

- 1 -

Active Disturbance Rejection Control System

Technical Field

The present invention relates, in general terms, to active disturbance rejection control systems.

Background

The active disturbance rejection control (ADRC) system provides insight for common methods to deal with both linear and nonlinear systems, time-varying and time-invariant systems. Despite the existence of valuable works concerning measurement noise suppression of ADRC systems, some practical issues in speed regulation systems have not been properly addressed in the prior art. Also, to ensure the required performance of feedforward compensation is achieved, precise modeling of the plant system is critical. A more precise plant model provides higher compensation in the feedforward manner. Also, a more progressive design in the feedback controller would be highly beneficial.

Though ADRC and its variants have been tried in many fields, they have a notable drawback, namely a complicated control structure with many tuning parameters. To solve this problem, linear ADRC (LADRC) was introduced, where only the bandwidth of the observer and the closed-loop system are tuned. However, considering the practical issues of LADRC, the well-known superiority of ADRC over classical controllers is completely subjected to sensor specifications.

Due to the quantization error caused by the position sensor in existing systems, the speed calculation is contaminated by the measurement noise. A low-pass filter (LPF) is often used to filter out high-frequency components of measurement noise. However, the LPF causes a phase lag between the filtered speed and actual speed. This is acceptable for systems where high dynamics are not

- 2 -

required. An alternative method to suppress measurement noise is to use the instantaneous speed observer (ISO). However, the bandwidth of the ISO has to be reduced in the lower speed range, leading to a deteriorated dynamic performance.

In addition, ADRC systems based on the phase-locking loop observer (PLL0) and those based on an extended state observer (ESO) have totally different disturbance rejection properties and measurement noise suppression performance compared to those based on ISO. Different combinations of speed feedback and disturbance feedforward result in different system dynamics even if the same structure of ESO is employed.

It is essential to clarify how the combination of speed feedback and disturbance feedforward affects the system dynamics. The conclusion obtained by the prior art is for the ADRC system to be based on the speed feedback, requiring the speed to be calculated first. For an ADRC system based on position feedback, the conclusion will be different since there is no measured speed in the system.

It would be desirable to overcome all or at least one of the above-described problems.

Summary

Disclosed herein is an active disturbance rejection control (ADRC) system, comprising:

- an extended state observer (ESO) module that is configured to:
 - receive input information comprising a torque reference signal for driving a motor, and an angular position that is a measured output of the motor;
 - determine, based on the input information, an estimated speed value and an estimated disturbance value; and

- 3 -

send the estimated speed value as a feedback control signal, and the estimated disturbance value as a feedforward control signal;

wherein the estimated speed value is based on a derivative of the angular position, and the estimated disturbance value is an output of a proportional-integral (PI) controller of the ESO module.

Disclosed herein is also an ADRC system, comprising:

an ESO module that is configured to:

receive input information comprising a torque reference signal for driving a motor, and a speed value that is a measured output of the motor passed to a first low-pass filter (LPF);

determine, based on the input information, an estimated speed value and an estimated disturbance value; and

send the estimated speed value as a feedback control signal, and the estimated disturbance value as a feedforward control signal;

wherein the estimated speed value is determined at least in part based on a second LPF that is of the same form as the first LPF, and the estimated disturbance value is selectable between an output of an integral controller and an output of a proportional-integral controller of the ESO module; or

wherein the estimated speed value is determined based on the output of a proportional-integral-derivative controller of the ESO module, and the estimated disturbance value is selectable between: an output of an integral controller of the ESO module; and an output of a proportional-integral controller of the ESO module; and the output of the proportional-integral-derivative controller of the ESO module.

In some embodiments, the estimated speed value may be determined at least in part based on the second LPF, and the second LPF may be of the same form as the first LPF, and the estimated disturbance value may be selectable between an output of the integral controller and the output of the proportional-integral controller of the ESO module.

- 4 -

In some embodiments, the ESO module may have a fourth-order characteristic polynomial.

In some embodiments, the ESO module may be characterized by the following transfer functions:

$$\begin{aligned}\Phi_r(s) &= \frac{\Omega(s)}{\Omega^*(s)} = \frac{\Delta_3(s+k_{ps})}{\Delta_3(s+k_{ps})+(k_b-1)sG_3(s)} \\ \Phi_d(s) &= \frac{\Omega(s)}{d_{ex}(s)} = \frac{k_b G_3(s)}{\Delta_3(s+k_{ps})+(k_b-1)sG_3(s)} \\ \Phi_n(s) &= \frac{\Omega(s)}{-s\delta_p(s)} = 1 - s\Phi_d(s)\end{aligned}$$

where $\Phi_r(s)$ is the transfer function from the command reference to the output; $\Phi_d(s)$ is the transfer function from the disturbance to the output; $\Phi_n(s)$ is the transfer function from the measurement noise to the output; and Δ_3 is the characteristic polynomial of ESO.

In some embodiments, a transfer function of the second LPF may be:

$$G_{LPF}(s) = \frac{\omega_{f2}}{s + \omega_{f1}}$$

where ω_{f1} and ω_{f2} are gains and s is the Laplace Operator.

In some embodiments, the second LPF may be set to be an integrator.

In some embodiments, the estimated speed value may be determined based on the output of the proportional-integral-derivative controller of the ESO module.

Disclosed herein is also an ADRC system comprising an ESO module that has a fourth-order characteristic polynomial.

- 5 -

The ESO module may be characterized by the following transfer functions:

$$\begin{aligned}\Phi_r(s) &= \frac{\Omega(s)}{\Omega^*(s)} = \frac{\Delta_3(s+k_{ps})}{\Delta_3(s+k_{ps})+(k_b-1)sG_3(s)} \\ \Phi_d(s) &= \frac{\Omega(s)}{d_{ex}(s)} = \frac{k_b G_3(s)}{\Delta_3(s+k_{ps})+(k_b-1)sG_3(s)} \\ \Phi_n(s) &= \frac{\Omega(s)}{-s\delta_p(s)} = 1 - s\Phi_d(s)\end{aligned}$$

where $\Phi_r(s)$ is the transfer function from the command reference to the output; $\Phi_d(s)$ is the transfer function from the disturbance to the output; $\Phi_n(s)$ is the transfer function from the measurement noise to the output; and Δ_3 is the characteristic polynomial of ESO.

Brief description of the drawings

Embodiments of the present invention will now be described, by way of non-limiting example, with reference to the drawings in which:

Figure 1 is a block diagram of the conventional ADRC speed-regulation system;

Figure 2 shows the effect of τ on the conventional ESO and the ADRC system;

Figure 3 is a block diagram of the modified ADRC speed-regulation system;

Figure 4 shows the effect of τ on the modified ESO and the modified ADRC system;

Figure 5 shows root locus as τ varies ($\omega_{o2} = 500$ rad/s);

Figure 6 is a block diagram of the proposed ESO-based ADRC system;

Figure 7 shows dynamic performance of the proposed ESO and the ESO-based ADRC system under different ω_{f1} ;

- 6 -

Figure 8a shows root locus of the ESO-based ADRC system when $\omega_{o3} = 1500$ rad/s;

Figure 8b shows root locus of the ESO-based ADRC system when $\omega_{o3} = 2000$ rad/s;

Figure 9 is a block diagram of the proposed PLLO-based ADRC system;

Figure 10 illustrates dynamic performance of the proposed PLLO and the PLLO-based ADRC system under different ω_{f1} ;

Figure 11a illustrates root locus of the PLLO-based ADRC system when $k_{ps} = 300$ rad/s, where $\omega_{o3} = 300$ rad/s;

Figure 11b illustrates root locus of the PLLO-based ADRC system when $k_{ps} = 300$ rad/s, where $\omega_{o3} = 600$ rad/s;

Figure 12a compares four ADRC systems, showing command tracking performance;

Figure 12b compares the four ADRC systems, showing disturbance rejection performance;

Figure 12c compares the four ADRC systems, showing measurement noise suppression performance;

Figure 13a illustrates a block diagram of a test bench;

Figure 13b illustrates configuration of a test bench;

Figure 14 shows step response of the four ADRC systems under different bandwidth of the LPF;

- 7 -

Figure 15 illustrates disturbance rejection of the four ADRC systems under different bandwidth of the LPF;

Figure 16 compares the four systems when tracking step speed reference and rejecting different kinds of disturbances;

Figure 17 compares the four systems when tracking sinusoidal speed reference;

Figure 18 shows robustness of the proposed ESO-based ADRC system to inertia variation;

Figure 19 illustrates robustness of the proposed PLL-based ADRC system to inertia variation;

Figures 20a to 20c illustrate block diagrams of the six systems based on the third order ESO with regard to $3^{\text{rd}}\text{ESO}_0$ (Figure 20a), $3^{\text{rd}}\text{ESO}_1 - 2$ (Figure 20b), $3^{\text{rd}}\text{ESO}_3 - 5$ (Figure 20c);

Figure 21 illustrates an equivalent block diagram of the ADRC system;

Figure 22 illustrates a block diagram of different ADRC systems in transfer function form;

Figure 23a illustrates a bode diagram of the six third order ESOs when observing speed with respect to disturbance rejection property;

Figure 23b illustrates a bode diagram of the six third order ESOs when observing speed with respect to measurement noise suppression performance;

- 8 -

Figure 24a illustrates a bode diagram of the six third order ESOs when observing disturbance with respect to disturbance rejection property;

Figure 24b illustrates a bode diagram of the six third order ESOs when observing disturbance with respect to measurement noise suppression performance;

Figure 25a illustrates a bode diagram of $G_{fdb_speed}(S)$;

Figure 25b illustrates a bode diagram of $G_{fdb_disturbance}(S)$;

Figure 26a illustrates a bode diagram of the six systems under the same p_0 with respect to disturbance rejection property;

Figure 26b illustrates a bode diagram of the six systems under the same p_0 with respect to measurement noise suppression performance;

Figure 27a illustrates a bode diagram of the six systems under different p_0 with respect to disturbance rejection property;

Figure 27b illustrates a bode diagram of the six systems under different p_0 with respect to measurement noise suppression performance;

Figures 28a to 28d illustrate block diagrams of the ten ADRC systems based on the fourth order ESO with regard to system $4^{th}ESO_0$ (Figure 28a), $4^{th}ESO_1 - 2$ (Figure 28b), $4^{th}ESO_3 - 5$ (Figure 28c); $4^{th}ESO_6 - 9$ (Figure 28d);

Figure 29a illustrates a bode diagram of the ten fourth order ESOs for observing speed with respect to disturbance rejection property;

Figure 29b illustrates a bode diagram of the ten fourth order ESOs for observing speed with respect to measurement noise suppression performance;

- 9 -

Figure 30a illustrates a bode diagram of the ten fourth order ESOs for observing disturbance with respect to disturbance rejection property;

Figure 30b illustrates a bode diagram of the ten fourth order ESOs for observing disturbance with respect to measurement noise suppression performance;

Figure 31a illustrates a bode diagram of the ten systems under the same p_0 with respect to disturbance rejection property;

Figure 31b illustrates a bode diagram of the ten systems under the same p_0 with respect to measurement noise suppression performance;

Figure 32a illustrates a bode diagram of the five systems under different p_0 with respect to disturbance rejection property;

Figure 32b illustrates a bode diagram of the five systems under different p_0 with respect to measurement noise suppression performance;

Figure 33 illustrates an equivalent block diagram of the system 4thESO₁ – 2;

Figures 34a to 34d illustrate effect of k_b on the system 4thESO₁ with regard to root locus (Figure 34a), tracking performance (Figure 34b), disturbance rejection ability (Figure 34c); measurement noise suppression (Figure 34d);

Figures 35a to 35d illustrate effect of k_b on the system 4thESO₄ with regard to root locus (Figure 35a), tracking performance (Figure 35b), disturbance rejection ability (Figure 35c); measurement noise suppression (Figure 35d);

- 10 -

Figures 36a to 36d illustrate effect of k_b on the system 4thESO_8 with regard to root locus (Figure 36a), tracking performance (Figure 36b), disturbance rejection ability (Figure 36c); measurement noise suppression (Figure 36d);

Figure 37a illustrates a block diagram of a test bench;

Figure 37b illustrates configuration of a test bench;

Figures 38a to 38f illustrate tracking performance of the six performance for step reference with regard to 3rdESO_1 (Figure 38a), 3rdESO_2 (Figure 38b), 3rdESO_3 (Figure 38c), 3rdESO_4 (Figure 38d), 3rdESO_5 (Figure 38e) and 4thESO_4 (Figure 38f);

Figures 39a to 39f illustrate the disturbance rejection property of the six systems for step load with regard to 3rdESO_1 (Figure 39a), 3rdESO_2 (Figure 39b), 3rdESO_3 (Figure 39c), 3rdESO_4 (Figure 39d), 3rdESO_5 (Figure 39e) and 4thESO_4 (Figure 39f);

Figures 40a to 40f illustrate the disturbance rejection property of the six systems for ramp load with regard to 3rdESO_1 (Figure 40a), 3rdESO_2 (Figure 40b), 3rdESO_3 (Figure 40c), 3rdESO_4 (Figure 40d), 3rdESO_5 (Figure 40e) and 4thESO_4 (Figure 40f);

Figures 41a to 41c illustrate the disturbance rejection property of the three systems for low-speed operation at 10 rpm regarding 3rdESO_1 (Figure 30a), at 10 rpm regarding 3rdESO_4 (Figure 30b) and at 10 rpm regarding 4thESO_4 (Figure 30c); and

Figure 42 shows tracking performance of the three systems for step speed reference under different k_b .

Detailed description

- 11 -

The present invention relates to active disturbance rejection controllers (ADRC). In a permanent magnet synchronous motor speed regulation system, the dynamic performance of the conventional ADRC system is deteriorated by using a low bandwidth speed filter. To solve this problem, two ADRC controllers that consider speed measurement noise are described. One proposed ADRC system is based on an extended state observer (ESO). Another proposed ADRC system is based on a phase-locking loop observer (PLLO). In the proposed two ADRC systems, an integrator is employed as the speed filter so that the measured position can be directly used for observing the speed and disturbance without calculating speed. Meanwhile, by using an integrator as the speed filter, the dynamic performance of the proposed ESO-based ADRC system is not affected since it is independent of the speed filter, and the proposed PLLO-based ADRC system has a better rejection ability for low-frequency disturbance.

Also disclosed are ADRC controllers based on a fourth-order extended state observer. In applications such as the elevator and the robotic arm control and measurement, high disturbance rejection ability is needed for low-frequency disturbance. In an ADRC system, the third-order ESO is generally adopted to estimate the speed and the disturbance simultaneously based on the position feedback. However, a classical ADRC system based on the third-order ESO can only reject the constant disturbance. Though a phase locked-loop observer can be employed for enhancing the rejection ability for low-frequency disturbance, i.e. rejecting the ramp disturbance, it deteriorates measurement noise suppression performance. To counter this, a fourth-order ESO is proposed for the ADRC system based on the position feedback so that better low-disturbance rejection ability and high-frequency measurement noise suppression performance can be achieved. The proposed ADRC system can reject both constant disturbance and ramp disturbance. Meanwhile, it has the same high frequency measurement noise suppression performance as an

- 12 -

ADRC system based on a classical third-order ESO. The effectiveness of the proposed method is experimentally verified.

Contributions of the present methods and systems include the following. First, a general LPF is built to regard the integrator as a special case of LPF, and the effect of the LPF in the conventional ADRC system and modified ADRC system is analyzed. Second, an ESO and PLL considering the speed filter are proposed to have a good observation of the speed and disturbance. The two proposed ADRC systems are compared with the conventional ADRC system and the modified ADRC system. Moreover, five possible combinations of speed feedback and disturbance feedforward affect for system dynamics are analyzed in ADRC systems based on position feedback. The proposed fourth-order ESO-based ADRC system shows both good low-frequency disturbance rejection ability and nice high-frequency measurement noise suppression performance. It is thus very suitable for applications such as in elevators and robotic arms, where high disturbance rejection ability for the low-frequency disturbance is required.

The present disclosure first discusses conventional ADRC speed regulation system. Among many topologies of electric machines, surface mounted permanent magnet synchronous motor (PMSM) has been widely used in servo applications because of its advantageous features including high efficiency, high-power density, large torque-to-inertia ratio, low noise, and free maintenance. The motion equation of the PMSM system can be given as:

$$\dot{\Omega} = \frac{-B\Omega + T_e - T_L}{J} \quad (1)$$

where Ω is the mechanical angular velocity, B is the viscous friction torque coefficient, J is the moment of inertia, T_e is the electromagnetic torque, and T_L is the load torque.

The feedback control law is designed to obtain the proper control quantity, i.e., the torque reference, so that the speed reference tracking error can attenuate

- 13 -

as desired. As a result, the motion equation should be modified as a function of the torque reference T_e^*

$$\dot{\Omega} = \frac{T_e^* - (T_e^* - T_e + B\Omega + T_L)}{J} = bT_e^* + d \quad (2)$$

where $b = 1/J$ is the control gain, $d = -(T_e^* - T_e + B\Omega + T_L)/J$ is the total disturbance caused by the electromagnetic torque tracking error, the viscous friction torque, and the load torque.

Defining the reference of mechanical angular velocity as Ω^* , then the tracking error of mechanical angular velocity can be expressed as $e_s = \Omega^* - \Omega$, resulting in:

$$\dot{e}_s = \dot{\Omega}^* - \dot{\Omega} = \dot{\Omega}^* - bT_e^* - d \quad (3)$$

Adopting a linear feedback control law:

$$\dot{e}_s = -k_{ps}e_s \quad (4)$$

where k_{ps} is the proportional gain. Combining the above two equations yields:

$$T_e^* = \frac{\dot{\Omega}^* + k_{ps}(\Omega^* - \Omega) - d}{b} \quad (5)$$

In the above equation, the mechanical angular velocity Ω and the disturbance d are generally unknown and usually substituted by their estimated value $\hat{\Omega}$ and $\hat{d}$, thus the control quantity of the system is modified as:

$$T_e^* = \frac{\dot{\Omega}^* + k_{ps}(\Omega^* - \hat{\Omega}) - \hat{d}}{b} \quad (6)$$

Considering that the actual system cannot provide infinite control output, saturation limit is usually applied as follows:

$$T_{esat}^* = \begin{cases} T_{emax}^* \text{sgn}(T_e^*), & |T_e^*| > T_{emax}^* \\ T_e^*, & |T_e^*| \leq T_{emax}^* \end{cases} \quad (7)$$

where T_{emax}^* is the torque limit.

In some embodiments, the estimated speed value may be determined based on the output of the proportional-integral-derivative controller of the ESO

- 14 -

module. Without considering the saturation of the torque, the output speed when using the modified control quantity can be expressed as:

$$\Omega(s) = \Omega^*(s) - \frac{k_{ps}[\hat{\Omega}(s) - \Omega(s)] + [\hat{d}(s) - d(s)]}{s + k_{ps}} \quad (8)$$

It shows that the output speed tracks the command speed perfectly if the estimated speed and the estimated disturbance converge to their actual value.

In speed control systems, good speed-reference tracking and excellent load-torque rejection ability cannot be obtained simultaneously by using a conventional one-degree-of-freedom controller. Two-degree-of-freedom (TDOF) systems can be used for robust control, such as disturbance/uncertainty estimation and attenuation (DUEA) techniques of which extended state observer (ESO) is one.

Generally, the rotor position is obtained from the position sensors such as the encoder or resolver, and the speed is calculated by the derivative of the position. Due to the quantization error in the measurement of the position, the speed calculated by the classical frequency method is contaminated by a lot of noise, especially in the low-speed range. Denoting δ_n as the speed measurement noise, thus the measured speed can be expressed as $\Omega^m = \Omega + \delta_n$. The analysis of the speed measurement noise caused by the quantization error can be found. In order to attenuate the measurement noise, a small bandwidth LPF is usually adopted in the speed feedback.

Suppose the feedback speed is filtered by a first-order LPF, whose transfer function can be expressed by:

$$G_{\text{LPF}}(s) = \frac{1}{\tau s + 1} = \frac{\omega_f}{s + \omega_f} \quad (9)$$

where τ and ω_f are the time constant and the cut-off frequency of the LPF, respectively.

- 15 -

In the conventional design process of the ESO, the speed filter is not considered. A classical linear second-order ESO for the system can be constructed as:

$$\begin{aligned}\tilde{\Omega} &= \Omega^m - \hat{\Omega} = \Omega + \delta_n - \hat{\Omega} \\ \dot{\hat{\Omega}} &= bT_e^* + \hat{d} + k_1\tilde{\Omega} \\ \dot{\hat{d}} &= k_2\tilde{\Omega}\end{aligned}\quad (10)$$

where $\tilde{\Omega}$ is the speed observation error, and k_1 and k_2 are the observer gains.

Linear active disturbance rejection controllers for PMSM speed regulation system are first discussed with reference to speed filtering. In particular, presently disclosed is an example ADRC system 100 as shown in Figure 1, comprising an ESO module 102. The ESO module 102 is configured to receive input information 104. The input information 104 comprises a torque reference signal for driving a motor, and an angular position that is a measured output of the motor. The ESO module 102 then determines, based on the input information, an estimated speed value 106 and an estimated disturbance value 108. The estimated speed value 106 is based on a derivative of the angular position, and the estimated disturbance value 108 is an output of a proportional-integral (PI) controller 110 of the ESO module 102. The estimated speed value 106 is then sent as a feedback control signal, and the estimated disturbance value 108 as a feedforward control signal.

The block diagram of the ADRC speed-regulation system is shown in Figure 1, where Ω_f^m is the filtered speed, and there is:

$$\Omega_f^m(s) = \frac{1}{\tau s + 1} \Omega^m(s) = G_{LPF}(s) \Omega^m(s) \quad (11)$$

From (10) and Figure 1, the estimated speed 106 and the estimated disturbance 108 can be deduced as:

$$\begin{cases} \hat{\Omega}(s) = \frac{\Delta_2 - s^2}{\Delta_2} \Omega_f^m(s) + \frac{s}{\Delta_2} bT_e^*(s) \\ \hat{d}(s) = \frac{k_2 s}{\Delta_2} \Omega_f^m(s) - \frac{k_2}{\Delta_2} bT_e^*(s) \end{cases} \quad (12)$$

- 16 -

where the characteristic polynomial $\Delta_2 = s^2 + k_1s + k_2$ can be set as $\Delta_2 = (s + \omega_{o2})^2$, ω_{o2} is the natural frequency of the second-order ESO, then the gains in the ESO can be calculated by $k_1 = 2\omega_{o2}$, $k_2 = \omega_{o2}^2$.

Substituting (5) into (12) yields:

$$\begin{cases} \hat{\Omega}(s) = \frac{\Delta_2 + \tau s^3}{\Delta_2(\tau s + 1)} [\Omega(s) + \delta_n(s)] - \frac{s}{\Delta_2} [d(s) + s\delta_n(s)] \\ \hat{d}(s) = -\frac{k_2 \tau s^2}{\Delta_2(\tau s + 1)} [\Omega(s) + \delta_n(s)] + \frac{k_2}{\Delta_2} [d(s) + s\delta_n(s)] \end{cases} \quad (13)$$

This shows that the adoption of the speed filter leads to an uncorrected speed estimation. Notably, the estimated disturbance is affected by the actual speed. However, the effect is very small because it is proportional to the second order derivative of the actual speed. When $\omega_{o2} = 800 \text{ rad/s}$, the speed estimation performance ($\hat{\Omega}/\Omega$), the disturbance estimation performance ($\hat{d}/d$), and the measurement noise suppression performance ($\hat{\Omega}/\delta_n$) of the conventional ESO are shown in Figure 2. It can be seen that the measurement noise suppression performance can be improved from -20 to -40 dB/dec, while the speed estimation performance is deteriorated.

Similarly, by substituting (13) into (8), the transfer functions from the speed reference to the output speed, from the disturbance to the output speed, and from the measurement noise to the output speed can be deduced as:

$$\begin{cases} \Phi_r(s) = \frac{\Omega(s)}{\Omega^*(s)} = \frac{\Delta_2(s + k_{ps})(\tau s + 1)}{\Delta_1^{\text{cl}}} \\ \Phi_d(s) = \frac{\Omega(s)}{d(s)} = \frac{s(s + k_1 + k_{ps})(\tau s + 1)}{\Delta_2^{\text{cl}}} \\ \Phi_n(s) = -\frac{\Omega(s)}{\delta_n(s)} = \frac{k_{ps}(\Delta_2 - s^2) + k_2 s}{\Delta_2^{\text{cl}}} \end{cases} \quad (14)$$

where Δ_2^{cl} is the characteristic polynomial of the closed-loop control system:

$$\Delta_2^{\text{cl}} = \tau s^3(s + k_1 + k_{ps}) + \Delta_2(s + k_{ps}) \quad (15)$$

- 17 -

For a smooth speed reference for which the derivative is not so large, e.g., the sinusoidal speed reference or the parabolic speed reference, the derivative feedforward can work normally. Therefore, the transfer function from the speed reference to the output speed is written as $\Phi_{r\text{-sin}}(s)$. For the step speed reference, however, the derivative feedforward is blocked due to the saturation of the control quantity, thus the transfer function from the step speed reference to the output speed is deduced as:

$$\Phi_{r\text{-step}}(s) = \frac{\Omega(s)}{\Omega^*(s)} = \frac{\Delta_2 k_{ps}(\tau s + 1)}{\Delta_2^{cl}} \quad (16)$$

When $k_{ps} = 300$ and $\omega_{o2} = 800\text{rad/s}$, the speed estimation performance (Ω/Ω), the disturbance estimation performance ($\hat{d}/d$), and the measurement noise suppression performance ($\hat{n}/\delta_n$) of the closed-loop control system are also shown in Figure 2. Here, the measurement noise suppression performance of the system can be improved from -40 to -60 dB/dec by adopting the speed filter, however, it leads to a deteriorated tracking performance for the medium-frequency command. As a result, the rejection property for the medium-frequency disturbance is weakened at the same time.

Now with regard to the modified ADRC: the estimated speed value may be determined at least in part based on the second LPF, and the second LPF may be of the same form as the first LPF, and the estimated disturbance value may be selectable between an output of the integral controller and the output of the proportional-integral controller of the ESO module. From (8) and (13), it is evident that the poor dynamic performance is caused by the wrong estimation of the speed.

Figure 3 shows the block diagram of the modified ADRC speed-regulation system. Presently disclosed is another example ADRC system 300 as shown in Figure 3. The ADRC system 300 includes an ESO module 302 configured to receive input information 304. The input information 304 includes a torque reference signal for driving a motor, and an angular position that is a measured

- 18 -

output of the motor. The ESO module 302 determines, based on the input information, an estimated speed value 306 and an estimated disturbance value 308. The estimated speed value 306 is based on a derivative of the angular position, and the estimated disturbance value 308 is an output of a proportional-integral (PI) controller 310 of the ESO module 302. Of those estimates, the estimated speed value 306 is sent as a feedback control signal, and the estimated disturbance value 308 as a feedforward control signal.

In the system, the modified ESO is expressed as:

$$\begin{cases} \tilde{\Omega} = \Omega^m - \hat{\Omega} \\ \dot{\tilde{\Omega}}_f = -\tilde{\Omega}_f + \tilde{\Omega} \\ \dot{\hat{\Omega}} = bT_e^* + \hat{d} + k_1\tilde{\Omega}_f \\ \dot{\hat{d}} = k_2\tilde{\Omega}_f \end{cases} \quad (17)$$

where $\tilde{\Omega}_f$ is the filtered observation error.

Similarly, the observed states can be deduced as:

$$\begin{cases} \hat{\Omega}(s) = \Omega(s) + \delta_n(s) - \frac{s(\tau s + 1)}{\Delta_3} [d(s) + s\delta_n(s)] \\ \hat{d}(s) = \frac{k_2}{\Delta_3} [d(s) + s\delta_n(s)] \end{cases} \quad (18)$$

where the characteristic polynomial $\Delta_3 = \tau s^3 + s^2 + k_1 s + k_2$.

It shows that the speed can be properly estimated, whereas the disturbance observation is affected by the LPF. When $\omega_{o2} = 800$ rad/s, the speed estimation performance (Ω/Ω), the disturbance estimation performance (d/d), and the measurement noise suppression performance (Ω/δ_n) of the conventional ESO are shown in Figure 4. Similarly, the suppression performance for the high-frequency measurement noise can be improved from -20 to -40 dB/dec.

By substituting (18) into (8), the transfer functions from the sinusoidal speed reference to the output speed, from the step speed reference to the output speed, from the disturbance to the output speed, and from the measurement noise to the output speed can be deduced as:

- 19 -

$$\begin{cases} \Phi_{r_sin}(s) = \frac{\Omega(s)}{\Omega^*(s)} = 1 \\ \Phi_{r_step}(s) = \frac{\Omega(s)}{\Omega^*(s)} = \frac{k_{ps}}{s+k_{ps}} \\ \Phi_d(s) = \frac{\Omega(s)}{d(s)} = \frac{k_{ps}s(\tau s+1)+\Delta_3-k_2}{(s+k_{ps})\Delta_3} \\ \Phi_n(s) = -\frac{\Omega(s)}{\delta_n(s)} = \frac{k_{ps}(k_1s+k_2)+k_2s}{(s+k_{ps})\Delta_3} \end{cases} \quad (19)$$

It can be seen that the tracking performance is not affected by the time constant of the LPF. When $k_{ps} = 300$, $\omega_{o2} = 800\text{rad/s}$, the disturbance rejection property (Ω/d), and the measurement noise suppression performance (Ω/δ_n) of the closed-loop control system are also shown in Figure 4. It shows that the measurement noise suppression performance is improved from -40 to -60 dB/dec. However, it is found that the medium-frequency disturbance is amplified. The larger the time constant of the LPF is, the larger the amplification is, which leads to the instability of the system.

According to the Routh-Hurwitz stability criterion, the condition for a stable operation is $\tau < k_1/k_2 = 2/\omega_{o2}$, i.e., $\omega_f > \omega_{o2}/2$ as verified by the root locus shown in Figure 5. Therefore, the bandwidth of the LPF and the bandwidth of the ESO should be carefully set to ensure a stable operation.

Here, a novel ADRC speed regulation system is described that considers the speed filter. In order to have a general discussion of the filter, the transfer function of the LPF is modified as:

$$G_{LPF}(s) = \frac{\omega_{f2}}{s + \omega_{f1}} \quad (20)$$

where ω_{f1} and ω_{f2} are the gains. When $\omega_{f1} = \omega_{f2} = \omega_f$, the LPF is the same first-order LPF as the one shown by (9). When $\omega_{f1} = 0$ and $\omega_{f2} = 1$, however, the LPF is equivalent to an integrator, which can be seen as a general LPF.

Extending the filtered speed Ω_f as a new state, then the state equation of the filtered speed can be expressed as:

- 20 -

$$\begin{cases} \dot{\Omega}_f = -\omega_{f1}\Omega_f + \omega_{f2}\Omega \\ \dot{\Omega} = bT_e^* + d \end{cases} \quad (21)$$

A classical linear ESO for the system expressed by (21) can be constructed as:

$$\begin{cases} \tilde{\Omega}_f = \Omega_f^m - \hat{\Omega}_f = \Omega_f + \delta_{nf} - \hat{\Omega}_f \\ \dot{\tilde{\Omega}}_f = -\omega_{f1}\tilde{\Omega}_f + \omega_{f2}(\hat{\Omega} + k_0\tilde{\Omega}_f) \\ \dot{\hat{\Omega}} = bT_e^* + \hat{d} + k_1\tilde{\Omega}_f \\ \dot{\hat{d}} = k_2\tilde{\Omega}_f \end{cases} \quad (22)$$

where $\Delta_3 = s^3 + \omega_{f1}s^2 + \omega_{f2}(k_0s^2 + k_1s + k_2)$ is the characteristic polynomial, k_0, k_1 , and k_2 are the observer gains.

The block diagram of the proposed ADRC system considering the speed filter is shown in Figure 6, which shows the block diagram of the modified ADRC speed-regulation system. Figure 6 shows an alternative ADRC system 600 that includes an ESO module 602. The ESO module 602 is configured to receive input information 604 comprising a torque reference signal for driving a motor, and a speed value that is a measured output of the motor passed to a first low-pass filter (LPF) 610. The ESO module 602 determines, based on the input information 604, an estimated speed value 606 and an estimated disturbance value 608. The estimated speed value 606 is sent as a feedback control signal, and the estimated disturbance value 608 as a feedforward control signal. In this embodiment the estimated speed value 606 can be determined based, at least in part, on a second LPF 612 that is of the same form as the first LPF 610, and the estimated disturbance value 608 is selectable between an output of an integral controller 614 and an output of a proportional-integral controller 616 of the ESO module 602. Alternatively, the estimated speed value 606 can be determined based on the output of a proportional-integral-derivative controller 618 of the ESO module 602, and the estimated disturbance value 608 is selectable between: an output of an integral controller 614 of the ESO module 602; and an output of a proportional-integral controller 616 of the ESO module 602; and the output of the proportional-integral-derivative controller 618 of the ESO module 602.

- 21 -

It can be seen that the difference between the proposed ADRC system and the modified ADRC system is the adoption of the gain k_0 . When $k_0 = 0$, the proposed ADRC system is equivalent to the modified ADRC system since the two LPF can be moved to the observation error path. As a result, the modified ADRC system is a subset of the proposed ESO-based ADRC.

From (22) and Figure 6, the estimated speed and the estimated disturbance can be deduced as:

$$\begin{cases} \hat{\Omega}(s) = \Omega(s) + \delta_n(s) - \frac{s(s + \omega_{f1} + k_0\omega_{f2})}{\Delta_3} [d(s) + s\delta_n(s)] \\ \hat{d}(s) = \frac{k_2\omega_{f2}}{\Delta_3} [d(s) + s\delta_n(s)] \end{cases} \quad (23)$$

Substituting (23) into (6), the transfer functions from the sinusoidal speed reference to the output speed, from the step speed reference to the output speed, from the disturbance to the output speed, and from the measurement noise to the output speed can be deduced as:

$$\begin{cases} \Phi_{r_sin}(s) = \frac{\Omega(s)}{\Omega^*(s)} = 1 \\ \Phi_{r_step}(s) = \frac{\Omega(s)}{\Omega^*(s)} = \frac{k_{ps}}{s + k_{ps}} \\ \Phi_d(s) = \frac{\Omega(s)}{d(s)} = \frac{(s + k_{ps})s(s + \omega_{f1} + k_0\omega_{f2}) + k_1\omega_{f2}s}{(s + k_{ps})\Delta_3} \\ \Phi_n(s) = -\frac{\Omega(s)}{\delta_n(s)} = \frac{k_{ps}\omega_{f2}(k_1s + k_2) + k_2\omega_{f2}s}{(s + k_{ps})\Delta_3} \end{cases} \quad (24)$$

By setting the characteristic polynomial of the observer as $\Delta_3 = (s + \omega_{o3})^3$, the gain in the ESO can be obtained as:

$$\begin{cases} k_0 = \frac{(3\omega_{o3} - \omega_{f1})}{\omega_{f2}} \\ k_1 = \frac{3\omega_{o3}^2}{\omega_{f2}} \\ k_2 = \frac{\omega_{o3}^3}{\omega_{f2}} \end{cases} \quad (25)$$

- 22 -

where ω_{o3} is the undamped natural frequency of the observer.

Consequently, the system output when the reference derivative feedforward works normally can be simplified as:

$$\Omega(s) = \Omega^*(s) - \delta_n(s) + \frac{(s + k_{ps})s(s + 3\omega_{o3}) + 3\omega_{o3}^2 s}{(s + k_{ps})\Delta_3} [d(s) + s\delta_n(s)] \quad (26)$$

From (26), it can be seen that the command tracking performance, the disturbance rejection ability, and the measurement noise suppression performance are all independent of the bandwidth of LPF, which is verified by the bode diagrams shown in Figure 7, where $k_{ns} = 300, \omega_{o3} = 800\text{rad/s}$. From (25), it can also be found that the gain k_0 should vary with the bandwidth of the LPF. However, k_0 in the modified ADRC system is always kept as zero.

It should be pointed out that the filtered speed is actually the integration of the speed or the mechanical angle θ_m when $\omega_{f1} = 0\text{rad/s}$ and $\omega_{f2} = 1\text{rad/s}$. Since the mechanical angle can be measured directly by using the position sensor, it is the best choice to set the LPF to be an integrator. In this case, the state equation of the PMSM system expressed by (21) is modified as:

$$\begin{cases} \dot{\theta}_m = \Omega \\ \dot{\Omega} = bT_e^* + d \end{cases} \quad (27)$$

and the gains in the ESO expressed by (25) is also simplified as:

$$\begin{cases} k_1 = 3\omega_{o3} \\ k_2 = 3\omega_{o3}^2 \\ k_3 = \omega_{o3}^3 \end{cases} \quad (28)$$

Considering the variation of system inertia, nominal inertia J_n and the nominal control gain b_n in the control system may be different from their actual value. Denoting $k_b = b_n/b = J/J_n$ as the control gain ratio, (24) can be modified as:

- 23 -

$$\begin{cases} \Phi_{r-\sin}(s) = \frac{\Omega(s)}{\Omega^*(s)} = \frac{\Delta_3(s+k_{ps})}{\Delta_3(s+k_{ps})+(k_b-1)sG_3(s)} \\ \Phi_{r-\text{step}}(s) = \frac{\Omega(s)}{\Omega^*(s)} = \frac{k_{ps}\Delta_3}{\Delta_3(s+k_{ps})+(k_b-1)sG_3(s)} \\ \Phi_d(s) = \frac{\Omega(s)}{d(s)} = \frac{k_b G_3(s)}{\Delta_3(s+k_{ps})+(k_b-1)sG_3(s)} \\ \Phi_n(s) = -\frac{\Omega(s)}{\delta_n(s)} = \frac{\Delta_3(s+k_{ps})-sG_3(s)}{\Delta_3(s+k_{ps})+(k_b-1)sG_3(s)} = 1 - s\Phi_d(s) \end{cases} \quad (29)$$

where the transfer function $G_3(s)$ is expressed as:

$$\begin{aligned} G_3(s) &= s[(s+k_{ps})(s+\omega_{f1}+k_0\omega_{f2})+\omega_{f2}k_1] \\ &= s[(s+k_{ps})(s+3\omega_{o3})+3\omega_{o3}^2] \end{aligned} \quad (30)$$

It can be seen that $G_3(s)$ is independent of the LPF bandwidth. The characteristic polynomial of the closed-loop control system is expressed as:

$$\Delta_3(s+k_{ps})+(k_b-1)sG_3(s)=0 \quad (31)$$

Rewriting the characteristic polynomial in the manner:

$$1+k_b \frac{sG_3(s)}{\Delta_3(s+k_{ps})-sG_3(s)}=0 \quad (32)$$

The condition for a stable operation can be deduced according to the Routh-Hurwitz stability criterion, as expressed by (33):

$$k_b > \frac{\omega_{o3}(3k_{ps}+\omega_{o3})^2}{(8k_{ps}^2+9k_{ps}\omega_{o3}+3\omega_{o3}^2)(k_{ps}+3\omega_{o3})} \quad (33)$$

When $k_{ps} = 300$, ω_{o3} in the ESO-based ADRC system is 1500 and 2000 rad/s, the condition for a stable operation is $k_b > 0.156$ and $k_b > 0.147$, respectively, which is also verified by the root locus shown in Figure 8.

From the above analysis, it can be seen that the LPF bandwidth has no effect on the stability and the dynamic performance of the proposed ESO-based ADRC system.

- 24 -

Now with reference to a PLLO-Based ADRC system that considers the speed filter: the ADRC system has different dynamics when different combinations of the speed feedback and disturbance feedforward are employed. Similarly, PLLO considering the speed filter is designed for observing the speed and the disturbance and the ADRC system based on the PLLO is analyzed.

The block diagram of the PLLO-based ADRC system is shown in Figure 9, which shows the block diagram of the modified ADRC speed-regulation system. Figure 9 shows an ADRC system 900 including a PLLO module 902 that is configured to receive input information 904 comprising a torque reference signal for driving a motor, and a speed value that is a measured output of the motor passed to a first low-pass filter (LPF) 910. The PLLO module 902 then determines, based on the input information 904, an estimated speed value 906 and an estimated disturbance value 908. The estimated speed value 906 is sent as a feedback control signal, and the estimated disturbance value 908 as a feedforward control signal. On the one hand, the estimated speed value 906 can be determined, at least in part, based on a second LPF 912 that is of the same form as the first LPF 910, and the estimated disturbance value 908 is selectable between an output of an integral controller 914 and an output of a proportional-integral controller 916 of the PLLO module 902. On the other hand, the estimated speed value 906 can be determined based on the output of a proportional-integral-derivative controller 918 of the PLLO module 902, and the estimated disturbance value 908 is selectable between: an output of an integral controller 914 of the PLLO module 902; and an output of a proportional-integral controller 916 of the PLLO module 902; and the output of the proportional-integral-derivative controller 918 of the PLLO module 902.

The difference between the PLLO-based ADRC system and the ESO-based ADRC system is the combination of speed feedback and disturbance feedforward. The state equation of the PLLO for the system expressed by (21) can be expressed as:

- 25 -

$$\begin{cases} \tilde{\Omega}_f = \Omega_f^m - \hat{\Omega}_f \\ \dot{\tilde{\Omega}}_f = -\omega_{f1}\tilde{\Omega}_f + \omega_{f2}\hat{\Omega} \\ \dot{\tilde{\Omega}} = bT_e^* + \hat{d} + k_0\tilde{\Omega}_f \\ \dot{\hat{d}} = k_1\tilde{\Omega}_f + k_2\tilde{\Omega}_f \end{cases} \quad (34)$$

It can be easily proved that the enhanced ADRC system has the same characteristic polynomial as the proposed ADRC system. The estimated speed and the estimated disturbance from the PLLO can be deduced as:

$$\begin{cases} \hat{\Omega}(s) = \Omega(s) + \delta_n(s) - \frac{s(s+\omega_{f1})}{\Delta_3} [d(s) + s\delta_n(s)] \\ \hat{d}(s) = \frac{(k_1s+k_2)\omega_{f2}}{\Delta_3} [d(s) + s\delta_n(s)] \end{cases} \quad (35)$$

Substituting (35) into (6), the transfer functions from the sinusoidal speed reference to the output speed, from the step speed reference to the output speed, from the disturbance to the output speed, and from the measurement noise to the output speed can be deduced as:

$$\begin{cases} \Phi_{r_sin}(s) = \frac{\Omega(s)}{\Omega^*(s)} = 1 \\ \Phi_{r_step}(s) = \frac{\Omega(s)}{\Omega^*(s)} = \frac{k_{ps}}{s + k_{ps}} \\ \Phi_d(s) = \frac{\Omega(s)}{d(s)} = \frac{(s + k_{ps})s(s + \omega_{f1}) + \omega_{f2}k_0s^2}{(s + k_{ps})\Delta_3} \\ \Phi_n(s) = -\frac{\Omega(s)}{\delta_n(s)} = \frac{k_{ps}[\Delta_3 - s^2(s + \omega_{f1})] + (k_1s + k_2)\omega_{f2}s}{(s + k_{ps})\Delta_3} \end{cases} \quad (36)$$

When $k_{ps} = 300, \omega_{o3} = 800\text{rad/s}$, the bode diagrams of the PLLO-based ADRC system under different ω_{f1} is shown in Figure 10. It can be seen that the system dynamic performance is affected by the bandwidth of the LPF. However, it is worth noting that the effect of the LPF in the PLLO-based ADRC system is different from that in the ESO-based ADRC system. The reduced LPF bandwidth degrades the high-frequency measurement noise suppression performance but enhances the low-frequency disturbance rejection ability in the PLLO-based ADRC system.

- 26 -

Specially, when $\omega_{f1} = 0\text{rad/s}$ and $\omega_{f2} = 1\text{rad/s}$, the system ability for rejecting the low-frequency disturbance can be enhanced from 20 to 40 dB/dec, i.e., the PLLO-based system is able to reject the ramp disturbance, as can see from (36). As a result, it is also the best choice to set the LPF to an integrator in the PLLO-based ADRC system.

Considering the variation of system inertia, the transfer functions expressed by (36) can be modified as the same as the those represented by (29), with the exception of the transfer function $G_3(s)$, which is expressed as:

$$\begin{aligned} G_3(s) &= s[(s + k_{ps})(s + \omega_{f1}) + \omega_{f2}k_0s] \\ &= s^2(s + 3\omega_{o3}) + k_{ps}s(s + \omega_{f1}) \end{aligned} \quad (37)$$

It shows that $G_3(s)$ is affected by the LPF bandwidth. When $\omega_{f1} = \omega_{f2} = \omega_f \neq 0$, the condition for a stable operation becomes:

$$\begin{cases} k_b > \frac{K_2(K_1K_0 - K_2)}{k_{ps}(\omega_{o3}^3K_0 - \omega_fK_2)K_0} \omega_f > \frac{\omega_{o3}^3K_0}{K_2} \\ k_b < \frac{K_2(K_1K_0 - K_2)}{k_{ps}(\omega_{o3}^3K_0 - \omega_fK_2)K_0} \omega_f \leq \frac{\omega_{o3}^3K_0}{K_2} \end{cases} \quad (38)$$

where K_0, K_1 , and K_2 are expressed as:

$$\begin{cases} K_0 = k_{ps} + \omega_f(k_0 + 1) = k_{ps} + 3\omega_{o3} \\ K_1 = \omega_f(k_{ps}k_0 + k_1) = k_{ps}(3\omega_{o3} - \omega_f) + 3\omega_{o3}^2 \\ K_2 = \omega_f(k_1k_{ps} + k_2) = \omega_{o3}^2(3k_{ps} + \omega_{o3}) \end{cases} \quad (39)$$

It should be pointed out that when the LPF bandwidth satisfies the following condition:

$$\frac{\omega_{o3}^3K_0}{K_2} < \omega_f < \frac{3\omega_{o3}(k_{ps} + \omega_{o3})K_0 - K_2}{K_0k_{ps}} \quad (40)$$

The condition for a stable operation is changed to $k_b > 0$, implying that the PLLO-based ADRC system is always stable regardless of the inertia variation.

- 27 -

When $\omega_{f1} = 0$ and $\omega_{f2} = 1\text{rad/s}$, the condition for a stable operation is a little bit different, i.e.:

$$k_b < \frac{8k_{ps}^2 + 9\omega_{o3}k_{ps} + 3\omega_{o3}^2}{k_{ps}(k_{ps} + 3\omega_{o3})} \quad (41)$$

When $k_{ps} = 300, \omega_{o3}$ in the PLL-based ADRC system is 300 and 600rad/s, respectively, the condition for a stable operation in different situations is shown in Table I, which states stable operating conditions for the PLL-based ADRC system in different situation. These results are verified by the root locus shown in Figure 11.

ω_{o3}	$\omega_n = \omega_{/2} = \omega_F = 2000$	$\omega_1 = \omega_{f2} = \omega_F = 1000$	$\omega_h = 0$ and $\omega_{/2} = 1$
300	$k_b > 0.294$	$k_b > -0.714$	$k_b < 5.00$
600	$k_b > -2.192$	$k_b > -22.143$	$k_b < 5.32$

Table I

From the above analysis, it can be seen that the LPF bandwidth affects both the dynamic performance and the stability of the proposed PLL-based ADRC system. Though the system rejection ability for low-frequency disturbance can be improved by employing an integrator as the LPF, the system's robustness and stability is deteriorated.

Presently, the conventional ADRC system ($\omega_f = 2000\text{rad/s}$), the modified ADRC system ($\omega_f = 2000\text{rad/s}$), the proposed ESO-based ADRC system ($\omega_{f1} = 0\text{rad/s}$ and $\omega_{f2} = 1\text{rad/s}$), and the conventional ADRC system ($\omega_{f1} = 0\text{rad/s}$ and $\omega_{f2} = 1\text{rad/s}$), are compared. Comparing Figure 10 with Figure 7, the present ESO-based ADRC system has a better measurement noise suppression performance than the present PLL-based ADRC system, while the latter performs better low-frequency disturbance rejection ability than the former.

As a result, in practice, the ESO's bandwidth is generally set to be higher than that of the PLLO.

When $k_{ps} = 300$, the undamped natural frequency in the four systems is set as 800, 800, 1000, and 300 rad/s, respectively, the bode diagram of the four ADRC systems are shown in Figure 12. It can be seen that the conventional ADRC system, the modified ADRC system, and the proposed ESO-based ADRC system have nearly the same disturbance rejection property and the measurement noise suppression performance. Nevertheless, the conventional ADRC system has a poor tracking performance. The proposed PLLO-based ADRC system has the best rejection ability for the low-frequency disturbance but the worst suppression performance for the medium-frequency disturbance and the high-frequency measurement noise.

Because both the proposed ESO-based ADRC system and the PLLO-based ADRC system have advantages, ADRC practitioners can choose between the two ADRC systems based on their application's requirements.

In order to verify the above analysis and the proposed methods, experimental results of the four ADRC systems are presented and analyzed. The specification of the servo motor is shown in Table II. As shown in Figure 13(a), the control strategy is based on space vector pulsewidth modulation control with $i_d^* = 0$. Decoupled PI regulators are employed in the current loop to control i_d and i_q , respectively, and different ADRC controllers are employed in the speed loop. The direct current (DC) bus voltage u_{dc} is 150 V. The saturation limit of q -axis reference current is 9 A. The bandwidths of d -axis and q -axis current-loops are 2000 rad/s. The resolution of the incremental encoder is 2500 pulses per revolution.

Symbol	Quantity	Symbol	Quantity
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- 29 -

Rated power P_N	0.75(kW)	Number of pole-pairs p_n	4
Rated voltage U_N	220(V)	D axis inductance L_d	8.0(mH)
Rated speed n_N	3000(rpm)	Q axis inductance L_q	8.0(mH)
Rated torque T_N	2.4(N · m)	Torque constant K_t	0.553(N · m/A)
Current Limit I_{smax}	10(A)	Motor inertia J_M	1.62 $\times 10^{-4}$ (kg · m ²)
Stator resistance R_s	1.1(Ω)	Motor system inertia J	4.44 $\times 10^{-4}$ (kg · m ²)

Table II

The configuration of the test bench is shown in Figure 13(b). A programmable dc power supply is used to provide 150 Vdc Bus voltage. There are two same PMSMs, one acts as a driving motor and the other acts as a generator. The driving inverter consists of dSPACE DS1103, Mitsubishi intelligent power module, current and voltage hall sensors, filtering and protection circuit, etc. Other circuitry may be used, depending on availability and application. The field-oriented control is employed for driving the PMSM.

Given a step speed reference from -1000 to 1000 rpm, when $k_{ps} = 300$, the undamped natural frequency of ESO in the conventional ADRC system and the modified ADRC are the same, i.e., $\omega_{o2} = 800\text{rad/s}$, while the one in the ESO-based ADRC system and the PLL0-based ADRC system is 1000 and 300 rad/s, respectively. The bandwidth of the LPF for speed feedback is set as 2000 and 500 rad/s, respectively. The speed responses of the four ADRC systems when tracking step speed reference and rejecting the load disturbance are shown in Figures 14 and 15, respectively. In order to have a clear understanding of the relationship between the observed speed and the actual speed, both the observed speed $n_{fdb_Observed}$ and the filtered speed n_{fdb_LPF} are given. Meanwhile,

- 30 -

step loading and ramp unloading are used to test the disturbance rejection ability under different kinds of load torque. The bandwidth of the LPF is set to 2000 rad/s so that the filtered speed can be treated as the actual speed in low dynamic situations.

It can be seen that with the decrease of the bandwidth of LPF, the conventional ADRC system is prone to oscillation. This verifies that the tracking performance of the conventional ADRC system is affected by the bandwidth of LPF. This further verifies that the modified ADRC system tends to be unstable, the estimated speed, the disturbance rejection property, and the measurement noise suppression of the proposed ESO-based ADRC system are all not affected by the bandwidth of the LPF. For the PLL-based ADRC system, the tracking performance is not affected by the bandwidth of the LPF. However, lower bandwidth is good for the disturbance rejection, while bad for the measurement noise suppression. Meanwhile, the higher the bandwidth is, the larger deviation between the observed speed and the actual speed is. All these results are identical to the theoretical results.

As analyzed before, the conventional ADRC system and the modified ADRC system also show a good dynamic performance when a high bandwidth ($\omega_f = 2000\text{rad/s}$) LPF is used. When $\omega_{f1} = 0\text{rad/s}$ and $\omega_{f2} = 1\text{rad/s}$, the proposed ESO-based ADRC system and the PLL-based ADRC system have their special advantages. Therefore, these four systems are compared in the experiments. The speed responses when tracking the step speed reference, rejecting the load disturbance, and rejecting the torque ripples in the low-speed range (30 and 100 rpm) are shown in Figure 16.

It can be seen from Figure 16 that all the four systems have a good tracking performance for step speed reference. The convention ADRC system, the modified ADRC system, and the proposed ESO-based ADRC system have nearly the same disturbance rejection property for all kinds of disturbances. When a ramp load disturbance is activated, all these three ADRC systems have

a constant steady-state tracking error, however, the PLL-based ADRC system has no steady-state tracking error. Hence, the PLL-based ADRC system shows a better rejection ability for the low-frequency disturbance than the other three systems. When a step load torque disturbance is activated, however, the PLL-based ADRC system shows a worse disturbance rejection ability than the other three systems.

The PLL-based ADRC system has a poor rejection ability for the medium-frequency disturbance, which can also be found in the constant speed operation experiments. When the speed is only 30 rpm, the frequency of the main component of the torque ripple is only 18π rad/s (9 Hz), the four systems have a comparable smooth speed. However, when the speed is increased to 100 rpm, the frequency of the main component of the torque ripple is only 60π rad/s (30 Hz), the PLL-based ADRC system has a larger speed pulsation. Moreover, in the low-speed range, the speed measurement noise in the PLL-based ADRC system is much larger than that in the other systems. Though smaller bandwidth PLL can be used to achieve a better measurement noise suppression, more serious speed oscillation can be expected.

Figure 17 shows the comparison of the sinusoidal command tracking performance. Two kinds of sinusoidal speed references are used for the test. When $n_{\text{ref}} = 100\sin 40\pi t$ rpm, the four ADRC systems have a comparable tracking performance. The tracking error is caused by the Coulomb friction torque. The effect of the Coulomb friction torque can be alleviated when a high-frequency sinusoidal speed reference is used. When $n_{\text{ref}} = 50\sin 200\pi t$ rpm, a speed response with larger amplitude can be found in the conventional ADRC system, which is identical to the theoretical results shown in Figure 12(a).

In Figure 12, delays caused by the current control loop and the digital system are not considered. Actually, these delays will cause the phase delay in the speed response. It can be seen that the modified ADRC system, the proposed ESO-based ADRC system, and the proposed PLL-based ADRC system show

- 32 -

better tracking performance for the high-frequency sinusoidal speed reference than the conventional ADRC system, which is consistent with the theoretical results.

In Figures 18 and 19, the robustness of the two proposed ADRC systems to inertia variations is evaluated, where $k_{ns} = 300$, the undamped natural frequency of ESO and PLLO is 1000 and 300 rad/s, respectively.

From Figure 18, it can be seen that the LPF bandwidth has no effect on the stability and the dynamic performance of the proposed ESO-based ADRC system. From Figure 19, it can be seen that when the LPF bandwidth is 2000 rad/s, the proposed PLLO-based ADRC system tends to be unstable as k_b decreases. The situation can be improved by lowering the LPF bandwidth. However, when $\omega_{f1} = 0$ rad/s and $\omega_{f2} = 1$ rad/s, the system is prone to be unstable as k_b increases. These results are all consistent to the theoretical ones.

It is worth pointing out that the LPF bandwidth affects the speed estimation of the proposed PLLO. As can be seen from the disturbance rejection property $\hat{\Omega}(s)/d(s)$ shown in Figure 10 the disturbance has larger effect on the estimated speed as the LPF bandwidth increases, which explains why, when using a higher LPF bandwidth, the estimated speed differs more from the filtered speed.

Presently, the conventional speed ADRC system and the modified ADRC system considering the speed filter are analyzed. The conventional ADRC system is prone to oscillation when the bandwidth of the LPF is decreased since the speed filter is not considered in the design of ESO. In the modified ADRC system, the LPF is used for filtering the observation error of ESO. Though the tracking performance is improved, the modified ADRC system tends to be unstable when a low bandwidth LPF is employed. To solve these problems, two ADRC systems considering the speed filter are proposed.

The proposed ESO-based ADRC system is independent of the bandwidth of the LPF, thus it is the best choice to use the integrator as the LPF. In this case, the position can be directly taken as an input of the observer and thus the speed calculation can be avoided, which simplifies the control algorithm. The proposed ESO-based ADRC system has a comparable disturbance rejection ability and measurement noise suppression performance with the conventional ADRC system and the modified ADRC system.

The proposed PLLO-based ADRC system is affected by the bandwidth of the LPF. When an integrator is used as the LPF, it has a better rejection ability for the low-frequency disturbance than the other three ADRC systems at the expense of deteriorated measurement noise suppression performance and robustness. Similarly, no speed calculation is needed for observing disturbance when compared with the PLLO-based ADRC system.

Presently, ten ADRCs based on the fourth-order GPIOs are presented for PMSM speed control system. In order to reveal the relationship between these ADRCs and the conventional ADRCs based on the third order ESO, six different third-order-ESO-based ADRCs are developed.

The feedback control law is designed to attenuate the speed reference tracking error as desired. According to the feedback control law, the control output, i.e., the torque reference, can be deduced. Therefore, the system model should be expressed as a function of the torque references.

$$\begin{cases} \dot{\theta}_m = \Omega \\ \dot{\Omega} = \frac{T_e^* - (T_e^* - T_e) - B\Omega - T_L}{J} = bT_e^* + d_n \end{cases} \quad (42)$$

where θ_m is the mechanical rotor angle, Ω is the mechanical angular velocity, T_e is the electromagnetic torque, T_e^* is the torque reference, B is the viscous friction torque coefficient, J is the moment of inertia, T_L is the load torque, $b = 1/J$ is the control gain, $d_n = -(T_e^* - T_e + B\Omega + T_L)/J$ is the nominal disturbance caused by the lumped disturbance torque with a known inertia.

- 34 -

Considering the uncertainties of the moment of inertia, the nominal value J_n is employed in the control system and the motion equation is modified as

$$\begin{cases} \dot{\theta}_m &= \Omega \\ \dot{\Omega} &= b_n T_e^* + (b - b_n) T_e^* + d_n \\ &= b_n T_e^* + d_{to} \end{cases} \quad (43)$$

where $d_{to} = (b - b_n) T_e^* + d_n$ is the total disturbance when considering the inertia mismatches.

Defining the reference of mechanical angular velocity as Ω^* , the tracking error of mechanical angular velocity can be expressed as $e_s = \Omega^* - \Omega$, resulting in:

$$\dot{e}_s = \dot{\Omega}^* - \dot{\Omega} = \dot{\Omega}^* - b_n T_e^* - d_{to} \quad (44)$$

Adopting a linear feedback control law

$$\dot{e}_s = k_p e_s \quad (45)$$

where k_{ps} is the proportional gain.

Substituting (45) into (44) yields

$$T_e^* = \frac{\dot{\Omega}^* + k_{ps}(\Omega^* - \Omega) - d_{to}}{b_n} \quad (46)$$

In (46), the mechanical angular velocity Ω and the disturbance d_{to} are generally unknown. These two states may be observed. When Ω and d_{to} are substituted by their estimated values $\hat{\Omega}$ and $\hat{d}_{to}$, the torque reference is modified as

$$T_e^* = \frac{\dot{\Omega}^* + k_{ps}(\Omega^* - \hat{\Omega}) - \hat{d}_{to}}{b_n} \quad (47)$$

In the practical system, the reference torque limit is applied as follows.

$$T_{esat}^* = \begin{cases} T_{em}^* \text{sign}(T_e^*), & |T_e^*| > T_{em}^* \\ T_e^*, & |T_e^*| \leq T_{em}^* \end{cases} \quad (48)$$

where T_{em}^* is the reference torque limit.

- 35 -

According to (43), the relationship between the torque reference and the total disturbance can be expressed by

$$b_n T_e^*(s) = s\Omega(s) - d_{to}(s) \quad (49)$$

Without considering the saturation of the torque reference, the system output can be obtained by substituting (49) into (47)

$$\begin{aligned} \Omega(s) &= \Omega^*(s) + \frac{k_{ps}}{s + k_{ps}} \tilde{\Omega}(s) + \frac{1}{s + k_{ps}} \tilde{d}_{to}(s) \\ &= \Omega^*(s) + G_{\text{fdb_speed}}(s) \tilde{\Omega}(s) + G_{\text{fdb_disturbance}}(s) \tilde{d}_{to}(s) \end{aligned} \quad (50)$$

where $\tilde{\Omega} = \Omega - \hat{\Omega}$ is the speed observation error and $\tilde{d}_{to} = d_{to} - \hat{d}_{to}$ is the observation error of the total disturbance, $G_{\text{fdb_speed}}(s)$ is a first order LPF with a bandwidth of k_{ps} , $G_{\text{fdb_disturbance}}(s)$ is a first order LPF with attenuated magnitude.

From (50), the dynamic performance of the closed-loop control system depends on the proportional gain k_{ps} , the speed observation error, and the disturbance observation error. The system has perfect command tracking performance and disturbance rejection properties. This is particularly the case if the speed and the total disturbance can be well observed.

The rotor position may be obtained from position sensors such as an encoder (e.g. rotary encoder) or resolver, and the speed is calculated by the derivative of the position. Due to the quantization error in the measurement of the position, the speed calculated by the classical frequency method is contaminated by measurement noise, especially in the low-speed range. Denoting δ_p as the position measurement noise, the measured position can be expressed as $\theta_m^m = \theta_m + \delta_p$.

To estimate the total disturbance, a classical third order linear ESO can be constructed for the system expressed by (43). For simplification, the six ADRC systems based on the six different third order ESOs are briefed as system

3rdESO_0, 3rdESO_1, 3rdESO_2, 3rdESO_3, 3rdESO_4, and 3rdESO_5, as shown in Figure 20. For these different ESOs, the state equations are different, as shown in Table III. In the state equations, $\hat{\theta}_m$ and $\tilde{\theta}_m$ are the estimated mechanical position and the position observation error, respectively.

Feedback/ Feedforward States		ESO Structure	Transfer functions
ESO_0	$\hat{\Omega}_0(s)$	$\begin{cases} \dot{\hat{\theta}}_m = \hat{\Omega}_1 + k_0 \tilde{\theta}_m + k_1 \int \tilde{\theta}_m dt \\ \dot{\hat{\Omega}}_1 = b T_e^* + \hat{d}_1 \\ \dot{\hat{d}}_1 = k_2 \tilde{\theta}_m \end{cases}$	$\begin{cases} G_1(s) = s + k_0 + k_1/s \\ G_2(s) = k_2 \\ G_3(s) = (s + k_{ps})(s^2 + k_0 s + k_1) \end{cases}$
	$\hat{d}_1(s)$		
ESO_1	$\hat{\Omega}_1(s)$	$\begin{cases} \dot{\hat{\theta}}_m = \hat{\Omega}_1 + k_0 \tilde{\theta}_m \\ \dot{\hat{\Omega}}_1 = b T_e^* + \hat{d}_1 + k_1 \tilde{\theta}_m \\ \dot{\hat{d}}_1 = k_2 \tilde{\theta}_m \end{cases}$	$\begin{cases} G_1(s) = s + k_0 \\ G_2(s) = k_2 \\ G_3(s) = (s + k_{ps})(s^2 + k_0 s + k_1 s) \end{cases}$
	$\hat{d}_1(s)$		
ESO_2	$\hat{\Omega}_1(s)$	$\begin{cases} \dot{\hat{\theta}}_m = \hat{\Omega}_1 + k_0 \tilde{\theta}_m \\ \dot{\hat{\Omega}}_1 = b T_e^* + \hat{d}_2 \\ \dot{\hat{d}}_2 = k_1 \tilde{\theta}_m + k_2 \tilde{\theta}_m \end{cases}$	$\begin{cases} G_1(s) = s + k_0 \\ G_2(s) = k_1 s + k_2 \\ G_3(s) = (s + k_{ps})(s^2 + k_0 s) \end{cases}$
	$\hat{d}_2(s)$		
ESO_3	$\hat{\Omega}_2(s)$	$\begin{cases} \dot{\hat{\theta}}_m = \hat{\Omega}_2 \\ \dot{\hat{\Omega}}_2 = b T_e^* + \hat{d}_1 + k_0 \tilde{\theta}_m + k_1 \dot{\tilde{\theta}}_m \\ \dot{\hat{d}}_1 = k_2 \tilde{\theta}_m \end{cases}$	$\begin{cases} G_1(s) = s \\ G_2(s) = k_2 \\ G_3(s) = (s + k_{ps})s^2 + k_0 s^2 + k_1 s \end{cases}$
	$\hat{d}_1(s)$		
ESO_4	$\hat{\Omega}_2(s)$	$\begin{cases} \dot{\hat{\theta}}_m = \hat{\Omega}_2 \\ \dot{\hat{\Omega}}_2 = b T_e^* + \hat{d}_2 + k_0 \tilde{\theta}_m \\ \dot{\hat{d}}_2 = k_1 \tilde{\theta}_m + k_2 \tilde{\theta}_m \end{cases}$	$\begin{cases} G_1(s) = s \\ G_2(s) = k_1 s + k_2 \\ G_3(s) = (s + k_{ps})s^2 + k_0 s^2 \end{cases}$
	$\hat{d}_2(s)$		
ESO_5	$\hat{\Omega}_2(s)$	$\begin{cases} \dot{\hat{\theta}}_m = \hat{\Omega}_2 \\ \dot{\hat{\Omega}}_2 = b T_e^* + \hat{d}_3 \\ \dot{\hat{d}}_3 = k_0 \tilde{\theta}_m + k_1 \dot{\tilde{\theta}}_m + k_2 \tilde{\theta}_m \end{cases}$	$\begin{cases} G_1(s) = s \\ G_2(s) = k_0 s^2 + k_1 s + k_2 \\ G_3(s) = (s + k_{ps})s^2 \end{cases}$
	$\hat{d}_3(s)$		

Table III

- 37 -

Each of Figures 20a-c shows an example ADRC system 2000 including an ESO module 2002 configured to receive input information 2004 comprising a torque reference signal for driving a motor, and an angular position that is a measured output of the motor. The ESO module 2002 then determines, based on the input information, an estimated speed value 2006 and an estimated disturbance value 2008. The estimated speed value 2006 is based on a derivative of the angular position, and the estimated disturbance value 2008 is an output of a proportional-integral (PI) controller of the ESO module 2002. The estimated speed value 2006 is then sent as a feedback control signal, and the estimated disturbance value 2008 as a feedforward control signal.

From the block diagrams, the observed speed and the observed disturbance can be expressed in the same form

$$\begin{cases} \tilde{\theta}_m(s) = \frac{s}{\Delta_3} [s\Omega^m(s) - b_n T_e^*(s)] \\ \hat{\Omega}(s) = \Omega^m(s) - G_1(s)\tilde{\theta}_m(s) \\ \hat{d}_{to}(s) = \frac{G_2(s)}{s} \tilde{\theta}_m(s) \end{cases} \quad (51)$$

where $\Delta_3 = s^3 + k_0 s^2 + k_1 s + k_2$ is the characteristic polynomial, $\Omega^m(s) = s\theta_m^m(s) = s\theta_m(s) + s\delta_p(s)$ is the virtual measured speed, the transfer functions $G_1(s)$ and $G_2(s)$ are employed for ease of calculation and they vary with the ESOs.

It is a common practice to set Δ_3 as $(s + p_0)^3$, where p_0 is the natural frequency of ESO. Hence, the gains in ESO can be calculated by $k_0 = 3p_0$, $k_1 = 3p_0^2$, $k_2 = p_0^3$. Substituting (49) into (51) yields

$$\begin{cases} \tilde{\theta}_m(s) = \frac{s}{\Delta_3} [s\delta_n(s) + d_{to}(s)] \\ \tilde{\Omega}(s) = \frac{sG_1(s)}{\Delta_3} d_{to}(s) - \frac{\Delta_3 - s^2 G_1(s)}{\Delta_3} \delta_n(s) \\ \tilde{d}_{to}(s) = \frac{\Delta_3 - G_2(s)}{\Delta_3} d_{to}(s) - \frac{sG_2(s)}{\Delta_3} \delta_n(s) \end{cases} \quad (52)$$

where $\delta_n(s) = s\delta_p(s)$ is the speed measurement noise, which is a colored noise instead of a white noise.

- 38 -

Equation (52) shows that the steady-state position observation error is proportional to the derivative of the total disturbance, thus the observed position should not be employed in the coordinates transformation where accurate position is required.

The observed disturbance can be viewed as the total disturbance filtered by a low pass filter (LPF). The LPF is decided by the characteristic polynomial Δ_3 and the transfer function $G_2(s)$. When $G_2(s) = k_2$, the LPF is a typical third order LPF. This means the low-frequency disturbance can be well observed. From (43), it can be seen that the internal disturbance is proportional to the control gain deviation $(b - b_n)$ and the torque reference. In the case of slow-varying speed, the internal disturbance is of low-frequency and thus, the total disturbance can be well observed. In the case of fast-varying speed, however, it is difficult to have a good observation of the total disturbance.

Substituting (51) into (47) yields

$$b_n T_e^*(s) = \frac{(s + k_{ps})\Omega^*(s) - k_{ps}\Omega^m(s) + k_{ps}sG_1(s) - G_2(s)}{\Delta_3} [s\Omega^m(s) - b_n T_e^*(s)] \quad (53)$$

According to (53), the control output can be obtained as

$$b_n T_e^*(s) = s\Omega^m(s) + \frac{\Delta_3(s + k_{ps})}{G_3(s)} [\Omega^*(s) - \Omega^m(s)] \quad (54)$$

where the transfer function $G_3(s)$ is expressed by

$$G_3(s) = \Delta_3 + k_{ps}G_1(s) - G_2(s) \quad (55)$$

According to (42), the relationship between the torque reference and the external disturbance can be expressed by

$$b_n T_e^*(s) = k_b b T_e^*(s) = k_b [s\Omega(s) - d_n(s)] \quad (56)$$

where $k_b = b_n/b = J/J_n$ is the control gain ratio.

- 39 -

Substituting (56) into (54), the transfer functions from the command reference to the output, from the disturbance to the output, and from the measurement noise to the output are

$$\begin{cases} \Phi_r(s) = \frac{\Omega(s)}{\Omega^*(s)} = \frac{\Delta_3(s + k_{ps})}{\Delta_3(s + k_{ps}) + (k_b - 1)sG_3(s)} \\ \Phi_d(s) = \frac{\Omega(s)}{d_n(s)} = \frac{k_b G_3(s)}{\Delta_3(s + k_{ps}) + (k_b - 1)sG_3(s)} \\ \Phi_n(s) = \frac{\Omega(s)}{-s\delta_p(s)} = \frac{\Delta_3(s + k_{ps}) - sG_3(s)}{\Delta_3(s + k_{ps}) + (k_b - 1)sG_3(s)} = 1 - s\Phi_d(s) \end{cases} \quad (57)$$

(57) shows that the system has a perfect command tracking performance only when $k_b = 1$, and any parameter mismatch leads to the deteriorated tracking performance. The speed tracking error caused by the disturbance is proportional to the control gain ratio k_b . The smaller k_b is, the better disturbance rejection ability is. However, the disturbance rejection ability contradicts the measurement noise suppression performance. Smaller k_b leads to poorer measurement noise suppression performance.

The frequency-domain analysis of ADRC system has been established, where the system is transformed to a transfer function-based 2DOF system first, then a traditional frequency analysis method can be used to evaluate the stability and dynamic performance. However, the expression of the closed-loop control system is complicated. The problem is presently solved using transfer functions $G_1(s)$, $G_2(s)$, and $G_3(s)$.

According to (51), the equivalent block diagram of the ADRC system based on the third order ESO is shown in Figure 21. It can be seen that the ESO is equivalent to DOB with a LPF of $G_2(s)/\Delta_3$ when $G_1(s) = 0$.

Figure 21 shows an example ADRC system 2100 including an ESO module 2102 configured to receive input information 2104 comprising a torque reference signal for driving a motor, and an angular position that is a measured output

- 40 -

of the motor. The ESO module 2102 then determines, based on the input information, an estimated speed value 2106 and an estimated disturbance value 2108. The estimated speed value 2106 is based on a derivative of the angular position, and the estimated disturbance value 2108 is an output of a proportional-integral (PI) controller of the ESO module 2102. The estimated speed value 2106 is then sent as a feedback control signal, and the estimated disturbance value 2108 as a feedforward control signal.

The block diagram of the ADRC system in transfer function form is shown in Figure 22, the system output can be expressed as

$$\Omega(s) = \frac{G_c(s)bF(s)}{s^2 + G_c(s)b} \Omega^*(s) + \frac{s d_n(s)}{s^2 + G_c(s)b} - \frac{G_c(s)b}{s^2 + G_c(s)b} s \delta_p(s) \quad (58)$$

Comparing (58) with (57), the transfer function $G_c(s)$ and $F(s)$ can be deduced as

$$G_c(s) = s \cdot \frac{1}{b_n} \left[\frac{\Delta_3(s + k_{ps})}{G_3(s)} - s \right] \quad (59)$$

$$F(s) = \frac{1}{s} \cdot \frac{\Delta_3(s + k_{ps})}{\Delta_3(s + k_{ps}) - s G_3(s)} \quad (60)$$

Without considering the output saturation, the open loop transfer function of the ADRC system can be expressed as

$$G_L(s) = \frac{G_c(s)b}{s^2} = \frac{1}{k_b} \left[\frac{\Delta_3(s + k_{ps})}{s G_3(s)} - 1 \right] \quad (61)$$

Nonetheless, in this scenario, it is difficult to analysis how the ESO affects the dynamic performance of the closed-loop control system. This problem is presently solved by frequency-domain analysis of ESO.

The transfer functions from the total disturbance and measurement noise to the speed observation error are

- 41 -

$$\begin{cases} \frac{\tilde{\Omega}(s)}{d_{to}(s)} = \frac{sG_1(s)}{\Delta_3} \\ \frac{\tilde{\Omega}(s)}{-\delta_n(s)} = \frac{\Delta_3 - s^2G_1(s)}{\Delta_3} \end{cases} \quad (62)$$

(62) shows that the speed observation error is only related to $G_1(s)$. According to Table III, the two $G_1(s)$ in ESO_1 and ESO_2 are the same, the three $G_1(s)$ in ESO_3, ESO_4, and ESO_5 are the same. When $p_o = 1000\text{rad/s}$, the bode diagrams of the six third order ESOs when observing speed are shown in Figure 23. It can be seen that ESO_0 has a disturbance rejection ability of 0 dB/dec but a measurement noise suppression performance of -60 dB/dec ESO_1 and ESO_2 have the same disturbance rejection ability ($+20\text{ dB/dec}$) and measurement noise suppression performance (-40 dB/dec) while ESO_3, ESO_4, and ESO_5 have the same disturbance rejection ability ($+40\text{ dB/dec}$) and measurement noise suppression performance (-20 dB/dec). The frequency analysis explains why ESO can be used as an instantaneous speed observer or state filter.

The transfer functions from the total disturbance and measurement noise to the disturbance observation error are:

$$\begin{cases} \frac{\tilde{d}_{to}(s)}{d_{to}(s)} = \frac{\Delta_3 - G_2(s)}{\Delta_3} \\ \frac{\tilde{d}_{to}(s)}{-\delta_n(s)} = \frac{sG_2(s)}{\Delta_3} \end{cases} \quad (63)$$

According to Table III, the three $G_2(s)$ in ESO_0, ESO_1, and ESO_3 are the same, the two $G_2(s)$ in ESO_2 and ESO_4 are the same, the $G_2(s)$ in ESO_5 is unique. When $p_o = 1000\text{rad/s}$, the bode diagrams of the six third order ESOs when observing disturbance are shown in Figure 24. It can be seen that ESO_0, ESO_1 and ESO_3 have the same low-frequency disturbance rejection ability of $+20\text{ dB/dec}$ and high-frequency measurement noise suppression performance of -40 dB/dec , ESO_2 and ESO_4 have the same low-frequency disturbance rejection ability of $+40\text{ dB/dec}$ and high-frequency measurement noise

- 42 -

suppression performance of -20 dB/dec, and ESO_5 has the best low-frequency disturbance rejection ability of $+60$ dB/dec and the worst high-frequency measurement noise suppression performance of 0 dB/dec.

When $k_{ps} = 300$ rad/s, $p_o = 1000$ rad/s, the bode diagrams of $G_{fdb_speed}(s)$ and $G_{fdb_disturbance}(s)$ are shown in Figure 25, the bode diagrams of the six systems are shown in Figure 26. The six ESOs have different dynamic performance for observing speed and observing disturbance, results in different dynamic performance of the closed-loop control systems.

Denoting S_{eso_sp} and S_{eso_dist} as the slope of the bode curves of ESO when observing speed and disturbance, S_{fdb_sp} and S_{fdb_dist} as the slope of the bode curves of the transfer functions $G_{fdb_speed}(s)$ and $G_{fdb_disturbance}(s)$, and S_{cl} as the slope of the bode curves of the closed-loop control system, respectively. According to the results in Figure 26, S_{cl} can be calculated by

$$S_{cl} = \begin{cases} \min(S_{eso_sp} + S_{fdb_sp}, S_{eso_dist} + S_{fdb_dist}) & \text{Low-frequency} \\ \min(S_{eso_sp} + S_{fdb_sp}, S_{eso_dist} + S_{fdb_dist}) & \text{High-frequency} \end{cases} \quad (64)$$

Take the system 3rd ESO_1 as an example. When a proportional feedback control is employed, in the low-frequency range, S_{eso_sp} and S_{eso_dist} of ESO_1 are both $+20$ dB/dec (see Figure 23 and Figure 24), S_{fdb_sp} and S_{fdb_dist} are both 0 dB/dec (see the solid line in Figure 25), then the closed-loop control system is expected to have a low-frequency disturbance rejection ability of $+20$ dB/dec. In the high-frequency range, $S_{eso_sp} = -20$ dB/dec because Δ_3 is one order higher than $sG_1(s)$, while $S_{eso_dist} = 0$ dB/dec due to the same order between Δ_3 and $\Delta_3 - G_2(s)$, S_{fdb_sp} and S_{fdb_dist} are both -20 dB/dec, then the closed-loop control system is expected to have a disturbance rejection ability of -20 dB/dec. The calculated results when using a proportional feedback control law are shown in TABLE IV, which illustrates dynamic performance of systems based on the third order ESO. These calculated results are consistent with the results shown in Figure 26.

Based on the above analysis:

- 1 The system 3rd ESO_0 has the best high-frequency measurement noise suppression performance, whereas it cannot reject a constant disturbance because a constant disturbance causes a steady-state speed estimation error.
- 2 The system 3rd ESO_2 has a better low-frequency disturbance rejection ability than the system 3rd ESO_1 because ESO_2 has a better low-frequency disturbance rejection ability than ESO_1 when observing disturbance.
- 3 The system 3rd ESO_3 has a better low-frequency disturbance rejection ability than the system 3rd ESO_1 because ESO_3 has a better low-frequency disturbance rejection ability than ESO_1 when observing speed.
- 4 The system 3rd ESO_4 has a low-frequency disturbance rejection ability of +40 dB/dec because ESO_4 has the same low-frequency disturbance rejection ability (+40 dB/dec) when observing speed and when observing disturbance.
- 5 The system 3rd ESO_5 has a better disturbance rejection ability than the system 3rd ESO_4 because ESO_5 has a better disturbance rejection ability than ESO_4 when observing disturbance.

These results cannot be obtained from conventional frequency-domain analysis.

The measurement noise caused by the quantization error is not a white noise. The measurement noise is distributed in the entire frequency domain. In this case, different systems should not be compared under the same bandwidth of ESO. Therefore, p_o in the six systems are set to be different so that some systems have the same low-frequency disturbance rejection ability or the same high-frequency measurement noise suppression performance. When $k_{ps} = 300 \text{ rad/s}$, p_o in the six systems based on the third order ESO (systems 3rd ESO_0 – 5) are 2300, 2000, 700, 1700, 600, and 320 rad/s respectively, the bode

- 44 -

diagrams of the six systems are shown in Figure 27. The comparison of different systems is drawn as follows:

- 1 When systems 3rd ESO_1, 3rd ESO_2, and 3rd ESO_3 have the same low-frequency disturbance rejection ability (20 dB/dec), the system 3rd ESO_1 has the best high-frequency measurement noise suppression performance (−60 dB/dec).
- 2 When systems 3rd ESO_4 and 3rd ESO_5 have the same low-frequency disturbance rejection ability (40 dB/dec), the system 3rd ESO_4 has a better high-frequency measurement noise suppression performance (−40 dB/dec).
- 3 When systems 3rd ESO_3 and 3rd ESO_4 have the same high-frequency measurement noise suppression performance (−40 dB/dec), the system 3rd ESO_4 has a better low-frequency disturbance rejection ability (40 dB/dec).

Low-frequency Disturbance Rejection Property (dB/dec)					High-frequency Measurement noise suppression performance (dB/dec)					
	S_{eso_sp}	S_{dib_sp}	S_{eso_dist}	S_{dib_dist}	S_{cl}	S_{eso_sp}	S_{dib_sp}	S_{eso_dist}	S_{dib_dist}	S_{cl}
ESO_0	0		+20		0	-60		-40		-60
ESO_1	+20		+20		+20	-40		-40		-60
ESO_2	+20	0(P)	+40	0(P)	+20	-40		-20		-40
ESO_3	+40	0(PI)	+20	+20(PI)	+20(P) +40(PI)	-20	-20	-40	-20	-40
ESO_4	+40		+40		+40	-20		-20		-40
ESO_5	+40		+60		+40	-20		0		-20

Table IV

In summary, systems 3rd ESO_1 and 3rd ESO_4 are the two best candidates for achieving good control performance. Compared with the system 3rd ESO_4, the system 3rd ESO_1 is better in suppressing the high-frequency measurement noise and rejecting medium-frequency disturbance but worse in rejecting low-frequency disturbance.

- 45 -

According to the previous analysis, there are two directions to improve the disturbance rejection ability, one is to improve the frequency-domain characteristics of $G_{\text{fdb_disturbance}}(s)$, the other is to enhance the frequency-domain characteristics of ESO. To improve the frequency-domain characteristics, there are also two ways. One way to improve the frequency-domain characteristics is to use nonlinear control, such as finite time control and sliding mode control. However, it is not easy to tune the gains in the nonlinear observer. The other way to improve the frequency-domain characteristics is to use higher order controller or observer, such as GPIC and GPIO.

To implement GPI control, an integral action is added to the feedback control law (45), giving (65)

$$\dot{e}_s = k_{ps}e_s - \int k_{is}e_s dt \quad (65)$$

where k_{is} is the integral gain.

As a result, (50) will be modified accordingly as

$$\begin{aligned} \Omega(s) &= \Omega^*(s) + \frac{k_{ps}s + k_{is}}{s^2 + k_{ps}s + k_{is}} \tilde{\Omega}(s) + \frac{s}{s^2 + k_{ps}s + k_{is}} \tilde{a}_{to}(s) \\ &= \Omega^*(s) + G_{\text{fdb_speed}}(s) \tilde{\Omega}(s) + G_{\text{fdb_disturbance}}(s) \tilde{a}_{to}(s) \end{aligned} \quad (66)$$

The bode diagrams of $G_{\text{fdb_speed}}(s)$ and $G_{\text{fdb_disturbance}}(s)$ are also shown in Figure 25 (dashed line). Compared with the proportional control scheme, the PI control scheme changes the characteristics of $G_{\text{fdb_disturbance}}(s)$ in the low-frequency range. The rejection ability for the closed-loop control system for low-frequency disturbance can therefore be improved. Especially, using the PI control scheme rather than the P control scheme, the rejection ability of the system 3rd ESO_3 for the low-frequency disturbance can be considerably improved (from +20 dB/dec to +40 dB/dec). In other words, the system 3rd ESO_3 can reject both constant disturbance and ramp disturbance by using the PI controller, while it can only reject a constant disturbance by using the P controller. It should be noted that though the PI controller can be utilized to

- 46 -

increase low-frequency disturbance rejection, the wind up of the integrator can also be an issue. As an alternative to GPIC, GPIO can be used.

Assuming the total disturbance is a ramp disturbance with a constant slope of d_d , then the state equation of the system can be expressed by:

$$\begin{cases} \dot{\theta}_m = \Omega \\ \dot{\Omega} = b_n T_e^* + d_{to} \\ \dot{d}_{to} = d_d \\ \dot{d}_d = 0 \end{cases} \quad (67)$$

It is worth pointing out that the constant disturbance is a special kind of ramp disturbance, i.e., the slope is zero. Therefore, the system with constant disturbance can also be expressed by (67). There are overall ten ESOs can be constructed for the system (67). One of the ten ESOs can be expressed as:

$$\begin{cases} \dot{\hat{\theta}}_m = \hat{\Omega} + k_0 \tilde{\theta}_m \\ \dot{\hat{\Omega}} = b_n T_e^* + \hat{d}_{to} + k_1 \tilde{\theta}_m \\ \dot{\hat{d}}_{to} = \hat{d}_d + k_2 \tilde{\theta}_m \\ \dot{\hat{d}}_d = k_3 \tilde{\theta}_m \end{cases} \quad (68)$$

The block diagram of the ten ADRC systems based on the fourth order ESO as shown in Figure 28. According to (68), the ESO output and the system output can be expressed in the same way as those in third order ESO-based systems by simply changing the transfer function $G_2(s)$ to $k_2 + k_3/s$. In this case, the characteristic polynomial is changed to $\Delta_3 = s^3 + k_0 s^2 + k_1 s + k_2 + k_3/s$. By setting the characteristic polynomial as $\Delta_3 = (s + p_o)^4/s$, the gains in the ESO can be calculated by $k_0 = 4p_o, k_1 = 6p_o^2, k_2 = 4p_o^3, k_3 = p_o^4$.

Figures 28a-d each shows an example ADRC system 2800 including an ESO module 2802 configured to receive input information 2804 comprising a torque reference signal for driving a motor, and an angular position that is a measured output of the motor. The ESO module 2802 then determines, based on the

- 47 -

input information, an estimated speed value 2806 and an estimated disturbance value 2808. The estimated speed value 2806 is based on a derivative of the angular position, and the estimated disturbance value 2808 is an output of a proportional-integral (PI) controller of the ESO module 2802. The estimated speed value 2806 is then sent as a feedback control signal, and the estimated disturbance value 2808 as a feedforward control signal.

When $p_o = 1000\text{rad/s}$, the bode diagrams of the ten fourth order ESOs when observing speed and disturbance are shown in Figure 29 and Figure 30, respectively. According to (64), the calculated results when using a proportional feedback control law are shown in Table V. These results are verified by the bode diagrams shown in Figure 31, where $k_{ps} = 300\text{rad/s}$ and $p_o = 1000\text{rad/s}$. Similarly, the systems 4thESO_1, 4thESO_4, and 4thESO_8 are the three best candidates among the ten systems. The system 4thESO_1 has a low-frequency disturbance rejection ability of +20 dB/dec while it has the best high-frequency measurement noise suppression performance (−80 dB/dec), the system 4thESO_8 has a high-frequency measurement noise suppression performance of −40 dB/dec whereas it has the best low-frequency disturbance rejection ability (+60 dB/dec), the system 4thESO_4, on the other hand, has a comprised dynamic performance, with a low-frequency disturbance rejection ability of +40 dB/dec and a high-frequency measurement noise suppression performance of −60 dB/dec.

Low-frequency Disturbance Rejection Property (dB/dec)					High-frequency Measurement noise suppression performance (dB/dec)								
$S_{eso\ sp}$		$S_{\hat{y}\ sp}$	$S_{eso\ dist}$		$S_{\hat{y}\ dist}$	S_{cl}	$S_{eso\ sp}$		$S_{\hat{y}\ sp}$	$S_{eso\ dist}$		$S_{\hat{y}\ dist}$	S_{cl}
ESO_0	0			+20		0	-80			-60			-80
ESO_1	+20			+20		+20	-60			-60			-80
ESO_2	+20			+40		+20	-60			-40			-60
ESO_3	+40			+20		+20(P) +40(PI)	-40			-60			-60
ESO_4	+40	0(P)		+40	0(P)	+40	-40			-40			-60
ESO_5	+40	0(PI)		+60	+20(PI)	+40	-40	-20		-20	-20		-40
ESO_6	+60			+20		+20(P) +40(PI)	-20			-60			-40
ESO_7	+60			+40		+40(P) +60(PI)	-20			-40			-40
ESO_8	+60			+60		+60	-20			-20			-40
ESO_9	+60			+80		+60	-20			0			-20

Table V

Presently, the five ADRC systems 3rdESO_1, 3rdESO_4, 4thESO_1, 4thESO_4, and 4thESO_8 are compared. When $k_b = 1$ and $k_{ps} = 300$ rad/s, p_o is 2000, 900, 2700, 1300, and 640 rad/s in the five systems 3rdESO_1, 3rdESO_4, 4thESO_1, 4thESO_4, and 4thESO_8, bode diagrams of the five systems are shown in Figure 32. From Figure 32 it is evident that:

1. The systems 4thESO_1 and 3rdESO_1 have the same low-frequency disturbances ability, while the system 4thESO_1 has a better high-frequency measurement noise suppression performance than the system 3rdESO_1. The same conclusion can be obtained when comparing the system 4thESO_4 with the system 3rdESO_4. These results show that ADRC systems based on the higher order ESO have a better high-frequency measurement noise suppression performance.
2. The systems 4thESO_8 and 3rdESO_4 have the same high-frequency measurement noise suppression performance, while the system 4thESO_8 has a better low-frequency disturbance rejection ability than the system 3rdESO_4. The same conclusion can be obtained when comparing the system 4thESO_4 with the system 3rdESO_1.

- 49 -

- 3 The system 4thESO_4 has the same low-frequency disturbance rejection ability as the system 3rdESO_4 and the same high-frequency measurement noise suppression performance as the system 3rdESO_1.

The system 4thESO_4 and 4thESO_8 can be obtained from the system 3rdESO_1 and by extending a higher order derivative of disturbance while the system 4thESO_4 can be obtained from the system 3rdESO_1 by extending a higher order integral of state.

In summary, the low-frequency disturbance rejection ability can be enhanced by extending a higher order derivative of disturbances, while the high-frequency measurement noise suppression performance can be improved by extending a higher order integral of states. The equivalent block diagram of the system 4thESO_1-2 is shown in Figure 33.

Figure 33 shows an example ADRC system 3300 including an ESO module 3302 configured to receive input information 3304 comprising a torque reference signal for driving a motor, and an angular position that is a measured output of the motor. The ESO module 3302 then determines, based on the input information, an estimated speed value 3306 and an estimated disturbance value 3308. The estimated speed value 3306 is based on a derivative of the angular position, and the estimated disturbance value 3308 is an output of a proportional-integral (PI) controller of the ESO module 3302. The estimated speed value 3306 is then sent as a feedback control signal, and the estimated disturbance value 3308 as a feedforward control signal.

Based on the flatness theory, the orders of ESO can be extended to be higher (five or more) by extending a higher order derivative of disturbance. It is, however, not the best option. Systems with better low-frequency disturbance rejection ability are more sensitive to the variation of control gain. When $k_{ps} = 300 \text{ rad/s}$, p_o is 2700, 1300, and 640 rad/s in systems 4thESO_1, 4thESO_4, and 4thESO_8. Effect of k_b on the system's stability and dynamic performances are

illustrated in Figure 34, Figure 35, Figure 36. As can be observed, the stable operating condition for systems 4thESO_1, 4thESO_4, and 4thESO_8 are $k_b > 0.258$, $k_b > 0.233$, and $k_b < 2.94$, respectively. Since $k_b < 1$ is harmful to the measurement noise suppression performance, smaller inertia is usually tried first in the controller. Additionally, the actual inertia of the motor system may be increased to ten times its nominal value when loading. Therefore, $k_b > 1$ is more common in real applications. As a result, the systems 4thESO_1 and 4thESO_4 can remain stable when the inertia is increased, while the system 4thESO_8 cannot. Compared with the system 4thESO_1, the system 4thESO_4 has a compromise low-frequency disturbance rejection ability and high-frequency measurement noise suppression performance. Therefore, among the ten ADRC systems based on the fourth-order ESO, only the system 4thESO_4 will be compared to the ADRC systems based on the third-order ESO.

To verify the proposed algorithm, MATLAB/Simulink simulation and experiment results are presented and analyzed in this section. The specification of the servo motor is shown in Table VI. As shown in Figure 37(a), the control strategy is based on space vector pulse width modulation (SVPWM) control with $i_d^* = 0$. Decoupled PI regulators are employed in the current-loop to control i_d and i_q respectively, and different ADRCs are employed in the speed-loop. The DC bus voltage u_{dc} is 150 V. The saturation limit of q -axis reference current is 9 A. The bandwidths of d -axis and q -axis current-loops are 2000rad/s.

Symbol	Quantity	Symbol	Quantity
Rated power P_N	0.75 (kW)	Number of pole-pairs p_n	4
Rated voltage U_N	220 (V)	D axis inductance L_d	8.0 (mH)
Rated speed n_N	3000 (rpm)	Q axis inductance L_q	8.0 (mH)
Rated torque T_N	2.4 (N·m)	Torque constant K_t	0.553 (N·m/A)
Current Limit $I_{\max}$	10 (A)	Motor inertia J_M	1.62×10^{-4} (kg·m ²)
Stator resistance R_s	1.1 (Ω)	Motor system inertia J	4.44×10^{-4} (kg·m ²)

Table VI

The configuration of the test bench is shown in Figure 37(b). A programmable DC power supply is used to provide 150 V DC Bus voltage. There are two same PMSMs, one acts as a driving motor driven by a self-made inverter, and the other acts as a generator. The self-made inverter consists of dSPACE DS1103, Mitsubishi intelligent power module (IPM), current and voltage hall sensors, filtering and protection circuit, etc. The field-oriented control is employed for driving the PMSM.

Presently, the five ADRC systems based on the 3rd order ESO and the sixth ADRC system 4thESO_4 are compared without considering the parameter uncertainties. When $k_{ps} = 100 \text{ rad/s}$, p_o in the six ESOs are 2000 rad/s, 700 rad/s, 1700 rad/s, 600 rad/s, 70 rad/s and 1300 rad/s, respectively. The tracking performance of the six systems for the step speed reference and the sinusoidal speed reference are shown in Figure 38. It can be seen that the tracking performance of the six systems for the step speed reference are the same, which is consistent with the theoretical results. The different thing is the noise in the current. Obviously, the system 3rdESO_5 has the poorest measurement noise suppression performance.

The disturbance rejection property of the six systems when loading a step load and unloading a ramp load are shown in Figure 39 and Figure 40, respectively. It should be noted that the estimated speed in systems 3rdESO_2 and 3rdESO_5 keeps constant when the load varies if the current is not saturated. Without considering the current saturation, the estimated speed in these two systems is not affected by the disturbance, which can be proved as follows. When $k_b = 1$, substituting (56) into (57) yields

$$\hat{\Omega}(s) = \Omega^*(s) + \frac{\Delta_3 - s^2 G_1(s) - G_2(s)}{\Delta_3} [s^2 \delta_p(s) + d_{to}(s)] \quad (69)$$

Specially, when $\Delta_3 = s^2 G_1(s) + G_2(s)$, the estimated speed equals to the reference speed and independent of the disturbance. To reveal the variation of

- 52 -

real speed, another speed in magenta, which is filtered by a first-order low pass filter with a bandwidth of 4000rad/s, is shown in the experimental results.

It can be seen from Figure 39 and Figure 40 that the former three systems 3rdESO_1, 3rdESO_2, and 3rdESO_3 can only reject the constant disturbance but cannot reject the ramp disturbance while the latter three systems 3rdESO_4, 3rdESO_5, and 4thESO_4 can reject both the constant disturbance and the ramp suppression performance, and the latter two systems are disturbance. In the former three systems, the system 3rdESO_1 promising for rejecting the low-frequency disturbance. For has the best measurement noise suppression performance. In the further comparison, the disturbance rejection ability of the three latter three systems, systems 3rdESO_4 and 4thESO_4 have better measurement noise suppression performance than the system 3rdESO_5.

In summary, in the six ADRC systems, the systems 3rdESO_1, 3rdESO_4 and 4thESO_4 have a good measurement noise suppression performance, and the latter two systems are promising for rejecting the low-frequency disturbance. For further comparison, the disturbance rejection ability of the three systems at the low speed of 10 rpm are compared, as shown in Figure 41. The speed calculated using the frequency method (4100) contains more noise than the estimated speed (4102). Clearly, the estimated speed closely tracks the speed calculated using the frequency method. The frequency p_o in the three systems are set to 1200 rad/s, 350 rad/s, and 800 rad/s so that the speed drops under the step load are 3rdESO_4, and 4thESO_4 have a good measurement noise and 800rad/s so that the speed drops under the step load are comparable. It can be seen that system 3rdESO 1 has the best step disturbance rejection ability, while the system 4thESO_4 has the best ramp disturbance rejection ability. Compared with the system 3rdESO_4, the system 4thESO_4 is better in both disturbance rejection and measurement noise suppression. These results are consistent with the theory.

- 53 -

Presently, the three systems 3rdESO_1, 3rdESO_4, and 4thESO_4 are compared considering the parameter uncertainties. When $k_{ps} = 300$ rad/s, p_o in the three systems 3rdESO_1, 3rdESO_4, and 4thESO_4 are 2000 rad/s, 600 rad/s, and 1300 rad/s respectively. It is noted that in theoretical analysis shown in Figure 32, p_o in the system 3rdESO_4 is 900 rad/s, while it is reduced to 600 rad/s in the real system due to noise. The step response of the three systems when k_b is 0.5, 1.0, and 2.0 are shown in Figure 42.

It shows that $k_b > 1$ is beneficial for the measurement noise suppression but causes the overshoot and oscillation in the step response, while $k_b < 1$ deteriorates the measurement noise suppression performance. In these three systems, the system 3rdESO_1 has the best robustness to inertia variations, whereas the system 4thESO_4 has the worst. As can see from Figure 42, amplification of medium-frequency references of the system causes the oscillation in the step response when $k_b = 2.0$. Therefore, the oscillation can be reduced by using a smooth speed reference, which can be realized by a tracking differentiator or other filters. To improve the system robustness to parameter variations, some nonlinear observers or adaptive ESO can be employed.

Presently, different ADR controllers based on the fourth order GPI observers are present. These controllers are compared with the ADR controllers based on the third order ESOs by frequency-domain analysis. Conclusions are drawn as follows:

1. There are overall six different ADR controllers based on the third order ESO, among which the systems 3rdESO_1 and 3rdESO_4 are the best two candidates. There are overall ten different ADR controllers based on the fourth order GPIO, among which the systems 4thESO_1, 4thESO_4, and 4thESO_8 are the three best candidates.
2. The systems 4thESO4 and 4thESO_8 can be deduced from the systems 3rdESO_1 and 3rdESO_4 respectively by extending the derivative of dis-

- 54 -

turbance, resulting in a better low-frequency disturbance rejection ability. However, the system 4thESO_1 can be deduced from the system 3rdESO_1 extending the integral state, resulting in a better high-frequency measurement noise suppression performance. The GPI observers based on the extension of derivative of disturbances can be transformed to those based on the extension of integral states.

3. The closed-loop control system's dynamic performance is decided by the dynamic performance of ESO when observing speed and the dynamic performance of ESO when observing disturbance, as well as the feedback control law. If a proportional feedback control law is used the system's low-frequency disturbance rejection ability is mostly determined by ESO's dynamic performance, while the high-frequency disturbance rejection ability is mostly determined by the feedback proportional gain. Different ESOs have different dynamic performances. Using a PI feedback control law instead of a proportional feedback control law improves the system's low-frequency disturbance rejection ability while having less effect on other dynamic performance. However, the saturation caused by the integrator should be carefully dealt with.

It will be appreciated that many further modifications and permutations of various aspects of the described embodiments are possible. Accordingly, the described aspects are intended to embrace all such alterations, modifications, and variations that fall within the spirit and scope of the appended claims.

Throughout this specification and the claims which follow, unless the context requires otherwise, the word "comprise", and variations such as "comprises" and "comprising", will be understood to imply the inclusion of a stated integer or step or group of integers or steps but not the exclusion of any other integer or step or group of integers or steps.

The reference in this specification to any prior publication (or information derived from it), or to any matter which is known, is not, and should not be

- 55 -

taken as an acknowledgment or admission or any form of suggestion that that prior publication (or information derived from it) or known matter forms part of the common general knowledge in the field of endeavor to which this specification relates.

Claims:

1. An active disturbance rejection control (ADRC) system, comprising:
 - an extended state observer (ESO) module that is configured to:
 - receive input information comprising a torque reference signal for driving a motor, and a speed value that is a measured output of the motor passed to a first low-pass filter (LPF);
 - determine, based on the input information, an estimated speed value and an estimated disturbance value; and
 - send the estimated speed value as a feedback control signal, and the estimated disturbance value as a feedforward control signal;
 - wherein the estimated speed value is determined at least in part based on a second LPF that is of the same form as the first LPF, and the estimated disturbance value is selectable between an output of an integral controller and an output of a proportional-integral controller of the ESO module; or
 - wherein the estimated speed value is determined based on the output of a proportional-integral-derivative controller of the ESO module, and the estimated disturbance value is selectable between:
 - an output of an integral controller of the ESO module; and
 - an output of a proportional-integral controller of the ESO module; andthe output of the proportional-integral-derivative controller of the ESO module.
2. The ADRC system of claim 1, wherein the estimated speed value is determined at least in part based on the second LPF, and the second LPF is of the same form as the first LPF, and the estimated disturbance value is selectable between an output of the integral controller and the output of the proportional-integral controller of the ESO module.
3. The ADRC system of claim 1 or 2, wherein the ESO module has a fourth-order characteristic polynomial.

4. The ADRC system according to claim 3, wherein the ESO module is characterized by the following transfer functions:

$$\begin{aligned}\Phi_r(s) &= \frac{\Omega(s)}{\Omega^*(s)} = \frac{\Delta_3(s+k_{ps})}{\Delta_3(s+k_{ps})+(k_b-1)sG_3(s)} \\ \Phi_d(s) &= \frac{\Omega(s)}{d_n(s)} = \frac{k_b G_3(s)}{\Delta_3(s+k_{ps})+(k_b-1)sG_3(s)} \\ \Phi_n(s) &= \frac{\Omega(s)}{-s\delta_p(s)} = 1 - s\Phi_d(s)\end{aligned}$$

where $\Phi_r(s)$ is the transfer function from the command reference to the output; $\Phi_d(s)$ is the transfer function from the disturbance to the output; $\Phi_n(s)$ is the transfer function from the measurement noise to the output; Δ_3 is the characteristic polynomial; Ω^* is a reference of mechanical angular velocity; Ω is a mechanical angular velocity; k_{ps} is a proportional gain; k_b is control gain ratio; s is a Laplace Operator; $s\delta_p(s)$ is a speed measurement noise; $G_3(s)$ is a transfer function based on s and angular velocity; and d_n is a disturbance.

5. The ADCR system of claim 1 or 2, wherein a transfer function of the second LPF is:

$$G_{\text{LPF}}(s) = \frac{\omega_{f2}}{s + \omega_{f1}}$$

where ω_{f1} and ω_{f2} are gains and s is the Laplace Operator.

6. The ADRC system of claim 5, wherein the second LPF is set to be an integrator.
7. The ADRC system of claim 1, wherein the estimated speed value is determined based on the output of the proportional-integral-derivative controller of the ESO module.