is simply set equal to the intended term to maturity. After the disbursement of the loan, the intended term to maturity is of no consequence and is, thus, not stored in the model.

3) The debtor selects an intended payment on the loan and maximum and minimum limits for the payments on the loan.

L_0 is determined analogously to the situation in which the debtor selects fixed payments on the loan, that is by applying (F) where $YD(0,L_0)$ is the intended payment on the loan. However, a similar formula must be derived for serial loans. The payments on the loan for the first year are given by

$$YD(0,L_0)=\left(nRG(0)-\frac{n(n-1)}{2}\frac{RG(0)}{n(L_0-\varphi)}\right)\frac{R^K}{n}+n\frac{RG(0)}{n(L_0-\varphi)}$$

which, solved for L_0 yields

$$L_0=\frac{RG(0)-RG(0)R^K\frac{n-1}{2n}}{YD(0,L_0)-RG(0)R^K}-\varphi$$

After the disbursement of the loan, the model suspends the intended payment on the loan, which, thus, is not stored in the model.

All inputs to the model (both the inner and the outer model) are entered in step A. These inputs are:

Inputs related to the funding

Number of financial instrument applied (and thus the refinancing percentage, coupon interest rate on each financial instrument applied, number and date of settlements each year and price of each of the financial instruments applied.

Inputs related to the debtor

Date of disbursement, principal, principle of redemption, number and date of payments on the loan each year, maximum and minimum limits for the payments on the loan, maximum and minimum limits for the term to maturity on the loan, and, optionally, a preferred payment on the loan or term to maturity.

In addition, also the values of inc and ϵ are input. Finally, the value of closing time is to be input which may depend on the refinancing percentage of the loan. Thus, closing time is a set of parameters comprising one element for each possible refinancing percentage (at present 9 elements)

When inputting the inputs, it is tested that

$$YD_J^{max} \geq YD_J^{min} \geq 0 \text{ and } L^{max} \geq L^{min} \geq 0$$

Since the loan is required to mature on a settlement date, it is required that the values of L^{max} and L^{min} entered comply with a settlement date. For instance, if the loan is disbursed on 30 June, it is required that L^{max} and L^{min} are indicated by an whole (integer) number of years +½.

Step B - Adjust L_M, Determine M

As in the type F model, the term to maturity is to be adjusted when the loan approaches maturity. However, it is not sufficient to adjust the term to maturity just in the last refinancing period before maturity. On the contrary, the term to maturity must be adjusted already when the funding period ÷ closing time stretches to the expected maturity of the loan, cf. section 3.1. On the other hand, the adjustment is facilitated by the fact that only a prolongation of the term to maturity to $\bar{L}_M$ is to be considered, whereas the term to maturity will not be shortened cf. section 3.1. Thus, the

adjustment needs not follow a step-by-step procedure as in the type F model.

In mathematical terms, the adjustment of the term to maturity is given by

For $J \geq M \div$closing time, the term to maturity is prolonged to $\bar{L}_J$, i.e., the nearest bond settlement date,

and otherwise not.

Inputs to the inner model is thus

L_J for $J < M \div$closing time or
$\bar{L}_J$ for $J \geq M \div$closing time and
L^{MAX} and L^{MIN}

Additionally, in step B the value of M is determined based on the term to maturity.

Step C - Calculate the adjustment of L_J

Based on the terms to maturity input, the model has calculated the payments on the loan. Step C calculates an adjustment ΔL_J of L_J based on the ratio between the payments on the loan (for a full payment period) on the one hand and the limits for the payments on the loan on the other hand. Depending on the time of the calculations, the first payment on the loan may be fractioned, in which case the first payment is calculated proportionally. Thus, the model allows the first payment after the disbursement of the loan to violate the band.

As in the type F model, the adjustment of the term to maturity is set to zero if either L^{MAX} or L^{MIN} is binding. Thus, if either

$YD(J.L^{MAX}) \geq YD_M^{max}$ or

$YD(J,L^{MIN}) \leq YD_M^{min}$ then

$\Delta L_J = 0$

If neither L^{MAX} nor L^{MIN} is binding, the model calculates an adjustment based on the limits for the payments on the loan by applying a function $F(L_J)$, which. like in the type F model, measures the deviaion of the payments from the band. However, there is a number of points in time for the calculation where the minimum limit (YD_J^{min}) is to be suspended.

If the funding period comprises the maturity of the loan, hence $J \geq M-m_0$, the term to maturity must not be shortened. To prevent the model from continuing the routine endlessly, YD_J^{min} is suspended.

Correspondingly, the minimum limit must be suspended for $J \geq M \div$closing time so that the term to maturity can be prolonged to the next bond settlement date. However, since closing time$\leq m_0$ this is comprised by the above-mentioned suspension of the minimum limit. Thus, when defining $F(L_J)$ it is not necessary to take closing time into consideration.

$F(L_J)$ is thus defined as, for $J<M-m_0$,

1) For $YD(J.L_J) > YD_J^{max}$

(i) $F(L_J) = YD(J,L_J) - YD_J^{max}$

2) For $YD_J^{min} \leq YD(J,L_J) \leq YD_J^{max}$

(ii) $F(L_J) = 0$

3) and for $YD(J.L_J) < YD_J^{min}$

(iii) $F(L_J)=YD(J,L_J)-YD_J^{min}$

and for $J \geq M-m_0$ (where L_J is replaced by $\bar{L}_J$ when the model reaches the closing time period)

1) For $YD(J,L_J)>YD_J^{max}$

(i) $F(L_J)=YD(J,L_J)-YD_J^{max}$

2) For $YD_J^{min} \leq YD(J,L_J) \leq YD_J^{max}$

(ii) $F(L_J)=0$

For a serial loan, $YD(J.L_J)$ is defined as the payments on the loan the first year of the refinancing period.

Based on the value of $F(L_J)$, an adjustment of L_J is to be calculated like in the type F model. The adjustment is calculated applying the reduced version of the Gauss-Newton algorithm.

If $F(L_J)=0$, the payment on the loan is within the band and thus L_J is not adjusted, corresponding to

$$\Delta L_J=0$$

If $F(L_J) \neq 0$, the adjustment is calculated applying

$$\Delta L_J = F(L_J) \frac{inc}{F(L_J)-F(L_J+inc)}$$

Step D - Has $F(L_J)$ converged ?
(identical with step D in the type F model)

In step D it is tested whether $F(L_J)$ has converged so that the payments on the loan are within the band defined by the maximum and minimum limits. If not, the model applies the calculated adjustment.

Whether $F(L_J)$ has converged is determined by testing the mathematical convergence of the function. This is done by evaluating the volume of the adjustment of the term to maturity which was determined in step C against the value of $F(L_J)$. If the adjustment is very small, cf. the conditions mentioned below, there is no reason to continue the iterative routine: The model can not get any closer.

Mathematical convergence is defined by the conditions

(i) $$\frac{|\Delta L_J|}{\epsilon |L_J|} < \epsilon$$

(ii) $$\left|\frac{[F(L_J)^2 - F(L_J + \Delta L_J)^2]}{F(L_J)^2}\right| < \epsilon$$

where ϵ is a very small figure.

If just one of the conditions is fulfilled L_J is accepted as the term to maturity at the Jth refinancing. In principle, the convergence may also be tested by applying

(iii) $$|F(L_J)| < \epsilon$$

where ϵ is fixed at 0,00001. Thus, a maximum deviation from the maximum or minimum limit of DKK 0,00001 is accepted.

The definition of $F(L_J)$ in step C implies that the convergence conditions are identical irrespective of the term to maturity for which the value of F is calculated.

Step E - Apply the adjustment of L_J
(identical with step E in the type F model)

$YD(J,L_J)$ was rejected in step D. Thus, L_J is to be adjusted, after which the inner model calculate the payments on the loan for $L_J+\Delta L_J$. First, however, the model must pass step B so that the term to maturity is adjusted for J=M.

Step F - Is $L_M<L^{min}$?
(identical with step F in the type F model)

In step F, it is checked that the full term to maturity L_M is not shorter than the minimum limit. If

$$L_M<L^{min}$$

the model proceeds in step G in which the payments on the loan are adjusted according to (B). Otherwise, the model will proceed in step F.

It should be noted that steps F and I are the only steps which are not applied for every value of J.

Step G - Calculate the payments on the loan for $L_J=L^{min}$
(identical with step G in the type F model)

If $L_M<L^{min}$, the payments on the loan are to be adjusted according to (B), which stipulated

(B)
$$YD(J,.)=YD(J,L_J) \quad \text{for } 0\leq J\leq J''$$
$$YD(J,.)=YD(J,L^{min}) \quad \text{for } J''\leq J\leq M$$

where J'' denotes the minimum value of J for which $YD(J,L_J)>YD(J,L^{min})$.

$YD(J,L_J)$ has already been calculated for J=0,1,2,…,M, whereas $YD(J,L^{min})$ must be calculated in the inner model. However, the link from step G to the inner model is not simple. At first, (B) is calculated for J=0. Depending on whether

$$YD(0,.)=YD(0,L_0) \text{ or } YD(0,.)=YD(0,L^{min})$$

the remaining debt at the end of the period may be determined. Based on the remaining debt, $YD(1,L^{min})$ is calculated and YD(1,.) can be determined and thus the remaining debt at the end of the period etc. Hence, the link from step G to the inner model is successive.

However, it will never be necessary to recalculate $YD(J,L_J)$ in a corresponding manner. From the point on where $YD(L^{min})$ determines the payments on the loan, this will be the case until the maturity.

Step H - Adjust the payments according to minimum condition
(identical with step H in the type F model)

Having determined the value of YD(J,.) for each J, it only remains to compound a payment profile. Information on funding, interest rate on the loan etc. are compounded in compliance with the payments on the loan to be applied, cf. (B).

When the payment profile in full has been determined, the model proceeds in step L.

Step I - Is $L_M>L^{max}$?
(identical with step I in the type F model)

Analogously to step F, it is tested that L_M does not exceed L^{max}. If

$L_M > L^{max}$,

the payments on the loan are to be adjusted in step J. Otherwise, the calculations are complete and the model can proceed in step L.

Step J - Calculate the payments on the loan for $L_J = L^{max}$
(identical with step J in the type F model)

This step is completely analogous to step G except from the fact that $L_M > L^{max}$, so that the payments on the loan are to be adjusted according to (A).

(A)

$$YD(J,.) = YD(J,L_J) \quad \text{for } 0 \le J \le J'$$

$$YD(J,.) = YD(J,L^{max}) \quad \text{for } J' \le J \le M$$

where J' denotes the minimum value of J for which $YD(J,L_J) < YD(J,L^{max})$. As in step G, the transition from step J to the inner model is successive.

Step K - Adjust the payments according to maximum condition
(identical with step K in the type F model)

Analogously to step H, a payment profile based on the payments on the loan for J=0,1,2,…,M determined in step J is to be compounded. Then, the model proceeds in step L.

Step L - The calculations are complete. The result may be applied
(identical with step L in the type F model)

A compounded payment profile and the term to maturity L_M have been calculated. In the inner model, the volumes of the financial instruments applied and the interest rate on the

loan have been determined. Thus, the computation of the loan is complete and the result may be applied.

3.2.2 The inner model for type P - the iterative solution

The inner model for type P is not simultaneous, but rather consits of an outer and an inner loop in a two-step procedure. Thus, together with the routine in the outer model, the total model comprises three levels.

In Fig. 24 a flow-chart for the model is shown. The two-step procedure does not appear clearly form the flow-chart and is, therefore, briefly outlined in the following.

The model begins with a guess at an interest rate on the loan. On the basis hereof, the payments on the loan and a first guess at the funding are determined. Then the model estimates the funding in an inner loop in step 7 to step 10. The estimation is performed in an iterative routine. The loop is left when the funding fulfils the requirement as to the part of the remaining debt that is refinanced, and the proceeds criterion.

However, a situation may arise for the model where it is not possible to observe the intended refinancing percentage and the proceeds criterion at the same time. If both criteria are not met within 30 iterations in the inner loop, it is presumed that a solution does not exist and the model leaves the inner loop. Then, the funding is adjusted to observe the proceeds criterion.

In the outer loop, it is tested that the interest rate on the loan corresponds to the yield to maturity of the funding portfolio, which, however is adjusted for possible differences in the number of payment dates on the debtor side and on the funding side, respectively. If this is not the

case, the guess at the interest rate is adjusted, and the model again recalculates a debtor payment profile etc. Thereby, the inner loop is also called again with the new interest rate.

Thus, the iterative procedure concerning the interest rate constitutes the outer loop.

Only when all the requirements have been fulfilled does the model leave the outer loop too, and the final result of the calculations in the inner model is available. Then the model returns to the outer model with information on funding, interest rate on the loan, and payment profile.

The same notation as in the inner type F model is applied. Thus, in the inner model for type P, J and L_J are fixed, whereas the individual years are indexed by j.

Step 0 - Determine m and TILT

Before actual calculations are performed, it must be input to the model whether the loan is disbursed in December with $m=m_0$ funding instruments or at any other time of the year with $m=m_0+1$ funding instruments.

This is done by assigning TILT $\epsilon\epsilon\{0,1\}$ a value.

Set TILT=1 and $m=m_0+1$	if the loan is disbursed in January-November
Set TILT=0 and m=m0	in all other cases.

The primary function of TILT is to indicate that an adjustment in the number of funding instruments at the disbursement of the loan is performed, where it will generally apply that

$$m=m_0+TILT$$

In connection with the maturity of the loan, m is also to be adjusted to ensure that no financial instruments with maturity later than the maturity of the loan are applied. Thus, m is assigned a value according to

$$m=\min[L_0-\sigma;m_0+TILT]$$

where $L_0-\sigma$ designates the remaining term to maturity of the loan.

Step 1 - Determine initial interest rate

In this step an initial interest rate on the loan is determined. In order to minimize the number of iterations to be carried out later on it is expedient that the determination of the initial interest rate on the loan is not arbitrary.

In the model, a guess is made that the interest rate on the loan is a weighted average of the yield on the individual funding instruments, thus

$$R^{\kappa}=\frac{\sum_{t=1}^{m}\frac{1}{m}t\,r(0,t)}{\sum_{t=1}^{m}\frac{1}{m}t}$$

as in step A.

Step 2 - Determine the payments on the loan

In this step the model begins the outer loop. When a guess at the interest rate has been made, the payment profile may be calculated.

Step 3 - Is m=1, m=2 or m>2?

The remaining term to maturity on the loan is decisive for the further operations in the model. The calculations in the model may proceed in three different ways.

If m=1, the funding and the interest rate are calculated rather simply in step 3a.

If m=2, the calculations proceed in step 3b, wherein the funding and the interest rate on the loan are calculated by a method which is related to the F model. LAIR type P50 will be calculated in this step apart from at the issue, wherein m may be 3 depending on the date of the disbursement of the loan. In addition, in the last year, the loan will be calculated in step 3a.

If m>2, calculations are continued in the model proper in step 4.

This means that in step 3 the type P loan is, in general, first calculated in the model proper, then in step 3b, and finally in step 3a.

Step 3a - Determine funding and interest rate for m=1

If m=1, the funding can be determined on the basis of the proceeds condition as

$$M(1)\cdot\frac{Fin(0)}{K(1)}$$

Since no more refinancing occur, it follows from the strict balance principle that

$$YD(m)\cdot[1+R^{N}(m)]H(m)$$

which determines an unambiguous interest rate on the loan. When the calculations in step 3a are complete, the calculations proceed in step 14.

Step 3b - Determine funding and interest rate for m=2

If m=2, the balance requirement in the first following year can be written as

$$(I) \quad YD(1)+REG^{D}\frac{RG(0)}{m_0}-(1+REG\,R^{N}(1))H(1)+REG\,R^{N}(2)H(2)$$

and the proceeds criterion as

$$(J) \quad M(1)K(1)+M(2)K(2)-Fin(0)$$

These are sufficient conditions for determining the funding analytically as the solution to two equations (I and J) with two unknown variables (M(1) and M(2)).

In order to do this, (I) and (J) are written into a matrix form in the following way

$$C\times M=D$$

Wherein

$M=(M(1),M(2))$ is a 2×1 vector and

$$D\left[REG^{D}\frac{RG(0)}{m_0}+YD(1)-(1+REG\,R^{N}(1))H(0,1)-REG\,R^{N}(2)H(0,1),Fin(1)\right]$$

is a 2×1 vector too, and

$$C\begin{pmatrix}1+REG\,R^{N}(1) & REG\,R^{N}(2)\\ K(1) & K(2)\end{pmatrix}$$

C is a square matrix and therefore can be inverted which yields the solution

$$M=\max[0;C^{-1}D]$$

as strictly negative funding volumes are not accepted.

The REG and REG^D factors only influence loans which are either disbursed with a term to maturity of 2 years or type P50,0 the first year.

The model continues in step 11, where it is checked that the funding criterion has been observed. This check is necessary if $C^{-1}D$ yields negative funding volumes. In this situation, the max-condition implies that the model is making an excess of funding when determining the funding.

If a funding principal is adjusted because it would otherwise be negative, a deviation in the current refinancing percentage arises in relation to the intended refinancing percentage. Therefore, the analytic solution does not provide certainty that the intended refinancing profile may be respected.

Step 4 - Define a trend function

In step 4, a trend function is defined, which trend function estimates the size of the refinancing amounts as a function of the time t. The trend function is shown in Fig. 25.

In principle, the trend function may have any functional form. A excellent estimate for the development of the remaining debt can be obtained by applying a polynomial of the (q-1)th degree.

Thus, the trend function has the form

$$a_0+a_1t+a_2t^2+\ldots a_{q-1}t^{q-1}$$

wherein $0 \le t \le m$. Furthermore, a limitation must be set on q so that the degree of the polynomial does not exceed the number of funding instruments minus 1, whereby the polynomial would have too many degrees of freedom. This means that

$$q \le m$$

Thus, for a LAIR type P20,0 a polynomial of at the most the 4th degree is estimated. If q=m, the trend function will estimate the refinancing amounts perfectly.

If the loan is disbursed with $m=m_0+1$ in the special TILT procedure, the degree of the polynomial is not increased. Therefore, the exact restriction on q can be expressed as

$$q \le m-TILT$$

Step 5 - Determine trend function coefficients

Then the coefficients of the trend function must be estimated. From step 2 the development of the remaining debt is known given the guess at an interest rate on the loan and given the term to maturity of the loan.

At times of interest rate adjustment t=1,2,..,m the trend function must correspond to the intended refinancing of the loan. At the same time negative marginal funding must be avoided. Therefore the trend function must be estimated so that the function value of each t corresponds to the maximum of either the intended refinancing or the funding already issued in the bond with maturity on the date t.

$$a_0+a_1t+a_2t^2+\ldots+a_{q-1}t^{q-1} \ge \max\left[REG^D \frac{RG(t)}{m_0}, H(0,t+1)\right]$$

for t={0,1,...,m-1}.

Thereby, the coefficients can be determined by the matrix equation

$$(a_0, a_1, \ldots, a_{q-1}) = [B^T B]^{-1} B^T \max\left[REG^D \frac{RG(t)}{m_0}, H(0, t-1)\right]$$

wherein $(a_0, a_1, \ldots, a_{q-1})$ is a q×1 vector
B is a m×q matrix and
max[.,.] is a m×1 vector

The matrix B is given by

$$B = \begin{bmatrix} 1 & t_0 & t_0^2 & . & . & t_0^{q-1} \\ 1 & t_1 & t_1^2 & . & . & t_1^{q-1} \\ . & . & . & . & . & . \\ 1 & t_{m-1} & t_{m-1}^2 & . & . & t_{m-1}^{q-1} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & . & 0 \\ 1 & 1 & 1 & 1 & . & 1 \\ 1 & 2 & 4 & 8 & . & 2^{q-1} \\ 1 & 3 & 9 & 27 & . & 3^{q-1} \\ . & . & . & . & . & . \\ 1 & m-1 & (m-1)^2 & . & . & (m-1)^{q-1} \end{bmatrix}$$

The matrix is produced by assigning values from 0 to m-1 to t in the trend function. The first row comprises the values of $t^0, t^1, t^2, \ldots, t^{q-1}$ for t=0. The second row comprises the similar values for t=1, etc. The expression $[B^T B]^{-1} B^T$ is an approximation of B^{-1} based on least squares. The approximation is necessary when B can not be inverted. B will always be of full rank. However, if q<m - as in the special TILT procedure - B will not be a square matrix and thus can not be inverted.

Step 6 - Guess at an increment to the two coefficients

The trend function must be adjusted in the following steps, so that the balance requirements correspond to the intended

refinancing and so that the proceeds criterion is fulfilled. Initially, however, the trend function must be adjusted.

The adjustment is generally performed by means of two factors G_0 and G_1 in the following way

$$G_0a_0+G_1a_1t+a_2t^2+\ldots a_{q-1}t^{q-1}$$

G_0 effects a parallel shift of the trend function up and down in the (H(j),j) plane, whereas G_1 influences the slope of the trend function.

Already before the iteration in the inner loop is started, the factors are assigned the values

$$G_0=1.25 \text{ and } G_0=1$$

corresponding to a shift upwards. The idea of the shift is to obtain better information concerning the relationship between the individual marginal funding principals.

In the example, H(0,4) is shown disproportionately large so that according to the trend function, M(4) will be 0. If the marginal funding volumes are generally increased in a later step, the model, however, does not have much information as to how much the volume of the other marginal funding instruments must be increased before M(4) is increased as well. By a parallel shift of the trend function upwards this information is obtained.

If TILT=1 and the model thus operates with an extra funding instrument, only one G-variable is introduced in step 6. That is, because of the special refinancing profile, the volume of the first funding instrument is explicitly estimated by a variable Z which is explained in detail in the next step. Provisionally, set Z=1.

Step 7 - The funding is determined

The inner loop of the inner model starts in this step.

On the basis of the trend function, the marginal funding is determined by

$$M(j) = \max[0;[G_0a_0+G_1a_1t+a_2t^2+ \ldots.+a_{q-1}t^{q-1}]-H(0,j)]$$

wherein $t=j-1=\{0,1,\ldots,m-1\}$ as before. At the disbursement of the loan, $H(j)=M(j)+H(0,j)$ and $H(0,j)=0$ are again applied.

That the loop starts in step 7 and not in step 5 or 6 implies that $a_0,a_1,\ldots,a_{q-1}$ are only estimated once for every guess at an interest rate. Therefore, in the following step $a_0,a_1,\ldots,a_{q-1}$ are constant. Correspondingly, $(G_0,G_1)=(1.25,1)$ is solely an initial guess.

For TILT=1 the volume of the first instrument M(1) is determined as

$$M(1)=Z-H(0,j)$$

This is necessary, since a polynomial would have difficulties in estimating M(1) which constitutes down to one fourth of the volume of the other funding instruments as a consequence of the reduced refinancing percentage.

When the model in the following steps estimates Z on the basis of the balance requirements and the proceeds criterion, M(1) is explicitly determined so that the criteria are fulfilled with certainty at the first occurring refinancing.

The other funding volumes are determined as above. Generally, therefore, the following applies

$$M(j)=\max[0;[Z,G_0a_0+a_1t+a_2t^2+ \ldots. +a_{q-1}t^{q-1}]-H(0,j)]$$

Analogously, the funding for

$$((G_0+inc,G_1),(G_0,G_1+inc))$$

is determined. From now on the notation $(G_0,G_1)+inc$ is simply applied. The standard is that inc has the value 0.00001.

Step 8 - Calculate the proceeds and balance criteria

In step 8 it is assessed whether the factors G_0, and G_1 are determined so that the balance requirement, given the intended refinancing profile, is observed and the proceeds criterion is fulfilled.

The balance requirement is given by

$$YD(1)+REG^D\frac{RG(0)}{m_0}-H(1)+REG\sum_{j=1}^{m}R^N(j)H(j)$$

and the proceeds criterion is given by

$$Fin(0)-\sum_{j=1}^{m}K(j)M(j)$$

Only the balance requirement for the first year must be checked. Of course, the funding must also be arranged taking into consideration the remaining years, but this is ensured by the initial determination of the trend function.

Step 9 - Calculate adjustment of increment

On the basis of the calculated values for the balance requirement and the proceeds criterion, respectively, an adjustment

of (G_0,G_1) is determined on the basis of the Gauss-Newton algorithm.

Already in step C in the type F model a general formula apparatus was set up. In the F model an adjustment to the term to maturity of the loan was determined, whereas in step 9 an adjustment to the factors in the trend function is determined - however, to a large extent, the method is the same.

The most important difference si the definition of the F function. F(.) is here defined as a function of (G_0,G_1)

$$F(G_0,G_1)=[Fin(0)-\sum_{j=1}^{m}K(j)M(j)\ ,$$

$$H(1)+REG\sum_{j=1}^{m}R^N(j)H(j)-YD(1)-REG^D\frac{1}{m_0}RG(0)]$$

Also here, the value of F(.) is determined for both (G_0,G_1) and $(G_0,G_1)+inc$. The adjustment of (G_0,G_1) is referred to as h and is defined as

$$h=\Delta(G_0,G_1)=\frac{[D^TD]^{-1}D^Tg}{j_v}$$

wherein D and g are defined as in the F model. It must, howver be noted that

$$J_a=\frac{1}{inc}\begin{bmatrix}F_1(G_0,G_1)-F_1(G_0+inc,G_1) & F_1(G_0,G_1)-F_1(G_0,G_1+inc)\\ F_2(G_0,G_1)-F_2(G_0+inc,G_1) & F_2(G_0,G_1)-F_2(G_0,G_1+inc)\end{bmatrix}$$

The subsign for F states whether it is the first or the second part of the expression for F which is estimated. I.e.,

$F_1(G_0,G_1)$ determines the value of the proceeds criterion for (G_0,G_1).

If TILT=1, an adjustment to (Z,G_0) is determined instead.

It must be noted, however, that step 9 only allows 30 attempts. This means that if the model during 30 attempts has not been able to estimate an adjustment of the trend function and thus the funding satisfactorily, the loop is abandoned, and the model continues in step 11. However, the model may hereafter return with a new guess at the interest rate.

Step 10 - Are the balance and the proceeds criteria fulfilled?

On the basis of the calculated adjustments, it is to be determined whether the trend function determines the volumes of the financial instruments applied in such a way that the balance requirement and the proceeds criterion have been fulfilled.

In step D of the type F model were introduced the criteria for mathematical convergence which are to be used here.

If h fulfils just one of the conditions

(i) $$\frac{|h|}{\varepsilon|G_0,G_1|}<\varepsilon$$

(ii) $$\left|\frac{F(G_0,G_1)^2-F((G_0,G_1)+h)}{F(G_0,G_1)^2}\right|<\varepsilon$$

h has converged, and (G_0,G_1) and the matching funding can be applied.

If none of the conditions has been fulfilled, the adjustment h is added to (G_0,G_1) and step 7 is repeated.

Also here, the requirement as to mathematical convergence can be replaced by the specific conditions - the proceeds criterion and the balance requirement. This means that the funding may be applied if

(iii) $$\left|Fin(0)-\sum_{j=1}^{m}K(j)M(j)\right|<\varepsilon$$

(iv) $$\left|Fin(1)-REG^{D}\frac{RG(0)}{m_0}\right|<\varepsilon$$

Step 11 - Is the proceeds criterion fulfilled?

Step 11 aims at the situation in which the model has not been able to determine a trend function satisfactorily. Thus, the funding does not fulfil both the balance requirement, or more specifically, the intended refinancing profile, and the proceeds criterion for the given interest rate on the loan.

It cannot be excluded that no solution which fulfils both requirements exists, so the model necessarily has to continue.

The proceeds condition is, however, an indispensable requirement. Therefore, it is checked whether the proceeds condition is fulfilled. This is done by the condition

$$\left|Fin(0)-\sum_{j=1}^{m}K(j)M(j)\right|<\varepsilon$$

If this is the case, then the model continues in step 13 - otherwise the volumes of the financial instruments applied are adjusted in step 12.

Step 12 - Adjust the funding

The model only reaches step 12 if it has not been possible in the inner loop at the same time to meet the intended refinancing profile and the proceeds criterion. Thus, the volumes of the financial instruments applied must be adjusted with the sole object of fulfilling the proceeds condition. Therefore, appropriate adjustment of each volume is performed.

In principle, three situations may occur

1) The model calculates an excess of funding, i.e. the funding volumes must be reduced

$$M \left(\frac{Fin(1)}{\sum_{j=1}^{m} K(j)M(j)} \right) M \qquad \text{for } \mathbf{M}=(M(1),M(2),\ldots,M(m)$$

2) The model makes a deficiency of funding, i.e. the funding volumes must be increased. Here M(1) is maintained so that the refinancing percentage at the next refinancing does not increase more than what is absolutely necessary. M(1) is determined as

$$M \left(1, \frac{Fin(1)-K(1)M(1)}{\sum_{j=2}^{m} K(j)M(j)} \right) M \qquad \text{for } \mathbf{M}=(M(1),M(2),\ldots,M(m)$$

3) Finally, the funding principals may sum up to zero. Here the entire funding is placed in the instrument with the shortest maturity, i.e.

$$M(1)=\frac{Fin(1)}{K(1)}$$ and M(j)=0 for

j={2,3,...,m}

After the adjustment, the proceeds condition is fulfilled, and the model continues in step 13.

Step 13 - Calculate an adjustment of the interest rate

In this step, an adjustment to the guess at an interest rate on the loan is to be calculated. The adjustment is calculated in relation to the yield of the portfolio of funding instruments, with correction, however, for the typically different numbers of settlements within each year on the debtor side and the funding side, respectively. Thus, the adjustment of the interest rate on the loan is calculated as the difference between the corrected interest rate on the loan and the yield to maturity of the funding portfolio.

Firstly, the function F(.) is defined as

$$F(R^K)=\sum_{i=1}^{n}\frac{R^K}{n}\frac{RG(i-1)}{RG(0)}-REG\,r^P$$

wherein

r^P is defined as the yield to maturity of the funding portfolio.

n is the number of debtor payment dates per year in the next refinancing period.

With the definition of $F(R^K)$ the Gauss-Newton algorithm may be applied.

$$\Delta R^K = \frac{\left[D^T D\right]^{-1} D^T g}{j_v}$$

The definitions are not to be repeated here. It should only be noted that

$$J_a = \frac{F(R^K) - F(R^K + inc)}{inc}$$

The algorithm can be simplified analogously to step C in the type F model.

Step 14 - Has the interest rate converged?

Step 14 determines whether the outer loop is to continue or whether the model has reached a satisfactory result. The criterion is the extent of the accordance between the interest rate on the loan and the yield on the portfolio of funding instruments.

The interest rate on the loan has converged and can thus be accepted if

$$\left| \sum_{i=1}^{n} \frac{R^K}{n} \frac{RG(i-1)}{RG(0)} - REG\, r^P \right| < e$$

wherein e in the model is set to be 0.00001,

is fulfilled. This means that the calculations in the inner model are finished, and can be finalized in step 16.

Otherwise, the calculations proceed in step 15, wherein the guess at the interest rate on the loan is adjusted.

Step 15 - Adjust the interest rate

The guess at the interest rate on the loan which was rejected in step 14 is adjusted with ΔR^K. Then the model continues in step 2, wherein the payments on the loan are determined for the new guess at the interest rate.

Step 16 - Calculations are complete. The result may be applied

The model has determined an interest rate and a number of positive funding volumes. Thus, the calculations are complete for the given value of the term to maturity.

The model may return to the outer model with information on the payments on the loan, the funding, and the interest rate on the loan.

3.2.3 The inner model for type P - the analytical solution

To a large extent, the analytical solution is based on the same structure as the iterative solution. Thus, the analytical solution divides the inner model into an inner and an outer loop, where the outer loop comprise an iterate routine targeting the interest rate on the loan, whereas the funding profile is determined in the inner loop.

Yet, the analytical solution of type P differs from the iterative solution by the fact that in the inner loop, the model does not determine the funding volumes by applying an iterative routine, but rather analytically. Primarily, this facilitates the computations in the inner model.

However, the analytical solution also implies that the model will always reach a solution. In the iterative solution there was a risk that the model would have to discontinue the

iteration after 30 guesses in which the model did not succeed in finding a solution fulfilling both the proceeds criterion and the intended refinancing at the same time. The model then adjusted the funding volumes in the outer loop (steps 11 and 12).

Thus, the fact that a solution will also be found has the consequence that a similar adjustment of the funding volumes will not be necessary in the outer loop. Apart from this, the outer loop is identical with the outer loop in the iterative solution.

The complete structure of the model is shown in Fig. 26. In the following, each step is described. For the sake of clearness, the steps that are identical with the iterative solution are included, with an indication of the identity.

The analytical solution implies that the inner loop comprises four steps.

First, (G_0,G_1) is determined in step 6 as the factors in the trend function which comply with both the proceeds criterion and the intended refinancing in the first year. Based on the adjusted trend function the funding volumes are calculated in step 7 as the difference between the value of the trend function and the funding volumes previously issued.

In advance it cannot be excluded that one or more of the calculated funding volumes are negative, which is not accepted, since it would imply that the debtor has to buy bonds. If it turns out in step 8 that one or more funding volumes are negative, these are set to 0 (zero) by means of an indicator function. The indicator function forms a m-dimensional vector, in which the jth element is zero if the jth funding volume is set to zero, and otherwise one. The adjustment of the indicator function is performed in step 9.

When the indicator function has been adjusted, the calculation of (G_0,G_1) is repeated based on the proceeds criterion and the intended refinancing in the first year, but with the reduced number of funding instruments. This process is performed until no further negative funding volumes are revealed in step 8.

It cannot be excluded that only one funding instrument is left at the end. In this situation, it will generally not be possible to observe both the proceeds criterion and the intended refinancing requirement for the first year. Thus, the latter must be suspended as the proceeds criterion is given the higher priority. In this situation, a value of G_0 is determined in compliance whit the proceeds criterion in step 6.

Correspondly, in the procedure it may often occur that the volume of the first funding instrument is set to 0 (zero), in which case it will generally not be possible to observe the intended refinancing requirement in the first year. Hence, the same procedure as above where only G_0 is determined in step 6 is applied.

Step 0 - Determine m and TILT
(identical with step 0 i the iterative model)

TILT is assigned a value according to the following rules.

Set TILT=1	if the loan is disbursed in January-November
Set TILT=0	in all other cases.

At the same time, the value of m is temporarily determined as

$$m=m_0+TILT$$

In connection with the maturity of the loan m is also to be adjusted to ensure that no financial instruments with maturity later than the maturity of the loan are applied. Thus, m is assigned a value according to

$$m=\min[L_0-\sigma;m_0+TILT]$$

where $L_0-\sigma$ designates the remaining term to maturity of the loan.

Step 1 - Determine initial interest rate
(identical with step 1 i the iterative model)

The initial interest rate on the loan is assigned according to

$$R^{K_m}\frac{\sum_{t=1}^{m}\frac{1}{m}t\,r(0,t)}{\sum_{t=1}^{m}\frac{1}{m}t}$$

Step 2 - Determine the payments on the loan
(identical with step 2 i the iterative model)

In this step, the model begins the outer loop calculating the payments on the loan given the interest rate on the loan.

Step 3 - Is m=1 or m>1?

The calculations in the model may proceed in one of two different ways depending on the value of m.

If m=1 the calculations are continued in step 3a, where the funding is determined very simply since only 1 funding instrument is applied.

If m>1, the calculations proceed in step 4 in the general model. There is no need to distinguish between the situations

where m=2 or m>2 as in the iterative model, since the determination of the funding volumes are based on the same principles in the analytical model. Thus, step 3b is left out of the model.

Step 3a - Determine funding and interest rate for m=1

If m=1, the funding can be determined on the basis of the proceeds condition as

$$M(1)\cdot\frac{Fin(0)}{K(1)}$$

Since no more refinancing occur, it follows from the strict balance principle that

$$YD(m)=[1+R^{N}(m)]H(m)$$

which determines an unambiguous interest rate on the loan. When the calculations in step 3a are finished, they proceed in step 14.

Step 4 - Define a trend function
(identical whit step 4 in the iterative model)

As in the iterative solution the trend function is defined as a polynomial of the degree q-1

$$a_0+a_1t+a_2t^2+\ldots a_{q-1}t^{q-1}$$

wherein $0\leq t\leq m$ and $q\leq m$-TILT so that the degree of the polynomial is not increased in the speciel TILT-procedure.

Step 5 - Determine trend function coefficients
(identical whit step 5 in the iterative model)

The coefficients in the trend function $a_0, a_1, \ldots, a_m$ are determined by applying

$$(a_0, a_1, \ldots, a_{q-1}) = [B^T B]^{-1} B^T \max\left[REG^D \frac{RG(t)}{m_0}, H(0, t-1)\right]$$

wherein $(a_0, a_1, \ldots, a_{q-1})$ is a q×1 vector
B is a m×q matrix and
max[.,.] is a m×1 vector

The matrix B is given by

$$B\begin{bmatrix} 1 & t_0 & t_0^2 & . & . & t_0^{q-1} \\ 1 & t_1 & t_1^2 & . & . & t_1^{q-1} \\ . & . & . & . & . & . \\ 1 & t_{m-1} & t_{m-1}^2 & . & . & t_{m-1}^{q-1} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & . & 0 \\ 1 & 1 & 1 & 1 & . & 1 \\ 1 & 2 & 4 & 8 & . & 2^{q-1} \\ 1 & 3 & 9 & 27 & . & 3^{q-1} \\ . & . & . & . & . & . \\ 1 & m-1 & (m-1)^2 & . & . & (m-1)^{q-1} \end{bmatrix}$$

Step 6 - Determine G_0 and G_1

In step 6 the trend function is adjusted in accordance with two conditions. For one thing, the funding must fulfil the proceeds criterion and for another, the funding must fulfil the intended refinancing in the first year given by the balance requirement.

In general, the adjustment is performed by introducing the parameters G_0 and G_1 so that the trend function appears as follows

$$G_0 a_0 + G_1 a_1 t + a_2 t^2 + \ldots + a_{q-1} t^{q-1}$$

The trend function may immediatly be divided into a variant part and an invariant part in regard to G_0 and G_1. The variant part is denoted X(j) and the invariant part Y(j), thus

$$X(j)=G_0a_0+G_1a_1t \quad \text{and} \quad Y(j)=a_2t^2+\ldots+a_{q-1}t^{q-1}$$

for $t=j-1$.

The marginal volume of the funding instrument with a term to maturity of j years, M(j), is determined as the value of the trend function ÷ the volume of the funding instrument already issued, H(0,j). However, the volume must not be negative, thus

$$M(j)=\max[0;G_0a_0+G_1a_1t+a_2t^2+\ldots+a_{q-1}t^{q-1}-H0,j)]$$

which by applying X(j) and Y(j) may be simplified into

$$M(j)=\max[0;X(j)+Y(j)-H(0,j)]$$

The expression must be further simplified. This is necessary, since G_0 and G_1 cannot be isolated as long as the expression comprises a max-function. Thus, an indicator function, I(j) j=1,2,…,m, is implemented with the value zero if the jth funding instrument is to be set to zero, and otherwise with the value one. Thereby, the indicator function forms an m-dimensional vector.

When step 6 is first applied the values of the indicator function is set according to

$$I(j) = (1,1,\ldots,1),$$

After this, the value of I(j) will be determined in step 9.

When implementing the indicator function, the marginal funding volumes are determined according to

(I) $$M(j)=I(j)[X(j)+Y(j)-H(0,j)]=I(j)X(j)+I(j)(Y(j)-H(0,j))$$

If both conditions

$$\sum_{j=1}^{m} I(j) \geq 2 \quad \text{and} \quad I(1)>0$$

are fulfilled, corresponding to the first funding volume exceeding 0 (zero) and, at the same time, the number of remaining funding volumes being greater than or equal to 2, the marginal funding must observe both the proceeds criterion and the intended refinancing in the first year.

Thus, G_0 and G_1 are calculated solving

(i) $$YD(1)+REG^{D}\frac{RG(0)}{m_0} - H(0,1)+M(1)+REG\sum_{j=1}^{m} R^{n}(j)\left[M(j)+H(0,j)\right]$$

and

(ii) $$Fin(0)-\sum_{j=1}^{m} K(j)M(j)$$

wherein (i) is the balance requirement and (ii) is the proceeds criterion.

The expression (I) may then be inserted into (i) for M(1) and M(j)

$$YD(1)+REG^{D}\frac{RG(0)}{m_0}-H(0,1)+I(1)X(1)+I(1)\left(Y(1)-H(0,1)\right)$$

$$+REG\sum_{j=1}^{m} R^{n}(j)\left[I(j)X(j)+I(j)\left(Y(j)-H(0,j)\right)+H(0,j)\right]$$

The expression is solved for X(j)

$$(J)\quad I(1)X(1)+REG\sum_{j=1}^{m} R^{N}(j)I(j)X(j)-YD(1)+REG^{D}\frac{RG(0)}{m_0}-H(0,1)$$

$$-I(1)(Y(1)-H(0,1))-REG\sum_{j=1}^{m} R^{n}(j)[I(j)(Y(j)-H(0,j))+H(0,j)]$$

In the following, the right-hand side of the expression is designated by the variable Z_1.

Analogously, the expression (I) may be inserted in the proceeds criterion. This leads to the expression

$$Fin(0)-\sum_{j=1}^{m} K(j)(I(j)X(j)+I(j)(Y(j)-H(0,j)))$$

The expression is solved for X(j)

$$(K)\quad \sum_{j=1}^{m} K(j)I(j)X(j)-Fin(0)-\sum_{j=1}^{m} K(j)I(j)(Y(j)-H(0,j))$$

The right-hand side of this expression is denoted Z_2.

Together, (J) and (K) define two equations with to unknown variables G_0 and G_1. However, the solution of the equation system depends on the value of TILT.

If <u>TILT equals 0</u>, the general solution is applied.

X(j) was defined as the part of the trend function variant as to G_0 and G_1. In a matrix form X(j) may, therefore, be described as

$$X=a\times G$$

wherein

X=(X(1),X(2),…,X(m)) is a 1×m vector and
G=(G_0,G_1) is a 2×1 vector and
a is a m×2 matrix given by

$$a = \begin{bmatrix} a_0 & 0 \\ a_0 & a_1 \\ a_0 & 2a_1 \\ \vdots & \vdots \\ a_0 & (m-1)a_1 \end{bmatrix}$$

A matrix K is then defined so that K×X and thus K×a×G forms the left-hand sides in (L) and (K).

$$K\begin{bmatrix} I(1)\left(1+REG\cdot R^N(1)\right) & I(2)REG\cdot R^N(2) & \cdots & I(m)REG\cdot R^N(m) \\ I(1)K(1) & I(2)K(2) & \cdots & I(m)K(m) \end{bmatrix}$$

Defining K, the two equations (L) and (K) may be written on a matrix form, utilizing that the right-hand side of (L) and (K) are given by Z_1 and Z_2.

$$Z=K\times a\times G$$

wherein $Z=(Z_1,Z_2)$ is a 2×1 vector. K×a forms a m×m matrix that may be inverted. G_0 and G_1 may thus be determined as

$$G=[K\times a]^{-1}Z$$

If, on the other hand <u>if TILT=1</u>, M(1) is determined explicitly as in the iterative solution. The marginal funding is determined in accordance with

$$M(j)=\max[0;(X(1);G_0a_0+a_1t+a_2t^2+\ldots+a_{q-1}t^{q-1})-H0,j)]$$

Again the trend function may be divided into a variant and an invariant part defined as

$$X(j)=\begin{cases} G_0 a_0 & \forall j>1 \\ X(1) & j=1 \end{cases} \quad \text{and} \quad Y(j)=a_1 t+a_2 t^2+\ldots+a_{q-1} t^{q-1}$$

for t=j-1

X(1) and G_0 must be determined in compliance with the proceeds criterion and the intended refinancing in the first year. This defines the equations (J) and (K) identical to above - X(j) and Y(j), however, being defined by the above definitions.

$$(J) \quad I(1)X(1)+REG\sum_{j=1}^{m} R^N(j)I(j)X(j)=YD(1)+REG^D\frac{RG(0)}{m_0}+H(0,1)$$

$$-I(1)\left(Y(1)-H(0,j)\right)-REG\sum_{j=1}^{m} R^n(j)\left[I(j)\left(Y(j)-H(0,j)\right)+H(0,j)\right]$$

$$(K) \quad \sum_{j=1}^{m} K(j)I(j)X(j)=Fin(0)+\sum_{j=1}^{m} K(j)I(j)\left(Y(j)-H(0,j)\right)$$

The right-hand sides of (J) and (K) are designated Z_1^* and Z_2^*, respectively.

X(j) may be written on the matrix form

$$X=a^*\times G^*$$

wherein $G^*=(G_0, X(1))$ and

$$a^* = \begin{bmatrix} 0 & 1 \\ a_0 & 0 \\ \vdots & \vdots \\ a_0 & 0 \end{bmatrix}$$

Analogously to the above, (J) and (K) lead to the expression

$$Z^* = K \times a^* \times G^*$$

wherein $Z^* = (Z_1^*, Z_2^*)$ and K is, unchanged, defined as

$$K = \begin{bmatrix} I(1)\left(1+REG\text{-}R^N(1)\right) & I(2)REG\text{-}R^N(2) & \cdots & I(m)REG\text{-}R^N(m) \\ I(1)K(1) & I(2)K(2) & \cdots & I(m)K(m) \end{bmatrix}$$

The matrix equation is solved by

$$G^* = [K \times a^*]^{-1} Z^*$$

If, on the other hand,

$$\sum_{j=1}^{m} I(j) < 2 \quad \text{or} \quad I(1) = 0$$

only the proceeds criterion may be fulfilled, and the balance requirement is suspended. In principle, then G_0 and G_1 must be determined on the basis of only one equation which yields an infinite number of solutions. Therefore in this situation, G_1 is assigned a fixed value which, of course, must be suitable so that the refinancing the first year does not differ to an unnecessarily large extent. Numerical analysis has shown that

$$G_1 = 1$$

leads to suitable results.

The fixing of G_1 implies that G_1 now is comprised by the invariant part of the trend function, thus

$$X(j) = G_0 a_0 \quad \text{and} \quad Y(j) = G_1 a_1 t + a_2 t^2 + \ldots + a_{q-1} t^{q-1}$$

It remains to determine G_0 in compliance with the proceeds criterion. The proceeds criterion is given as

$$Fin(0) - \sum_{j=1}^{m} K(j)\,(I(J)X(j) + I(j)\,(Y(j) - H(0,j)))$$

Solved for X(j) this leads to the expression

$$(K) \quad \sum_{j=1}^{m} K(j)I(j)X(j) - Fin(0) - \sum_{j=1}^{m} K(j)I(j)\,(Y(j) - H(0,j))$$

When the right-hand side of (K) is denoted Z_2^{**}, G_0 is determined by solving

$$G^{**} = [K^{**} \times a^{**}]^{-1} Z^{**}$$

wherein

$G^{**} = G_0$ and

$K^{**} = [I(1)K(1), I(2)K(2), \ldots, I(m)K(m)]$ and

$$a^{*} = \begin{bmatrix} 0 \\ a_0 \\ \vdots \\ a_0 \end{bmatrix} \quad \text{and}$$

$$Z^{**} = Z_2^{**}$$

On the other hand, it may also be utilized that $X(j) = G_0 a_0$, which inserted in (K) yields

$$G_0 a_0 \sum_{j-1}^{m} K(j) I(j) - Fin(0) - \sum_{j01}^{m} K(j) I(j) (Y(j) - H(0,j))$$

Solved for G_0

$$G_0 - \frac{Fin(0) - \sum_{j-1}^{m} K(j) I(j) (Y(j) - H(0,j))}{a_0 \sum_{j-1}^{m} K(j) I(j)}$$

and thus G_0 is determined. By insertion of G_0 in the balance requirement, the refinancing in the first year may be calculated residually.

When both G_0 as well as G_1 (or X(1) for TILT=1) have been determined, the calculations in the model continue in step 7.

Step 7 - The funding is determined

In step 7 the marginal volumes of the funding instruments are determined as the difference between the value of the trend function for each respective j and the volumes previously issued. The marginal funding volumes are determined either as

$$M(j) = G_0 a_0 + G_1 a_1 t + a_2 t^2 + \ldots + a_{q-1} t^{q-1} - H0, j)$$

for TILT=0 or as

$$M(j) = (X(1); G_0 a_0 + G_1 a_1 t + a_2 t^2 + \ldots + a_{q-1} t^{q-1}) - H0, j)$$

The implementation of the indicator function is the reason why the marginal funding volumes are determined without applying a max-condition which would involve that negative funding volumes are set to 0 (zero) in this step. If M(j') is

negative, I(j') is set to zero, and thus the next time the step is applied, M(j') equals zero.

Step 8 - Are all funding volumes positive?

In step 8 it is determined whether the calculated funding profile may be applied, or whether it is necessary to perform the calculations in the inner loop once again.

If

$$M(j) \geq 0 \qquad \forall j \in (1,2,\ldots,m)$$

the model moves on to step 10 i the outer loop.

On the other hand, if

$$\exists j \in (1,2,\ldots,m) \qquad M(j) < 0$$

is fulfilled, the model proceeds in step 9.

Step 9 - Adjust indicator function

If the model reaches step 9, one or more of the marginal funding volumes are negative, which is indicated by assigning the indicator function the value 0 (zero) for the particular j. Thus, I(j) is adjusted as follows

$$I(j)=0 \qquad \text{for all j where } M(j)<0$$

$$I(j)=1 \qquad \text{otherwise.}$$

Alternatively, I(j) might be set to 0 (zero) for one funding instrument at a time. If so, the particular funding instrument should be the one with the numerically greatest volume among the negative funding volumes. When a negative

funding volume is deleted, the proceeds criterion implies that fewer bonds are sold in total. Thus, it is only in speciel cases that the marginal funding will not decrease for every j when a negative funding volume is set to 0 (zero). Thus, if the marginal volume of a particular instrument is negative, it is not very likely that subsequent calculations will yield a positive funding volume for the particular instrument.

When the adjustment has been performed, the calculation of G_0 and G_1 is repeated in step 6.

Step 10 - Calculate an adjustment of the interest rate
(identical with step 13 in the iterative model)

Based on the extent of convergence relative to the yield to maturity of the funding portfolio, an adjustment of the interest rate on the loan is calculated in step 10.

A function $F(R^K)$ is defined as

$$F(R^K)=\sum_{i=1}^{n}\frac{R^K}{n}\frac{RG(i-1)}{RG(0)}-REG\,r^P$$

The function is applied in the Gauss-Newton algorithm, thus

$$\Delta R^K=\frac{\left[D^TD\right]^{-1}D^Tg}{j_v}$$

After this, the model continues in step 11

Step 11 - Has the interest rate converged?
(identical with step 14 in the iterative model)

If the adjusted interest rate on the loan equals the yield to maturity of the funding portfolio, the interest rate on the

loan has converged and may be applied. The criterion for this is

$$\left| \sum_{i=1}^{n} \frac{R^K}{n} \frac{RG(i\text{-}1)}{RG(0)} - REG\ r^p \right| < \epsilon$$

wherein ϵ is assigned the value 0.00001.

If the criterion is met, the iterative routine in the inner loop may end, and the model returns, *via* step 13, to the outer loop. Otherwise, the model proceeds to step 12.

Step 12 - Adjust the interest rate
(identical with step 15 in the iterative model)

The interest rate on the loan is adjusted applying the adjustment calculated in step 10, and the model continues in step 2.

Step 13 - Calculations are complete. The result may be applied
(identical with step 16 in the iterative model)

The model returns to the outer model with information on the funding profile, the interest rate on the loan and the payments on the loan.

3.3 General comments on LAIR type P

From the outset, it is not certain that the model is able to respect the intended refinancing percentage each year, even though the model determines the funding according to a long-term strategy as mentioned in the introduction.

The long perspective in the determination of the funding is introduced via the adjustment of the trend function. Firstly,

the trend function is estimated on the basis of the intended refinancing profile for the whole funding period. This is, however, not a very good strategy for the funding, since the model can easily end up in a situation wherein too many bonds with maturity in a single year have been issued. In the inner loop of the model, however, an adjustment is made to a_1, which controls the slope of the function. In Fig. 27 is shown an example of the adjustment of the funding. Line 78 is the initial trend function, whereas line 80 is the adjusted trend function.

Thus, only a part of the future refinancing amount is funded now, the rest of the funding will take place later pari passu with the development in the interest rates.

It is exactly the determination of the trend function that justifies the special procedure applied when the loan matures. The adjustment of the trend function is based on the refinancing percentage in the first year and the proceeds criterion. Without applying a special procedure there would be a risk that the term to maturity on the loan is determined as, e.g., 13 months on the loan side, whereas payments according to a term to maturity of two years are funded on the bond side. The consequence would be a substantial rise in the interest rate on the loan in the last year before maturity. By introducing the closing time parameter, a term to maturity of two years is calculated with also on the loan side.

Appendix 1 Detailed description of the matrix in the Gauss-Newton algorithm

$$F(x)=(f_1,f_2)_{1\times 2}$$

The suffixed subsigns (1×2) designate the size of the matrix (number of rows × number of columns)

$$J=\begin{bmatrix} \frac{f_1-f_1'(1)}{inc} & \frac{f_1-f_1'(2)}{inc} \\ \frac{f_2-f_2'(1)}{inc} & \frac{f_2-f_2'(2)}{inc} \end{bmatrix}$$

$$F(x+inc)=(f_1'(1),f_2'(2))_{1\times 2},\ \text{for } x+inc=(x(1)+inc,x(2))$$

$$=(f_1'(2),f_2'(2))_{1\times 2},\ \text{for } x+inc=(x(1),x(2)+inc)$$

$$J^{TJ}=\begin{bmatrix} J^2(1,1)+(J^T(1,2)\cdot J(2,1)) & w \\ w & J^2(2,2)+(J^T(2,1)\cdot J(1,2)) \end{bmatrix}_{2\times 2}$$

wherein

$$w=J^T(2,1)\cdot J(1,1)+J^T(2,2)\cdot J(2,1).$$

$$g=(g_1,g_2)_{1\times 2}$$

$$g_1=J^T(1,1)f_1+J^T(1,2)f_2$$

$$g_2=J^T(2,1)f_1+J^T(2,2)f_2$$

$$J_v=(J_v(1),J_v(2))_{1\times 2}$$

$$J_v(1)=\sqrt{J^TJ(1,1)}$$

$$J^V(2)=\sqrt{J^TJ(2,2)}$$

$$J_v J_v T = \begin{bmatrix} J_v(1)^2 & x \\ x & J_v(2)^2 \end{bmatrix} \quad , \text{ for } x = J_v(1) \quad J_v(2)$$

EXAMPLE OF LAIR WITH LIMITS FOR THE PAYMENTS ON THE LOAN

In the following, an example of calculation of a LAIR type P with limits for the payments on the loan and limits for the term to maturity is given. The calculations in the model are based on the inputs entered, these are listed below. In addition, the model comprises a number of parameters that have fixed values in all the calculations performed. In principle, these values may be altered from one calculation to the other, but for practical reasons they are kept constant. The parameters are also listed below.

Fixed value parameters:

Maximum difference in balance (e)	: DKK 0.0000001
Maximum difference in proceeds (e)	: DKK 0.0000001
Maximum difference in interest rates (e)	: 0.00001%
inc	: 0.0001

Input values:

Principal of the loan	: DKK 1,000,000
Intended term to maturity	: 20 years
Maximum limit for the payments	: DKK 23,000
Minimum limit for the payments	: DKK 20,000
Maximum limit for the term (L^{max})	: 30 years
Minimum limit for the term (L^{min})	: 0 years
Technical maximum limit for the term (L^{MAX})	: 100 years
Technical minimum limit for the term (L^{MIN})	: 0 years
Number of debtor payments pr. year	: 4
Number of bond settlements pr. year	: 1
Repayment principle	: annuity
Type of LAIR	: P20 (20 per cent of the remaining

debt is refinanced each year)

Date of the disbursement of the loan : 01.01.1997

Closing time : 2 years

Shift in yield structure in : year 5 (rise in interest rates of 3 percentage points)

In addition, the model requires information on the bonds applied for the funding of the loan. Since the loan is a LAIR type P20 where 20 per cent of the remaining debt is refinanced each year, the loan is funded in six bonds. The relevant information about the six bonds is listed in table 1.

Table 1

Date of maturity	Coupon interest rate	Price
01.01.1998	6	101.92
01.01.1999	6	102.62
01.01.2000	6	103.42
01.01.2001	7	106.28
01.01.2002	7	106.23

To better illustrate the understanding of the functionality of the model, a permanent rise in interest rates in year 5 (subsequent to the 4th refinancing - before the 5th refinancing) is assumed in the example. The rise in interest rates causes the yield to maturity of all bonds to rise 3 percentage points and remain at this level until the loan matures. This implies that the valid bond prices subsequent to the 4th refinancing are as stated in table 2.

Table 2

Coupon interest rate	Price
6	99.07
6	97.13
6	98.06
7	95.98
7	93.94

The yield structure entered into the model when the loan is disbursed is applied both for the disbursement and for the refinancing until the year in which the yield structure is altered as stated above.

Subsequent to the second refinancing of the loan, a new bond with the term to maturity of 5 years is required in the model. As the price of this bond is not known when the loan is disbursed, the model "issues" a bond with a term to maturity of 5 years with data identical to the bond with a term to maturity of 5 years at the disbursement of the loan. In other words, the model reuses the bond data entered at the time of disbursement of the loan until the year in which the yield structure is shifted. From then on, the model reuses the new bond data in the remaining term to maturity.

The results of the model may be related to the debtor side and the bond side, respectively, the debtor side and the bond side being linked together by the balance requirement and the proceeds criterion.

Repayment of the loan:

The debtor side:

The model outputs a complete amortization profile in accordance with the repayment principle entered. In addition, the amortization profile observes the limits for the payments on the loan which are given a lower priority than the limits for the term to maturity.

The amortization profile includes the payments on the loan comprising interest payments and repayments, and the remaining debt at the end of each year and the interest rate on the loan.

The bond side:

Correspondingly, the model outputs coupon payments on the bonds, bond redemptions, and remaining debt at the beginning of each settlement period.

The amortization profile is shown in table 3.

Table 3

	The bond side			The debtor side				
payment number	remaining debt	coupon payments	Redemp-tions	remaining debt	interest payments	repayment	payment on the loan	quarterly interest rate
1	961,441	15,485	0	992,910	13,279	7,090	20,369	1.3279
2	961,441	15,485	0	985,725	13,185	7,184	20,369	1.3279
3	961,441	15,485	0	978,445	13,089	7,280	20,369	1.3279
4	941,905	15,485	19,536	971,069	12,993	7,377	20,369	1.3279
5	934,898	15,226	0	963,550	12,760	7,519	20,279	1.3140
6	934,898	15,226	0	955,932	12,661	7,618	20,279	1.3140
7	934,898	15,226	0	948,214	12,561	7,718	20,279	1.3140
8	914,685	15,226	20,213	940,395	12,460	7,819	20,279	1.3140
9	903,911	15,176	0	932,552	12,591	7,843	20,434	1.3389
10	903,911	15,176	0	924,604	12,486	7,948	20,434	1.3389
11	903,911	15,176	0	916,550	12,380	8,054	20,434	1.3389
12	882,878	15,176	21,033	908,388	12,272	8,162	20,434	1.3389
13	872,671	15,094	0	900,169	12,314	8,219	20,533	1.3556
14	872,671	15,094	0	891,839	12,203	8,330	20,533	1.3556

	The bond side			The debtor side				
payment number	remaining debt	coupon payments	Redemp-tions	remaining debt	interest payments	repayment	payment on the loan	quarterly interest rate
15	872,671	15,094	0	883,396	12,090	8,443	20,533	1.3556
16	850,914	15,094	21,757	874,838	11,975	8,558	20,533	1.3556
17	841,282	14,610	0	866,198	11,955	8,640	20,595	1.3665
18	841,282	14,610	0	857,440	11,837	8,758	20,595	1.3665
19	841,282	14,610	0	848,563	11,717	8,878	20,595	1.3665
20	817,344	14,610	23,938	839,564	11,596	8,999	20,595	1.3665
21	826,735	14,347	0	831,152	13,510	8,412	21,923	1.6092
22	826,735	14,347	0	822,604	13,375	8,548	21,923	1.6092
23	826,735	14,347	0	813,919	13,237	8,685	21,923	1.6092
24	796,434	14,347	30,300	805,094	13,098	8,825	21,923	1.6092
25	805,339	13,973	0	796,579	14,286	8,515	22,800	1.7744
26	805,339	13,973	0	787,913	14,135	8,666	22,800	1.7744
27	805,339	13,973	0	779,093	13,981	8,820	22,800	1.7744
28	770,030	13,973	35,309	770,117	13,824	8,976	22,800	1.7744
29	778,528	13,517	0	761,658	14,541	8,459	23,000	1.8882
30	778,528	13,517	0	753,040	14,382	8,618	23,000	1.8882

	The bond side			The debtor side				
payment number	remaining debt	coupon payments	Redemp-tions	remaining debt	interest payments	repayment	payment on the loan	quarterly interest rate
31	778,528	13,517	0	744,259	14,219	8,781	23,000	1.8882
32	740,595	13,517	37,933	735,312	14,053	8,947	23,000	1.8882
33	748,759	13,004	0	726,787	14,475	8,525	23,000	1.9685
34	748,759	13,004	0	718,093	14,307	8,693	23,000	1.9685
35	748,759	13,004	0	709,229	14,136	8,864	23,000	1.9685
36	708,773	13,004	39,986	700,190	13,961	9,039	23,000	1.9685
37	716,321	12,428	0	691,356	14,166	8,834	23,000	2.0232
38	716,321	12,428	0	682,344	13,988	9,012	23,000	2.0232
39	716,321	12,428	0	673,149	13,805	9,195	23,000	2.0232
40	674,035	12,428	42,286	663,768	13,619	9,381	23,000	2.0232
41	681,074	11,806	0	654,457	13,689	9,311	23,000	2.0623
42	681,074	11,806	0	644,954	13,497	9,503	23,000	2.0623
43	681,074	11,806	0	635,255	13,301	9,699	23,000	2.0623
44	636,299	11,806	44,775	625,356	13,101	9,899	23,000	2.0623
45	642,806	11,134	0	615,431	13,076	9,924	23,000	2.0909
46	642,806	11,134	0	605,299	12,868	10,132	23,000	2.0909

	The bond side			The debtor side				
payment number	remaining debt	coupon payments	Redemp-tions	remaining debt	interest payments	repayment	payment on the loan	quarterly interest rate
47	642,806	11,134	0	594,956	12,656	10,344	23,000	2.0909
48	595,342	11,134	47,464	584,396	12,440	10,560	23,000	2.0909
49	601,285	10,405	0	573,743	12,347	10,653	23,000	2.1128
50	601,285	10,405	0	562,865	12,122	10,878	23,000	2.1128
51	601,285	10,405	0	551,757	11,892	11,108	23,000	2.1128
52	550,904	10,405	50,382	540,414	11,658	11,342	23,000	2.1128
53	556,029	9,596	0	528,916	11,501	11,499	23,000	2.1282
54	556,029	9,596	0	517,172	11,256	11,744	23,000	2.1282
55	556,029	9,596	0	505,178	11,006	11,994	23,000	2.1282
56	502,415	9,596	53,614	492,930	10,751	12,249	23,000	2.1282
57	506,927	8,732	0	480,491	10,561	12,439	23,000	2.1425
58	506,927	8,732	0	467,785	10,295	12,705	23,000	2.1425
59	506,927	8,732	0	454,807	10,022	12,978	23,000	2.1425
60	449,855	8,732	57,071	441,552	9,744	13,256	23,000	2.1425
61	453,589	7,792	0	428,069	9,517	13,483	23,000	2.1554
62	453,589	7,792	0	414,296	9,227	13,773	23,000	2.1554

	The bond side			The debtor side				
payment number	remaining debt	coupon payments	Redemp-tions	remaining debt	interest payments	repayment	payment on the loan	quarterly interest rate
63	453,589	7,792	0	400,225	8,930	14,070	23,000	2.1554
64	392,756	7,792	60,833	385,852	8,626	14,374	23,000	2.1554
65	395,606	6,764	0	371,217	8,365	14,635	23,000	2.1680
66	395,606	6,764	0	356,265	8,048	14,952	23,000	2.1680
67	395,606	6,764	0	340,989	7,724	15,276	23,000	2.1680
68	330,664	6,764	64,942	325,381	7,393	15,607	23,000	2.1680
69	332,501	5,642	0	309,472	7,091	15,909	23,000	2.1792
70	332,501	5,642	0	293,216	6,744	16,256	23,000	2.1792
71	332,501	5,642	0	276,606	6,390	16,610	23,000	2.1792
72	263,070	5,642	69,431	259,634	6,028	16,972	23,000	2.1792
73	264,364	4,450	0	242,368	5,734	17,266	23,000	2.2085
74	264,364	4,450	0	224,720	5,353	17,647	23,000	2.2085
75	264,364	4,450	0	206,683	4,963	18,037	23,000	2.2085
76	190,162	4,450	74,202	188,248	4,565	18,435	23,000	2.2085
77	191,012	3,163	0	169,560	4,312	18,688	23,000	2.2906
78	191,012	3,163	0	150,444	3,884	19,116	23,000	2.2906

	The bond side			The debtor side				
payment number	remaining debt	coupon payments	Redemp-tions	remaining debt	interest payments	repayment	payment on the loan	quarterly interest rate
79	191,012	3,163	0	130,890	3,446	19,554	23,000	2.2906
80	111,665	3,163	79,346	110,888	2,998	20,002	23,000	2.2906
81	112,494	1,828	0	98,113	2,571	12,775	15,346	2.3181
82	112,494	1,828	0	85,042	2,274	13,071	15,346	2.3181
83	112,494	1,828	0	71,668	1,971	13,374	15,346	2.3181
84	58,426	1,828	54,068	57,984	1,661	13,684	15,346	2.3181
85	58,635	910	0	44,107	1,692	13,877	15,568	2.9174
86	58,635	910	0	29,825	1,287	14,282	15,568	2.9174
87	58,635	910	0	15,127	870	14,698	15,568	2.9174
88	0	910	58,635	0	441	15,127	15,568	2.9174

The debtor side as well as the bond side are specified in detail in two additional tables.

The debtor side:

In table 4 the refinancing percentage, the interest rate on the loan, the remaining term to maturity, the total term to maturity, the actual payments on the loan, and the maximum and minimum limits for the payments on the loan are shown on a yearly basis. Based on the table it may be verified whether the model observes the requirements and specifications entered and, in addition, how the model adjusts the term to maturity to comply with the limits for the payments on the loan.

Table 4

year	type of LAIR	refinancing percentage	interest rate on the loan pr. year	remaining term to maturity	total term to maturity	actual quarterly payment on the loan	maximum limit for the payment	minimum limit for the payment
1	P20,0	20	5.3118	20	20	20,369	23,000	20,000
2	P20,0	20	5.2559	19	20	20,279	23,000	20,000
3	P20,0	20	5.3557	18	20	20,434	23,000	20,000
4	P20,0	20	5.4225	17	20	20,533	23,000	20,000
5	P20,0	20	5.4660	16	20	20,595	23,000	20,000
6	P20,0	20	6.4369	15	20	21,923	23,000	20,000
7	P20,0	20	7.0975	14	20	22,800	23,000	20,000
8	P20,0	20	7.5528	13.3688	20.3688	23,000	23,000	20,000
9	P20,0	20	7.8740	12.7278	20.7278	23,000	23,000	20,000
10	P20,0	20	8.0928	11.9435	20.9435	23,000	23,000	20,000
11	P20,0	20	8.2492	11.0747	21.0747	23,000	23,000	20,000
12	P20,0	20	8.3638	10.1543	21.1543	23,000	23,000	20,000
13	P20,0	20	8.4511	9.2029	21.2029	23,000	23,000	20,000
14	P20,0	20	8.5129	8.2299	21.2299	23,000	23,000	20,000
15	P20,0	20	8.5698	7.2487	21.2487	23,000	23,000	20,000

year	type of LAIR	refinancing percentage	interest rate on the loan pr. year	remaining term to maturity	total term to maturity	actual quarterly payment on the loan	maximum limit for the payment	minimum limit for the payment
16	P20,0	20	8.6218	6.2612	21.2612	23,000	23,000	20,000
17	P20,0	20	8.6722	5.2696	21.2696	23,000	23,000	20,000
18	P20,0	20	8.7169	4.2744	21.2744	23,000	23,000	20,000
19	P20,0	20	8.8341	3.2818	21.2818	23,000	23,000	20,000
20	P20,0	20	9.1625	2.2918	21.2918	23,000	23,000	20,000
21	P20,0	20	9.2723	2	22	15,346	23,000	20,000
22	P20,0	0	11.6695	1	22	15,568	23,000	20,000

Table 4 shows that the intended refinancing profile is complied with each year.

Based on the intended term to maturity the model calculates the payments on the loan. Given that the payments on the loan comply with the limits entered, the model continues to calculate with this term to maturity until the model finds that the limits for the payment can no longer be observed. Table 4 shows that at the disbursment of the loan, the intended term to maturity of 20 years corresponds to payments on the loan within the maximum and minimum limits of DKK 23,000 and DKK 20,000, respectively. Thus, the model continues to use the term to maturity of 20 years as long as the limits for the payments on the loan are observed.

The interest rate column shows that the model applies the new bond data from the 5th refinancing and onwards. A LAIR type P20 is characterised by the fact that the full impact of a rise in interest rates takes 5 years. This is recognized in table 4 where the interest rate on the loan has in fact risen about 3 percentage points during a 5-year period.

Furthermore, table 4 shows that before the full impact of the rise in interest rates has manifested itself, it is necessary for the model to prolong the term to maturity beyond the intended 20 years. In year 8, the term to maturity is prolonged to 20.3688 years in order to reduce the payment on the loan to the level of the maximum limit of DKK 23,000. As the interest rate on the loan rises further after year 8, the term to maturity must be continuously prolonged so as to maintain the payments on the loan within the band. The terms calculated by the model during the term to maturity of the loan are designated as technical terms to maturity, since it is only the term to maturity in the last year that indicates how long the loan actually exists.

This process continues until the closing time period is reached. By definition, the loan is required to mature on a bond settlement date, thus, when the loan is refinanced the last time, the remaining term to maturity must be exactly one year. This leaves a choice of rounding an odd term to maturity either up or down to a whole year. To avoid a situation where bonds issued mature later than the loan, the term to maturity of a LAIR type P is always rounded up.

Closing time designates the number of years prior to the maturity of the loan where the remaining term to maturity is to be rounded up. In this particular case, closing time is 2 years. Table 4 shows that the total term to maturity in year 20 is 21,2918 years. In year 21, the model reaches the closing time period, and the term to maturity is rounded up to 22 years. The procedure described may also be perceived from the point of the remaining term to maturity.

Because of this procedure of rounding up the term to maturity, it may become necessary to suspend the minimum limit for the payment on the loan. This is shown in table 4 where the payment on the loan is lower than the minimum limit in the last two years (the closing time period)

The bond side

The model outputs the initial funding and the prices of the funding instruments applied. Table 5 shows that the model initially funds the loan in 6 bonds with prices according to the input. The columns in table 5 show data for the bonds with a maturity of 1, 2, 3, 4, 5, or 6 years, respectively, for each refinancing. The rows in table 5 show, for each refinancing, the volume of bonds already issued of the bond with the term to maturity in question, the price of the bonds, and the marginal volume issued at the time of the refinancing in question.

Table 5

	1 year term bond	2 year term bond	3 year term bond	4 year term bond	5 year term bond	6 year term bond
disbursement						
previously issued volume	0	0	0	0	0	0
bond price	101.92	102.62	103.42	106.28	106.23	104.92
marginal issued volume	219,535.99	161,258.79	155,159.42	148,729.56	141,951.28	134,805.73
1st refinancing						
previously issued volume	161,258.79	155,159.42	148,729.56	141,951.28	134,805.73	
bond price	101.92	102.62	103.42	106.28	106.23	
marginal issued volume	53,168.11	45,881.59	38,596.69	31,313.73	24,032.98	
2nd refinancing						
previously issued volume	201,041.01	187,326.25	173,265.01	158,838.71	0.00	
bond price	101.92	102.62	103.42	106.28	106.23	
marginal issued volume	8,070.52	5,286.35	4,164.15	8,686.52	157,232.26	

	1 year term bond	2 year term bond	3 year term bond	4 year term bond	5 year term bond	6 year term bond
3rd refinancing						
previously issued volume	192,612.60	177,429.16	167,525.23	157,232.26	0.00	
bond price	101.92	102.62	103.42	106.28	106.23	
marginal issued volume	10,821.52	7,590.83	4,721.71	4,311.08	150,426.20	
4th refinancing						
previously issued volume	185,019.99	172,246.94	161,543.34	150,426.20	0.00	
bond price	101.92	102.62	103.42	106.28	106.23	
marginal issued volume	13,886.07	9,500.40	5,114.73	2,903.25	140,641.04	
5th refinancing						
previously issued volume	181,747.34	166,658.07	153,329.45	140,641.04	0.00	
bond price	99.07	97.13	98.06	95.98	93.94	
marginal issued volume	16,465.76	12,366.19	8,607.67	9,363.54	137,555.54	
6th refinancing						

	1 year term bond	2 year term bond	3 year term bond	4 year term bond	5 year term bond	6 year term bond
previously issued volume	179,024.26	161,937.12	150,004.58	137,555.54	0.00	
bond price	99.07	97.13	98.06	95.98	93.94	
marginal issued volume	17,303.09	13,259.72	9,216.34	6,082.57	130,955.29	
7th refinancing						
previously issued volume	175,196.84	159,220.92	143,638.11	130,955.29	0.00	
bond price	99.07	97.13	98.06	95.98	93.94	
marginal issued volume	16,759.84	13,003.24	9,246.63	6,010.59	124,496.45	
8th refinancing						
previously issued volume	172,224.16	152,884.74	136,965.88	124,496.45	0.00	
bond price	99.07	97.13	98.06	95.98	93.94	
marginal issued volume	14,824.35	12,095.07	9,365.79	6,636.52	119,266.01	
9th refinancing						
previously issued volume	164,979.81	146,331.67	131,132.97	119,266.01	0.00	

	1 year term bond	2 year term bond	3 year term bond	4 year term bond	5 year term bond	6 year term bond
bond price	99.07	97.13	98.06	95.98	93.94	
marginal issued volume	17,344.42	13,428.54	9,512.65	5,596.77	108,728.47	
10th refinancing						
previously issued volume	159,760.21	140,645.62	124,862.78	108,728.47	0.00	
bond price	99.07	97.13	98.06	95.98	93.94	
marginal issued volume	17,768.30	13,804.37	9,840.44	5,876.52	99,787.16	
11th refinancing						
previously issued volume	154,449.99	134,703.22	114,604.99	99,787.16	0.00	
bond price	99.07	97.13	98.06	95.98	93.94	
marginal issued volume	18,085.50	14,187.57	10,289.64	6,391.71	90,306.71	
12th refinancing						
previously issued volume	148,890.79	124,894.63	106,178.87	90,306.71	0.00	
bond price	99.07	97.13	98.06	95.98	93.94	

	1 year term bond	2 year term bond	3 year term bond	4 year term bond	5 year term bond	6 year term bond
marginal issued volume	18,370.01	14,604.79	10,839.56	7,074.34	80,125.45	
13th refinancing						
previously issued volume	139,499.42	117,018.43	97,381.05	80,125.45	0.00	
bond price	99.07	97.13	98.06	95.98	93.94	
marginal issued volume	22,197.68	16,820.53	11,443.38	6,066.23	65,476.83	
14th refinancing						
previously issued volume	133,838.96	108,824.43	86,191.68	65,476.83	0.00	
bond price	99.07	97.13	98.06	95.98	93.94	
marginal issued volume	21,818.46	16,998.94	12,179.42	7,359.91	54,238.31	
15th refinancing						
previously issued volume	125,823.37	98,371.10	72,836.74	54,238.31	0.00	
bond price	99.07	97.13	98.06	95.98	93.94	
marginal issued volume	23,320.41	18,148.35	12,976.30	7,804.24	40,070.22	

	1 year term bond	2 year term bond	3 year term bond	4 year term bond	5 year term bond	6 year term bond
16th refinancing						
previously issued volume	116,519.45	85,813.04	62,042.55	40,070.22	0.00	
bond price	99.07	97.13	98.06	95.98	93.94	
marginal issued volume	25,593.09	19,722.92	13,852.75	7,982.57	24,009.67	
17th refinancing						
previously issued volume	105,535.96	75,895.30	48,052.79	24,009.67	0.00	
bond price	99.07	97.13	98.06	95.98	93.94	
marginal issued volume	28,971.11	21,892.61	14,814.10	7,735.59	5,593.74	
18th refinancing						
previously issued volume	97,787.91	62,866.89	31,745.26	5,593.74		
bond price	99.07	97.13	98.06	95.98		
marginal issued volume	28,340.73	20,508.68	12,676.62	4,844.56		
19th refinancing						

	1 year term bond	2 year term bond	3 year term bond	4 year term bond	5 year term bond	6 year term bond
previously issued volume	83,375.57	44,421.88	10,438.30			
bond price	99.07	97.13	98.06			
marginal issued volume	33,620.39	17,592.04	1,563.68			
20th refinancing						
previously issued volume	62,013.92	12,001.98				
bond price	99.07	97.13				
marginal issued volume	14,231.98	24,246.28				
21th refinancing						
previously issued volume	36,248.26					
bond price	99.07					
marginal issued volume	22,386.90					

Table 5 shows that when disbursed, the loan is funded as:

H(1):	219,535.99
H(2):	161,258.79
H(3):	155,159.42
H(4):	148,729.56
H(5):	141,951.28
H(6):	134,805.73

Thus, given the bond prices the proceeds criterion is fulfilled:

{(219,535.99*101.92)+(161,258.79*102.62)+(155, 159.42*103.42)+(148,729.56*106.28)+(141,951.28* 106.23)+(134,805.73*104.92)}/100=1,000,000.00

Then, information on the marginal funding issued at each refinancing is shown. In the particular example, the funding at the first refinancing comprises:

H(1)=at the disbursement H(2):	53,168.11
H(2)=at the disbursement H(3):	45.881,59
H(3)=at the disbursement H(4):	38.596,69
H(4)=at the disbursement H(5):	31.313,73
H(5)=at the disbursement H(6):	24.032,98

Thus, when the first refinancing of the loan has been performed, the total funding volume is:

H(1):	(161,258.79+53,168.11)=214,426.90
H(2):	(155,159.42+45,881.59)=201,041.01
H(3):	(148,729.56+38,596.69)=187,326.25
H(4):	(141,951.28+31,313.73)=173,265.01
H(5):	(134,805.73+24,032.98)=158,838.71

At the second refinancing, the first funding instrument is redeemed and, thus, is not be found in table 5. At this

refinancing a new bond with a term to maturity of 5 years is required. The model uses the same price for this bond as for the bond with a term to maturity of 5 years when the loan was disbursed. Table 5 also shows that the volume previously issued is zero for this bond.

The funding proceeds (the refinancing amount) may be calculated at each refinancing. When the loan is refinanced for the first time, the refinancing amount is

{(53,168.11*101.92)+(45,881.59*102.62)+(38,596.69*103.42)+(31,313.73*106.28)+(24,032.98*106.23)}/100=200,000

This amount corresponds to 1/5 of the remaining debt being refinanced. At the next refinancing the remaining debt has been reduced and the refinancing amount is DKK 194,213.75. Still, 1/5 of the remaining debt, at this point DKK 971,068.75 cf. the amortization profile, is refinanced.

Debtor side and bond side:

It is also possible to list the total payments on the debtor side as well asd on the bond side to check that the balance condition is fulfilled.

The model is solved by calculating the interest rate that equals the payments on the debtor side with the payments on the bond side, respectively, at the same time as all other requirements are met. Given an interest rate on the loan of 5.31 per cent and a term to maturity of 20 years, the first year's payments on the loan are DKK 81,477.32 cf. table 1.

On the bond side, the payments on the loan must equal the coupon payments and the bond repayments. These are shown in table 1, but may also be calculated explicitly on the basis of the other result tables. The bond repayment in the first

year is equal to the volumes of bonds redeemed at the first refinancing of the loan minus the bonds issued at this refinancing. Based on table 5 the bond repayments may be calculated as

219,535.99-(53,168.11*101.92+45,881.59*102.62+
38,596.69*103.42+31,313.73*106.28+24,032,98*106.23)/100
=19,536.00

The coupon payments in the first year is also calculated on the basis of table 5. The total coupon payments are calculated by multiplying the volume of bonds issued and the matching coupon interest rates.

219,535.99*6%+161,258.79*6%+155,159.42*6%+
148,729.56*7%+141,951.28*7%+134,805.73*7%=61,941.32

When adding the total bond repayments and coupon payments in the first year, it appears that the total payments match the payments on the loan in the first year exactly.

19,536.00+61,941.32=81,477.32

Thus the balance condition is fulfilled.

CLAIMS

1. A method for determining the type, the number, and the volume of financial instruments for the funding of a loan with equivalent proceeds to a debtor as well as the term to maturity and the payment profile on the loan by means of a first computer system, the loan being designed to be at least partially refinanced during the remaining term to maturity,

- requirements having been made to the effect that
  - the term to maturity of the loan is not longer than a predetermined maximum limit or shorter than a predetermined minimum limit,
  - debtor's payments on the loan are within predetermined limits,
    - rules having been made as to how the two above-mentioned requirements are mutually prioritized,
- requirements having been made as to a maximum permissible difference in balance between, on the one hand, payments on the loan and refinancing amounts and, on the other hand, net payments to the owner of the financial instruments applied for the funding,
- requirements having been made as to a maximum permissible difference in proceeds between, on the one hand, the sum of the market price of the volume of the financial instruments applied for the funding of the loan and, on the other hand, the principal of the loan,
- and requirements having been made as to a maximum permissible difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding,

which method comprises

(a) inputting and storing, in a memory or a storage medium of the computer system, a first set of data indicating the parameters: principal of the loan and repayment profile of the loan,

(b) inputting and storing, in a memory or a storage medium of the computer, a second set of data indicating

(i) a maximum and a minimum limit for debtor's payments on the loan for each of a number of periods which together cover the term to maturity of the loan,

(ii) a maximum and a minimum limit for the term to maturity of the loan, and

(iii) rules for the mutual prioritization of, on the one hand, the limits for debtor's payments on the loan input under (i) and, on the other hand, the limits for the term to maturity of the loan input under (ii)

(iv) and optionally a desired/intended payment on the loan or a desired/intended term to maturity when there is not equivalence between the maximum and the minimum limit for the payment on the loan during the first period (i) or when there is not equivalence between the maximum and the minimum limit for the term to maturity (ii),

(c) inputting and storing, in a memory or a storage medium of the computer system, a third set of data indicating a desired/intended refinancing profile, such as one or more point(s) in time at which refinancing is to take place, and indicating the amount of the remaining debt to be refinanced at said point(s) in time,

and/or said set of data indicating a desired/intended funding profile such as the desired/intended number of financial instruments applied for the funding, with their type and volume,

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(d) inputting and storing, in a memory or a storage medium of the computer system, a fourth set of data indicating a maximum permissible difference in balance within a predetermined period, a maximum permissible difference in proceeds and optionally a maximum permissible difference in interest rates equivalent to the difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding,

(e) determining and storing, in a memory or a storage medium of the computer system, a fifth set of data indicating a selected number of financial instruments with inherent characteristics such as type, price/market price, and date of the price/market price,

(f) determining and storing, in a memory or a storage medium of the computer system, a sixth set of data representing a first interest rate profile and either a first term to maturity profile or a first payment profile on the loan,

(g) calculating and storing, in a memory or a storage medium of the computer system, a seventh set of data representing

- a first term to maturity profile or a first payment profile (depending on what was determined under (f)) corresponding to interest and repayment for debtor
- as well as a first remaining debt profile,

the term to maturity profile or payment profile and the remaining debt profile being calculated on the basis of

- the principal of the loan and the repayment profile input under (a),
- the set of data input under (b),

- the refinancing profile and/or the funding profile input under (c),
- and the interest rate profile and either the payment profile or the term to maturity profile determined under (f),

(h) selecting a number of financial instruments among the financial instruments stored under (e), and calculating and storing an eight set of data indicating said selected financial instruments with their volumes to be applied in the funding of the loan, which eight set of data is calculated on the basis of

- the payment profile determined under (f) or calculated under (g) and
- the remaining debt profile calculated under (g),
- the refinancing profile input under (c) and/or the funding profile input under (c),
- the set of data input under (b),
- the requirements input under (d), and
- when the calculation is for a refinancing where financial instruments from an earlier funding have not matured yet, the type, the number and the volume of these instruments,

if necessary performing one or more recalculations, including, if necessary, selection of a new number of the financial instruments stored under (e),

after each recalculation storing, in a memory or a storage medium of the computer system

- the recalculated interest rate profile,
- the recalculated term to maturity profile,
- the recalculated payment profile,
- the recalculated remaining debt profile, and

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- the selected financial instruments with their calculated volumes,

until all conditions stated under (b) and (d) have been fulfilled,

after which, if desired, the thus determined combination of the type, the number and the volume of financial instruments for funding the loan

- together with the calculated term to maturity,
- together with the calculated payment profile,
- preferably together with the calculated interest rate, and
- preferably together with the calculated remaining debt profile,

is read out, transferred to a storage medium or sent to another computer system.

2. The method according to claim 1, wherein calculations are performed for all future financing periods until the maturity of the loan.

3. The method according to claim 2, wherein the result or results of the calculation for one or more later financing periods is/are used when calculating for one or more previous financing periods.

4. The method according to claim 1, 2 or 3, wherein the recalculations are performed on two or more levels in which the results of an "inner" model are used as a part of an "outer" model.

5. The method according to claim 4, wherein, starting from a first term to maturity profile, the recalculations indicated are performed in an outer model, the term to maturity being

adjusted in substantially each recalculation, until the payment on the loan for each financing period is within the under limits stipulated in (b)(i),

the type, the number and the volume of the financial instruments applied for the funding of the loan being calculated in an inner model for each outer iteration, in which inner model recalculations are performed until the relevant variables related to the type, the number and the volume of the financial instruments are determined observing the other requirements/ conditions/desires

after which, if the term to maturity at which the payment profile is within the limits stipulated for the term to maturity is not within the limits for the term to maturity stipulated under (b)(ii), the set of rules stipulated in (b)(iii) is applied for determining how the requirement as to the term to maturity and the requirement as to the payment on the loan are prioritized, whereupon the results are recalculated observing the prioritization, preferably by applying the inner model one or more times.

6. The method according to claim 5, wherein the term to maturity

in the case where the limits for the term to maturity stipulated under (b)(ii) are given a higher priority than the limits for the payment on the loan stipulated under (b)(i) and the term to maturity at which the payment profile is within the limits stipulated under (b)(i) is not within the limits for the term to maturity stipulated under (b)(ii),

is determined as either

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(1) the term to maturity which is compatible with a recalculated fixed payment on the loan until the maturity, the recalculated fixed payment on the loan being the smallest possible

(2) the limit for the term to maturity, among the limits for the term to maturity stipulated in (b)(ii), which would otherwise be exceeded,

(3) the term to maturity that is the shorter of either

the term to maturity at which the payment profile is within the limits stipulated therefor under (b)(i)

or

the limit for the term to maturity, among the limits for the term to maturity stipulated under (b)(ii), which would otherwise be exceeded.

7. The method according to any of the claims 1-6, wherein the financial instruments in (e) are determined in such a way that at least one financial instrument is an instrument on which payment falls due within the first period in which a maximum permissible difference in balance applies.

8. The method according to any of the claims 1-7, wherein the requirement as to maximum permissible difference in proceeds is given by a convergence condition for difference in proceeds, and/or the requirement as to maximum permissible difference in interest rates is given by a convergence condition for the difference in interest rates, and/or the requirement as to maximum permissible difference in balance is given by a convergence condition for the difference in balance.

9. The method according to any of the claims 1-8, wherein, in the calculation, adjustment is made for any difference

between, on the one hand, the time of disbursement of the loan and/or the repayment date and, on the other hand, the payment date of the financial instruments by adjusting proportionally for the already expired part or the remaining part of the disbursement period and the repayment period, respectively.

10. The method according to any of the claims 1-9, wherein the term to maturity

when the set of data under (c) indicates that calculation is to be made for the case where a full refinancing of the remaining debt is to be performed periodically with a predetermined period which is shorter than the term to maturity of the loan, and the remaining term to maturity of the loan is shorter than the period which according to (c) passes between two consecutive refinancings, and the remaining term to maturity does not correspond to the maturity of the last maturing instrument among the financial instruments selected under (h), but it is desired that the loan matures at the same time as the last maturing financial instrument selected under (h)

is determined as

(i) the term to maturity of the loan prolonged as little as possible to a date of maturity of one or more of the selected financial instruments provided the payment profile will not thereby be below the minimum limit for the payment on the loan stated under (b)(i), or

(ii) the term to maturity of the loan shortened as little as possible to a date of maturity of one or more of the selected financial instruments provided the payment profile will not thereby be above the maximum limit for

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the payment on the loan stated under (b)(i) and provided the condition under (i) is not met, or

(iii) the term to maturity of the loan prolonged as little as possible to a date of maturity of one or more of the selected financial instruments provided none of the conditions stated under (i) or (ii) are met.

11. The method according to any of the claims 1-10, wherein the set of data under (c) indicates that calculation is to be made for the case where a full refinancing of the remaining debt with a predetermined period is to be performed periodically with a predetermined period which is shorter than the term to maturity of the loan, which method, for determination, in the inner model, of the volumes of financial instruments indicated in step (h), comprises calculation of the difference in proceeds for the calculated volumes of the financial instruments applied for the funding, and/or calculation of an adjustment of the interest rate, said adjustment of the interest rate preferably being calculated taking into consideration the calculated difference in proceeds, it being calculated whether the adjustment of the interest rate is so small that the interest rate fulfils the requirement as to maximum permissible difference in interest rates or a convergence condition for the difference in interest rates, or whether the adjustment of the interest rate is so small that the requirement as to maximum permissible difference in proceeds or a convergence condition for the difference in proceeds is fulfilled.

12. The method according to claim 11, wherein, in case the requirements or conditions as to the difference in proceeds or the difference in the interest rates are not fulfilled, the recalculations comprise one or more interest rate iterations, each interest rate iteration comprising

calculating and storing, in a memory or a storage medium of the computer, data indicating a new interest rate

which is preferably based on the previous interest rate on the loan and the calculated adjustment of the interest rate,

calculating and storing, in a memory or a storage medium of the computer, data indicating a new payment profile and a new remaining debt profile for debtor, which payment profile and remaining debt profile are calculated taking into consideration the new interest rate on the loan, the principal of the loan, the repayment profile input under (a) and the refinancing profile and/or the funding profile input under (c), and the term to maturity, and

calculating and storing, in a memory or a storage medium of the computer system, data indicating a new set of volumes for the financial instruments applied for the funding.

13. The method according to claim 11 or 12, wherein the interest rate iteration is made by applying a numeric optimization algorithm or by "grid search".

14. The method according to claim 13, wherein the optimization algorithm is a Gauss-Newton algorithm.

15. The method according to any of the claims 10-14, which method, when the relevant requirement(s) as to maximum permissible difference in proceeds and/or the requirement as to maximum permissible difference in interest rates are fulfilled, further comprises

determining whether all the calculated volumes of financial instruments are positive, and

in case the calculated set of volumes comprises at least one negative volume, further comprises

either

i) selecting a new number of financial instruments among the financial instruments stored under (e), one or more of the instruments in the new number of instruments being determined in such a way that the payments on this/these fall

due relatively later in relation to the original number of financial instruments, followed by performing a recalculation according to any of the claims 11-14, or

ii) putting the negative volume or volumes equal to 0, followed by performing a recalculation in accordance with any of the claims 11-14.

16. The method according to any of the claims 1-8, wherein the term to maturity

when the set of data under (c) indicates that calculation is to be made for the case where a partial refinancing of the remaining debt is to be performed periodically with a predetermined period which is shorter than the term to maturity of the loan, e.g. so that the refinancing is equivalent to a fixed fraction of the remaining debt of the loan, and the remaining term to maturity of the loan is shorter than or equal to a fixed value, and it is intended that the loan matures no later than the time of maturity indicated under (e) for one or more of the financial instruments applied for refinancing of the loan,

is determined as the term to maturity prolonged as little as possible to a date of maturity of one or more financial instruments.

17. The method according to any of the claims 1-8 and 16, in which the set of data (c) indicates that calculation is to be performed for the case where partial refinancing of the remaining debt is performed periodically with a predetermined period which is shorter than the term to maturity of the loan, e.g. so that the refinancing is equivalent to a fixed fraction of the remaining debt of the loan, by which method the volumes of some of or all of the financial instruments applied for the funding are calculated in such a way in the

inner model in the first calculation in step (h) that they substantially reflect a shifted level remaining debt profile, whereupon, if necessary, recalculations are performed, until all the requirements mentioned under (d) are fulfilled.

18. The method according to claim 17, wherein, in step (h), the volumes of some of or all of the financial instruments are calculated using a function adjusted to a shifted level remaining debt profile.

19. The method according to claim 18, wherein, in one or more of the recalculations optionally performed in step (h), the volumes of some of or all of the financial instruments are calculated using a function adjusted to a shifted level remaining debt profile.

20. The method according to claim 18 or 19, wherein the function is a polynomial function with a maximum degree which is one (1) less than the number of financial instruments applied.

21. The method according to claim 20, wherein the polynomial function is calculated using a statistical curve fit method.

22. The method according to claim 21, wherein the statistic curve fit method is the least squares' method.

23. The method according to any of the claims 17-22, wherein recalculation of all of or some of the data mentioned in (g) and (h), and/or one or more function coefficients to the function representing the shifted level remaining debt profile and/or the interest rate is performed using iteration carried out by applying numeric optimization algorithms or by grid search.



24. The method according to claim 23, wherein the optimization algorithm is a Gauss-Newton algorithm.

25. The method according to any of the claims 17-24, wherein, in case the requirements or conditions as to the difference in proceeds and/or the difference in interest rates and/or the difference in balance calculated taking into consideration the refinancing profile input under (c) are not fulfilled, the recalculations comprise one or more iterations, each iteration comprising

calculating and storing data indicating a new interest rate and/or

calculating and storing data indicating a new payment profile and remaining debt profile for debtor, which payment profile and remaining debt profile are calculated taking into consideration the new interest rate on the loan, the principal of the loan and the repayment profile input under (a), the refinancing profile and/or the funding profile input under (c), and the term to maturity and/or

calculating and storing data indicating a new set of function coefficients for the function which is adjusted to the shifted level remaining debt profile, and/or

calculating and storing data indicating a new set of volumes of the financial instruments applied for the funding, which new set of volumes is calculated on the basis of the financial instruments already determined for the funding, the new payment profile and the new remaining debt profile.

26. The method according to any of the claims 17-25, which method in step (h) comprises determination of whether the calculated volumes of financial instruments fulfil at least two of two or more predetermined convergence conditions which are preferably calculated taking into consideration a calculated difference in proceeds and a difference in balance calculated taking into consideration the refinancing profile input under (c), and in case the calculated volumes of



financial instruments do not fulfil these conditions, the recalculations comprise one or more iterations of the function coefficients for the function which is adapted to the shifted level remaining debt profile, each iteration comprising

calculating and storing data indicating two or more new function coefficients for the function representing the shifted level remaining debt profile,

calculating and storing data indicating a new set of volumes for the financial instruments applied for the funding, which new set of volumes is calculated taking into consideration the new function representing the shifted level remaining debt profile,

determining whether the new set of calculated volumes of financial instruments fulfils the at least two or more predetermined convergence conditions,

until the new set of calculated volumes of financial instruments fulfils these conditions.

27. The method according to claim 26, wherein the new function coefficient(s) is/are calculated taking into consideration the calculated difference in proceeds and a difference in balance calculated taking into consideration the refinancing profile input under (c).

28. The method according to claim 26 or 27, which method comprises calculating the difference between the interest rate on the loan and the yield to maturity of the calculated volumes of the financial instruments, it being calculated whether the difference in interest rates is so small that it fulfils the requirement as to maximum permissible difference in interest rates or a convergence condition for the difference in interest rates.

29. The method according to claim 28, wherein, in case the requirements or conditions as to the difference in interest



rates are not fulfilled, then the recalculations include one or more interest rate iterations, each interest rate iteration including

calculating and storing an adjustment of the interest rate, the adjustment of the interest rate preferably being calculated taking into consideration the difference between the interest rate on the loan and the yield to maturity of the calculated volumes of the financial instruments, e.g. by use of a Gauss-Newton algorithm,

calculating and storing data indicating a new interest rate which is preferably based on the previous interest rate and the calculated adjustment of the interest rate on the loan,

calculating and storing data indicating a new payment profile and a new remaining debt profile for debtor, which payment profile and remaining debt profile are calculated taking into consideration the new interest rate, the principal of the loan and the repayment profile input under (a), the refinancing profile and/or the funding profile input under (c), and the term to maturity,

calculating and storing data indicating a new set of function coefficients for the function adjusted to the shifted level remaining debt profile, and

calculating and storing data indicating a new set of volumes for the financial instruments applied for the funding.

30. The method according to any of the claims 17-25, which method comprises the determination of whether the calculated volumes of financial instruments fulfil at least three of three or more predetermined convergence conditions which are preferably calculated taking into consideration a calculated difference in proceeds, a difference in balance calculated taking into consideration the refinancing profile input under (c), and a maximum permissible difference in interest rates, and in case the calculated volumes of financial instruments,

the calculated payment profile and the calculated term to maturity do not fulfil these conditions, then the recalculations comprise one or more iterations, each iteration comprising

calculating and storing an adjustment of the interest rate, the adjustment of the interest rate being preferably calculated taking into consideration the difference between the interest rate on the loan and the yield to maturity of the financial instruments,

calculating and storing data indicating a new interest rate which is preferably based on the previous interest rate and the calculated adjustment of the interest rate on the loan,

calculating and storing data indicating a new payment profile and a new remaining debt profile for debtor, which payment profile and remaining debt profile are calculated taking into consideration the new interest rate, the principal of the loan, and the repayment profile input under (a), the refinancing profile and/or the funding profile input under (c), and the term to maturity, and

calculating and storing data indicating a new set of function coefficients for the function adjusted to the shifted level remaining debt profile, and

calculating and storing data indicating a new set of volumes for the financial instruments applied for the funding, which new set of volumes is calculated taking into consideration the new function representing the shifted level remaining debt profile,

determining whether the new set of calculated volumes of financial instruments fulfils the at least three or more predetermined convergence conditions.

31. The method according to any of the claims 17-30, wherein, in case the calculated set of volumes comprises at least one negative volume, the negative volume(s) is/are put equal to

0, whereupon the calculations continue on the basis of the thus determined volumes of financial instruments.

32. The method according to claim 31, which method in the inner model comprises calculation of whether the volume of financial instruments fulfills the requirement as to maximum permissible difference in proceeds, and in case the calculated volumes do not meet this requirement, one or more adjustments of the previously calculated positive volumes of financial instruments are performed, adjustments being performed until the new set of volumes of financial instruments meets the requirement as to maximum permissible difference in proceeds and maximum permissible difference in balance.

33. The method according to any of the claims 17-22, wherein the determination of one or more of the function coefficients for the function adjusted to the shifted level remaining debt profile is performed analytically.

34. The method according to claim 33, wherein recalculation of all of or some of the data mentioned in (g) and (h) and/or the interest rate is performed by iteration carried out using numeric optimization algorithms or by grid search.

35. The method according to claim 34, wherein the optimization algorithm is a Gauss-Newton algorithm.

36. The method according to any of the claims 33-35, wherein, in case the requirements or conditions as to the difference in interest rates and/or the difference in proceeds and/or the difference in balance calculated taking into consideration the refinancing profile input under (c) are not fulfilled, the recalculation comprises one or more iterations, each iteration comprising

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calculating and storing data indicating a new interest rate and/or

calculating and storing data indicating a new payment profile and a new remaining debt profile for debtor, which payment profile and remaining debt profile are calculated taking into consideration the new interest rate on the loan, the principal of the loan, and repayment profile input under (a), the refinancing profile and/or the funding profile input under (c), and the term to maturity and/or

calculating and storing data indicating a new set of function coefficients for the function which is adjusted to the shifted level remaining debt profile, and/or

calculating and storing data indicating a new set of volumes of the financial instruments applied for the funding, which new set of volumes is calculated on the basis of the financial instruments already determined for the funding, the new payment profile and the new remaining debt profile.

37. The method according to claim 36, wherein the new function coefficient(s) is/are calculated taking into consideration the calculated difference in proceeds and a difference in balance calculated taking into consideration the refinancing profile input under (c).

38. The method according to claim 36 or 37, which method comprises the calculation of the difference between the interest rate on the loan and the yield to maturity of the calculated volumes of the financial instruments, it being calculated whether the difference in interest rates is so small that it fulfils the requirement as to maximum permissible difference in interest rates or a convergence condition for the difference in interest rates.

39. The method according to claim 38, wherein, in case the requirements as to the difference in interest rates are not fulfilled, then the recalculations include one or more

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interest rate iterations, each interest rate iteration including

calculating and storing an adjustment of the interest rate, the adjustment of the interest rate preferably being calculated taking into consideration the difference between the interest rate on the loan and the yield to maturity of the calculated volumes of the financial instruments, e.g. by use of a Gauss-Newton algorithm,

calculating and storing data indicating a new interest rate which is preferably based on the previous interest rate and the calculated adjustment of the interest rate on the loan,

calculating and storing data indicating a new payment profile and a new remaining debt profile for debtor, which payment profile and remaining debt profile are calculated taking into consideration the new interest rate, the principal of the loan, and the repayment profile input under (a), the refinancing profile and/or the funding profile input under (c), and term to maturity,

calculating and storing data indicating a new set of function coefficients for the function adjusted to the shifted level remaining debt profile, and

calculating and storing data indicating a new set of volumes for the financial instruments applied for the funding.

40. The method according to any of the claims 33-39, which method in step (h) comprises determining the volume of the financial instruments by analytical calculation of one or more of the function coefficients for the function adjusted to the shifted level remaining debt profile, so that the requirements under (d) as to maximum permissible difference in proceeds and maximum permissible difference in balance, taking the refinancing profile input under (c) into consideration, are fulfilled.

41. The method according to any of the claims 33-40, wherein, in case the calculated volumes of the financial instruments applied comprise at least one negative volume, the negative volume(s) is/are put equal to 0 jointly or one at a time, the calculations being continued, after one such or each such operation of assigning 0-value(s), on the basis of the thus determined volumes of the financial instruments applied.

42. The method according to claim 41, wherein the calculated function coefficients for the function adjusted to the shifted level remaining debt profile are regulated by an indicator function which indicates either that a financial instrument is to be applied or that an instrument is not to be applied, which indicator function is adjusted at each operation of assigning 0-value(s), and wherein the function coefficients are calculated in such a way that both the proceeds criterion and the balance criterion, calculated taking the refinancing profile under (c) into consideration, are fulfilled in the case where the indicator function indicates that two or more financial instruments are to applied of which at least one instrument matures within the first predetermined period for which it is indicated that a partial refinancing of the remaining debt is to be performed, and both the proceeds criterion and the balance criterion are fulfilled in all other cases.

43. The method according to any of the claims 17-42, wherein the volume of one or more financial instruments, especially the first to mature and/or the last to mature, is not calculated by use of the function representing the remaining debt profile, but is determined separately in order to ensure that the actual refinancing profile corresponds to the one input in (c).

44. The method according to any of the claims 1-9, which method comprises the calculation in the inner model of



whether the volume of financial instruments in the funding profile indicated under (c) fulfils the requirement as to maximum permissible difference in proceeds, and in case the volumes indicated do not fulfil this requirement, then one or more adjustments of the previously indicated volumes of financial instruments is/are performed, adjustments being performed until the new set of volumes of financial instruments fulfils the requirement as to maximum permissible difference in proceeds.

45. The method according to claim 44, which method comprises the calculation of whether the resulting set of volumes of financial instruments fulfils the requirement as to maximum permissible difference in proceeds, and in case the resulting volumes do not fulfil this requirement, then one or more calculations of new volumes for at least one of the financial instruments which does not fulfil the requirement as to maximum permissible difference in balance is/are performed.

46. The method according to claim 45, wherein new volumes are calculated for one or more financial instruments to which repayments are to be performed in a period wherein the requirement as to maximum permissible difference in balance is not fulfilled.

47. The method according to claim 45 or 46, wherein new volumes are calculated for one or more financial instruments to which repayments are to be performed in the last period wherein the requirement as to maximum permissible difference in balance is not fulfilled.

48. The method according to claim 46 or 31, wherein the new volumes are calculated on the basis of the difference in balance for the periods in which the corresponding, previously found volumes do not fulfil the requirement as to maximum permissible difference in balance.

49. The method according to any of the claims 45-48, wherein calculation is carried out as to whether the new set of volumes fulfils the requirement as to maximum permissible difference in proceeds, and in case the volumes indicated do not fulfil this requirement, then one or more adjustments of the volumes indicated are performed until the new set of volumes fulfils the requirement as to maximum permissible difference in proceeds, and in case the new set of volumes does not fulfil the requirements as to maximum permissible difference in balance, new volumes are calculated for at least one of the financial instruments which does not fulfil the requirement as to maximum permissible difference in balance, the adjustment and calculation of new sets of volumes being performed until the requirements as to both maximum permissible difference in proceeds and maximum permissible difference in balance are fulfilled.

50. The method according to any of the claims 45-48, which method comprises calculation of the difference in proceeds for the calculated volumes of the financial instruments applied for the funding and/or calculation of an adjustment of the interest rate on the loan, the adjustment of the interest rate preferably being calculated taking into consideration the calculated difference in proceeds, it being calculated whether the adjustment of the interest rate is so small that the interest rate on the loan fulfils the requirement as to maximum permissible difference in interest rates or a convergence condition for the interest rate difference, or whether the adjustment of the interest rate is so small that the requirement as to maximum permissible difference in proceeds or a convergence condition for the difference in proceeds is fulfilled.

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51. The method according to any of the claims 44-50, wherein, in case the requirements or conditions as to the difference in proceeds or the difference in interest rates are not fulfilled, then the recalculation comprises one or more interest rate iterations, each interest rate iteration comprising

calculating and storing data indicating a new interest rate which is preferably based on the previous interest rate and the calculated adjustment of the interest rate,

calculating and storing data indicating a new payment profile and remaining debt profile for debtor, which payment profile and remaining debt profile are calculated taking into consideration the new interest rate, the principal of the loan and the repayment profile input under (a), the refinancing profile and/or the funding profile input under (c), and the term to maturity, and

calculating and storing data indicating a new set of volumes for the financial instruments applied for the funding, which new set of volumes is calculated on the basis of the financial instruments already determined for the funding, the new payment profile and the new remaining debt profile as well as the requirement as to the maximum permissible difference in balance.

52. A method for determining the type, the number, and the volume of financial instruments for the funding of a loan with equivalent proceeds to a debtor by means of a first computer system, the loan being designed to be at least partially refinanced during the remaining term to maturity,

- requirements having been made as to a maximum permissible difference in balance between, on the one hand, payments on the loan and refinancing amounts and, on the other hand, net payments to the owner of the financial instruments applied for the funding,
- requirements having been made as to a maximum permissible difference in proceeds between, on the one

hand, the sum of the market price of the volume of the financial instruments applied for the funding of the loan, and on the other hand, the principal of the loan, and requirements having been made as to a maximum permissible difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding,

which method comprises

(a) inputting and storing, in a memory or a storage medium of the computer system, a first set of data indicating the parameters: principal of the loan and repayment profile of the loan,

(b) inputting and storing, in a memory or a storage medium of the computer, a second set of data indicating the term to maturity of the loan,

(c) inputting and storing, in a memory or a storage medium of the computer system, a third set of data indicating a desired/intended refinancing profile, such as one or more point(s) in time at which refinancing is to take place, and indicating the amount of the remaining debt to be refinanced at said point(s) in time,

(d) inputting and storing, in a memory or a storage medium of the computer system, a fourth set of data indicating a maximum permissible difference in balance within a predetermined period, a maximum permissible difference in proceeds and a maximum permissible difference in interest rates equivalent to the difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding,



(e) determining and storing, in a memory or a storage medium of the computer system, a fifth set of data indicating a selected number of financial instruments with inherent characteristics such as type, price/market price, and date of the price/market price,

(f) determining and storing, in a memory or a storage medium of the computer system, a sixth set of data representing a first interest rate profile,

(g) calculating and storing, in a memory or a storage medium of the computer system, a seventh set of data representing

- a first payment profile corresponding to interest and repayment for debtor
- as well as a first remaining debt profile,

the payment profile and the remaining debt profile being calculated on the basis of

- the principal of the loan and the repayment profile input under (a),
- the set of data input under (b),
- the refinancing profile and/or the funding profile input under (c),
- and the interest rate profile determined under (f),

(h) selecting a number of financial instruments among the financial instruments stored under (e), and calculating and storing an eight set of data indicating said selected financial instruments with their volumes to be applied in the funding of the loan, which eight set of data is calculated on the basis of

- the payment profile calculated under (g) and



- the remaining debt profile calculated under (g),
- the refinancing profile input under (c) and/or the funding profile input under (c),
- the set of data input under (b),
- the requirements input under (d), and
- when the calculation is for a refinancing where financial instruments from an earlier funding have not matured yet, the type, the number and the volume of these instruments,

if necessary performing one or more recalculations, including, if necessary, selection of a new number of the financial instruments stored under (e),

after each recalculation storing, in a memory or a storage medium of the computer system

- the recalculated interest rate profile,
- the recalculated payment profile,
- the recalculated remaining debt profile, and
- the selected financial instruments with their calculated volumes,

until all conditions stated under (b) and (d) have been fulfilled,

after which, if desired, the thus determined combination of the type, the number and the volume of financial instruments for funding the loan

- together with the calculated payment profile,
- preferably together with the calculated interest rate, and
- preferably together with the calculated remaining debt profile,

is read out, transferred to a storage medium or sent to another computer system,

the set of data (c) indicating that calculation is to be performed for the case where partial refinancing of the remaining debt is performed periodically with a predetermined period, which period is shorter than the term to maturity of the loan, e.g. in such a way that the refinancing is equivalent to a fixed fraction of the remaining debt of the loan, some of or all of the financial instruments applied for the funding in the first calculation in step (h) being calculated in such a way that they substantially reflect a shifted level remaining debt profile, whereupon, if necessary, recalculations are performed until all the requirements mentioned under (d) are fulfilled,

the volume of some of or all of the financial instruments being calculated, in the calculation in step (h), by applying a function adjusted to a shifted level remaining debt profile, the determination of one or more of the function coefficients for the function adjusted to the shifted level remaining debt profile being performed analytically.

53. The method according to claim 52, wherein the volume of some of or all of the financial instruments in one or more of the recalculations optionally performed in step (h) are calculated by applying a function adjusted to a shifted level remaining debt profile, the determination of one or more of the function coefficients for the function adjusted to the shifted level remaining debt profile being performed analytically.

54. The method according to claim 52 or 53, wherein the function is a polynomial function with a maximum degree of 1 (one) less than the number of financial instruments applied.

55. The method according to claim 54, wherein the polynomial function is calculated by use of a statistic curve fit method.

56. The method according to claim 55, wherein the statistic curve fit method is the least squares' method.

57. The method according to any of the claims 52-56, wherein recalculation of all of or some of the data mentioned in (g) and (h) and/or recalculation of the interest rate is performed by iteration carried out using numeric optimization algorithms or by grid search

58. The method according to claim 57, wherein the optimization algorithm is a Gauss-Newton algorithm.

59. The method according to any of the claims 52-58, wherein, in case the requirements or conditions as to the difference in interest rates and/or the difference in proceeds and/or the difference in balance calculated taking into consideration the refinancing profile input under (c) are not fulfilled, the recalculation comprises one or more iterations, each iteration comprising

calculating and storing data indicating a new interest rate and/or

calculating and storing data indicating a new payment profile and a new remaining debt profile for debtor, which payment profile and remaining debt profile are calculated taking into consideration the new interest rate on the loan, the principal of the loan and the repayment profile input under (a), the refinancing profile and/or the funding profile input under (c), and the term to maturity and/or

calculating and storing data indicating a new set of function coefficients for the function which is adjusted to the shifted level remaining debt profile, and/or

calculating and storing data indicating a new set of volumes of the financial instruments applied for the funding, which new set of volumes is calculated on the basis of the financial instruments already determined for the funding, the new payment profile and the new remaining debt profile.

60. The method according to claim 59, wherein the new function coefficient(s) is/are calculated taking into consideration the calculated difference in proceeds and a difference in balance calculated taking into consideration the refinancing profile input under (c).

61. The method according to claim 59 or 60, which method comprises calculation of the difference between the interest rate on the loan and the yield to maturity of the calculated volumes of the financial instruments, it being calculated whether the difference in interest rates is so small that it fulfils the requirement as to maximum permissible difference in interest rates or a convergence condition for the difference in interest rates.

62. The method according to claim 61, wherein, in case the requirements or conditions as to the difference in interest rates are not fulfilled, then the recalculations include one or more interest rate iterations, each interest rate iteration including

calculating and storing an adjustment of the interest rate, the adjustment of the interest rate preferably being calculated taking into consideration the difference between the interest rate on the loan and the yield to maturity of the calculated volumes of the financial instruments, e.g. by use of a Gauss-Newton algorithm,

calculating and storing data indicating a new interest rate which is preferably based on the previous interest rate and the calculated adjustment of the interest rate on the loan,

calculating and storing data indicating a new payment profile and a new remaining debt profile for debtor, which payment profile and remaining debt profile are calculated taking into consideration the new interest rate, the principal of the loan and the repayment profile input under (a), the refinancing profile and/or the funding profile input under (c), and the term to maturity,

calculating and storing data indicating a new set of function coefficients for the function adjusted to the shifted level remaining debt profile, and

calculating and storing data indicating a new set of volumes for the financial instruments applied for the funding.

63. The method according to any of the claims 52-62, wherein the determination of the volume of the financial instruments by analytical calculation of one or more of the function coefficients for the function adjusted to a shifted level remaining debt profile is performed so that the requirements under (d) as to maximum permissible difference in proceeds and maximum permissible difference in balance, taking the refinancing profile input under (c) into consideration, are fulfilled.

64. The method according to any of the claims 52-63, wherein, in case the calculated volumes of the financial instruments applied comprise at least one negative volume, the negative volumes are set equal to 0 jointly or one at a time, the calculations being continued, after one such or each such operation of assigning 0-value(s), on the basis of the thus determined volumes of the financial instruments applied.

65. The method according to claim 64, wherein the calculated function coefficients for the function adjusted to the shifted level remaining debt profile is regulated by an indicator function which indicates either that a financial

instrument is to be applied or that an instrument is not to be applied, which indicator function is adjusted at each operation of assigning 0-value(s), and wherein the function coefficients are calculated in such a way that both the proceeds criterion and the balance criterion, calculated taking the refinancing profile under (c) into consideration, are fulfilled in the case where the indicator function indicates that two or more financial instruments are to applied of which at least one instrument matures within the first predetermined period for which it is indicated that a partial refinancing of the remaining debt is to be performed, and both the proceeds criterion and the balance criterion are fulfilled in all other cases.

66. The method according to any of the claims 52-65, wherein the volume of one or more financial instruments, especially the first to mature and/or the last to mature, is not calculated by use of the function representing the remaining debt profile, but is determined separately in order to ensure that the actual refinancing profile is equivalent to the one input in (c).

67. The method according to any of the claims 52-66, wherein the financial instruments in (e) are determined in such a way that at least one financial instrument is an instrument on which payment falls due within the first period in which a maximum permissible difference in balance applies.

68. The method according to any of the claims 52-67, wherein the requirement as to maximum permissible difference in proceeds is given by a convergence condition for difference in proceeds, and/or the requirement as to maximum permissible difference in interest rates is given by a convergence condition for the difference in interest rates, and/or the requirement as to maximum permissible difference in balance

is given by a convergence condition for the difference in balance.

69. The method according to any of the claims 52-68, wherein, in the calculation, adjustment is made for any difference between, on the one hand, the time of disbursement of the loan and/or the repayment date and, on the other hand, the payment date of the financial instruments by adjusting proportionally for the already expired part or the remaining part of the disbursement period and the repayment period, respectively.

70. A computer system for determining the type, the number, and the volume of financial instruments for the funding of a loan with equivalent proceeds to a debtor as well as the term to maturity and the payment profile on the loan, the loan being designed to be at least partially refinanced during the remaining term to maturity,

- requirements having been made to the effect that
  - the term to maturity of the loan is not longer than a predetermined maximum limit or shorter than a predetermined minimum limit,
  - debtor's payments on the loan are within predetermined limits,
    - rules having been made as to how the two above-mentioned requirements are mutually prioritized,
- requirements having been made as to a maximum permissible difference in balance between, on the one hand, payments on the loan and refinancing amounts and, on the other hand, net payments to the owner of the financial instruments applied for the funding,

- requirements having been made as to a maximum permissible difference in proceeds between, on the one hand, the sum of the market price of the volume of the financial instruments applied for the funding of the loan and, on the other hand, the principal of the loan,
- and requirements having been made as to a maximum permissible difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding,

which computer system comprises

(a) means, typically input means and a memory or a storage medium, for inputting and storing a first set of data indicating the parameters: principal of the loan and repayment profile of the loan,

(b) means, typically input means and a memory or a storage medium, for inputting and storing a second set of data indicating

(i) a maximum and a minimum limit for debtor's payments on the loan for each of a number of periods which together cover the term to maturity of the loan,

(ii) a maximum and a minimum limit for the term to maturity of the loan, and

(iii) rules for the mutual prioritization of, on the one hand, the limits for debtor's payments on the loan input under (i) and, on the other hand, the limits for the term to maturity of the loan input under (ii)

(iv) and optionally a desired/intended payment on the loan or a desired/intended term to maturity when there is not equivalence between the maximum and the minimum limit for the payment on the loan during the first period (i) or when there is not equivalence between the maximum and the minimum limit for the term to maturity (ii),

(c) means, typically input means and a memory or a storage medium, for inputting and storing a third set of data indicating a desired/intended refinancing profile, such as one or more point(s) in time at which refinancing is to take place, and indicating the amount of the remaining debt to be refinanced at said point(s) in time,

and/or said set of data indicating a desired/intended funding profile such as the desired/intended number of financial instruments applied for the funding, with their type and volume,

(d) means, typically input means and a memory or a storage medium, for inputting and storing a fourth set of data indicating a maximum permissible difference in balance within a predetermined period, a maximum permissible difference in proceeds and optionally a maximum permissible difference in interest rates equivalent to the difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding,

(e) means, typically input means and a memory or a storage medium, for determining and storing a fifth set of data indicating a selected number of financial instruments with inherent characteristics such as type, price/market price, and date of the price/market price,

(f) means, typically input means and/or calculation means and a memory or a storage medium, for determining and storing a sixth set of data representing a first interest rate profile and either a first term to maturity profile or a first payment profile on the loan,

(g) means, typically calculation means and a memory or a storage medium, for calculating and storing a seventh set of data representing

- a first term to maturity profile or a first payment profile (depending on what was determined under (f)) corresponding to interest and repayment for debtor
- as well as a first remaining debt profile,

the term to maturity profile or payment profile and the remaining debt profile being calculated on the basis of

- the principal of the loan and the repayment profile input under (a),
- the set of data input under (b),
- the refinancing profile and/or the funding profile input under (c),
- and the interest rate profile and either the payment profile or the term to maturity profile determined under (f),

(h) means, typically calculation means and a memory or a storage medium, for selecting a number of financial instruments among the financial instruments stored under (e), and calculating and storing an eight set of data indicating said selected financial instruments with their volumes to be applied in the funding of the loan, which eight set of data is calculated on the basis of

- the payment profile determined under (f) or calculated under (g) and
- the remaining debt profile calculated under (g),
- the refinancing profile input under (c) and/or the funding profile input under (c),
- the set of data input under (b),
- the requirements as to maximum difference in balance, maximum difference in proceeds and/or maximum difference in interest rates input under (d), and
- when the calculation is for a refinancing where financial instruments from an earlier funding have not



matured yet, the type, the number and the volume of these instruments,

the means being adapted to perform, if necessary, one or more recalculations, including, if necessary, selection of a new number of the financial instruments stored under (e),

the means being further adapted to store, after each recalculation,

- the recalculated interest rate profile,
- the recalculated term to maturity profile,
- the recalculated payment profile,
- the recalculated remaining debt profile, and
- the selected financial instruments with their calculated volumes,

until all conditions stated under (b) and (d) have been fulfilled,

output means for outputting the thus determined combination of the type, the number and the volume of financial instruments for funding the loan

- together with the calculated term to maturity,
- together with the calculated payment profile,
- preferably together with the calculated interest rate, and
- preferably together with the calculated remaining debt profile,

or means for transferring the combination to a storage medium or for sending it to another computer system.

71. A computer system for determining the type, the number, and the volume of financial instruments for the funding of a

loan with equivalent proceeds to a debtor by means of a first computer system, the loan being designed to be at least partially refinanced during the remaining term to maturity,

- requirements having been made as to a maximum permissible difference in balance between, on the one hand, payments on the loan and refinancing amounts and, on the other hand, net payments to the owner of the financial instruments applied for the funding,
- requirements having been made as to a maximum permissible difference in proceeds between, on the one hand, the sum of the market price of the volume of the financial instruments applied for the funding of the loan, and on the other hand, the principal of the loan,
- and requirements having been made as to a maximum permissible difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding,

which computer system comprises

(a) means, typically input means and a memory or a storage medium, for inputting and storing a first set of data indicating the parameters: principal of the loan and repayment profile of the loan,

(b) means, typically input means and a memory or a storage medium, for inputting and storing a second set of data indicating the term to maturity of the loan,

(c) means, typically input means and a memory or a storage medium, for inputting and storing a third set of data indicating a desired/intended refinancing profile, such as one or more point(s) in time at which refinancing is to take place, and indicating the amount of the remaining debt to be refinanced at said point(s) in time,

(d) means, typically input means and a memory or a storage medium, for inputting and storing a fourth set of data indicating a maximum permissible difference in balance within a predetermined period, a maximum permissible difference in proceeds and a maximum permissible difference in interest rates equivalent to the difference between the interest rate on the loan and the yield to maturity of the financial instruments applied for the funding,

(e) means, typically input means and a memory or a storage medium, for determining and storing a fifth set of data indicating a selected number of financial instruments with inherent characteristics such as type, price/market price, and date of the price/market price,

(f) means, typically input means and/or calculation means and a memory or storage means, for determining and storing a sixth set of data representing a first interest rate profile,

(g) means, typically calculation means and a memory or a storage means, for calculating and storing a seventh set of data representing

- a first payment profile corresponding to interest and repayment for debtor
- as well as a first remaining debt profile,

the payment profile and the remaining debt profile being calculated on the basis of

- the principal of the loan and the repayment profile input under (a),
- the set of data input under (b),
- the refinancing profile and/or the funding profile input under (c),
- and the interest rate profile determined under (f),

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(h) means, typically calculation means and a memory or a storage medium, for selecting a number of financial instruments among the financial instruments stored under (e), and calculating and storing an eight set of data indicating said selected financial instruments with their volumes to be applied in the funding of the loan, which eight set of data is calculated on the basis of

- the payment profile calculated under (g) and
- the remaining debt profile calculated under (g),
- the refinancing profile input under (c) and/or the funding profile input under (c),
- the set of data input under (b),
- the requirements input under (d), and
- when the calculation is for a refinancing where financial instruments from an earlier funding have not matured yet, the type, the number and the volume of these instruments,

the means being adapted to perform, if necessary, one or more recalculations, including, if necessary, selection of a new number of the financial instruments stored under (e),

the means being further designed to store, after each recalculation,

- the recalculated interest rate profile,
- the recalculated payment profile,
- the recalculated remaining debt profile, and
- the selected financial instruments with their calculated volumes,

until all conditions stated under (b) and (d) have been fulfilled,

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and to thereafter outputting the thus determined combination of the type, the number and the volume of financial instruments for funding the loan

- together with the calculated payment profile,
- preferably together with the calculated interest rate, and
- preferably together with the calculated remaining debt profile,

transferring it to a storage medium or sending it to another computer system,

the set of data (c) indicating that calculation is to be performed for the case where partial refinancing of the remaining debt is performed periodically with a predetermined period, which period is shorter than the term to maturity of the loan, e.g. in such a way that the refinancing is equivalent to a fixed fraction of the remaining debt of the loan, some of or all of the financial instruments applied for the funding in the first calculation in step (h) being calculated in such a way that they substantially reflect a shifted level remaining debt profile, whereupon, if necessary, recalculations are performed until all the requirements mentioned under (d) are fulfilled,

the volume of some of or all of the financial instruments being calculated, in the calculation in step (h), by applying a function adjusted to a shifted level remaining debt profile, the determination of one or more of the function coefficients for the function adjusted to the shifted level remaining debt profile being performed analytically.

72. A computer system according to claim 70 or 71, wherein the input means comprise one or more of the following:

- a keyboard, a computer mouse, a touch screen or touch plate, a microphone, means for voice recognition, a scanner, or any other means for human interaction,

- interfacing means which electronically connect the computer system to a data network which is designed to transmit data from a storage medium or a data transmitting system to the computer system,

- means for reading data from one or more storage media.

73. A computer system according to any of the claims 70-72, wherein the storage media comprise one or more of the following:

- electronic memories such as ROM, PROM, EPROM, EEPROM or RAM

- one or more erasable or non-erasable disc storage media such as optical or magnetic disc storage media, or

- any other storage media.

74. A computer system according to any of the claims 70-73, wherein the output means comprise one or more of the following:

- one or more data screens
- one or more printers,
- one or more telefax machines,
- one or more voice generating machines
- interfacing means which electronically connect the computer system to a data network designed to transfer data from the computer system to a data transmission system that comprises or is connected to one or more output means of the above-mentioned types.

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75. A computer system according to any of the claims 70-74, wherein the data network constitutes or is a part of a local network such as a Local Area Network.

76. A computer system according to claim 75, wherein the local network is a part of or is connected to a wide network, such as a Wide Area Network.

77. A computer system according to any of the claims 72-76, wherein the data network comprises one or more ISDN connections, one or more telecommunication connections connected to the computer system by means of at least one telecommunication connection means such as a modem, the telecommunication connections comprising one or more telephone connections, wireless connections or other means of data connection.

78. A computer system according to any of the claims 70-77, wherein the calculation means comprise one or more electronic calculation circuits such as microprocessors, as an integrated part of the computer system.

79. A computer system according to any of the claims 70-77, wherein the means for calculation comprise one or more electronic calculation circuits such as micro processors, which are connected to the computer system via the data network.

